

for some j and Y . If we consider going from $abcde$ backwards to reach S , we can imagine starting with tree (3), and “pruning off” its handle; then prune off the handle (“ e ”) of the resulting tree, and so on until only the root S is left. This process of pruning the handle at each step corresponds exactly to derivation (5) in reverse. The reader may easily verify, in fact, that “handle pruning” always produces, in reverse, the derivation obtained by replacing the *rightmost* intermediate character at each step, and this may be regarded as an alternative way to define the concept of handle. During the pruning process, all leaves to the right of the handle are terminals, if we begin with all terminal leaves.

We are interested in algorithms for parsing, and thus we want to be able to recognize the handle when only the leaves of the tree are given. Number the productions of the grammar $1, 2, \dots, s$ in some arbitrary fashion. Suppose $\alpha = X_1 \cdots X_n \cdots X_t$ is a sentential form, and suppose there is a derivation tree in which the handle is $X_{r+1} \cdots X_n$, obtained by application of the p th production. ($0 \leq r \leq n \leq t, 1 \leq p \leq s$.) We will say (n, p) is a *handle* of α .

A grammar is said to be *translatable from left to right with bound k* (briefly, an “LR(k) grammar”) under the following circumstances. Let $k \geq 0$, and let “ \dashv ” be a new character not in $I \cup T$. A k -sentential form is a sentential form followed by k “ \dashv ” characters. Let $\alpha = X_1 X_2 \cdots X_n X_{n+1} \cdots X_{n+k} Y_1 \cdots Y_u$ and $\beta = X_1 X_2 \cdots X_n X_{n+1} \cdots X_{n+k} Z_1 \cdots Z_v$ be k -sentential forms in which $u \geq 0, v \geq 0$ and in which none of $X_{n+1}, \dots, X_{n+k}, Y_1, \dots, Y_u, Z_1, \dots, Z_v$ is an intermediate character. If (n, p) is a handle of α and (m, q) is a handle of β , we require that $m = n, p = q$. In other words, a grammar is LR(k) if and only if any handle is always uniquely determined by the string to its left and the k terminal characters to its right.

This definition is more readily understandable if we take a particular value of k , say $k = 1$. Suppose we are constructing a derivation sequence such as (5) in reverse, and the current string (followed by the delimiter \dashv for convenience) has the form $X_1 \cdots X_n X_{n+1} \alpha \dashv$, where the tail end “ $X_{n+1} \alpha \dashv$ ” represents part of the string we have not yet examined; but all possible reductions have been made at the left of the string so that the right boundary of the handle must be at position X_r for $r \geq n$. We want to know, by looking at the next character X_{n+1} , if there is in fact a handle whose right boundary is at position X_n ; if so, we want this handle to correspond to a unique production, so we can reduce the string and repeat the process; if not, we know we can move to the right