Agricultural Index Insurance Model

José I. Velarde Morales

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1 Overview

The goal of insurance is to protect farmers against catastrophic events. We want to make sure that the worse case scenario for farmers doesn't enter them into a poverty trap. The most relevant feature of the problem are the price constraint (ie the premium constraint) and the effects of spatial correlations on prices in the multi-zone case. I think that what is causing the most problems is the constraint that $I(\theta)$ is piecewise linear, this seems to be causing most of our problems with non-convexity, however, having it be linear seems to perform worse than the baseline.

2 Original Simplified Model

The first model we proposed aimed to minimize the probability that a farmer's net loss would exceed an exogenously given threshold. This was subject to a maximum premium constraint and a constraint specifying a piecewise linear structure for the payout function. In the model below, l is the farmer's loss, π is the premium he pays, $I(\theta)$ is the insurance payout, and $\bar{l}$ is the loss threshold we don't want the farmer to exceed. In other words, we don't want the farmer to have net losses higher than $\bar{l}$ with high probability. The model is:

Single Zone

$$
\begin{aligned}
\min_{a,b,\pi} \; & P(l + \pi - I(\theta) \geq \bar{l}) && (1)\\
\text{s.t. } \; & I(\theta) = (a\theta - b)^+ && (2)\\
& \pi = E[I(\theta)] && (3)\\
& b \geq 0 && (4)\\
& \pi \leq \bar{\pi} && (5)
\end{aligned}
$$

Multiple Zone

In the multiple zone case, there is an additional cost coming from the capital requirements to insure the entire portfolio. In the model below, Z is the number of insured zones, $\bar{pi}$ is the maximum premium, c_k is the cost of capital, and β_z is a measure of the relative riskiness of zone z

$$\min_{t,a,b,e,\pi} \max_{z} P(l_z - \pi_z + I_z(\theta_z) \geq \bar{l}) \tag{6}$$

$$\text{s.t. } I_z(\theta_z) = (a_z\theta_z + b_z)^+, \forall z \tag{7}$$

$$K^P = CVaR_{1-\epsilon}\left(\sum_z I_z(\theta_z)\right) - Z\bar{\pi} \tag{8}$$

$$\pi_z = E[I_z(\theta_z)] + c_k K^P \beta_z, \forall z \tag{9}$$

$$\pi_z \leq \bar{\pi}, \forall z \tag{10}$$

3 Minimum CVaR Model

Since probabilitic constraints are generally non-convex, we also considered trying to minimize the conditional value at risk (CVaR) of the farmer's net loss instead. I think this is a good objective for our context because it minimizes the expected value of the loss in the worse case scenarios. It also has the advantage of having been well studied in the literature, and there are tractable reformulations for having $CV@R$ in the objective and constraints for general loss distributions. This model minimizes the $CV@R$ of the farmer's net loss subject to a constraint on the premium. The premium constraints are expressed as a fraction of the full insured amount. So, if $\bar{\pi}$ is the maximum premium share and K is the full insured amount, then the constraint on the premium would be: $\pi \leq K\bar{\pi}$.

Single Zone Model

Model Parameters

- ϵ: This defines the CV@R objective. $\epsilon = 0.1$ means that our objective is on the expected value of the loss given that it is above the 90^{th} percentile.
- $\bar{\pi}$: This is the maximum value of the premium.
- K: maximum insured amount

Model

$$\min_{a,b \geq 0} CV@R_{1-\epsilon}\left(\ell - \min\left\{(a\theta + b), K\right\}\right) \tag{11}$$

$$\text{s.t. } a\mathbb{E}\left[\theta\right] + b \leq K\bar{\pi} \tag{12}$$

$$a\theta + b \geq 0 \tag{13}$$

We reformulated the problem in the following way. In the model below, p_k is the probability of event k, and k indexes the possible realizations of θ, l.

$$\min_{a,b,\gamma,t} \quad t + \frac{1}{\epsilon}\sum_k p_k\gamma_k \tag{14}$$

$$\text{s.t. } \gamma_k \geq l^k - \min\left\{(a\hat{l}(\theta^k) + b), K\right\} - t, \forall k \tag{15}$$

$$\gamma_k \geq 0, \forall k \tag{16}$$

$$0 \leq a\hat{l}(\theta^k) + b, \forall k \tag{17}$$

$$K\bar{\pi} \geq a\mathbb{E}[\hat{l}(\theta)] + b \tag{18}$$

Original Multiple Zone Model

Model Parameters

- ϵ: This defines the CV@R objective. $\epsilon = 0.1$ means that our objective is on the expected value of the loss given that it is above the 90^{th} percentile.
- $\bar{\pi}$: This is the maximum value of the premium.
- $\underline{\pi}$: This is the minimum value of the premium.
- β_z: This is a measure of the relative riskiness of zone z.
- ϵ_P: This is the epsilon corresponding to the $CV@R$ of the entire portfolio. This is used to determine the required capital for the portfolio. Values I've seen used are $\epsilon_P = 0.01$ and $\epsilon_P = 0.05$.
- Z: number of insured zones.
- c_k: cost of capital
- K_z: maximum insured amount of zone z.

