Subjective Logic

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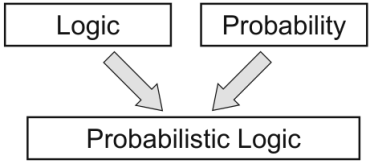
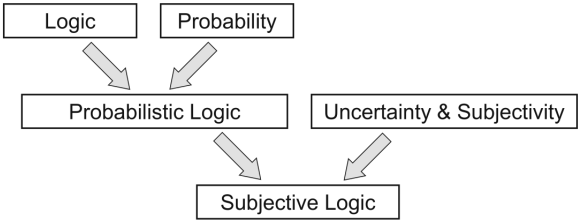
Probabilistic Logic



- It is an extension of binary logic.
- Propositions get assigned probabilities.
- Formulas of probability calculus replace truth tables.

Subjective Logic



- Uncertainty: A *subjetive opinion* can have *uncertainty about probabilities*.
- Subjectivity: *Subjective belief ownership* can be explicitly expressed

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Suppose we have an urn with balls that can be red, green or blue. The domain $\mathbb{X}$ will be:

$$\mathbb{X} = \{red, green, blue\} \tag{1}$$

Hyperdomain

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Let $\mathbb{X}$ be a domain.

The *hyperdomain* denoted by $\mathscr{R}(\mathbb{X})$ is

$$\text{Hyperdomain: } \mathscr{R}(\mathbb{X}) = \mathscr{P}(\mathbb{X}) \setminus \{\mathbb{X}, \emptyset\} \tag{2}$$

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The hyperdomain of our urn is:

$$\begin{aligned} \mathscr{R}(\mathbb{X}) = \{ \quad & \{red\}, \{green\}, \{blue\}, \\ & \{red, green\}, \{red, blue\}, \{green, blue\} \quad \} \end{aligned} \tag{3}$$

Belief Mass Distribution and Uncertainty Mass

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The belief mass distribution and uncertainty mass will be:

$$
\begin{aligned}
\mathbf{b}_X(\{red\}) &= 0.1 \\
\mathbf{b}_X(\{green\}) &= 0 \\
\mathbf{b}_X(\{blue\}) &= 0.2 \\
\mathbf{b}_X(\{red, green\}) &= 0 \\
\mathbf{b}_X(\{red, blue\}) &= 0 \\
\mathbf{b}_X(\{green, blue\}) &= 0.3 \\
u_X &= 0.4
\end{aligned}
\tag{4}
$$

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The hyper-opinion about our urn is

$$\omega_X = \left(\begin{array}{lll} \mathbf{b}_X(\{red\}) & = 0.1, & \mathbf{a}_X \\ \mathbf{b}_X(\{green\}) & = 0, & \\ \mathbf{b}_X(\{blue\}) & = 0.2, & \\ \mathbf{b}_X(\{red, green\}) & = 0, & \\ \mathbf{b}_X(\{red, blue\}) & = 0, & \\ \mathbf{b}_X(\{green, blue\}) & = 0.3, & \\ u_X & = 0.4. & \end{array} \right) \tag{5}$$

Comparison with Dempster-Shafer Belief Theory

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Dempster-Shafer Belief Theory (DST) is a general framework for reasoning with uncertainty.

Dempster-Shafer Belief Theory	Subjective Logic
DST uses the term 'frame of discernment'	SL uses domain.
DST uses *basic belief assignment* denoted by $\mathbf{m}(x)$	SL uses *belief mass distribution* and *uncertainty mass*
Basic belief can be assigned to the frame	We can't observe evidence about the domain. SL uses uncertainty mass instead.
Dempster's rule	Belief constraint fusion operator.

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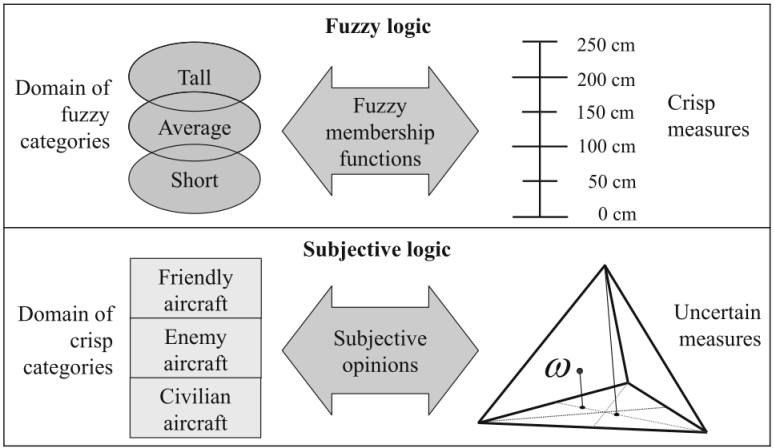

Fig. 5.1 Difference between fuzzy membership functions and subjective opinions.

Comparison with Kleene's Three-Valued Logic

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In subjective logic, a conjunction of large number of opinions 'I don't know' will say that $(x \wedge y \wedge \cdots \wedge z)$ is likely FALSE.

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We studied

- Basic definitions about Subjective Logic
- Opinion representations
- Decision making

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Next

- Entropy and conflict between opinions
- Subjective Logic operators