Supplementary Document to XXXXXXXX

General Deformation

The theory of deformations that was proposed in the main manuscript for GR, the metric in Einstein-Hilbert action is replaced with a function of $h_{ab}$and $\pounds_t h_{ab}$,

$$h_{ab} \to H_{ab}\left(h_{cd}, \pounds_t h_{cd}\right)$$

The issue with such a deformation is the lack of new coordinates to swallow up the acceleration tensor field,$\ddot{h}_{ab}$, which can be responsible for producing Ostrogradskian instabilities. According to Buchbinder *et.al* [**?**], there are an infinite number of canonically equivalent ways of going about choosing this new coordinate. The condition is that its time derivative is the only term in the lagrangian that will contain $\ddot{h}_{ab}$. This limits us to two choices.

Choice a): The "Greater Metric" $H_{ab}\left(h_{cd}, \pounds_t h\right)$

In this case, the new coordinate will be the metric H_{ab} itself. This means that h_{ab} will only appear in the theory due to contractions of any other tensor. This choice is not appropriate as the resulting theory will not be infintessimally connected to GR in some perturbative expansion of $H_{ab} = h_{ab} + \epsilon f\left(h_{ab}, \pounds_t h_{ab}\right) + \sum_{i=2}^{\infty} \epsilon^i f_{(i)}\left(h_{ab}, \pounds_t h_{ab}\right)$, where ϵ is a small parameter. This means that one will not reduce back to GR when the deformations are surpressed. This will result in a radically different set of equations of motions where a the usual gravitational dynamics are "locked" in the dynamics of H_{ab}. Therefore such a choice of a new coordinate is not physically relevant.

Choice b): The aspect of H_{ab} that depends on $\pounds_t h_{ab}$

The main isue is that a coordinate J_{ab} is required such that its time-derivative is as below,

$$\dot{J}_{ab} = \frac{\partial H_{ab}}{\partial \dot{h}_{cd}} \ddot{h}_{cd}$$

However the only way that such a term appears is through the time-derivative of H_{ab}, which was ruled out as a choice of coordinate in "Choice a" . The only resolution left to us is to think of the form that H_{ab} comes in. A taylor expansion of it has the form,

$$\begin{aligned} H_{ab}\left(h_{cd}, \pounds_t h_{cd}\right) &= h_{ab} + a\pounds_t h_{ab} + b\pounds_t h_{ab} h^{ab} + ch h_{ab} + \cdots \\ &= \left(h_{ab} + ch h_{ab} + \mathcal{O}\left(h^3\right)\right) + \left(a\pounds_t h_{ab} + b\pounds_t h_{ab} h^{ab} + \mathcal{O}\left(\pounds h^2\right)\right) \\ &= H^1_{ab}\left(h_{cd}\right) + H^2_{ab}\left(h_{cd}, \pounds_t h_{cd}\right) \end{aligned}$$

where the parts of H_{ab} that contain time-derivative contributions are subtracted off in function space to give the functions above. We can then define the new coordinate to be

$$J_{ab} = H^2_{ab}\left(h_{cd}, \pounds_t h_{cd}\right)$$

Throughout the expression, there are terms like

$$\frac{\partial H_{ab}}{\partial h_{cd}} = \frac{\partial H^1_{ab}}{\partial h_{cd}} + \frac{\partial J_{ab}}{\partial h_{cd}}$$

The other term, $H^1_{ab}\left(h_{cd}\right)$ will always be linear to h_{ab}, but have terms that are proportional to some spacetime function. Hence it represents a conformal scaling of h_{ab} and can be decomposed to h_{ab} itself and the rest of the conformal scaling, which will be denoted as $\tilde{H}_{ab}\left(h_{cd}\right)$,

$$H^1_{ab} = h_{ab} + \tilde{H}_{ab}\left(h, h_{ab}\right)$$

Derivation of the Hamiltonian Density

To obtain the general constraints a bunch of work must be done with the deformed theory. We start with the Lagrangian $\int d^4x \sqrt{-g}R$, and suppose the variable transformation

$$h_{ab} \to H_{ab}\left(h_{cd}, \pounds_t h_{cd}\right) = h_{ab} + \tilde{H}_{ab}\left(h_{cd}\right) + \tilde{J}_{ab}\left(h_{cd}, \pounds_t h_{cd}\right)$$

Inserting this into the Gauss-Codassi equations, and not the volume element[1] $\sqrt{-g} = N\sqrt{h}$, is the next step,

$$R = R^{(3)} + K_{ab}K^{ab} - K^2 - 2\nabla_\alpha\left(n^\alpha{}_{;\beta}n^\beta - n^\beta{}_{;\alpha}n^\alpha\right)$$

Reduction 1): Determine the 3-Ricci Scalar term

The 3-metric will depend on h_{ab} and $\pounds_t h_{ab}$ through H_{ab}, as seen below

$$\begin{aligned} R^{(3)}\left(h_{ab}\right) &\to \tilde{R}^{(3)}\left(H_{ab}\right) \\ &= 2H^{ab}\left(\Gamma^c{}_{a[b,c]} + \Gamma^d_{a[b}\Gamma^c_{c]d}\right) \\ &= R_0^{(3)}\left(h_{ab}\right) + R_1^{(3)}\left(h_{ab}, J_{ab}\right) \end{aligned}$$

The Christoffel symbols are as below, where there will be those defined purely in terms of either h_{ab},$\tilde{H}_{ab}$, J_{ab} or a mix of them,

$$\begin{aligned} \Gamma^a{}_{bc} &= \frac{1}{2}H^{ae}\left(H_{eb,c} + H_{ec,b} - H_{bc,e}\right) \\ &= \frac{1}{2}\left(h^{ae} + \tilde{H}^{ae} + J^{ae}\right)\left(h_{eb,c} + h_{ec,b} - h_{bc,e} + \tilde{H}_{eb,c} + \tilde{H}_{ec,b} - \tilde{H}_{bc,e} + J_{eb,c} + J_{ec,b} - J_{bc,e}\right) \\ &= \Gamma^a{}_{bc} + \frac{1}{2}\left(\tilde{H}^a{}_{b,c} + \tilde{H}^a{}_{c,b} - \tilde{H}_{bc}{}^a\right) + \frac{1}{2}\left(J^a{}_{b,c} + J^a{}_{c,b} - J_{bc}{}^a\right) \\ &+ \tilde{\Gamma}^a{}_{bc} + \tilde{H}^{ae}\left(h_{eb,c} + h_{ec,b} - h_{bc,e}\right) + \tilde{H}^{ae}\left(J_{eb,c} + J_{ec,b} - J_{bc,e}\right) \\ &+ \left(\Gamma^J\right)^a{}_{bc} + J^{ae}\left(h_{eb,c} + h_{ec,b} - h_{bc,e}\right) + J^{ae}\left(\tilde{H}_{eb,c} + \tilde{H}_{ec,b} - \tilde{H}_{bc,e}\right) \\ &= \Gamma^a{}_{bc} + \left(\Gamma^H\right)^a{}_{bc} + \left(\Gamma^J\right)^a{}_{bc} \end{aligned}$$

$$\begin{aligned} \left(\Gamma^H\right)^a{}_{bc} &= \tilde{\Gamma}^a{}_{bc} + \frac{1}{2}\left(\tilde{H}^a{}_{b,c} + \tilde{H}^a{}_{c,b} - \tilde{H}_{bc}{}^a\right) + \tilde{H}^{ae}\left(h_{eb,c} + h_{ec,b} - h_{bc,e}\right) \\ \left(\Gamma^J\right)^a{}_{bc} &= \left(\Gamma^J\right)^a{}_{bc} + \frac{1}{2}\left(J^a{}_{b,c} + J^a{}_{c,b} - J_{bc}{}^a\right) + \tilde{H}^{ae}\left(J_{eb,c} + J_{ec,b} - J_{bc,e}\right) \\ &\quad + J^{ae}\left(h_{eb,c} + h_{ec,b} - h_{bc,e}\right) + J^{ae}\left(\tilde{H}_{eb,c} + \tilde{H}_{ec,b} - \tilde{H}_{bc,e}\right) \end{aligned}$$

These terms will provide gradient like contributions to the Lagrangians for both J and h.

Reduction 2): Extrinsic Curvature Terms

The extrinsic curvature changes according to the deformation in the following way,

[1] Deforming the volume element would imply some deeper geometric that can change the speed of light, which is not desired.

$$
\begin{aligned}
K_{ab} &\rightarrow \frac{1}{2N}\left(\pounds_t H_{ab}\left(h_{ab}, \pounds_t h_{ab}\right) - \nabla_a N_b - \nabla_b N_a\right) \\
&= \frac{1}{2N}\left(\frac{\partial H_{ab}}{\partial h_{cd}}\pounds_t h_{cd} + \frac{\partial H_{ab}}{\partial \pounds_t h_{cd}}\pounds_t\pounds_t h_{cd} - \nabla_a N_b - \nabla_b N_a\right) \\
&= \frac{1}{2N}\left(\frac{\partial H_{ab}}{\partial h_{cd}}\pounds_t h_{cd} + \frac{\partial H_{ab}}{\partial \pounds_t h_{cd}}\pounds_t\pounds_t h_{cd} - \nabla_a N_b - \nabla_b N_a\right) \\
&= \frac{1}{2N}\left(\pounds_t h_{ab} - \nabla_a N_b - \nabla_b N_a\right) \\
&+\frac{1}{2N}\left(\frac{\partial \tilde{H}_{ab}}{\partial h_{cd}}\pounds_t h_{cd}\right) \\
&+\frac{1}{2N}\left(\frac{\partial J_{ab}}{\partial h_{cd}}\pounds_t h_{cd} + \frac{\partial J_{ab}}{\partial \pounds_t h_{cd}}\pounds_t\pounds_t h_{cd}\right) \\
&=\frac{1}{2N}\left(K_{ab} + \frac{\partial \tilde{H}_{ab}}{\partial h_{cd}}\pounds_t h_{cd} + \frac{\partial J_{ab}}{\partial h_{cd}}\pounds_t h_{cd} + \frac{\partial J_{ab}}{\partial \pounds_t h_{cd}}\pounds_t\pounds_t h_{cd}\right)
\end{aligned}
$$

$$
\begin{aligned}
\tilde{K}^{ab} &\rightarrow \frac{1}{2N}\left(h^{ac}h^{bd}\pounds_t H_{cd}\left(h, \pounds_t h\right) - \nabla^a N^b - \nabla^b N^a\right) \\
&\rightarrow \frac{1}{2N}\left(h^{ac}h^{bd}\frac{\partial H_{cd}}{\partial h_{ef}}\pounds_t h_{ef} + h^{ac}h^{bd}\frac{\partial H_{ab}}{\partial \pounds_t h_{ef}}\pounds_t\pounds_t h_{ef} - \nabla^a N^b - \nabla^b N^a\right) \\
&= \frac{1}{2N}\left(\pounds_t h^{ab} - \nabla^a N^b - \nabla^b N^a\right) \\
&+ \frac{1}{2N}\left(h^{ac}h^{bd}\frac{\partial \tilde{H}_{cd}}{\partial h_{ef}}\pounds_t h_{ef}\right) \\
&+ \frac{1}{2N}\left(h^{ac}h^{bd}\frac{\partial J_{cd}}{\partial h_{ef}}\pounds_t h_{ef} + h^{ac}h^{bd}\frac{\partial J_{cd}}{\partial \pounds_t h_{ef}}\pounds_t\pounds_t h_{ef}\right)
\end{aligned}
$$

which gives the K^2 terms,

$$
\begin{aligned}
\tilde{K}_{ab}\tilde{K}^{ab} &= \frac{1}{4N^2}\left[\left(2NK_{ab} + \frac{\partial \tilde{H}_{ab}}{\partial h_{gh}}\pounds_t h_{gh} + \frac{\partial J_{ab}}{\partial h_{gh}}\pounds_t h_{gh} + \frac{\partial J_{ab}}{\partial \pounds_t h_{gh}}\pounds_t\pounds_t h_{gh}\right)\left(2NK^{ab} + h^{ac}h^{bd}\frac{\partial \tilde{H}_{cd}}{\partial h_{ef}}\pounds_t h_{ef} + h^{ac}h^{bd}\frac{\partial J_{cd}}{\partial h_{ef}}\pounds_t h_{ef} + h^{ac}h^{bd}\frac{\partial J_{cd}}{\partial \pounds_t h_{ef}}\pounds_t\pounds_t h_{ef}\right)\right] \\
&= K_{ab}K^{ab} \\
&+\frac{1}{4N^2}\left[+2N\frac{\partial \tilde{H}_{ab}}{\partial h_{gh}}\pounds_t h_{gh}K^{ab} + 2N\frac{\partial J_{ab}}{\partial h_{gh}}\pounds_t h_{gh}K^{ab} + 2N\frac{\partial J_{ab}}{\partial \pounds_t h_{gh}}\pounds_t\pounds_t h_{gh}K^{ab}\right] \\
&+\frac{1}{4N^2}\left[2NK_{ab}\frac{\partial \tilde{H}^{ab}}{\partial h_{ef}}\pounds_t h_{ef} + \frac{\partial \tilde{H}^{cd}}{\partial h_{gh}}\frac{\partial \tilde{H}_{cd}}{\partial h_{ef}}\pounds_t h_{gh}\pounds_t h_{ef} + \frac{\partial J_{ab}}{\partial h_{gh}}\frac{\partial \tilde{H}^{ab}}{\partial h_{ef}}\pounds_t h_{ef}\pounds_t h_{gh} + \frac{\partial \tilde{H}_{ab}}{\partial h_{gh}}\pounds_t h_{gh}\frac{\partial J_{ab}}{\partial \pounds_t h_{gh}}\pounds_t\pounds_t h_{gh}\right] \\
&+\frac{1}{4N^2}\left[2NK^{ab}\frac{\partial J_{ab}}{\partial h_{ef}}\pounds_t h_{ef} + \frac{\partial \tilde{H}^{ab}}{\partial h_{gh}}\frac{\partial J_{ab}}{\partial h_{ef}}\pounds_t h_{ef}\pounds_t h_{gh} + h^{ac}h^{bd}\frac{\partial J_{ab}}{\partial h_{gh}}\frac{\partial J_{cd}}{\partial h_{ef}}\pounds_t h_{ef}\pounds_t h_{gh} + h^{ac}h^{bd}\frac{\partial J_{ab}}{\partial \pounds_t h_{gh}}\frac{\partial J_{cd}}{\partial h_{ef}}\pounds_t h_{ef}\pounds_t\pounds_t h_{gh}\right] \\
&+\frac{1}{4N^2}\left[2NK^{ab}\frac{\partial J_{ab}}{\partial \pounds_t h_{ef}}\pounds_t\pounds_t h_{ef} + \frac{\partial \tilde{H}^{ab}}{\partial h_{gh}}\frac{\partial J_{ab}}{\partial \pounds_t h_{ef}}\pounds_t\pounds_t h_{ef}\pounds_t h_{gh} + \frac{\partial J^{ab}}{\partial \pounds_t h_{gh}}\pounds_t\pounds_t h_{gh}\frac{\partial J_{ab}}{\partial h_{gh}}\pounds_t h_{gh} + \frac{\partial J^{ab}}{\partial \pounds_t h_{gh}}\pounds_t\pounds_t h_{gh}\frac{\partial J_{ab}}{\partial \pounds_t h_{gh}}\pounds_t\pounds_t h_{gh}\right]
\end{aligned}
$$

The Trace is deformed in the following way

$$
\tilde{K} = h^{ab}\tilde{K}_{ab} = K + \frac{1}{2N}\left(\frac{\partial \tilde{H}_a{}^a}{\partial h_{cd}}\pounds_t h_{cd} + \frac{\partial J}{\partial h_{cd}}\pounds_t h_{cd} + \frac{\partial J}{\partial \pounds_t h_{cd}}\pounds_t\pounds_t h_{cd}\right)
$$

