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Transformaciones Lineales

1.- Verificar que la transformación $T: R^2 \to R^2$ tal que $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -x \\ -y \end{pmatrix}$ cumple la definición de una transformación lineal

Para verificar se deben cumplir las dos propiedades por definición

* $T(u+v) = T(u) + T(v)$
* $T(\lambda u) = \lambda T(u)$

1.- $T(u+v)$ $u=(x_1, y_1)$ $v=(x_2, y_2)$

$$T(u+v) = T\left(\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}\right) = T\begin{pmatrix} x_1+x_2 \\ y_1+y_2 \end{pmatrix} = \left(\begin{matrix} -(x_1+x_2) \\ -(y_1+y_2) \end{matrix}\right) = \begin{pmatrix} -x_1-x_2 \\ -y_1-y_2 \end{pmatrix}$$

$$= \begin{pmatrix} -x_1 \\ -y_1 \end{pmatrix} + \begin{pmatrix} -x_2 \\ -y_2 \end{pmatrix} = T\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + T\begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = T(u) + T(v)$$

Se cumple

2.- $\lambda \in R$ $u = \begin{pmatrix} x \\ y \end{pmatrix}$

$$T(\lambda u) = T\left(\lambda\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}\right) = T\left(\begin{pmatrix} \lambda x \\ \lambda y \end{pmatrix}\right) = \begin{pmatrix} -\lambda x \\ -\lambda y \end{pmatrix} = \lambda\begin{pmatrix} -x \\ -y \end{pmatrix} = \lambda T\begin{pmatrix} x \\ y \end{pmatrix} = \lambda T(u)$$

se cumple

$\therefore \left(T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -x \\ -y \end{pmatrix}\right)$ es una transformación lineal

2.- Dados los vectores $u_1=(2,-1)$, $u_2=(1,1)$ $u_3=(-1,-4)$
$V_1=(1,3)$, $V_2=(2,3)$, $V_3=(-5,-6)$

Jutifique si existe una transformación lineal $A: R^2 \to R^2$ tal que $Au_1=V_1$, $Au_2=V_2$ y $Au_3=V_3$

$A=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $AU_1=V_1$ $AU_2=V_2$ $AU_3=V_3$

Para AV_1

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \quad \begin{matrix} 2a-b=1 \\ 2c-d=3 \end{matrix}$$

Para AV_2=

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad \begin{matrix} a+b=2 \\ c+d=3 \end{matrix}$$

Para AV_3 =

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} -1 \\ -4 \end{pmatrix} = \begin{pmatrix} -5 \\ -6 \end{pmatrix} \quad \begin{matrix} -a-4b=-5 \\ -c-4d=-6 \end{matrix}$$

Ocupando AV_1 y AV_2

$$\begin{array}{r} 2a-b=1 \\ + \quad a+b=2 \\ \hline 3a \quad =3 \\ a=1 \end{array} \qquad \begin{array}{r} 2c-d=3 \\ c+d=3 \\ \hline 3c \quad =6 \\ c=2 \end{array}$$

AV_1

$2(1)-b=1$
$2-b=1$
$b=1$
$2(2)-d=3$
$4-d=3$
$d=1$

sust. AV_2

$1+1=2$ ✓
$2+1=3$ ✓

Reflejando los valores para Av_3

$a=1 \quad b=1 \quad c=2 \quad d=1$ comprobando

$-a-4b=-5$
$-1-4(1)=-5$
$-5=-5$
$-c-4d=-6$
$-2-4(1)=-6$
$-2-4=-6$
$-6=-6$

$A=\begin{pmatrix}1&1\\2&1\end{pmatrix}$

$Av_2=V_2$

$\begin{pmatrix}1&1\\2&1\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}2\\3\end{pmatrix}$

$\begin{pmatrix}1+1\\2+1\end{pmatrix}=\begin{pmatrix}2\\3\end{pmatrix}$

$\begin{pmatrix}2\\3\end{pmatrix}=\begin{pmatrix}2\\3\end{pmatrix}$

$Av_1=v_2$

$\begin{pmatrix}1&1\\2&1\end{pmatrix}\begin{pmatrix}2\\-1\end{pmatrix}=\begin{pmatrix}1\\3\end{pmatrix}$

$\begin{pmatrix}2-1\\4-1\end{pmatrix}=\begin{pmatrix}1\\3\end{pmatrix}\Rightarrow\begin{pmatrix}1\\3\end{pmatrix}=\begin{pmatrix}1\\3\end{pmatrix}$

