

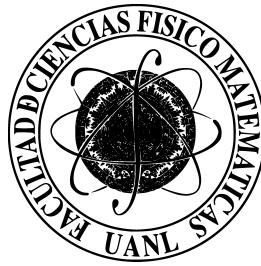
Universidad Autónoma de Nuevo León

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Investigación de operaciones | Group: 032

The N-Queens Problem

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Abstract

This document addresses the N-queens problem, which consists of determining how to place N queens on a chessboard of NxN squares such that none can attack each other. It is posed as an Operations Research problem, defining an objective function based on a maximized sum up to N, together with constraints that guarantee the exact placement of N queens without conflict. The mathematical model considers the necessary conditions on rows, columns and diagonals to ensure an optimal solution.

Keywords: Constraints, N-Queens, Optimal Solutions, Dynamic Programming, Mathematical Model

1 Introduction

The N-queens problem is a classic in the field of computation and mathematics, with roots dating back to the 19th century. It was first proposed in 1848 by the German chess player Max Bezzel, who posed the original problem of placing eight queens on an 8x8 chessboard in such a way that none of them attack each other. This implies that no two queens can be on the same row, column or diagonal. Later, in 1850, the mathematician Franz Nauck generalized the problem for boards of size $n \times n$, giving rise to what is known today as the N-queens problem. With the development of modern computing, the N-queens problem continues to be an excellent pedagogical tool for introducing concepts of optimization, graph theory and data structures. In addition, it has been used as a basis for challenges in programming competitions and as a starting point for more complex problems within constraint analysis and Boolean satisfiability. Over time, this problem has proven its value not only as an intellectual challenge, but also as an enduring example of how a seemingly simple problem can have profound implications for computational science.

In the present work, dynamic programming was chosen to solve the N-Queens problem which is an algorithmic technique used to solve complex optimization problems. This technique consists of dividing a complex problem into several smaller subproblems, storing their solutions to avoid repetitions in the calculations and it is commonly used to reduce time and simplify coding processes. N-Queens is a combinatorial and decision problem, where a valid placement of the N queens is sought. These decisions are made sequentially, by placing one queen after another depending on the positions already used. Each decision affects the system's state and influences subsequent decisions. Dynamic programming stores results of partial configurations to avoid redundant exploration. In this way, dynamic programming memorizes the results of combinations of states, in order to avoid repeating explorations that are not useful.

2 Objectives

2.1 Main objective

Design and implement an algorithm that solves the problem of N-Queens using dynamic programming techniques.

2.2 Specific objectives

- Develop a mathematical model that accurately represents the constraints of the N-Queens problem.
- Create a dynamic programming-based solution that systematically explores and stores previous results to improve performance and reduce redundant calculations.
- Find all valid (optimal) solutions that satisfy the problem's conditions.
- Evaluate the time and space complexity of the implemented algorithm, and compare its performance with other traditional methods such as backtracking.
- Visually represent the obtained solutions to enhance their interpretation and analysis.

3 Methods

3.1 Identification of Model Elements

- Known parameters of the problem (input data):
 - The size of the board and the number of queens.
- Decision variables:
 - Matrix $[x_{i,j}]$: binary matrix indicating if a queen is at that position or not.

$$x_{i,j} = \begin{cases} 1 & \text{if the position is occupied} \\ 0 & \text{if the position is available} \end{cases}$$

- Objective function(s): Maximize the number of feasible solutions satisfying the conditions:

$$Z = \sum_{i=1}^s X_k(\mathbf{x}_{ij})$$

where

$$\mathbf{X}_k = \begin{cases} 1 & \text{if the configuration is valid} \\ 0 & \text{if the configuration is not valid} \end{cases}$$

- \mathbf{X}_k represents a matrix of size $n \times n$ for iteration k .
- Each element $x_{ij} \in \{0, 1\}$, where $x_{ij} = 1$ if a queen is placed at row i and column j , and $x_{ij} = 0$ otherwise.
- s is the total number of possible configurations.

- **Constraints:**

- No more than one queen in the same row.

$$\sum_{j=1}^n x_{i,j} = 1; \forall j = \{1, \dots, n\}$$

- There should not be more than one queen in the same column.

$$\sum_{i=1}^n x_{i,j} = 1; \forall j = \{1, \dots, n\}$$

- There should not be more than one queen on the same main diagonal.

$$\sum_{i=1}^n \sum_{j=1}^n x_{i,j} \leq 1; \forall k = \{1 - n, \dots, n - 1\}$$

where $k = i - j$

- There should not be more than one queen on the same secondary diagonal.

$$\sum_{i=1}^n \sum_{j=1}^n x_{i,j} \leq 1; \forall k = \{0, \dots, 2n - 2\}$$

where $k = i + j$

- Additional restrictions are added with each feasible solution found.

$$x_{a,b} + x_{c,d} + x_{e,f} + \dots + x_{\alpha,\delta} < n$$

where n is the number of queens to be placed, and the sum represents the cells where a solution was found.

3.2 Implementation of the model formulated in python

The mathematical model was implemented in gurobi and, in the following code, we define the variables where each cell has a binary variable $x[i,j]$ which ensures that all decision variables are defined as binary. We also define the constraints and objective function of the model.

```
ampl = AMPL()
ampl.set_option('solver', 'gurobi')
#ampl.set_option('gurobi_options', 'outputFlag=0')

model = f"""
set I := 0..{n-1};
set J := 0..{n-1};

var x{{i in I, j in J}}, binary;

s.t. row{{i in I}}: sum{{j in J}} x[i,j] = 1;
s.t. column{{j in J}}: sum{{i in I}} x[i,j] = 1;
s.t. diagPos{{d in {n+1..n-1}}}: sum{{i in I, j in J: i-j = d}} x[i,j] <= 1;
s.t. diagNeg{{d in 0..(2*n-2)}}: sum{{i in I, j in J: i+j = d}} x[i,j] <= 1;

maximize obj: sum{{i in I, j in J}} x[i,j];
"""

ampl.eval(model)
```

Figure 1: Mathematical model

By defining I and J in this way, valid indexes are created for the rows and columns of the board.

```

set I := 0..{n-1};
set J := 0..{n-1};

```

Figure 2: Set generation

For each row i , the sum of the variables must be exactly 1. This prevents more than one queen or none at all in the same row. The same applies for the columns.

```

s.t. row{{i in I}}: sum{j in J} x[i,j] = 1;
s.t. column{{j in J}}: sum{i in I} x[i,j] = 1;

```

Figure 3: Constraints columns and rows

It is ensured that for each positive diagonal, there is at least one queen. The same applies for the negative diagonal.

```

s.t. diagPos{{d in {n+1}..{n-1}}}: sum{i in I, j in J: i-j = d} x[i,j] <= 1;
s.t. diagNeg{{d in 0..{2*n-2}}}: sum{i in I, j in J: i+j = d} x[i,j] <= 1;

```

Figure 4: Constraints diagonals

Maximizing the number of queens placed, as well as adapting the row and column constraints, the solution will always have queens.

```

maximize obj: sum{{i in I, j in J}} x[i,j];

```

Figure 5: Objective function

Consistency in a model is critical to ensure its validity and robustness. In our case, this consistency guarantees that the model faithfully represents the problem domain and that the applied algorithm can function correctly based on well-founded principles and accurate information. This coherence provides a solid foundation for further analysis, testing and practical application.

After all the analysis and development stages described above, we can affirm that our model is consistent. This means that all the elements and components defined within the model operate in harmony, without internal contradictions or logical conflicts. Each element contributes meaningfully to the overall structure, and all definitions are complete and well integrated.

3.2.1 Adjusting the model parameters and development

When the model finds a valid solution, it may be the case that it returns the same solution in the following iterations. For this reason, we have a block of code where a new constraint is created and adapted to the current items.

```
new_restriction = f'{` + `join([f'x[{i}][{j}]' for i,j in queens_at_sorted])} <= {n-1}'
#print("ADDED NEW RESTRICTION: ", new_restriction)
ampl.eval(f"s.t. sol{counter}: {new_restriction};")
counter += 1
```

Figure 6: Adaptive constraint

This forces the model to find another solution in the next compilation; the process continues until it is no longer possible to find a different solution.

To generate all equivalent forms of a solution of the N-Queens problem, we have rotations and reflections. This way, each solution is transformed into all possible forms.

```
def reflejar_horizontal(tablero):
    return tablero[::-1]

def reflejar_vertical(tablero):
    return [fila[::-1] for fila in tablero]

def rot90(tablero):
    return [list(row) for row in zip(*tablero[::-1])]

def rot180(tablero):
    return [fila[::-1] for fila in tablero[::-1]]

def rot270(tablero):
    return [list(row) for row in zip(*tablero)][::-1]

def generar_equivalecetes(tablero):
    """Devuelve un conjunto de soluciones equivalentes por rotación"""
    horizontal, vertical = reflejar_horizontal(tablero), reflejar_vertical(tablero)
    rotaciones = [
        tablero,
        rot90(tablero),
        rot180(tablero),
        rot270(tablero),
        horizontal,
        rot90(horizontal),
        rot180(horizontal),
        rot270(horizontal),
        vertical,
        rot90(vertical),
        rot180(vertical),
        rot270(vertical)
    ]
    return set(tuple(tuple(fila) for fila in rot) for rot in rotaciones)
```

Figure 7: Rotations and reflections

In addition, the solutions are stored in a json file.

Generating a JSON file to store all solutions to the N-Queens problem is essential for preserving results in a structured and reusable format. JSON offers a readable and standardized way to represent each solution, making it easy to analyze, visualize and share. By saving the output, we avoid recompilation, ensure interoperability with other tools or languages and facilitate debugging and validation.

```
def save_solutions(n, solutions_set, dir_name="solutions"):
    os.makedirs(dir_name, exist_ok=True) # Create directory if it doesn't exist
    file_path = os.path.join(dir_name, f'n_{n}_solutions.json')

    # Convert set of tuples to list of lists for JSON serialization
    serializable_solutions = [[[cell for cell in row] for row in board] for board in solutions_set]

    with open(file_path, "w") as f:
        json.dump(serializable_solutions, f, indent=2)

    print(f"Saved {len(solutions_set)} solutions for n={n} in {file_path}")
```

Figure 8: Saving solutions in format json

4 Results

The subsequent table illustrates the execution times obtained by our model when N assumes values from 1 to 11.

Number of Queens	Number of solutions	Time (seconds)
1	1	0.0476
2	0	0.0377
3	0	0.0457
4	2	0.0687
5	10	0.0746
6	4	0.0706
7	40	0.3703
8	92	0.3429
9	352	1.2722
10	724	5.4401
11	2680	36.4468

Table 1: Results of the N-Queens Problem

As can be seen from the case of 7 queens, the execution time starts to increase exponentially, mainly due to the increase in the number of possible solutions. Our approach for this problem is to optimize an objective function that maximizes the number of queens that can be placed on a chessboard of size $N \times N$. To solve the model, the Gurobi solver was used through AMPL, since it was considered the most suitable for the approach taken; however, there could be other solvers with better performance. During the development of the model, functions that generate rotations and reflections of a solution were incorporated, which allow the algorithm to reduce the computation time by avoiding the problem of having to find all the symmetrical configurations by itself. Despite these improvements, the model still presents optimization opportunities to reduce the use of resources and improve its efficiency.

The following images demonstrate the code when requesting solutions for a specific number of queens. The solutions for $n=1$ through $n=4$ are listed.

```
Ingrese la cantidad de reinas (N) a posicionar en un tablero NxN: 1
Solving time: 0.02397465705871582

Se encontraron 1 para 1 reinas!!!
Para ver todas las soluciones ingrese 's'.... .
¿Deseas ver todas las soluciones? (s/n): 
```

Figure 9: Select n=1

```
Found 1 different solutions for 1 queens
***** Solution 1 *****
Q
-----
```

Figure 10: Solutions to n=1

```
Ingrese la cantidad de reinas (N) a posicionar en un tablero NxN: 2
Solving time: 0.02858448028564453
```

```
NO SOLUTIONS WERE FOUND
```

Figure 11: Select n=2

```
Ingrese la cantidad de reinas (N) a posicionar en un tablero NxN: 3
Solving time: 0.03973245620727539
```

```
NO SOLUTIONS WERE FOUND
```

Figure 12: Select n=3

```
Ingrese la cantidad de reinas (N) a posicionar en un tablero NxN: 4
Solving time: 0.05458950996398926
```

```
Se encontraron 2 para 4 reinas!!!
Para ver todas las soluciones ingrese 's'.....
```

```
¿Deseas ver todas las soluciones? (s/n): 
```

Figure 13: Select n=4

```
Found 2 different solutions for 4 queens
***** Solution 1 *****
. . Q .
Q . . .
. . . Q
. Q . .

-----
***** Solution 2 *****
. Q . .
. . . Q
Q . . .
. . Q .
```

Figure 14: Solutions to n=4

5 Conclusion

This project addressed the classic N-Queens problem, which consists of placing N queens on an $N \times N$ chessboard so that no two queens threaten each other. The main objective was to develop a mathematical model and computational approach capable of finding all valid configurations under the given constraints.

We formulated the problem implementing the solution using the Gurobi solver through AMPL. To improve performance and avoid redundant computations, we incorporated mechanisms to generate all symmetrical variations (rotations and reflections) of each solution and filter duplicates. This not only optimized the solution-finding process but also provided a comprehensive understanding of the problem's structure.

The model successfully identified all solutions for various board sizes, with increasing computational time as the number of queens grew, reflecting the exponential complexity of the problem. Execution time and system resource usage were acceptable for lower values of N , but optimizations would be necessary for larger instances.

As a future improvement, alternative solvers or parallel processing techniques could be explored to enhance performance. Additionally, refining the mathematical model to further reduce the solution space without missing valid configurations could be a valuable direction.

Overall, this project allowed us to apply key concepts from Operations Research and Linear Programming while strengthening our problem-solving and algorithmic design skills. It demonstrated how mathematical modeling and computational tools can be effectively combined to tackle complex combinatorial problems.

6 Bibliographic References

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7 Annexes

The following images illustrate various results of the program developed.

7.1 Solution to 1 Queen



7.2 Solution to 2 Queens

There's no solution.

7.3 Solution to 3 Queens

There's no solution.

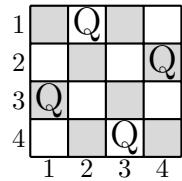


Figure 15: Solution 1

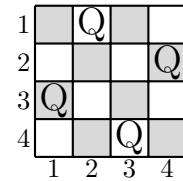


Figure 16: Solution 2

7.4 Solution to 4 Queens

7.5 Solution to 5 Queens

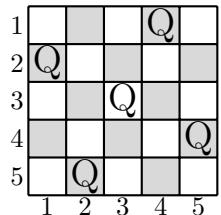


Figure 17: Solution 1

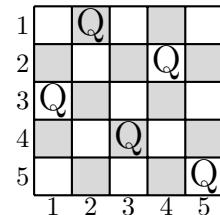


Figure 18: Solution 2

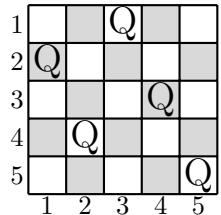


Figure 19: Solution 3

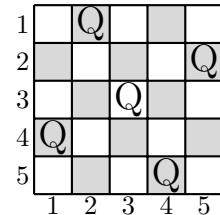


Figure 20: Solution 4

1	Q			
2				Q
3		Q		
4				Q
5		Q		

Figure 21: Solution 5

1				Q
2			Q	
3	Q			
4				Q
5		Q		

Figure 22: Solution 6

1				Q
2		Q		
3				Q
4	Q			
5		Q		

Figure 23: Solution 7

1				Q
2		Q		
3				Q
4			Q	
5	Q			

Figure 24: Solution 8

1	Q			
2		Q		
3				Q
4		Q		
5			Q	

Figure 25: Solution 9

1			Q	
2				Q
3		Q		
4				Q
5	Q			

Figure 26: Solution 10

7.6 Solution to 6 Queens

1		Q				
2			Q			
3				Q		
4	Q					
5		Q				
6			Q			

Figure 27: Solution 1

1			Q			
2						Q
3		Q				
4					Q	
5	Q					
6			Q			

Figure 28: Solution 2

1				Q		
2	Q					
3					Q	
4		Q				
5					Q	
6			Q			

Figure 29: Solution 3

1						Q
2			Q			
3					Q	
4						Q
5	Q					
6			Q			

Figure 30: Solution 4

7.7 Solution to 7 Queens

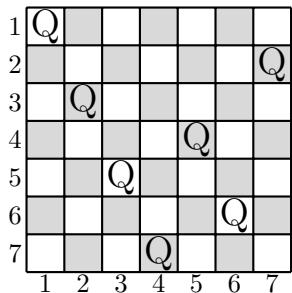


Figure 31: Solución 1

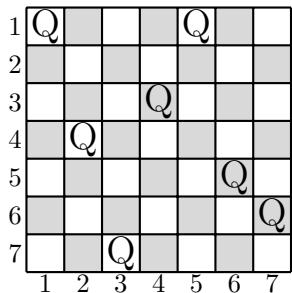


Figure 32: Solución 2

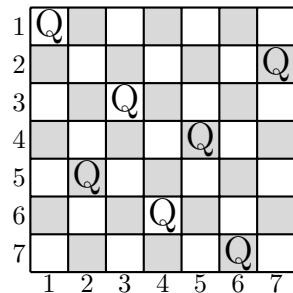


Figure 33: Solución 3

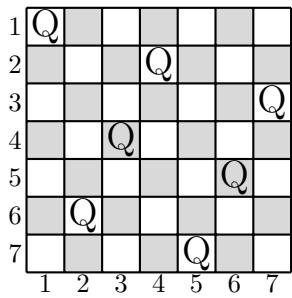


Figure 34: Solución 4

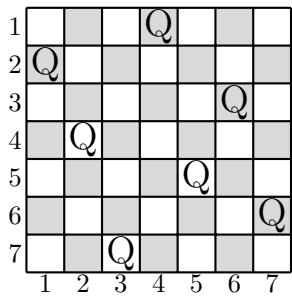


Figure 35: Solución 5

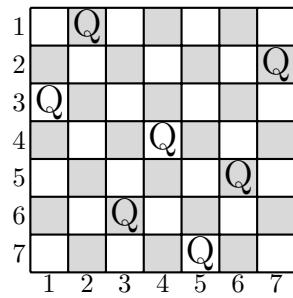


Figure 36: Solución 6

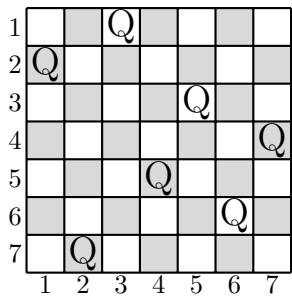


Figure 37: Solución 7

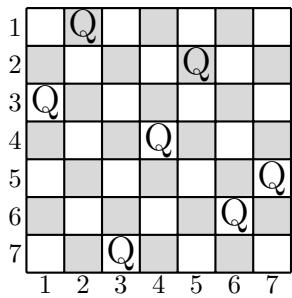


Figure 38: Solución 8

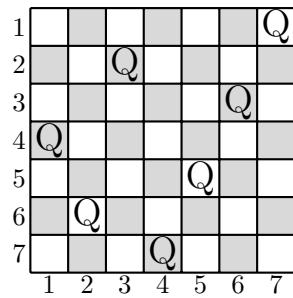


Figure 39: Solución 9

1				Q		
2	Q					
3					Q	
4			Q			
5	Q					
6						Q
7		Q				

Figure 40: Solution 10

1	Q					
2						Q
3				Q		
4	Q					
5					Q	
6			Q			
7				Q		

Figure 41: Solution 11

1			Q			
2						Q
3		Q				
4			Q			
5					Q	
6	Q					
7			Q			

Figure 42: Solution 12

1				Q		
2	Q					
3						Q
4		Q				
5	Q					
6				Q		
7					Q	

Figure 43: Solution 13

1				Q		
2		Q				
3						Q
4			Q			
5	Q					
6					Q	
7						Q

Figure 44: Solution 14

1				Q		
2						Q
3						Q
4	Q					
5						Q
6	Q					
7		Q				

Figure 46: Solution 16

1				Q		
2						Q
3						Q
4		Q				
5						Q
6	Q					
7			Q			

Figure 47: Solution 17

1				Q		
2		Q				
3						Q
4						Q
5	Q					
6					Q	
7	Q					

Figure 49: Solution 19

1					Q	
2	Q					
3		Q				
4			Q			
5					Q	
6	Q					
7			Q			

Figure 50: Solution 20

1	Q					
2						Q
3		Q				
4				Q		
5					Q	
6	Q					
7			Q			

Figure 51: Solution 21

1		Q				
2						Q
3	Q					
4				Q		
5	Q					
6			Q			
7					Q	

Figure 52: Solution 22

1		Q				
2						Q
3	Q					
4				Q		
5						Q
6			Q			
7	Q					

Figure 53: Solution 23

1			Q			
2	Q					
3		Q				
4				Q		
5						Q
6			Q			
7	Q					

Figure 54: Solution 24

1		Q				
2						Q
3			Q			
4	Q					
5			Q			
6				Q		
7	Q					

Figure 55: Solution 25

1		Q				
2						Q
3			Q			
4	Q					
5			Q			
6				Q		
7	Q					

Figure 56: Solution 26

1			Q			
2		Q				
3						
4	Q					
5		Q				
6			Q			
7	Q					

Figure 57: Solution 27

1		Q				
2	Q					
3			Q			
4				Q		
5	Q					
6			Q			
7				Q		

Figure 58: Solution 28

1			Q			
2		Q				
3						
4			Q			
5	Q					
6			Q			
7	Q					

Figure 59: Solution 29

1				Q		
2						Q
3	Q					
4		Q				
5				Q		
6	Q					
7			Q			

Figure 60: Solution 30

1				Q		
2			Q			
3						
4				Q		
5	Q					
6			Q			
7	Q					

Figure 61: Solution 31

1			Q			
2						Q
3	Q					
4		Q				
5				Q		
6	Q					
7			Q			

Figure 62: Solution 32

1	Q					
2						Q
3						
4			Q			
5						Q
6	Q					
7			Q			

Figure 63: Solution 33

1				Q			
2							Q
3		Q					
4					Q		
5	Q						
6			Q				
7						Q	

Figure 64: Solution 34

1					Q		
2				Q			
3							Q
4	Q						
5						Q	
6			Q				
7					Q		

Figure 65: Solution 35

1						Q	
2	Q						
3						Q	
4				Q			
5						Q	
6			Q				
7					Q		

Figure 66: Solution 36

1					Q		
2			Q				
3							Q
4	Q						
5							Q
6	Q						
7			Q				

Figure 67: Solution 37

1	Q						
2				Q			
3							Q
4					Q		
5		Q					
6						Q	
7			Q				

Figure 68: Solution 38

1					Q		
2							Q
3						Q	
4				Q			
5			Q				
6						Q	
7	Q						

Figure 69: Solution 39

1		Q					
2					Q		
3							Q
4	Q						
5			Q				
6						Q	
7			Q				

Figure 70: Solution 40

7.8 Solution to 8 Queens

1		Q					
2				Q			
3		Q					
4	Q						
5						Q	
6			Q				
7				Q			
8	Q						

Solution 1

1			Q				
2	Q						
3						Q	
4		Q					
5				Q			
6						Q	
7			Q				
8	Q						

Solution 4

1		Q					
2				Q			
3	Q						
4						Q	
5					Q		
6			Q				
7					Q		
8	Q						

Solution 2

1			Q				
2						Q	
3	Q						
4						Q	
5	Q						
6			Q				
7	Q						
8				Q			

Solution 5

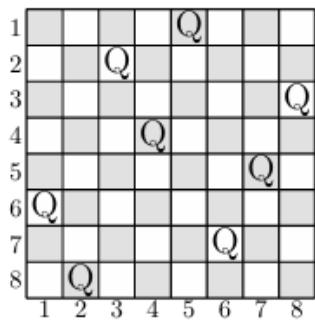
1				Q			
2	Q						
3				Q			
4						Q	
5	Q						
6				Q			
7			Q				
8	Q						

Solution 3

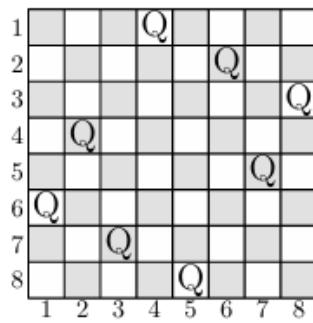
1					Q		
2	Q						
3						Q	
4	Q						
5						Q	
6			Q				
7	Q						
8		Q					