Its square is determined to be,

$$
\begin{aligned}
&\tilde{K}^2 \\
&= \left(K + \frac{1}{2N}\left(\frac{\partial \tilde{H}_a{}^a}{\partial h_{ef}}\pounds_t h_{ef} + \frac{\partial J}{\partial h_{ef}}\pounds_t h_{ef} + \frac{\partial J}{\partial \pounds_t h_{ef}}\pounds_t\pounds_t h_{ef}\right)\right)\left(K + \frac{1}{2N}\left(\frac{\partial \tilde{H}_a{}^a}{\partial h_{cd}}\pounds_t h_{cd} + \frac{\partial J}{\partial h_{cd}}\pounds_t h_{cd} + \frac{\partial J}{\partial \pounds_t h_{cd}}\pounds_t\pounds_t h_{cd}\right)\right) \\
&= K^2 + \frac{1}{2N}\left(K\frac{\partial \tilde{H}_a{}^a}{\partial h_{cd}}\pounds_t h_{cd} + K\frac{\partial J}{\partial h_{cd}}\pounds_t h_{cd} + K\frac{\partial J}{\partial \pounds_t h_{cd}}\pounds_t\pounds_t h_{cd}\right) \\
&+ \left(\frac{1}{2N}K\frac{\partial \tilde{H}_a{}^a}{\partial h_{ef}}\pounds_t h_{ef} + \frac{1}{4N^2}\left(\frac{\partial \tilde{H}}{\partial h_{cd}}\frac{\partial \tilde{H}}{\partial h_{ef}}\pounds_t h_{ef}\pounds_t h_{cd} + \frac{\partial J}{\partial h_{cd}}\frac{\partial \tilde{H}}{\partial h_{ef}}\pounds_t h_{ef}\pounds_t h_{cd} + \frac{\partial J}{\partial \pounds_t h_{cd}}\frac{\partial \tilde{H}}{\partial h_{ef}}\pounds_t h_{ef}\pounds_t\pounds_t h_{cd}\right)\right) \\
&+ \left(\frac{1}{2N}K\frac{\partial J}{\partial h_{ef}}\pounds_t h_{ef} + \frac{1}{4N^2}\left(\frac{\partial \tilde{H}}{\partial h_{cd}}\frac{\partial J}{\partial h_{ef}}\pounds_t h_{ef}\pounds_t h_{cd} + \frac{\partial J}{\partial h_{cd}}\frac{\partial J}{\partial h_{ef}}\pounds_t h_{ef}\pounds_t h_{cd} + \frac{\partial J}{\partial \pounds_t h_{cd}}\frac{\partial J}{\partial h_{ef}}\pounds_t h_{ef}\pounds_t\pounds_t h_{cd}\right)\right) \\
&+ \frac{K}{2N}\frac{\partial J}{\partial \pounds_t h_{ef}}\pounds_t\pounds_t h_{ef} + \frac{1}{4N^2}\left(\frac{\partial \tilde{H}_a{}^a}{\partial h_{cd}}\frac{\partial J}{\partial \pounds_t h_{ef}}\pounds_t\pounds_t h_{ef}\pounds_t h_{cd} + \frac{\partial J}{\partial h_{cd}}\frac{\partial J}{\partial \pounds_t h_{ef}}\pounds_t\pounds_t h_{ef}\pounds_t h_{cd} + \frac{\partial J}{\partial \pounds_t h_{cd}}\frac{\partial J}{\partial \pounds_t h_{ef}}\pounds_t\pounds_t h_{ef}\pounds_t\pounds_t h_{cd}\right)
\end{aligned}
$$

Reduction 3): State full Lagrangian form in terms of h_{ab} and $\pounds_t h_{ab}$

The result of the computations in Reduction 3 is displayed below,

$$
\begin{aligned}
R &= R^{(3)} + K_{ab}K^{ab} - K^2 - 2\nabla_\alpha\left(n^\alpha{}_{;\beta}n^\beta - n^\beta{}_{;\alpha}n^\alpha\right) + 16\pi\mathcal{L}_{\mathrm{MG}}\left(h, \dot{h}, \ddot{h}\right) \\
&= R^{(3)} + \left(h^{ac}h^{bd} - h^{ab}h^{cd}\right)K_{ab}K_{cd} + 16\pi\mathcal{L}_{\mathrm{MG}}\left(h, \dot{h}, \ddot{h}\right) - 2\nabla_\alpha\left(n^\alpha{}_{;\beta}n^\beta - n^\beta{}_{;\alpha}n^\alpha\right)
\end{aligned}
$$

where the modifications due to the deformation are written below. We will at this point begin to note which terms are related to $\dot{J}_{ab}$. Recall that the choice of $\dot{J}_{ab}$ as the new cordinate is so that its time derivative absorbs every $\ddot{h}_{ab}$. The derivative is defined as

$$\dot{J}_{ab} = \frac{\partial J_{ab}}{\partial h_{cd}}\dot{h}_{cd} + \frac{\partial J_{ab}}{\partial \dot{h}_{cd}}\ddot{h}_{cd}$$

Therefore the modified Lagrangian is,

$$16\pi\mathcal{L}_{\mathrm{MG}}\left(h,\dot{h},\ddot{h}\right) = R^{(\tilde{H},J)} + \frac{1}{2N}\left(K\frac{\partial \tilde{H}_a{}^a}{\partial h_{cd}}\dot{h}_{cd} + K\underbrace{\left(\frac{\partial J}{\partial h_{cd}}\dot{h}_{cd} + \frac{\partial J}{\partial \dot{h}_{cd}}\ddot{h}_{cd}\right)}_{\dot{J}} + K\frac{\partial \tilde{H}_a{}^a}{\partial h_{ef}}\dot{h}_{ef} + K\underbrace{\left(\frac{\partial J}{\partial h_{ef}}\dot{h}_{ef} + \frac{\partial J}{\partial \dot{h}_{ef}}\ddot{h}_{ef}\right)}_{\dot{J}}\right)$$

$$+ \frac{1}{4N^2}\left(\frac{\partial \tilde{H}}{\partial h_{cd}}\frac{\partial \tilde{H}}{\partial h_{ef}}\dot{h}_{ef}\dot{h}_{cd} + \frac{\partial \tilde{H}}{\partial h_{ef}}\dot{h}_{ef}\underbrace{\left(\frac{\partial J}{\partial h_{cd}}\dot{h}_{cd} + \frac{\partial J}{\partial \pounds_t h_{cd}}\ddot{h}_{cd}\right)}_{\dot{J}}\right)$$

$$+ \frac{1}{4N^2}\left(\frac{\partial \tilde{H}}{\partial h_{cd}}\dot{h}_{cd}\underbrace{\left(\frac{\partial J}{\partial h_{ef}}\dot{h}_{ef} + \frac{\partial J}{\partial \pounds_t h_{ef}}\ddot{h}_{ef}\right)}_{\dot{J}}\right)$$

$$+ \frac{1}{4N^2}\left(\underbrace{\frac{\partial J}{\partial h_{cd}}\frac{\partial J}{\partial h_{ef}}\dot{h}_{ef}\dot{h}_{cd} + \frac{\partial J}{\partial \pounds_t h_{cd}}\frac{\partial J}{\partial h_{ef}}\dot{h}_{ef}\ddot{h}_{cd} + \frac{\partial J}{\partial h_{cd}}\frac{\partial J}{\partial \pounds_t h_{ef}}\ddot{h}_{ef}\dot{h}_{cd} + \frac{\partial J}{\partial \pounds_t h_{cd}}\frac{\partial J}{\partial \pounds_t h_{ef}}\ddot{h}_{ef}\ddot{h}_{cd}}_{\dot{J}^2}\right)$$

$$+ \frac{1}{4N^2}\left[2N\frac{\partial \tilde{H}_{ab}}{\partial h_{gh}}\dot{h}_{gh}K^{ab} + 2NK^{ab}\underbrace{\left(\frac{\partial J_{ab}}{\partial h_{gh}}\dot{h}_{gh} + \frac{\partial J_{ab}}{\partial \pounds_t h_{gh}}\dot{h}_{gh}\right)}_{\dot{J}_{ab}}\right]$$

$$+ \frac{1}{4N^2}\left[2NK_{ab}\frac{\partial \tilde{H}^{ab}}{\partial h_{ef}}\dot{h}_{ef} + \frac{\partial \tilde{H}^{cd}}{\partial h_{gh}}\frac{\partial \tilde{H}_{cd}}{\partial h_{ef}}\dot{h}_{gh}\dot{h}_{ef} + \frac{\partial \tilde{H}^{ab}}{\partial h_{ef}}\dot{h}_{ef}\underbrace{\left(\frac{\partial J_{ab}}{\partial h_{gh}}\dot{h}_{gh} + \frac{\partial J_{ab}}{\partial \pounds_t h_{gh}}\ddot{h}_{gh}\right)}_{\dot{J}_{ab}}\right]$$

$$+ \frac{1}{4N^2}\left[+\frac{\partial \tilde{H}^{ab}}{\partial h_{gh}}\pounds_t h_{gh}\underbrace{\left(\frac{\partial J_{ab}}{\partial h_{ef}}\dot{h}_{ef} + \frac{\partial J_{ab}}{\partial \pounds_t h_{ef}}\ddot{h}_{ef}\right)}_{\dot{J}_{ab}} + 2NK^{ab}\underbrace{\left(\frac{\partial J_{ab}}{\partial h_{ef}}\dot{h}_{ef} + \frac{\partial J_{ab}}{\partial \pounds_t h_{ef}}\ddot{h}_{ef}\right)}_{\dot{J}_{ab}}\right]$$

$$+ \frac{1}{4N^2}\left[\underbrace{\frac{\partial J^{ab}}{\partial h_{ef}}\dot{h}_{ef}\frac{\partial J_{ab}}{\partial h_{gh}}\dot{h}_{gh} + \frac{\partial J^{ab}}{\partial h_{ef}}\dot{h}_{ef}\frac{\partial J_{ab}}{\partial \pounds_t h_{gh}}\ddot{h}_{gh} + \frac{\partial J^{ab}}{\partial \pounds_t h_{gh}}\ddot{h}_{gh}\frac{\partial J_{ab}}{\partial h_{gh}}\dot{h}_{gh} + \frac{\partial J^{ab}}{\partial \pounds_t h_{gh}}\ddot{h}_{gh}\frac{\partial J_{ab}}{\partial \pounds_t h_{gh}}\ddot{h}_{gh}}_{h^{ac}h^{bd}\dot{J}_{ab}\dot{J}_{cd}}\right]$$

$$= R^{(\tilde{H},J)}$$

$$+\dot{J}\left(\frac{K}{N} + \frac{1}{4N^2}\left(2\frac{\partial \tilde{H}}{\partial h_{ef}}\dot{h}_{ef}\right)\right) + \frac{1}{4N^2}\dot{J}^2$$

$$+\frac{1}{2N^2}\left(2NK^{ab} + \frac{\partial \tilde{H}^{ab}}{\partial h_{ef}}\dot{h}_{ef}\right)\dot{J}_{ab} + \frac{1}{4N^2}h^{ac}h^{bd}\dot{J}_{ab}\dot{J}_{cd}$$

$$+\dot{h}_{cd}\left(\frac{K}{N}\frac{\partial \tilde{H}}{\partial h_{cd}} + \frac{1}{N}\frac{\partial \tilde{H}_{ab}}{\partial h_{cd}}K^{ab}\right) + \frac{1}{4N^2}\left(\frac{\partial \tilde{H}}{\partial h_{cd}}\frac{\partial \tilde{H}}{\partial h_{ef}} + \frac{\partial \tilde{H}^{cd}}{\partial h_{cd}}\frac{\partial \tilde{H}_{cd}}{\partial h_{ef}}\right)\dot{h}_{ef}\dot{h}_{cd}$$

Reduction 4): Define Conjugate Momenta

For geometric reasons, we keep the original definition of $\dot{h}_{ab}$ in terms of the old extrinsic curvature K_{ab}, in the Legendre transformation in (**??**)

$$\begin{aligned}\dot{h}_{ab} &= 2NK_{ab} + N_{a|b} + N_{b|a} \\ &= 2NK_{ab} + N_{a|b} + N_{b|a}\end{aligned}$$

The new momenta conjugate to £h_{ab}, is,

$$\begin{aligned}\pi^{ab} &= \frac{1}{16\pi}\frac{\partial\left(16\pi\sqrt{-g}\mathcal{L}_G\right)}{\partial\dot{h}_{ab}} = \frac{1}{16\pi}\frac{\partial\left(16\pi\sqrt{-g}\mathcal{L}_G\right)}{\partial K_{cd}}\frac{\partial\left(K_{cd}\right)}{\partial\dot{h}_{ab}} \\ &= \frac{1}{16\pi}N\sqrt{h}\left(\left(h^{ac}h^{bd} - h^{ab}h^{cd}\right)K_{ab} + \frac{\partial 16\pi\mathcal{L}_{\text{MG}}\left(h,\dot{h},\ddot{h}\right)}{\partial\dot{h}_{ij}}\frac{\partial\dot{h}_{ij}}{\partial K_{cd}}\right)\frac{\partial\left(K_{cd}\right)}{\partial\dot{h}_{ab}} \\ &= \frac{1}{16\pi}N\sqrt{h}\left(\left(K^{cd} - h^{cd}K\right) + \frac{\partial 16\pi\mathcal{L}_{\text{MG}}\left(h,\dot{h},\ddot{h}\right)}{\partial\dot{h}_{ab}}\right)\end{aligned}$$

where the modified Lagrangian is

$$\begin{aligned}16\pi\mathcal{L}_{\text{MG}}\left(h,\dot{h},\ddot{h}\right) &= R^{\left(\tilde{H},J\right)} \\ &\quad+\dot{J}\left(\frac{K}{N} + \frac{1}{4N^2}\left(2\frac{\partial\tilde{H}}{\partial h_{ef}}\dot{h}_{ef}\right)\right) + \frac{1}{4N^2}\dot{J}^2 \\ &\quad+\frac{1}{2N^2}\left(2NK^{ab} + \frac{\partial\tilde{H}^{ab}}{\partial h_{ef}}\dot{h}_{ef}\right)\dot{J}_{ab} + \frac{1}{4N^2}h^{ac}h^{bd}\dot{J}_{ab}\dot{J}_{cd} \\ &\quad+\dot{h}_{cd}\left(\frac{K}{N}\frac{\partial\tilde{H}}{\partial h_{cd}} + \frac{1}{N}\frac{\partial\tilde{H}_{ab}}{\partial h_{cd}}K^{ab}\right) + \frac{1}{4N^2}\left(\frac{\partial\tilde{H}}{\partial h_{cd}}\frac{\partial\tilde{H}}{\partial h_{ef}} + \frac{\partial\tilde{H}^{cd}}{\partial h_{gh}}\frac{\partial\tilde{H}_{cd}}{\partial h_{ef}}\right)\dot{h}_{ef}\dot{h}_{cd}\end{aligned}$$