Para $Av_3=V_3$

$\begin{pmatrix}1&1\\2&1\end{pmatrix}\begin{pmatrix}-1\\-4\end{pmatrix}=\begin{pmatrix}-5\\-6\end{pmatrix}$

$\begin{pmatrix}-1-4\\-2-4\end{pmatrix}=\begin{pmatrix}-5\\-6\end{pmatrix}$

$\begin{pmatrix}-5\\-6\end{pmatrix}=\begin{pmatrix}-5\\-6\end{pmatrix}$

3. Determine la matriz del operador $A: R^2 \to R^2$ de modo que transforme los vectores $v=(1,2)$ y $v=(3,4)$ a $Au=(1,1)$ y $Av=(2,2)$

Conociendo $Av=v$

$A=\begin{pmatrix}a&b\\c&d\end{pmatrix}$ $\quad Au=v_A=\begin{pmatrix}a&b\\c&d\end{pmatrix}\begin{pmatrix}1\\2\end{pmatrix}=\begin{pmatrix}1\\1\end{pmatrix}$ $\quad a+2b=1$, $c+2d=1$

$Av=u_A=\begin{pmatrix}a&b\\c&d\end{pmatrix}\begin{pmatrix}3\\4\end{pmatrix}=\begin{pmatrix}2\\2\end{pmatrix}$ $\quad 3a+4b=2$, $3c+4d=2$

Au con Av

$\left(\begin{array}{c|c}a+2b&1\\3a+4b&2\end{array}\right)\to\left(\begin{array}{c|c}a+2b&1\\0-2b&-1\end{array}\right)\to\left(\begin{array}{c|c}a-0&0\\0-2b&-1\end{array}\right)\to \begin{matrix}a=0\\b=\frac{1}{2}\end{matrix}$

$\left(\begin{array}{c|c}c+2d&1\\3c+4d&2\end{array}\right)\to\left(\begin{array}{c|c}c+2d&1\\0-2d&-1\end{array}\right)\to\left(\begin{array}{c|c}c-0&0\\0-2d&-1\end{array}\right)\to \begin{matrix}c=0\\d=\frac{1}{2}\end{matrix}$

Comprobamos

$\begin{pmatrix}0&\frac{1}{2}\\0&\frac{1}{2}\end{pmatrix}\begin{pmatrix}1\\2\end{pmatrix}=\begin{pmatrix}1\\1\end{pmatrix}\Rightarrow\begin{pmatrix}0+1\\0+1\end{pmatrix}=\begin{pmatrix}1\\1\end{pmatrix}\Rightarrow\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}1\\1\end{pmatrix}$

$\begin{pmatrix}0&\frac{1}{2}\\0&\frac{1}{2}\end{pmatrix}\begin{pmatrix}3\\4\end{pmatrix}=\begin{pmatrix}2\\2\end{pmatrix}\Rightarrow\begin{pmatrix}0+2\\0+2\end{pmatrix}=\begin{pmatrix}2\\2\end{pmatrix}\Rightarrow\begin{pmatrix}2\\2\end{pmatrix}=\begin{pmatrix}2\\2\end{pmatrix}$

$A=\begin{pmatrix}0&\frac{1}{2}\\0&\frac{1}{2}\end{pmatrix}$

4.- Dada la expresión general de una transformación lineal $f: \mathbb{R}^3 \to \mathbb{R}^2$ tal que $f(x,y,z) = ax + by + cz$. Determinar la transformación lineal que asocie los vectores $u=(1,2,3)$ con 1, $v=(-1,2,3)$ con 0 y $u=(1,-2,3)$ con 0

dada $f(x,y,z) = ax + bx + cz$

$A = (a,b,c)$

$AV = U$

Para $v=(-1,2,3)=0$

$AV = 0$

$(a,b,c)(-1,2,3) = 0$

$-a+2b+3c=0$

Para $V=(1,-2,3)=0$

$Av = 0$

$(a,b,c)(1,-2,3)=0$

$a-2b+3c=0$

Para $u=(1,2,3)=1$

$Av=u$

$(a,b,c)(1,2,3)=1$

$a+2b+3c=1$

$$\begin{pmatrix} a+2b+3c=1 \\ -a+2b+3c=0 \\ a-2b+3c=0 \end{pmatrix} = \left(\begin{array}{ccc|c} 1&2&3&1 \\ -1&2&3&0 \\ 1&-2&3&0 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1&2&3&1 \\ 0&4&6&1 \\ 0&-4&0&-1 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1&2&3&1 \\ 0&-4&0&-1 \\ 0&4&6&1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1&2&3&1 \\ 0&-4&0&-1 \\ 0&0&6&0 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1&2&0&1 \\ 0&1&0&+1/4 \\ 0&0&1&0 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1&0&0&1/2 \\ 0&1&0&1/4 \\ 0&0&1&0 \end{array}\right)$$

