Generalized decomposition methods

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General form

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$$\zeta = \sum \kappa_i$$

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- Ready to decompose!

Three different strategies

Generalized methods interchangeable

- stepwise replacement (Andreev)
- difference-scaled derivatives (LTRE, Caswell)
- continuous perturbation (Horiuchi)

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- Unless you want to purge swap order bias

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- needs extra function for derivatives
- better w *analytic* derivs

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- Tricks to ensure bounded parameters

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- All in `DemoDecomp`

Post decomposition

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- Summarize results

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- Make sense of results
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- Let's do this