Subjective Logic

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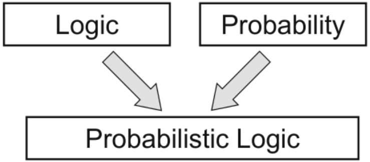
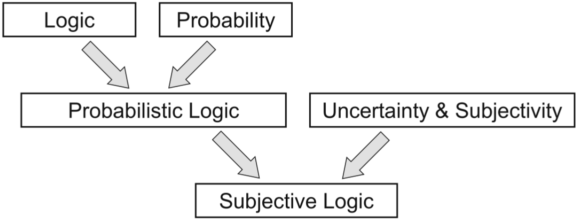
Probabilistic Logic



- It is an extension of binary logic.
- Propositions get assigned probabilities.
- Formulas of probability calculus replace truth tables.

Subjective Logic



- Uncertainty: A *subjetive opinion* can have *uncertainty about probabilities*.
- Subjectivity: *Subjective belief ownership* can be explicitly expressed

Domain

Definition

The *domain* is the set of values that represents possible states (or events) of a variable situation. These events are mutually exclusive.

Suppose we have an urn with balls that can be red, green or blue. The domain $\mathbb{X}$ will be:

$$\mathbb{X} = \{red, green, blue\} \tag{1}$$

Hyperdomain

Definition

Let $\mathbb{X}$ be a domain.

The *hyperdomain* denoted by $\mathscr{R}(\mathbb{X})$ is

$$\text{Hyperdomain: } \mathscr{R}(\mathbb{X}) = \mathscr{P}(\mathbb{X}) \setminus \{\mathbb{X}, \emptyset\} \tag{2}$$

The hyperdomain of our urn is:

$$\begin{aligned} \mathscr{R}(\mathbb{X}) = \{ \quad & \{\textit{red}\}, \{\textit{green}\}, \{\textit{blue}\}, \\ & \{\textit{red}, \textit{green}\}, \{\textit{red}, \textit{blue}\}, \{\textit{green}, \textit{blue}\} \quad \} \end{aligned} \tag{3}$$

Base rate distribution

Definition

Let $\mathbb{X}$ be a domain, and let X be a random variable in $\mathbb{X}$.

The *base rate distribution* $\mathbf{a}_X$ assigns base rate probability to possible values of $X \in \mathbb{X}$.

It is probability distribution used in case we *do not have evidences* about the events, a *prior probability*.

For all examples, the base rate distribution is uniform.

Belief Mass Distribution and Uncertainty Mass

Definition

Let $\mathbb{X}$ be a domain with corresponding $\mathscr{R}(\mathbb{X})$, and let X be a hypervariable over $\mathscr{R}(\mathbb{X})$.

A *belief mass distribution* denoted $\mathbf{b}_X$ assigns belief mass to possible values of the hypervariable X.

The *belief mass distribution* is subadditive: $\sum_{x \in \mathscr{R}(\mathbb{X})} \mathbf{b}_X(x) \leq 1$.

The sub-additivity of belief mass distributions is complemented by *uncertainty mass* denoted u_X.

Belief Mass Distribution and Uncertainty Mass

Now, I told you that our urn has 10 colored balls. One ball is red, 2 are blue, and 3 are green or blue. Therefore, you know nothing about 4 balls.

The belief mass distribution and uncertainty mass will be:

$$\begin{aligned}
\mathbf{b}_X(\{red\}) &= 0.1 \\
\mathbf{b}_X(\{green\}) &= 0 \\
\mathbf{b}_X(\{blue\}) &= 0.2 \\
\mathbf{b}_X(\{red, green\}) &= 0 \\
\mathbf{b}_X(\{red, blue\}) &= 0 \\
\mathbf{b}_X(\{green, blue\}) &= 0.3 \\
u_X &= 0.4
\end{aligned} \tag{4}$$

Hyper-opinion

Definition

A hyper-opinion on the hypervariable X is the ordered triplet $x = (\mathbf{b}_X, u_X, \mathbf{a}_X)$.

