José David Pazos Ricaurte - 1004535394

Solución Parcial #2

2.1 $x(t) = |6\sin(3t+\pi/4)|^2 = 36\sin^2(3t+\pi/4)$

con la propiedad $\sin^2(\theta) = \frac{1}{2} - \frac{1}{2}\cos(2\theta)$

Tenemos que:

$$36\left[\frac{1}{2} - \frac{1}{2}\cos(2[3t+\pi/4])\right) = 18 - 18\cos(6t+\pi/2) = x(t)$$

Ahora,

$$\cos(\theta + \pi/2) = -\sin(\theta)$$

entonces mi señal queda:

$$x(t) = 18 + 18\sin(6t)$$

Forma trigonométrica

$$x(t) = a_0 + \sum_{n=-N}^{N} a_n\cos(n\omega_0 t) + b_n\sin(n\omega_0 t)$$

ya que $x(t)$ corresponde a una función seno y el seno representa una función con simetría impar, entonces $x(t) = -x(-t)$

Entonces:

$$\boxed{a_n = 0}$$

Finalmente:

$$x(t) = 18 + 18\sin(6t) = a_0 + \sum_{n=-N}^{N} b_n\sin(n\omega_0 t)$$

$$a_0 = C_0 = \frac{1}{t_F - t_i}\int_{t_i}^{t_F} x(t)\,dt$$

$$a_0 = \frac{1}{\pi-(-\pi)} \int_{-\pi}^{\pi} 78 + 78\sin(6t)\,dt$$

$$= \frac{78}{2\pi}\int_{-\pi}^{\pi} dt + \frac{78}{2\pi}\int_{-\pi}^{\pi} \sin(6t)\,dt$$

$$= \frac{78t}{2\pi}\Big|_{-\pi}^{\pi} + \frac{78}{12\pi}\left(-\cos(6t)\right)\Big|_{-\pi}^{\pi}$$

$$= \frac{78t}{2\pi}\Big|_{-\pi}^{\pi} + \frac{78}{12\pi}\overbrace{\Big[\cancel{\underbrace{-\cos(6\pi)}_{-1} + \underbrace{\cos(6(-\pi))}_{1}}\Big]}^{0}$$

$$= \frac{78\pi}{2\pi} - \frac{-78\pi}{2\pi} = 78$$

$$\boxed{a_0 = 78}$$

Ahora, para bn:

$$b_n = \frac{2}{t_F - t_i}\int_{t_i}^{t_F} x(t)\sin(n\omega_0 t)\,dt$$

$$b_n = \frac{2}{2\pi}\int_{-\pi}^{\pi} 78 + 78\sin(6t)\cdot\sin(n\omega_0 t)\,dt$$

$$b_n = \frac{2}{2\pi}\left[\int_{-\pi}^{\pi} 78\sin(n\omega_0 t)\,dt + \int_{-\pi}^{\pi} 78\sin(6t)\sin(n\omega_0 t)\,dt\right]$$

Por identidad:

$$\sin(\theta)\sin(\alpha) = \frac{\cos(\theta-\alpha) - \cos(\theta+\alpha)}{2}$$

entonces:

$$\sin(\theta) = \sin(6t) \quad ; \quad \sin(\alpha) = \sin(n\omega_0 t)$$

$$= \frac{\cos(6t - n\omega_0 t) - \cos(6t + n\omega_0 t)}{2}$$

$$= \frac{\cos((6-n\omega_0)t) - \cos((6+n\omega_0)t)}{2}$$

$\omega_0 = \frac{2\pi}{T}$; $T = 2\pi \Rightarrow \omega_0 = \frac{2\pi}{2\pi} \Rightarrow \omega_0 = 1 \,(rad/s)$

$$b_n = \frac{2}{2\pi}\Bigg[\underbrace{\int_{-\pi}^{\pi} 18\sin(nt)\,dt}_{1} + \underbrace{\int_{-\pi}^{\pi} 18\,\frac{\cos((6-n)t) - \cos(6+n)t)}{2}\,dt}_{2}$$

Para 1

$$\frac{2}{2\pi}\int_{-\pi}^{\pi} 18\sin(nt)\,dt = \frac{18}{\pi n}\left[\cancel{-\cos(nt)\Big|_{-\pi}^{\pi}}^{\,0}\right] = 0$$