Solution 6

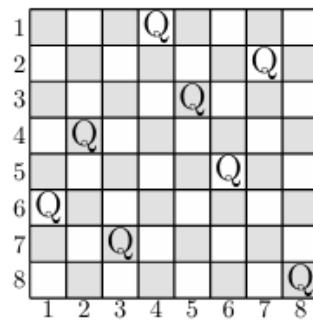
Figure 71: Solutions 1 to 6



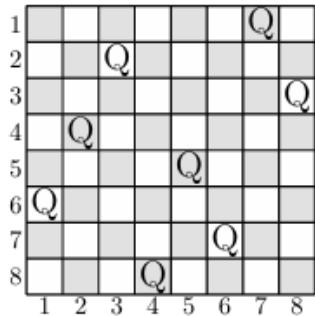
Solution 13



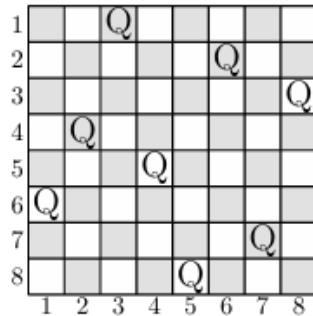
Solution 14



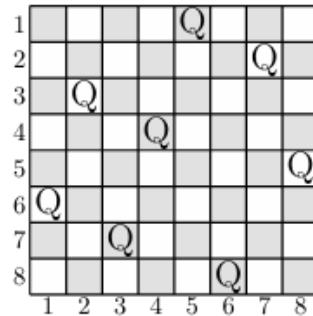
Solution 15



Solution 16

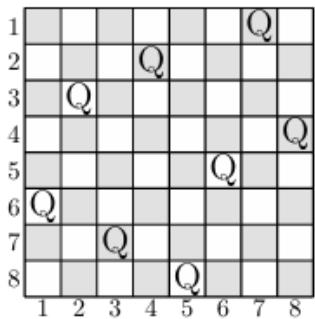


Solution 17

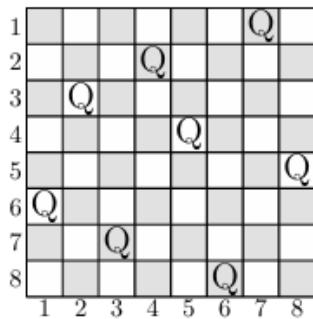


Solution 18

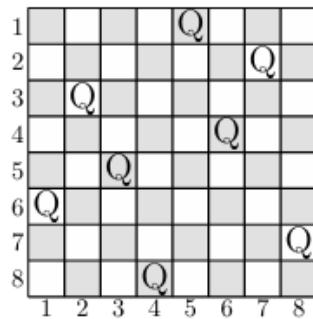
Figure 72: Solutions 13 to 18



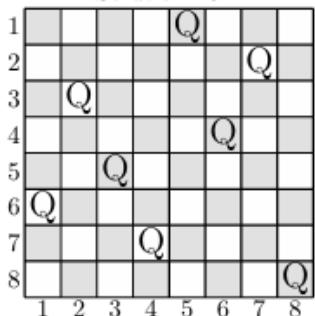
Solution 19



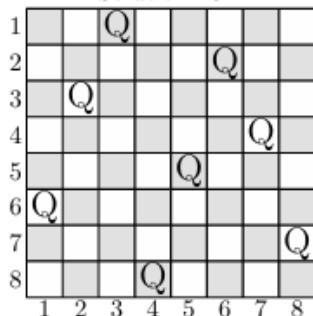
Solution 20



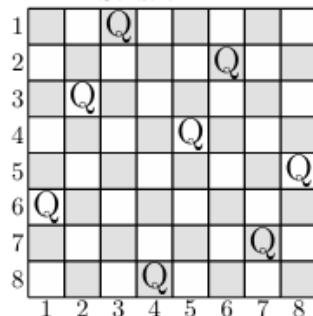
Solution 21



Solution 22



Solution 23



Solution 24

Figure 73: Solutions 19 to 24

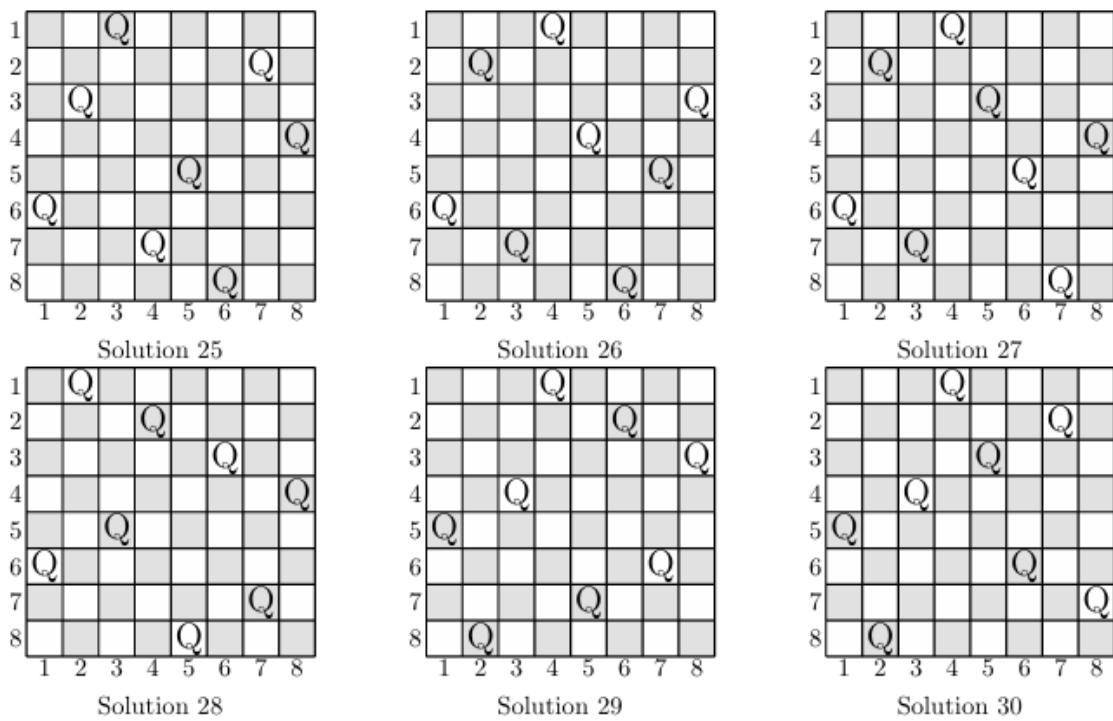


Figure 74: Solutions 25 to 30

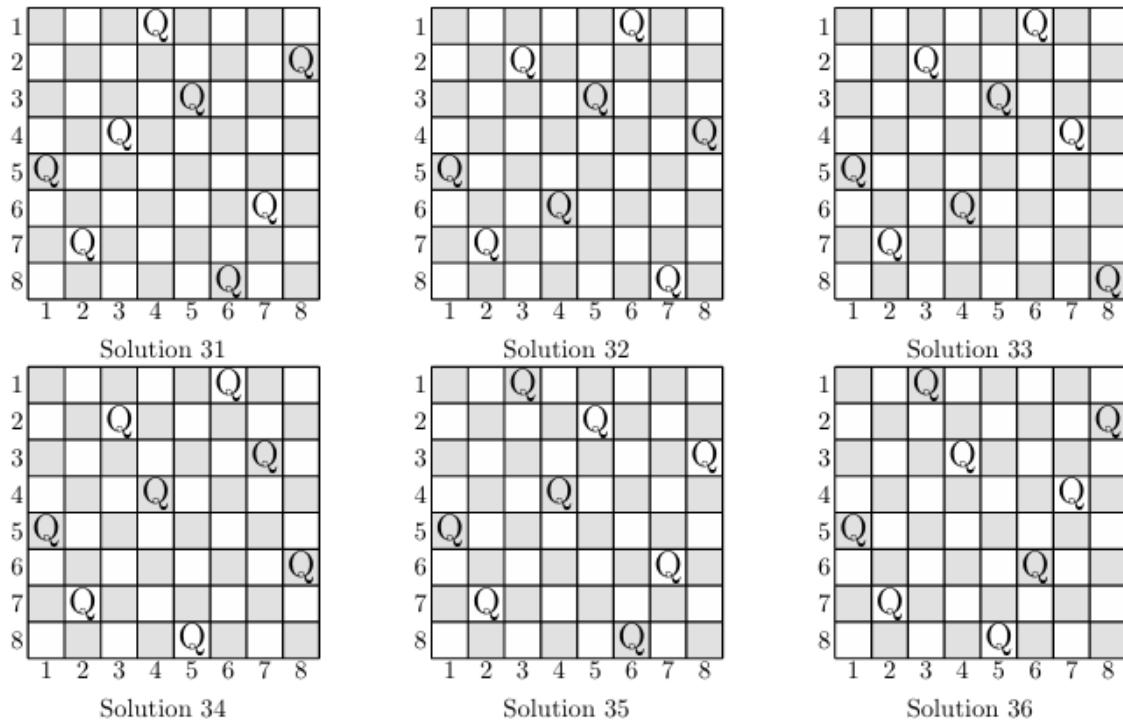


Figure 75: Solutions 31 to 36

1					Q		
2							Q
3	Q						
4			Q				
5	Q						
6						Q	
7					Q		
8		Q					

Solution 37

1			Q				
2					Q		
3	Q						
4							Q
5	Q						
6						Q	
7					Q		
8						Q	

Solution 38

1				Q			
2		Q					
3							Q
4							Q
5	Q						
6		Q					
7					Q		
8		Q					

Solution 39

1				Q			
2	Q						
3							Q
4			Q				
5	Q						
6							Q
7		Q					
8				Q			

Solution 40

1			Q				
2	Q						
3							Q
4			Q				
5	Q						
6		Q					
7					Q		
8		Q					

Solution 41

1							Q
2	Q						
3							Q
4			Q				
5	Q						
6		Q					
7					Q		
8		Q					

Solution 42

Figure 76: Solutions 37 to 42

1						Q	
2	Q						
3							Q
4		Q					
5	Q						
6		Q					
7					Q		
8		Q					

Solution 43

1				Q			
2	Q						
3							Q
4			Q				
5	Q						
6		Q					
7					Q		
8		Q					

Solution 44

1	Q						
2				Q			
3							Q
4		Q					
5	Q						
6		Q					
7					Q		
8		Q					

Solution 46

1					Q		
2						Q	
3			Q				
4	Q						
5		Q					
6					Q		
7		Q					
8				Q			

Solution 47

1	Q						
2						Q	
3							Q
4						Q	
5	Q						
6							Q
7						Q	
8					Q		

Solution 45

1		Q					
2						Q	
3							Q
4	Q						
5		Q					
6						Q	
7		Q					
8				Q			

Solution 48

Figure 77: Solutions 43 to 48

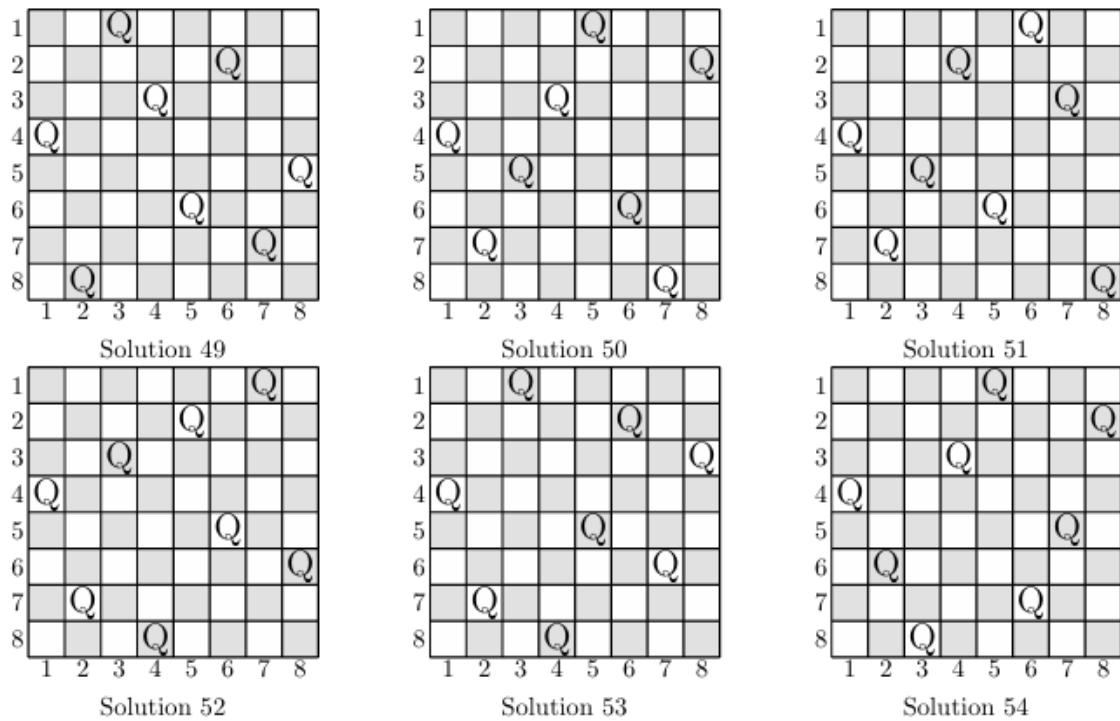


Figure 78: Solutions 49 to 54

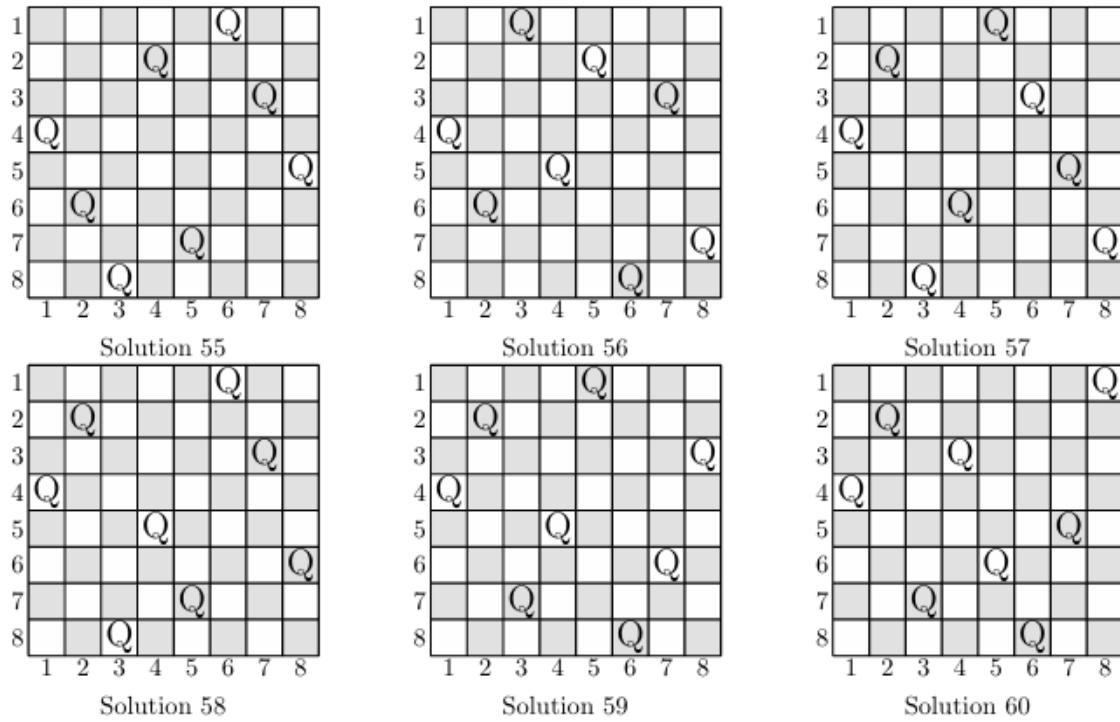


Figure 79: Solutions 55 to 60

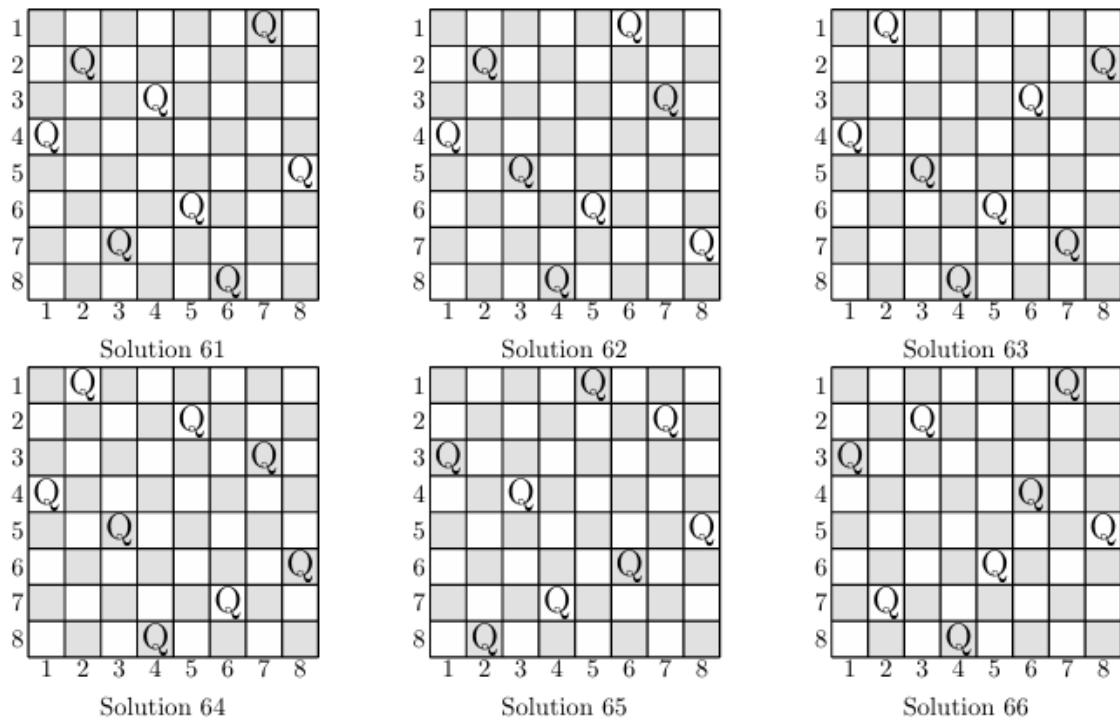


Figure 80: Solutions 61 to 66

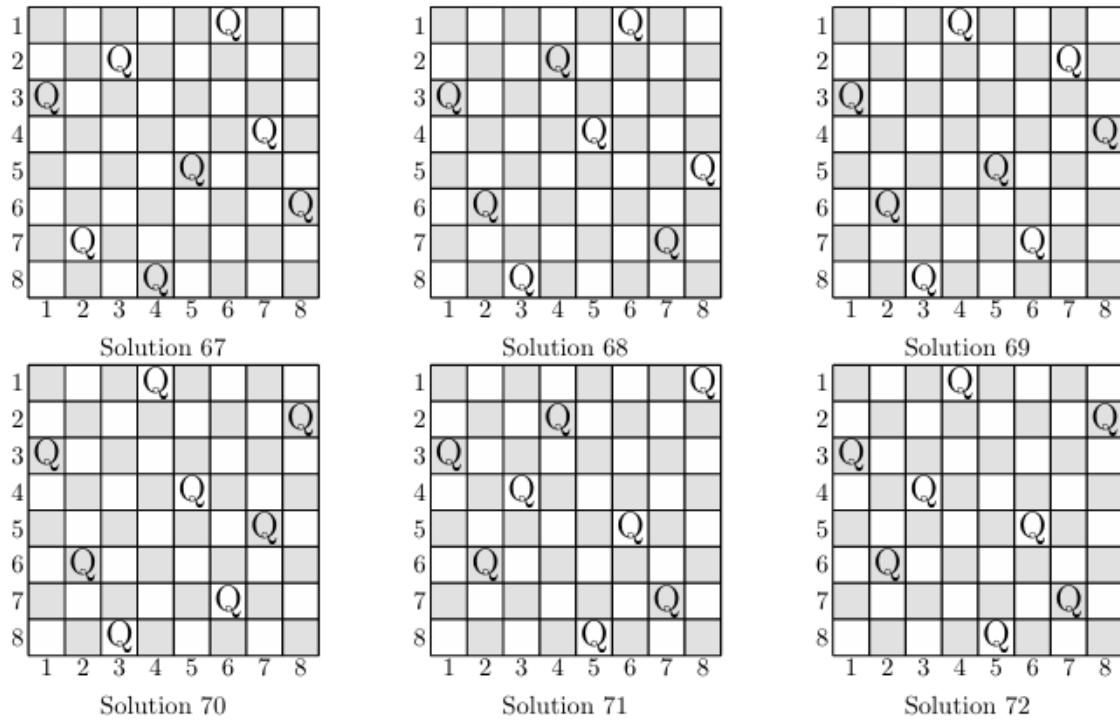
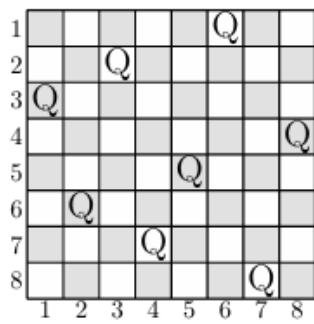
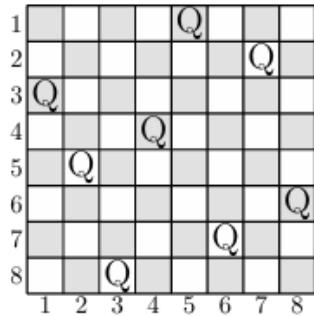


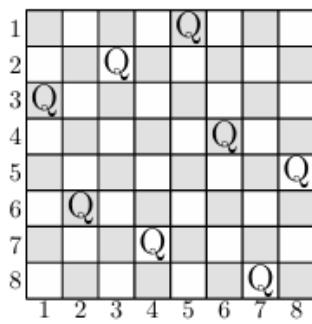
Figure 81: Solutions 67 to 72



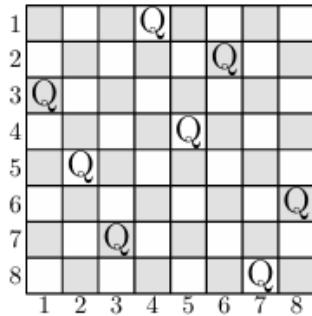
Solution 73



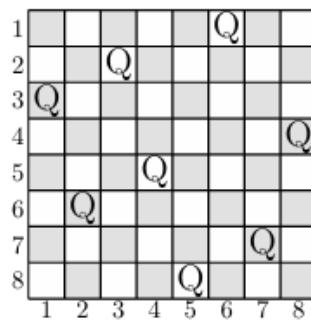
Solution 76



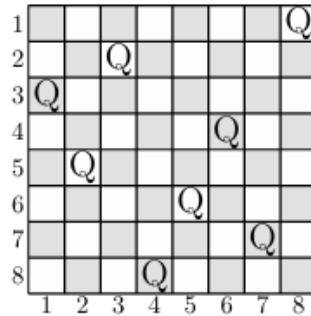
Solution 74



Solution 77

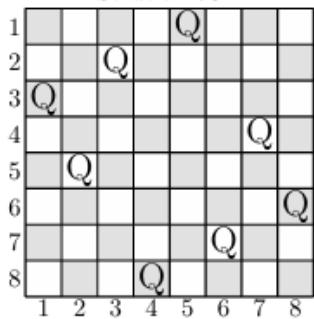


Solution 75

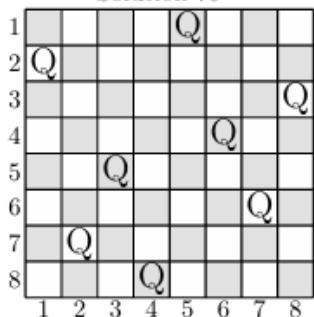


Solution 78

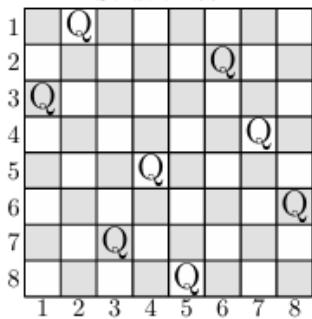
Figure 82: Solutions 73 to 78



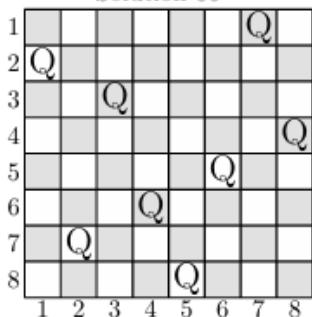
Solution 79



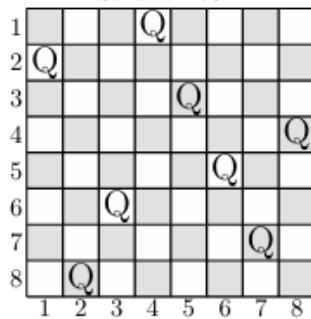
Solution 82



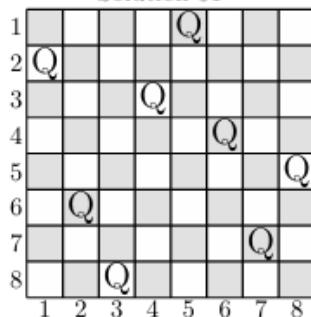
Solution 80



Solution 83



Solution 81



Solution 84

Figure 83: Solutions 79 to 84

1			Q				
2	Q						
3						Q	
4				Q			
5						Q	
6		Q					
7			Q				
8				Q			

Solution 85

1					Q		
2	Q						
3							Q
4				Q			
5		Q					
6						Q	
7		Q					
8				Q			

Solution 86

1						Q	
2	Q						
3							Q
4				Q			
5		Q					
6						Q	
7		Q					
8				Q			

Solution 87

1							Q
2	Q						
3				Q			
4		Q					
5							Q
6		Q					
7				Q			
8		Q					

Solution 88

1	Q						
2							Q
3			Q				
4				Q			
5							Q
6		Q					
7			Q				
8		Q					

Solution 89

1	Q						
2							Q
3			Q				
4				Q			
5							Q
6		Q					
7			Q				
8		Q					

Solution 91

1	Q						
2							Q
3						Q	
4							Q
5		Q					
6				Q			
7						Q	
8		Q					

Solution 92

Figure 84: Solutions 85 to 92

7.9 Solution to 9 Queens

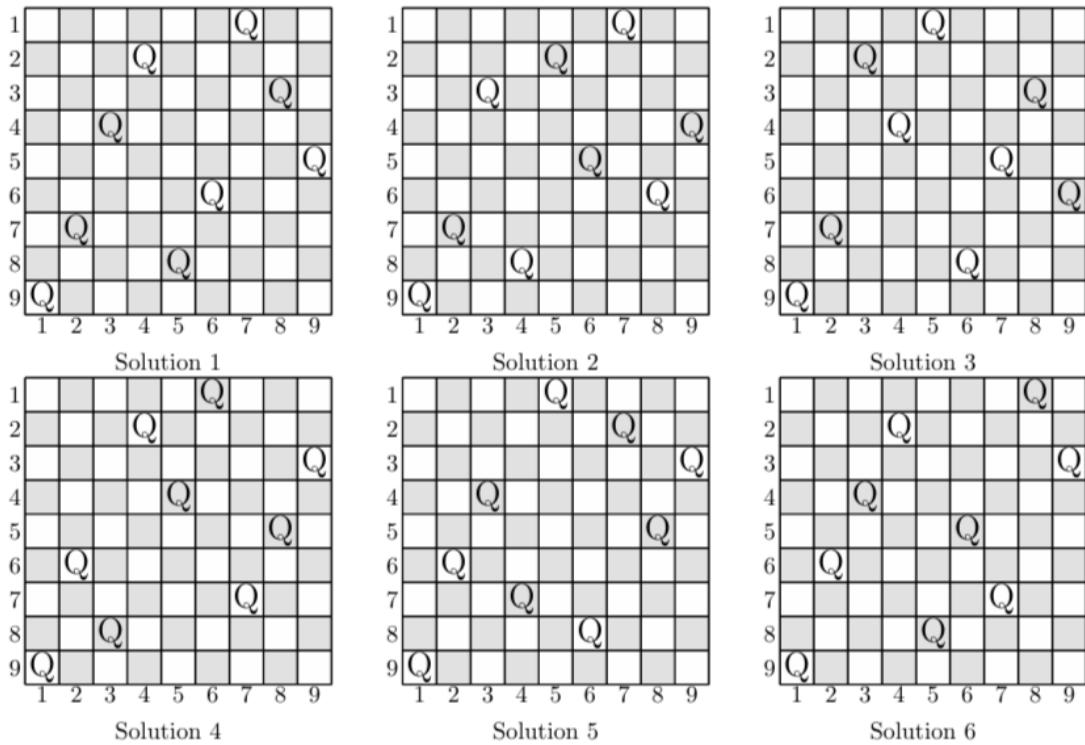


Figure 85: Solutions 1 to 6

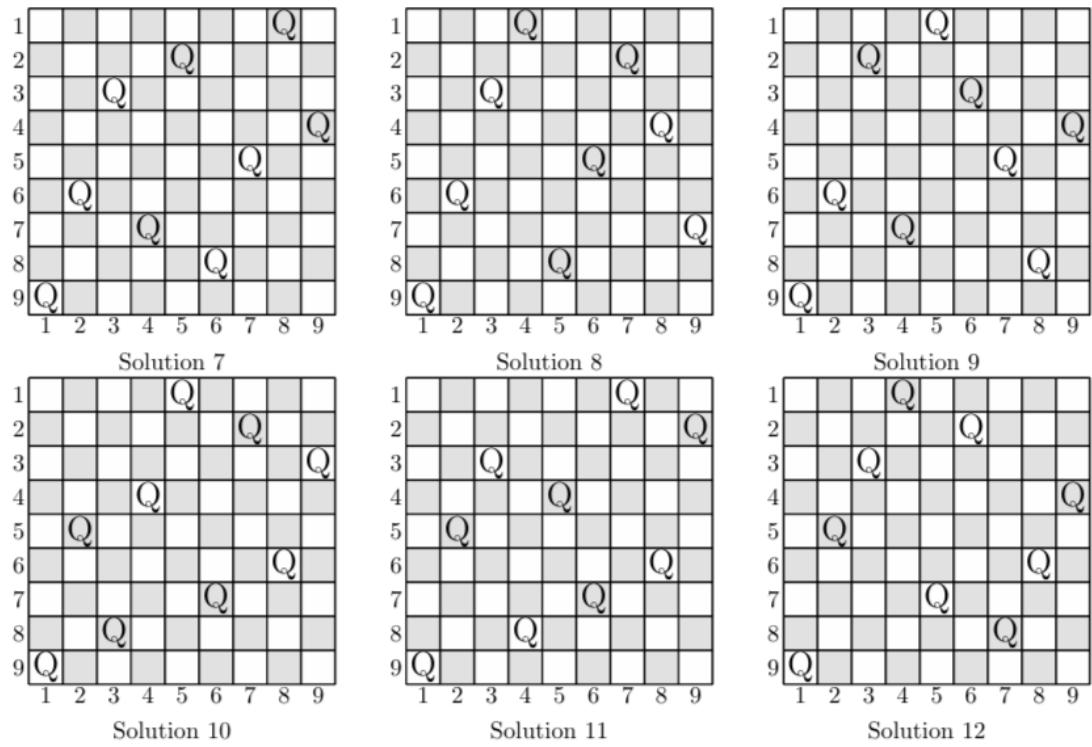


Figure 86: Solutions 7 to 12

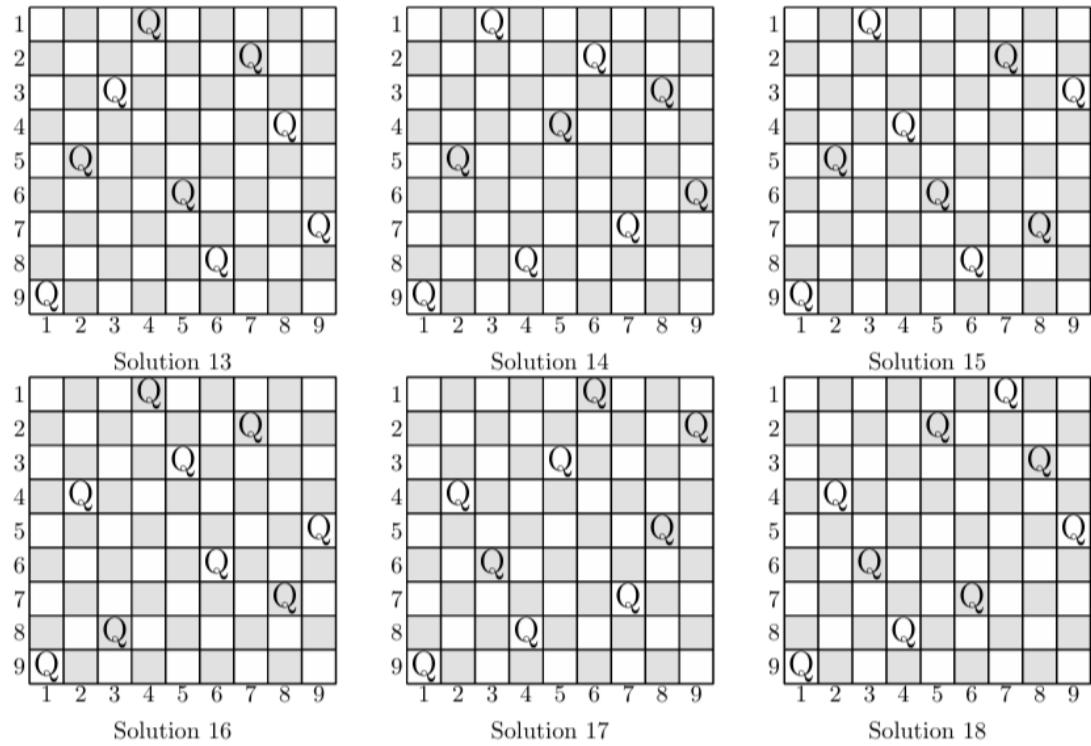


Figure 87: Solutions 13 to 18

1								Q
2		Q						
3			Q					
4	Q							
5							Q	
6					Q			
7			Q					
8				Q				
9	Q							

Solution 19

1			Q					
2								Q
3	Q							
4			Q					
5						Q		
6				Q				
7					Q			
8				Q				
9	Q							

Solution 22

1								Q
2				Q				
3	Q							
4							Q	
5		Q						
6							Q	
7					Q			
8				Q				
9	Q							

Solution 25

1				Q				
2	Q							
3			Q					
4							Q	
5	Q							
6							Q	
7			Q					
8				Q				
9	Q							

Solution 28

1								Q
2		Q						
3			Q					
4	Q							
5			Q					
6				Q				
7					Q			
8				Q				
9	Q							

Solution 20

1								Q
2			Q					
3	Q							
4								Q
5							Q	
6				Q				
7					Q			
8				Q				
9	Q							

Solution 23

1								Q
2			Q					
3	Q							
4								Q
5							Q	
6				Q				
7					Q			
8				Q				
9	Q							

Solution 21

Solution 24

1								Q
2		Q						
3			Q					
4				Q				
5					Q			
6		Q						
7					Q			
8				Q				
9	Q							

Solution 26

1								Q
2			Q					
3				Q				
4					Q			
5						Q		
6		Q						
7						Q		
8	Q							
9				Q				

Solution 27

1								Q
2				Q				
3	Q							
4							Q	
5							Q	
6		Q						
7							Q	
8	Q							
9				Q				

Solution 30

Figure 88: Solutions 19 to 24

Figure 89: Solutions 25 to 30

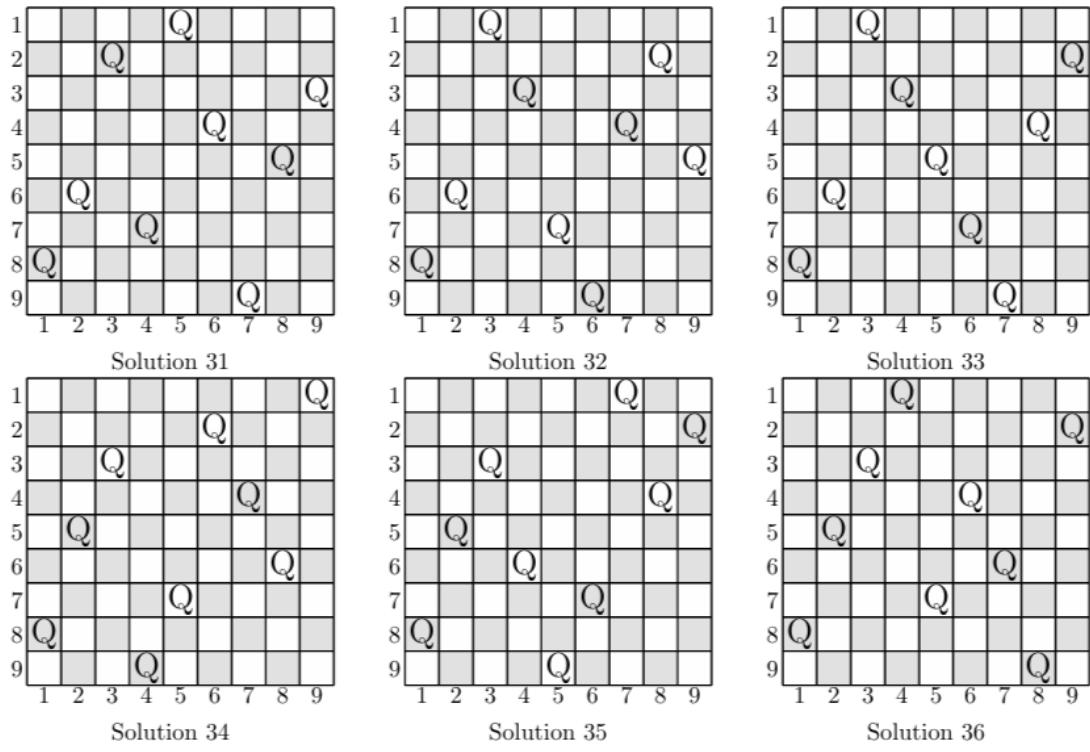


Figure 90: Solutions 31 to 36

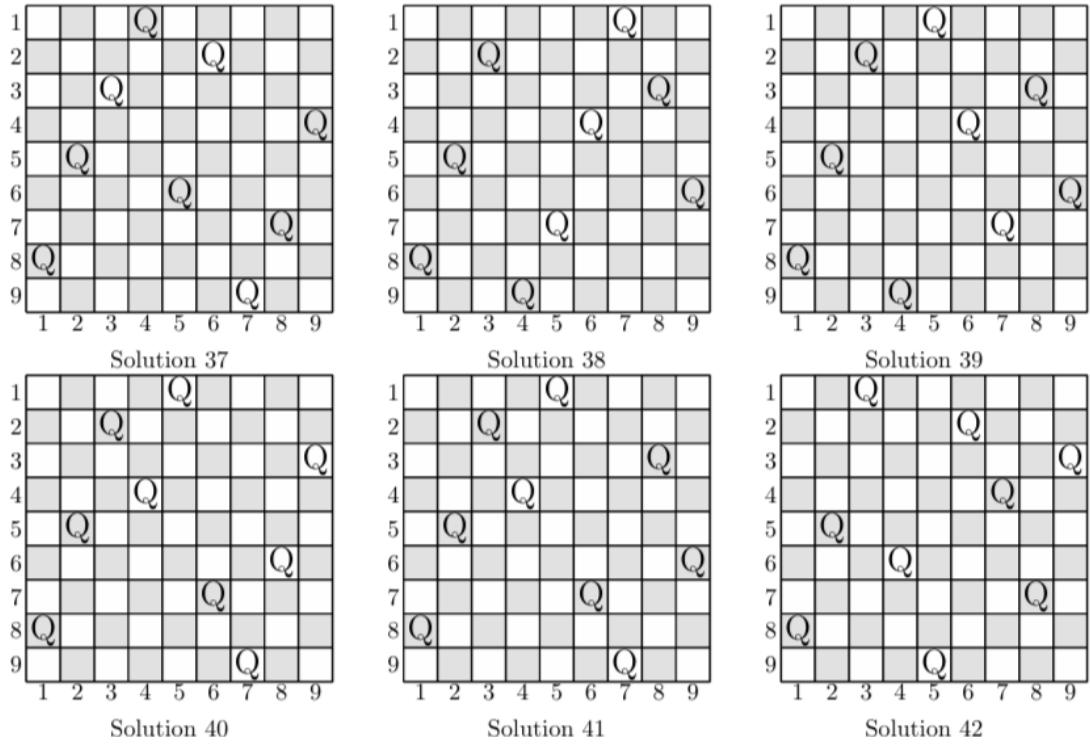
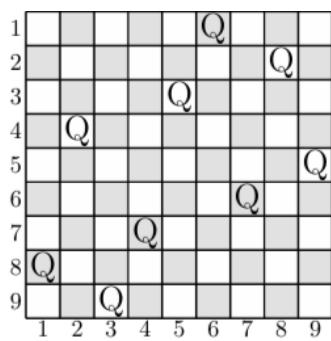
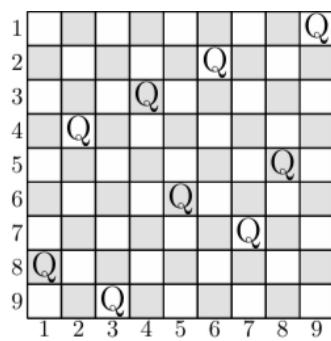