Its derivative with respect to $\dot{h}_{ab}$ is,

$$
\begin{aligned}
\frac{\partial 16\pi\mathcal{L}_{\mathrm{MG}}\left(h,\dot{h},\ddot{h}\right)}{\partial\dot{h}_{ab}} &= h^{cd}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}_{ab}}\left(\frac{K}{N}+\frac{1}{4N^2}\left(2\frac{\partial\tilde{H}}{\partial h_{ef}}\dot{h}_{ef}\right)\right)+\dot{J}\left(\frac{h^{cd}}{N}\frac{\partial K_{cd}}{\partial\dot{h}_{ab}}+\frac{1}{4N^2}\left(2\frac{\partial\tilde{H}}{\partial h_{ab}}\right)\right)\\
&+\frac{1}{4N^2}\left[h^{cd}h^{ef}\left(\frac{\partial\dot{J}_{cd}}{\partial\dot{h}_{ab}}\dot{J}_{ef}+\dot{J}_{cd}\frac{\partial\dot{J}_{ef}}{\partial\dot{h}_{ab}}\right)\right]\\
&+\frac{1}{2N^2}\left(2NK^{cd}+\frac{\partial\tilde{H}^{cd}}{\partial h_{ef}}\dot{h}_{ef}\right)\frac{\partial\dot{J}_{cd}}{\partial\dot{h}_{ab}}+\frac{1}{2N^2}\left(2N\frac{\partial K^{cd}}{\partial\dot{h}_{ab}}+\frac{\partial\tilde{H}^{cd}}{\partial h_{ab}}\right)\dot{J}_{ab}\\
&+\frac{1}{4N^2}h^{ce}h^{df}\left(\frac{\partial\dot{J}_{cd}}{\partial\dot{h}_{ab}}\dot{J}_{ef}+\dot{J}_{cd}\frac{\partial\dot{J}_{ef}}{\partial\dot{h}_{ab}}\right)\\
&+\left(\frac{K}{N}\frac{\partial\tilde{H}}{\partial h_{ab}}+\frac{1}{N}\frac{\partial\tilde{H}_{cd}}{\partial h_{ab}}K^{cd}\right)+\dot{h}_{cd}\left(\frac{1}{N}h^{ef}\frac{\partial K_{ef}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}}{\partial h_{cd}}+\frac{1}{N}\frac{\partial\tilde{H}_{ef}}{\partial h_{cd}}\frac{\partial K^{ef}}{\partial\dot{h}_{ab}}\right)\\
&+\frac{1}{4N^2}\left(\frac{\partial\tilde{H}}{\partial h_{cd}}\frac{\partial\tilde{H}}{\partial h_{ef}}+\frac{\partial\tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial\tilde{H}_{\tilde{c}\tilde{d}}}{\partial h_{ef}}\right)\left(\delta^a_e\delta^b_f\dot{h}_{cd}+\dot{h}_{ef}\delta^a_c\delta^b_d\right)\\
&=\frac{1}{2N^2}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\left(2NK+\frac{\partial\tilde{H}}{\partial h_{ef}}\dot{h}_{ef}\right)+\frac{1}{2N^2}\dot{J}\left(h^{ab}+\left(\frac{\partial\tilde{H}}{\partial h_{ab}}\right)+\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\right)\\
&+\frac{1}{2N^2}\dot{h}_{cd}\left(h^{ab}\frac{\partial\tilde{H}}{\partial h_{cd}}+\frac{\partial\tilde{H}^{ab}}{\partial h_{cd}}+\frac{\partial\tilde{H}}{\partial h_{cd}}\frac{\partial\tilde{H}}{\partial h_{ab}}+\frac{\partial\tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial\tilde{H}_{\tilde{c}\tilde{d}}}{\partial h_{ab}}\right)\\
&+\frac{1}{2N^2}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}_{ab}}\left(2NK^{cd}+\frac{\partial\tilde{H}^{cd}}{\partial h_{ef}}\dot{h}_{ef}\right)+\frac{1}{2N^2}\left(\dot{J}_{ab}+\frac{\partial\tilde{H}^{cd}}{\partial h_{ab}}\dot{J}_{cd}+\frac{\partial\dot{J}_{cd}}{\partial\dot{h}_{ab}}\dot{J}^{cd}\right)\\
&+\frac{1}{2N^2}\left(2NK\frac{\partial\tilde{H}}{\partial h_{ab}}+2NK^{cd}\frac{\partial\tilde{H}_{cd}}{\partial h_{ab}}\right)
\end{aligned}
$$

This finally gives,

$$
\begin{aligned}
\pi^{ab} &= \underbrace{\frac{1}{16\pi}\sqrt{h}\left(K^{cd}-h^{cd}K\right)}_{\pi_0^{ab}}\\
&+\frac{\sqrt{h}}{32\pi N}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}_{ab}}\left(2NKh^{cd}+2NK^{cd}\right)+\frac{\sqrt{h}}{32\pi N}\dot{J}\left(h^{ab}+\left(\frac{\partial\tilde{H}}{\partial h_{ab}}\right)+\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\right)\\
&+\frac{\sqrt{h}}{32\pi N}\dot{h}_{cd}\left(h^{ab}\frac{\partial\tilde{H}}{\partial h_{cd}}+\frac{\partial\tilde{H}^{ab}}{\partial h_{cd}}+\frac{\partial\tilde{H}}{\partial h_{cd}}\frac{\partial\tilde{H}}{\partial h_{ab}}+\frac{\partial\tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial\tilde{H}_{\tilde{c}\tilde{d}}}{\partial h_{ab}}\right)\\
&+\frac{\sqrt{h}}{32\pi N}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}_{ab}}\left(h^{cd}\frac{\partial\tilde{H}}{\partial h_{ef}}\dot{h}_{ef}+\frac{\partial\tilde{H}^{cd}}{\partial h_{ef}}\dot{h}_{ef}\right)+\frac{\sqrt{h}}{32\pi N}\left(\dot{J}_{ab}+\frac{\partial\tilde{H}^{cd}}{\partial h_{ab}}\dot{J}_{cd}+\frac{\partial\dot{J}_{cd}}{\partial\dot{h}_{ab}}\dot{J}^{cd}\right)\\
&+\frac{\sqrt{h}}{32\pi N}\left(2NK\frac{\partial\tilde{H}}{\partial h_{ab}}+2NK^{cd}\frac{\partial\tilde{H}_{cd}}{\partial h_{ab}}\right)
\end{aligned}
$$

The momentum conjugate to J^{ab} is a bit easier to calculate and can be determined almost from inspection,

$$
\begin{aligned}
\pi_J^{ab} &= \frac{1}{16\pi}\frac{\partial\left(16\pi\sqrt{-g}\mathcal{L}_G\right)}{\partial\dot{J}_{ab}}=\frac{1}{16\pi}\frac{\partial\left(16\pi\sqrt{-g}\mathcal{L}_{\mathrm{MG}}\right)}{\partial\dot{J}_{ab}}\\
&=\frac{\sqrt{h}}{32\pi N}\left(h^{ab}\frac{\partial\tilde{H}}{\partial h_{ef}}\dot{h}_{ef}+\frac{\partial\tilde{H}^{ab}}{\partial h_{ef}}\dot{h}_{ef}\right)\\
&+\frac{\sqrt{h}}{32\pi N}2N\left(K^{ab}+h^{ab}K\right)+\frac{\sqrt{h}}{32\pi N}\left(\dot{J}^{ab}+h^{ab}\dot{J}\right)
\end{aligned}
$$

Reduction 5): Represent the Hamiltonian form.

To proceed, we must gather all the terms. We have that

$$\begin{aligned}
\frac{32\pi N^2}{N\sqrt{h}} N\sqrt{h}\mathcal{L}_{\mathrm{MG}}\left(h,\dot{h},\ddot{h}\right) = {} & 2N^2 R^{(\tilde{H},J)} \\
& + \left[\dot{J}\left(2NK + \frac{\partial \tilde{H}}{\partial h_{ef}}\dot{h}_{ef}\right) + \frac{1}{2}\dot{J}^2\right] \\
& + \left[\left(2NK^{ab} + \frac{\partial \tilde{H}^{ab}}{\partial h_{ef}}\dot{h}_{ef}\right)\dot{J}_{ab} + \frac{1}{2}h^{ac}h^{bd}\dot{J}_{ab}\dot{J}_{cd}\right] \\
& + \left[\dot{h}_{cd}\left(2NK\frac{\partial \tilde{H}}{\partial h_{cd}} + 2N\frac{\partial \tilde{H}_{ab}}{\partial h_{cd}}K^{ab}\right) + \frac{1}{2}\left(\frac{\partial \tilde{H}}{\partial h_{cd}}\frac{\partial \tilde{H}}{\partial h_{ef}} + \frac{\partial \tilde{H}^{cd}}{\partial h_{gh}}\frac{\partial \tilde{H}_{cd}}{\partial h_{ef}}\right)\dot{h}_{ef}\dot{h}_{cd}\right]
\end{aligned}$$

The hamiltonian density is then

$$\mathcal{H}_G = \pi^{ab}\dot{h}_{ab} + \pi_J^{ab}\dot{J}_{ab} - \underbrace{N\sqrt{h}}_{\sqrt{-g}}\mathcal{L}_G$$

$$\begin{aligned}
\frac{32\pi N}{\sqrt{h}}\mathcal{H}_G = {} & \frac{32\pi N}{\sqrt{h}}\pi^{ab}\dot{h}_{ab} + \frac{32\pi N}{\sqrt{h}}\pi_J^{ab}\dot{J}_{ab} - 32\pi N^2\mathcal{L}_G \\
= {} & -32\pi N^2\mathcal{L}_G^0 + \frac{32\pi N}{\sqrt{h}}\pi_0^{ab}\dot{h}_{ab} \\
& - 2N^2 R^{(\tilde{H},J)} \\
& - \left[\dot{J}\left(2NK + \frac{\partial \tilde{H}}{\partial h_{ef}}\dot{h}_{ef}\right) + \frac{1}{2}\dot{J}^2\right] \\
& - \left[\left(2NK^{ab} + \frac{\partial \tilde{H}^{ab}}{\partial h_{ef}}\dot{h}_{ef}\right)\dot{J}_{ab} + \frac{1}{2}h^{ac}h^{bd}\dot{J}_{ab}\dot{J}_{cd}\right] \\
& - \left[\dot{h}_{cd}\left(2NK\frac{\partial \tilde{H}}{\partial h_{cd}} + 2N\frac{\partial \tilde{H}_{ab}}{\partial h_{cd}}K^{ab}\right) + \frac{1}{2}\left(\frac{\partial \tilde{H}}{\partial h_{cd}}\frac{\partial \tilde{H}}{\partial h_{ef}} + \frac{\partial \tilde{H}^{cd}}{\partial h_{gh}}\frac{\partial \tilde{H}_{cd}}{\partial h_{ef}}\right)\dot{h}_{ef}\dot{h}_{cd}\right] \\
& + \frac{\partial \dot{J}_{cd}}{\partial \dot{h}_{ab}}\left(2NKh^{cd} + 2NK^{cd}\right)\dot{h}_{ab} + \dot{J}\left(h^{ab} + \left(\frac{\partial \tilde{H}}{\partial h_{ab}}\right) + \frac{\partial \dot{J}}{\partial \dot{h}_{ab}}\right)\dot{h}_{ab} \\
& + \dot{h}_{cd}\left(h^{ab}\frac{\partial \tilde{H}}{\partial h_{cd}} + \frac{\partial \tilde{H}^{ab}}{\partial h_{cd}} + \frac{\partial \tilde{H}}{\partial h_{cd}}\frac{\partial \tilde{H}}{\partial h_{ab}} + \frac{\partial \tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial \tilde{H}_{\tilde{c}\tilde{d}}}{\partial h_{ab}}\right)\dot{h}_{ab} \\
& + \frac{\partial \dot{J}_{cd}}{\partial \dot{h}_{ab}}\left(h^{cd}\frac{\partial \tilde{H}}{\partial h_{ef}}\dot{h}_{ef} + \frac{\partial \tilde{H}^{cd}}{\partial h_{ef}}\dot{h}_{ef}\right)\dot{h}_{ab} + \left(\dot{J}_{ab} + \frac{\partial \tilde{H}^{cd}}{\partial h_{ab}}\dot{J}_{cd} + \frac{\partial \dot{J}_{cd}}{\partial \dot{h}_{ab}}\dot{J}^{cd}\right)\dot{h}_{ab} \\
& + \left(2NK\frac{\partial \tilde{H}}{\partial h_{ab}} + 2NK^{cd}\frac{\partial \tilde{H}_{cd}}{\partial h_{ab}}\right)\dot{h}_{ab} \\
& + \left(h^{ab}\frac{\partial \tilde{H}}{\partial h_{ef}}\dot{h}_{ef} + \frac{\partial \tilde{H}^{ab}}{\partial h_{ef}}\dot{h}_{ef}\right)\dot{J}_{ab} \\
& + \left(2NK^{ab} + h^{ab}2NK\right)\dot{J}_{ab} + \left(\dot{J}^{ab} + h^{ab}\dot{J}\right)\dot{J}_{ab}
\end{aligned}$$

The issue with this Hamiltonian density is that it is a function of $\dot{h}$ and $\dot{J}$ and not the momenta. This makes it challenging to determine what actually depends on the lapse and shift when when one wants to vary the hamiltonian with respect to the them when getting the constraints. Furthermore, Hamilton's equations require the Hamiltonian density to be described in terms of the conjugate variables. To rectify this, we must try to write these in terms of $\pi_J^{ab}\pi_{J,ab}$ and $\pi^{ab}\pi_{ab}$. In the general deformation case, this is exceedingly long, but has been done for the reader in Reduction 6.

Reduction 6): Get $\pi_J^{ab}\pi_{Jab}$ and $\pi_{ab}\pi^{ab}$

The momenta are defined as follows,

$$\begin{aligned}
\pi^{ab}\left(\frac{32\pi N}{\sqrt{h}}\right) &= 2N\left(K^{ab} - h^{ab}K\right) \\
&+\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}\left(2NKh^{ij} + 2NK^{ij}\right) \\
&\quad + \dot{J}\left(h^{ab} + \left(\frac{\partial \tilde{H}}{\partial h_{ab}}\right) + \frac{\partial \dot{J}}{\partial \dot{h}_{ab}}\right) \\
&+\dot{h}_{ij}\left(h^{ab}\frac{\partial \tilde{H}}{\partial h_{ij}} + \frac{\partial \tilde{H}^{ab}}{\partial h_{ij}}\right) \\
&+\dot{h}_{ij}\left(\frac{\partial \tilde{H}}{\partial h_{ij}}\frac{\partial \tilde{H}}{\partial h_{ab}} + \frac{\partial \tilde{H}^{k\ell}}{\partial h_{ij}}\frac{\partial \tilde{H}_{k\ell}}{\partial h_{ab}}\right) \\
&+\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}\left(h^{ij}\frac{\partial \tilde{H}}{\partial h_{k\ell}}\dot{h}_{k\ell} + \frac{\partial \tilde{H}^{ij}}{\partial h_{k\ell}}\dot{h}_{k\ell}\right) \\
&+\left(\dot{J}_{ab} + \frac{\partial \tilde{H}^{ij}}{\partial h_{ab}}\dot{J}_{ij} + \frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}\dot{J}^{ij}\right) \\
&+\left(2NK\frac{\partial \tilde{H}}{\partial h^{ab}} + 2NK^{ij}\frac{\partial \tilde{H}_{ij}}{\partial h_{ab}}\right)
\end{aligned}$$

and

$$\begin{aligned}
\frac{32\pi N}{\sqrt{h}}\pi_J^{ab} &= \left(h^{ab}\frac{\partial \tilde{H}}{\partial h_{ef}}\dot{h}_{ef} + \frac{\partial \tilde{H}^{ab}}{\partial h_{ef}}\dot{h}_{ef}\right) \\
&+\left(2NK^{ab} + h^{ab}2NK\right) \\
&+\left(\dot{J}^{ab} + h^{ab}\dot{J}\right)
\end{aligned}$$

