

$$\begin{array}{ll}
 [S; \dashv \dashv] \rightarrow [A; \dashv \dashv] & [C; \dashv \dashv] \rightarrow [B; e \dashv] \\
 [S; \dashv \dashv] \rightarrow A[D; \dashv \dashv] & [C; \dashv \dashv] \rightarrow B[E; \dashv \dashv] \\
 [S; \dashv \dashv] \rightarrow AD \dashv \dashv [1] & [C; \dashv \dashv] \rightarrow BE \dashv \dashv [4] \\
 [A; \dashv \dashv] \rightarrow a[C; \dashv \dashv] & [B; e \dashv] \rightarrow bcde \dashv \dashv [3] \\
 [A; \dashv \dashv] \rightarrow aC \dashv \dashv [2] & [E; \dashv \dashv] \rightarrow e \dashv \dashv [6] \\
 [D; \dashv \dashv] \rightarrow \dashv \dashv [5]
 \end{array} \tag{16}$$

It is, of course, unnecessary to list productions which cannot be reached from  $[S; \dashv \dashv]$ . Condition (15) is immediate; one may see an intimate connection between (16) and the tree (3).

Our second method for testing the  $LR(k)$  condition is related to the first but it is perhaps more natural and at the same time it gives a method for parsing the grammar  $G$  if it is indeed  $LR(k)$ . The parsing method is complicated by the appearance of  $\epsilon$  in the grammar, when it becomes necessary to be very careful deciding when to insert an intermediate symbol  $A$  corresponding to the production  $A \rightarrow \epsilon$ . To treat this condition properly we will define  $H'_k(\sigma)$  to be the same as  $H_k(\sigma)$  except omitting all derivations that contain a step of the form

$$A\omega \rightarrow \omega,$$

i.e., when an intermediate as the *initial character* is replaced by  $\epsilon$ . This means we are avoiding derivation trees whose handle is an empty string at the extreme left. For example, in the grammar

$$S \rightarrow BC \dashv \dashv, B \rightarrow Ce, B \rightarrow \epsilon, C \rightarrow D, C \rightarrow Dc, D \rightarrow \epsilon, D \rightarrow d$$

we would have

$$\begin{aligned}
 H_3(S) = & \{ \dashv \dashv, c \dashv \dashv, ce \dashv, cec, ced, d \dashv \dashv, dce, \\
 & de \dashv, dec, ded, e \dashv \dashv, ec \dashv, ed \dashv, edc \}
 \end{aligned}$$

$$H'_3(S) = \{ dce, de \dashv, dec, ded \}.$$

As before we assume the productions of  $G$  are written in the form (11). We will also change  $G$  by introducing a new intermediate  $S_0$  and adding a "zeroth" production

$$S_0 \rightarrow S \dashv^k \tag{16}$$