Comprobamos a $(1/2, 1/4, 0)$

$(1/2, 1/4, 0)(1,2,3) = 1$

$1/2 + 1/2 = 1$

$1 = 1$

$(1/2, 1/4, 0)(-1,2,3) = 0$

$-1/2 + 1/2 = 0$

$0 = 0$

$(1/2, 1/4, 0)(1,-2,3) = 0$

$1/2 - 1/2 + 0 = 0$

$0 = 0$

5.- Determinar la transformación $T: \mathbb{R}^3 \to \mathbb{R}^3$ tal que $T(1,0,1) = (1,0,0)$, $T(0,1,1) = (0,1,0)$, $T(0,0,1) = (1,1,1)$

$T = M_{3\times 3}$

$A(V_1) = U_1$ $\begin{pmatrix} a&b&c \\ d&e&f \\ g&h&i \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} a+0+c \\ d+0+f \\ g+0+i \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$AV_2 = U_2$ $\begin{pmatrix} a&b&c \\ d&e&f \\ g&h&i \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 0+b+c \\ 0+e+f \\ 0+h+i \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$AV_3 = U_3$ $\begin{pmatrix} a&b&c \\ d&e&f \\ g&h&i \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} c \\ f \\ i \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$c=1$ $\quad a+1=1$ $\quad a=0$ $\quad b+1=0$ $\quad b=-1$

$f=1$ $\quad d+1=1$ $\quad d=0$ $\quad e+1=1$ $\quad e=0$

$i=1$ $\quad g+1=1$ $\quad g=0$ $\quad h+1=0$ $\quad h=-1$

$$T = \begin{pmatrix} 0&-1&1 \\ 0&0&1 \\ 0&-1&1 \end{pmatrix}$$

Transformaciones y Cambios de Base

Sea $T: \mathbb{R}^2 \to \mathbb{R}^2$ tal que $T(x,y) = (x-2y, 3x+y)$ y sea $B = \{(1,1),(0,3)\}$ y $\Gamma = \{(-1,2),(-1,0)\}$

- Calcular la matriz asociada a T, $[T]^{\Gamma}_{B}$
- Calcular los vectores $[u]_B$ y $[Tu]_\Gamma$ con $u=(1,1)$
- Verificar que $[T]^{\Gamma}_{B}[u]_B = [Tu]_\Gamma$

1.- Aplicamos T a B

$T(1,1) = (1-2(1), 3(1)+(1)) = (-1,4)$

$T(0,3) = (0-2(3), 3(0)+3) = (-6,3)$

$$[T]^{\Gamma}_{B} = \left(\begin{array}{cc|cc} -1 & -1 & -1 & -6 \\ 2 & 0 & 4 & 3 \end{array}\right) \Rightarrow \left(\begin{array}{cc|cc} -1 & -1 & -1 & -6 \\ 0 & -2 & 2 & -9 \end{array}\right) \Rightarrow \left(\begin{array}{cc|cc} -1 & -1 & -1 & -6 \\ 0 & 1 & -1 & 9/2 \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{cc|cc} -1 & 0 & -2 & -3/2 \\ 0 & 1 & -1 & 9/2 \end{array}\right) \Rightarrow \left(\begin{array}{cc|cc} 1 & 0 & 2 & 3/2 \\ 0 & 1 & -1 & 9/2 \end{array}\right) \quad \begin{pmatrix} 2 & 3/2 \\ -1 & 9/2 \end{pmatrix}$$

2.- $[u]_B$ y $[Tu]_\Gamma$ con $u=(1,1)$

$$[u]_B = \left(\begin{array}{cc|c} 1 & 0 & 1 \\ 1 & 3 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{cc|c} 1 & 3 & 1 \\ 1 & 0 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{cc|c} 1 & 3 & 1 \\ 0 & -3 & 0 \end{array}\right) = \left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 0 \end{array}\right)$$

