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Solución Parcial # 2

$$2.7 \quad x(t) = |6 \sin(3t + \pi/4)|^2 = 36 \sin^2(3t + \pi/4)$$

con la propiedad $\sin^2(\theta) = \frac{1}{2} - \frac{1}{2} \cos(2\theta)$

Tenemos que:

$$36 \left[\frac{1}{2} - \frac{1}{2} \cos(2[3t + \pi/4]) \right] = 18 - 18 \cos(6t + \pi/2) = x(t)$$

Ahora, $\cos(\theta + \pi/2) = -\sin(\theta)$

Entonces mi señal queda:

$$x(t) = 18 + 18 \sin(6t)$$

Forma trigonométrica

$$x(t) = a_0 + \sum_{n=-N}^N a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)$$

ya que $x(t)$ corresponde a una función seno y el seno representa una función con simetría impar, entonces $x(t) = -x(-t)$

Entonces:

$$a_n = 0$$

Finalmente:

$$x(t) = 18 + 18 \sin(6t) = a_0 + \sum_{n=-N}^N b_n \sin(n\omega_0 t)$$

$$a_c = C_0 = \frac{1}{t_F - t_i} \int_{t_i}^{t_F} x(t) dt$$

$$\begin{aligned}
 a_0 &= \frac{1}{\pi - (-\pi)} \int_{-\pi}^{\pi} 18 + 18 \sin(6t) dt \\
 &= \frac{18}{2\pi} \int_{-\pi}^{\pi} dt + \frac{18}{2\pi} \int_{-\pi}^{\pi} \sin(6t) dt \\
 &= \frac{18t}{2\pi} \Big|_{-\pi}^{\pi} + \frac{18}{2\pi} \left[-\cos(6t) \right]_{-\pi}^{\pi} \\
 &= \frac{18t}{2\pi} \Big|_{-\pi}^{\pi} + \frac{18}{2\pi} \left[-\cos(6\pi) + \cos(6(-\pi)) \right] \\
 &= \frac{18\pi}{2\pi} - \frac{-18\pi}{2\pi} = 18
 \end{aligned}$$

$$a_0 = 18$$

Ahora, para b_n :

$$\begin{aligned}
 b_n &= \frac{2}{t_f - t_i} \int_{t_i}^{t_f} x(t) \sin(n\omega_0 t) dt \\
 b_n &= \frac{2}{2\pi} \int_{-\pi}^{\pi} 18 + 18 \sin(6t) \cdot \sin(n\omega_0 t) dt \\
 b_n &= \frac{2}{2\pi} \left[\int_{-\pi}^{\pi} 18 \sin(n\omega_0 t) dt + \int_{-\pi}^{\pi} 18 \sin(6t) \sin(n\omega_0 t) dt \right]
 \end{aligned}$$

Por identidad:

$$\sin(\theta) \sin(\alpha) = \frac{\cos(\theta - \alpha) - \cos(\theta + \alpha)}{2}$$

entonces:

$$\sin(\theta) = \sin(6t) \quad ; \quad \sin(\alpha) = \sin(n\omega_0 t)$$

$$= \frac{\cos(6t - n\omega_0 t) - \cos(6t + n\omega_0 t)}{2}$$

$$= \frac{\cos((6-n\omega_0)t) - \cos((6+n\omega_0)t)}{2}$$

$$\omega_0 = \frac{2\pi}{T} \quad ; T = 2\pi \Rightarrow \omega_0 = \frac{2\pi}{2\pi} \Rightarrow \omega_0 = 1 \text{ (rad/s)}$$

$$b_n = \frac{2}{2\pi} \left[\underbrace{\int_{-\pi}^{\pi} 18 \sin(nt) dt}_{\textcircled{1}} + \int_{-\pi}^{\pi} 18 \underbrace{\frac{\cos((6-n)t) - \cos((6+n)t)}{2}}_{\textcircled{2}} dt \right]$$

Para $\textcircled{1}$

$$\frac{2}{2\pi} \int_{-\pi}^{\pi} 18 \sin(nt) dt = \frac{18}{\pi n} \left[-\cos(nt) \right]_{-\pi}^{\pi} = 0$$

Para $\textcircled{2}$

$$\begin{aligned} & \frac{2}{2\pi} \left[\int_{-\pi}^{\pi} 9 \cos((6-n)t) dt - \int_{-\pi}^{\pi} 9 \cos((6+n)t) dt \right] \\ &= \frac{2}{2\pi} \left[\frac{9}{6-n} \sin((6-n)t) \Big|_{-\pi}^{\pi} - \frac{9}{6+n} \sin((6+n)t) \Big|_{-\pi}^{\pi} \right] \\ &= 18 \frac{\sin((6-n)\pi) - \sin((6-n)(-\pi))}{2\pi(6-n)} - 18 \frac{\sin((6+n)\pi) - \sin((6+n)(-\pi))}{2\pi(6+n)} = b_n \end{aligned}$$

Para $n \neq 6$, b No obstante, para $n=6$ debemos calcular el límite y aproximar la indeterminación $\frac{0}{0}$.