The squares of these are as below. First the conjugate momentum for J,

$$
\begin{aligned}
\pi_J^{ab}\pi_{Jab}\left(\frac{h}{32^2\pi^2N^2}\right)^{-1} &= h\frac{\partial\tilde{H}}{\partial h_{cd}}\dot{h}_{cd}\frac{\partial\tilde{H}}{\partial h_{ef}}\dot{h}_{ef} + \frac{\partial\tilde{H}}{\partial h_{cd}}\dot{h}_{cd}\frac{\partial\tilde{H}}{\partial h_{ef}}\dot{h}_{ef} + \frac{\partial\tilde{H}}{\partial h_{cd}}\dot{h}_{cd}\frac{\partial\tilde{H}}{\partial h_{ef}}\dot{h}_{ef} + \frac{\partial\tilde{H}_{ab}}{\partial h_{ef}}\frac{\partial\tilde{H}^{ab}}{\partial h_{cd}}\dot{h}_{cd}\dot{h}_{ef} \\
&\quad + 2NK\frac{\partial\tilde{H}}{\partial h_{cd}}\dot{h}_{cd} + h\frac{\partial\tilde{H}}{\partial h_{cd}}\dot{h}_{cd}2NK + 2NK_{ab}\frac{\partial\tilde{H}^{ab}}{\partial h_{cd}}\dot{h}_{cd} + \frac{\partial\tilde{H}}{\partial h_{cd}}\dot{h}_{cd}2NK \\
&\quad + \dot{J}\frac{\partial\tilde{H}}{\partial h_{ef}}\dot{h}_{ef} + h\frac{\partial\tilde{H}}{\partial h_{ef}}\dot{h}_{ef}\dot{J} + \dot{J}_{ab}\frac{\partial\tilde{H}^{ab}}{\partial h_{ef}}\dot{h}_{ef} + \frac{\partial\tilde{H}}{\partial h_{ef}}\dot{h}_{ef}\dot{J} \\
&\quad + \left(2NK\frac{\partial\tilde{H}}{\partial h_{ef}}\dot{h}_{ef} + 2NK^{ab}\frac{\partial\tilde{H}_{ab}}{\partial h_{ef}}\dot{h}_{ef} + h2NK\frac{\partial\tilde{H}}{\partial h_{ef}}\dot{h}_{ef} + \frac{\partial\tilde{H}}{\partial h_{ef}}2NK\dot{h}_{ef}\right) \\
&\quad + \left(4N^2K^{ab}K_{ab} + 4N^2K^2\right) \\
&\quad + \left(2NK^{ab}\dot{J}_{ab} + 2NK\dot{J} + \dot{J}2NK + h2NK\dot{J}\right) \\
&\quad + \left(\dot{J}\frac{\partial\tilde{H}}{\partial h_{ef}}\dot{h}_{ef} + \dot{J}^{ab}\frac{\partial\tilde{H}_{ab}}{\partial h_{ef}}\dot{h}_{ef} + h\dot{J}\frac{\partial\tilde{H}}{\partial h_{ef}}\dot{h}_{ef} + \frac{\partial\tilde{H}}{\partial h_{ef}}\dot{J}\dot{h}_{ef}\right) \\
&\quad + \left(2NK_{ab}\dot{J}^{ab} + \dot{J}2NK + 2NK\dot{J} + h\dot{J}2NK\right) \\
&\quad + \dot{J}^{ab}\dot{J}_{ab} + \dot{J}^2 + \dot{J}^2 + h\dot{J}^2 \\
&= \dot{J}^{ab}\dot{J}_{ab} + \dot{J}^2 + \dot{J}^2 + h\dot{J}^2 \\
&\quad + h\frac{\partial\tilde{H}}{\partial h_{cd}}\dot{h}_{cd}\frac{\partial\tilde{H}}{\partial h_{ef}}\dot{h}_{ef} + \frac{\partial\tilde{H}}{\partial h_{cd}}\dot{h}_{cd}\frac{\partial\tilde{H}}{\partial h_{ef}}\dot{h}_{ef} + \frac{\partial\tilde{H}}{\partial h_{cd}}\dot{h}_{cd}\frac{\partial\tilde{H}}{\partial h_{ef}}\dot{h}_{ef} + \frac{\partial\tilde{H}_{ab}}{\partial h_{ef}}\frac{\partial\tilde{H}^{ab}}{\partial h_{cd}}\dot{h}_{cd}\dot{h}_{ef} \\
&\quad + \left(4N^2K^{ab}K_{ab} + 4N^2K^2\right) \\
&\quad + 2\left(\dot{J}\frac{\partial\tilde{H}}{\partial h_{ef}}\dot{h}_{ef} + h\frac{\partial\tilde{H}}{\partial h_{ef}}\dot{h}_{ef}\dot{J} + \dot{J}_{ab}\frac{\partial\tilde{H}^{ab}}{\partial h_{ef}}\dot{h}_{ef} + \frac{\partial\tilde{H}}{\partial h_{ef}}\dot{h}_{ef}\dot{J}\right) \\
&\quad + 2\left(2NK\frac{\partial\tilde{H}}{\partial h_{cd}}\dot{h}_{cd} + h\frac{\partial\tilde{H}}{\partial h_{cd}}\dot{h}_{cd}2NK + 2NK_{ab}\frac{\partial\tilde{H}^{ab}}{\partial h_{cd}}\dot{h}_{cd} + \frac{\partial\tilde{H}}{\partial h_{cd}}\dot{h}_{cd}2NK\right) \\
&\quad + 2\left(2NK^{ab}\dot{J}_{ab} + 2NK\dot{J} + \dot{J}2NK + h2NK\dot{J}\right)
\end{aligned}
$$

Now the square of the conjugate momentum for h_{ab},[2]

$$\pi^{ab}\pi_{ab}\left(\frac{32\pi N}{\sqrt{h}}\right)^2$$

$$=$$

======== 1 ========

$$4N^2\left[K^{ab}K_{ab} - 2K^2 + hK\right]$$

$$+\left(KK^{ab}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}h^{cd} - KK^{cd}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}h^{ab} + K^{ab}K^{cd}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}} - K^2\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}h^{ab}h^{cd}\right)$$

$$+2N\dot{J}\left(K^{ab}\right)\left(K^{ab}\frac{\partial \tilde{H}}{\partial h^{ab}} - K\frac{\partial \tilde{H}}{\partial h^{ab}}h^{ab} + K^{ab}\frac{\partial \dot{J}}{\partial \dot{h}^{ab}} - K\frac{\partial \dot{J}}{\partial \dot{h}^{ab}}h^{ab}\right)$$

$$+2N\left(K^{ab}\frac{\partial H_{ab}}{\partial h_{cd}}\dot{h}_{cd} - K\frac{\partial H_{ab}}{\partial h_{cd}}h^{ab}\dot{h}_{cd} + K\frac{\partial H}{\partial h_{cd}}\dot{h}_{cd} - K\frac{\partial H}{\partial h_{cd}}\dot{h}_{cd}h\right)$$

$$+2N\left(K^{ab}\frac{\partial \tilde{H}}{\partial h_{cd}}\frac{\partial \tilde{H}}{\partial h^{ab}}\dot{h}_{cd} + K^{ab}\frac{\partial \tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial \tilde{H}_{\tilde{c}\tilde{d}}}{\partial h^{ab}}\dot{h}_{cd} - K\frac{\partial \tilde{H}}{\partial h_{cd}}\frac{\partial \tilde{H}}{\partial h^{ab}}h^{ab}\dot{h}_{cd} - K\frac{\partial \tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial \tilde{H}_{\tilde{c}\tilde{d}}}{\partial h^{ab}}h^{ab}\dot{h}_{cd}\right)$$

$$+2N\left(K^{ab}\frac{\partial \dot{J}}{\partial \dot{h}^{ab}}\frac{\partial \tilde{H}}{\partial h_{ef}}\dot{h}_{ef} + K^{ab}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}\frac{\partial \tilde{H}^{cd}}{\partial h_{ef}}\dot{h}_{ef} - K\frac{\partial \dot{J}}{\partial \dot{h}^{ab}}\frac{\partial \tilde{H}}{\partial h_{ef}}\dot{h}_{ef}h^{ab} - K\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}\frac{\partial \tilde{H}^{cd}}{\partial h_{ef}}\dot{h}_{ef}h^{ab}\right)$$

$$+2NKK^{ab}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}h^{cd} - KK^{cd}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}h^{ab} + 2NKK^{ab}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}h^{cd} - KK^{cd}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}h^{ab}$$

$$+2N\left(K^{ab}\dot{J}^{ab} + K^{ab}\dot{J}_{cd}\frac{\partial \tilde{H}^{cd}}{\partial h^{ab}} + K^{ab}\dot{J}^{cd}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}} - K\dot{J}^{ab}h^{ab} - K\dot{J}_{cd}\frac{\partial \tilde{H}^{cd}}{\partial h^{ab}}h^{ab} - K\dot{J}^{cd}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}h^{ab}\right)$$

$$+4N^2\left(KK^{ab}\frac{\partial \tilde{H}}{\partial h_{ab}} - KK\frac{\partial \tilde{H}}{\partial h_{ab}}h^{ab} + K^{cd}K^{ab}\frac{\partial \tilde{H}_{cd}}{\partial h^{ab}} - KK^{cd}\frac{\partial \tilde{H}_{cd}}{\partial h^{ab}}h^{ab}\right)$$

============ 2 ==========

$$+4N^2\left(Kh^{ij}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}K_{ab} - K^2h^{ij}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}h_{ab} + \frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}K_{ab}K^{ij} - \frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}h_{ab}KK^{ij}\right)$$

$$+4N^2\left(K^2\frac{\partial \dot{J}}{\partial \dot{h}^{ab}}\frac{\partial \dot{J}}{\partial \dot{h}_{ab}} + KK^{cd}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}\frac{\partial \dot{J}}{\partial \dot{h}_{ab}} + KK^{ij}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}\frac{\partial \dot{J}}{\partial \dot{h}^{ab}} + K^{cd}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}K^{ij}\right)$$

$$+\dot{J}2N\left(K\frac{\partial \dot{J}}{\partial \dot{h}_{ab}}h_{ab} + K\frac{\partial \dot{J}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}}{\partial h^{ab}} + K\frac{\partial \dot{J}}{\partial \dot{h}_{ab}}\frac{\partial \dot{J}}{\partial \dot{h}^{ab}} + h_{ab}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}K^{ij} + \frac{\partial \tilde{H}}{\partial h^{ab}}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}K^{ij} + \frac{\partial \dot{J}}{\partial \dot{h}^{ab}}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}K^{ij}\right)$$

$$+2N\left(K\frac{\partial \tilde{H}}{\partial h_{cd}}\frac{\partial \dot{J}}{\partial \dot{h}_{ab}}\dot{h}_{cd}h_{ab} + K\frac{\partial \tilde{H}_{ab}}{\partial h_{cd}}\frac{\partial \dot{J}}{\partial \dot{h}_{ab}}\dot{h}_{cd} + K^{ij}\frac{\partial \tilde{H}}{\partial h_{cd}}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}h_{ab}\dot{h}_{cd} + K^{ij}\frac{\partial \tilde{H}_{ab}}{\partial h_{cd}}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}\dot{h}_{cd}\right)$$

$$+2N\left(K\frac{\partial \dot{J}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}}{\partial h_{cd}}\frac{\partial \tilde{H}}{\partial h^{ab}}h^{ij}\dot{h}_{cd} + K\frac{\partial \dot{J}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial \tilde{H}_{\tilde{c}\tilde{d}}}{\partial h^{ab}}\dot{h}_{cd} + K^{ij}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}}{\partial h_{cd}}\frac{\partial \tilde{H}}{\partial h^{ab}}\dot{h}_{cd} + K^{ij}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial \tilde{H}_{\tilde{c}\tilde{d}}}{\partial h^{ab}}\dot{h}_{cd}\right)$$

$$+2N\left(K\frac{\partial \dot{J}}{\partial \dot{h}^{ab}}\frac{\partial \dot{J}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}}{\partial h_{ef}}\dot{h}_{ef} + K\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}\frac{\partial \dot{J}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}^{cd}}{\partial h_{ef}}\dot{h}_{ef} + K^{ij}\frac{\partial \dot{J}}{\partial \dot{h}^{ab}}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}}{\partial h_{ef}}\dot{h}_{ef} + K^{ij}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}^{cd}}{\partial h_{ef}}\dot{h}_{ef}\right)$$

$$+2N\left(K\dot{J}^{ab}\frac{\partial \dot{J}}{\partial \dot{h}_{ab}} + K\dot{J}_{cd}\frac{\partial \dot{J}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}^{cd}}{\partial h^{ab}} + K\dot{J}^{cd}\frac{\partial \dot{J}}{\partial \dot{h}_{ab}}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}} + K^{ij}\dot{J}^{ab}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}} + K^{ij}\dot{J}_{cd}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}^{cd}}{\partial h^{ab}} + K^{ij}\dot{J}^{cd}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}\right)$$

$$+4N^2\left(KK\frac{\partial \dot{J}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}}{\partial h_{ab}} + K^{cd}K\frac{\partial \dot{J}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}_{cd}}{\partial h^{ab}} + KK^{ij}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}}{\partial h_{ab}} + K^{cd}K^{ij}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}_{cd}}{\partial h^{ab}}\right)$$

[2] The =========n========== dividers represent which tensor contraction I did, and is useful when trying to sort the terms.

============= 3 =============

$$+2N\dot{J}\left(K-hk+K_{ab}\frac{\partial\tilde{H}}{\partial h_{ab}}-K\frac{\partial\tilde{H}}{\partial h_{ab}}h_{ab}+K_{ab}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}-K\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}h_{ab}\right)$$

$$+2N\dot{J}\left(K\frac{\partial\dot{J}}{\partial\dot{h}^{ab}}h^{ab}+K^{cd}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}h^{ab}+K\frac{\partial\dot{J}}{\partial\dot{h}^{ab}}\frac{\partial\tilde{H}}{\partial h_{ab}}+K^{cd}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}\frac{\partial\tilde{H}}{\partial h_{ab}}+K\frac{\partial\dot{J}}{\partial\dot{h}^{ab}}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}h^{cd}+K^{cd}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\right)$$

$$+\dot{J}\dot{J}\left(h+\frac{\partial\tilde{H}}{\partial h_{ab}}\frac{\partial\tilde{H}}{\partial h^{ab}}+2\frac{\partial\tilde{H}}{\partial h_{ab}}h_{ab}+2\frac{\partial\tilde{H}}{\partial h_{ab}}\frac{\partial\dot{J}}{\partial\dot{h}^{ab}}+2\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}h_{ab}+\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\frac{\partial\dot{J}}{\partial\dot{h}^{ab}}\right)$$

$$+\dot{J}\left(\frac{\partial\tilde{H}}{\partial h_{cd}}h\dot{h}_{cd}+\frac{\partial\tilde{H}}{\partial h_{cd}}\dot{h}_{cd}+\frac{\partial\tilde{H}}{\partial h_{cd}}\frac{\partial\tilde{H}}{\partial h_{ab}}\dot{h}_{cd}h_{ab}+\frac{\partial\tilde{H}_{ab}}{\partial h_{cd}}\frac{\partial\tilde{H}}{\partial h_{ab}}\dot{h}_{cd}+\frac{\partial\tilde{H}}{\partial h_{cd}}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\dot{h}_{cd}h_{ab}+\frac{\partial\tilde{H}_{ab}}{\partial h_{cd}}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\dot{h}_{cd}\right)$$

$$+\dot{J}\left(\frac{\partial\tilde{H}}{\partial h_{cd}}\frac{\partial\tilde{H}}{\partial h^{ab}}h^{ab}\dot{h}_{cd}+\frac{\partial\tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial\tilde{H}_{\tilde{c}\tilde{d}}}{\partial h^{ab}}h^{ab}\dot{h}_{cd}+\frac{\partial\tilde{H}}{\partial h_{cd}}\frac{\partial\tilde{H}}{\partial h^{ab}}\frac{\partial\tilde{H}}{\partial h_{ab}}\dot{h}_{cd}\right)$$

$$+\dot{J}\left(\frac{\partial\tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial\tilde{H}_{\tilde{c}\tilde{d}}}{\partial h^{ab}}\frac{\partial\tilde{H}}{\partial h_{ab}}\dot{h}_{cd}+\frac{\partial\tilde{H}}{\partial h_{cd}}\frac{\partial\tilde{H}}{\partial h^{ab}}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\dot{h}_{cd}+\frac{\partial\tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial\tilde{H}_{\tilde{c}\tilde{d}}}{\partial h^{ab}}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\dot{h}_{cd}\right)$$

$$+\dot{J}\left(\frac{\partial\tilde{H}}{\partial h_{ef}}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}h^{ab}h^{cd}\dot{h}_{ef}+\frac{\partial\tilde{H}^{cd}}{\partial h_{ef}}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}h^{ab}\dot{h}_{ef}+\frac{\partial\tilde{H}}{\partial h_{ef}}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}\frac{\partial\tilde{H}}{\partial h_{ab}}h^{cd}\dot{h}_{ef}\right)$$

$$+\dot{J}\left(\frac{\partial\tilde{H}^{cd}}{\partial h_{ef}}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}\frac{\partial\tilde{H}}{\partial h_{ab}}\dot{h}_{ef}+\frac{\partial\tilde{H}}{\partial h_{ef}}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}h^{cd}\dot{h}_{ef}+\frac{\partial\tilde{H}^{cd}}{\partial h_{ef}}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\dot{h}_{ef}\right)$$

$$+\left(\dot{J}^2+\dot{J}\dot{J}_{cd}\frac{\partial\tilde{H}^{cd}}{\partial h^{ab}}h^{ab}+\dot{J}\dot{J}^{cd}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}h^{ab}+\dot{J}^{ab}\dot{J}\frac{\partial\tilde{H}}{\partial h_{ab}}+\dot{J}_{cd}\dot{J}\frac{\partial\tilde{H}^{cd}}{\partial h^{ab}}\frac{\partial\tilde{H}}{\partial h_{ab}}\right)$$

$$+\left(\dot{J}^{cd}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}+\dot{J}^{ab}\dot{J}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}+\dot{J}_{cd}\dot{J}\frac{\partial\tilde{H}^{cd}}{\partial h^{ab}}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}+\dot{J}^{cd}\dot{J}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\right)$$

$$+2N\dot{J}\left(K\frac{\partial\tilde{H}}{\partial h_{ab}}h^{ab}+K^{cd}\frac{\partial\tilde{H}_{cd}}{\partial h^{ab}}h^{ab}+K\frac{\partial\tilde{H}}{\partial h_{ab}}\frac{\partial\tilde{H}}{\partial h_{ab}}+K^{cd}\frac{\partial\tilde{H}_{cd}}{\partial h^{ab}}\frac{\partial\tilde{H}}{\partial h_{ab}}+K\frac{\partial\tilde{H}}{\partial h_{ab}}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}+K^{cd}\frac{\partial\tilde{H}_{cd}}{\partial h^{ab}}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\right)$$