$[Tu]_\Gamma = T(1,1) = (-1,4)$

$$= 2\,T = \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$$

3.- $[T]^{\Gamma}_{B}[u]_B = [Tu]_\Gamma$

$$[T]^{\Gamma}_{B}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 & 3/2 \\ -1 & 9/2 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$ se cumple

2.- Sea $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ y $B = \{(1,1),(-1,3)\}$ y $\Gamma = \{(0,-2),(-5,1)\}$

- Calcular la matriz cambio de ~~base~~ base $Q_{B\to\Gamma}$
- Calcular la matriz asociada a T $[T]^{B}_{B}$
- Calcular la matriz asociada a T $[T]_\Gamma$

1.- Para $(1,1)$

$$\left(\begin{array}{cc|c} 0 & -5 & 1 \\ -2 & 1 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{cc|c} -2 & 1 & 1 \\ 0 & -5 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{cc|c} 2 & -1 & -1 \\ 0 & 1 & -1/5 \end{array}\right)$$

$$\left(\begin{array}{cc|c} 2 & 0 & -6/5 \\ 0 & 1 & -1/5 \end{array}\right) \Rightarrow \left(\begin{array}{cc|c} 1 & 0 & -3/5 \\ 0 & 1 & -1/5 \end{array}\right)$$

Para $(-1,3)$

$$\left(\begin{array}{cc|c} 0 & -5 & -1 \\ -2 & 1 & 3 \end{array}\right) \Rightarrow \left(\begin{array}{cc|c} 1 & 0 & 1/5 \\ 0 & 1 & -7/5 \end{array}\right)$$

$$Q_{B\to\Gamma} = \begin{pmatrix} -3/5 & -7/5 \\ -1/5 & 1/5 \end{pmatrix}$$

2.- $A=\begin{pmatrix}0&1\\-1&0\end{pmatrix}$

$AB_1=\begin{pmatrix}0&1\\-1&0\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}1\\-1\end{pmatrix}$

Para $(1,-1)$

$\left(\begin{array}{cc|c}1&-1&1\\1&3&-1\end{array}\right)\Rightarrow\left(\begin{array}{cc|c}1&-1&1\\0&4&-2\end{array}\right)\Rightarrow\left(\begin{array}{cc|c}1&0&-1/2\\0&1&-1/2\end{array}\right)$

$AB_2=\begin{pmatrix}0&1\\-1&0\end{pmatrix}\begin{pmatrix}-1\\3\end{pmatrix}=\begin{pmatrix}3\\1\end{pmatrix}$

Para $(3,1)$

$\left(\begin{array}{cc|c}1&-1&3\\1&3&1\end{array}\right)\Rightarrow\left(\begin{array}{cc|c}1&0&2\\0&1&-1\end{array}\right)$

$[T]_B^B=\begin{pmatrix}-1/2&2\\-1/2&-1\end{pmatrix}$

3.- $Q_{\Gamma\to B}[T]_B^B Q_{B\to\Gamma}$

$Q_{B\to\Gamma}=Q_{\Gamma\to B}^{-1}$ $\left(\begin{array}{cc|cc}-3/5&-7/5&1&0\\-1/5&1/5&0&1\end{array}\right)\Rightarrow\left(\begin{array}{cc|cc}1&0&5/4&7/4\\0&1&5/4&-3/4\end{array}\right)$

$\begin{pmatrix}-3/5&-7/5\\-1/5&1/5\end{pmatrix}\begin{pmatrix}-1/2&2\\-1/2&-1\end{pmatrix}\begin{pmatrix}5/4&7/4\\5/4&-3/4\end{pmatrix}\Rightarrow\begin{pmatrix}-1&0\\0&-1\end{pmatrix}$

Sea $A=\begin{pmatrix}1&2\\-1&0\end{pmatrix}$ sea $T_v=A_v$ y sean $B=\{(1,0),(0,1)\}$ $\Gamma=\{(1,1),(1,-1)\}$

- Calcular $[T_A]_B$
- Calculal la matriz $Q_{B\to\Gamma}$
- Calcular $[T]$.

