(1-1) To prove that $f_n \xrightarrow{u} f$, let $\varepsilon > 0$ be given. Our goal is to find some N such that if $n > N$ and $c \in [0, \infty)$, then $\frac{1}{n} < \varepsilon$. It follows that if $N = \frac{1}{\varepsilon}$, then

$$n > N \implies \frac{1}{n} < \frac{1}{N} = \varepsilon$$

Then $f_n \xrightarrow{u} f$.

(1-2) Consider the left side. We have

$$\begin{aligned}
\lim_{n\to\infty} \int_0^\infty f_n \mathrm{d}x &= \lim_{n\to\infty} \lim_{s\to\infty} \int_0^s \frac{1}{n} \mathrm{d}x \\
&= \lim_{n\to\infty} \lim_{s\to\infty} \left.\frac{x}{n}\right|_0^s \\
&= \lim_{n\to\infty} \lim_{s\to\infty} \\
&= \infty
\end{aligned}$$

While the right side is

$$\int_0^\infty f \mathrm{d}x = 0$$

So that the left and right sides are not equal to each other.

(2-1) To prove that $f_n \xrightarrow{u} f$, let $\varepsilon > 0$ be given. Our goal is to find some N such that if $n > N$ and $c \in [-1, 1]$, then $|f_n(c) - 0| < \varepsilon$. Pick $N = \frac{1}{2\varepsilon}$. Then for all $n > N$ and for all $c \in [-1, 1]$, we have

$$\begin{aligned}
|f_n(c) - 0| &= \left|\frac{c}{1 + n^2c^2}\right| \\
&= \frac{|c|}{|1 + n^2c^2|}
\end{aligned}$$

Since $(nc - 1)^2 \geq 0$, then $1 + n^2c^2 \geq 2n|c|$. Then:

$$\begin{aligned}
&\leq \frac{|c|}{2n|c|} \\
&= \frac{1}{2n} \\
&< \varepsilon
\end{aligned}$$

Then $f_n \xrightarrow{u} f$.

(2-2) By the Algebraic Differentiability Theorem, we have for all $n \geq 1$:

$$f_n'(x) = \frac{1 - x^2n^2}{(1 + x^2n^2)^2}$$

To prove that $f_n' \xrightarrow{p} g$, consider any $c \in [-1, 1]$. We consider two cases:

(a) If $c = 0$, then $(f_n'(c)) = (1)$, which obviously converges to 1.

(b) Suppose $0 < |c| \leq 1$. Then

$$(f_n'(c)) = \frac{1 - c^2n^2}{(1 + c^2n^2)^2}$$

By the Algebraic Limit Theorem, we have:

$$\begin{aligned}\lim_{n\to\infty}(f_n'(c)) &= \lim_{n\to\infty} \frac{1 - c^2n^2}{(1 + c^2n^2)^2}\\ &= \lim_{n\to\infty} \frac{n^{-2} - c^2}{(n^{-1} + c^2n)^2}\\ &= 0\end{aligned}$$

Then $f_n' \xrightarrow{p} g$.

(2-3) Since g is not a continuous function, then f_n' does not converge to g uniformly.