not contain useless productions, i.e., for any A in I there are terminal strings α, β, γ such that $S \equiv> \alpha A\gamma \Rightarrow \alpha\beta\gamma$.

The first method of testing is to construct another grammar $\mathfrak{F}$ which reflects all possible configurations of a handle and k characters to its right. The intermediate symbols of $\mathfrak{F}$ will be $[A;\alpha]$, where α is a k-letter string on $T \cup \{\dashv\}$; and also $[p]$, where p is the number of production in $\mathcal{G}$. The terminal symbols of $\mathfrak{F}$ will be $I \cup T \cup \{\dashv\}$.

For convenience we define $H_k(\sigma)$ to be the set of all k-letter strings β over $T \cup \{\dashv\}$ such that $\sigma \equiv> \beta\gamma$ with respect to $\mathcal{G}$ for some γ; this is the set of all possible initial strings of length k derivable from σ.

Let the pth production of $\mathcal{G}$ be

$$A_p \rightarrow X_{p1} \cdots X_{pn_p},\ 1 \leqq p \leqq s,\ n_p \geqq 0. \tag{11}$$

We construct all productions of the following form:

$$[A_p\,;\alpha] \rightarrow X_{p1} \cdots X_{p(j-1)}[X_{pj}\,;\beta] \tag{12}$$

where $1 \leqq j \leqq n_p$, X_{pj} is intermediate, and α, β are k-letter strings over $T \cup \{\dashv\}$ with β in $H_k(X_{p(j+1)} \cdots X_{pn_p}\alpha)$. Add also the productions

$$[A_p\,;\alpha] \rightarrow X_{p1} \cdots X_{pn_p}\alpha[p] \tag{13}$$

It is now easy to see that with respect to $\mathfrak{F}$,

$$[S;\ \dashv^k] \Rightarrow X_1 \cdots X_n X_{n+1} \cdots X_{n+k}[p] \tag{14}$$

if and only if there exists a k-sentential form $X_1 \cdots X_n X_{n+1} \cdots X_{n+k} Y_1 \cdots Y_u$ with handle (n, p) and with $X_{n+1} \cdots Y_u$ not intermediates. Therefore by definition, $\mathcal{G}$ will be LR(k) if and only if $\mathfrak{F}$ satisfies the following property:

$$[S;\ \dashv^k] \Rightarrow \theta[p] \text{ and } [S;\ \dashv^k] \Rightarrow \theta\varphi[q] \text{ implies } \varphi = \epsilon \text{ and } p = q. \tag{15}$$

But $\mathfrak{F}$ is a *regular* grammar, and well-known methods exist for testing condition (15) in regular grammars. (Basically one first transforms $\mathfrak{F}$ so that all of its productions have the form $Q_i \rightarrow aQ_j$, and then if $Q_0 = [S;\ \dashv^k]$, one can systematically prepare a list of all pairs (i, j) such that there exists a string α for which $Q_0 \Rightarrow \alpha Q_i$ and $Q_0 \Rightarrow \alpha Q_j$.)

When $k = 2$, the grammar $\mathfrak{F}$ corresponding to (2) is