The hyper-opinion about our urn is

$$x = \begin{pmatrix} \mathbf{b}_X(\{red\}) & = 0.1, & \mathbf{a}_X(\{red\}) & = 1/3, \\ \mathbf{b}_X(\{green\}) & = 0, & \mathbf{a}_X(\{green\}) & = 1/3, \\ \mathbf{b}_X(\{blue\}) & = 0.2, & \mathbf{a}_X(\{blue\}) & = 1/3, \\ \mathbf{b}_X(\{red, green\}) & = 0, & \mathbf{a}_X(\{red, green\}) & = 2/3, \\ \mathbf{b}_X(\{red, blue\}) & = 0, & \mathbf{a}_X(\{red, blue\}) & = 2/3, \\ \mathbf{b}_X(\{green, blue\}) & = 0.3, & \mathbf{a}_X(\{green, blue\}) & = 2/3, \\ u_X & = 0.4. & & \end{pmatrix} \tag{5}$$

Comparison with Dempster-Shafer Belief Theory

Dempster-Shafer Belief Theory (DST) is a general framework for reasoning with uncertainty.

Dempster-Shafer Belief Theory	Subjective Logic
DST uses the term ‘frame of discernment’	SL uses domain.
DST uses *basic belief assignment* denoted by $\mathbf{m}(x)$	SL uses *belief mass distribution* and *uncertainty mass*
Basic belief can be assigned to the frame	We can’t observe evidence about the domain. SL uses uncertainty mass instead.
Dempster’s rule	Belief constraint fusion operator.

Comparison with Imprecise Probabilities

In Subjective Logic, an hyperopinion can be represented as a Dirichlet PDF.

The Imprecise Dirichlet Model (IDM) is a method for determine upper and lower probabilities produced by setting the minimum and maximum base rates in the Dirichlet PDF.

Let E_X^+ an E_X^- be upper and lower probabilities, as defined at the IDM.

$$u_X = \mathrm{E}_X^+ - \mathrm{E}_X^- \tag{6}$$

Comparison with Fuzzy Logic

In Fuzzy Logic, truth values of variables may be *any real number between 0 and 1*.
There are levels of truth in the interval that overlaps.

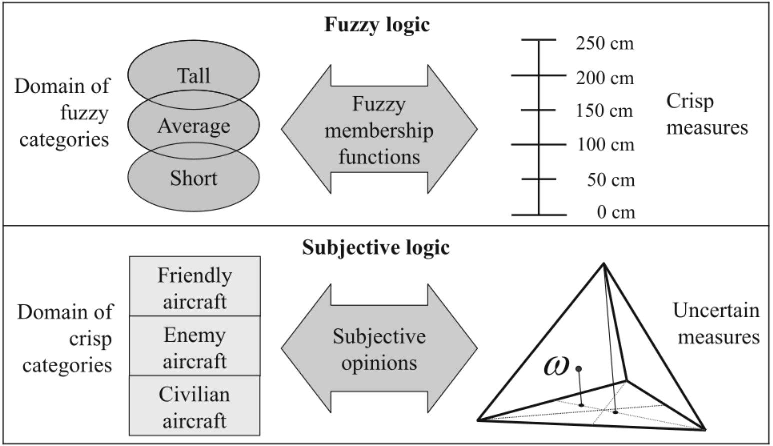


Fig. 5.1 Difference between fuzzy membership functions and subjective opinions.

Comparison with Kleene's Three-Valued Logic

Propositions can be assigned one of three truth-values specified as TRUE, FALSE and UNKNOWN.

$$(x \wedge y \wedge \cdots \wedge z) = \mathrm{UNKNOWN} \tag{7}$$

In subjective logic, a conjunction of large number of opinions 'I don't know' will say that $(x \wedge y \wedge \cdots \wedge z)$ is likely FALSE.

Subjective Logic as a Generalisation of Probabilistic Logic

An opinion with $|\mathbb{X}| = 2$, all belief mass assigned to one value and no uncertainty is a boolean value. Binary logic applies.

An opinion with no uncertainty is a probability distribution. Probabilistic logic applies.