Model

$$\min_{a,b,K^P} \max_{z} \quad CV@R_{1-\epsilon}(\ell_z - \min\left\{(a_z\hat{\ell}_z(\theta_z) + b_z), K_z\right\}) \tag{19}$$

$$\text{s.t. } K\bar{\pi} \geq a_z\mathbb{E}[\hat{\ell}_z(\theta_z^k)] + b_z + c_k\beta_z K^P, \forall k, \forall z \tag{20}$$

$$0 \leq a_z\hat{\ell}_z(\theta_z^k) + b_z, \forall k, \forall z \tag{21}$$

$$K^P + Z\underline{\pi} \geq CV@R_{1-\epsilon_P}\left(\sum_z a_z\hat{\ell}_z(\theta_z) + b_z\right) \tag{22}$$

The reformulation is:

$$\min_{a,b,\gamma,t,m,K^P} \quad m \tag{23}$$

$$\text{s.t.} \quad t_z + \frac{1}{\epsilon}\sum_k p_k \gamma_z^k \leq m, \forall z \tag{24}$$

$$\gamma_z^k \geq \ell^k - \min\left\{(a_z \hat{\ell}_z(\theta_z^k) + b_z), K_z\right\} - t_z, \forall k, \forall z \tag{25}$$

$$\gamma_z^k \geq 0, \forall k, \forall z \tag{26}$$

$$t_p + \frac{1}{\epsilon_p}\sum_k p_k \gamma_P^k \leq K^P + Z\underline{\pi} \tag{27}$$

$$\gamma_P^k \geq \sum_z a_z \hat{\ell}_z(\theta_z^k) + b_z - t_p, \forall k \tag{28}$$

$$\gamma_P^k \geq 0, \forall k \tag{29}$$

$$K_z \bar{\pi} \geq a_z \mathbb{E}[\hat{\ell}_z(\theta_z)] + b_z + c_k \beta_z K^P, \forall z \tag{30}$$

$$0 \leq a_z \hat{\ell}_z(\theta_z^k) + b_z, \forall k, \forall z \tag{31}$$

Modified Multiple Zone Model

The original multiple zone model led to unsatisfactory results. One of the problems was that the model excluded functions of the form: $f(x) = \max\{0, a\theta + b\}$, which is the function space we ideally want to optimize over. We ideally want to optimize over functions of the form: $f(x) = \min\{\max\{0, a\theta + b\}, K\}$. We want to optimize over this function space because these types of functions are what is traditionally used in index insurance contracts. The reason they are popular is that they start paying out once the loss is deeemed big enough. The problem with the original model is that it was constrained to start paying out whenever the predicted loss was non-negative. This meant the model was forced to grant payouts even in cases where the predicted loss was small, this in turn affected the size of the payouts it could give in cases of large losses. To approximate the program over the class of functions we are interested in, we modified the program in the following way. We also changed the premium constraint into an overall budget constraint, to make comparisons with the status quo more straightforward:

Model

$$\min_{a,b,K^P} \max_z \quad CV@R_{1-\epsilon}(\ell_z - \min\left\{(a_z \hat{\ell}_z(\theta_z) + b_z), K_z\right\}) \tag{32}$$

$$\text{s.t.} \ B \geq \sum_z \max\left\{0, a_z \hat{\ell}_z(\theta_z) + b_z\right\} + c_k K^P \tag{33}$$

$$K^P + Z\pi_{SQ} \geq CV@R_{1-\epsilon_P}\left(\sum_z \max\left\{0, a_z \hat{\ell}_z(\theta_z) + b_z\right\}\right) \tag{34}$$

The reformulation is:

$$
\begin{aligned}
\min_{a,b,\alpha,\gamma,t,m,K^P} \quad & m && (35)\\
\text{s.t.} \quad & t_z + \frac{1}{\epsilon}\sum_k p_k \gamma_z^k \le m, \forall z && (36)\\
& \gamma_z^k \ge \ell^k - \min\left\{(a_z \hat{\ell}_z(\theta_z^k) + b_z), K_z\right\} - t_z, \forall k, \forall z && (37)\\
& \gamma_z^k \ge 0, \forall k, \forall z && (38)\\
& B \ge \sum_k \sum_z \alpha_z^k + c_k K^P && (39)\\
& t_p + \frac{1}{\epsilon_p}\sum_k p_k \gamma_P^k \le K^P + Z\pi_{SQ} && (40)\\
& \gamma_P^k \ge \sum_z \alpha_z^k - t_p, \forall k && (41)\\
& \gamma_P^k \ge 0, \forall k && (42)\\
& \alpha_z^k \ge a_z \hat{\ell}_z(\theta_z^k) + b_z, \forall k, \forall z && (43)\\
& \alpha_z^k \ge 0, \forall k, \forall z && (44)
\end{aligned}
$$