1.- Dado $T_v=A_v$

$[T_A]_B=A=\begin{pmatrix}1&2\\-1&0\end{pmatrix}$

2.- $Q_{B\to\Gamma}$

$\left(\begin{array}{cc|c}1&1&1\\1&-1&0\end{array}\right)\Rightarrow\left(\begin{array}{cc|c}1&1&1\\0&-2&-1\end{array}\right)\Rightarrow\left(\begin{array}{cc|c}1&0&1/2\\0&1&1/2\end{array}\right)$ $\begin{pmatrix}1/2&1/2\\1/2&-1/2\end{pmatrix}$

$\left(\begin{array}{cc|c}1&1&0\\1&-1&1\end{array}\right)\Rightarrow\left(\begin{array}{cc|c}1&0&1/2\\0&1&-1/2\end{array}\right)$

3.- $[T_A]^{-1}$

$[T_A]=\left(\begin{array}{cc|cc}1&2&1&0\\-1&0&0&1\end{array}\right)\Rightarrow\left(\begin{array}{cc|cc}1&2&1&0\\0&2&1&1\end{array}\right)\Rightarrow\left(\begin{array}{cc|cc}1&0&0&-1\\0&1&1/2&1/2\end{array}\right)[T_A]^{-1}=\begin{pmatrix}0&-1\\1/2&1/2\end{pmatrix}$

4: Considere las Bases de $\mathbb{R}^3$ y $B = \{(1,0,0),(0,1,0),(0,0,1)\}$ y $\Gamma = \{(1,0,1),(2,1,2),(1,2,2)\}$ $A \begin{vmatrix} 1 & 3 & -2 \\ 2 & -4 & 1 \\ 3 & -1 & 2 \end{vmatrix}$

- Encontrar la matriz cambio de base $Q_{B \to \Gamma}$
- $[T]_B$
- $[T]_\Gamma$

1: Dado que B esta en la base canonica

$$Q_{B \to \Gamma} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 1 & 1 & 2 \end{bmatrix}$$

2: $[T]_B$ B es la base canonica xd

$$[T]_B = \begin{pmatrix} 1 & 3 & -2 \\ 2 & -4 & 1 \\ 3 & -1 & 2 \end{pmatrix}$$

3: Calculamos la inversa de $Q_{B \to \Gamma}$

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & -1 & 1 & -1 & 0 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 3 & -1 & 1 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & -3 & 1 & -2 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 3 & -1 & 1 & 1 \end{array}\right) \Rightarrow$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -1 & 1 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 3 & -1 & 1 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -1 & 1 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1/3 & 1/3 & 1/3 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 2/3 & 1/3 & -2/3 \\ 0 & 0 & 1 & -1/3 & 1/3 & 1/3 \end{array}\right)$$

$$Q_{B \to \Gamma}^{-1} \Rightarrow \begin{pmatrix} 0 & -1 & 1 \\ 2/3 & 1/3 & -2/3 \\ -1/3 & 1/3 & 1/3 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -1 & 1 \\ 2/3 & 1/3 & -2/3 \\ 1/3 & 1/3 & 1/3 \end{pmatrix} \begin{pmatrix} 1 & 3 & -2 \\ 2 & -4 & 1 \\ 3 & -1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 1 & 1 & 2 \end{pmatrix} \Rightarrow \begin{pmatrix} 7 & -17 & -10 \\ -5 & 11 & -6 \\ -1 & -1 & 0 \end{pmatrix}$$

5.- Considere el operador $G: \mathbb{R}^2 \to \mathbb{R}^2$, $G(x,y) = (2x-7y, 4x+3y)$
y $S = \{(1,3),(2,5)\}$

- Calcular la matriz cambio de base
- Usando $Q_{C \to S}$ encontrar $[G]_S$
- Comprobar que $[Gv]_S = [G]_S [v]_S$

1.- $S = \{(1,3),(2,5)\}$

$\mathbb{R}^2 = \{(1,0),(0,1)\}$ $\quad \left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 3 & 5 & 0 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{cc|cc} 1 & 0 & 5 & 5 \\ 0 & 1 & -2 & -2 \end{array}\right)$

$Q_{C \to S} = \begin{pmatrix} 5 & 5 \\ -2 & -2 \end{pmatrix}$

2.- $G(x,y) = (2x-7y, 4x+3y)$

$G(1,3) = (2(1)-7(3), 4(1)+3(3)) = (-19, 13)$
$G(2,5) = (2(2)-7(5), 4(2)+3(5)) = (-27, 23)$

$\left(\begin{array}{cc|cc} 1 & 2 & -19 & -27 \\ 3 & 5 & 13 & 23 \end{array}\right) \Rightarrow \left(\begin{array}{cc|cc} 1 & 0 & -11 & -13 \\ 0 & 1 & -4 & -7 \end{array}\right)$ $\quad [G]_S = \begin{pmatrix} -11 & -13 \\ -4 & -7 \end{pmatrix}$