Para 2

$$\frac{2}{2\pi}\left[\int_{-\pi}^{\pi} 9\cos((6-n)t)\,dt - \int_{-\pi}^{\pi} 9\cos((6+n)t)\,dt\right]$$

$$= \frac{2}{2\pi}\left[\frac{9}{6-n}\operatorname{sen}(6-n)t\Big|_{-\pi}^{\pi} - \frac{9}{6+n}\operatorname{sen}(6+n)t\Big|_{-\pi}^{\pi}\right]$$

$$= 18\,\frac{\sin(6-n)\pi - \sin(6-n)(-\pi)}{2\pi(6-n)} - 18\,\frac{\sin(6+n)\pi - \sin(6+n)(-\pi)}{2\pi(6+n)} = b_n$$

Para $n \neq 6$, b No obstante, para $n=6$ debemos calcular el Límite y aproximar la indeterminación $\frac{0}{0}$

$$b_6 = 18 \lim_{n\to 6} \frac{\frac{d}{dn}\,\sin(6-n)\pi - \sin(6-n)(-\pi)}{\frac{d}{dn}\,2\pi(6-n)}$$

$$b_6 = 78 \lim_{n \to 6} \frac{\cos((6-n)\pi)(-\pi) - \cos(-(6-n)\pi)(\pi)}{-2\pi}$$

$$b_6 = 78 \, \frac{\cos(0)(-\pi) - \cos(0)\pi}{-2\pi} = 78 \, \frac{\cancel{-2\pi}}{\cancel{-2\pi}} = 78$$

$$\boxed{b_6 = 78}$$

Por tanto,

$$\boxed{a_n = \begin{cases} 78 & n = 0 \\ 0 & \forall n / \{0\} \end{cases}}$$

$$\boxed{b_n = \begin{cases} 78 & n = 6 \\ -78 & n = -6 \\ 0 & \forall n / \{6, -6\} \end{cases}}$$

Forma exponencial:

$$c_0 = a_0 = 78$$

y

$$c_n = \frac{a_n - j b_n}{2} \qquad C_{-6} = j9$$

$$c_n = \frac{0 - j78}{2} = \frac{-j78}{2} = -j9$$

Se obtiene:

$$\boxed{c_n = \begin{cases} 78 & n = 0 \\ -9j & n = 6 \\ 9j & n = -6 \\ 0 & \forall n / \{0, 6, -6\} \end{cases}}$$

Para construir la señal:

$$X(t) = \sum_{n=-N}^{N} C_n e^{jnt}$$

$$x(t) = C_{-6} e^{-j6t} + C_0 e + C_6 e^{j6t}$$

$$= 9j(\cos(6t) - 9j^2 \operatorname{sen}(6t) + 78 - 9j\cos(6t) - 9j^2 \operatorname{sen}(6t)$$

$$x(t) = 78 + 78 \operatorname{sen}(6t)$$

2.2) $c(t) = A_c \cos(2\pi F_c t)$, $A_c, F_c \in \mathbb{R}$ $y(t) = \left(1 + \frac{m(t)}{A_c}\right) c(t)$

$F\{c(t)\} + F\left\{\frac{m(t)c(t)}{A_c}\right\}$

$F\{A_c \cos(2\pi F_c t)\} = A_c \cdot F\left\{\frac{e^{j2\pi F_c t} + e^{-j2\pi F_c t}}{2}\right\}$

$A_c \left[F\left\{\frac{e^{j2\pi F_c t}}{2}\right\} + F\left\{\frac{e^{-j2\pi F_c t}}{2}\right\}\right]$

$\frac{A_c}{\cancel{2}}\left[\cancel{2}\pi\delta(\omega - 2\pi F_c) + \cancel{2}\pi\delta(\omega + 2\pi F_c)\right]$

$A_c\pi\delta(\omega - 2\pi F_c) + A_c\pi\delta(\omega + 2\pi F_c) \Rightarrow$

$\Rightarrow C(\omega) = A_c\pi\delta\left[(\omega - 2\pi F_c) + (\omega + 2\pi F_c)\right]$

$F\left\{\frac{m(t)(\cancel{A_c}\cos(2\pi F_c t))}{\cancel{A_c}}\right\} = F\{\cos(2\pi F_c t)m(t)\} =$

$= F\left\{\frac{m(t)\, e^{j2\pi F_c t}}{2}\right\} + F\left\{\frac{m(t)\, e^{-j2\pi F_c t}}{2}\right\}$

$\frac{M(\omega - 2\pi F_c t)}{2} + \frac{M(\omega + 2\pi F_c t)}{2} = \frac{1}{2} M\left[(\omega - 2\pi F_c t) + (\omega + 2\pi F_c t)\right]$

$Y(\omega) = A_c\pi\delta\left[(\omega - 2\pi F_c t) + (\omega + 2\pi F_c t)\right] + \frac{1}{2} M\left[(\omega - 2\pi F_c t) + (\omega + 2\pi F_c t)\right]$