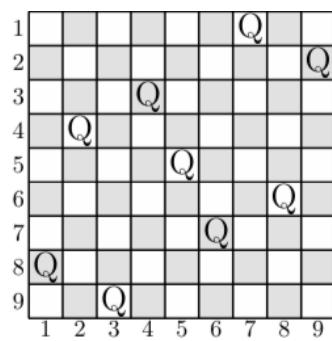
Figure 91: Solutions 37 to 42



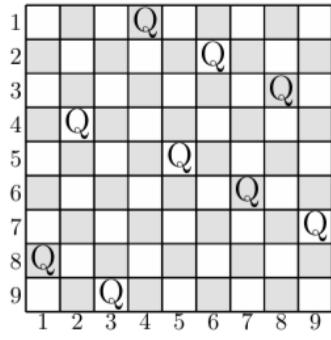
Solution 43



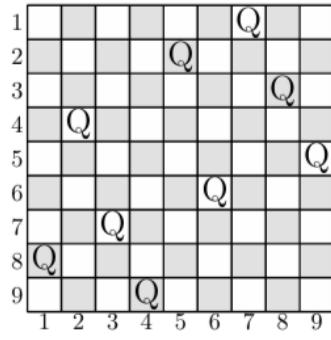
Solution 44



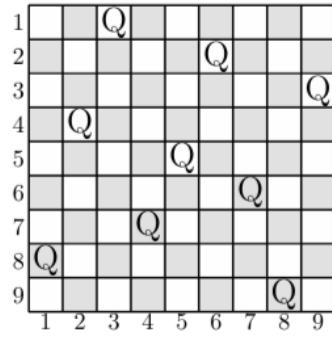
Solution 45



Solution 46

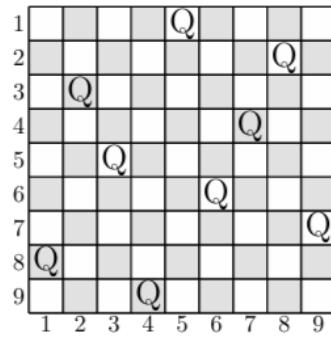


Solution 47

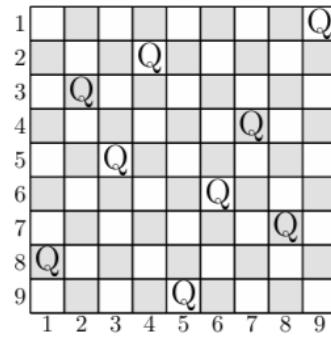


Solution 48

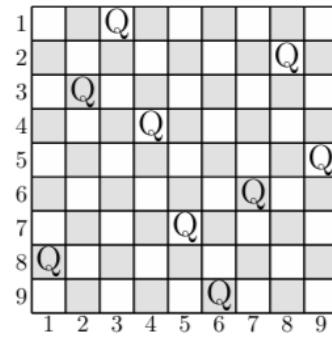
Figure 92: Solutions 43 to 48



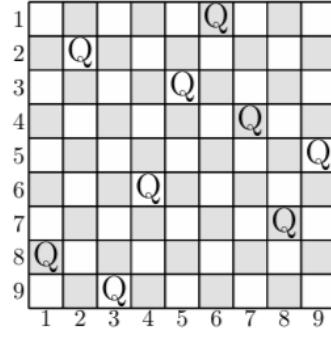
Solution 49



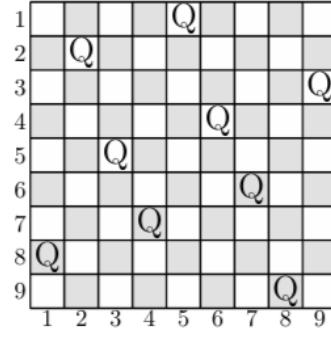
Solution 50



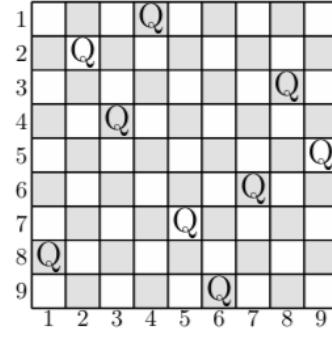
Solution 51



Solution 52

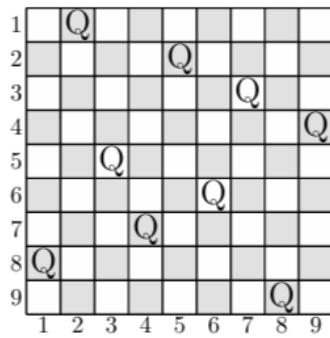


Solution 53

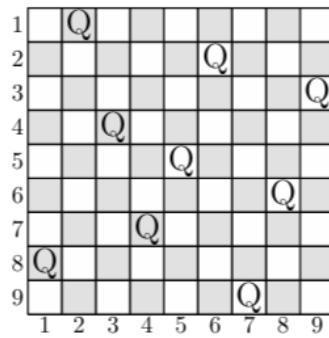


Solution 54

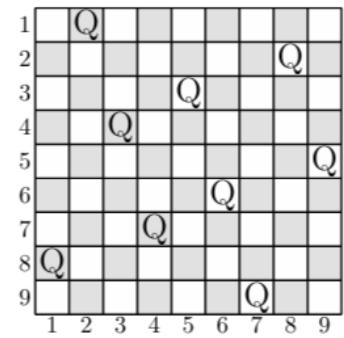
Figure 93: Solutions 49 to 54



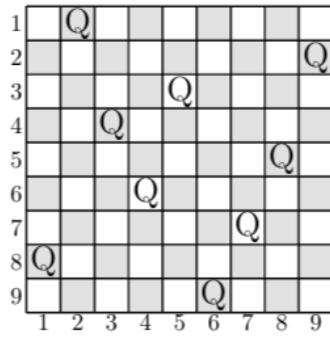
Solution 55



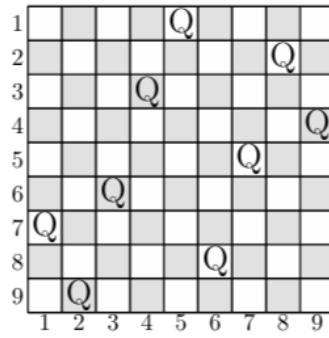
Solution 56



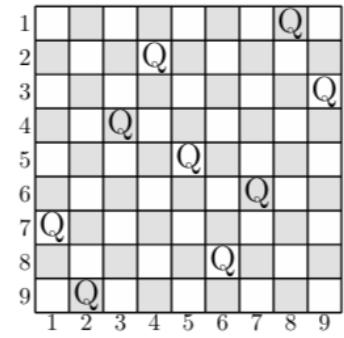
Solution 57



Solution 58

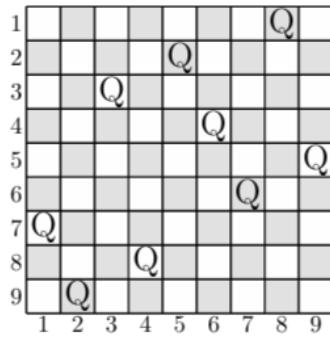


Solution 59

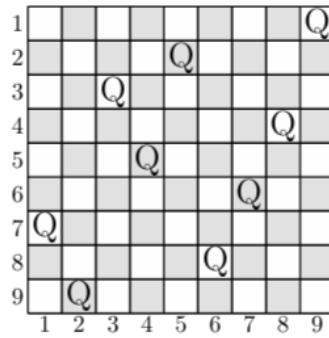


Solution 60

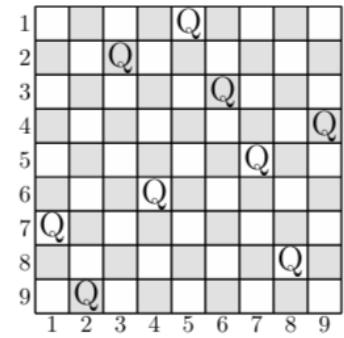
Figure 94: Solutions 55 to 60



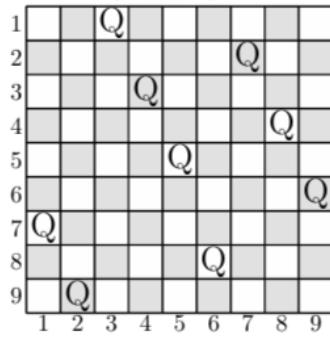
Solution 61



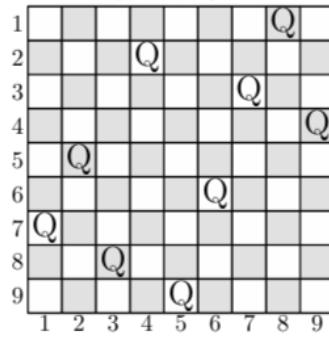
Solution 62



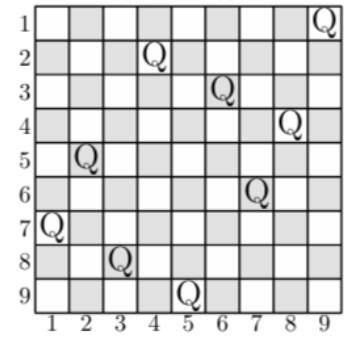
Solution 63



Solution 64

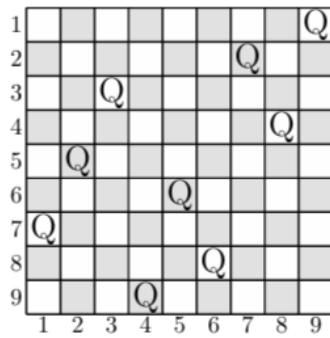


Solution 65

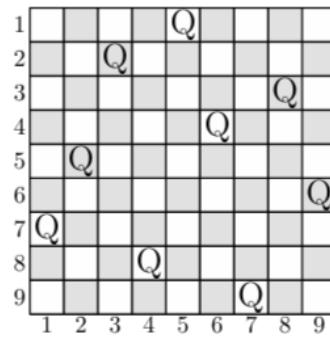


Solution 66

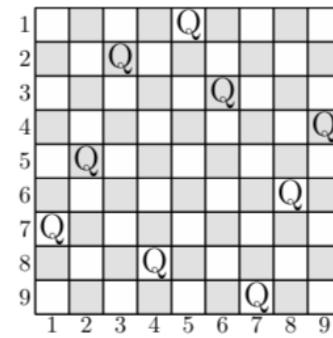
Figure 95: Solutions 61 to 66



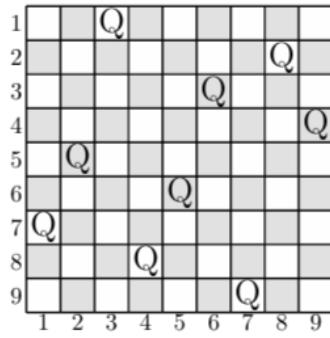
Solution 67



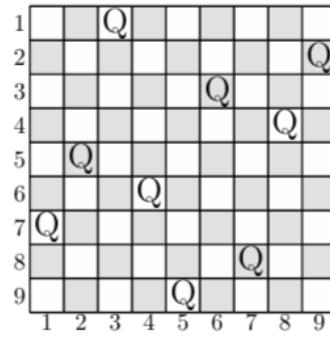
Solution 68



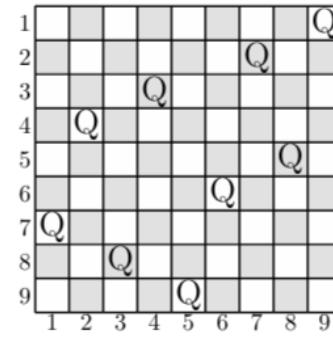
Solution 69



Solution 70

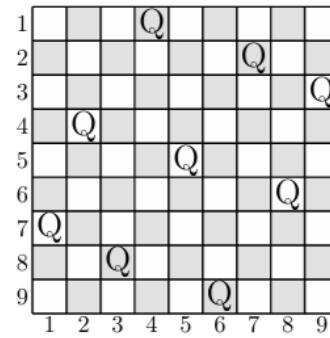


Solution 71

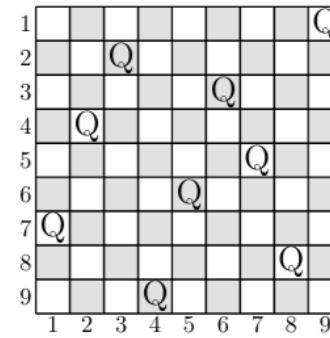


Solution 72

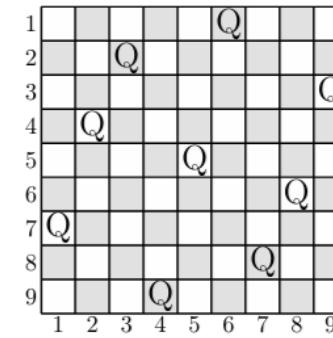
Figure 96: Solutions 67 to 72



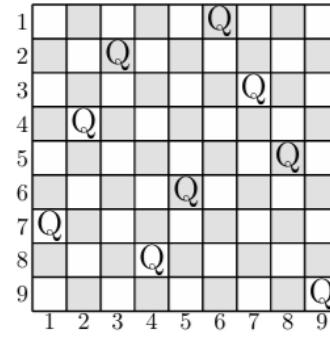
Solution 73



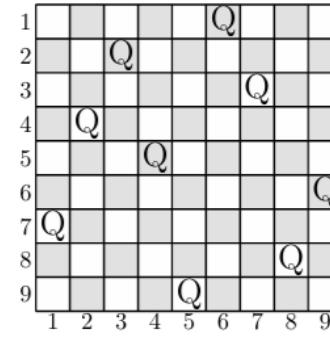
Solution 74



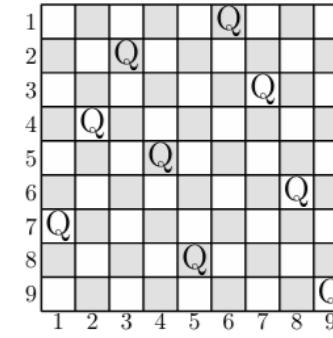
Solution 75



Solution 76



Solution 77



Solution 78

Figure 97: Solutions 73 to 78

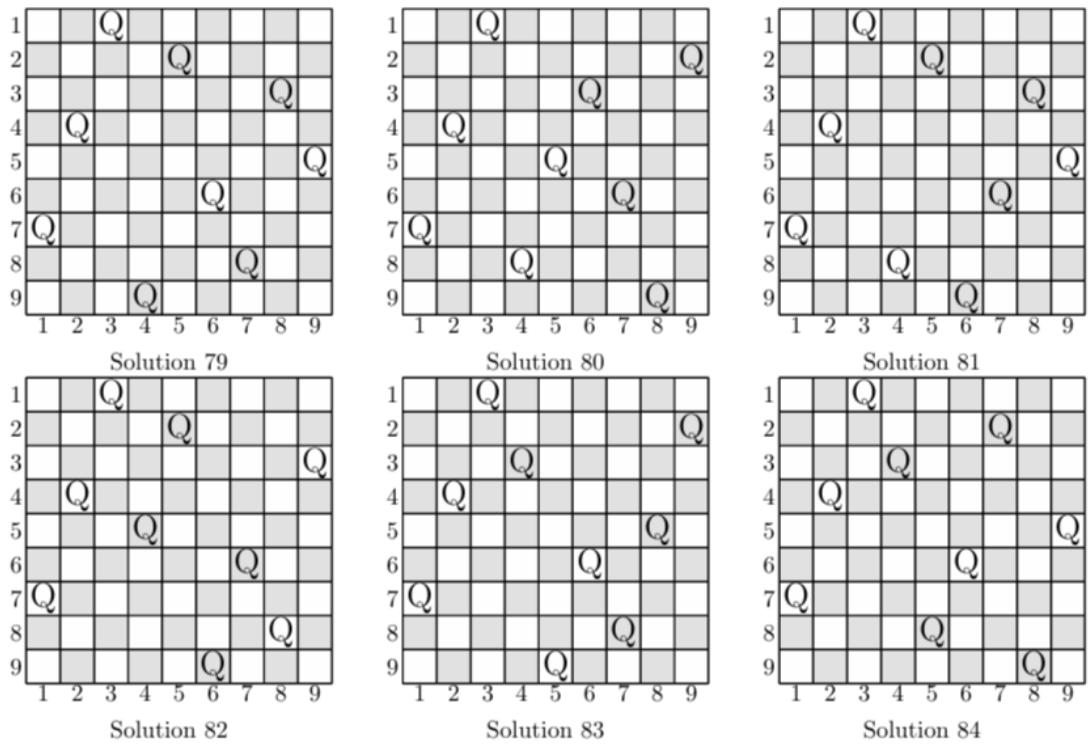


Figure 98: Solutions 79 to 84

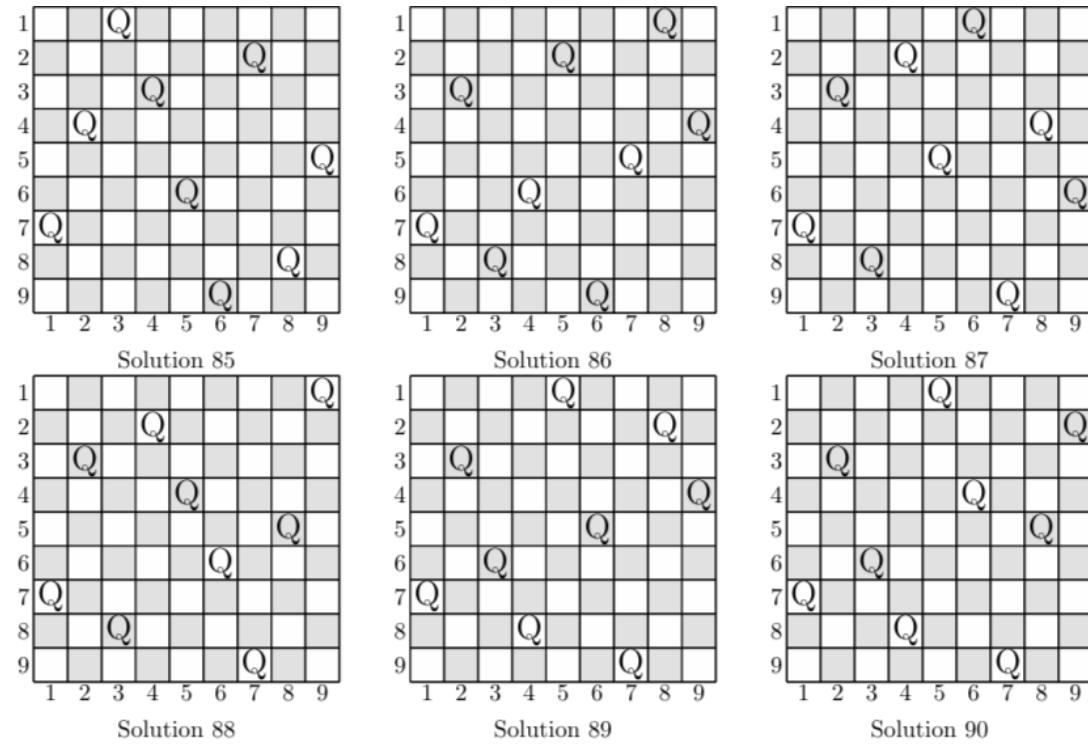


Figure 99: Solutions 85 to 90

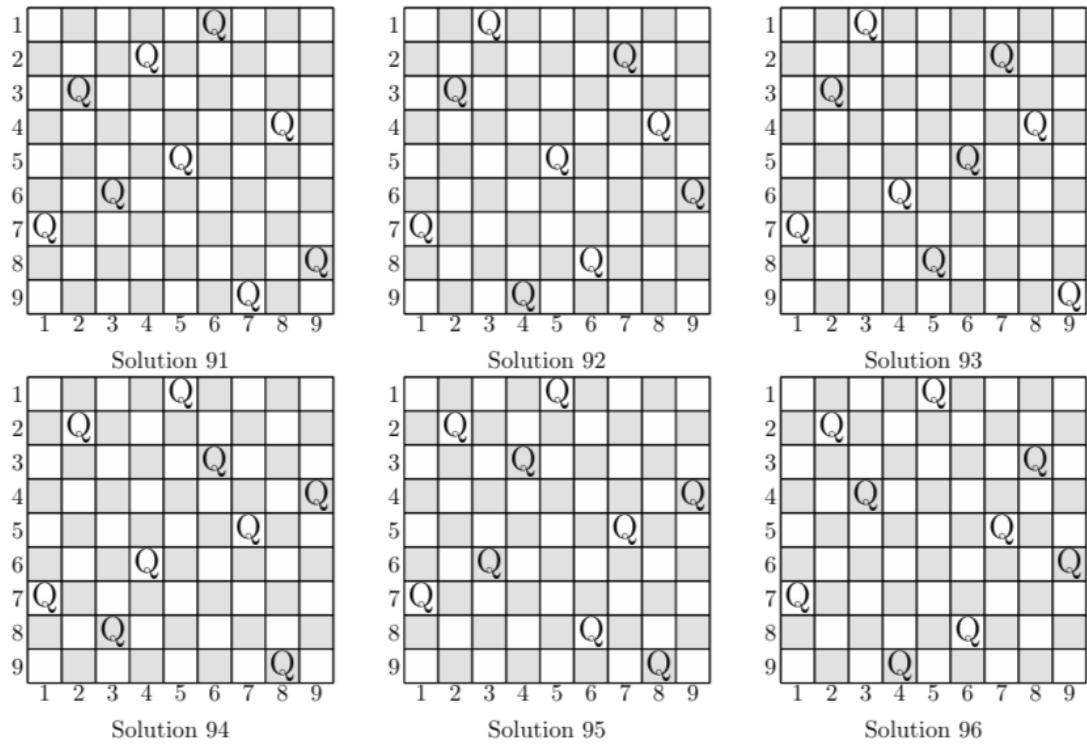


Figure 100: Solutions 91 to 96

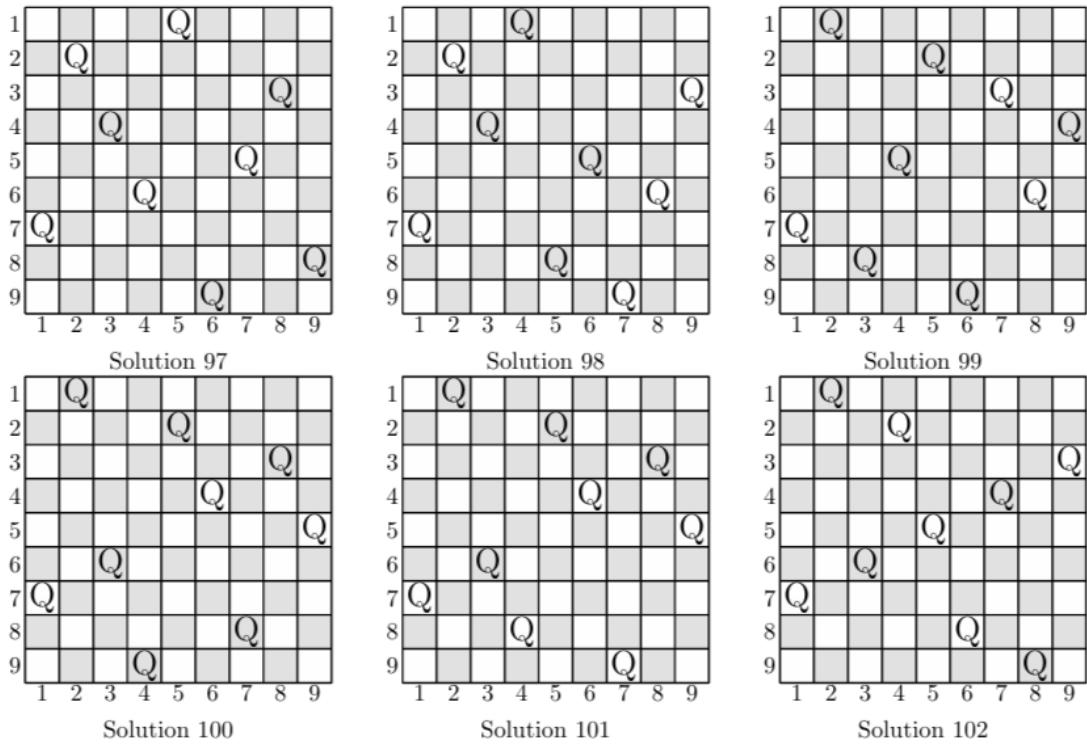


Figure 101: Solutions 97 to 102

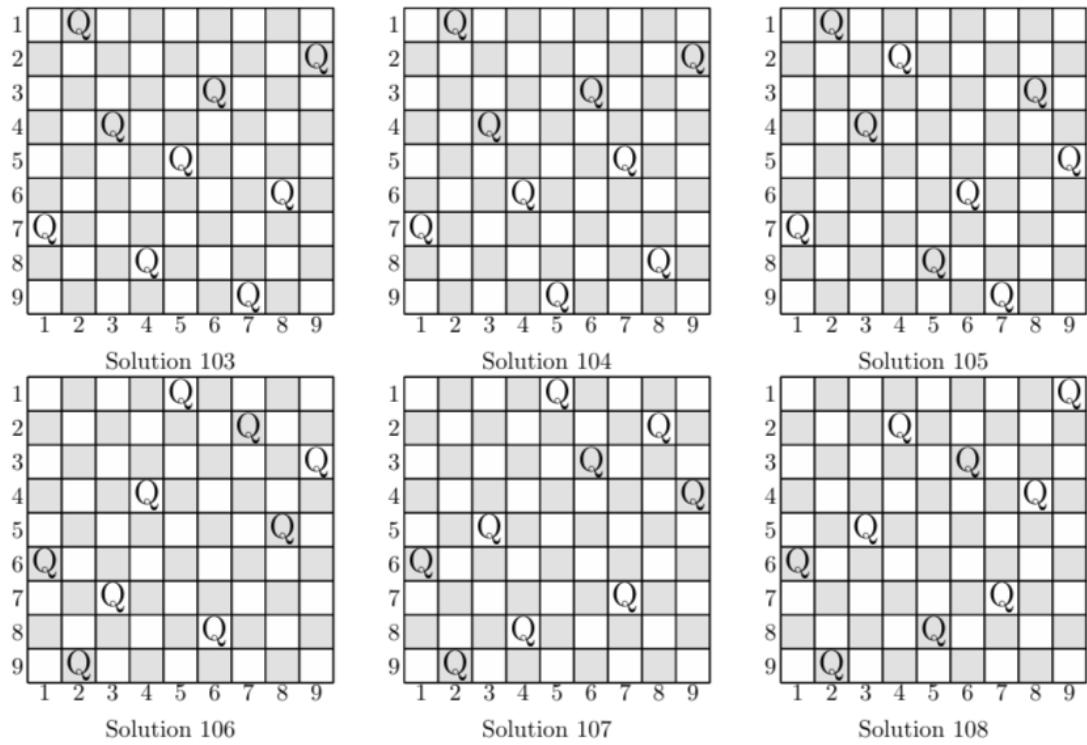


Figure 102: Solutions 103 to 108

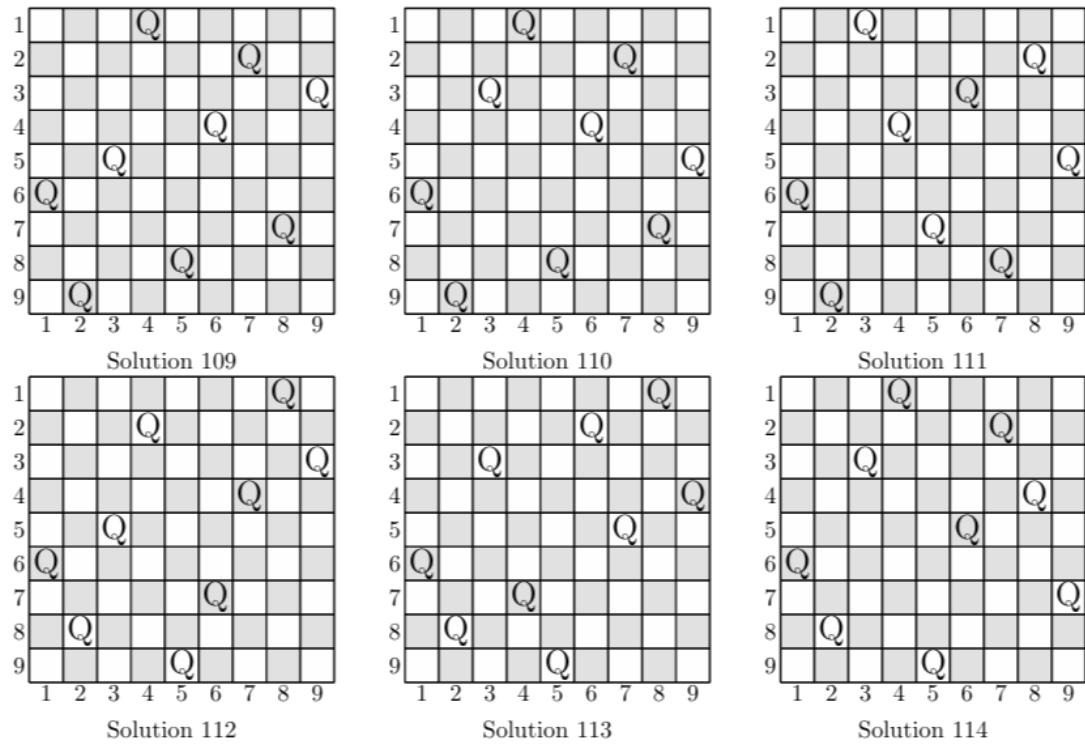
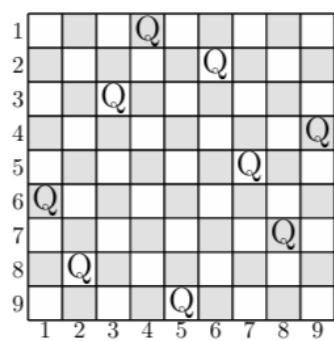
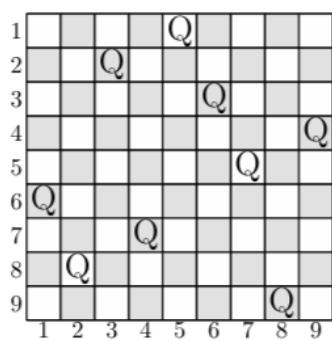


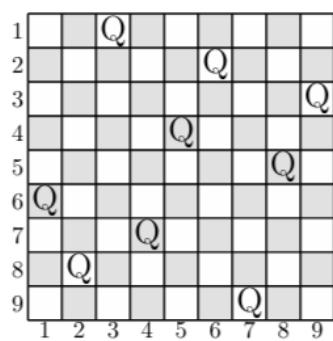
Figure 103: Solutions 109 to 114



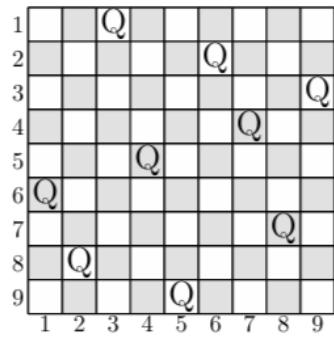
Solution 115



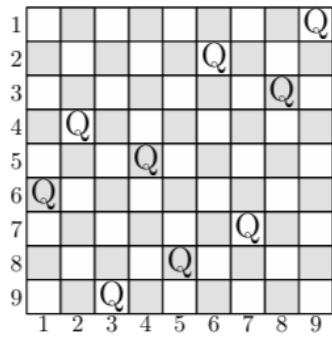
Solution 116



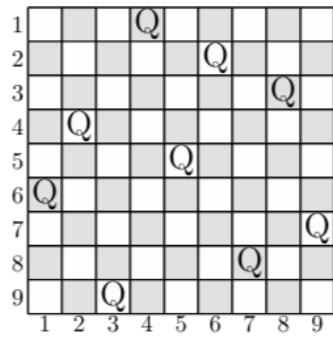
Solution 117



Solution 118

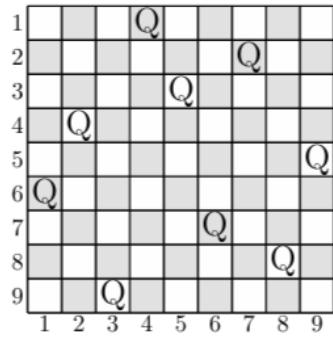


Solution 119

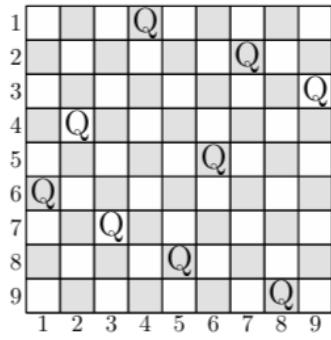


Solution 120

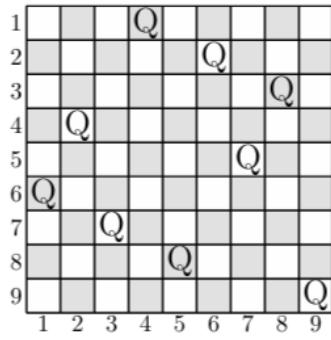
Figure 104: Solutions 115 to 120



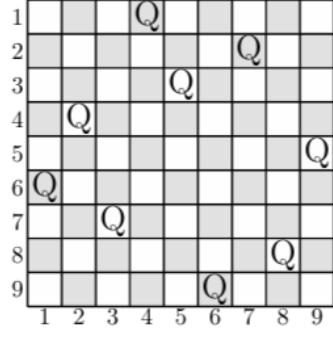
Solution 121



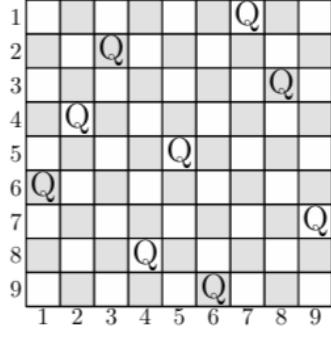
Solution 122



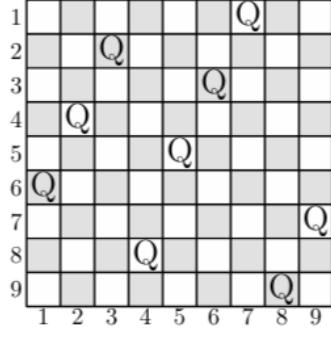
Solution 123



Solution 124



Solution 125



Solution 126

Figure 105: Solutions 12Solutions26

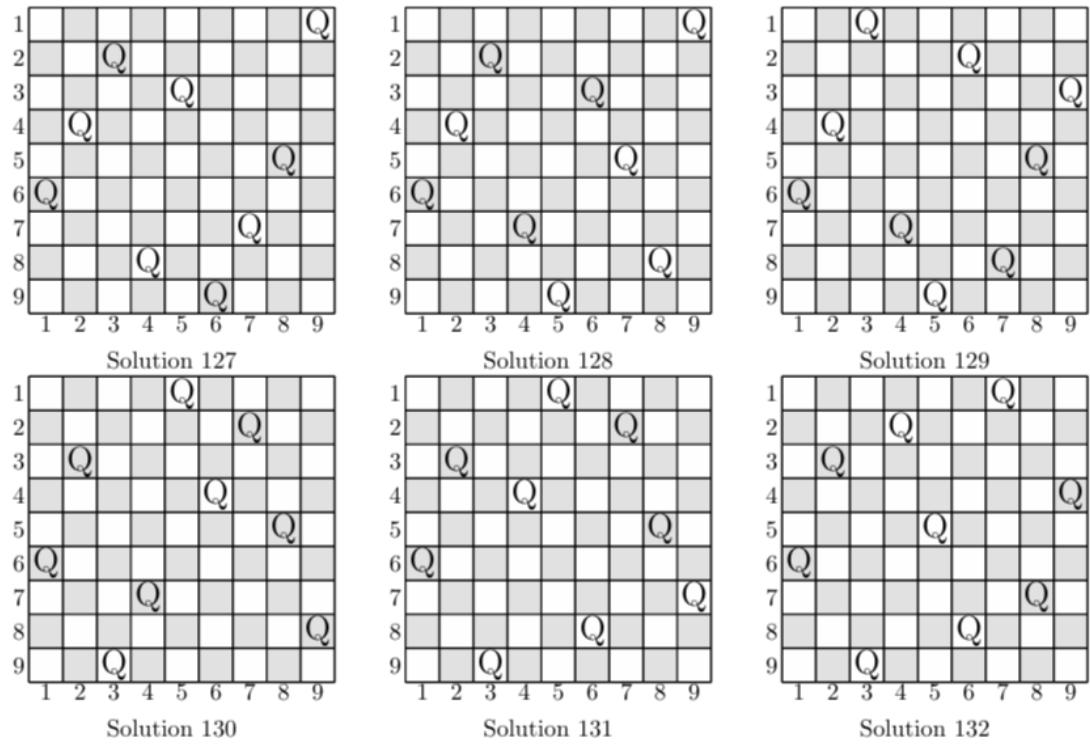


Figure 106: Solutions 127 to 132

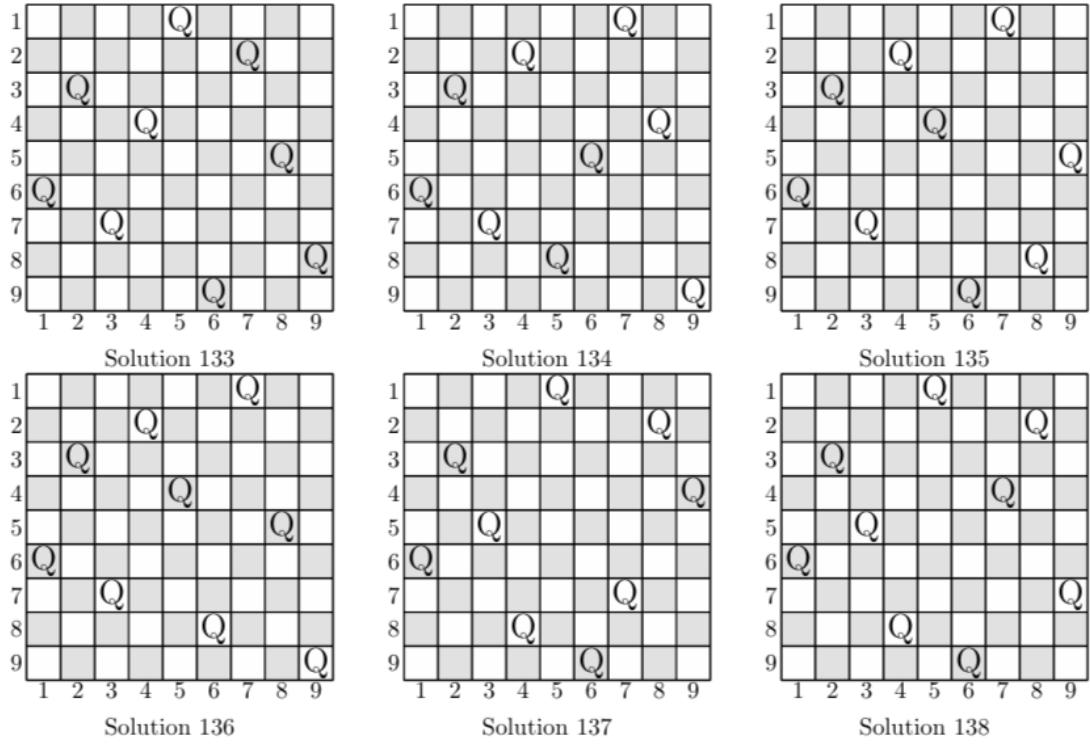
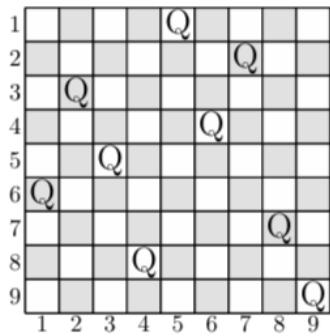
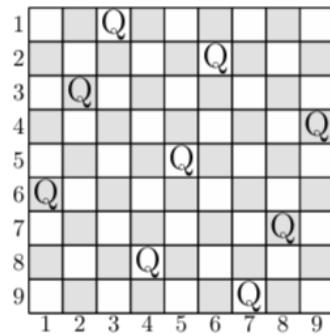