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$$+2N\left(K\frac{\partial\tilde{H}}{\partial h_{ij}}\dot{h}_{ij}-K\frac{\partial\tilde{H}}{\partial h_{ij}}h\dot{h}_{ij}+K_{ab}\frac{\partial\tilde{H}^{ab}}{\partial h_{ij}}\dot{h}_{ij}-K\frac{\partial\tilde{H}}{\partial h_{ij}}\dot{h}_{ij}\right)$$

$$+2N\left(K\frac{\partial\tilde{H}}{\partial h_{ij}}\frac{\partial\dot{J}}{\partial\dot{h}^{ab}}h^{ab}\dot{h}_{ij}+K^{cd}\frac{\partial\tilde{H}}{\partial h_{ij}}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}h^{ab}\dot{h}_{ij}+K\frac{\partial\dot{J}}{\partial\dot{h}^{ab}}\frac{\partial\tilde{H}^{ab}}{\partial h_{ij}}\dot{h}_{ij}+K^{cd}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}\frac{\partial\tilde{H}^{ab}}{\partial h_{ij}}\dot{h}_{ij}\right)$$

$$+\dot{J}\left(\frac{\partial\tilde{H}}{\partial h_{ij}}h\dot{h}_{ij}+\frac{\partial\tilde{H}}{\partial h^{ab}}\frac{\partial\tilde{H}}{\partial h_{ij}}h^{ab}\dot{h}_{ij}+\frac{\partial\dot{J}}{\partial\dot{h}^{ab}}\frac{\partial\tilde{H}}{\partial h_{ij}}h^{ab}\dot{h}_{ij}+\frac{\partial\tilde{H}}{\partial h_{ij}}\dot{h}_{ij}+\frac{\partial\tilde{H}}{\partial h^{ab}}\frac{\partial\tilde{H}^{ab}}{\partial h_{ij}}\dot{h}_{ij}+\frac{\partial\dot{J}}{\partial\dot{h}^{ab}}\frac{\partial\tilde{H}^{ab}}{\partial h_{ij}}\dot{h}_{ij}\right)$$

$$+\left(\frac{\partial\tilde{H}}{\partial h_{cd}}\frac{\partial\tilde{H}}{\partial h_{ij}}h\dot{h}_{ij}\dot{h}_{cd}+\frac{\partial\tilde{H}}{\partial h_{cd}}\frac{\partial\tilde{H}}{\partial h_{ij}}\dot{h}_{ij}\dot{h}_{cd}+\frac{\partial\tilde{H}}{\partial h_{cd}}\frac{\partial\tilde{H}}{\partial h_{ij}}\dot{h}_{ij}\dot{h}_{cd}+\frac{\partial\tilde{H}_{ab}}{\partial h_{cd}}\frac{\partial\tilde{H}^{ab}}{\partial h_{ij}}\dot{h}_{ij}\dot{h}_{cd}\right)$$

$$\left((2+1)\frac{\partial\tilde{H}}{\partial h_{cd}}\frac{\partial\tilde{H}}{\partial h_{ij}}\dot{h}_{ij}\dot{h}_{cd}+\frac{\partial\tilde{H}}{\partial h_{cd}}\frac{\partial\tilde{H}}{\partial h_{ij}}h\dot{h}_{ij}\dot{h}_{cd}+\frac{\partial\tilde{H}_{ab}}{\partial h_{cd}}\frac{\partial\tilde{H}^{ab}}{\partial h_{ij}}\dot{h}_{ij}\dot{h}_{cd}\right)$$

$$+\frac{\partial\tilde{H}}{\partial h_{cd}}\frac{\partial\tilde{H}}{\partial h^{ab}}\frac{\partial\tilde{H}}{\partial h_{ij}}h^{ab}\dot{h}_{ij}\dot{h}_{cd}+\frac{\partial\tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial\tilde{H}_{\tilde{c}\tilde{d}}}{\partial h^{ab}}\frac{\partial\tilde{H}}{\partial h_{ij}}h^{ab}\dot{h}_{ij}\dot{h}_{cd}+\frac{\partial\tilde{H}}{\partial h_{cd}}\frac{\partial\tilde{H}}{\partial h^{ab}}\frac{\partial\tilde{H}^{ab}}{\partial h_{ij}}\dot{h}_{ij}\dot{h}_{cd}+\frac{\partial\tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial\tilde{H}_{\tilde{c}\tilde{d}}}{\partial h^{ab}}\frac{\partial\tilde{H}^{ab}}{\partial h_{ij}}\dot{h}_{ij}\dot{h}_{cd}$$

$$+\left(\frac{\partial\tilde{H}}{\partial h_{ef}}\frac{\partial\tilde{H}}{\partial h_{ij}}\frac{\partial\dot{J}}{\partial\dot{h}^{ab}}\dot{h}_{ij}h^{ab}\dot{h}_{ef}+\frac{\partial\tilde{H}^{cd}}{\partial h_{ef}}\frac{\partial\tilde{H}}{\partial h_{ij}}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}h^{ab}\dot{h}_{ij}\dot{h}_{ef}+\frac{\partial\tilde{H}}{\partial h_{ef}}\frac{\partial\tilde{H}^{ab}}{\partial h_{ij}}\frac{\partial\dot{J}}{\partial\dot{h}^{ab}}\dot{h}_{ij}\dot{h}_{ef}+\frac{\partial\tilde{H}^{cd}}{\partial h_{ef}}\frac{\partial\tilde{H}^{ab}}{\partial h_{ij}}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}\dot{h}_{ij}\dot{h}_{ef}\right)$$

$$+\left(\dot{J}\frac{\partial\tilde{H}}{\partial h_{ij}}\dot{h}_{ij}+\dot{J}_{cd}\frac{\partial\tilde{H}}{\partial h_{ij}}\frac{\partial\tilde{H}^{cd}}{\partial h^{ab}}h^{ab}\dot{h}_{ij}+\dot{J}^{cd}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}\frac{\partial\tilde{H}}{\partial h_{ij}}h^{ab}\dot{h}_{ij}+\dot{J}^{ab}\frac{\partial\tilde{H}^{ab}}{\partial h_{ij}}\dot{h}_{ij}+\dot{J}_{cd}\frac{\partial\tilde{H}^{cd}}{\partial h^{ab}}\frac{\partial\tilde{H}^{ab}}{\partial h_{ij}}\dot{h}_{ij}+\dot{J}^{cd}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}\frac{\partial\tilde{H}^{ab}}{\partial h_{ij}}\dot{h}_{ij}\right)$$

$$+2N\left(K\frac{\partial\tilde{H}}{\partial h_{ab}}\frac{\partial\tilde{H}}{\partial h_{ij}}h^{ab}\dot{h}_{ij}+K^{cd}\frac{\partial\tilde{H}_{cd}}{\partial h^{ab}}\frac{\partial\tilde{H}}{\partial h_{ij}}h^{ab}\dot{h}_{ij}+K\frac{\partial\tilde{H}}{\partial h_{ab}}\frac{\partial\tilde{H}^{ab}}{\partial h_{ij}}\dot{h}_{ij}+K^{cd}\frac{\partial\tilde{H}_{cd}}{\partial h^{ab}}\frac{\partial\tilde{H}^{ab}}{\partial h_{ij}}\dot{h}_{ij}\right)$$

$$+2N\left(K_{ab}\frac{\partial\tilde{H}}{\partial h_{ij}}\frac{\partial\tilde{H}}{\partial h_{ab}}\dot{h}_{ij}-K\frac{\partial\tilde{H}}{\partial h_{ij}}\frac{\partial\tilde{H}}{\partial h_{ab}}h_{ab}\dot{h}_{ij}+K_{ab}\frac{\partial\tilde{H}^{k\ell}}{\partial h_{ij}}\frac{\partial\tilde{H}_{k\ell}}{\partial h_{ab}}\dot{h}_{ij}-K\frac{\partial\tilde{H}^{k\ell}}{\partial h_{ij}}\frac{\partial\tilde{H}_{k\ell}}{\partial h_{ab}}\dot{h}_{ij}h_{ab}\right)$$

$$+2N\left(K\frac{\partial\dot{J}}{\partial\dot{h}^{ab}}\frac{\partial\tilde{H}}{\partial h_{ij}}\frac{\partial\tilde{H}}{\partial h_{ab}}\dot{h}_{ij}+K^{cd}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}\frac{\partial\tilde{H}}{\partial h_{ij}}\frac{\partial\tilde{H}}{\partial h_{ab}}\dot{h}_{ij}+K\frac{\partial\dot{J}}{\partial\dot{h}^{ab}}\frac{\partial\tilde{H}^{k\ell}}{\partial h_{ij}}\frac{\partial\tilde{H}_{k\ell}}{\partial h_{ab}}\dot{h}_{ij}+K^{cd}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}\frac{\partial\tilde{H}^{k\ell}}{\partial h_{ij}}\frac{\partial\tilde{H}_{k\ell}}{\partial h_{ab}}\dot{h}_{ij}\right)$$

$$+\dot{J}\left(\frac{\partial\tilde{H}}{\partial h_{ij}}\frac{\partial\tilde{H}}{\partial h_{ab}}\dot{h}_{ij}h_{ab}+\frac{\partial\tilde{H}}{\partial h^{ab}}\frac{\partial\tilde{H}}{\partial h_{ij}}\frac{\partial\tilde{H}}{\partial h_{ab}}\dot{h}_{ij}+\frac{\partial\dot{J}}{\partial\dot{h}^{ab}}\frac{\partial\tilde{H}}{\partial h_{ij}}\frac{\partial\tilde{H}}{\partial h_{ab}}\dot{h}_{ij}\right)$$

$$+\dot{J}\left(\frac{\partial\tilde{H}^{k\ell}}{\partial h_{ij}}\frac{\partial\tilde{H}_{k\ell}}{\partial h_{ab}}\dot{h}_{ij}h_{ab}+\frac{\partial\tilde{H}}{\partial h^{ab}}\frac{\partial\tilde{H}^{k\ell}}{\partial h_{ij}}\frac{\partial\tilde{H}_{k\ell}}{\partial h_{ab}}\dot{h}_{ij}+\frac{\partial\dot{J}}{\partial\dot{h}^{ab}}\frac{\partial\tilde{H}^{k\ell}}{\partial h_{ij}}\frac{\partial\tilde{H}_{k\ell}}{\partial h_{ab}}\dot{h}_{ij}\right)$$

$$+\left(\frac{\partial\tilde{H}}{\partial h_{ij}}\frac{\partial\tilde{H}}{\partial h_{ab}}\frac{\partial\tilde{H}}{\partial h_{cd}}h_{ab}\dot{h}_{ij}\dot{h}_{cd}+\frac{\partial\tilde{H}}{\partial h_{ij}}\frac{\partial\tilde{H}}{\partial h_{ab}}\frac{\partial\tilde{H}_{ab}}{\partial h_{cd}}\dot{h}_{ij}\dot{h}_{cd}+\frac{\partial\tilde{H}^{k\ell}}{\partial h_{ij}}\frac{\partial\tilde{H}_{k\ell}}{\partial h_{ab}}\frac{\partial\tilde{H}}{\partial h_{cd}}h_{ab}\dot{h}_{ij}\dot{h}_{cd}+\frac{\partial\tilde{H}^{k\ell}}{\partial h_{ij}}\frac{\partial\tilde{H}_{k\ell}}{\partial h_{ab}}\frac{\partial\tilde{H}_{ab}}{\partial h_{cd}}\dot{h}_{ij}\dot{h}_{cd}\right)$$

$$+\left(\frac{\partial\tilde{H}}{\partial h_{cd}}\frac{\partial\tilde{H}}{\partial h^{ab}}\frac{\partial\tilde{H}}{\partial h_{ij}}\frac{\partial\tilde{H}}{\partial h_{ab}}\dot{h}_{ij}\dot{h}_{cd}+\frac{\partial\tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial\tilde{H}_{\tilde{c}\tilde{d}}}{\partial h^{ab}}\frac{\partial\tilde{H}}{\partial h_{ij}}\frac{\partial\tilde{H}}{\partial h_{ab}}\dot{h}_{ij}\dot{h}_{cd}\right)$$

$$+\left(\frac{\partial\tilde{H}}{\partial h_{cd}}\frac{\partial\tilde{H}}{\partial h^{ab}}\frac{\partial\tilde{H}^{k\ell}}{\partial h_{ij}}\frac{\partial\tilde{H}_{k\ell}}{\partial h_{ab}}\dot{h}_{ij}\dot{h}_{cd}+\frac{\partial\tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial\tilde{H}_{\tilde{c}\tilde{d}}}{\partial h^{ab}}\frac{\partial\tilde{H}^{k\ell}}{\partial h_{ij}}\frac{\partial\tilde{H}_{k\ell}}{\partial h_{ab}}\dot{h}_{ij}\dot{h}_{cd}\right)$$

$$+\left(\frac{\partial\dot{J}}{\partial\dot{h}^{ab}}\frac{\partial\tilde{H}}{\partial h_{ef}}\frac{\partial\tilde{H}}{\partial h_{ij}}\frac{\partial\tilde{H}}{\partial h_{ab}}\dot{h}_{ij}\dot{h}_{ef}+\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}\frac{\partial\tilde{H}^{cd}}{\partial h_{ef}}\frac{\partial\tilde{H}}{\partial h_{ij}}\frac{\partial\tilde{H}}{\partial h_{ab}}\dot{h}_{ij}\dot{h}_{ef}\right)$$

$$+\left(\frac{\partial\dot{J}}{\partial\dot{h}^{ab}}\frac{\partial\tilde{H}}{\partial h_{ef}}\frac{\partial\tilde{H}^{k\ell}}{\partial h_{ij}}\frac{\partial\tilde{H}_{k\ell}}{\partial h_{ab}}\dot{h}_{ij}\dot{h}_{ef}+\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}\frac{\partial\tilde{H}^{cd}}{\partial h_{ef}}\frac{\partial\tilde{H}^{k\ell}}{\partial h_{ij}}\frac{\partial\tilde{H}_{k\ell}}{\partial h_{ab}}\dot{h}_{ij}\dot{h}_{ef}\right)$$

$$+\left(\dot{J}^{ab}\frac{\partial\tilde{H}}{\partial h_{ij}}\frac{\partial\tilde{H}}{\partial h_{ab}}\dot{h}_{ij}+\dot{J}_{cd}\frac{\partial\tilde{H}^{cd}}{\partial h^{ab}}\frac{\partial\tilde{H}}{\partial h_{ij}}\frac{\partial\tilde{H}}{\partial h_{ab}}\dot{h}_{ij}+\dot{J}^{cd}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}\frac{\partial\tilde{H}}{\partial h_{ij}}\frac{\partial\tilde{H}}{\partial h_{ab}}\dot{h}_{ij}\right)$$

$$+\left(\dot{J}^{ab}\frac{\partial\tilde{H}^{k\ell}}{\partial h_{ij}}\frac{\partial\tilde{H}_{k\ell}}{\partial h_{ab}}\dot{h}_{ij}+\dot{J}_{cd}\frac{\partial\tilde{H}^{cd}}{\partial h^{ab}}\frac{\partial\tilde{H}^{k\ell}}{\partial h_{ij}}\frac{\partial\tilde{H}_{k\ell}}{\partial h_{ab}}\dot{h}_{ij}+\dot{J}^{cd}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}\frac{\partial\tilde{H}^{k\ell}}{\partial h_{ij}}\frac{\partial\tilde{H}_{k\ell}}{\partial h_{ab}}\dot{h}_{ij}\right)$$

$$+2N\left(K\frac{\partial\tilde{H}}{\partial h_{ab}}\frac{\partial\tilde{H}}{\partial h_{ij}}\frac{\partial\tilde{H}}{\partial h_{ab}}\dot{h}_{ij}+K^{cd}\frac{\partial\tilde{H}_{cd}}{\partial h^{ab}}\frac{\partial\tilde{H}}{\partial h_{ij}}\frac{\partial\tilde{H}}{\partial h_{ab}}\dot{h}_{ij}+K\frac{\partial\tilde{H}}{\partial h_{ab}}\frac{\partial\tilde{H}^{k\ell}}{\partial h_{ij}}\frac{\partial\tilde{H}_{k\ell}}{\partial h_{ab}}\dot{h}_{ij}+K^{cd}\frac{\partial\tilde{H}_{cd}}{\partial h^{ab}}\frac{\partial\tilde{H}^{k\ell}}{\partial h_{ij}}\frac{\partial\tilde{H}_{k\ell}}{\partial h_{ab}}\dot{h}_{ij}\right)$$

