Aula 12

12 Dia 12: Limites e crescimento de funções

Exercício 12.1. Calcule cada um dos limites abaixo.

(a) $\displaystyle\lim_{x\to+\infty} \frac{1}{x^2 + x^{-2}}$

(b) $\displaystyle\lim_{x\to-\infty} \frac{2x+1}{x-3} =$

(c) $\displaystyle\lim_{x\to+\infty} \frac{x^2 + 3x + 5}{x^2 - x + 2} =$

(d) $\displaystyle\lim_{x\to-\infty} \frac{5x^3 - x^2 + 1}{x^3 + 4x + 7} =$

(e) $\displaystyle\lim_{x\to+\infty} \frac{e^x}{1 + x^2} =$

(f) $\displaystyle\lim_{x\to+\infty} \frac{\ln(x)}{x} =$

(g) $\displaystyle\lim_{x\to-\infty} \frac{1}{1 + e^x} =$

(h) $\displaystyle\lim_{x\to+\infty} \frac{x^3 + 2x}{x^4 + 1} =$

(i) $\displaystyle\lim_{x\to+\infty} \frac{\sin(x)}{x}$

(j) $\displaystyle\lim_{x\to+\infty} \frac{x^5 + x^2}{2x^5 - 3x^4} =$

(k) $\displaystyle\lim_{x\to-\infty} \frac{3x^4 - 7x + 4}{x^4 - 2x^2 + 1} =$

(l) $\displaystyle\lim_{x\to+\infty} \left(\frac{1}{x^2} + \frac{3}{x^3} \right) =$

(m) $\displaystyle\lim_{x\to-\infty} \frac{x^2 + 2}{x - 1} =$

(n) $\displaystyle\lim_{x\to+\infty} \frac{x^4 - 2x^3}{x^5 + 4x^2} =$

(o) $\displaystyle\lim_{x\to+\infty} \frac{e^{-x}}{x^2 + e^{-x}} =$

(p) $\displaystyle\lim_{x\to+\infty} \frac{3x + 5}{\ln(x)} =$

(q) $\displaystyle\lim_{x\to+\infty} e^{x - x^2} =$

(r) $\displaystyle\lim_{x\to+\infty} \frac{5x^2 + x\sin(x)}{x^2 + 1} =$

(s) $\displaystyle\lim_{x\to+\infty} \frac{\ln(x)}{x^2} =$

(t) $\displaystyle\lim_{x\to+\infty} \left(x - \sqrt{x^2 + x} \right) =$

(u) $\displaystyle\lim_{x\to-\infty} \frac{x^3 + x^2}{x + 1} =$

(v) $\displaystyle\lim_{x\to+\infty} \frac{e^x}{x!} =$

(w) $\displaystyle\lim_{x\to-\infty} \frac{1}{x + 2} =$

(x) $\displaystyle\lim_{x\to-\infty} \sqrt{x^2 + 1} - |x| =$

Dica: Para (v), procure pela *Fórmula de Stirling*.

Exercício 12.2. Quais das estimativas abaixo estão corretas quando $x \to +\infty$? Para as que não estão corretas, corrija-as.

(a) $2x^2 + 1 = O(x^2)$

(b) $3x^4 - 547x^3 + 1 = O(x^4)$

(c) $x^5 + x\ln x = O(x\ln x)$

(d) $47x^4 - x^3 + \dfrac{1}{x} = O(x^4)$

(e) $400x^{20} + e^{-x} = O(e^{-x})$

(f) $\dfrac{x}{1+\mathrm{e}^x} = O(x)$

(g) $\dfrac{x}{1+\mathrm{e}^x} = O(\mathrm{e}^{-x})$

(h) $\dfrac{x}{1+\mathrm{e}^x} = O(1)$

(i) $x^4 + \mathrm{e}^x = O(x^4)$

(j) $x^4 + \mathrm{e}^x = O(\mathrm{e}^x)$

(k) $x^4 + \mathrm{e}^{-x} = O(\mathrm{e}^{-x})$

(l) $x^4 + \mathrm{e}^{-x} = O(x^4)$

(m) $\mathrm{e}^{x^2} + \mathrm{e}^x = O(\mathrm{e}^{x^2})$

(n) $\mathrm{e}^{x^2} + \mathrm{e}^x = O(\mathrm{e}^x)$

(o) $\dfrac{x^{3/2}}{1 + x\ln x} = O(x^{3/2})$

(p) $\dfrac{x^{3/2}}{1 + x\ln x} = O(x)$

(q) $\dfrac{x^{3/2}}{1 + x\ln x} = O(x\ln x)$