

3.5 Multinomial Opinions

José C. Oliveira

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Summary

3.5.1 The Multinomial Option Representation

3.5.2 The Dirichlet Multinomial Model

3.5.1 The Multinomial Opinion Representation I

Definition 3.4 (Multinomial Opinion)

Let \mathbb{X} be a domain larger than binary, i.e. so that $k = |\mathbb{X}| > 2$. Let X be a random variable in \mathbb{X} . A multinomial opinion over the random variable X is the ordered triplet $\omega_X = (\mathbf{b}_X, u_X, \mathbf{a}_X)$ where

- ▶ \mathbf{b}_X is a belief mass distribution over X ,
- ▶ u_X is the uncertainty mass which represents the vacuity of evidence,
- ▶ \mathbf{a}_X is a base rate distribution over \mathbb{X} ,

and the multinomial additivity requirement of Eq.(2.6) is satisfied.

A multinomial opinion has $(2k - 1)$ degrees of freedom.

3.5.1 The Multinomial Opinion Representation II

The projected probability distribution of multinomial opinions is defined by:

$$\mathbf{P}_X(x) = \mathbf{b}_X(x) + \mathbf{a}_X(x)u_X, \quad \forall x \in \mathbb{X}. \quad (3.12)$$

The variance of multinomial opinions is expressed as

$$\text{Var}_X = \frac{\mathbf{P}_X(x)(1 - \mathbf{P}_X(x)u_X)}{W + u_X}, \quad (3.13)$$

where W denotes non-informative prior weight, which must be set to $W = 2$.

3.5.2 The Dirichlet Multinomial Model