Single Zone Evaluation

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1 Setup

Data Generating Process

We use the following data generating processes for the toy examples. In order to make the comparison with the status quo more straightforward, we model losses instead of overall wealth.

$$
\begin{aligned}
l &= \beta\theta + \epsilon \\
\theta &\sim \mathcal{N}(5, 2) \\
\epsilon &\sim \mathcal{N}(0, 1) \\
\beta &= 3
\end{aligned}
$$

I draw 100 training samples from the above model to train the prediction model and to use as input for the optimization programs. I then evaluate both methods using 1000 samples drawn from the same model.

2 Status Quo

We will be comparing our proposed approach to the method developed by Chantarat et al. (2013), which, to the extent of our knowledge is what is currently being used for Kenya's Index Based Livestock Insurance (IBLI) program. The method is as follows:

- Use a clustering algorithm to group locations into clusters
- Fit a separate herd mortality function for each cluster. They use a linear regression model to predict herd mortality.
- Contracts are of the form: $I(\theta) = \max(\hat{M}(\theta) - M^*, 0) \times TLU \times P_{TLU}$ where TLU is the number of insured livestock units, and P_{TLU} is the price per insured livestock unit. In other words, their contract pays farmers for the full predicted loss beyond a threshold, M^*. This threshold, M^* is the contract's strike value.
- They choose the strike value that would explain the highest share of insurable losses in the historical data. In other words, they run the following regression: $y_s = \beta_s \hat{y_s} + \epsilon$ where y_s is the actual insured losses at strike value s and $\hat{y_s}$ is the predicted insured losses at strike value s. They choose the strike value $s = \arg\max_s \beta_s$.

To mimick this in our toy example, we set the status quo contracts to be $I(\theta) = \max(\hat{l}(\theta) - l^*, 0)$, since we are already assuming that l is the total loss suffered. For the toy example, we fit a (correctly specified) linear regression model to predict losses: $l = \beta\theta + \epsilon \implies \hat{l}(\theta) = \hat{\beta}\theta$.

3 Optimization Approach

In order to separate the effect of contract design from the effect of prediction quality, we will be basing our contracts on the same predictions used by the status quo method. In other words, we will use the status quo method to estimate a model that predicts loss based on theta, $\hat{l}(\theta)$, and our payout function will use that as input instead of θ. In other words, our model will define payout functions $I(\hat{l}(\theta))$, where $\hat{l}(\theta)$ is the same prediction function used by the status quo method.

Minimum CVaR Model

This model minimizes the $CVaR$ of the farmer's net loss subject to a constraint on the premium. The premium constraints are expressed as a fraction of the full insured amount.

Model Parameters

- ϵ: This defines the CVaR objective. $\epsilon = 0.1$ means that our objective is on the expected value of the loss given that it is above the 90^{th} percentile.
- $\bar{\pi}$: This is the maximum value of the premium.
- y: maximum insured amount

Single Zone Model

$$\min_{a,b \geq 0} \; CV@R_{1-\epsilon} \left(\ell - \min \left\{ (a\theta + b), K \right\} \right) \tag{1}$$

$$\text{s.t.} \quad a\mathbb{E}\left[\theta\right] + b \leq \bar{\pi} \tag{2}$$

$$a\theta + b \geq 0 \tag{3}$$

We reformulated the problem in the following way. In the model below, p_k is the probability of event k, and k indexes the possible realizations of θ, l.

$$\min_{a,b,\pi,\gamma,t} \quad t + \frac{1}{\epsilon} \sum_k p_k \gamma_k \tag{4}$$

$$\text{s.t.} \; \gamma_k \geq l^k - \min \left\{ (a\hat{l}(\theta^k) + b), K \right\} - t, \forall k \tag{5}$$

