

$$\begin{array}{ll}
[S; \vdash \vdash] \rightarrow [A; \vdash \vdash] & [C; \vdash \vdash] \rightarrow [B; e \vdash] \\
[S; \vdash \vdash] \rightarrow A[D; \vdash \vdash] & [C; \vdash \vdash] \rightarrow B[E; \vdash \vdash] \\
[S; \vdash \vdash] \rightarrow AD \vdash \vdash [1] & [C; \vdash \vdash] \rightarrow BE \vdash \vdash [4] \\
[A; \vdash \vdash] \rightarrow a[C; \vdash \vdash] & [B; e \vdash] \rightarrow bcde \vdash [3] \\
[A; \vdash \vdash] \rightarrow aC \vdash \vdash [2] & [E; \vdash \vdash] \rightarrow e \vdash \vdash [6] \\
[D; \vdash \vdash] \rightarrow \vdash \vdash [5]
\end{array} \tag{16}$$

It is, of course, unnecessary to list productions which cannot be reached from $[S; \vdash \vdash]$. Condition (15) is immediate; one may see an intimate connection between (16) and the tree (3).

Our second method for testing the $LR(k)$ condition is related to the first but it is perhaps more natural and at the same time it gives a method for parsing the grammar \mathcal{G} if it is indeed $LR(k)$. The parsing method is complicated by the appearance of ϵ in the grammar, when it becomes necessary to be very careful deciding when to insert an intermediate symbol A corresponding to the production $A \rightarrow \epsilon$. To treat this condition properly we will define $H'_k(\sigma)$ to be the same as $H_k(\sigma)$ except omitting all derivations that contain a step of the form

$$A\omega \rightarrow \omega,$$

i.e., when an intermediate as the *initial character* is replaced by ϵ . This means we are avoiding derivation trees whose handle is an empty string at the extreme left. For example, in the grammar

$$S \rightarrow BC \vdash \vdash \vdash, B \rightarrow Ce, B \rightarrow \epsilon, C \rightarrow D, C \rightarrow Dc, D \rightarrow \epsilon, D \rightarrow d$$

we would have

$$\begin{aligned}
H_3(S) = \{ & \vdash \vdash \vdash, c \vdash \vdash, ce \vdash, cec, ced, d \vdash \vdash, dce, \\
& de \vdash, dec, ded, e \vdash \vdash, ec \vdash, ed \vdash, edc \}
\end{aligned}$$

$$H'_3(S) = \{dce, de \vdash, dec, ded\}.$$

As before we assume the productions of \mathcal{G} are written in the form (11). We will also change \mathcal{G} by introducing a new intermediate S_0 and adding a "zeroth" production

$$S_0 \rightarrow S \vdash^k \tag{16}$$