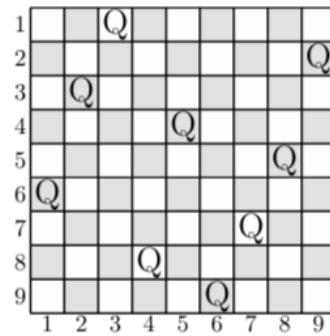
Figure 107: Solutions 133 to 138



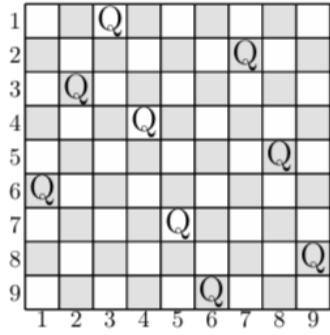
Solution 139



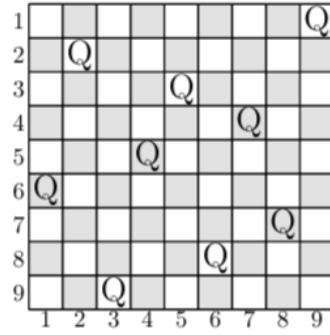
Solution 140



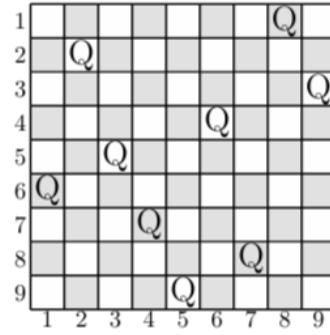
Solution 141



Solution 142

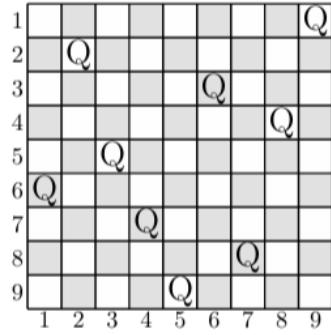


Solution 143

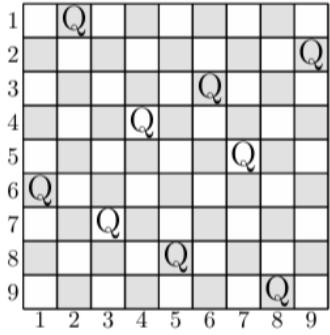


Solution 144

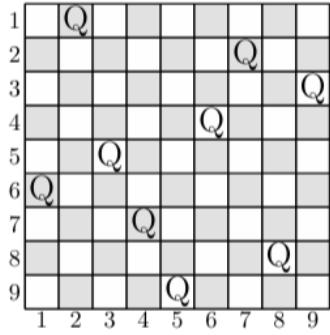
Figure 108: Solutions 139 to 144



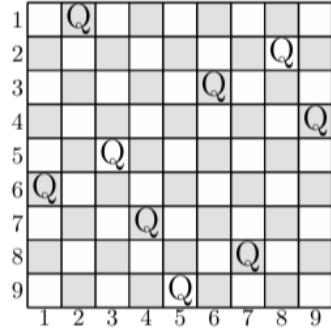
Solution 145



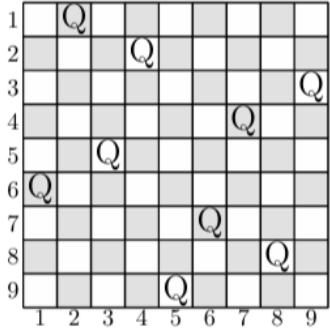
Solution 146



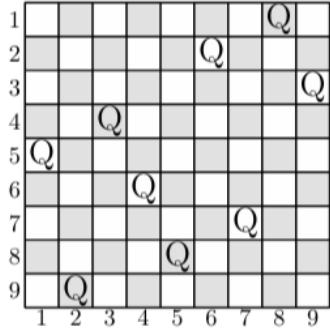
Solution 147



Solution 148



Solution 149



Solution 150

Figure 109: Solutions 145 to 150

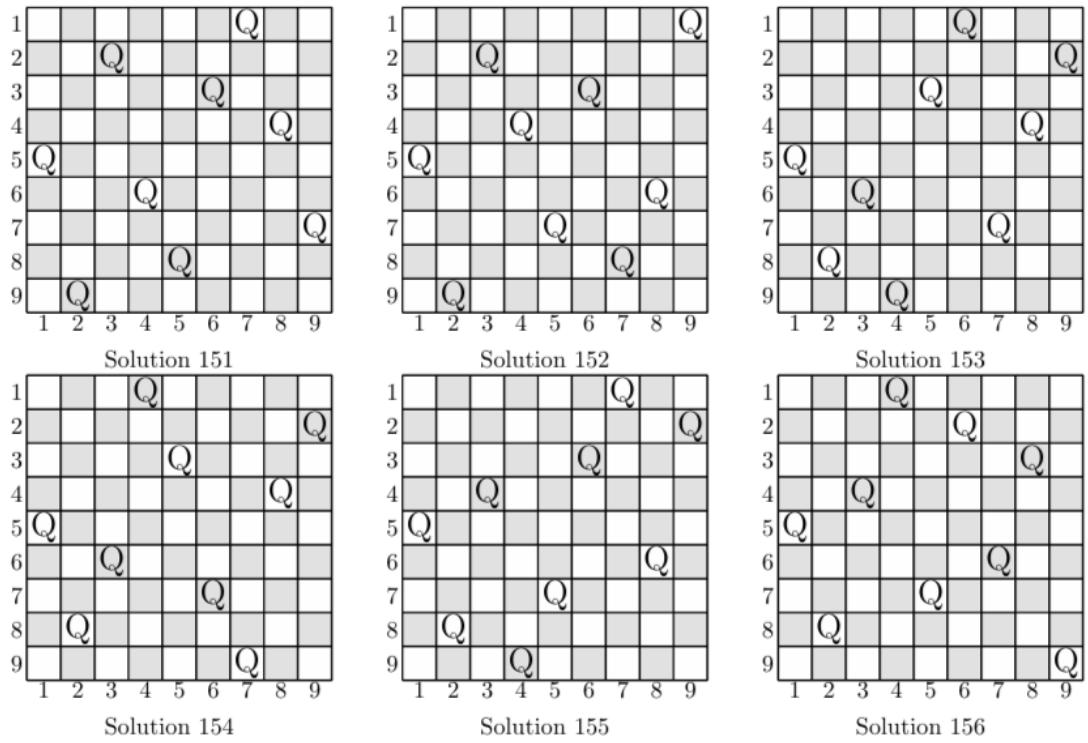


Figure 110: Solutions 15Solutions56

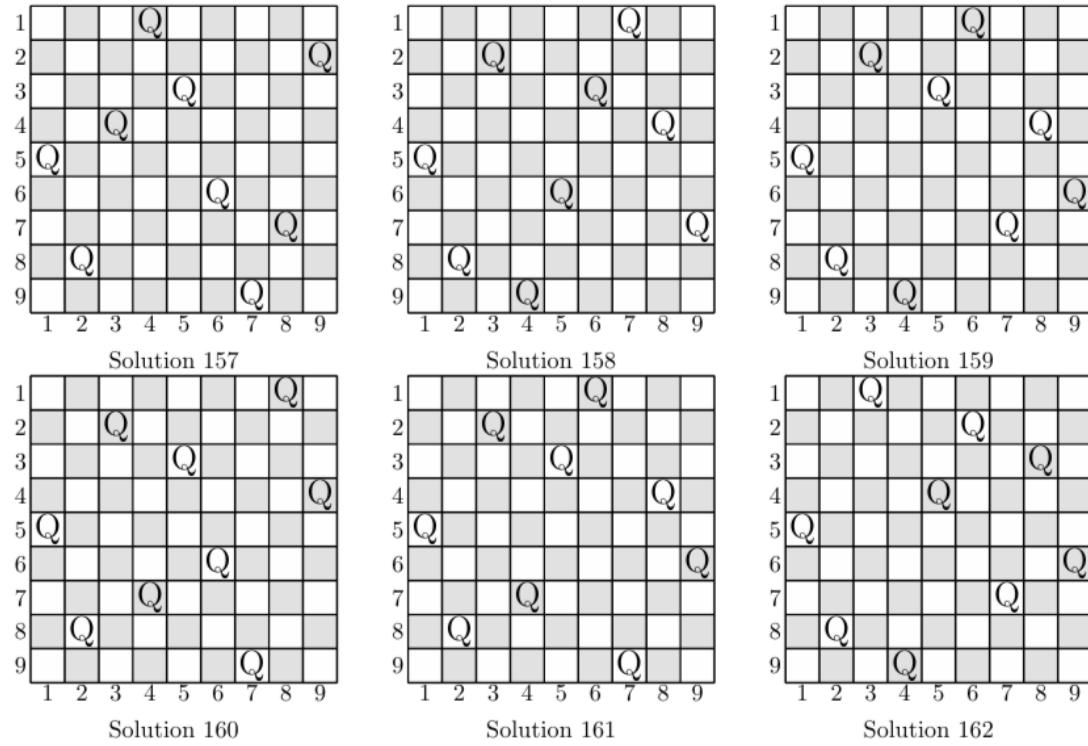


Figure 111: Solutions 157 to 162

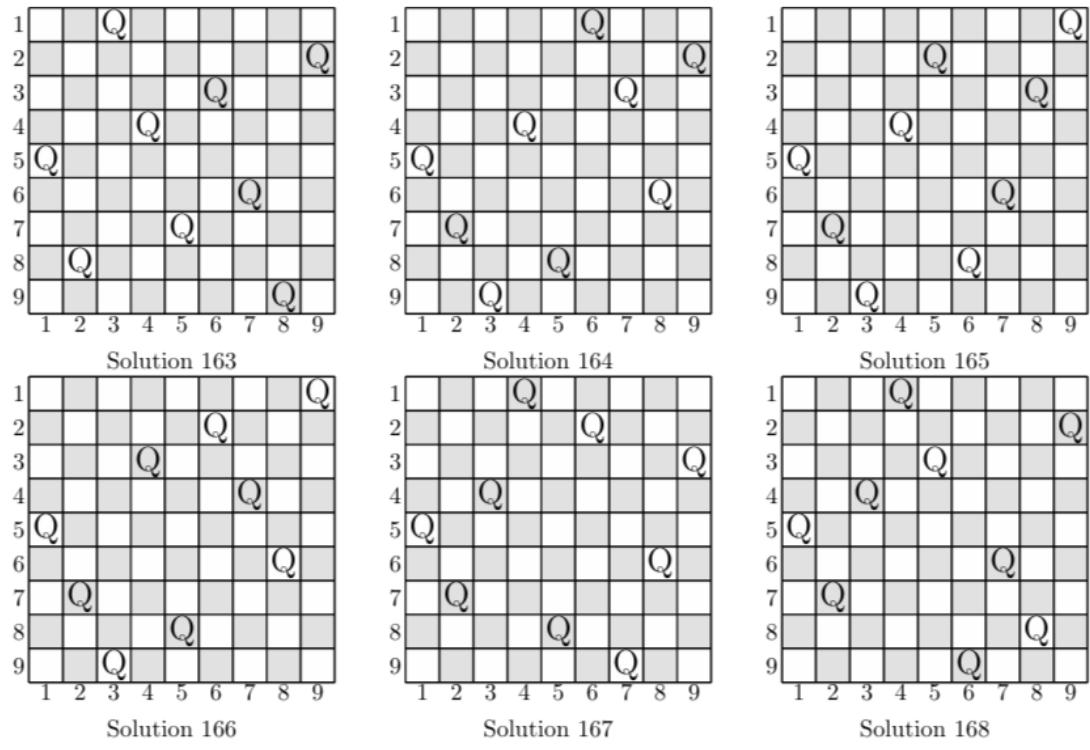


Figure 112: Solutions 163 to 168

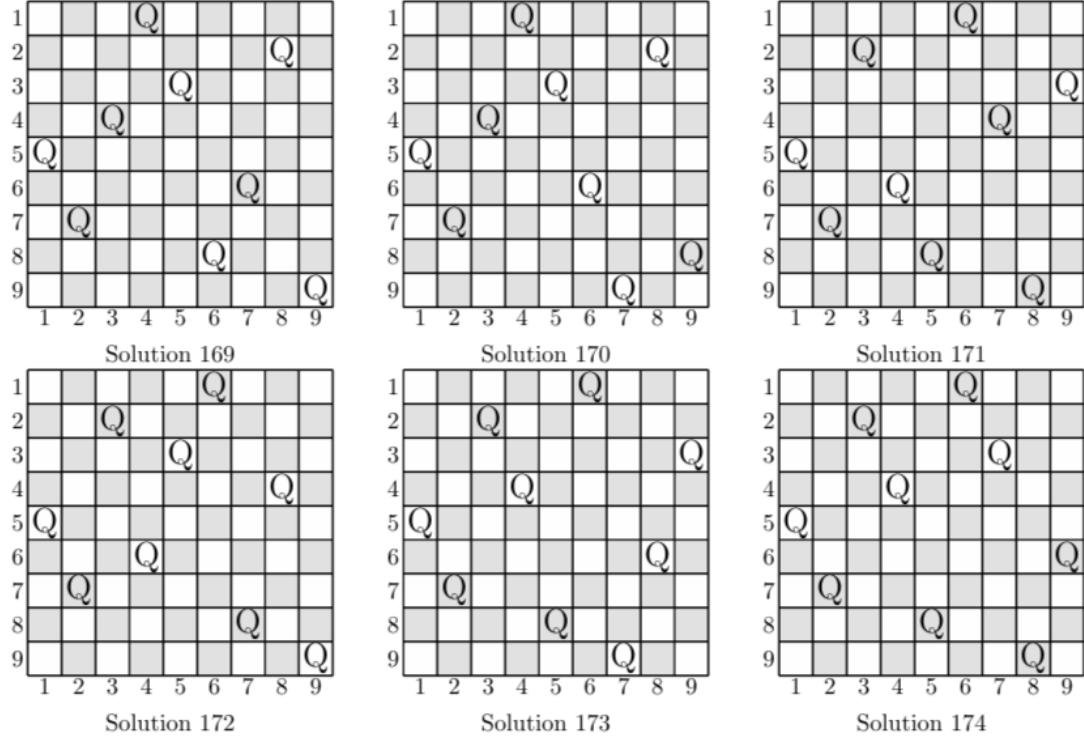
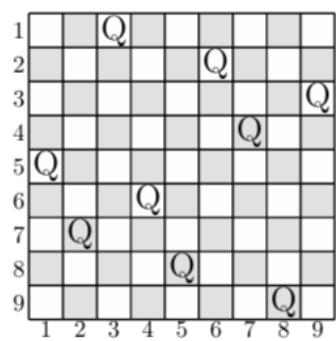
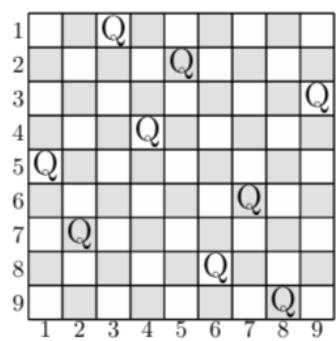