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$$+2N\left(K\frac{\partial\dot{J}}{\partial\dot{h}^{ab}}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}}{\partial h_{k\ell}}\dot{h}_{k\ell}+K_{ab}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}}{\partial h_{k\ell}}\dot{h}_{k\ell}-K\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}}{\partial h_{k\ell}}\dot{h}_{k\ell}h_{ab}+K_{ab}\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}^{ij}}{\partial h_{k\ell}}\dot{h}_{k\ell}-K\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}^{ij}}{\partial h_{k\ell}}\dot{h}_{k\ell}h_{ab}\right)$$

$$+2N\left(K^{cd}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}}{\partial h_{k\ell}}\dot{h}_{k\ell}+K\frac{\partial\dot{J}}{\partial\dot{h}^{ab}}\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}^{ij}}{\partial h_{k\ell}}\dot{h}_{k\ell}+K^{cd}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}^{ij}}{\partial h_{k\ell}}\dot{h}_{k\ell}\right)+\dot{J}\left(\frac{\partial\dot{J}}{\partial\dot{h}^{ab}}\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}^{ij}}{\partial h_{k\ell}}\dot{h}_{k\ell}\right)$$

$$+\dot{J}\left(\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}}{\partial h_{k\ell}}\dot{h}_{k\ell}h_{ab}+\frac{\partial\tilde{H}}{\partial h^{ab}}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}}{\partial h_{k\ell}}\dot{h}_{k\ell}+\frac{\partial\dot{J}}{\partial\dot{h}^{ab}}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}}{\partial h_{k\ell}}\dot{h}_{k\ell}+h_{ab}\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}^{ij}}{\partial h_{k\ell}}\dot{h}_{k\ell}+\frac{\partial\tilde{H}}{\partial h^{ab}}\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}^{ij}}{\partial h_{k\ell}}\dot{h}_{k\ell}\right)$$

$$+\left(\frac{\partial\tilde{H}}{\partial h_{cd}}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}}{\partial h_{k\ell}}\dot{h}_{k\ell}h_{ab}\dot{h}_{cd}+\frac{\partial\tilde{H}_{ab}}{\partial h_{cd}}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}}{\partial h_{k\ell}}\dot{h}_{k\ell}\dot{h}_{cd}+\frac{\partial\tilde{H}}{\partial h_{cd}}\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}^{ij}}{\partial h_{k\ell}}\dot{h}_{k\ell}h_{ab}\dot{h}_{cd}+\frac{\partial\tilde{H}_{ab}}{\partial h_{cd}}\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}^{ij}}{\partial h_{k\ell}}\dot{h}_{k\ell}\dot{h}_{cd}\right)$$

$$+\left(\frac{\partial\tilde{H}}{\partial h_{cd}}\frac{\partial\tilde{H}}{\partial h^{ab}}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}}{\partial h_{k\ell}}\dot{h}_{k\ell}\dot{h}_{cd}+\frac{\partial\tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial\tilde{H}_{\tilde{c}\tilde{d}}}{\partial h^{ab}}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}}{\partial h_{k\ell}}\dot{h}_{k\ell}\dot{h}_{cd}+\frac{\partial\tilde{H}}{\partial h_{cd}}\frac{\partial\tilde{H}}{\partial h^{ab}}\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}^{ij}}{\partial h_{k\ell}}\dot{h}_{k\ell}\dot{h}_{cd}+\frac{\partial\tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial\tilde{H}_{\tilde{c}\tilde{d}}}{\partial h^{ab}}\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}^{ij}}{\partial h_{k\ell}}\dot{h}_{k\ell}\dot{h}_{cd}\right)$$

$$+\left(\frac{\partial\dot{J}}{\partial\dot{h}^{ab}}\frac{\partial\tilde{H}}{\partial h_{ef}}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}}{\partial h_{k\ell}}\dot{h}_{k\ell}\dot{h}_{ef}+\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}\frac{\partial\tilde{H}^{cd}}{\partial h_{ef}}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}}{\partial h_{k\ell}}\dot{h}_{k\ell}\dot{h}_{ef}+\frac{\partial\dot{J}}{\partial\dot{h}^{ab}}\frac{\partial\tilde{H}}{\partial h_{ef}}\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}^{ij}}{\partial h_{k\ell}}\dot{h}_{k\ell}\dot{h}_{ef}+\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}\frac{\partial\tilde{H}^{cd}}{\partial h_{ef}}\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}^{ij}}{\partial h_{k\ell}}\dot{h}_{k\ell}\dot{h}_{ef}\right)$$

$$+\left(\dot{J}^{ab}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}}{\partial h_{k\ell}}\dot{h}_{k\ell}+\dot{J}_{cd}\frac{\partial\tilde{H}^{cd}}{\partial h^{ab}}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}}{\partial h_{k\ell}}\dot{h}_{k\ell}+\dot{J}^{cd}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}}{\partial h_{k\ell}}\dot{h}_{k\ell}+\dot{J}^{ab}\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}^{ij}}{\partial h_{k\ell}}\dot{h}_{k\ell}+\dot{J}_{cd}\frac{\partial\tilde{H}^{cd}}{\partial h^{ab}}\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}^{ij}}{\partial h_{k\ell}}\dot{h}_{k\ell}\right.$$

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$$+ \dot{J}^{cd}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}^{ij}}{\partial h_{k\ell}}\dot{h}_{k\ell}\Big)$$

$$+2N\left(K\frac{\partial \tilde{H}}{\partial h_{ab}}\frac{\partial \dot{J}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}}{\partial h_{k\ell}}\dot{h}_{k\ell} + K^{cd}\frac{\partial \tilde{H}_{cd}}{\partial h^{ab}}\frac{\partial \dot{J}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}}{\partial h_{k\ell}}\dot{h}_{k\ell} + K\frac{\partial \tilde{H}}{\partial h_{ab}}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}^{ij}}{\partial h_{k\ell}}\dot{h}_{k\ell} + K^{cd}\frac{\partial \tilde{H}_{cd}}{\partial h^{ab}}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}^{ij}}{\partial h_{k\ell}}\dot{h}_{k\ell}\right)$$

$$+2N\left(\dot{J}^{ab}K_{ab} - \dot{J}K + K_{ab}\dot{J}_{ij}\frac{\partial \tilde{H}^{ij}}{\partial h_{ab}} - K\dot{J}_{ij}\frac{\partial \tilde{H}^{ij}}{\partial h_{ab}}h_{ab} + K_{ab}\dot{J}^{ij}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}} - K\dot{J}^{ij}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}h_{ab}\right)$$

$$+2N\left(K\dot{J}^{ab}\frac{\partial \dot{J}}{\partial \dot{h}^{ab}} + \dot{J}^{ab}K^{cd}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}} + K\dot{J}_{ij}\frac{\partial \tilde{H}^{ij}}{\partial h_{ab}}\frac{\partial \dot{J}}{\partial \dot{h}^{ab}} + K^{cd}\dot{J}_{ij}\frac{\partial \tilde{H}^{ij}}{\partial h_{ab}}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}} + K\dot{J}^{ij}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}\frac{\partial \dot{J}}{\partial \dot{h}^{ab}} + K^{cd}\dot{J}^{ij}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}\right)$$

$$+\dot{J}\left(\dot{J} + \dot{J}^{ab}\frac{\partial \tilde{H}}{\partial h^{ab}} + \dot{J}^{ab}\frac{\partial \dot{J}}{\partial \dot{h}^{ab}} + \dot{J}_{ij}\frac{\partial \tilde{H}^{ij}}{\partial h_{ab}}h_{ab} + \dot{J}_{ij}\frac{\partial \tilde{H}^{ij}}{\partial h_{ab}}\frac{\partial \tilde{H}}{\partial h^{ab}} + \dot{J}_{ij}\frac{\partial \tilde{H}^{ij}}{\partial h_{ab}}\frac{\partial \dot{J}}{\partial \dot{h}^{ab}} + \dot{J}^{ij}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}h_{ab} + \dot{J}^{ij}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}}{\partial h^{ab}} + \dot{J}^{ij}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}\frac{\partial \dot{J}}{\partial \dot{h}^{ab}}\right)$$

$$+\left(\dot{J}^{ab}\frac{\partial \tilde{H}}{\partial h_{ij}}\dot{h}_{ij}h^{ab} + \dot{J}_{cd}\frac{\partial \tilde{H}}{\partial h_{ij}}\frac{\partial \tilde{H}^{cd}}{\partial h^{ab}}\dot{h}_{ij}h^{ab} + \dot{J}^{cd}\frac{\partial \tilde{H}}{\partial h_{ij}}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}\dot{h}_{ij}h^{ab} + \dot{J}^{ab}\frac{\partial \tilde{H}^{ab}}{\partial h_{ij}}\dot{h}_{ij} + \dot{J}_{cd}\frac{\partial \tilde{H}^{ab}}{\partial h_{ij}}\frac{\partial \tilde{H}^{cd}}{\partial h^{ab}}\dot{h}_{ij} + \dot{J}^{cd}\frac{\partial \tilde{H}^{ab}}{\partial h_{ij}}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}\dot{h}_{ij}\right)$$

$$+\left(\dot{J}^{ab}\frac{\partial \tilde{H}}{\partial h_{cd}}\frac{\partial \tilde{H}}{\partial h^{ab}}\dot{h}_{cd} + \dot{J}^{ab}\frac{\partial \tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial \tilde{H}_{\tilde{c}\tilde{d}}}{\partial h^{ab}}\dot{h}_{cd} + \dot{J}_{ij}\frac{\partial \tilde{H}^{ij}}{\partial h_{ab}}\frac{\partial \tilde{H}}{\partial h_{cd}}\frac{\partial \tilde{H}}{\partial h^{ab}}\dot{h}_{cd} + \dot{J}_{ij}\frac{\partial \tilde{H}^{ij}}{\partial h_{ab}}\frac{\partial \tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial \tilde{H}_{\tilde{c}\tilde{d}}}{\partial h^{ab}}\dot{h}_{cd} + \dot{J}^{ij}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}}{\partial h_{cd}}\frac{\partial \tilde{H}}{\partial h^{ab}}\dot{h}_{cd}\right)$$

$$+\left(\dot{J}^{ab}\frac{\partial \dot{J}}{\partial \dot{h}^{ab}}\frac{\partial \tilde{H}}{\partial h_{ef}}\dot{h}_{ef} + \dot{J}^{ab}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}\frac{\partial \tilde{H}^{cd}}{\partial h_{ef}}\dot{h}_{ef} + \dot{J}_{ij}\frac{\partial \tilde{H}^{ij}}{\partial h_{ab}}\frac{\partial \dot{J}}{\partial \dot{h}^{ab}}\frac{\partial \tilde{H}}{\partial h_{ef}}\dot{h}_{ef} + \dot{J}_{ij}\frac{\partial \tilde{H}^{ij}}{\partial h_{ab}}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}\frac{\partial \tilde{H}^{cd}}{\partial h_{ef}}\dot{h}_{ef} + \dot{J}^{ij}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}\frac{\partial \dot{J}}{\partial \dot{h}^{ab}}\frac{\partial \tilde{H}}{\partial h_{ef}}\dot{h}_{ef}\right)$$

$$+\left(\dot{J}^{ab}\dot{J}_{ab} + 2\dot{J}^{ab}\dot{J}_{cd}\frac{\partial \tilde{H}^{cd}}{\partial h^{ab}} + \dot{J}^{ab}\dot{J}^{cd}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}} + \dot{J}_{cd}\dot{J}_{ij}\frac{\partial \tilde{H}^{ij}}{\partial h_{ab}}\frac{\partial \tilde{H}^{cd}}{\partial h^{ab}} + \dot{J}^{cd}\dot{J}_{ij}\frac{\partial \tilde{H}^{ij}}{\partial h_{ab}}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}} + \dot{J}^{ab}\dot{J}^{ij}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}} + \dot{J}_{cd}\dot{J}^{ij}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}^{cd}}{\partial h^{ab}}\right)$$

$$+2N\left(K\dot{J}^{ab}\frac{\partial \tilde{H}}{\partial h_{ab}} + K^{cd}\dot{J}^{ab}\frac{\partial \tilde{H}_{cd}}{\partial h^{ab}} + K\dot{J}_{ij}\frac{\partial \tilde{H}^{ij}}{\partial h_{ab}}\frac{\partial \tilde{H}}{\partial h_{ab}} + K^{cd}\dot{J}_{ij}\frac{\partial \tilde{H}^{ij}}{\partial h_{ab}}\frac{\partial \tilde{H}_{cd}}{\partial h^{ab}} + K\dot{J}^{ij}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}}{\partial h_{ab}} + K^{cd}\dot{J}^{ij}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}_{cd}}{\partial h^{ab}}\right)$$

$$+\left(\dot{J}^{cd}\dot{J}^{ij}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}} + \dot{J}^{ij}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}\frac{\partial \tilde{H}^{cd}}{\partial h_{ef}}\dot{h}_{ef} + \dot{J}^{ij}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial \tilde{H}_{\tilde{c}\tilde{d}}}{\partial h^{ab}}\dot{h}_{cd}\right)$$

$$+4N^2\left(K_{ab}K\frac{\partial \tilde{H}}{\partial h^{ab}} - KK\frac{\partial \tilde{H}}{\partial h^{ab}}h^{ab} + K_{ab}K^{ij}\frac{\partial \tilde{H}_{ij}}{\partial h_{ab}} - KK^{ij}\frac{\partial \tilde{H}_{ij}}{\partial h_{ab}}h_{ab}\right)$$

$$+4N^2\left(K^2\frac{\partial \tilde{H}}{\partial h^{ab}}\frac{\partial \dot{J}}{\partial \dot{h}^{ab}} + K^{cd}K\frac{\partial \tilde{H}}{\partial h^{ab}}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}} + KK^{ij}\frac{\partial \tilde{H}_{ij}}{\partial h_{ab}}\frac{\partial \dot{J}}{\partial \dot{h}^{ab}} + K^{cd}K^{ij}\frac{\partial \tilde{H}_{ij}}{\partial h_{ab}}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}\right)$$

