Reverse Survival x5

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Reverse Survival x5 version

In this short note we present the differences between the original version and the x5 version of the Reverse Survival method implemented in the package *fertestr*.

The original version (let's call it x1) reconstructs the TFRs and birth counts (we call it B^t) for 15 points in time preceeding the date of inquiry each one corresponding to each single age of children's population count input (for ages 0 to 14). In the x5 version, we reconstruct the TFRs and B^t for each five year period before the inquiry based on population counts of children reported in five-year age groups (0-4, 5-9, 10-14). Therefore, the main difference of these two approaches lies in the reconstruction of B^t through reverse survival.

In the x1 approach, the inputs for computing B^t are: 1) population of children from 0-14 in single age groups, 2) a 'standard' survival function in single ages from ages 0 to 15 (lx), 3) estimates of child mortality probability for each three five year period preceding the inquiry (${}_5q_0^{t-2.5}$, ${}_5q_0^{t-7.5}$, ${}_5q_0^{t-12.5}$). The original version of the x1 approach (Moultrie et al. 2013) uses the logit transformation of the standard survival function and the mortality probabilities of prior periods to retrieve survival functions and from them compute the "proportion of births occurring x to $x+1$ years earlier that survive to the time of the inquiry, ${}^cL_x$." Details of this approach can be viewed in the spreadsheet attached to the method's explanation in http://demographicestimation.iussp.org/content/estimation-fertility-reverse-survival.

For the x5 alternative version, the inputs for computing B^t are similar: 1) population counts of children reported in five-year age groups, 2) a 'standard' survival function for ages 5, 10 and 15 (l_5, l_{10}, l_{15} - lx radix l_0 set to 1), 3) estimates of child mortality probability for each three five year period preceding the inquiry (${}_5q_0^{t-2.5}$, ${}_5q_0^{t-7.5}$, ${}_5q_0^{t-12.5}$). The estimation procedure of ${}^cL_x$ (proportion of births occurring x to $x+5$ years earlier that survive to the time of the inquiry) for periods $x = 2.5$ (0-4 years earlier), $x = 7.5$ (5-9 years earlier) and $x = 12.5$ (10-14 years earlier) follow the same strategy of the the original x1 version of the method, and will be described bellow.

Step 1: From ${}_5q_0$, compute α^t

We compute the intercept α^t of the linear relation between logits of observed (${}_5q_0$) and standard survival function (l_5) for each period preceding the inquiry. As Moultrie et al. (2013), we set the slope values as $\beta = 1$. Therefore,

$$Yobs_x^t = \alpha^t + \beta * Ystd_x$$
$$\alpha^t = Yobs_x^t - Ystd_x.$$

Step 2: from standard l_x and α, compute l_x^t

Then, by using the estimated α values, we can compute a set of survival functions l_5, l_{10} and l_{15} for each period earlier than the date of inquiry:

$$l_x^t = \frac{1}{1+\exp[2(\alpha^t + Y std_x)]}.$$

Step 3: compute ${}_nL_x$

Using estimated values for l_5^t, l_{10}^t and l_{15}^t and $l_0^t = 1$, we compute the person-years lived between ages x and $x+5$, for $x = 0$, 5 and 10:

$${}_nL_x^t = 5 * l_{x+5}^t +_n a_x * (l_x^t - l_{x+5}^t),$$

where ${}_5a_0 = 0.50$, ${}_5a_5 = 2$ and ${}_5a_{10} = 2.5$ by default (these values can be modified in the function by the user).

Step 4: compute survivorship ratios ${}_nP_x^t$

$${}_5P_0^t = \frac{{}_5L_0^t}{5l_0}$$

$${}_5P_5^t = \frac{{}_5L_5^t}{{}_5L_0^t}$$

$${}_5P_{10}^t = \frac{{}_5L_{10}^t}{{}_5L_5^t}$$

Step 5: define ${}_5S_x^t$ and compute cohort ${}^cS_x$[1]

We compute ${}_5S_x^t$ as the survivorship ratio between ages x and $x+5$ in periods t equal 0-4, 5-9 and 10-14 before the inquiry.

Then,

$${}_5S_0^{0-4} =_5 P_0^{0-4}$$

$${}_5S_0^{5-9} =_5 P_0^{5-9}$$

$${}_5S_0^{10-14} =_5 P_0^{10-14}$$

$${}_5S_5^{0-4} =_5 P_5^{0-4}$$

$${}_5S_5^{5-9} =_5 P_5^{5-9}$$

$${}_5S_{10}^{0-4} =_5 P_{10}^{0-4},$$

then:

$${}_5^cS_x = {}_5S_0^x {}_5S_5^{x-5} ... {}_5S_x^0,$$

[1] We prefer to call the Moultrie et al. (2013) version ${}^cL_x$ as ${}_5^cS_x$ to highlight the five-year period and to avoid confusing it with life table ${}_nL_x$

and

$${}^{c}_{5}S_0 = {}_5S_0^{0-4} = {}_5P_0^{0-4}$$

$${}^{c}_{5}S_5 = {}_5S_5^{0-4}\,{}_5S_0^{5-9} = {}_5P_5^{0-4}\,{}_5P_0^{5-9}$$

$${}^{c}_{5}S_{10} = {}_5S_{10}^{0-4}\,{}_5S_5^{5-9}\,{}_5S_0^{10-14} = {}_5P_{10}^{0-4}\,{}_5P_5^{5-9}\,{}_5P_0^{10-14}.$$

Step 6: reconstruct B^t

$$B^{x-2.5} = \frac{{}_5P_{0-4}}{5{}^{c}_{5}S_0}$$

$$B^{x-7.5} = \frac{{}_5P_{5-9}}{5{}^{c}_{5}S_5}$$

$$B^{x-12.5} = \frac{{}_5P_{10-14}}{5{}^{c}_{5}S_{10}}$$

Next steps

- Update ${}_na_x$ to ${}_na_x^t$, e.g., allow the average years lived for those who died within ages x and $x+n$ to vary in time by taking values from life tables of periods 0-4, 5-9 and 10-14 years earlier than the inquiry;
- Estimate a life table for the five years before the Census based on the five years life tables of WPP, without requiring the inputs of ${}_5q_0$ and ${}_{15}q_{45}$ and the calculation of *alpha* from a standard survival function.

References

Moultrie, Tom, Rob Dorrington, Allan Hill, Kenneth Hill, Ian Timæus, and Basia Zaba. 2013. *Tools for Demographic Estimation*. Paris: International Union for the Scientific Study of Population (IUSSP).