$$\gamma_k \geq 0 \tag{6}$$

$$0 \leq a\hat{l}(\theta^k) + b, \forall k \tag{7}$$

$$K\bar{\pi} \geq a\mathbb{E}[\hat{l}(\theta)] + b \tag{8}$$

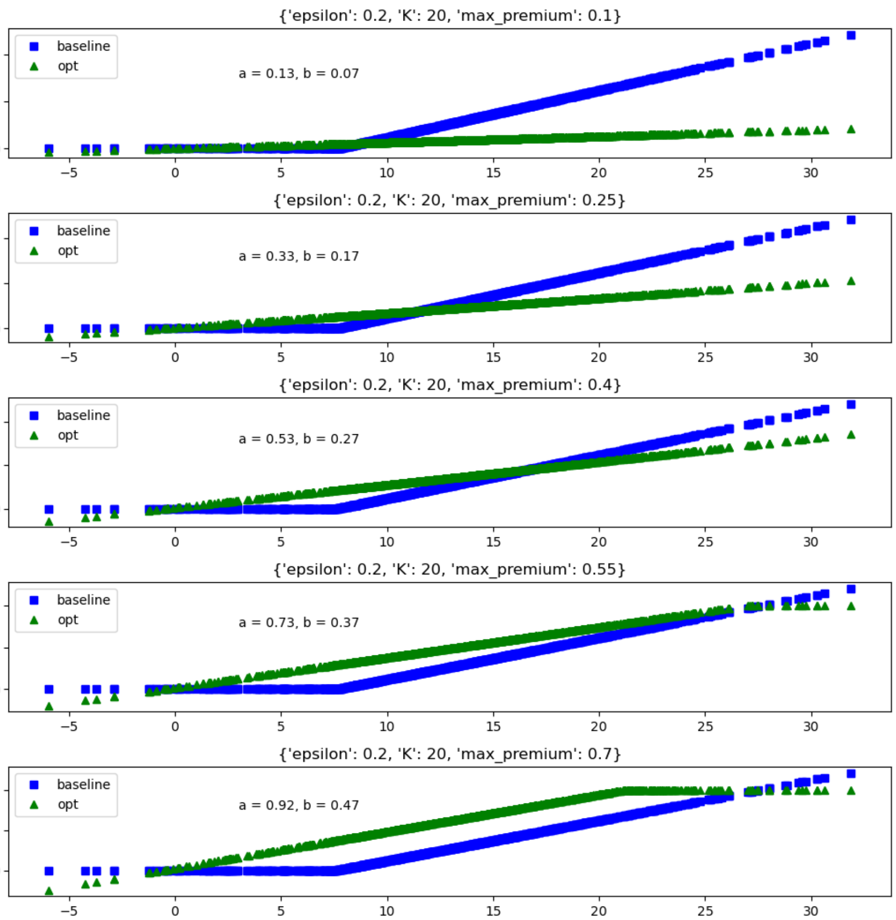
4 Results

The model mostly behaves as expected. Increasing the payment cap, K increases both a and b. Similarly, increasing the maximum premium, $\bar{\pi}$ increases the payout rate as well. However, a and b didn't seem sensitive to ϵ at all.

CVaR Model

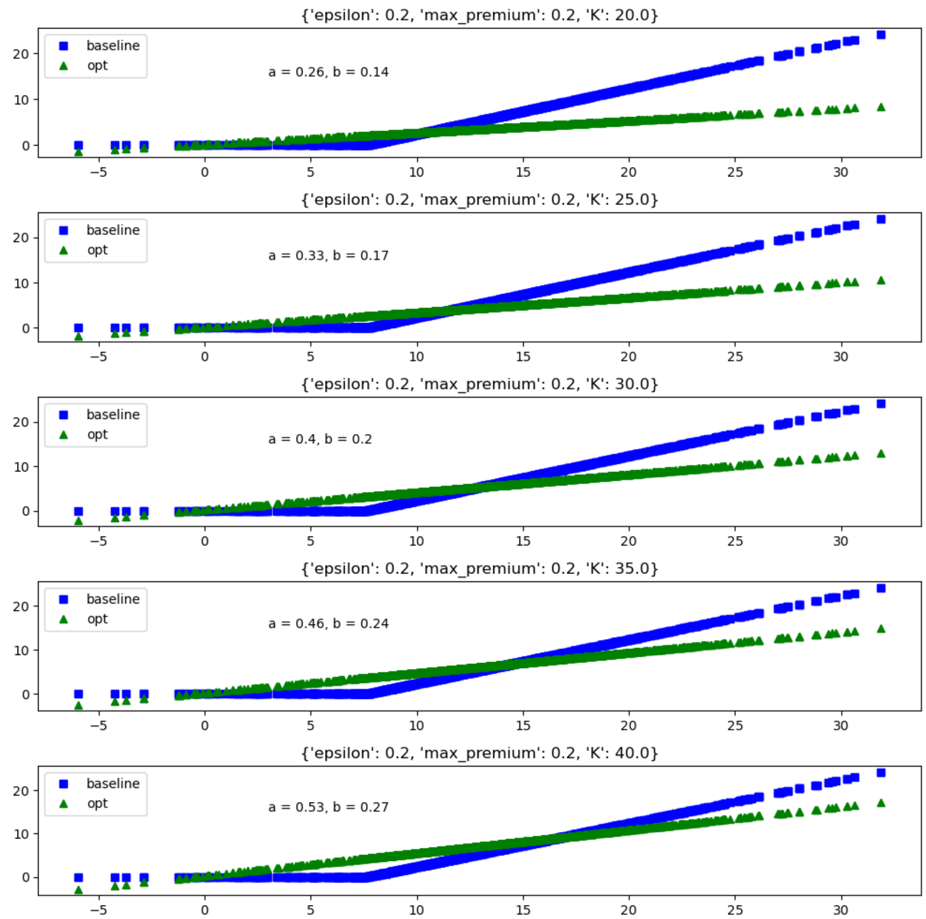
Max Premium Exploration

Figure 1: Max Premium Exploration



K Exploration

Figure 2: K Exploration



Epsilon Exploration

Figure 3: Epsilon Exploration