Subjective Logic

José C. Oliveira

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1 Introduction

One of the major goals of our research is to develop a quantitative logic for reasoning about belief in social networks. Subjective logic is a logic that may has the expressiveness that we look for improve our influence graph and the belief state of an agent.

Subjective logic is an extension of probabilistic logic. In probabilistic logic, we can express the truth value of a proposition by a probability distribution over a domain with disjoint events or states and reason by the axioms of probability. When we have two states of a domain $\mathbb{X}$ and the probability of a state $x \in \mathbb{X}$ can be $P(x) = 0$ or $P(x) = 1$, we have binary logic. The expression $x \wedge y$ is expressed in probabilistic logic as $P(x \wedge y) = P(x)P(y)$. Probabilistic logic is an extension of binary logic.

Subjective logic extends probabilistic logic by adding *uncertainty* and *subjectivity*. We can't express *'we don't know'* with probabilistic logic by an uniform distribution because it says that we know that the distribution over the domain is uniform. Subjective logic can express that *uncertainty* about the distribution. The *subjectivity* comes from the fact that we can assign an opinion about a proposition to an agent.

By the investigation we are doing about subjective logic, our main question is: Can we use subjective logic to improve our model of social networks? If yes, how?

This document is an introduction about subjective logic and it will present elementary definitions, types of opinions and an overview about computational trust. The main reference of this documents if the book *Subjective Logic: A Formalism for Reasoning Under Uncertainty* by Audun Jøsang.

2 Opinion representation

The main object of subjective logic is the *opinion*. There are many equivalent ways for opinion representation. Here I'm going to present the most used. We represent an opinion by ω_X^A, where A is the an agent and X is a random variable or *hypervariable*. This section presents the elementary definitions that compose an opinion.

2.1 Elementary definitions

In subjective logic, a domain is a state space consisting of a set of values which can also be called states, events, outcomes, hypotheses or propositions. Those values are assumed to be exclusive and exhaustive. Let $k = |\mathbb{X}|$ be the cardinality of $\mathbb{X}$.

Suppose we have a box have balls that can red, green, or blue. Then, the domain that represents all possible outcomes is

$$\mathbb{X} = \{red, green, blue\}. \tag{2.1}$$

Definition 2.1. *(Hyperdomain)* Let $\mathbb{X}$ be a domain, and let $\mathscr{P}(\mathbb{X})$ denote the powerset of $\mathbb{X}$. The powerset contains all subsets of $\mathbb{X}$, including the empty set $\emptyset$, and the domain $\mathbb{X}$ itself. The *hyperdomain* denoted $\mathscr{R}(\mathbb{X})$ is the reduced powerset of $\mathbb{X}$, i.e. the powerset excluding the empty-set $\varnothing$ and the domain value $\mathbb{X}$. The hyperdomain is expressed as

$$\text{Hyperdomain: } \mathscr{R}(\mathbb{X}) = \mathscr{P} \setminus \{\mathbb{X}, \emptyset\} \tag{2.2}$$

The hyperdomain of the box is

$$\begin{aligned} \mathscr{R}(\mathbb{X}) = \{ \quad & \{red\}, \{green\}, \{blue\}, \\ & \{red, green\}, \{red, blue\}, \{green, blue\} \quad \}. \end{aligned} \tag{2.3}$$

Let $\kappa = |\mathscr{R}(\mathbb{X})| = 2^k - 2.$ be the cardinality of $\mathscr{R}(\mathbb{R})$.

Every value of the hyperdomain with one value is called *singleton*. Every value with more the one value is called *composite value*. The interpretation of a composite value being TRUE, is that one and only one of the constituent singletons is TRUE. The set of all composite values is called *composite set*.

A *hypervariable* X is an random variable that its values from $\mathscr{R}(\mathbb{X})$. For example, if a hypervariable takes value from the composite value $\{blue, red\} \in \mathscr{R}(\mathbb{X})$, it means that we either draw a ball blue or red.

2.2 Binomial opinions

2.3 Multinomial opinions

2.4 Hypernomial opinions

- Elementary definitions

  - Domain and Hyperdomain
  - Base-rate distribution (prior)
  - Belief mass distribution and uncertainty
  - Projected probability distribution (posterior)

- Binomial opinion and example
- Multinomial option and example
- Hypernomial opinion and example

3 Computational trust

This is an overview. Nothing formal. (Maybe it will be if I have time.)

- Definition of trust. (Influence?)
- Trust transitivity. (Update function?)
- Belief fusion. (Overall update function?)

4 Next questions

When I was reviewing the book, I found something that I may have got wrong about trust transitivity. I'm not sure if the example I told at the meeting is possible in SL.

- Suppose that an agent A trusts an agent B (ω_B^A), and B trusts X (ω_X^A). Can the carnality of the domain of ω_X^A be greater than 2? (At the meeting, assumed yes. Now I'm not sure.)
- Is there a way (an operator) to consider the trust of an agent A when they obtain another trust opinion (from trust transitivity)?
  - If yes, can this operator have the same properties as the rational belief update?
  - If not, can we create an operator that has the same properties as the rational belief update?

References

[1] Audun Jøsang. *Subjective logic.* Springer, 2016.