MB202 cheatsheet

Goniometrické funkce

$$\sin^2 x = \frac{1}{2} \cdot [1 - \cos(2x)]$$

$$\cos^2 x = \frac{1}{2} \cdot [1 + \cos(2x)]$$

$$\sin(2x) = 2\sin(x) \cdot \cos(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

Pro liché funkce:

$$\tan\frac{x}{2} = t : s$$

$$\sin(x) = \frac{2t}{1+t^2}$$

$$\cos(x) = \frac{1-t^2}{1+t^2}$$

$$dx = \frac{2dt}{1+t^2}$$

Pro sudé funkce:

$$\tan(x) = t :$$

$$\sin^2 x = \frac{t^2}{1+t^2}$$

$$\cos^2 x = \frac{1}{1+t^2}$$

$$dx = \frac{dt}{1+t^2}$$

$$\sin^2 x = \frac{1}{2} \cdot [1 - \cos(2x)]$$

$$\cos^2 x = \frac{1}{2} \cdot [1 + \cos(2x)]$$

Limity

$$x^a << \beta^x << x! << x^x$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e \quad \lim_{x\to\infty}\left(1+\frac{a}{x}\right)^x = e^a$$

$$\lim_{x\to 0}\frac{e^x-1}{x} = 1 \quad \lim_{x\to 0}\frac{\arcsin(x)}{x} = 1$$

$$\lim_{x\to 0}\frac{\tan(x)}{x} = 1 \quad \lim_{x\to 0}\frac{\sin(x)}{x} = 1$$

L'Hospitalovo pravidlo:

$$\lim_{x\to n}\frac{f(x)'}{g(x)'}$$

Taylorův polynom

$$T_n(x) = \sum_{i=0}^{n}\frac{f^{(i)}(x_0)}{i!} \cdot (x-x_0)^i$$

$$R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!} \cdot (x-x_0)^{n+1}$$

$$f(x) = T_n(x) + R_n(x)$$

Derivace

$$(e^x)' = e^x$$

$$(a^x)' = a^x \ln(a)$$

$$(\ln x)' = \frac{1}{x}$$

$$(\log_a x)' = \frac{1}{x\ln a}$$

$$(\sin x)' = \cos x$$

$$(\cos x)' = -\sin x$$

$$(\tan x)' = \frac{1}{\cos^2 x}$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

$$(\arctan x)' = \frac{1}{1+x^2}$$

Integrály

$$\int dx = x + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int e^{ax} dx = \frac{1}{a} \cdot e^{ax} + C$$

$$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + C$$

$$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + C$$

$$\int \frac{1}{\cos^2(ax)}dx = \frac{1}{a} \cdot \tan(ax) + C$$

$$\int \frac{1}{\sin^2(ax)}dx = -\frac{1}{a} \cdot \cot(ax) + C$$

$$\int \frac{1}{\sqrt{a^2-x^2}}dx = \arcsin\frac{a}{a} + C$$

$$\int \frac{1}{\sqrt{a^2+x^2}}dx = \ln\left|x + \sqrt{x^2+a}\right| + C$$

$$\int \frac{1}{a^2+x^2}dx = \frac{1}{a} \cdot \arctan\frac{x}{a} + C$$

$$\int \frac{1}{a^2-x^2}dx = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + C$$

$$\int \frac{1}{x+a}dx = \ln|x+a| + C$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + C$$

$$\int \frac{dx}{(x-x_0)^2+a^2} = \frac{1}{a} \cdot \frac{x-x_0}{a} + C$$

Substituce:

$$\int f(\varphi(x)) \cdot \varphi'(x) dx = \int f(y) dx$$
$$y = \varphi(x) \to dy = \varphi'(x) \cdot dx$$

Pér partes:

$$\int uv' = uv - \int u'v$$

Vícenásobné kořeny v $\mathbb{C}$:

$$K_{n+1}(x_0, a) = \frac{1}{a^2} \cdot \left[\frac{2n-1}{2n} \cdot K_n(x_0, a) + \frac{1}{2n} \cdot \frac{x - x_0}{\left((x-x_0)^2 + a^2\right)^n} \right]$$

Zbytek

$$\sqrt[n]{n!} = \sqrt[2n]{2\pi n} \left(\frac{n}{e}\right)$$
$$\sinh(x) := \frac{e^x - e^{-x}}{2}$$
$$\cosh(x) := \frac{e^x + e^{-x}}{2} = (\sinh(x))'$$