3.- $[Gv]_S = [G]_S [v]_S$

$[Gv]_S = \left(\begin{array}{cc|c} 1 & 2 & x \\ 3 & 5 & y \end{array}\right) \Rightarrow \left(\begin{array}{cc|c} 1 & 0 & -11x-13y \\ 0 & 1 & -4x-7y \end{array}\right)$

$[G]_S [v_S] = \begin{pmatrix} -11x-13y \\ -4x-7y \end{pmatrix} = [Gv]_S \Rightarrow \begin{pmatrix} -11x-13y \\ -4x-7y \end{pmatrix}$

4.- Espacios de una Transformación lineal

a) Para $T: R^3 \to R^3$ definida como

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x-2y \\ y+3z \\ 2x-3y+3z \end{pmatrix}$$

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x-2y \\ y+3z \\ 2x-3y+3z \end{pmatrix} = x\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + y\begin{pmatrix} -2 \\ 1 \\ -3 \end{pmatrix} + z\begin{pmatrix} 0 \\ 3 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 3 \\ 2 & -3 & 3 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} x-2y \\ y+3z \\ 0 \end{pmatrix}$$

$y = -3z$

$x-(3z)2 = x+6z$

$x = -6z$

nulidad (1)

$$V = z\begin{pmatrix} -6 \\ -3 \\ z \end{pmatrix}$$

Nulidad = 1

Rango = 2

$$\left\{\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}\right\}$$

b) Para $T: R^4 \to R^4$ definida como

$$T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} x_1-x_4 \\ x_2-x_3 \\ x_1+x_2-x_3-x_4 \end{pmatrix} \Rightarrow x_1\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + x_2\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + x_3\begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix} + x_4\begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 1 & 1 & -1 & -1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} =$$

$x_1 - x_4 = 0 \quad x_1 = x_4$

$x_2 - x_3 = 0 \quad x_2 = x_3$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} x_4 \\ x_3 \\ x_3 \\ x_4 \end{pmatrix} = x_3\begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} + x_4\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$V = x_3\begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} + x_4\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \left\{\begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}\right\} \quad nul(T) = 2$$

$$\text{Rango} = \left\{\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}\right\} = 2$$

2.- En el espacio vectorial de funciones continuas sea $W = \{g \mid sen, cos\}$
Determinar la nulidad de $\varphi : W \to \mathbb{R}$ dado por:

a) $\varphi(f) = \int_0^{\pi} f(s)\,ds$

b) $\varphi(f) = f'(0)$

a) $f(x) = a\,sen(x) + b\cos(x)$

$$\varphi(f) = \int_0^{\pi} (a\,sen(s) + b\cos(s))\,ds = a\int_0^{\pi} sen(s)\,ds + b\int_0^{\pi} b\cos(s)\,ds$$

$-\cos(\pi) + \cos(0) = 2$ $\qquad$ $sen(\pi) - sen(0) = 0$

$\varphi(f) = a \cdot 2 + b \cdot 0 = 2a$ $\qquad$ para que $\varphi(f)$ sea cero $a = 0$
de modo que $f(x) = b\cos(x)$ es un espacio nulo

$nul(\varphi(f)) = 1$

b) $\varphi(f) = f'(0)$

Para $f(x) = a\,sen(x) + b\cos(x)$, la derivada de f es:

$f'(x) = a\cos(x) - b\,sen(x)$

de modo que al evaluarla en cero

$a\cos(0) - b\,sen(0) = a \cdot 1 - 0 = a$

Por lo tanto para que $\varphi(f) = 0$ necesitamos que $a = 0$ por lo tanto tambien incluy las funciones de la forma $b\cos(x)$ por lo tanto su nulidad es 1

$nul(f'(0)) = 1$

3.- Considerando la base canonica en $\mathbb{R}^3$ describir al espacio nulo para la transformacion y la imagen

$T(x, y, z) = (x - 2y + z,\ 2x - 3y + z,\ x + y - 2z)$

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x - 2y + z \\ 2x - 3y + z \\ x + y - 2z \end{pmatrix}$$

$$T = \begin{pmatrix} 1 & -2 & 1 \\ 2 & -3 & 1 \\ 1 & 1 & -2 \end{pmatrix} \qquad \ker(T) = \left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 2 & -3 & 1 & 0 \\ 1 & 1 & -2 & 0 \end{array}\right) \Rightarrow \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$x - 2y + z = 0$
$y - z = 0$