$$+\dot{J}2N\left(K\frac{\partial \tilde{H}}{\partial h^{ab}}h_{ab} + K\frac{\partial \tilde{H}}{\partial h^{ab}}\frac{\partial \tilde{H}}{\partial h^{ab}} + K\frac{\partial \tilde{H}}{\partial h^{ab}}\frac{\partial \dot{J}}{\partial \dot{h}^{ab}} + K^{cd}\frac{\partial \tilde{H}_{cd}}{\partial h^{ab}}h_{ab} + K^{cd}\frac{\partial \tilde{H}_{cd}}{\partial h^{ab}}\frac{\partial \tilde{H}}{\partial h^{ab}} + K^{cd}\frac{\partial \tilde{H}_{cd}}{\partial h^{ab}}\frac{\partial \dot{J}}{\partial \dot{h}^{ab}}\right)$$

$$+2N\left(K\frac{\partial \tilde{H}}{\partial h^{ab}}\frac{\partial \tilde{H}}{\partial h_{cd}}h_{ab}\dot{h}^{ab} + K\frac{\partial \tilde{H}}{\partial h^{ab}}\frac{\partial \tilde{H}_{ab}}{\partial h_{cd}}\dot{h}_{cd} + K^{ij}\frac{\partial \tilde{H}_{ij}}{\partial h_{ab}}\frac{\partial \tilde{H}}{\partial h_{cd}}h_{ab} + K^{ij}\frac{\partial \tilde{H}_{ij}}{\partial h_{ab}}\frac{\partial \tilde{H}_{ab}}{\partial h_{cd}}\dot{h}_{cd}\right)$$

$$+2N\left(K\frac{\partial \tilde{H}}{\partial h^{ab}}\frac{\partial \tilde{H}}{\partial h_{cd}}\frac{\partial \tilde{H}}{\partial h^{ab}}\dot{h}_{cd} + K\frac{\partial \tilde{H}}{\partial h^{ab}}\frac{\partial \tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial \tilde{H}_{\tilde{c}\tilde{d}}}{\partial h^{ab}}\dot{h}_{cd} + K^{ij}\frac{\partial \tilde{H}_{ij}}{\partial h_{ab}}\frac{\partial \tilde{H}}{\partial h_{cd}}\frac{\partial \tilde{H}}{\partial h^{ab}}\dot{h}_{cd} + K^{ij}\frac{\partial \tilde{H}_{ij}}{\partial h_{ab}}\frac{\partial \tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial \tilde{H}_{\tilde{c}\tilde{d}}}{\partial h^{ab}}\dot{h}_{cd}\right)$$

$$+2N\left(K\frac{\partial \tilde{H}}{\partial h^{ab}}\frac{\partial \dot{J}}{\partial \dot{h}^{ab}}\frac{\partial \tilde{H}}{\partial h_{ef}}\dot{h}_{ef} + K\frac{\partial \tilde{H}}{\partial h^{ab}}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}\frac{\partial \tilde{H}^{cd}}{\partial h_{ef}}\dot{h}_{ef} + K^{ij}\frac{\partial \tilde{H}_{ij}}{\partial h_{ab}}\frac{\partial \dot{J}}{\partial \dot{h}^{ab}}\frac{\partial \tilde{H}}{\partial h_{ef}}\dot{h}_{ef} + K^{ij}\frac{\partial \tilde{H}_{ij}}{\partial h_{ab}}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}\frac{\partial \tilde{H}^{cd}}{\partial h_{ef}}\dot{h}_{ef}\right)$$

$$+2N\left(\dot{J}^{ab}K\frac{\partial \tilde{H}}{\partial h^{ab}} + \dot{J}_{cd}K\frac{\partial \tilde{H}}{\partial h^{ab}}\frac{\partial \tilde{H}^{cd}}{\partial h^{ab}} + \dot{J}^{cd}K\frac{\partial \tilde{H}}{\partial h^{ab}}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}} + \dot{J}^{ab}K^{ij}\frac{\partial \tilde{H}_{ij}}{\partial h_{ab}} + \dot{J}_{cd}K^{ij}\frac{\partial \tilde{H}_{ij}}{\partial h_{ab}}\frac{\partial \tilde{H}^{cd}}{\partial h^{ab}} + \dot{J}^{cd}K^{ij}\frac{\partial \tilde{H}_{ij}}{\partial h_{ab}}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}\right)$$

$$+4N^2\left(KK\frac{\partial \tilde{H}}{\partial h^{ab}}\frac{\partial \tilde{H}}{\partial h_{ab}} + K^{cd}K\frac{\partial \tilde{H}}{\partial h^{ab}}\frac{\partial \tilde{H}_{cd}}{\partial h^{ab}} + KK^{ij}\frac{\partial \tilde{H}_{ij}}{\partial h_{ab}}\frac{\partial \tilde{H}}{\partial h_{ab}} + K^{cd}K^{ij}\frac{\partial \tilde{H}_{ij}}{\partial h_{ab}}\frac{\partial \tilde{H}_{cd}}{\partial h^{ab}}\right)$$

Their sum, organized, is

$$\left(\pi_J^{ab}\pi_{J,ab} + \pi^{ab}\pi_{ab}\right)\left(\frac{32\pi N}{\sqrt{h}}\right)^2$$

$$= +2\dot{J}^{ab}\dot{J}_{ab} + 4\dot{J}^2 + 2h\dot{J}^2$$

$$+ \dot{J}^2\left(\frac{\partial\tilde{H}}{\partial h_{ab}}\frac{\partial\tilde{H}}{\partial h^{ab}} + 2\frac{\partial\tilde{H}}{\partial h_{ab}}h_{ab} + 2\frac{\partial\tilde{H}}{\partial h_{ab}}\frac{\partial\dot{J}}{\partial\dot{h}^{ab}} + 2\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}h_{ab} + \frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\frac{\partial\dot{J}}{\partial\dot{h}^{ab}}\right)$$

$$+ \dot{J}^{ab}\dot{J}_{cd}\frac{\partial\tilde{H}^{cd}}{\partial h^{ab}} + \dot{J}^{ab}\dot{J}^{cd}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}} + \dot{J}^{ab}\dot{J}_{ij}\frac{\partial\tilde{H}^{ij}}{\partial h_{ab}} + \dot{J}_{cd}\dot{J}_{ij}\frac{\partial\tilde{H}^{ij}}{\partial h_{ab}}\frac{\partial\tilde{H}^{cd}}{\partial h^{ab}}$$

$$+ \dot{J}^{cd}\dot{J}_{ij}\frac{\partial\tilde{H}^{ij}}{\partial h_{ab}}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}} + \dot{J}^{ab}\dot{J}^{ij}\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}} + \dot{J}_{cd}\dot{J}^{ij}\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}^{cd}}{\partial h^{ab}} + \dot{J}^{cd}\dot{J}^{ij}\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}$$

$$4N^2\left(3K^{ab}K_{ab} + K^2 + hK^2\right)$$

$$+4N^2\left(2K^{ab}K^{cd}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}} + 2K^{ab}K^{ij}\frac{\partial\tilde{H}_{ij}}{\partial h^{ab}} + K^{cd}K^{ij}\frac{\partial\tilde{H}_{ij}}{\partial h_{ab}}\frac{\partial\tilde{H}_{cd}}{\partial h^{ab}} + K^{cd}K^{ij}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}} + K^{cd}K^{ij}\frac{\partial\tilde{H}_{ij}}{\partial h_{ab}}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}} + K^{cd}K^{ij}\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}_{cd}}{\partial h^{ab}}\right)$$

$$+4N^2\left(K^2\frac{\partial\tilde{H}}{\partial h^{ab}}\frac{\partial\tilde{H}}{\partial h_{ab}} + 2K^2\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}}{\partial h_{ab}} - 2K^2\frac{\partial\tilde{H}}{\partial h^{ab}}h^{ab} + K^2\frac{\partial\dot{J}}{\partial\dot{h}^{ab}}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}} - 2K^2\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}h^{ab}h^{cd} +\right)$$

$$+4N^2\left(2KK^{cd}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}} - 2KK^{cd}\frac{\partial\tilde{H}_{cd}}{\partial h^{ab}}h^{ab} + 2KK^{ab}\frac{\partial\tilde{H}}{\partial h_{ab}} + 2KK^{ab}\frac{\partial\dot{J}}{\partial\dot{h}^{ab}}h^{cd} - 2KK^{cd}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}h^{ab}\right)$$

$$+4N^2\left(+2K^{cd}K\frac{\partial\tilde{H}}{\partial h^{ab}}\frac{\partial\tilde{H}_{cd}}{\partial h_{ab}} + 2K^{cd}K\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}_{cd}}{\partial h^{ab}} + 2KK^{ij}\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}}{\partial h_{ab}}\right)$$

$$+2N\dot{J}\left(2K^{ab}\frac{\partial\tilde{H}}{\partial h^{ab}} + 2K^{ab}\frac{\partial\dot{J}}{\partial\dot{h}^{ab}} +\right)$$

$$+\dot{J}2N\left(2K - 2hK + 2K\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}}{\partial h^{ab}} + 2K\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\frac{\partial\dot{J}}{\partial\dot{h}^{ab}} + 2h_{ab}\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}}K^{ij} + 2K^{ij}\frac{\partial\tilde{H}}{\partial h^{ab}}\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}} + 2\frac{\partial\dot{J}}{\partial\dot{h}^{ab}}\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}}K^{ij}\right)$$

$$+2N\dot{J}\left(+2K^{cd}\frac{\partial\tilde{H}_{cd}}{\partial h^{ab}}h^{ab} + 2K\frac{\partial\tilde{H}}{\partial h_{ab}}\frac{\partial\tilde{H}}{\partial h_{ab}} + 2K^{cd}\frac{\partial\tilde{H}_{cd}}{\partial h^{ab}}\frac{\partial\tilde{H}}{\partial h_{ab}} + 2K\frac{\partial\tilde{H}}{\partial h^{ab}}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}} + 2K^{cd}\frac{\partial\tilde{H}_{cd}}{\partial h^{ab}}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\right)$$

$$+2N\left(+2h\frac{\partial\tilde{H}}{\partial h_{cd}}\dot{h}_{cd}K+4K_{ab}\frac{\partial\tilde{H}^{ab}}{\partial h_{cd}}\dot{h}_{cd}+2K\frac{\partial\tilde{H}}{\partial h_{cd}}\dot{h}_{cd}\right)$$
$$+2N\left(+2K\frac{\partial\tilde{H}}{\partial h^{ab}}\frac{\partial\tilde{H}_{ab}}{\partial h_{cd}}\dot{h}_{cd}+2K^{ij}\frac{\partial\tilde{H}_{ij}}{\partial h_{ab}}\frac{\partial\tilde{H}}{\partial h_{cd}}h_{ab}+2K^{ij}\frac{\partial\tilde{H}_{ij}}{\partial h_{ab}}\frac{\partial\tilde{H}_{ab}}{\partial h_{cd}}\dot{h}_{cd}\right)$$
$$+2N\left(2K^{ab}\frac{\partial\tilde{H}}{\partial h_{cd}}\frac{\partial\tilde{H}}{\partial h^{ab}}\dot{h}_{cd}+2K^{ab}\frac{\partial\tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial\tilde{H}_{\tilde{c}\tilde{d}}}{\partial h^{ab}}\dot{h}_{cd}-2K\frac{\partial\tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial\tilde{H}_{\tilde{c}\tilde{d}}}{\partial h^{ab}}h^{ab}\dot{h}_{cd}\right)$$
$$+2N\left(2K^{ab}\frac{\partial\tilde{H}}{\partial h_{ef}}\frac{\partial\dot{J}}{\partial\dot{h}^{ab}}\dot{h}_{ef}+2K^{ab}\frac{\partial\tilde{H}^{cd}}{\partial h_{ef}}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}\dot{h}_{ef}-2K\frac{\partial\tilde{H}^{cd}}{\partial h_{ef}}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}h^{ab}\dot{h}_{ef}\right)$$
$$+2N\left(+2K\frac{\partial\tilde{H}_{ab}}{\partial h_{cd}}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\dot{h}_{cd}+2K^{ij}\frac{\partial\tilde{H}}{\partial h_{cd}}\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}}h_{ab}\dot{h}_{cd}+2K^{ij}\frac{\partial\tilde{H}_{ab}}{\partial h_{cd}}\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}}\dot{h}_{cd}\right)$$
$$+2N\left(2\dot{J}hK+4K^{ab}\dot{J}^{ab}+2K^{ab}\dot{J}_{cd}\frac{\partial\tilde{H}^{cd}}{\partial h^{ab}}+2K\dot{J}-2K\dot{J}_{cd}\frac{\partial\tilde{H}^{cd}}{\partial h^{ab}}h^{ab}+2K\dot{J}^{ab}\frac{\partial\tilde{H}}{\partial h_{ab}}+2K^{cd}\dot{J}^{ab}\frac{\partial\tilde{H}_{cd}}{\partial h^{ab}}\right)$$
$$+2N\left(-2K\dot{J}^{cd}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}h^{ab}+2K^{ab}\dot{J}^{cd}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}+2K\dot{J}^{ab}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}+2K\dot{J}_{cd}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}^{cd}}{\partial h^{ab}}+2K\dot{J}^{cd}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}+2K^{ij}\dot{J}^{ab}\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}}\right)$$
$$+2N\left(2K^{ij}\dot{J}_{cd}\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}^{cd}}{\partial h^{ab}}+2K^{ij}\dot{J}^{cd}\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}\right)$$
$$+2N\left(2K\dot{J}_{ij}\frac{\partial\tilde{H}^{ij}}{\partial h_{ab}}\frac{\partial\tilde{H}}{\partial h_{ab}}+2K^{cd}\dot{J}_{ij}\frac{\partial\tilde{H}^{ij}}{\partial h_{ab}}\frac{\partial\tilde{H}_{cd}}{\partial h^{ab}}+2K\dot{J}^{ij}\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}}{\partial h_{ab}}+2K^{cd}\dot{J}^{ij}\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}_{cd}}{\partial h^{ab}}\right)$$
$$+2N\left(2K\frac{\partial\dot{J}}{\partial\dot{h}^{ab}}\frac{\partial\tilde{H}^{k\ell}}{\partial h_{ij}}\frac{\partial\tilde{H}_{k\ell}}{\partial h_{ab}}\dot{h}_{ij}+2K^{ij}\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial\tilde{H}_{\tilde{c}\tilde{d}}}{\partial h^{ab}}\dot{h}_{cd}+2K\frac{\partial\tilde{H}}{\partial h^{ab}}\frac{\partial\tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial\tilde{H}_{\tilde{c}\tilde{d}}}{\partial h^{ab}}\dot{h}_{cd}+2K^{ij}\frac{\partial\tilde{H}_{ij}}{\partial h_{ab}}\frac{\partial\tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial\tilde{H}_{\tilde{c}\tilde{d}}}{\partial h^{ab}}\dot{h}_{cd}\right)$$
$$+2N\left(2K\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}}{\partial h_{cd}}\frac{\partial\tilde{H}}{\partial h^{ab}}\dot{h}_{cd}+2K^{ij}\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}}{\partial h_{cd}}\frac{\partial\tilde{H}}{\partial h^{ab}}\dot{h}_{cd}+2K\frac{\partial\dot{J}}{\partial\dot{h}^{ab}}\frac{\partial\tilde{H}}{\partial h_{ij}}\frac{\partial\tilde{H}}{\partial h_{ab}}\dot{h}_{ij}\right)$$
$$+2N\left(2K^{cd}\frac{\partial\tilde{H}_{cd}}{\partial h^{ab}}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}}{\partial h_{k\ell}}\dot{h}_{k\ell}+2K^{cd}\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}_{cd}}{\partial h^{ab}}\frac{\partial\tilde{H}^{ij}}{\partial h_{k\ell}}\dot{h}_{k\ell}+2K\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}}{\partial h_{ab}}\frac{\partial\tilde{H}^{ij}}{\partial h_{k\ell}}\dot{h}_{k\ell}\right)$$
$$+2N\left(2K\frac{\partial\dot{J}}{\partial\dot{h}^{ab}}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}}{\partial h_{ef}}\dot{h}_{ef}+2K\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}\frac{\partial\dot{J}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}^{cd}}{\partial h_{ef}}\dot{h}_{ef}+2K^{ij}\frac{\partial\dot{J}}{\partial\dot{h}^{ab}}\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}}{\partial h_{ef}}\dot{h}_{ef}+2K^{ij}\frac{\partial\dot{J}_{cd}}{\partial\dot{h}^{ab}}\frac{\partial\dot{J}_{ij}}{\partial\dot{h}_{ab}}\frac{\partial\tilde{H}^{cd}}{\partial h_{ef}}\dot{h}_{ef}\right)$$
$$+2N\left(2K^{cd}\frac{\partial\tilde{H}_{cd}}{\partial h^{ab}}\frac{\partial\tilde{H}}{\partial h_{ij}}\frac{\partial\tilde{H}}{\partial h_{ab}}\dot{h}_{ij}+2K\frac{\partial\tilde{H}}{\partial h^{ab}}\frac{\partial\tilde{H}}{\partial h_{cd}}\frac{\partial\tilde{H}}{\partial h^{ab}}\dot{h}_{cd}\right)$$