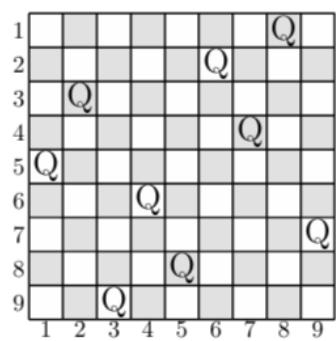
Figure 113: Solutions 169 to 174



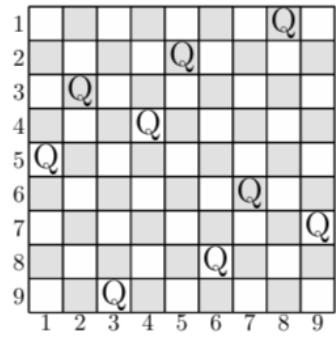
Solution 175



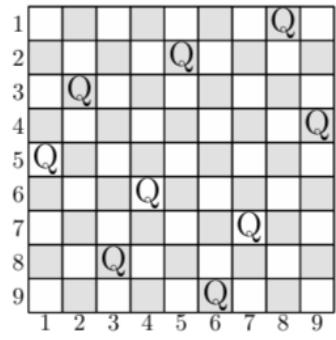
Solution 176



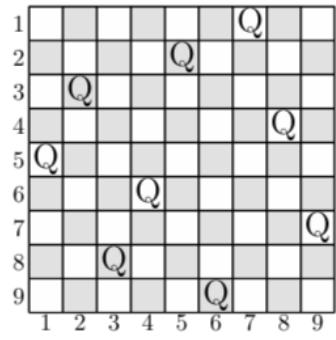
Solution 177



Solution 178

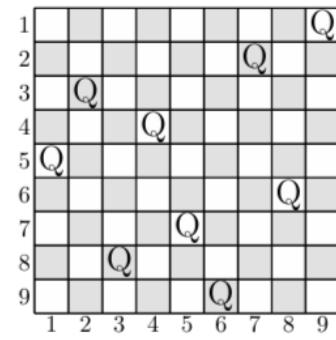


Solution 179

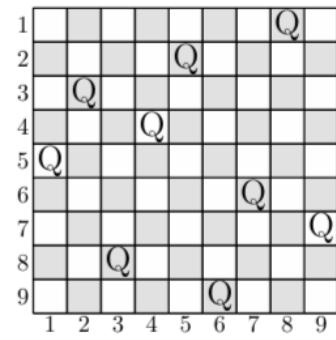


Solution 180

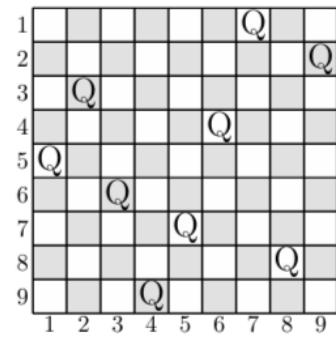
Figure 114: Solutions 175 to 180



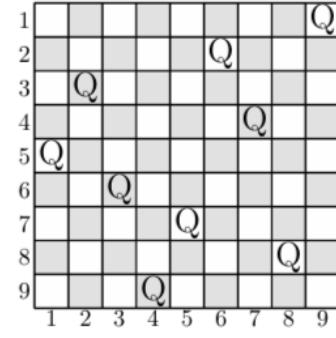
Solution 181



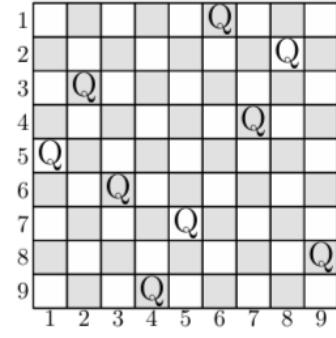
Solution 182



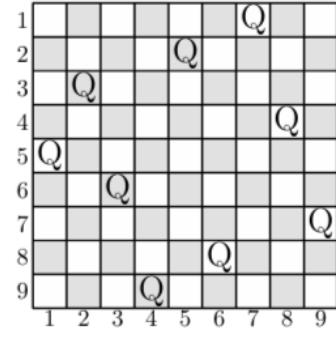
Solution 183



Solution 184



Solution 185



Solution 186

Figure 115: Solutions 18Solutions86

1		Q						
2			Q					
3	Q							
4				Q				
5	Q							
6					Q			
7		Q						
8			Q					
9						Q		

Solution 187

1		Q						
2				Q				
3	Q							
4					Q			
5	Q							
6						Q		
7			Q					
8				Q				
9							Q	

Solution 188

1		Q						
2				Q				
3	Q							
4					Q			
5	Q							
6						Q		
7			Q					
8				Q				
9							Q	

Solution 189

1								Q
2	Q							
3		Q						
4			Q					
5	Q							
6				Q				
7					Q			
8			Q					
9	Q							

Solution 190

1								Q
2	Q							
3		Q						
4			Q					
5	Q							
6				Q				
7					Q			
8			Q					
9	Q							

Solution 191

1								Q
2	Q							
3		Q						
4			Q					
5	Q							
6				Q				
7					Q			
8			Q					
9	Q							

Solution 192

Figure 116: Solutions 187 to 192

1								Q
2	Q							
3		Q						
4			Q					
5	Q							
6				Q				
7					Q			
8			Q					
9	Q							

Solution 193

1								Q
2	Q							
3		Q						
4			Q					
5	Q							
6				Q				
7					Q			
8			Q					
9	Q							

Solution 194

1								Q
2	Q							
3		Q						
4			Q					
5	Q							
6				Q				
7					Q			
8			Q					
9	Q							

Solution 196

1								Q
2	Q							
3		Q						
4			Q					
5	Q							
6				Q				
7					Q			
8			Q					
9	Q							

Solution 197

1								Q
2	Q							
3		Q						
4			Q					
5	Q							
6				Q				
7					Q			
8			Q					
9	Q							

Solution 195

1								Q
2	Q							
3		Q						
4			Q					
5	Q							
6				Q				
7					Q			
8			Q					
9	Q							

Solution 198

Figure 117: Solutions 193 to 198

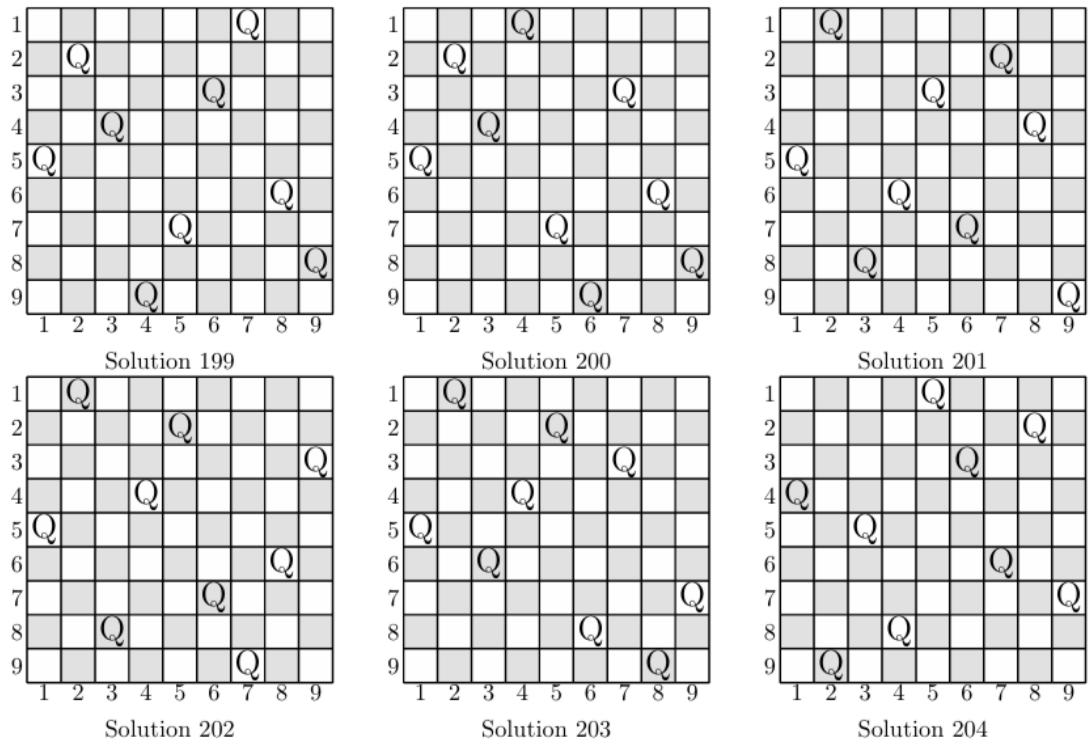


Figure 118: Solutions 199 to 204

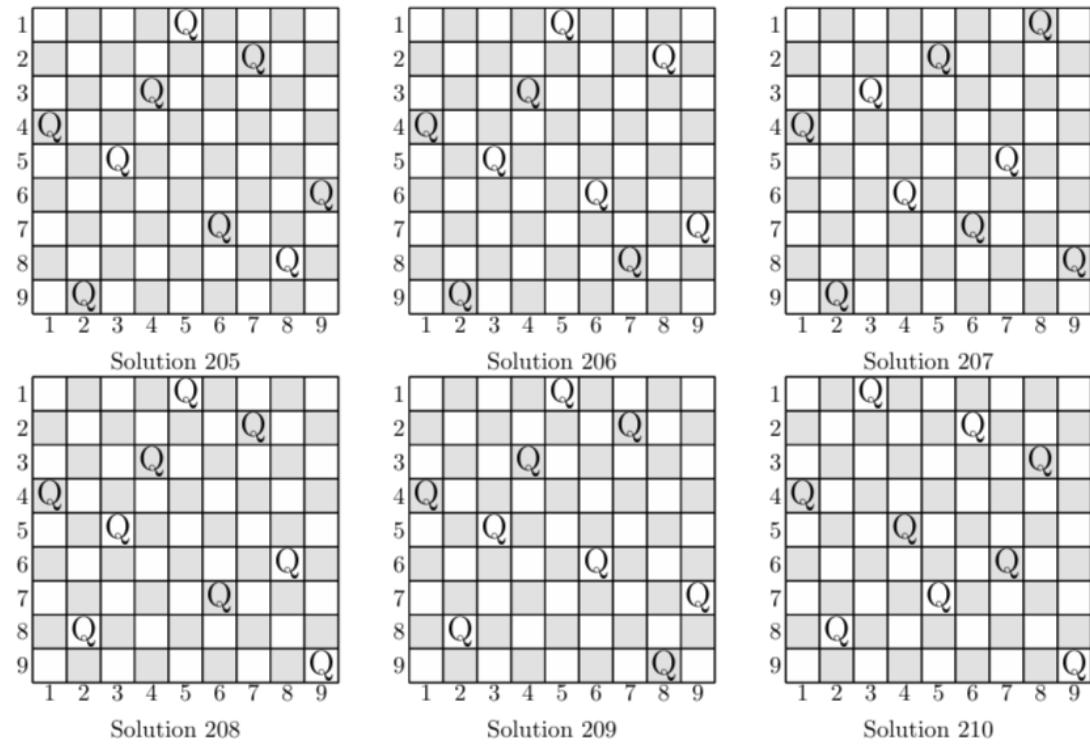


Figure 119: Solutions 205 to 211

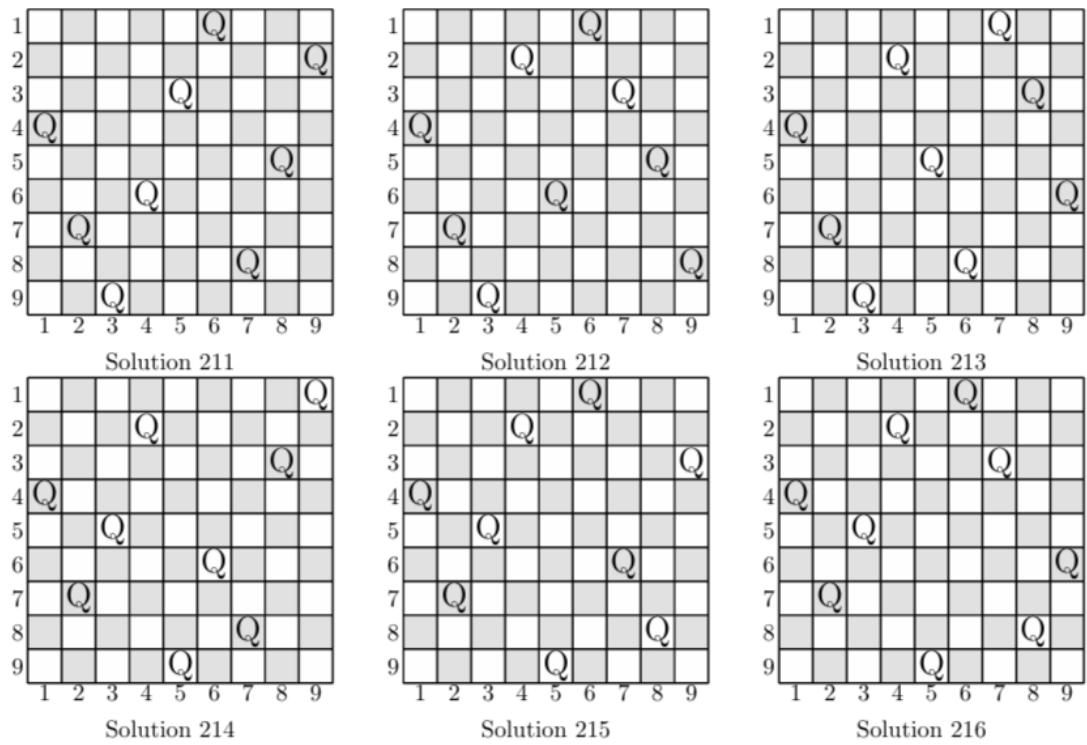


Figure 120: Solutions 211 to 216

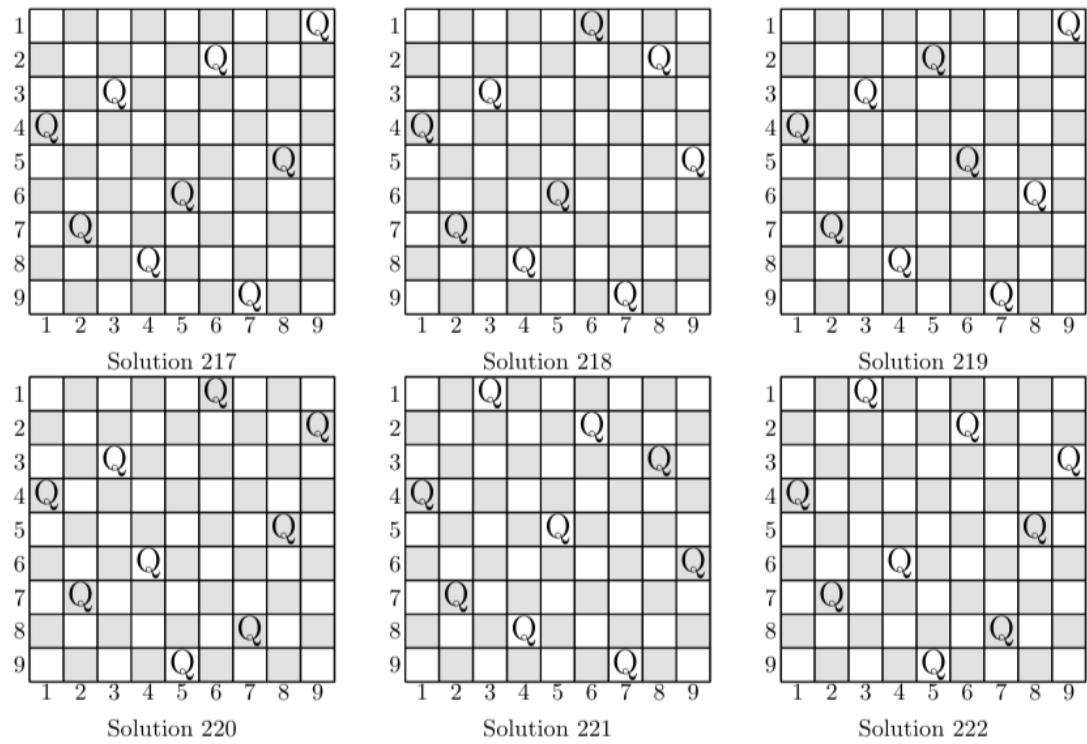


Figure 121: Solutions 217 to 222

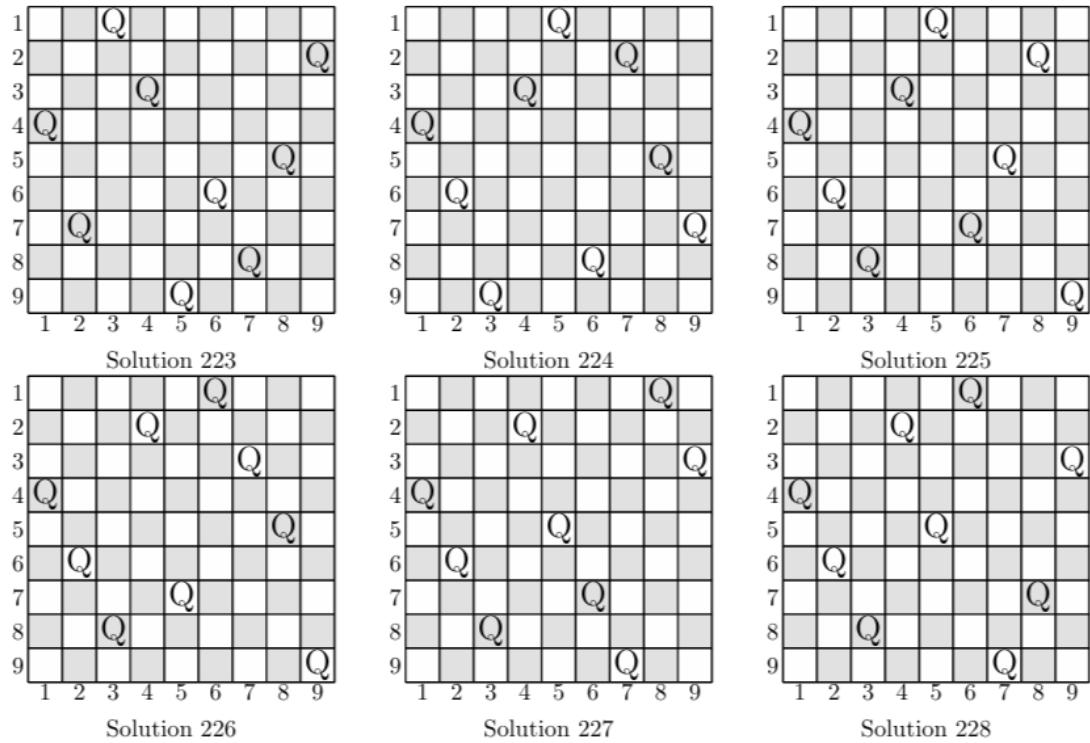


Figure 122: Solutions 223 to 228

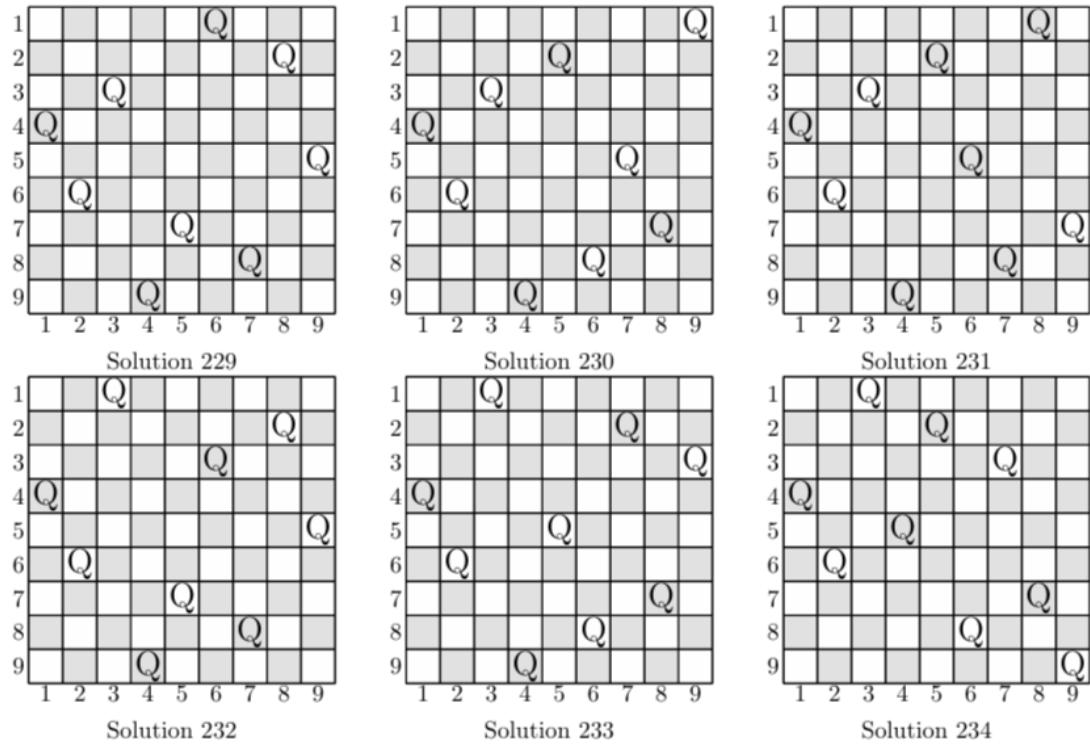


Figure 123: Solutions 229 to 234

1				Q				
2	Q							
3							Q	
4	Q							
5			Q					
6					Q			
7							Q	
8				Q				
9		Q						

Solution 235

1							Q	
2	Q							
3			Q					
4	Q							
5						Q		
6					Q			
7							Q	
8					Q			
9		Q						

Solution 236

1								Q
2	Q							
3							Q	
4	Q							
5							Q	
6							Q	
7					Q			
8						Q		
9		Q						

Solution 237

1				Q				
2	Q							
3							Q	
4	Q							
5						Q		
6						Q		
7		Q						
8			Q					
9			Q					

Solution 238

1					Q			
2	Q							
3							Q	
4	Q							
5						Q		
6						Q		
7		Q						
8			Q					
9			Q					

Solution 239

1								Q
2	Q							
3							Q	
4	Q							
5							Q	
6							Q	
7		Q						
8			Q					
9			Q					

Solution 240

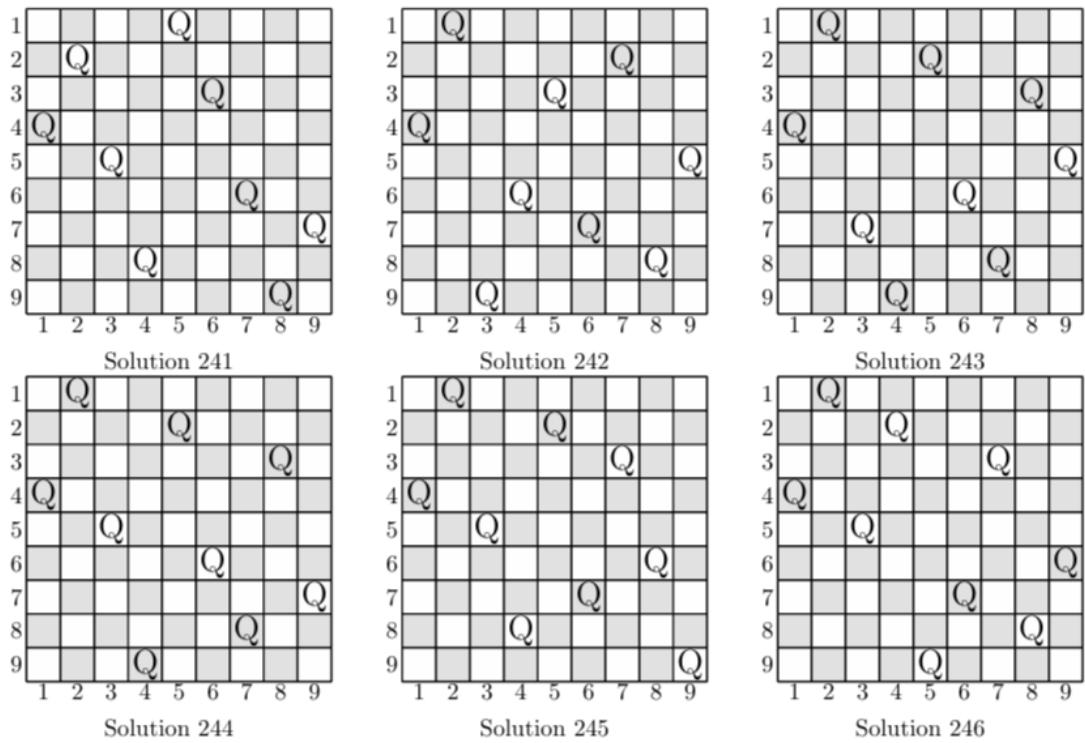


Figure 125: Solutions 241 to 246

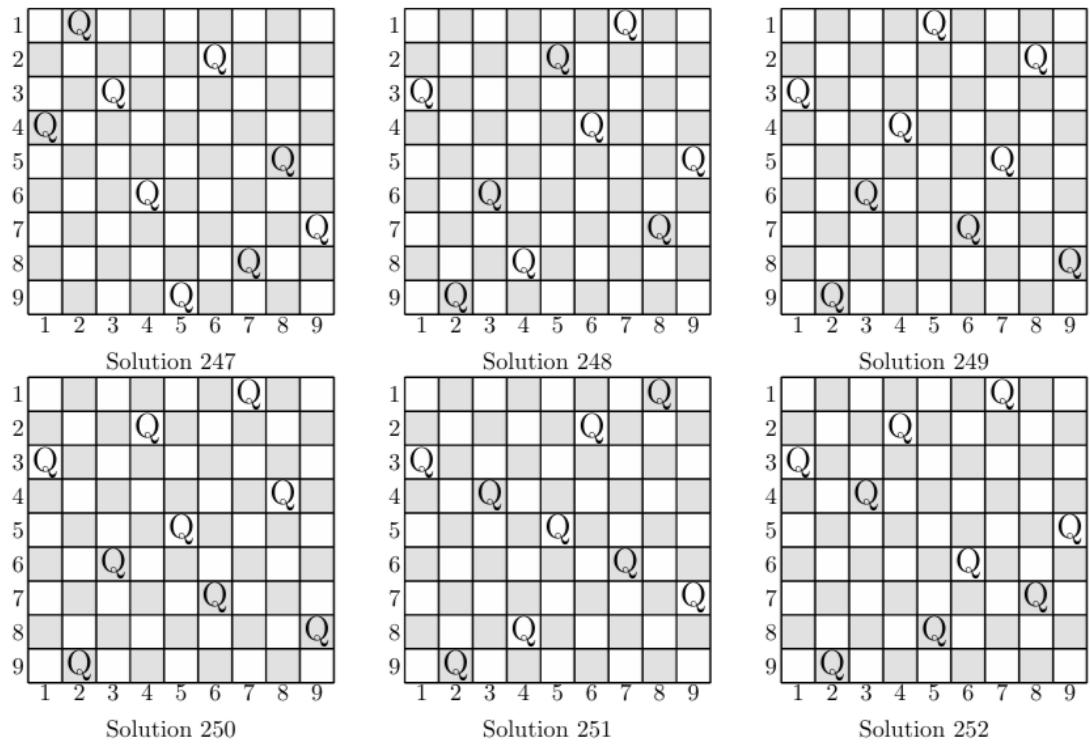


Figure 126: Solutions 247 to 252

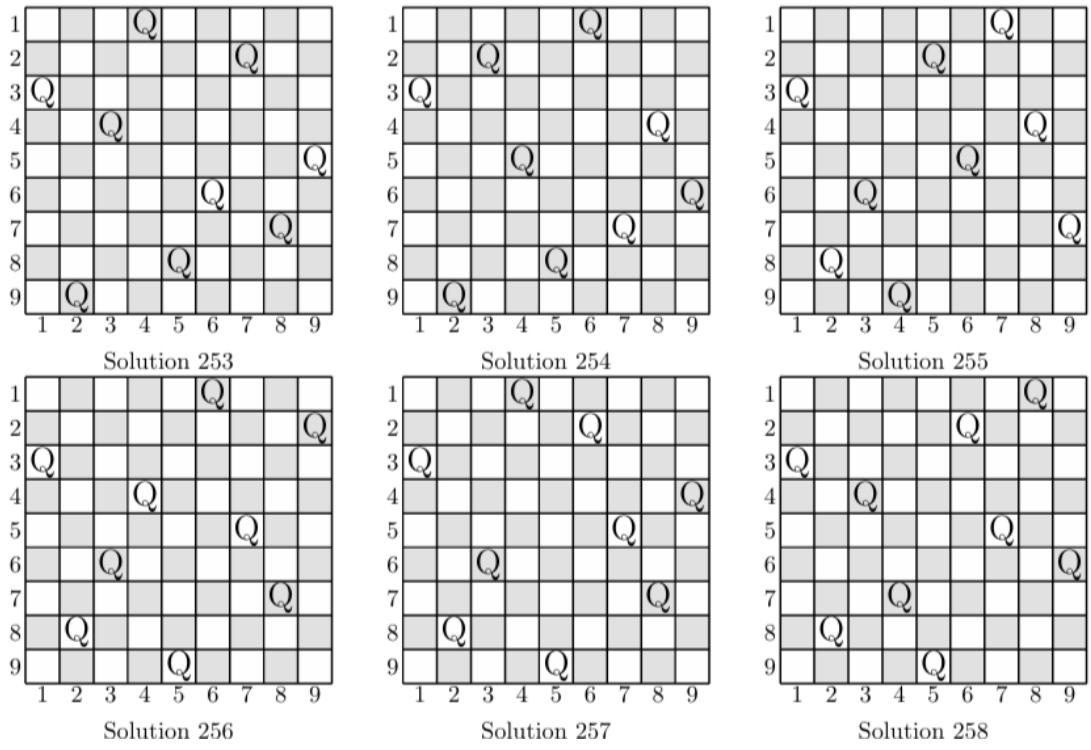


Figure 127: Solutions 253 to 258

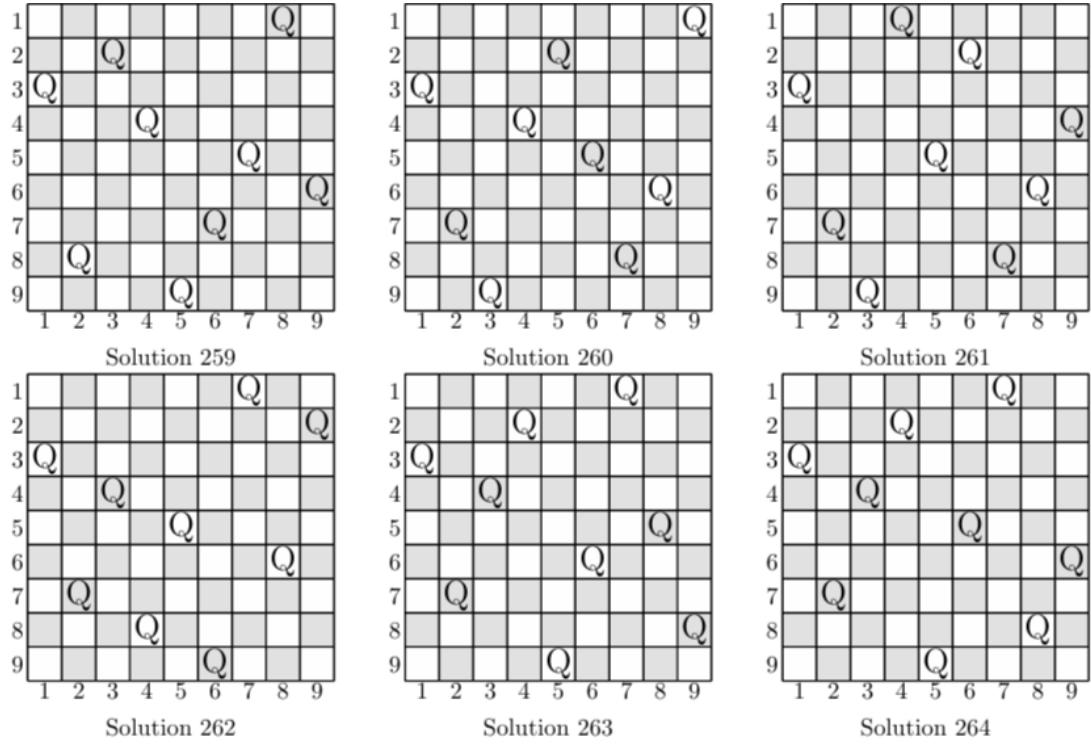
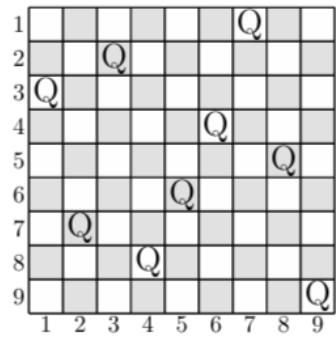
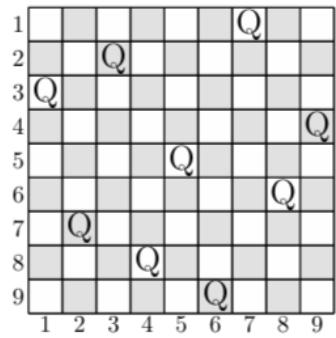


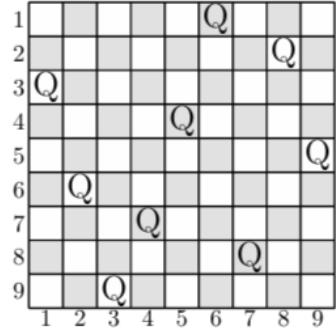
Figure 128: Solutions 259 to 264



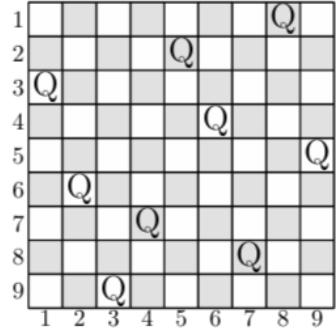
Solution 265



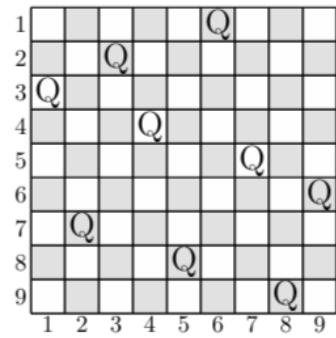
Solution 266



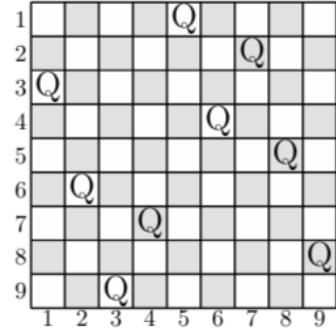
Solution 268



Solution 269

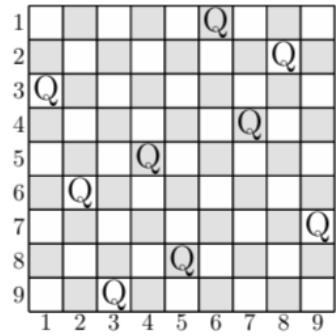


Solution 267

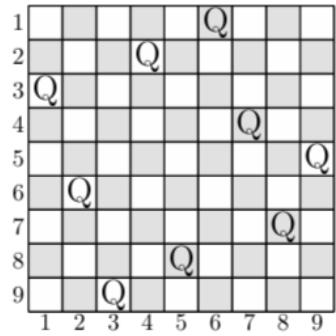


Solution 270

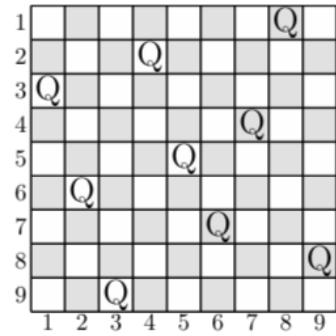
Figure 129: Solutions 265 to 270



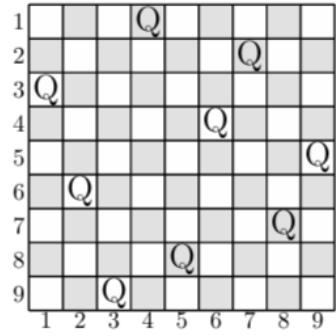
Solution 271



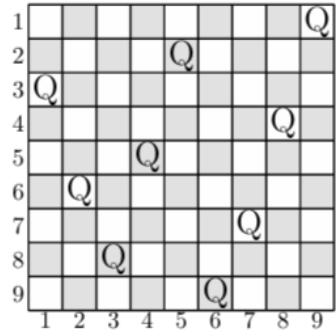
Solution 272



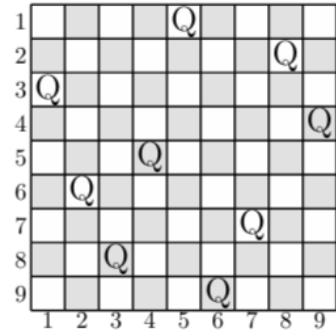
Solution 273



Solution 274

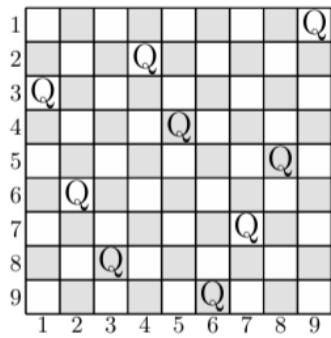


Solution 275

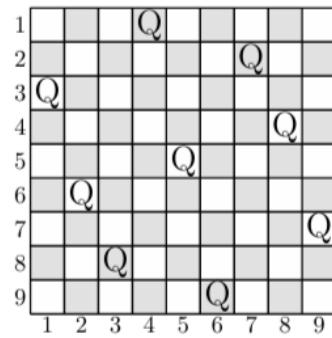


Solution 276

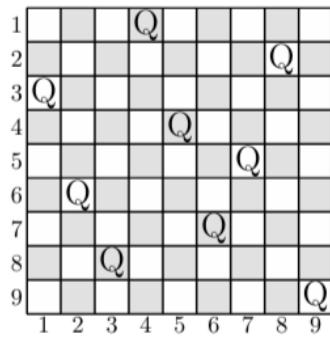
Figure 130: Solutions 271 to 276



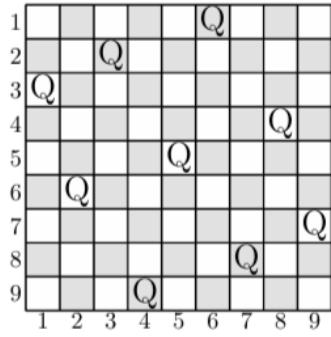
Solution 277



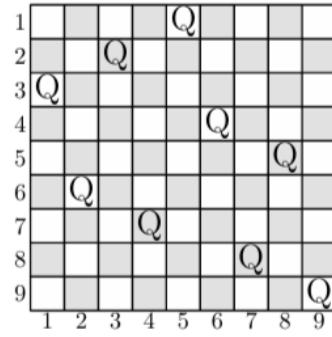
Solution 278



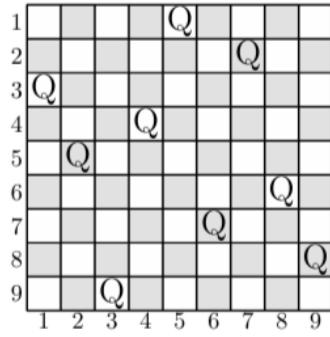
Solution 279



Solution 280

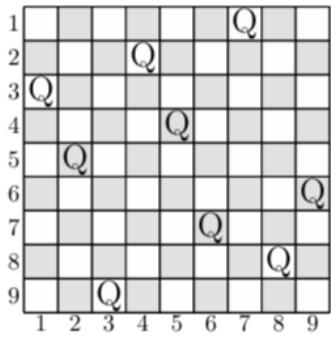


Solution 281

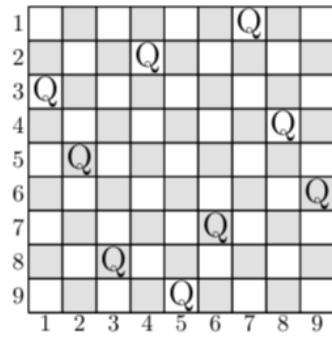


Solution 282

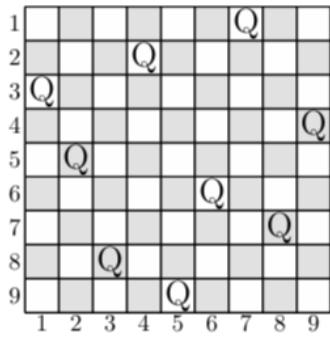
Figure 131: Solutions 277 to 282



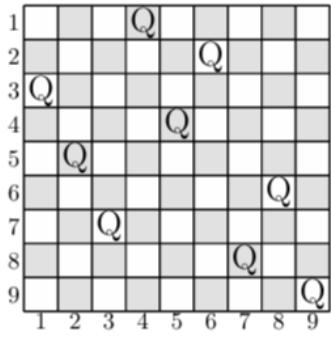
Solution 283



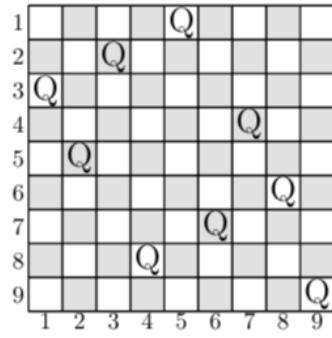
Solution 284



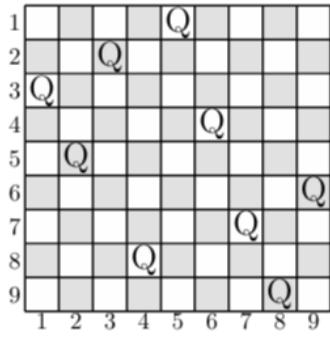
Solution 285



Solution 286



Solution 287



Solution 288

Figure 132: Solutions 283 to 288

1	Q							
2				Q				
3	Q							
4							Q	
5			Q					
6						Q		Q
7			Q					
8					Q			
9		Q						

Solution 289

1	Q							
2							Q	
3	Q							
4			Q					
5						Q		
6								Q
7					Q			
8				Q				
9						Q		

Solution 290

1	Q							
2							Q	
3	Q							
4			Q					
5						Q		
6								Q
7					Q			
8				Q				
9						Q		

Solution 291

1	Q							
2			Q					
3	Q							
4				Q				
5							Q	
6					Q			
7		Q						
8			Q					
9						Q		

Solution 292

1	Q							
2			Q					
3	Q							
4				Q				
5							Q	
6					Q			
7		Q						
8			Q					
9						Q		

Solution 293

1	Q							
2				Q				
3	Q							
4					Q			
5							Q	
6			Q					
7				Q				
8						Q		
9							Q	

Solution 294

Figure 133: Solutions 289 to 294

1						Q		
2	Q							
3				Q				
4			Q					
5						Q		
6		Q						
7			Q					
8							Q	
9	Q							

Solution 295

1						Q		
2	Q							
3			Q					
4				Q				
5						Q		
6		Q						
7			Q					
8							Q	
9	Q							

Solution 296

1						Q		
2	Q							
3			Q					
4				Q				
5		Q						
6						Q		
7			Q					
8					Q			
9	Q							

Solution 298

1						Q		
2	Q							
3			Q					
4				Q				
5						Q		
6		Q						
7			Q					
8					Q			
9	Q							

Solution 299

1						Q		
2	Q							
3			Q					
4				Q				
5						Q		
6		Q						
7			Q					
8					Q			
9	Q							

Solution 300

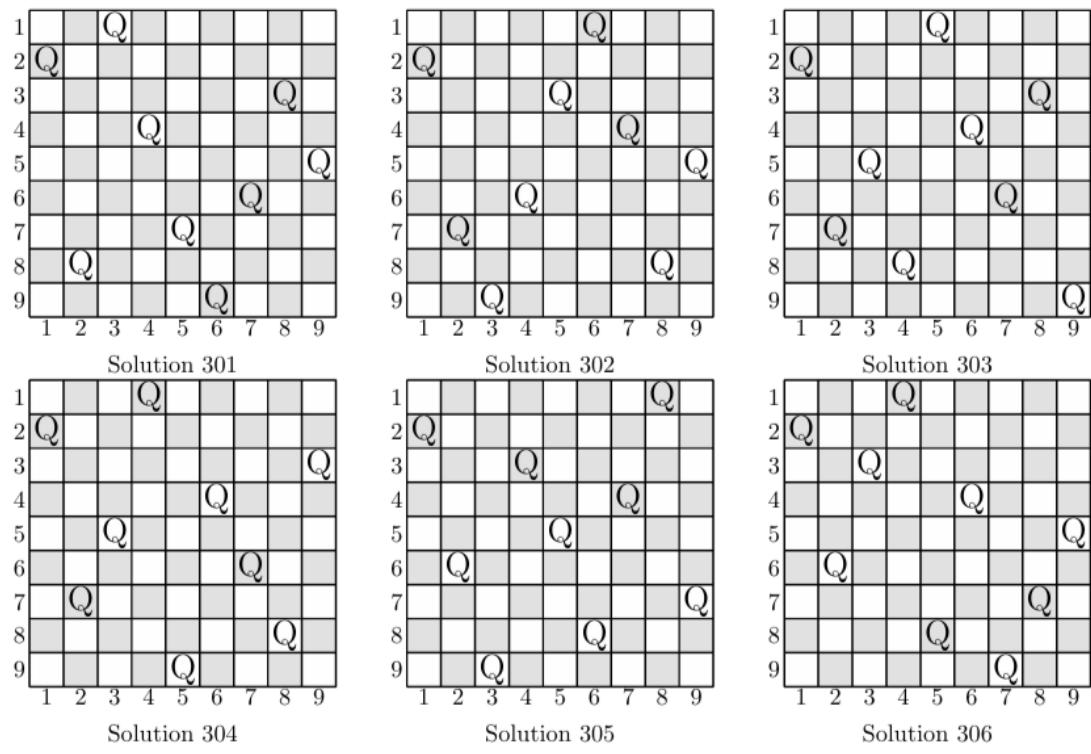


Figure 135: Solutions 301 to 306

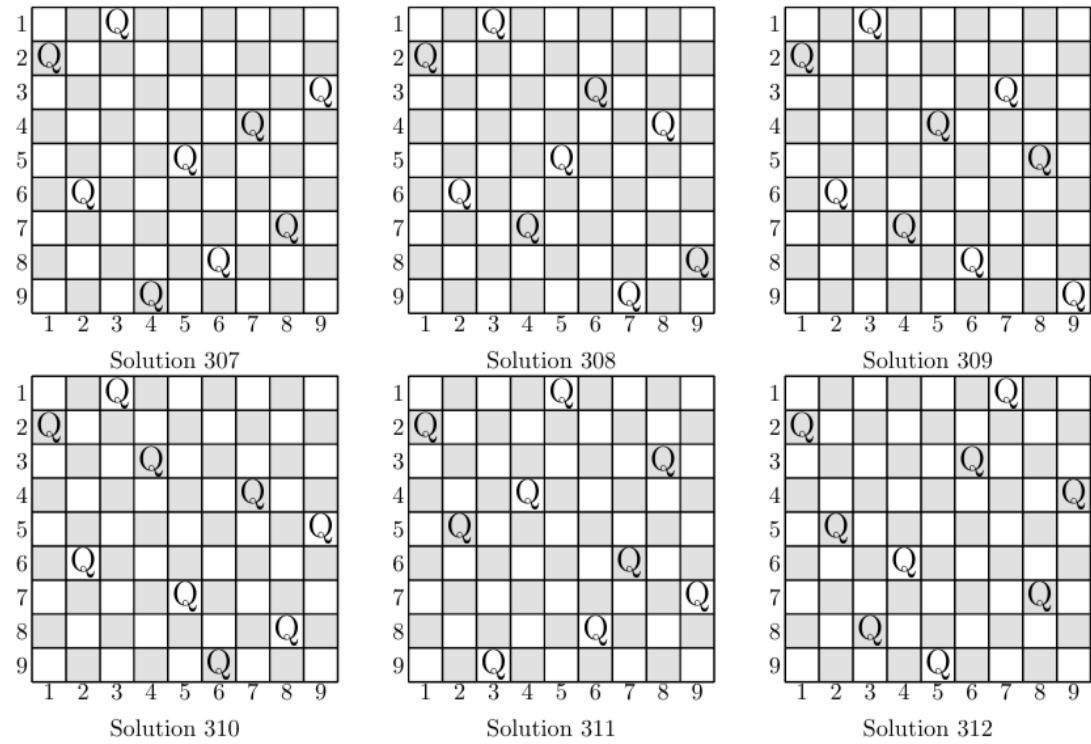


Figure 136: Solutions 307 to 312

1									Q		
2	Q										
3									Q		
4										Q	
5		Q									
6			Q								
7											Q
8			Q								
9				Q							
1	2	3	4	5	6	7	8	9			

Solution 313

1									Q		
2	Q										
3									Q		
4										Q	
5		Q									
6			Q								
7											Q
8			Q								
9				Q							
1	2	3	4	5	6	7	8	9			

Solution 314

1									Q		
2	Q										
3									Q		
4										Q	
5		Q									
6			Q								
7										Q	
8			Q								
9				Q							
1	2	3	4	5	6	7	8	9			

Solution 315

1									Q		
2	Q										
3									Q		
4										Q	
5		Q									
6			Q								
7										Q	
8			Q								
9				Q							
1	2	3	4	5	6	7	8	9			

Solution 316

1									Q		
2	Q										
3									Q		
4										Q	
5		Q									
6			Q								
7										Q	
8			Q								
9				Q							
1	2	3	4	5	6	7	8	9			

Solution 317

Solution 318

Figure 137: Solutions 313 to 318

1									Q		
2	Q										
3									Q		
4										Q	
5		Q									
6			Q								
7										Q	
8			Q								
9				Q							
1	2	3	4	5	6	7	8	9			

Solution 319

1									Q		
2	Q										
3									Q		
4										Q	
5		Q									
6			Q								
7										Q	
8			Q								
9				Q							
1	2	3	4	5	6	7	8	9			

Solution 320

1									Q		
2	Q										
3									Q		
4										Q	
5		Q									
6			Q								
7										Q	
8			Q								
9				Q							
1	2	3	4	5	6	7	8	9			

Solution 321

1									Q		
2	Q										
3									Q		
4										Q	
5		Q									
6			Q								
7										Q	
8			Q								
9				Q							
1	2	3	4	5	6	7	8	9			

Solution 322

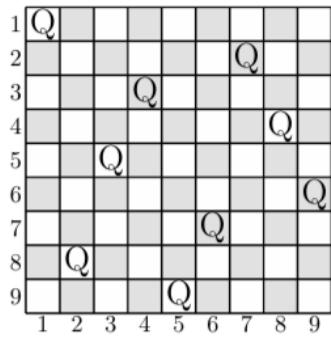
1									Q		
2	Q										
3									Q		
4										Q	
5		Q									
6			Q								
7										Q	
8			Q								
9				Q							
1	2	3	4	5	6	7	8	9			

Solution 323

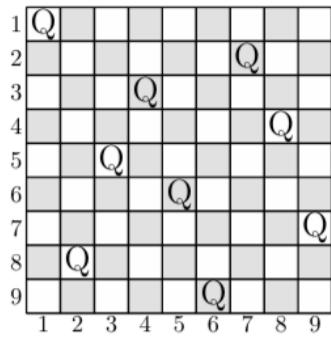
1									Q		
2	Q										
3									Q		
4										Q	
5		Q									
6			Q								
7										Q	
8			Q								
9				Q							
1	2	3	4	5	6	7	8	9			