$y = z$

$x - 2z + z = 0$
$x - z = 0$
$x = z$

$= z\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ $\qquad \ker(T) = \left\{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\right\}$

nulidad$(T) = 1$

$$Imagen(T) = \left\{\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix}\right\}$$

Rango$(T) = 2$

Diagonalización

Determinar si $\begin{pmatrix}3&2\\-2&-1\end{pmatrix}$ es diagonalizable

$$\begin{pmatrix}3-\lambda & 2\\ -2 & -1-\lambda\end{pmatrix} = -3-3\lambda+\lambda+\lambda^2+4 = \lambda^2-2\lambda+1 = (\lambda-1)^2$$

$$\lambda = 1$$

$$\begin{pmatrix}3-1 & 2\\ -2 & -1-1\end{pmatrix} = \begin{pmatrix}2 & 2\\ -2 & -2\end{pmatrix} = \begin{pmatrix}2 & 2\\ 0 & 0\end{pmatrix} = x=-y \quad x=\begin{pmatrix}-1\\ 1\end{pmatrix}$$

dado que unicamente se cuenta con un vector propio no es diagonalizable

Explicar para que la matriz $\begin{pmatrix}5&2\\2&4\end{pmatrix}$ es diagonalizable

$$\begin{pmatrix}5-\lambda & 2\\ 2 & 4-\lambda\end{pmatrix} = \lambda^2-9\lambda+16 \qquad \lambda_1 = \frac{9+\sqrt{17}}{2} \quad \lambda_2 = \frac{9-\sqrt{17}}{2}$$

Sabiendo que tiene dos valores propios, esta misma es diagonalizable dado que se generan dos vectores propios

Explicar porque $\begin{pmatrix}5&-2&6\\0&-1&9\\0&0&3\end{pmatrix}$ es diagonalizable

$$\begin{pmatrix}5-\lambda & -2 & 6\\ 0 & -1-\lambda & 9\\ 0 & 0 & 3-\lambda\end{pmatrix} = 5-\lambda(-1-\lambda(3-\lambda)-0) = (5-\lambda)(-\lambda-1)(3-\lambda)$$

$$\lambda=5 \quad \lambda=1 \quad \lambda=3$$

dado que tiene 3 λ diferentes es diagonalizable al ser li

Determinar si la matriz $A=\begin{pmatrix}1&3&0\\2&2&1\\-4&0&-2\end{pmatrix}$ es diagonalizable

$$\begin{pmatrix}1-\lambda & 3 & 0\\ 2 & 2-\lambda & 1\\ -4 & 0 & -2-\lambda\end{pmatrix} =$$

$$(1-\lambda)((2-\lambda)(-2-\lambda)-0)-3(-4-2\lambda+4)$$

$$(1-\lambda)(2-\lambda)^2-3(-2\lambda) = (1-\lambda)(2-\lambda^2)+6\lambda$$

$$(1-\lambda)(4-2\lambda+\lambda^2)-(6\lambda) = 4-2\lambda+\lambda^2-4\lambda+2\lambda^2-\lambda$$

$$4-12\lambda+3\lambda^2-\lambda^3$$

$$(\lambda-2)(\lambda^2+\lambda+14)$$

$\lambda=2$

$\lambda=\dfrac{-1+\sqrt{-55}}{2}$

$\lambda=\dfrac{-1-\sqrt{-55}}{2}$

es diagonalizable

Encontrar los valores propios y vectores propios de

$$\begin{pmatrix}1&-2\\-2&1\end{pmatrix} = \begin{pmatrix}1-\lambda&-2\\-2&1-\lambda\end{pmatrix} = (1-\lambda(1-\lambda))-(4) = 1-2\lambda+\lambda^2-4$$

$$= -3-2\lambda+\lambda^2$$

$$= (\lambda-3)(\lambda+1)$$

$\lambda_1 = 3$ $\lambda_2 = -1$ valores P.

