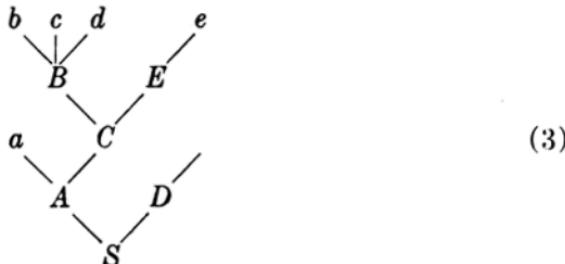


For example, the grammar

$$S \rightarrow AD, A \rightarrow aC, B \rightarrow bcd, C \rightarrow BE, D \rightarrow \epsilon, E \rightarrow e \quad (2)$$

defines the language consisting of the single string "abcde". Any sentential form in a grammar may be given at least one representation as the leaves of a *derivation tree* or "parse diagram"; for example, there is but one derivation tree for the string abcde in the grammar (2), namely,



(The root of the derivation tree is  $S$ , and the branches correspond in an obvious manner to applications of productions.) A grammar is said to be *unambiguous* if every sentential form has a unique derivation tree. The grammar (2) is clearly unambiguous, even though there are several different sequences of derivations possible, e.g.

$$S \rightarrow AD \rightarrow aCD \rightarrow aBED \rightarrow abcdED \rightarrow abcdeD \rightarrow abcde \quad (4)$$

$$S \rightarrow AD \rightarrow A \rightarrow aC \rightarrow aBE \rightarrow aBe \rightarrow abcde \quad (5)$$

In order to avoid the unimportant difference between sequences of derivations corresponding to the same tree, we can stipulate a particular order, such as insisting that we always substitute for the leftmost intermediate (as done in (4)) or the rightmost one (as in (5)).

In practice, however, we must start with the terminal string  $abcde$  and try to reconstruct the derivation leading back to  $S$ , and that changes our outlook somewhat. Let us define the *handle* of a tree to be the leftmost set of adjacent leaves forming a complete branch; in (3) the handle is  $bcd$ . In other words, if  $X_1, X_2, \dots, X_k$  are the leaves of the tree (where each  $X_i$  is an intermediate or terminal character or  $\epsilon$ ), we look for the smallest  $k$  such that the tree has the form