Solution 324

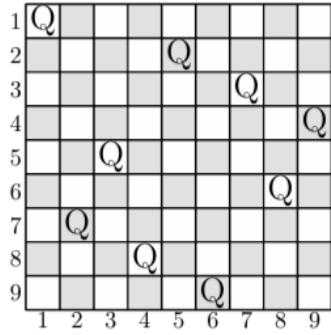
Figure 138: Solutions 319 to 324



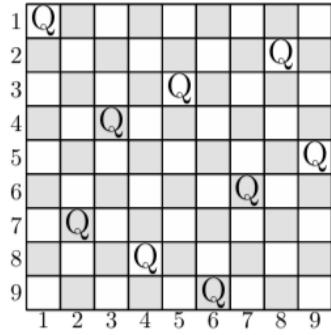
Solution 325



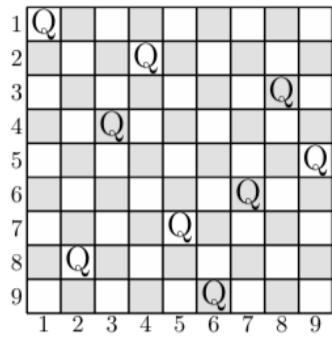
Solution 326



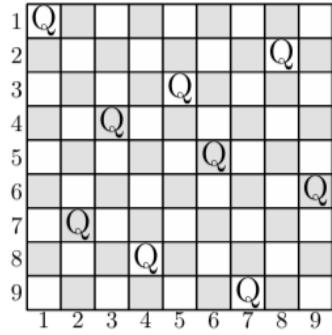
Solution 328



Solution 329

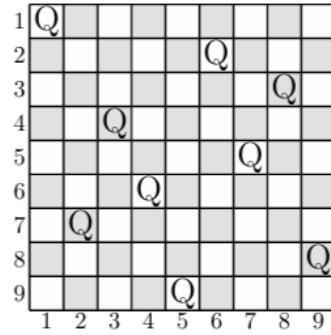


Solution 327

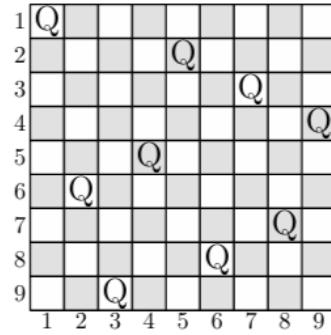


Solution 330

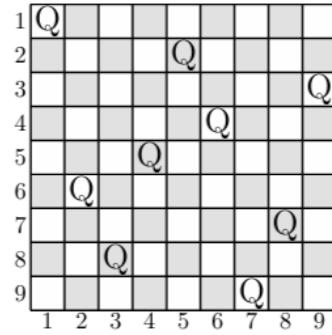
Figure 139: Solutions 325 to 330



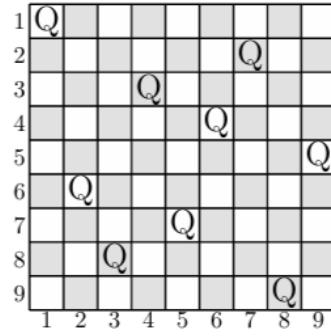
Solution 331



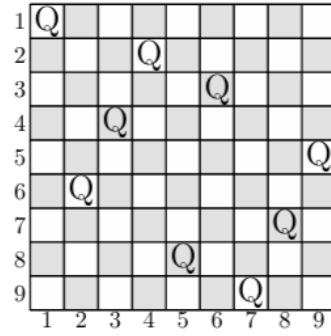
Solution 332



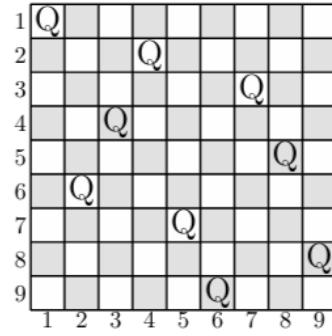
Solution 333



Solution 334

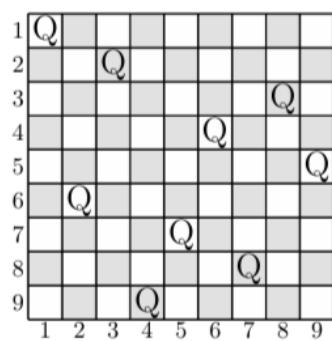


Solution 335

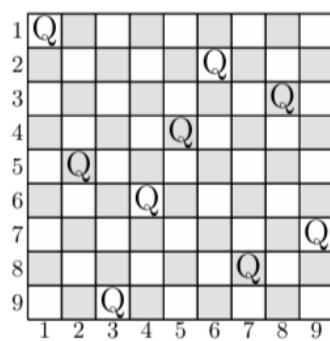


Solution 336

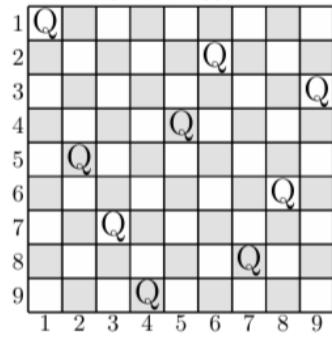
Figure 140: Solutions 331 to 336



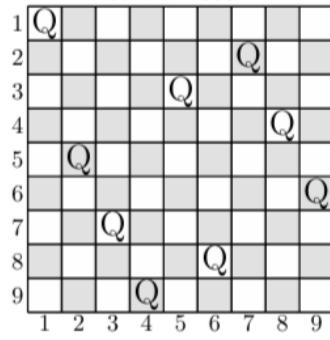
Solution 337



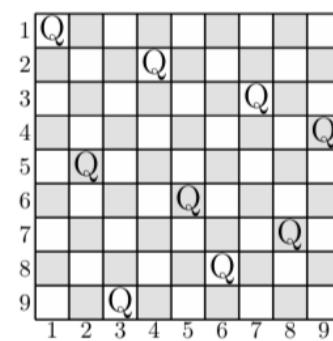
Solution 338



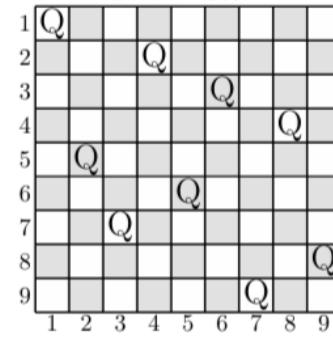
Solution 340



Solution 341

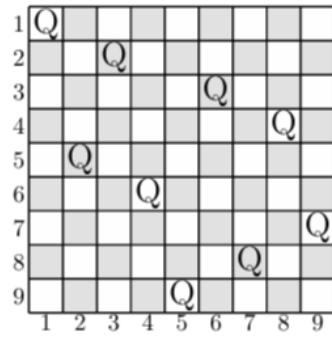


Solution 339

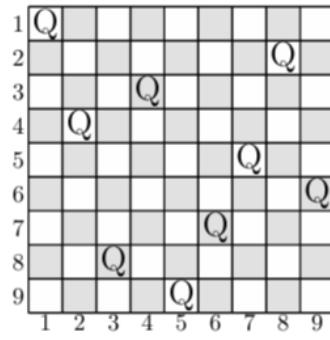


Solution 342

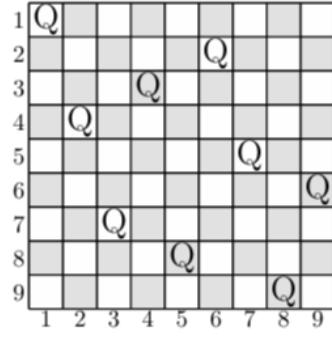
Figure 141: Solutions 337 to 342



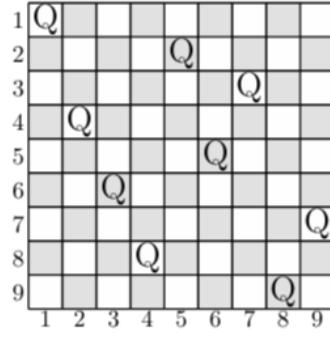
Solution 343



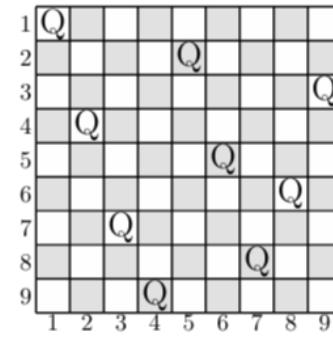
Solution 344



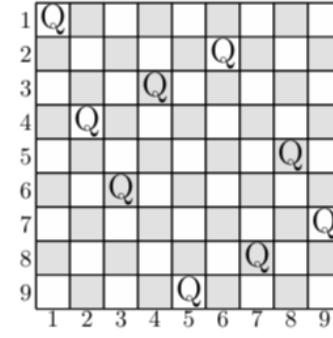
Solution 346



Solution 347



Solution 345



Solution 348

Figure 142: Solutions 343 to 348

1	Q							
2		Q						
3				Q				
4		Q						
5						Q		
6			Q					
7							Q	
8			Q					
9				Q				

Solution 349

1	Q							
2						Q		
3		Q						
4								Q
5							Q	
6				Q				
7							Q	
8			Q					
9				Q				

Solution 350

1	Q							
2						Q		
3		Q						
4								Q
5							Q	
6				Q				
7							Q	
8			Q					
9				Q				

Solution 351

1	Q							
2				Q				
3		Q						
4				Q				
5							Q	
6			Q					
7					Q			
8			Q					
9				Q				

Solution 352

Figure 143: Solutions 349 to 352

7.10 Solution to 10 Queens

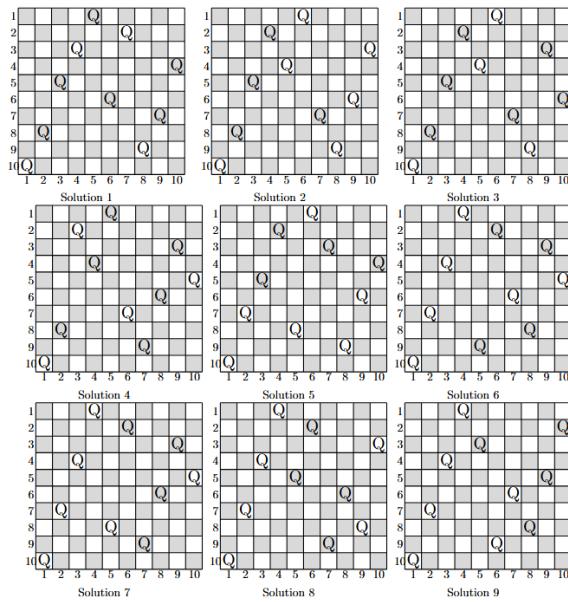


Figure 144: Solutions 1 to 9

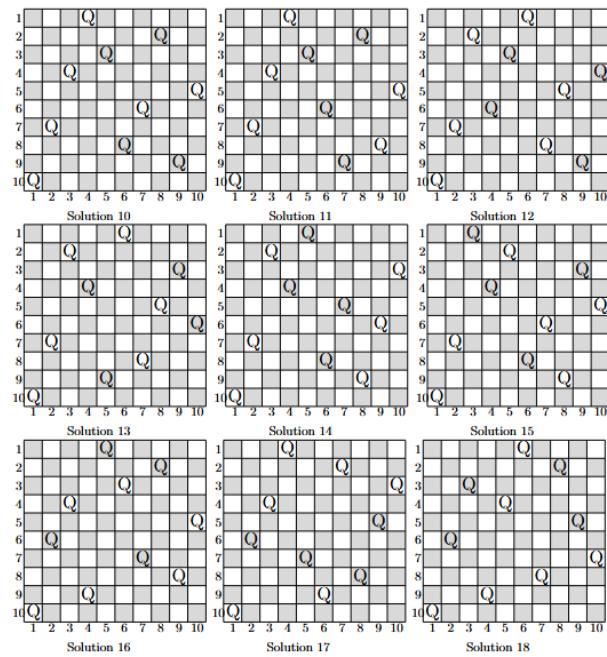


Figure 145: Solutions 10 to 18

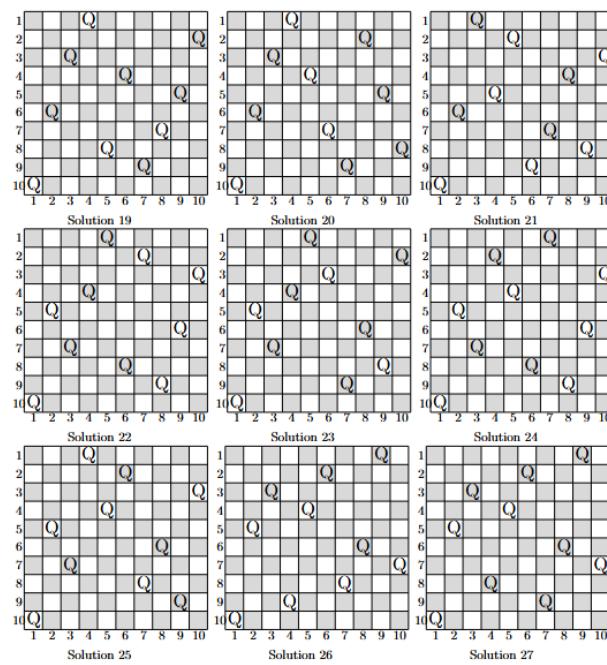


Figure 146: Solutions 19 to 27

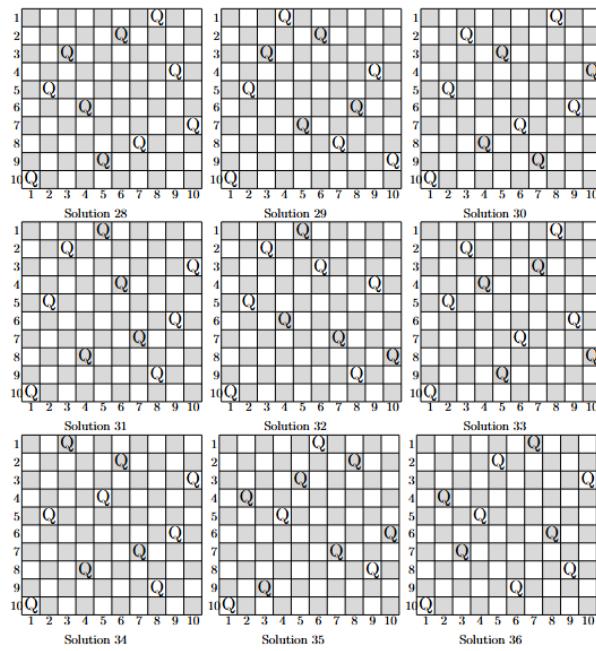


Figure 147: Solutions 28 to 36

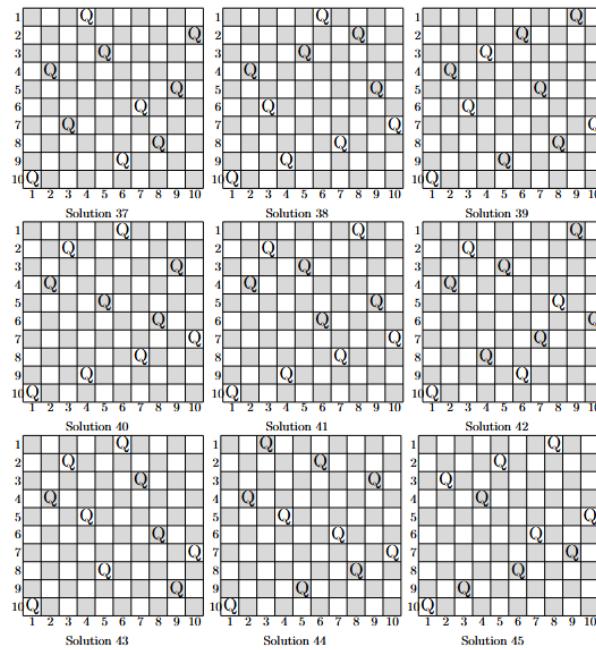


Figure 148: Solutions 37 to 45

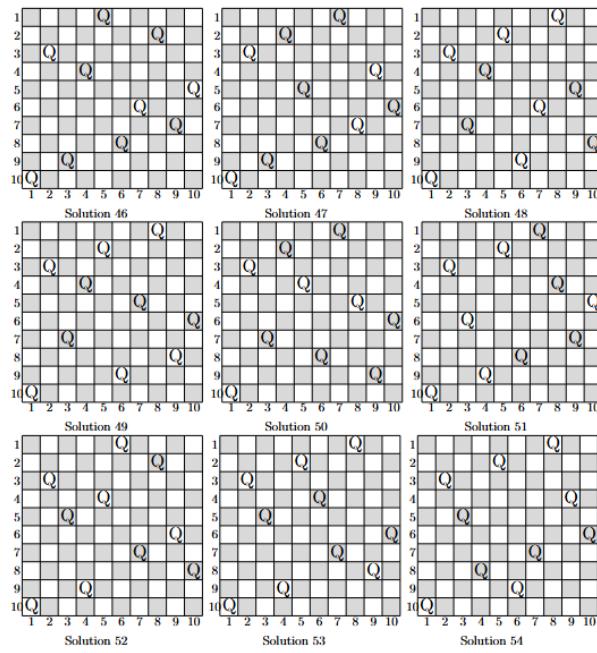


Figure 149: Solutions 46 to 54

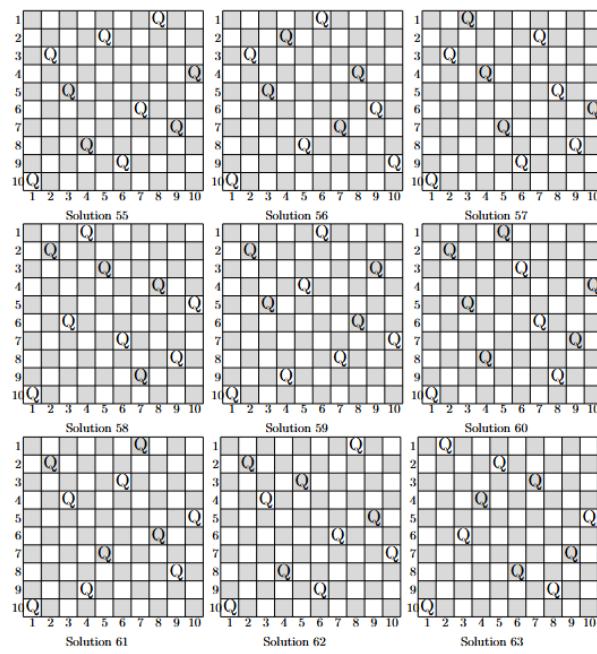


Figure 150: Solutions 55 to 63

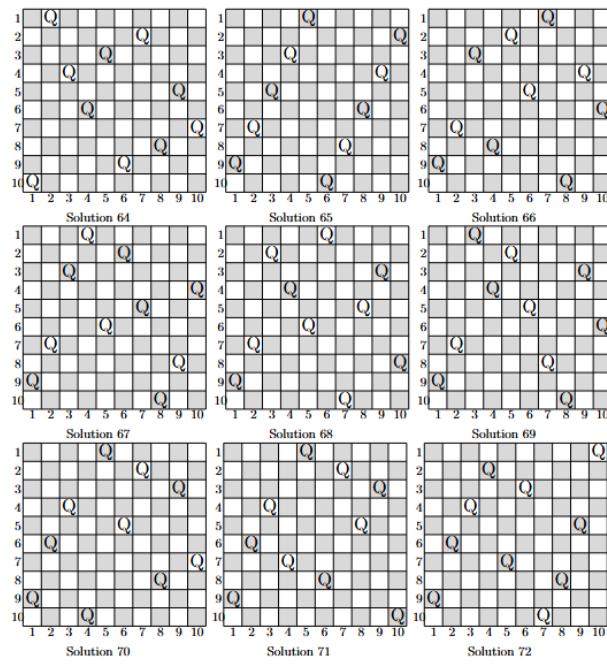


Figure 151: Solutions 64 to 72

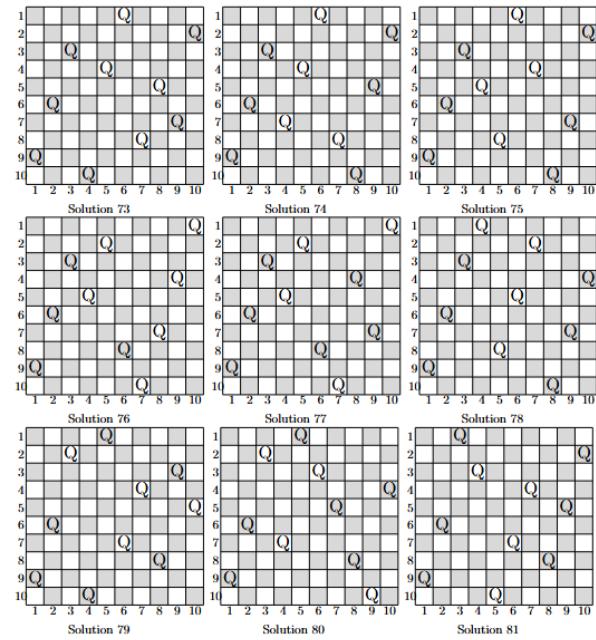


Figure 152: Solutions 73 to 81

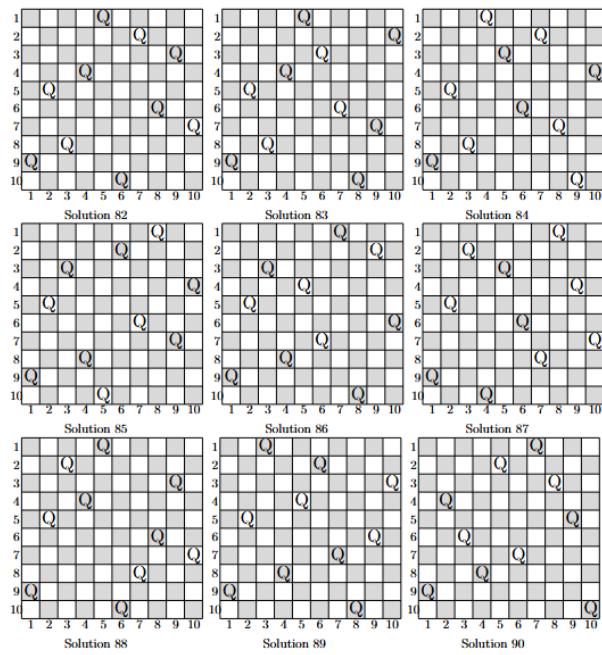


Figure 153: Solutions 82 to 90

