ME581 Homework 7
Due 12/09 2025

Instructions:
- **The following problems are to be documented, solved, and presented in Jupyter notebooks.**
- **Save the notebook as a single PDF, then upload and submit the PDF in Gradescope. Assign your pages to the specific problem they correspond to.**
- **Implement your own code for the specified problems.**

Problem 1

Consider a 1-D heat equation, the h here can represent temperature:
$$\frac{\partial h}{\partial t} = k \frac{\partial^2 h}{\partial x^2}$$

Solve the equation over the domain $x \in [0,100]$, with the boundary conditions: $h(0, t) = 0$, $h(100, t) = \sin\left(\frac{\pi}{10} t\right)$ and initial condition: $h(x, 0) = 0$. Use steps $\Delta x = 1$ and $\Delta t = 0.01$.

(a) Choose your preferred numerical method to discretize the PDE. State your method and write down the discretized version.

(b) Solve this partial differential equation with $k = 50$ and determine $h(x, t)$ at different time $t = 0, 4, \ldots \ldots, 16, 20, 30, 50$. Plot the results with both axes and legend.

(c) Plot the temperature value h at $x = 80$, $t \in [0, 50]$, with different diffusion rates $k = 5, 50, 500$ (Adjust the time step size if the solution diverges).

(d) What did you observe from the results of (b) and (c)? How does the k value affect the output? Explain this physical phenomena.

Problem 2

Consider the wave equation:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

$$0 \leq x \leq L, t \geq 0$$

Initial Conditions: Suppose the initial wave shape is a Gaussian centered at $x = 0.5$ with some width:

$$u(x, 0) = e^{-100(x-0.5)^2}$$

$$\frac{\partial u}{\partial t}(x, 0) = 0$$

Boundary Conditions: Assume Dirichlet boundary conditions where the displacement is zero at the boundaries:

$$u(0, t) = 0$$

$$u(L, t) = 0$$

For simplicity, let $c = 1$, $L = 1$. Use the spatial step and the time step as $\Delta x = 0.01$, $\Delta t = 0.005$.

(a) Choose your preferred numerical method to discretize the PDE. State your method and write down the discretized version.

(b) Calculate and plot $u(x, t)$ at different time:

$$t = 0.00 \ s, 0.25 \ s, 0.50 \ s, 0.75 \ s, 1.00 \ s, 5.00 \ s, 7.50 \ s.$$

(c) Plot the value u at $x = 0.25 \ and \ x = 0.5, t \in [0, 10]$.