$$
\begin{aligned}
&+2\dot{J}\left(4\frac{\partial \tilde{H}}{\partial h_{ef}}\dot{h}_{ef}+2h\frac{\partial \tilde{H}}{\partial h_{ef}}\dot{h}_{ef}\right)\\
&+\dot{J}\left(2\dot{J}^{ab}\frac{\partial \dot{J}}{\partial \dot{h}^{ab}}+2\dot{J}^{ij}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}}{\partial h^{ab}}+2\dot{J}^{ij}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}\frac{\partial \dot{J}}{\partial \dot{h}^{ab}}+2\dot{J}_{ij}\frac{\partial \tilde{H}^{ij}}{\partial h_{ab}}\frac{\partial \dot{J}}{\partial \dot{h}^{ab}}\right)\\
&+\dot{J}\left(2\dot{J}^{cd}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}h^{ab}+2\dot{J}^{ab}\frac{\partial \tilde{H}}{\partial h_{ab}}+2\dot{J}_{cd}\frac{\partial \tilde{H}^{cd}}{\partial h^{ab}}\frac{\partial \tilde{H}}{\partial h_{ab}}\right)\\
&+\dot{J}\left(2\frac{\partial \tilde{H}}{\partial h^{ab}}\frac{\partial \tilde{H}}{\partial h_{ij}}h^{ab}\dot{h}_{ij}+4\frac{\partial \dot{J}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}}{\partial h_{k\ell}}\dot{h}_{k\ell}h_{ab}+2\frac{\partial \dot{J}}{\partial \dot{h}^{ab}}\frac{\partial \tilde{H}^{ab}}{\partial h_{ij}}\dot{h}_{ij}\right)\\
&+\dot{J}\left(3\frac{\partial \tilde{H}}{\partial h_{cd}}\dot{h}_{cd}+2\frac{\partial \tilde{H}}{\partial h_{cd}}h\dot{h}_{cd}+2\dot{J}_{ij}\frac{\partial \tilde{H}^{ij}}{\partial h_{ab}}h_{ab}+2\frac{\partial \tilde{H}}{\partial h_{cd}}\frac{\partial \tilde{H}}{\partial h_{ab}}\dot{h}_{cd}h_{ab}+\frac{\partial \tilde{H}}{\partial h_{cd}}\frac{\partial \tilde{H}}{\partial h_{ab}}h\dot{h}_{cd}h_{ab}+2\frac{\partial \tilde{H}_{ab}}{\partial h_{cd}}\frac{\partial \tilde{H}}{\partial h_{ab}}\dot{h}_{cd}\right)\\
&+\dot{J}\left(2\frac{\partial \tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial \tilde{H}_{\tilde{c}\tilde{d}}}{\partial h^{ab}}h^{ab}\dot{h}_{cd}+2\frac{\partial \tilde{H}}{\partial h_{cd}}\frac{\partial \tilde{H}}{\partial h^{ab}}\frac{\partial \tilde{H}}{\partial h_{ab}}\dot{h}_{cd}\right)\\
&+\dot{J}\left(2\frac{\partial \tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial \tilde{H}_{\tilde{c}\tilde{d}}}{\partial h^{ab}}\frac{\partial \tilde{H}}{\partial h_{ab}}\dot{h}_{cd}+2\frac{\partial \tilde{H}}{\partial h_{cd}}\frac{\partial \tilde{H}}{\partial h^{ab}}\frac{\partial \dot{J}}{\partial \dot{h}_{ab}}\dot{h}_{cd}+2\frac{\partial \tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial \tilde{H}_{\tilde{c}\tilde{d}}}{\partial h^{ab}}\frac{\partial \dot{J}}{\partial \dot{h}_{ab}}\dot{h}_{cd}\right)\\
&+\dot{J}\left(+2\frac{\partial \tilde{H}^{cd}}{\partial h_{ef}}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}\frac{\partial \dot{J}}{\partial \dot{h}_{ab}}\dot{h}_{ef}+2\frac{\partial \dot{J}}{\partial \dot{h}^{ab}}\frac{\partial \tilde{H}}{\partial h_{ef}}\frac{\partial \tilde{H}}{\partial h_{ab}}\dot{h}_{ef}\right)\\
&+\dot{J}\left(2\frac{\partial \tilde{H}^{cd}}{\partial h_{ef}}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}\frac{\partial \tilde{H}}{\partial h_{ab}}\dot{h}_{ef}+2\frac{\partial \tilde{H}^{cd}}{\partial h_{ef}}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}h^{ab}\dot{h}_{ef}\right)\\
&+\dot{J}\left(2\frac{\partial \dot{J}}{\partial \dot{h}^{ab}}\frac{\partial \dot{J}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}}{\partial h_{k\ell}}\dot{h}_{k\ell}\right)
\end{aligned}
$$

$$+\left(4\frac{\partial \tilde{H}}{\partial h_{cd}}\frac{\partial \tilde{H}}{\partial h_{ij}}\dot{h}_{ij}\dot{h}_{cd}+2\frac{\partial \tilde{H}}{\partial h_{cd}}\frac{\partial \tilde{H}}{\partial h_{ij}}h\dot{h}_{ij}\dot{h}_{cd}+2\frac{\partial \tilde{H}_{ab}}{\partial h_{cd}}\frac{\partial \tilde{H}^{ab}}{\partial h_{ij}}\dot{h}_{ij}\dot{h}_{cd}\right)$$

$$+2\frac{\partial \tilde{H}}{\partial h_{cd}}\frac{\partial \tilde{H}}{\partial h^{ab}}\frac{\partial \tilde{H}}{\partial h_{ij}}h^{ab}\dot{h}_{ij}\dot{h}_{cd}+2\frac{\partial \tilde{H}}{\partial h_{cd}}\frac{\partial \tilde{H}}{\partial h^{ab}}\frac{\partial \tilde{H}^{ab}}{\partial h_{ij}}\dot{h}_{ij}\dot{h}_{cd}$$

$$+2\frac{\partial \tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial \tilde{H}_{\tilde{c}\tilde{d}}}{\partial h^{ab}}\frac{\partial \tilde{H}^{ab}}{\partial h_{ij}}\dot{h}_{ij}\dot{h}_{cd}+2\frac{\partial \tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial \tilde{H}_{\tilde{c}\tilde{d}}}{\partial h^{ab}}\frac{\partial \tilde{H}}{\partial h_{ij}}h^{ab}\dot{h}_{ij}\dot{h}_{cd}$$

$$+2\dot{J}^{ab}\frac{\partial \tilde{H}}{\partial h_{ij}}\frac{\partial \tilde{H}}{\partial h_{ab}}\dot{h}_{ij}+2\dot{J}_{cd}\frac{\partial \tilde{H}^{cd}}{\partial h^{ab}}\frac{\partial \tilde{H}^{ab}}{\partial h_{ij}}\dot{h}_{ij}$$

$$+2\dot{J}^{cd}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}\frac{\partial \tilde{H}}{\partial h_{ij}}h^{ab}\dot{h}_{ij}+2\dot{J}^{cd}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}\frac{\partial \tilde{H}^{ab}}{\partial h_{ij}}\dot{h}_{ij}+2\dot{J}^{ab}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}\frac{\partial \tilde{H}^{cd}}{\partial h_{ef}}\dot{h}_{ef}$$

$$+2\dot{J}^{ab}\frac{\partial \dot{J}}{\partial \dot{h}^{ab}}\frac{\partial \tilde{H}}{\partial h_{ef}}\dot{h}_{ef}+\dot{J}_{cd}\frac{\partial \tilde{H}}{\partial h_{ij}}\frac{\partial \tilde{H}^{cd}}{\partial h^{ab}}\dot{h}_{ij}h^{ab}$$

$$+2\dot{J}^{ab}\frac{\partial \tilde{H}}{\partial h_{ij}}\dot{h}_{ij}h^{ab}+4\dot{J}^{ab}\frac{\partial \tilde{H}^{ab}}{\partial h_{ij}}\dot{h}_{ij}$$

$$+2\dot{J}_{cd}\frac{\partial \tilde{H}^{cd}}{\partial h^{ab}}\frac{\partial \dot{J}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}}{\partial h_{k\ell}}\dot{h}_{k\ell}+2\dot{J}^{cd}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}\frac{\partial \dot{J}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}}{\partial h_{k\ell}}\dot{h}_{k\ell}+2\dot{J}_{cd}\frac{\partial \tilde{H}^{cd}}{\partial h^{ab}}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}^{ij}}{\partial h_{k\ell}}\dot{h}_{k\ell}+2\dot{J}^{cd}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}^{ij}}{\partial h_{k\ell}}\dot{h}_{k\ell}$$

$$+2\dot{J}_{cd}\frac{\partial \tilde{H}^{cd}}{\partial h^{ab}}\frac{\partial \tilde{H}}{\partial h_{ij}}\frac{\partial \tilde{H}}{\partial h_{ab}}\dot{h}_{ij}+2\dot{J}^{cd}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}\frac{\partial \tilde{H}}{\partial h_{ij}}\frac{\partial \tilde{H}}{\partial h_{ab}}\dot{h}_{ij}$$

$$+2\dot{J}^{ab}\frac{\partial \tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial \tilde{H}_{\tilde{c}\tilde{d}}}{\partial h^{ab}}\dot{h}_{cd}+2\dot{J}_{ij}\frac{\partial \tilde{H}^{ij}}{\partial h_{ab}}\frac{\partial \tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial \tilde{H}_{\tilde{c}\tilde{d}}}{\partial h^{ab}}\dot{h}_{cd}+2\dot{J}^{ij}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial \tilde{H}_{\tilde{c}\tilde{d}}}{\partial h^{ab}}\dot{h}_{cd}$$

$$+2\frac{\partial \tilde{H}}{\partial h_{ef}}\frac{\partial \tilde{H}}{\partial h_{ij}}\frac{\partial \dot{J}}{\partial \dot{h}^{ab}}\dot{h}_{ij}h^{ab}\dot{h}_{ef}+2\frac{\partial \tilde{H}^{cd}}{\partial h_{ef}}\frac{\partial \tilde{H}}{\partial h_{ij}}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}h^{ab}\dot{h}_{ij}\dot{h}_{ef}+2\frac{\partial \tilde{H}}{\partial h_{ef}}\frac{\partial \tilde{H}^{ab}}{\partial h_{ij}}\frac{\partial \dot{J}}{\partial \dot{h}^{ab}}\dot{h}_{ij}\dot{h}_{ef}+2\frac{\partial \tilde{H}^{cd}}{\partial h_{ef}}\frac{\partial \tilde{H}^{ab}}{\partial h_{ij}}\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}\dot{h}_{ij}\dot{h}_{ef}$$

$$+2\frac{\partial \dot{J}}{\partial \dot{h}^{ab}}\frac{\partial \tilde{H}}{\partial h_{ef}}\frac{\partial \tilde{H}}{\partial h_{ij}}\frac{\partial \tilde{H}}{\partial h_{ab}}\dot{h}_{ij}\dot{h}_{ef}+2\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}\frac{\partial \tilde{H}^{cd}}{\partial h_{ef}}\frac{\partial \tilde{H}}{\partial h_{ij}}\frac{\partial \tilde{H}}{\partial h_{ab}}\dot{h}_{ij}\dot{h}_{ef}+2\frac{\partial \dot{J}}{\partial \dot{h}^{ab}}\frac{\partial \tilde{H}}{\partial h_{ef}}\frac{\partial \tilde{H}^{k\ell}}{\partial h_{ij}}\frac{\partial \tilde{H}_{k\ell}}{\partial h_{ab}}\dot{h}_{ij}\dot{h}_{ef}+2\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}\frac{\partial \tilde{H}^{cd}}{\partial h_{ef}}\frac{\partial \tilde{H}^{k\ell}}{\partial h_{ij}}\frac{\partial \tilde{H}_{k\ell}}{\partial h_{ab}}\dot{h}_{ij}\dot{h}_{ef}$$

$$+\frac{\partial \tilde{H}}{\partial h_{cd}}\frac{\partial \tilde{H}}{\partial h^{ab}}\frac{\partial \tilde{H}}{\partial h_{ij}}\frac{\partial \tilde{H}}{\partial h_{ab}}\dot{h}_{ij}\dot{h}_{cd}+\frac{\partial \tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial \tilde{H}_{\tilde{c}\tilde{d}}}{\partial h^{ab}}\frac{\partial \tilde{H}}{\partial h_{ij}}\frac{\partial \tilde{H}}{\partial h_{ab}}\dot{h}_{ij}\dot{h}_{cd}$$

$$+\frac{\partial \tilde{H}}{\partial h_{cd}}\frac{\partial \tilde{H}}{\partial h^{ab}}\frac{\partial \tilde{H}^{k\ell}}{\partial h_{ij}}\frac{\partial \tilde{H}_{k\ell}}{\partial h_{ab}}\dot{h}_{ij}\dot{h}_{cd}+\frac{\partial \tilde{H}^{\tilde{c}\tilde{d}}}{\partial h_{cd}}\frac{\partial \tilde{H}_{\tilde{c}\tilde{d}}}{\partial h^{ab}}\frac{\partial \tilde{H}^{k\ell}}{\partial h_{ij}}\frac{\partial \tilde{H}_{k\ell}}{\partial h_{ab}}\dot{h}_{ij}\dot{h}_{cd}$$

$$+\left(\frac{\partial \dot{J}}{\partial \dot{h}^{ab}}\frac{\partial \tilde{H}}{\partial h_{ef}}\frac{\partial \dot{J}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}}{\partial h_{k\ell}}\dot{h}_{k\ell}\dot{h}_{ef}+\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}\frac{\partial \tilde{H}^{cd}}{\partial h_{ef}}\frac{\partial \dot{J}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}}{\partial h_{k\ell}}\dot{h}_{k\ell}\dot{h}_{ef}+\frac{\partial \dot{J}}{\partial \dot{h}^{ab}}\frac{\partial \tilde{H}}{\partial h_{ef}}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}^{ij}}{\partial h_{k\ell}}\dot{h}_{k\ell}\dot{h}_{ef}+\frac{\partial \dot{J}_{cd}}{\partial \dot{h}^{ab}}\frac{\partial \tilde{H}^{cd}}{\partial h_{ef}}\frac{\partial \dot{J}_{ij}}{\partial \dot{h}_{ab}}\frac{\partial \tilde{H}^{ij}}{\partial h_{k\ell}}\dot{h}_{k\ell}\dot{h}_{ef}\right)$$

Concluding Remarks

In the paper, a subset of the theory was considered, namely that of the one where the conformal part is surpressed $\tilde{H}=0$ and that the trace of J^{ab} and $\dot{J}^{ab}$ vanishes. However with the steps outlined in this supplementary material, it should be easier for the reader to consider cases that are more general.