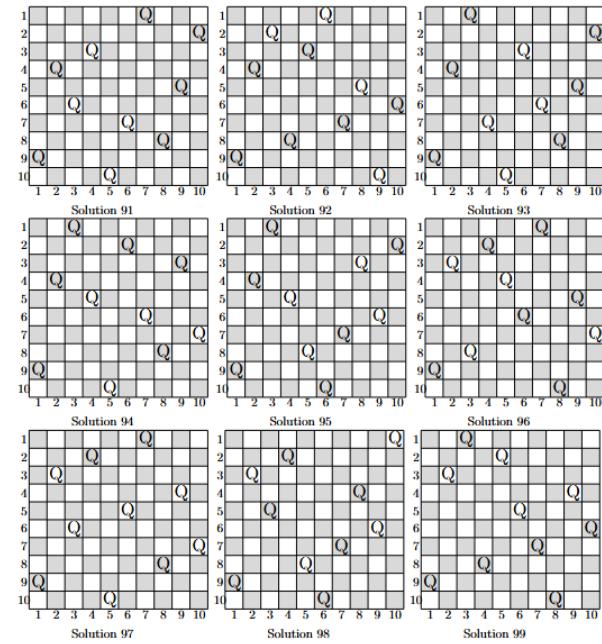


Figure 154: Solutions 91 to 99

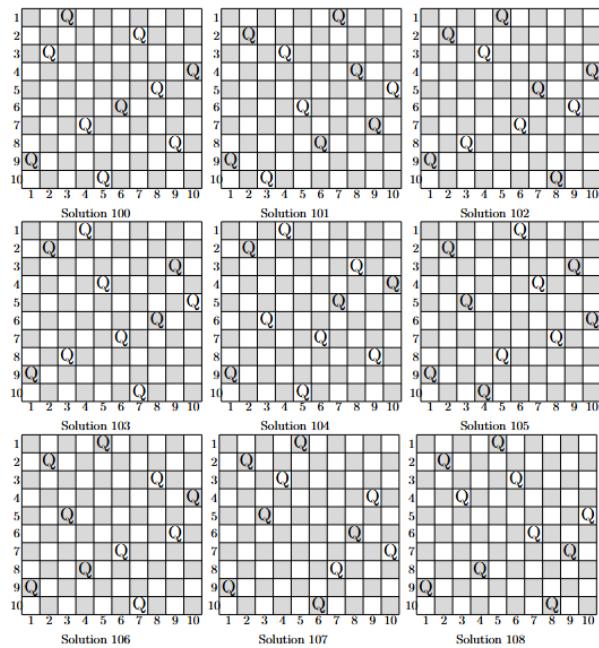


Figure 155: Solutions 100 to 108

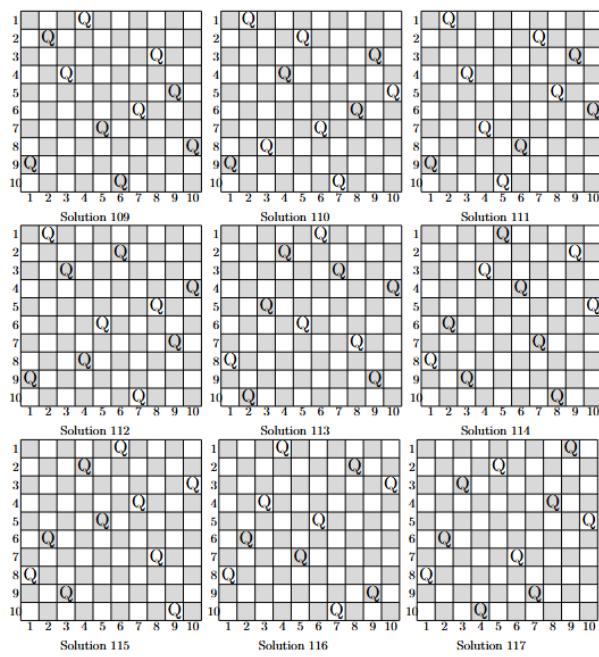


Figure 156: Solutions 109 to 117

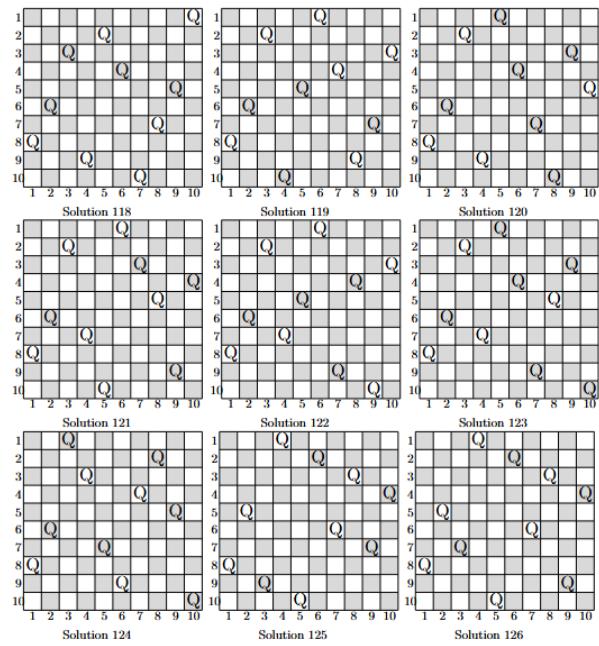


Figure 157: Solutions 118 to 126

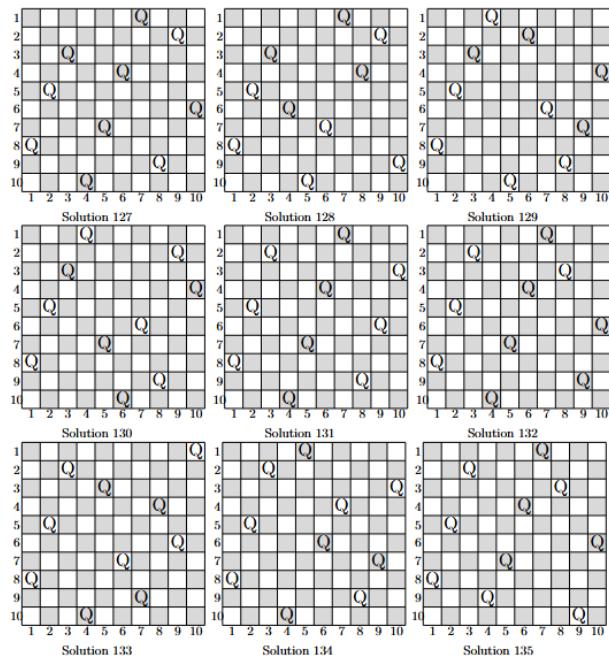


Figure 158: Solutions 127 to 135

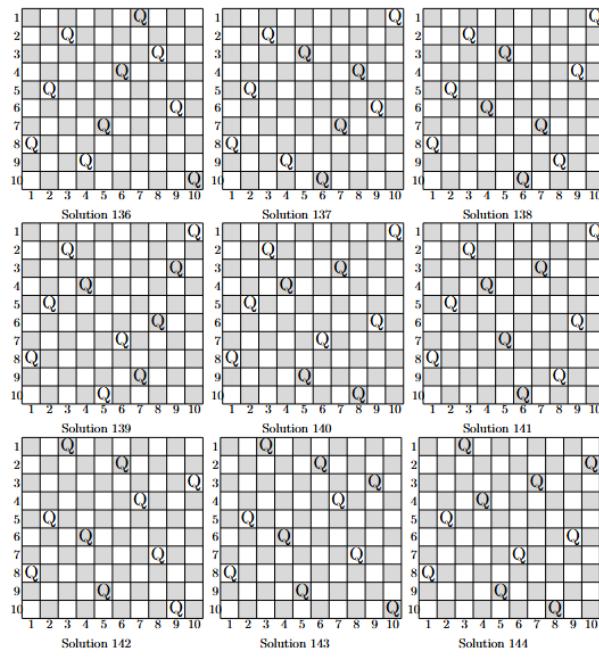


Figure 159: Solutions 136 to 144

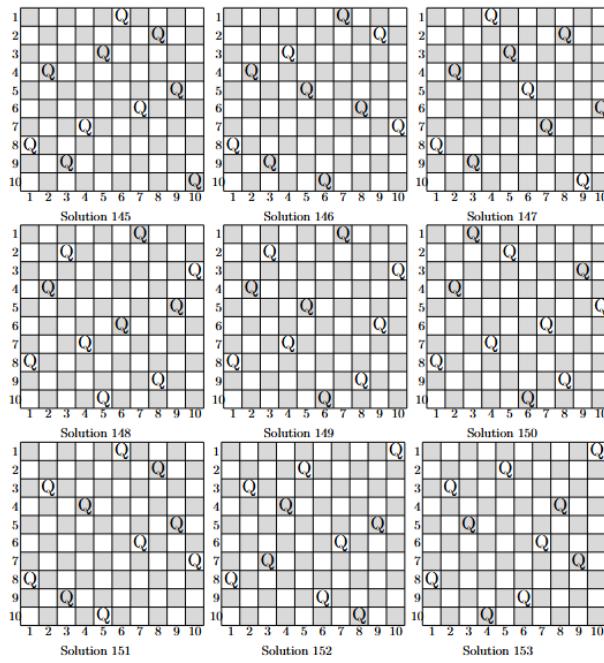


Figure 160: Solutions 145 to 153

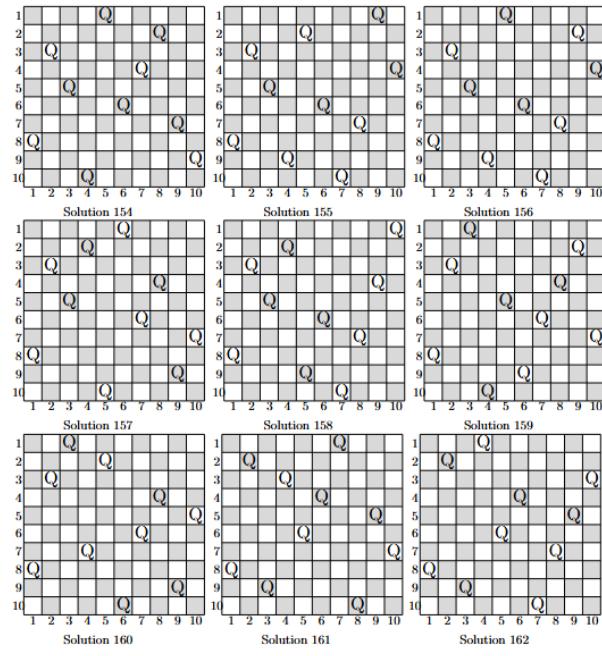


Figure 161: Solutions 154 to 162

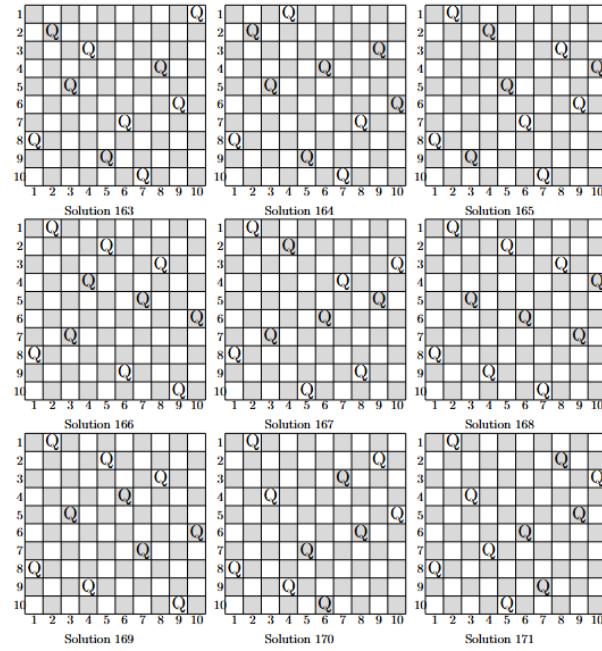


Figure 162: Solutions 163 to 171

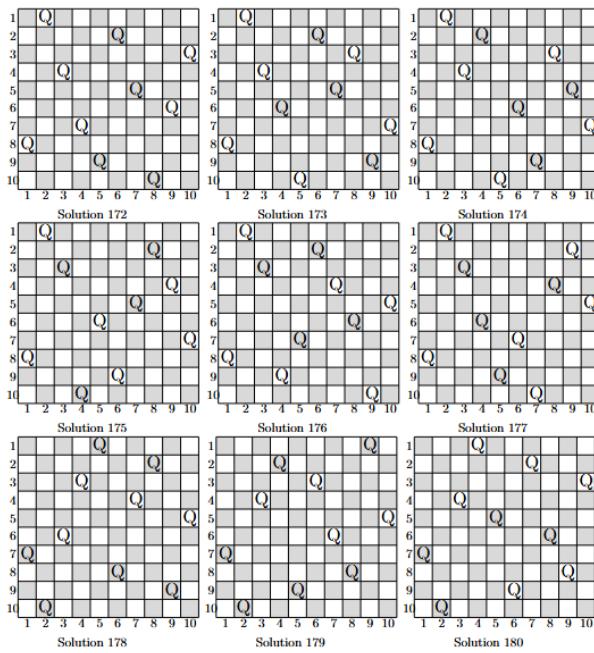


Figure 163: Solutions 172 to 180

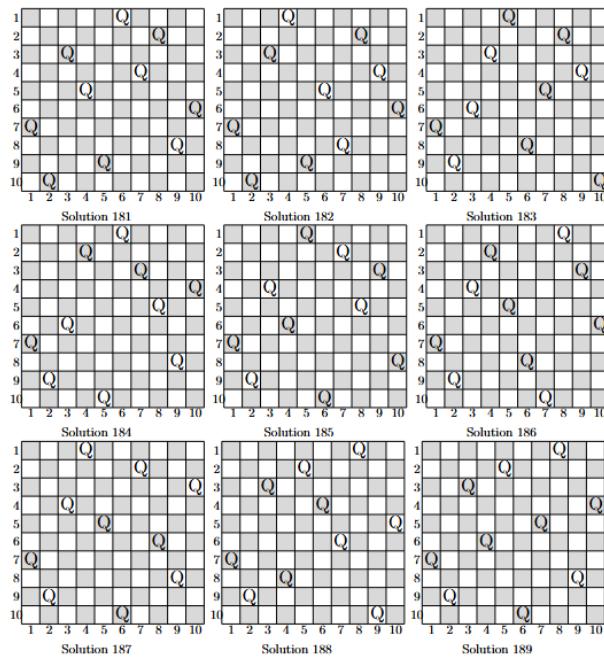


Figure 164: Solutions 181 to 189

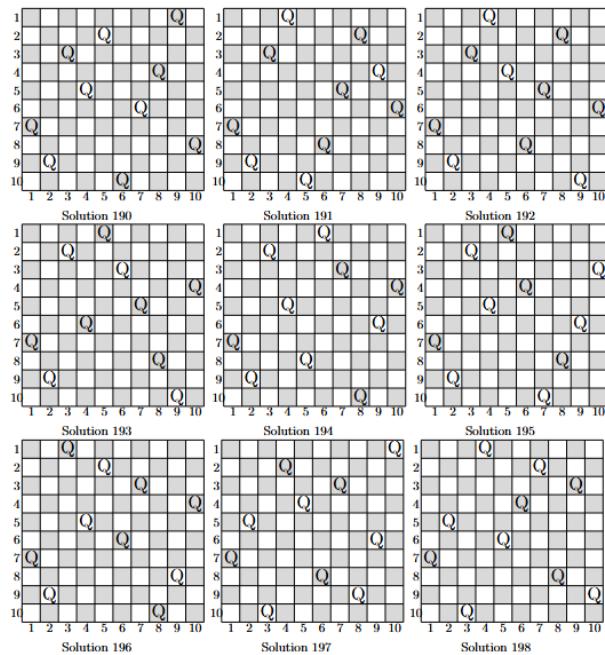


Figure 165: Solutions 190 to 198

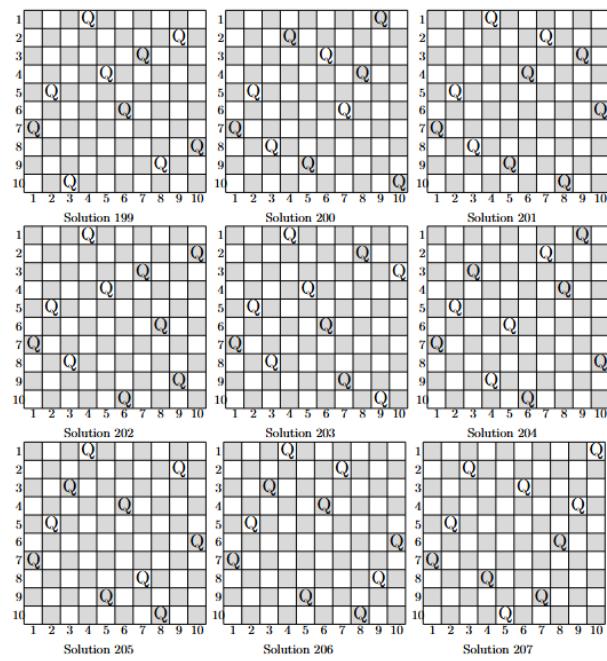


Figure 166: Solutions 199 to 207

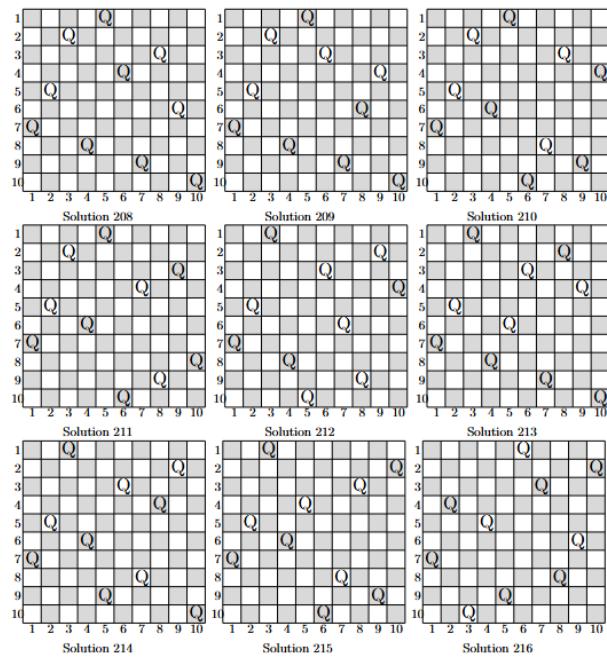


Figure 167: Solutions 208 to 216

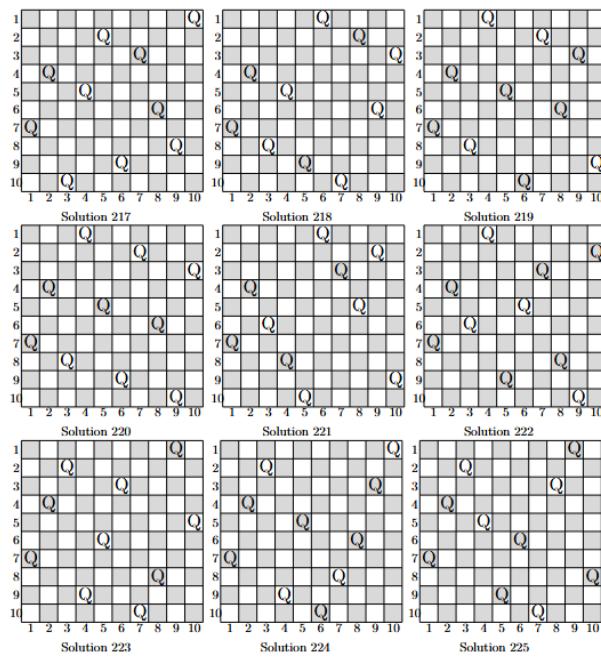


Figure 168: Solutions 217 to 225

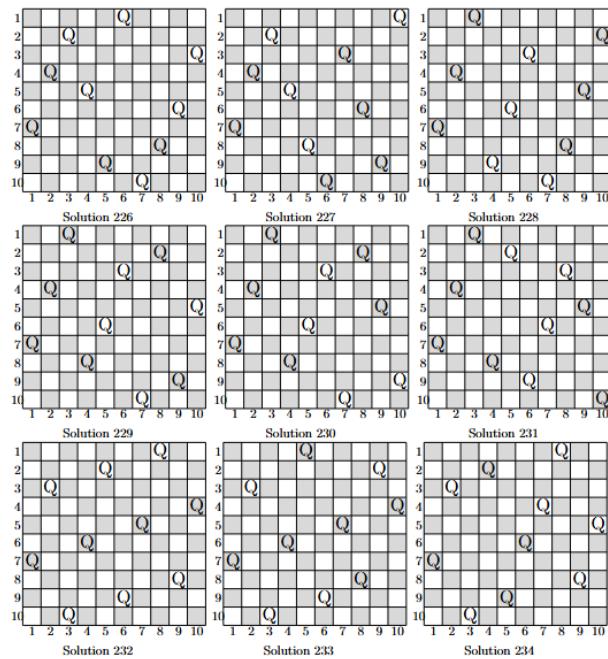


Figure 169: Solutions 226 to 234

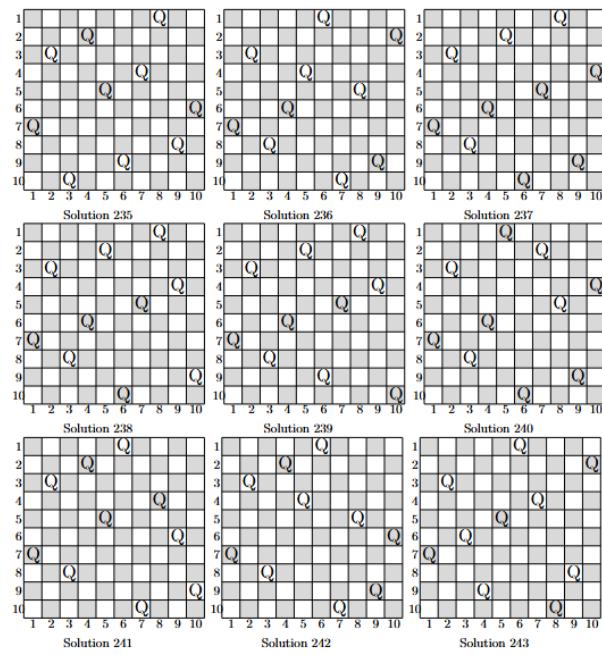


Figure 170: Solutions 235 to 243

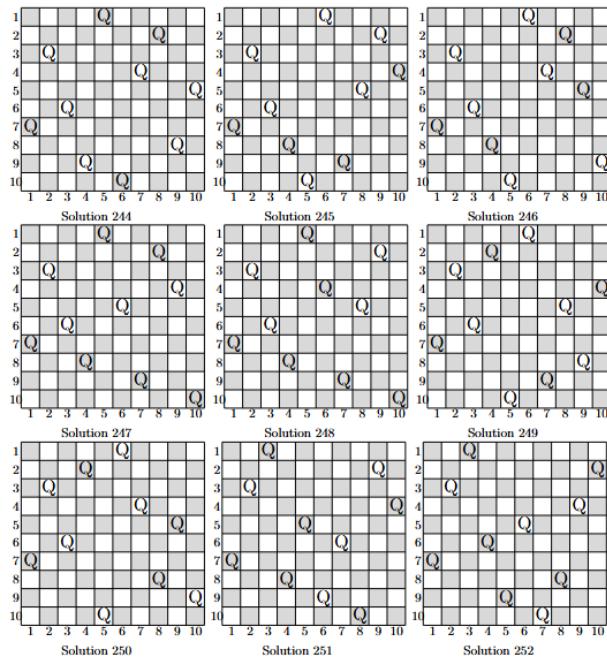


Figure 171: Solutions 244 to 252

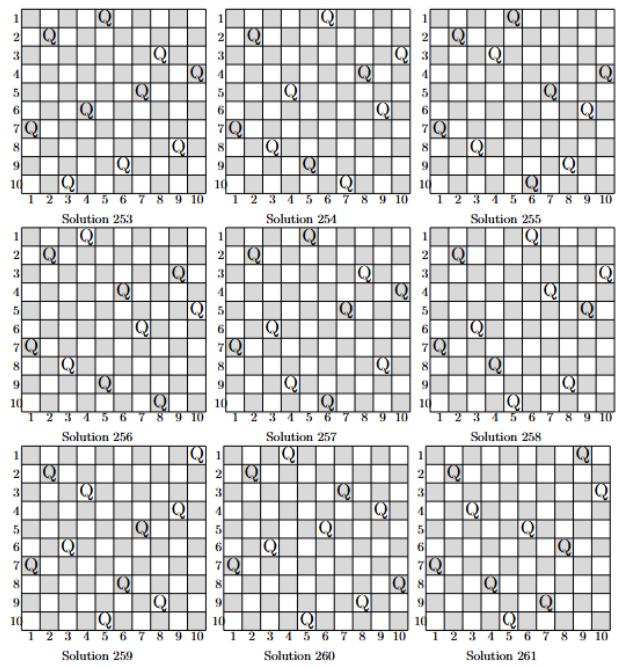


Figure 172: Solutions 253 to 261

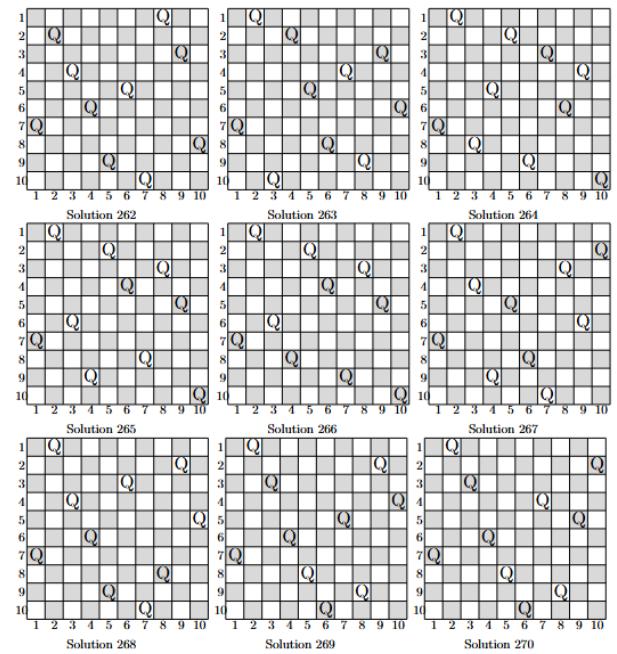


Figure 173: Solutions 262 to 270

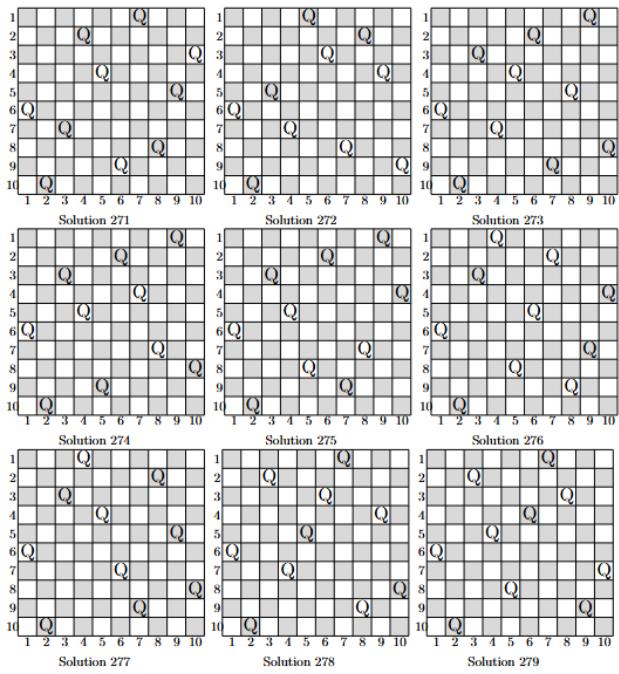


Figure 174: Solutions 271 to 279

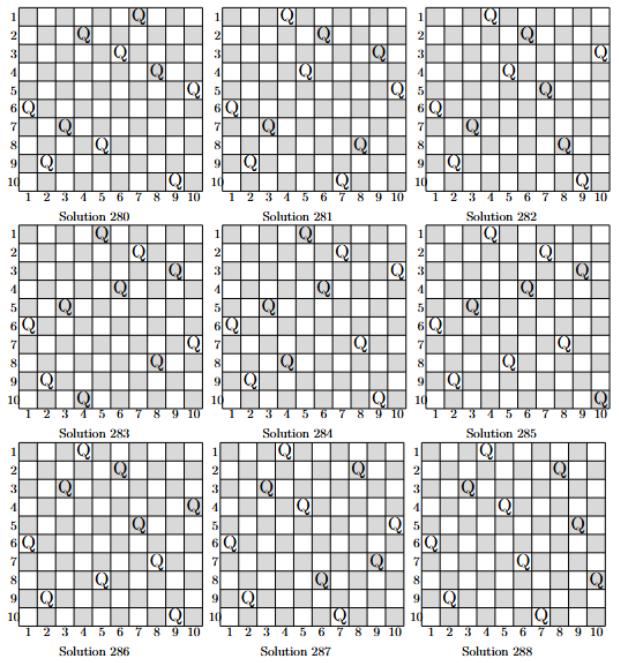


Figure 175: Solutions 280 to 288

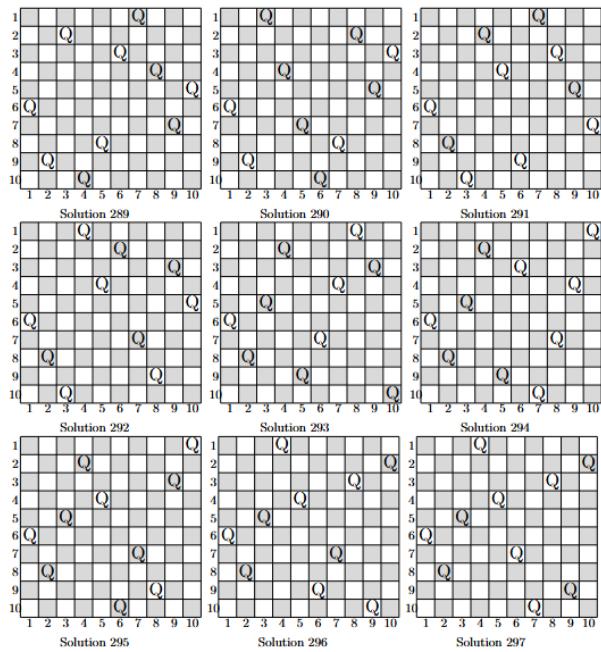


Figure 176: Solutions 289 to 297

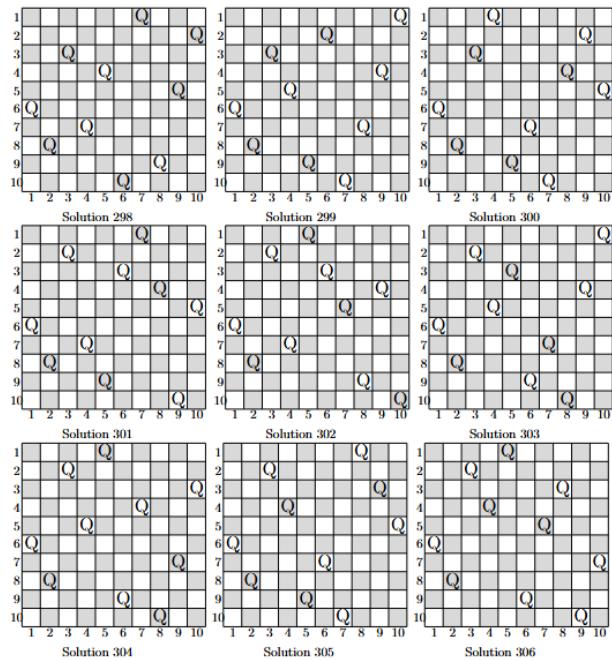


Figure 177: Solutions 298 to 306

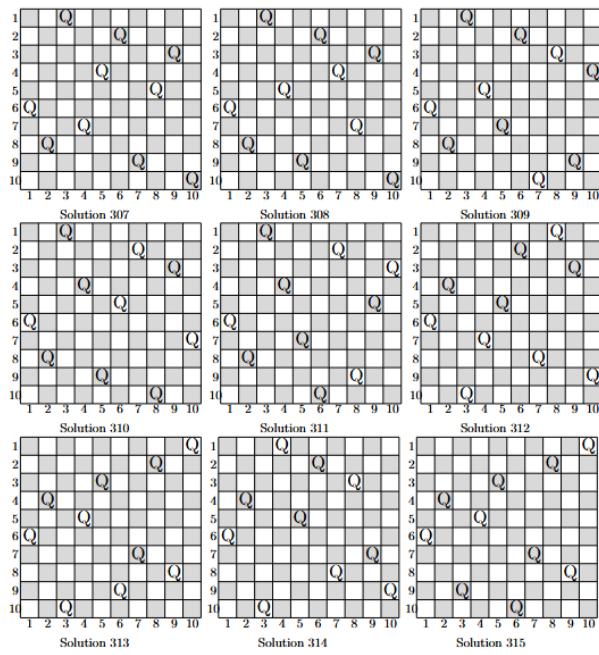


Figure 178: Solutions 307 to 315

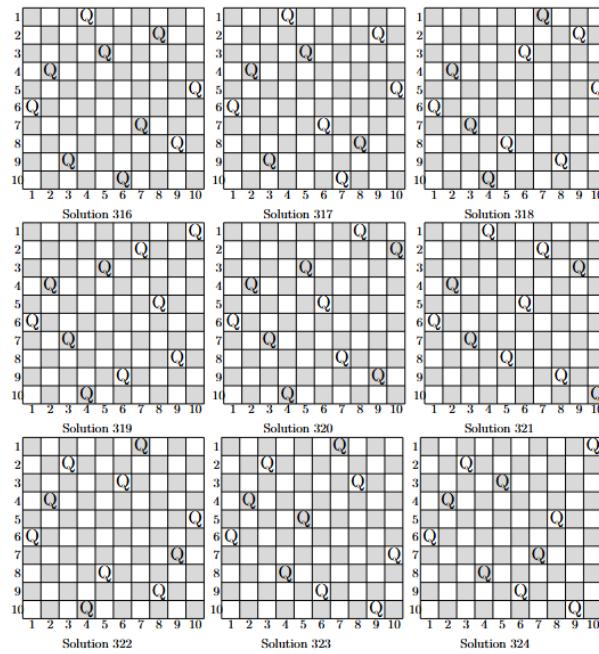


Figure 179: Solutions 316 to 324

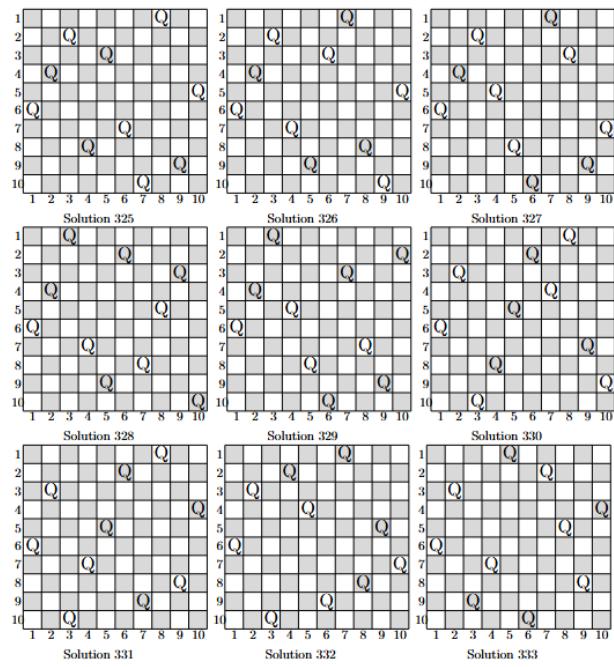


Figure 180: Solutions 325 to 333

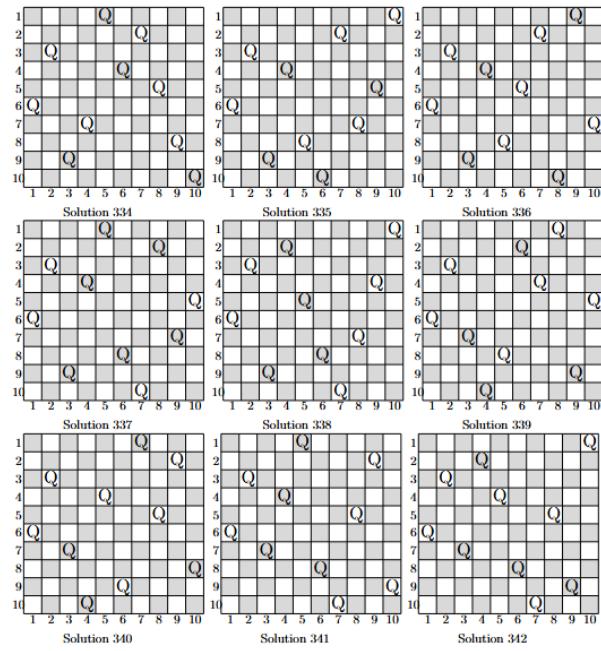


Figure 181: Solutions 334 to 342

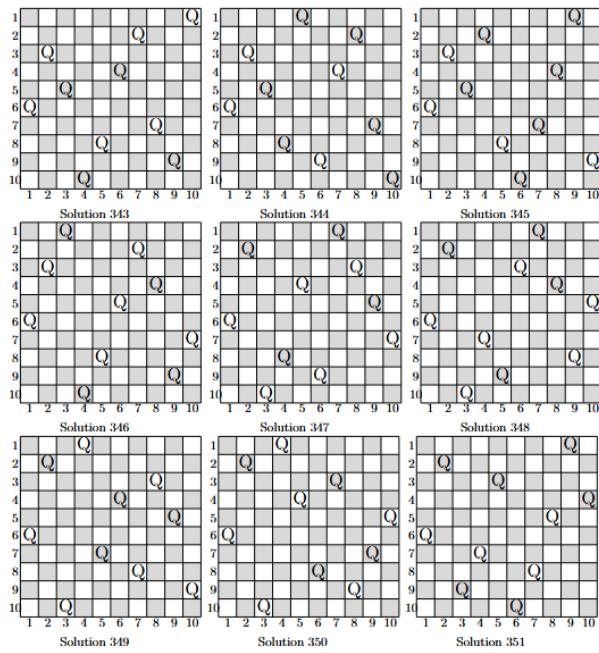


Figure 182: Solutions 343 to 351

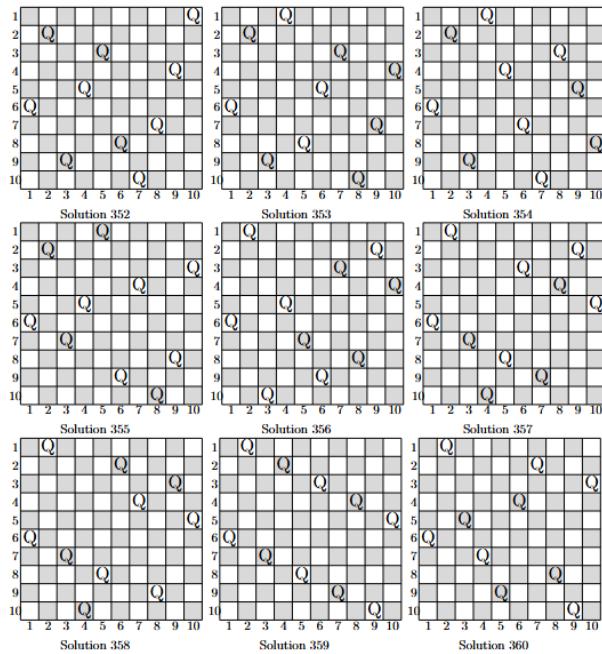


Figure 183: Solutions 352 to 360

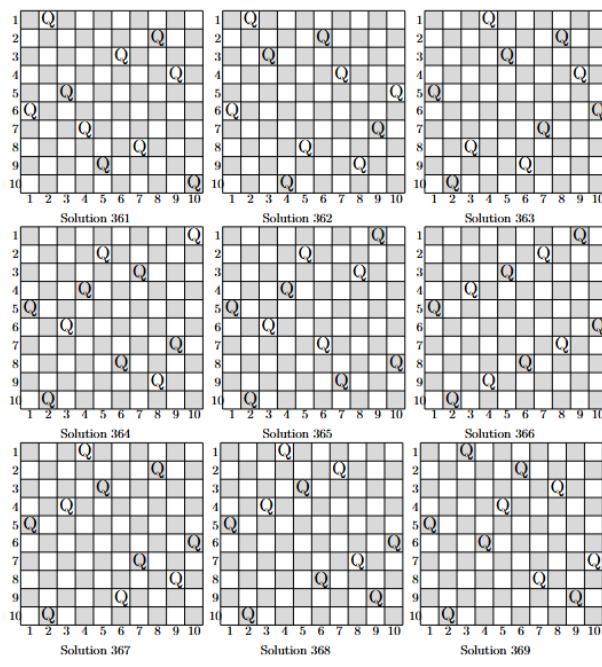


Figure 184: Solutions 361 to 369

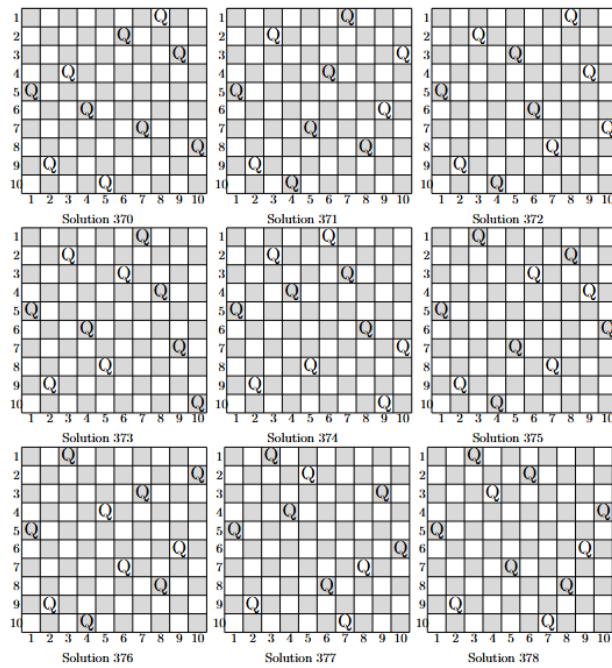


Figure 185: Solutions 370 to 378

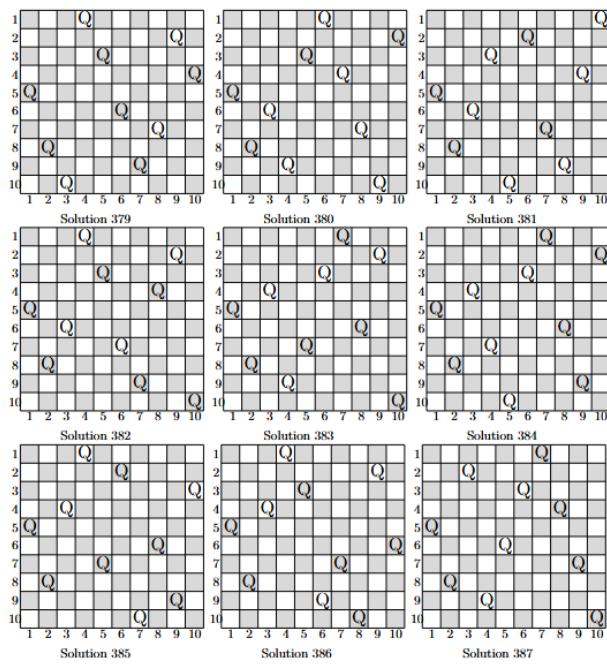


Figure 186: Solutions 379 to 387

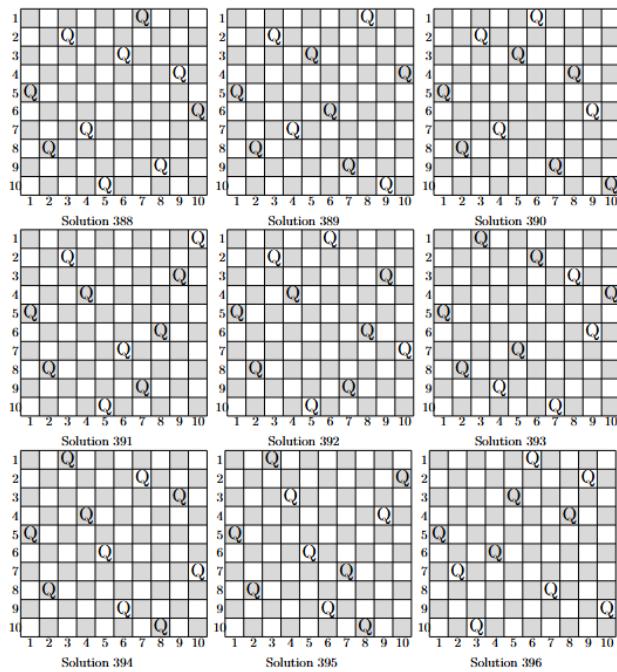


Figure 187: Solutions 388 to 396

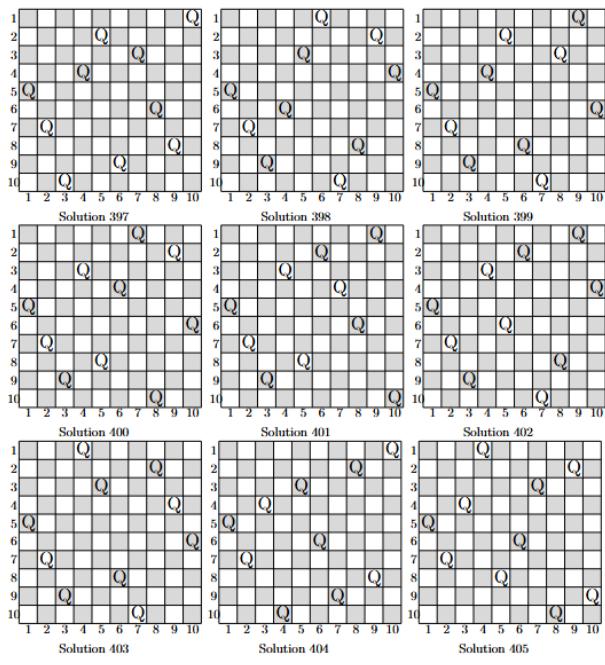


Figure 188: Solutions 397 to 405

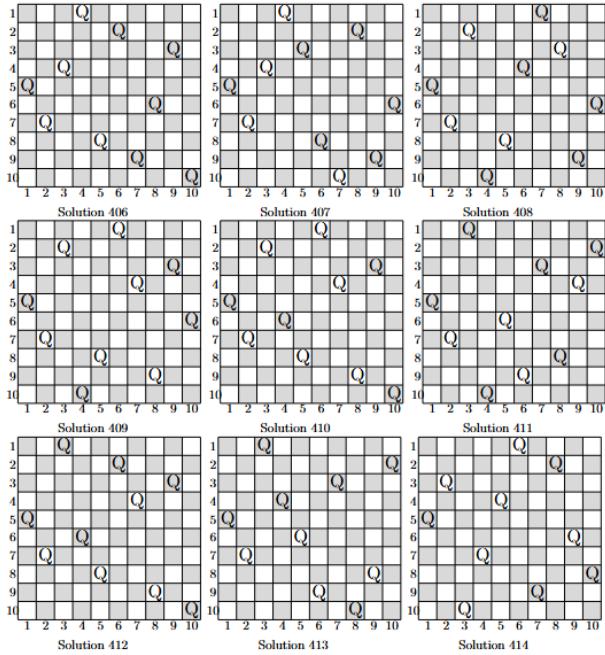