$$b_6 = 18 \lim_{n \rightarrow 6} \frac{\frac{d}{dn} \sin((6-n)\pi) - \sin((6-n)(-\pi))}{\frac{d}{dn} 2\pi(6-n)}$$

$$b_6 = 78 \lim_{n \rightarrow 6} \frac{\cos((6-n)\pi)(-\pi) - \cos(-(6-n)\pi)(\pi)}{-2\pi}$$

$$b_6 = 78 \frac{\cos(0)(-\pi) - \cos(0)\pi}{-2\pi} = 78 \frac{-\pi - \pi}{-2\pi} = 78$$

$$\boxed{b_6 = 78}$$

Por tanto,

$$a_n = \begin{cases} 78 & n=0 \\ 0 & \forall n / \{0\} \end{cases}$$

$$b_n = \begin{cases} 78 & n=6 \\ -78 & n=-6 \\ 0 & \forall n / \{6, -6\} \end{cases}$$

Forma exponencial:

$$c_0 = a_0 = 78$$

y

$$c_n = \frac{a_n - j b_n}{2}$$

$$c_{-6} = j9$$

$$c_n = \frac{0 - j78}{2} = \frac{-j78}{2} = -j9$$

Se obtiene:

$$c_n = \begin{cases} 78 & n=0 \\ -j9 & n=6 \\ j9 & n=-6 \\ 0 & \forall n / \{0, 6, -6\} \end{cases}$$

Para construir la señal:

$$X(t) = \sum_{n=-N}^N C_n e^{jnt}$$

$$X(t) = C_{-6} e^{-j6t} + C_0 e^0 + C_6 e^{j6t}$$

$$= 9j(\cos(6t) - 9j^2 \sin(6t)) + 18 - 9j\cos(6t) - 9j^2 \sin(6t)$$

$$X(t) = 18 + 18 \sin(6t)$$

El error relativo se calcula

$$\epsilon_x[\%] = \left[1 - \frac{1}{P_x} \sum_{n=-N}^N |c_n|^2 \right] 100\%$$

Para $P_x =$

$$P_x = \frac{1}{T} \int_{t_i}^{t_f} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\pi}^{\pi} |18 + 18 \sin(6t)|^2 dt$$

$$P_x = \frac{1}{2\pi} \left[\int_{-\pi}^{\pi} 18^2 dt + \int_{-\pi}^{\pi} 2(18)(18 \sin(6t)) dt + \int_{-\pi}^{\pi} 18^2 \sin^2(6t) dt \right]$$

$$P_x = \frac{1}{2\pi} \left[\int_{-\pi}^{\pi} 324 dt + \int_{-\pi}^{\pi} 648 \sin(6t) dt + 18^2 \int_{-\pi}^{\pi} \left(\frac{1}{2} - \frac{\cos(12t)}{2} \right) dt \right]$$

$$P_x = \frac{1}{2\pi} \left[324t \Big|_{-\pi}^{\pi} - \frac{648}{6} \cos(6t) \Big|_{-\pi}^{\pi} + 324 \left[\frac{1}{2}t \Big|_{-\pi}^{\pi} - \frac{1}{24} \sin(12t) \Big|_{-\pi}^{\pi} \right] \right]$$

$$P_x = \frac{1}{2\pi} \left[324\pi - 324(-\pi) - 108 \left[\cos(6\pi) - \cos(6(-\pi)) \right] + \right.$$

$$\left. 324 \left[\frac{1}{2}(\pi) - \frac{1}{2}(-\pi) \right] - \left[\frac{\sin(12\pi)}{24} - \frac{\sin(12(-\pi))}{24} \right] \right]$$

$$P_x = \frac{1}{2\pi} [648\pi + 0 + 324\pi] = \frac{972\pi}{2\pi} = 486$$

$$P_x = 486$$

$$\epsilon_x = \left[1 - \frac{(-9)^2 + (-18)^2 + (9)^2}{486} \right] \times 100\% = 0\%$$

2.2)

$$c(t) = A_c \cos(2\pi F_c t), A_c F_c \in \mathbb{R} \quad y(t) = \left(1 + \frac{m(t)}{A_c}\right) c(t)$$

$$F\{c(t)\} + F\left\{\frac{m(t)c(t)}{A_c}\right\}$$

$$F\{A_c \cos(2\pi F_c t)\} = A_c \cdot F\left\{\frac{e^{j2\pi F_c t} + e^{-j2\pi F_c t}}{2}\right\}$$

$$A_c \cdot \left[F\left\{\frac{e^{j2\pi F_c t}}{2}\right\} + F\left\{\frac{e^{-j2\pi F_c t}}{2}\right\}\right]$$

$$\frac{A_c}{2} [2\pi \delta(\omega - 2\pi F_c) + 2\pi \delta(\omega + 2\pi F_c)]$$

$$A_c \pi \delta(\omega - 2\pi F_c) + A_c \pi \delta(\omega + 2\pi F_c) \Rightarrow$$

$$\Rightarrow C(\omega) = A_c \pi \delta[(\omega - 2\pi F_c) + (\omega + 2\pi F_c)]$$

$$F\left\{\frac{m(t)(A_c \cos(2\pi F_c t))}{A_c}\right\} = F\{\cos(2\pi F_c t) m(t)\} =$$

$$= F\left\{\frac{m(t)}{2} e^{j2\pi F_c t}\right\} + F\left\{\frac{m(t)}{2} e^{-j2\pi F_c t}\right\}$$

$$\frac{M(\omega - 2\pi F_c)}{2} + \frac{M(\omega + 2\pi F_c)}{2} = \frac{1}{2} M[(\omega - 2\pi F_c) + (\omega + 2\pi F_c)]$$

$$y(\omega) = A_c \pi \delta[(\omega - 2\pi F_c) + (\omega + 2\pi F_c)] + \frac{1}{2} M[(\omega - 2\pi F_c) + (\omega + 2\pi F_c)]$$