Subjective Logic

Associativity of Cumulative Belief Fusion

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At the page 227 of the book [1], section 12.3.1, the author says the cumulative belief fusion operator is associative.

Let $\mathbb{X} = \{b, d\}$ be a domain and X be a random variable over $\mathbb{X}$. Let A and B be sources which have the following opinions about X.

$$\omega_X^A = \begin{pmatrix} \mathbf{b}_X^A(b) & = 1 & \mathbf{a}_X^A(b) & = 0.5 \\ \mathbf{b}_X^A(d) & = 0 & \mathbf{a}_X^A(d) & = 0.5 \\ u_X^A & = 0 & & \end{pmatrix} \tag{0.1}$$

$$\omega_X^B = \begin{pmatrix} \mathbf{b}_X^B(b) & = 0 & \mathbf{a}_X^B(b) & = 0.5 \\ \mathbf{b}_X^B(d) & = 1 & \mathbf{a}_X^B(d) & = 0.5 \\ u_X^B & = 0 & & \end{pmatrix} \tag{0.2}$$

Lets make a test with

$$(\omega_X^A \oplus \omega_X^A) \oplus \omega_X^B \quad \text{and} \quad \omega_X^A \oplus (\omega_X^A \oplus \omega_X^B) \tag{0.3}$$

$$\begin{aligned} (\omega_X^A \oplus \omega_X^A) \oplus \omega_X^B &= \begin{pmatrix} \mathbf{b}_X^A(b) & = 1 & \mathbf{a}_X^A(b) & = 0.5 \\ \mathbf{b}_X^A(d) & = 0 & \mathbf{a}_X^A(d) & = 0.5 \\ u_X^A & = 0 & & \end{pmatrix} \oplus \begin{pmatrix} \mathbf{b}_X^B(b) & = 0 & \mathbf{a}_X^B(b) & = 0.5 \\ \mathbf{b}_X^B(d) & = 1 & \mathbf{a}_X^B(d) & = 0.5 \\ u_X^B & = 0 & & \end{pmatrix} \\ &= \begin{pmatrix} \mathbf{b}_X^B(b) & = 0.5 & \mathbf{a}_X^B(b) & = 0.5 \\ \mathbf{b}_X^B(d) & = 0.5 & \mathbf{a}_X^B(d) & = 0.5 \\ u_X^B & = 0 & & \end{pmatrix} \end{aligned} \tag{0.4}$$

$$\omega_X^A \oplus (\omega_X^A \oplus \omega_X^B) = \begin{pmatrix} \mathbf{b}_X^A(b) & = 1 & \mathbf{a}_X^A(b) & = 0.5 \\ \mathbf{b}_X^A(d) & = 0 & \mathbf{a}_X^A(d) & = 0.5 \\ u_X^A & = 0 & & \end{pmatrix} \oplus \begin{pmatrix} \mathbf{b}_X^B(b) & = 0.5 & \mathbf{a}_X^B(b) & = 0.5 \\ \mathbf{b}_X^B(d) & = 0.5 & \mathbf{a}_X^B(d) & = 0.5 \\ u_X^B & = 0 & & \end{pmatrix}$$
$$= \begin{pmatrix} \mathbf{b}_X^B(b) & = 0.75 & \mathbf{a}_X^B(b) & = 0.5 \\ \mathbf{b}_X^B(d) & = 0.25 & \mathbf{a}_X^B(d) & = 0.5 \\ u_X^B & = 0 & & \end{pmatrix} \tag{0.5}$$

We see that $(\omega_X^A \oplus \omega_X^A) \oplus \omega_X^B \neq \omega_X^A \oplus (\omega_X^A \oplus \omega_X^B)$, therefore aleatory cumulative belief fusion is not associative.

References

[1] Audun Jøsang. *Subjective logic.* Springer, 2016.