Figure 189: Solutions 406 to 414

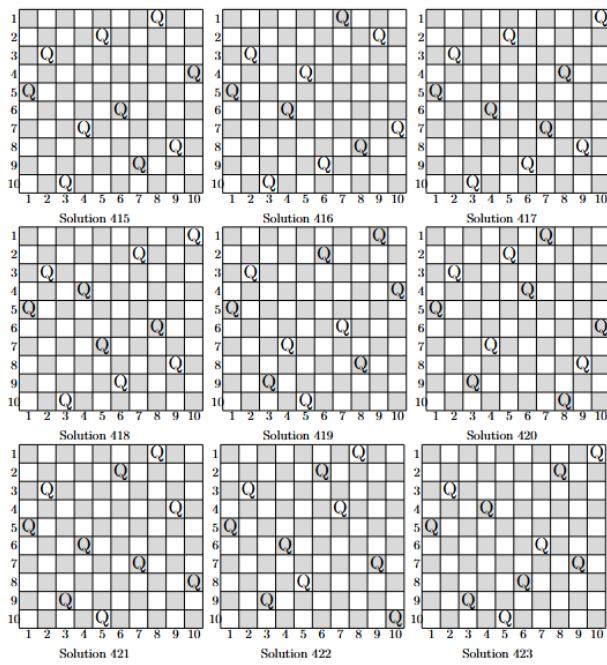


Figure 190: Solutions 415 to 423

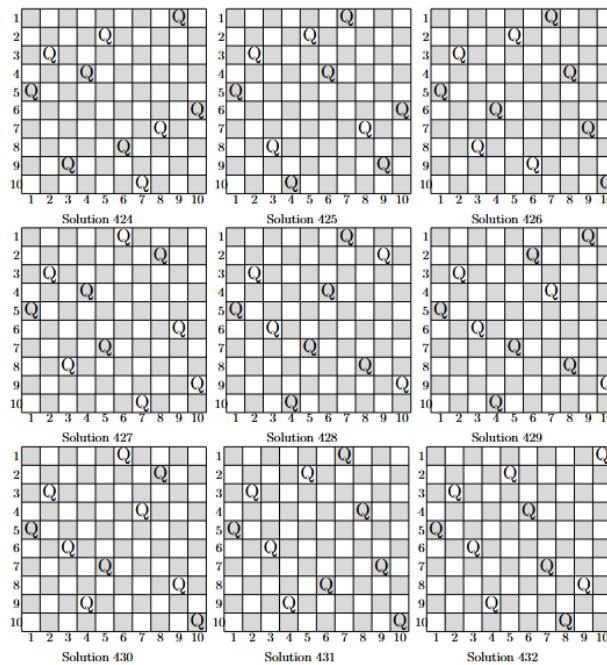


Figure 191: Solutions 424 to 432

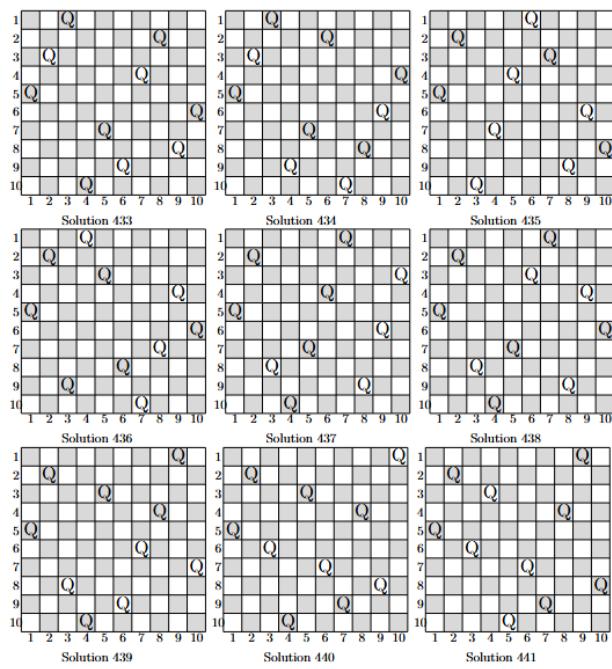


Figure 192: Solutions 433 to 441

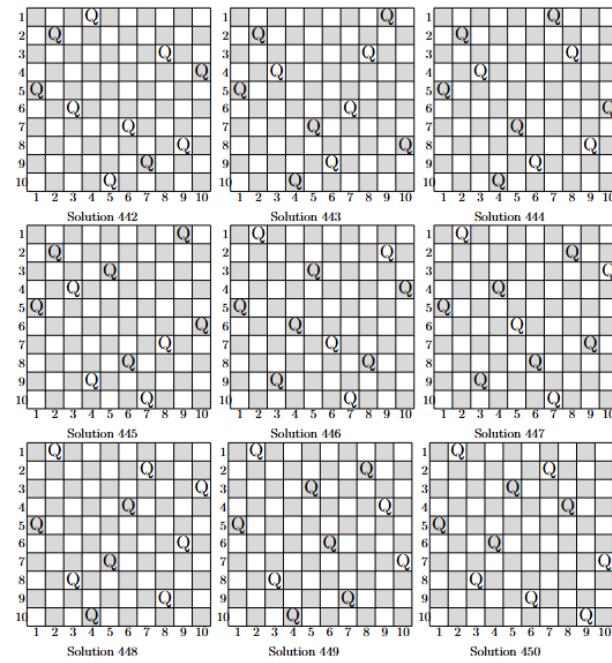


Figure 193: Solutions 442 to 450

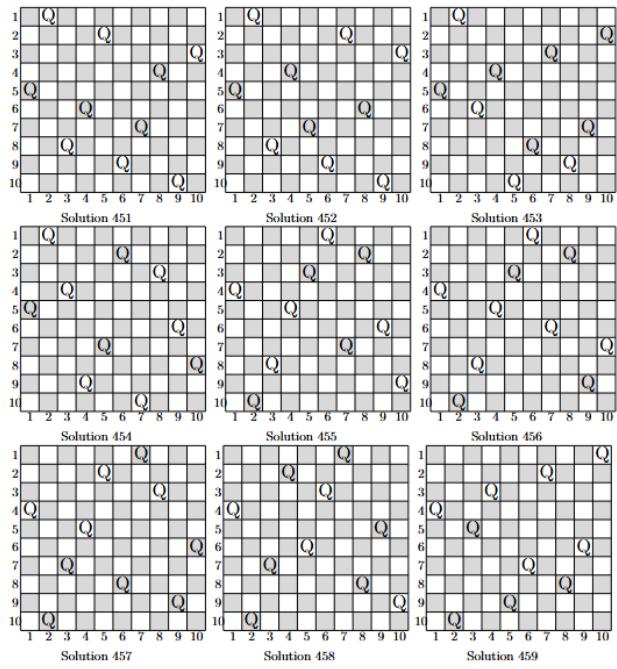


Figure 194: Solutions 451 to 459

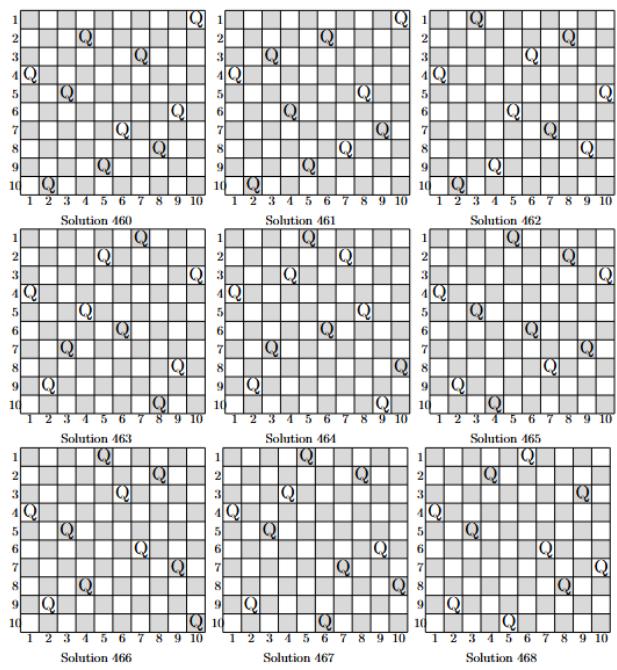


Figure 195: Solutions 460 to 468

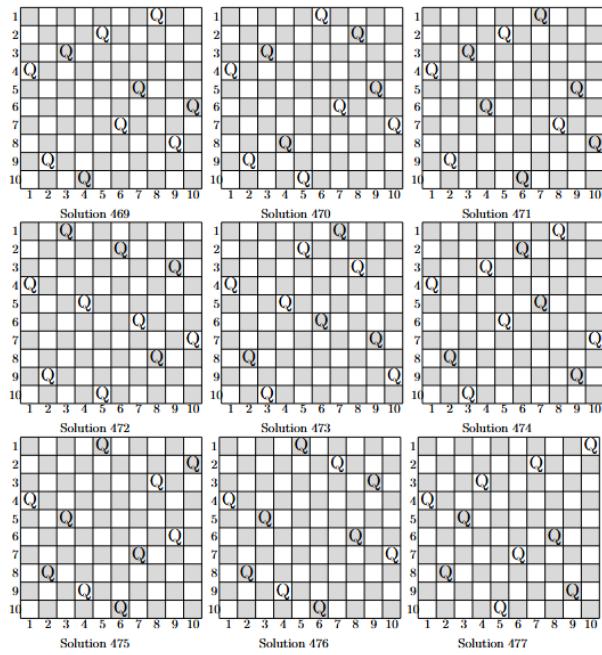


Figure 196: Solutions 469 to 477

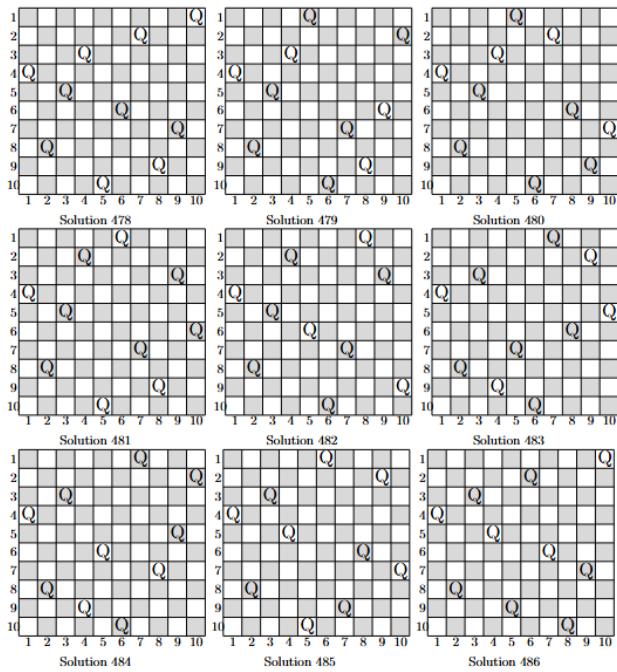


Figure 197: Solutions 478 to 486

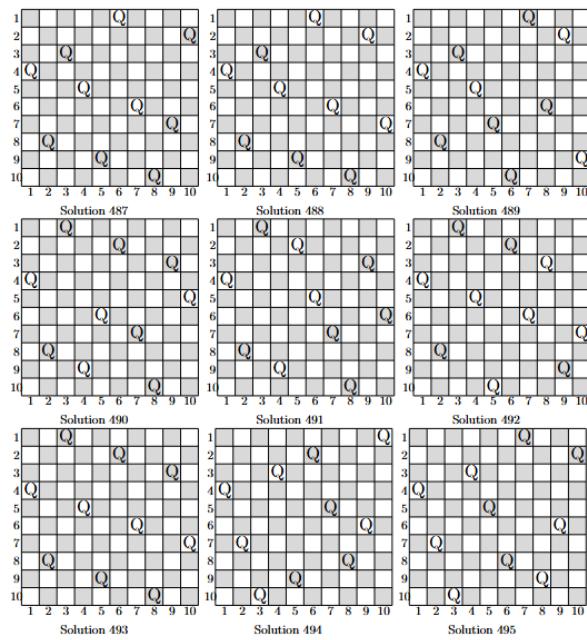


Figure 198: Solutions 487 to 495

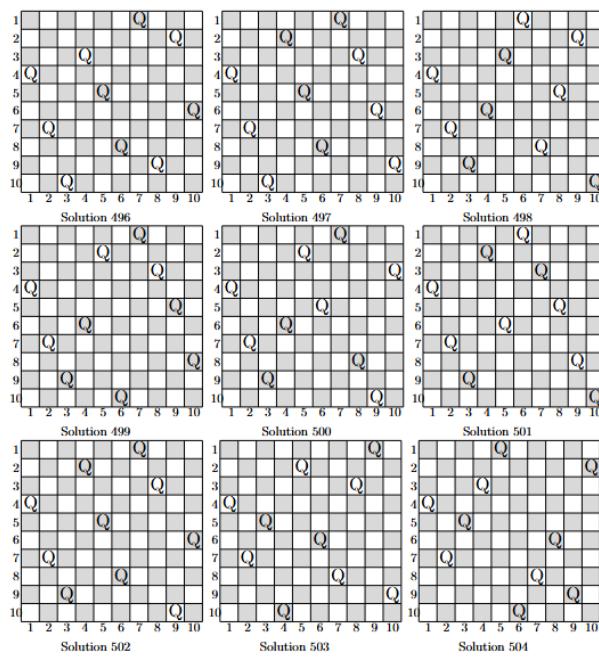


Figure 199: Solutions 496 to 504

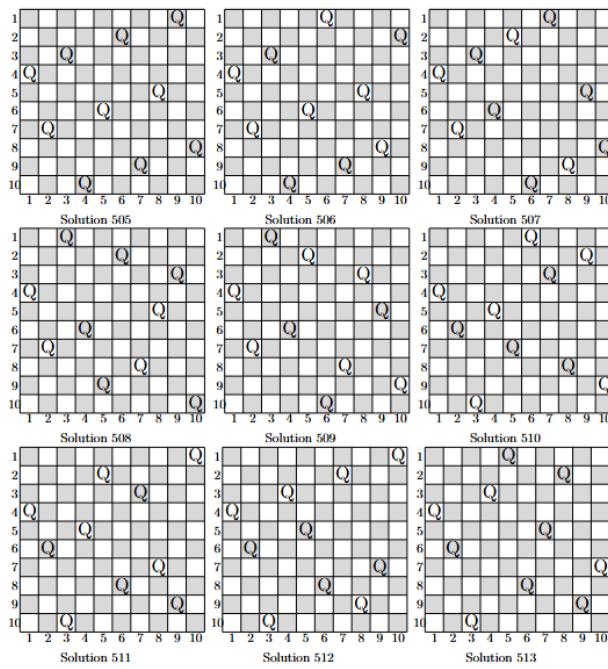


Figure 200: Solutions 505 to 513

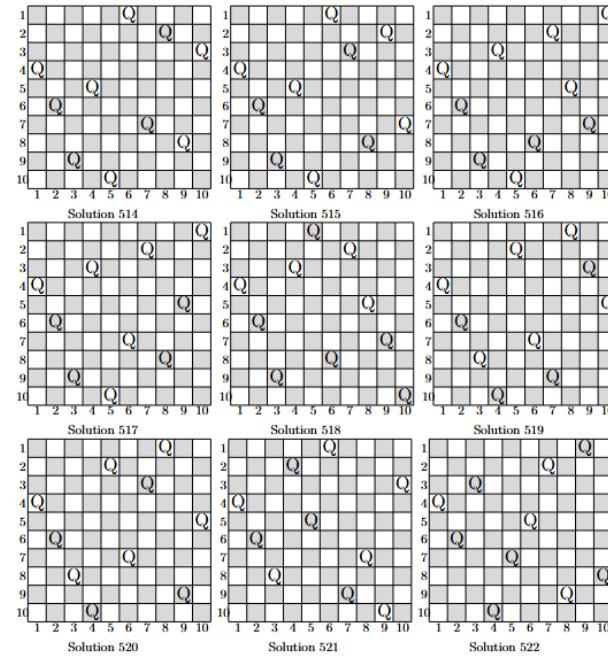


Figure 201: Solutions 514 to 522

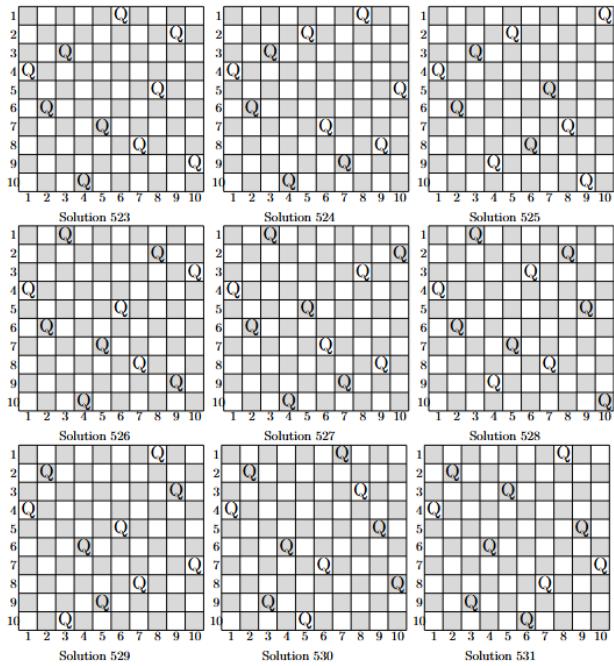


Figure 202: Solutions 523 to 531

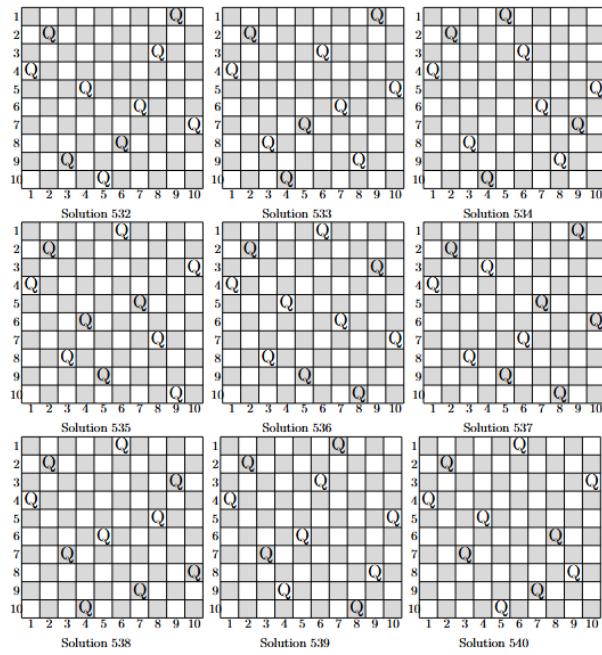


Figure 203: Solutions 532 to 540

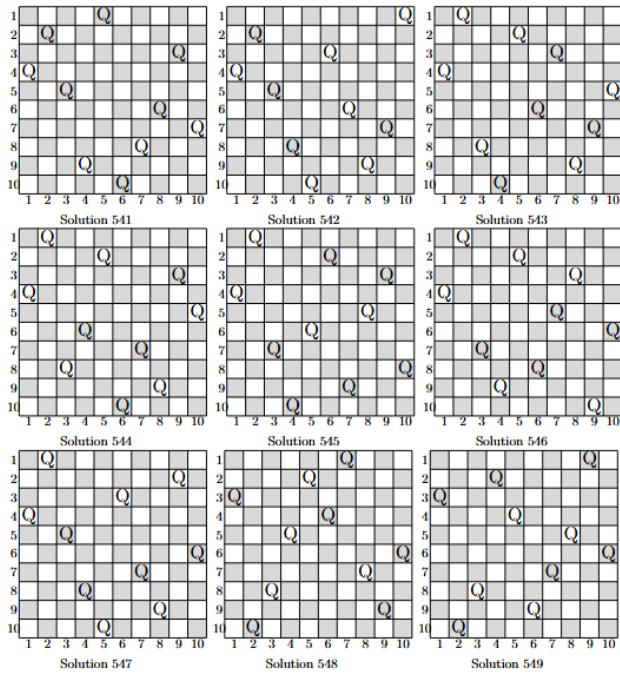


Figure 204: Solutions 541 to 549

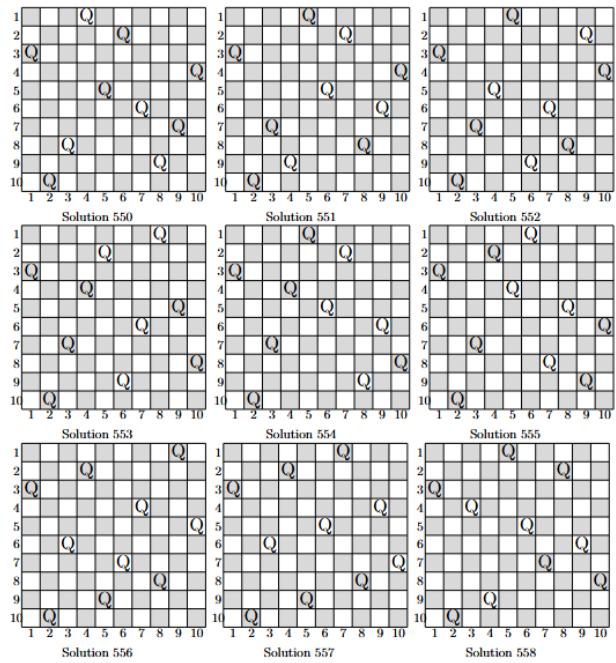


Figure 205: Solutions 550 to 558

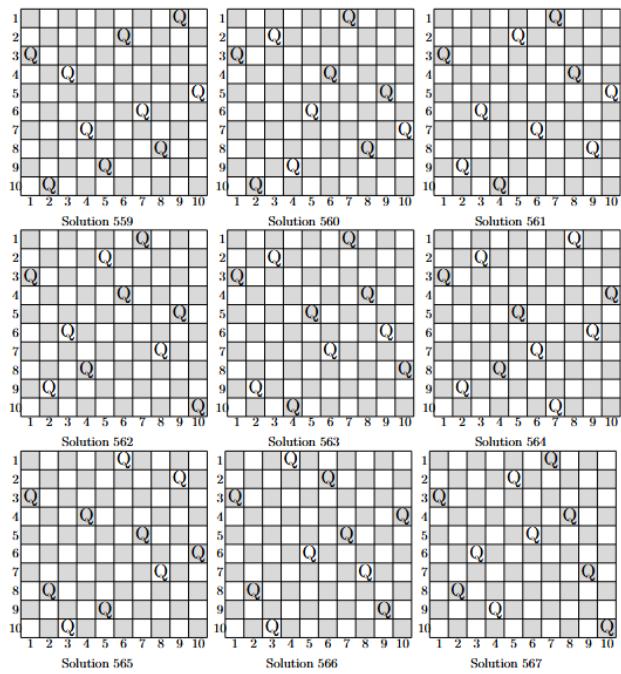


Figure 206: Solutions 559 to 567

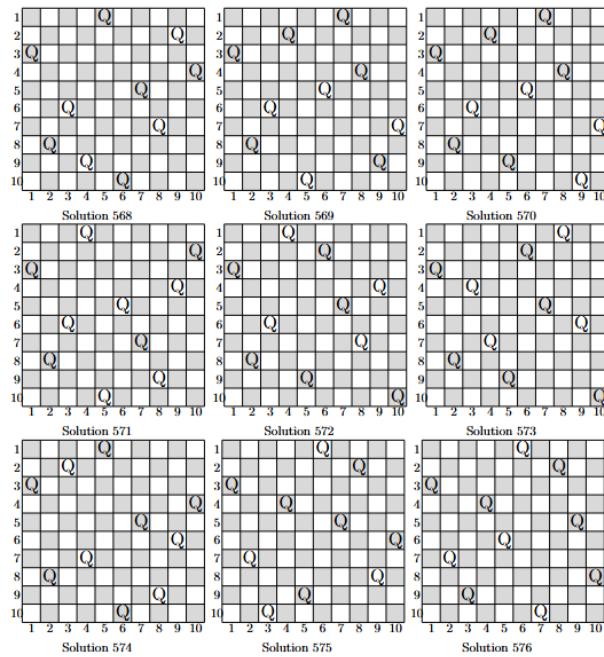


Figure 207: Solutions 568 to 576

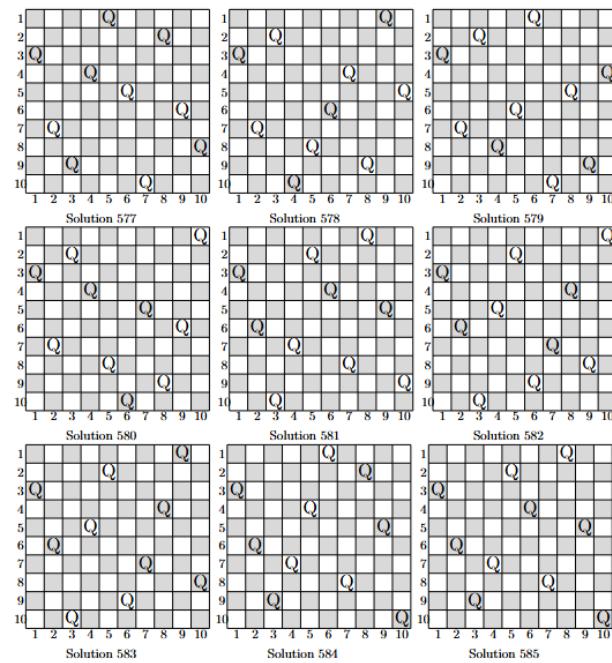


Figure 208: Solutions 577 to 585

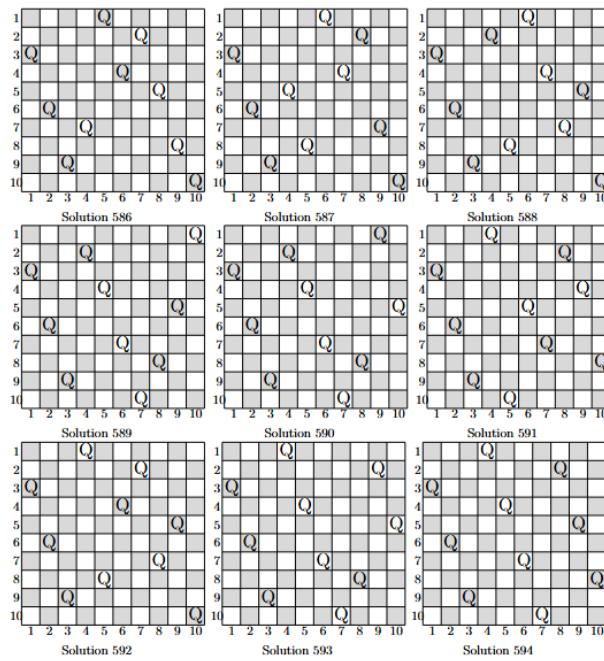


Figure 209: Solutions 586 to 594

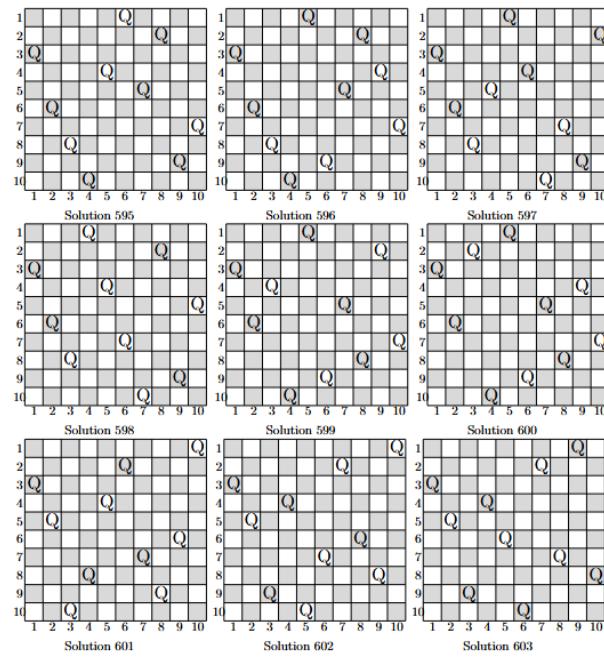


Figure 210: Solutions 595 to 603

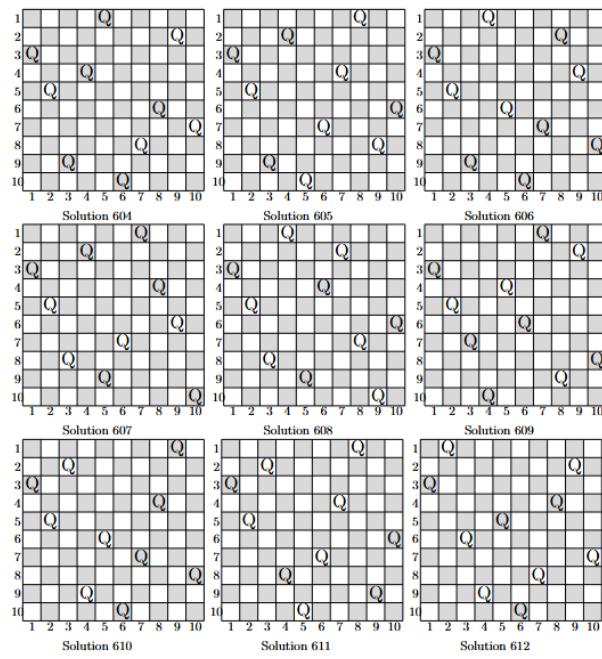


Figure 211: Solutions 604 to 612

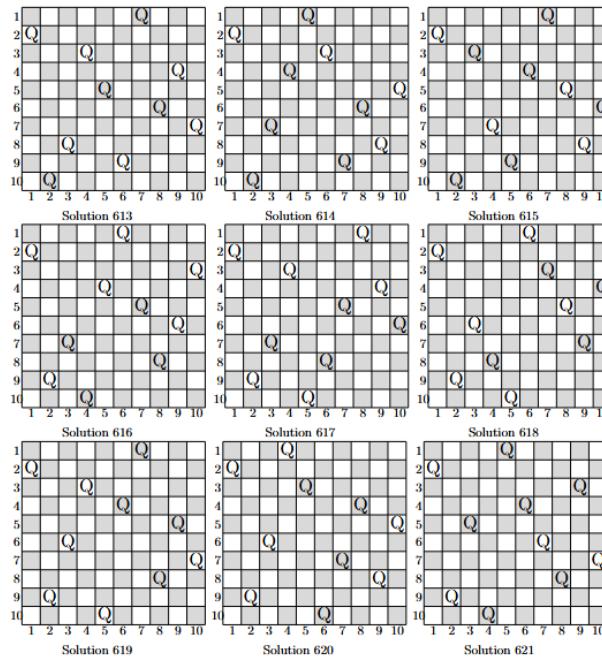


Figure 212: Solutions 613 to 621

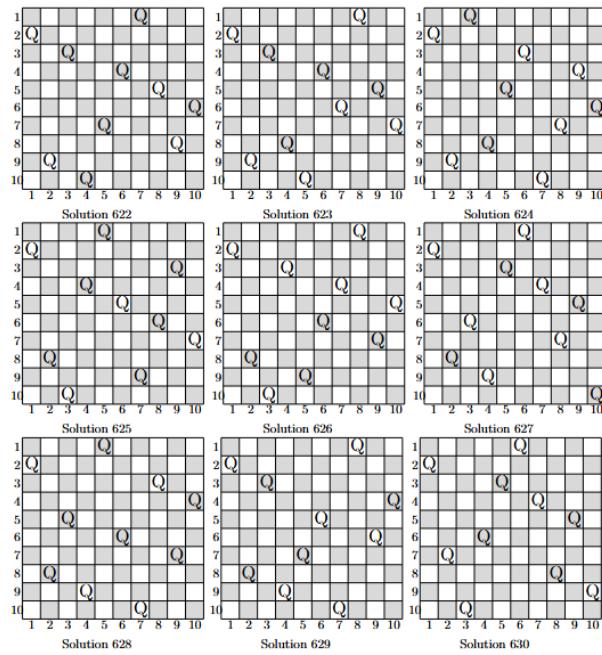


Figure 213: Solutions 622 to 630

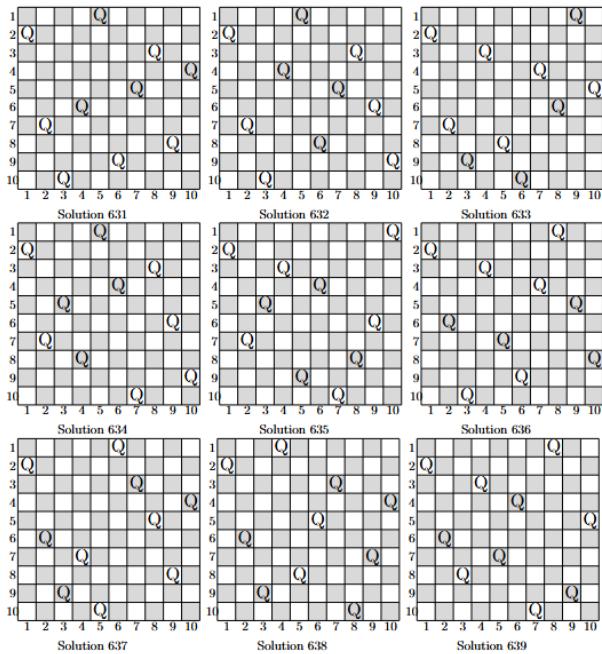


Figure 214: Solutions 631 to 639

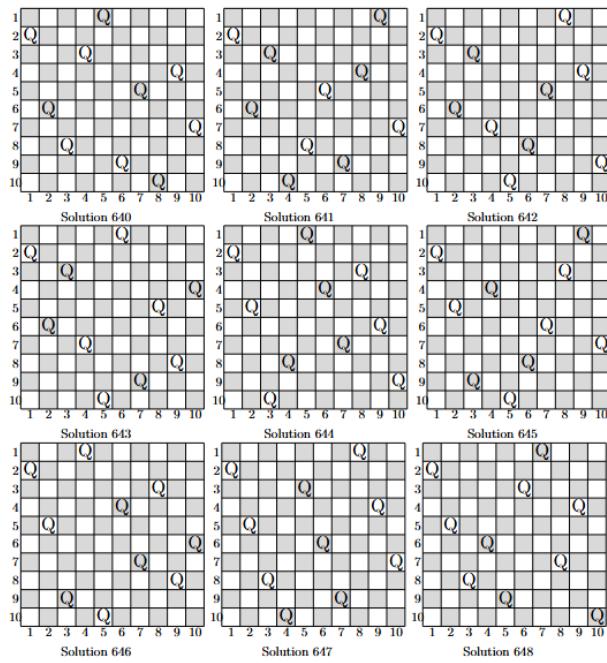


Figure 215: Solutions 640 to 648

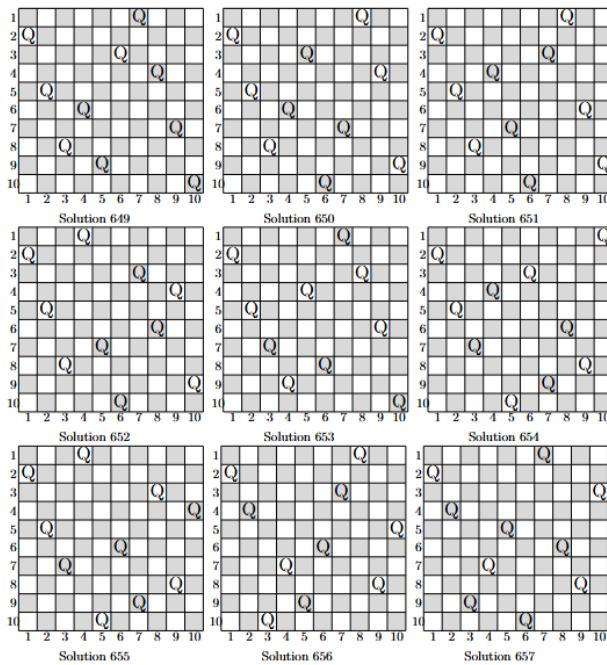


Figure 216: Solutions 649 to 657

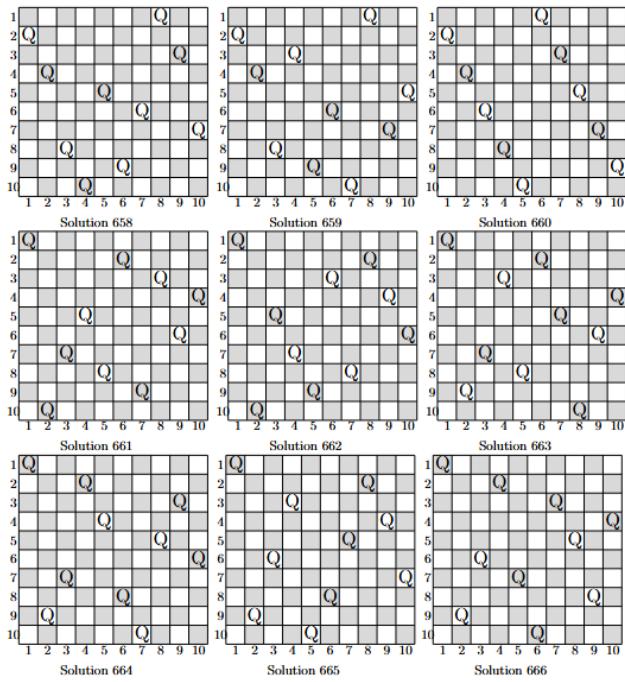


Figure 217: Solutions 658 to 666

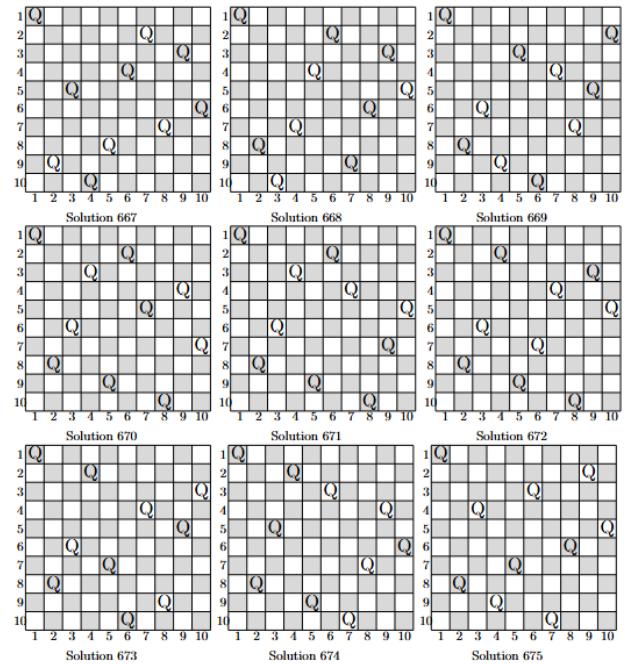


Figure 218: Solutions 667 to 675

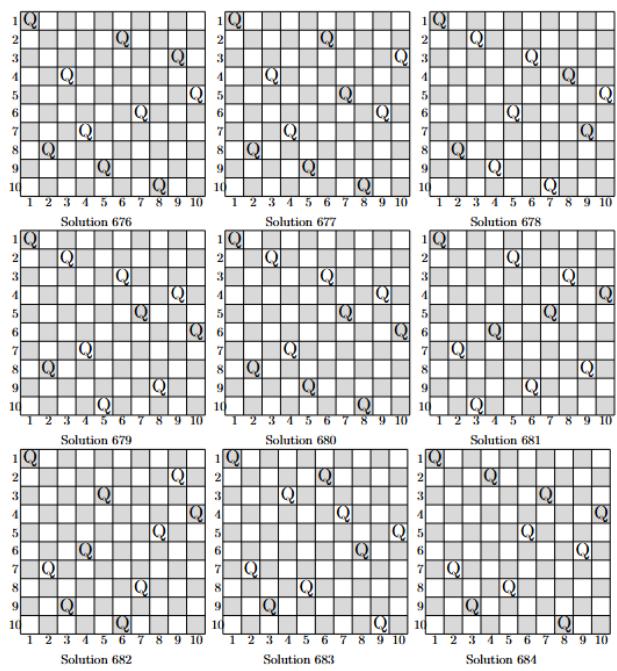


Figure 219: Solutions 676 to 684

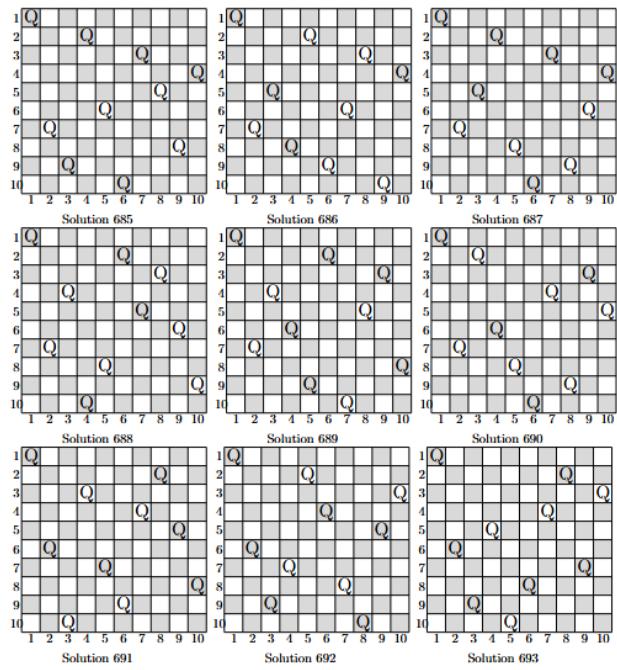


Figure 220: Solutions 685 to 693

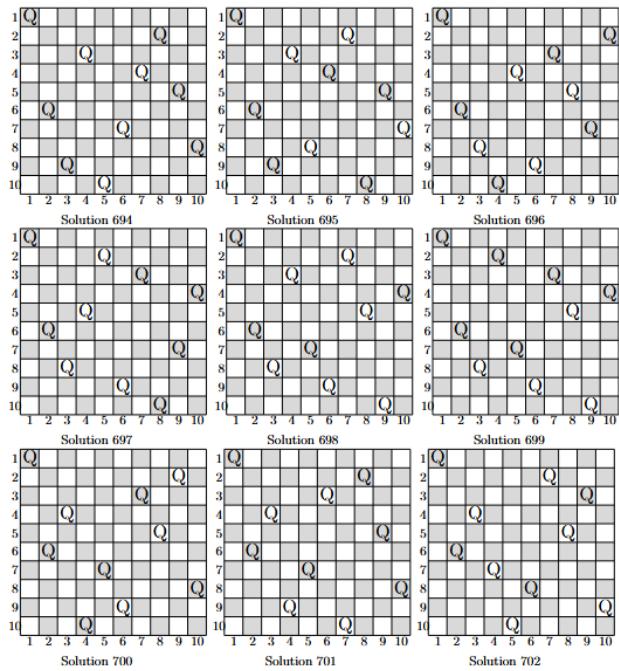


Figure 221: Solutions 694 to 702

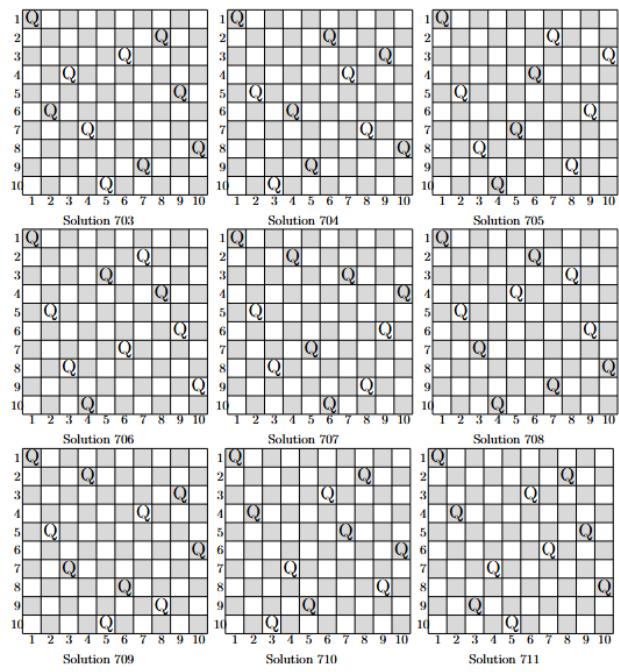


Figure 222: Solutions 703 to 711

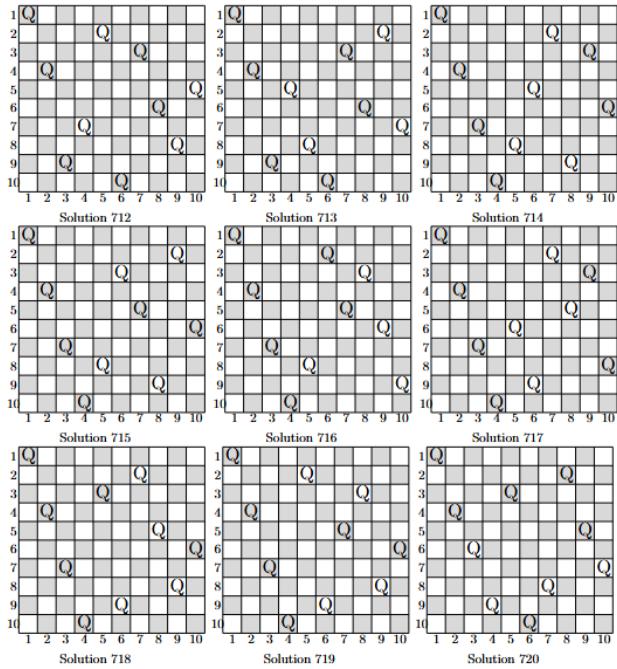


Figure 223: Solutions 712 to 720

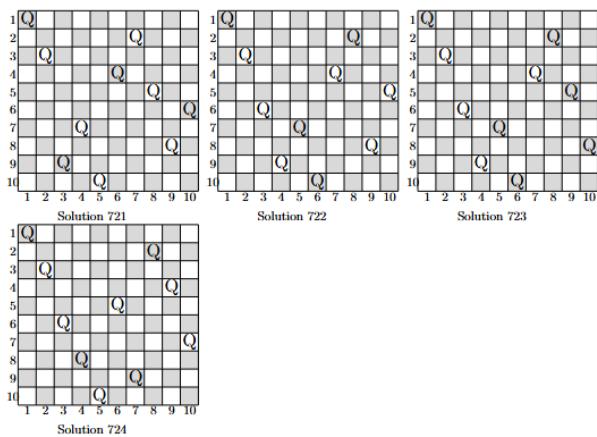


Figure 224: Solutions 721 to 724