Vectores P.

$$\begin{pmatrix}1-3&-2\\-2&1-3\end{pmatrix} = \begin{pmatrix}-2&-2\\-2&-2\end{pmatrix} = \begin{pmatrix}-2&-2\\0&0\end{pmatrix} = \quad -2x-2y= \quad x=-y \quad x\begin{pmatrix}1\\-1\end{pmatrix}$$

$$\begin{pmatrix}1+1&-2\\-2&1+1\end{pmatrix} = \begin{pmatrix}2&-2\\-2&2\end{pmatrix} = \begin{pmatrix}2&-2\\0&0\end{pmatrix} = x=y \quad x\begin{pmatrix}1\\1\end{pmatrix}$$

$$\begin{pmatrix}3-\lambda&-1&0\\-1&2-\lambda&-1\\0&-1&3-\lambda\end{pmatrix} = \det(A) = (3-\lambda)(\lambda^2-5\lambda+6) = (3-\lambda)(\lambda-3)(\lambda-2)$$

$\lambda=3$ $\lambda=2$

$$\begin{pmatrix}3-3&-1&0\\-1&2-3&-1\\0&-1&3-3\end{pmatrix} = \begin{pmatrix}0&-1&0\\-1&-1&-1\\0&-1&0\end{pmatrix} = \left|\begin{array}{l}y=0\\-x-y-z=0\\-y=0\end{array}\right.$$

$x+z=0 \Rightarrow x=-z$

$y=0$

$z\begin{pmatrix}-1\\0\\1\end{pmatrix}$

$$\begin{pmatrix}3-2&-1&0\\-1&2-2&-1\\0&-1&3-2\end{pmatrix} = \begin{pmatrix}1&-1&0\\-1&0&-1\\0&-1&1\end{pmatrix} = \begin{array}{l}x-y=0\\-x-z=0\\-y+z=0\end{array}$$

$x=y$

$x=z$

$y=z$

$z\begin{pmatrix}1\\1\\1\end{pmatrix}$

$$\cancel{\begin{pmatrix}1-\lambda&4&0\\2&-2&2\\-3/2&1&-2\lambda\end{pmatrix}} = \cancel{\det \; (1-\lambda)(\lambda^2+4\lambda+2)-28-8\lambda = \lambda^3+4\lambda^2-6\lambda-28}$$

5 En el espacio vectorial de las funciones continuas sea $W = G(e^x, e^{-x}, x)$ sea $T: W \to W$ dado por
$T(f) = f''(x) - x$

- $f''(x) = e^x$, $T(e^x) = e^x - x$
- $f''(x) = e^{-x}$, $T(e^{-x}) = e^{-x} - x$
- $f''(x) = 0$, $T(x) = -x$

Imagen $(T) = \{(e^x - x, e^{-x} - x, -x\} \Rightarrow (e^x - x, e^{-x})$

$a(e^x - x) + b(e^{-x} - x) = 0$

$ae^x + be^{-x} - (a+b)x = 0$ Rango $(T) = 2$

$\ker(T) = \left(\begin{array}{cc|c} e^x & e^{-x} & 0 \\ x & x & 0 \\ -x & -x & 0 \end{array}\right) = \left(\begin{array}{cc|c} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 0 \end{array}\right)$ nulidad $(T) = 1$

Valores y Vectores Propios

1: Determinar el polinomio característico

$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1-\lambda & 2 & 3 \\ 0 & 1-\lambda & 2 \\ 0 & 0 & 1-\lambda \end{pmatrix}$ det $= 1-\lambda(1-\lambda(1-\lambda) - 0) + 0 + 0$
$= 1-\lambda(1-2\lambda+\lambda^2) = 1-2\lambda+\lambda^2-\lambda+2\lambda^2-\lambda^3$
$= 1-3\lambda+3\lambda-\lambda^3$

$\begin{pmatrix} 1 & 1 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & -1 \end{pmatrix} = \begin{pmatrix} 1-\lambda & 1 & 0 \\ -1 & 1-\lambda & 1 \\ 0 & 1 & -1-\lambda \end{pmatrix}$ det $= 1-\lambda(1-\lambda(-1-\lambda) - 1) - 1(-1(1+\lambda)) + 0$
$1-\lambda(-\lambda^2) - 1(-1-\lambda) = -\lambda^2 + \lambda^3 + 1 + \lambda$

$\begin{pmatrix} 1 & -4 & 0 \\ 2 & -2 & -2 \\ -3/2 & 1 & -2 \end{pmatrix} = \begin{pmatrix} 1-\lambda & -4 & 0 \\ 2 & -2-\lambda & -2 \\ -3/2 & 1 & -2-\lambda \end{pmatrix} = 1-\lambda(\cancel{+2}+\lambda+\cancel{2}) - 4(2+3-3/2\lambda)$
$-\lambda+\lambda^2+4+6\lambda = 4+5\lambda+\lambda^2$