1.2 Regularized Polynomial Regression

◦) Matrix-Form

$$E(w) = \frac{1}{N} \sum_{i=1}^{N} \left(y_i - \sum_{k=0}^{10} x_i^k w_k \right)^2 + \alpha^2 \sum_{k=0}^{10} w_k^2$$

$$\alpha^2 \sum_{h=0}^{10} w_k^2 = \alpha^2_{1\times N} \cdot \vec{w}^T \cdot \vec{w}$$

$$E(w) = \frac{1}{N} \underbrace{\left(\overset{N\times 1}{\vec{y}} - \overset{N\times 11}{X} \cdot \overset{11\times 1}{\vec{w}} \right)^T}_{1\times N} \cdot \left(\vec{y} - X \cdot \vec{w} \right)$$

$$X = \begin{pmatrix} x_0^0 & x_0^1 & \dots & x_0^{10} \\ x_1^0 & x_1^1 & \dots & x_1^{10} \\ \vdots & & & \\ x_N^0 & x_N^1 & \dots & x_N^{10} \end{pmatrix}$$

Dim

◦) Weight-Vector

$$E'(w) = 0$$

$$E'(w) = \frac{1}{N} \cdot 2 \left(\vec{y} - X\vec{w} \right)^T \cdot (-X) + \alpha^2 \cdot 2 \cdot \vec{w}^T$$

$$\cancel{2} \cdot \alpha^2 \vec{w}^T = \frac{\cancel{2}}{N} \left(\vec{y} - X\vec{w} \right)^T \cdot X$$

$$\alpha^2 \vec{w} = \frac{1}{N} X^T \left(\vec{y} - X\vec{w} \right)$$

$$\alpha^2 \vec{w} = \frac{1}{N} \left(X^T \cdot \vec{y} - X^T \cdot X \cdot \vec{w} \right)$$

$$\alpha^2 \cdot I \cdot \vec{w} + X^T \cdot X \vec{w} \cdot \frac{1}{N} = \frac{1}{N} X^T \cdot \vec{y}$$

$$\left(\alpha^2 \cdot I + \frac{1}{N} \cdot X^T \cdot X \right) \cdot \vec{w} = \frac{1}{N} \cdot X^T \cdot \vec{y}$$

$$\vec{w} = \left(\alpha^2 \cdot I + \frac{1}{N} \cdot X^T \cdot X \right)^{-1} \cdot \frac{1}{N} \cdot X^T \cdot \vec{y}$$