Agricultural Index Insurance: An Optimization Approach

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The Problem of Agricultural Risk

- Farmers face a lot of risk, and the lack of risk management tools forces them to use coping strategies that hurt their long term welfare.
- Traditional insurance is prohibitively costly in most developing countries due to lack of data and high verification costs.
- Moral hazard, adverse selection, and the presence of large covariate shocks make the problem of agricultural insurance especially hard.

A Proposed Solution: Index Insurance

- In index insurance, an index (or statistic) is created using easily observable quantities (e.g. rainfall), and it is used to determine whether the insured party suffered an adverse event.
- If the index falls below a pre-determined threshold, the insurance company automatically issues out payments to the insured.
- This allows the insurance company to circumvent the issue of verification and moral hazard, since the actions of individual farmers cannot affect the index.

Index Insurance in Practice

- Since it was first proposed, index insurance programs have been implemented in many countries including India, Mexico, Tanzania, Malawi, Kenya, and many others (Jensen and Barrett (2017)).
- Today, tens of millions of farmers worldwide are covered by index insurance programs (Greatrex et al. (2015)).
- However, in most of these cases, the insurance has to be heavily subsidized by governments due to high cost and low demand (Greatrex et al. (2015)).

Project Overview

- Traditionally, the contract for each insured zone is designed independently of all other zones.
- The goal of this project is to make insurance less costly by improving the design of the insurance contracts.
- Our method simultaneously determines the contract parameters for different areas, while taking into account the correlation between the areas, reducing risk for the insurer.

What I would like feedback on

- Evaluation (what metrics to report, evaluation with observational and synthetic data, etc.)

Index Insurance Literature

- **Impact of Index Insurance:** Overall, there is evidence that index insurance reduces reliance on detrimental risk-coping strategies, increases investment, and leads to riskier, but more profitable production decisions (Jensen and Barrett (2017); Cole et al. (2013); Mobarak and Rosenzweig (2013); Karlan et al. (2014)).
- **Demand for Index Insurance:** Demand for index insurance tends to be low and highly price sensitive (Jensen and Barrett (2017); Cole et al. (2013); Cai, De Janvry, and Sadoulet (2020),Casaburi and Willis (2018)).
- **Design of Index Insurance:** There has been relatively little research studying the design of index insurance. The method developed by Chantarat et al. (2013) is the most commonly used in academic publications (Jensen, Stoeffler, et al. (2019); Flatnes, Carter, and Mercovich (2018)).

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Index Insurance: General Context

- Index insurance is a named-peril insurance.
- It is often tied to access to credit.
- Offered at different levels (micro level, meso level, and macro level)
- Often based on public-private partnerships. Research institutions usually design the product, and insurance companies (or governments) sell it.

Index Insurance: Design

Index insurance design generally consists of three steps:

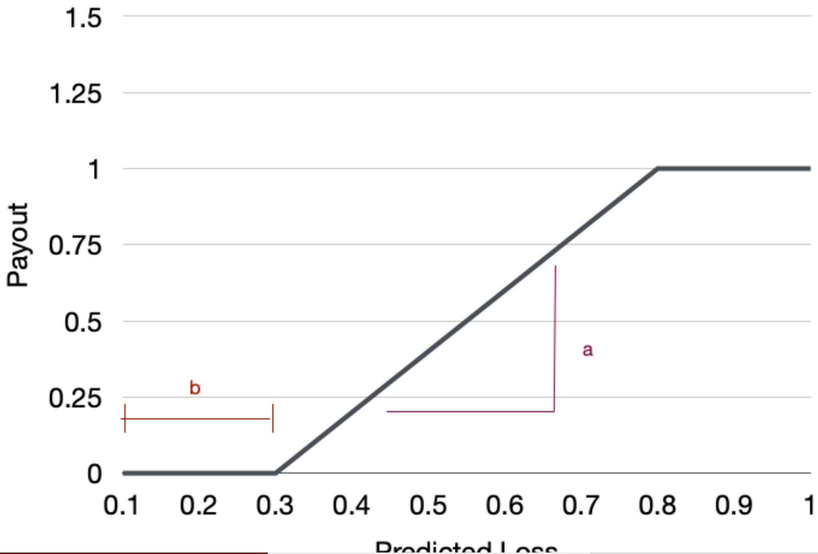
1. Prediction: building a model to predict loss.
2. Contract design: designing contracts specifying payouts based on model predictions.
3. Pricing: product pricing

This project focuses on the second step. Currently, a research team usually designs the insurance product, and the insurance company then prices the product.

Index Insurance: Definition and Parameters

- Index insurance uses a signal, θ, that is used to predict agricultural loss, $\hat{\ell}(\theta)$
- Index insurance contracts normally have the form: $I(\theta) = \min\left\{\max\left\{a\hat{\ell}(\theta) - b, 0\right\}, 1\right\}$, and a, b are the contract parameters.
- The expected cost, $C(I(\theta))$, of an insurance contract, $I(\theta)$ for an insurer is: $C(I(\theta)) = \mathbb{E}[I(\theta)] + c_\kappa K(I(\theta))$, where c_κ is the cost of holding capital, and K is the amount of capital required to insure the contract.
- The formula for K is $K(I(\theta)) = CVaR_{1-\epsilon_P}(I(\theta)) - \mathbb{E}[I(\theta)]$.

Example of Index Insurance Contract



Practitioner Interviews

- We conducted interviews with researchers and practitioners that had implemented index insurance programs in several countries (Malawi, Kenya, Senegal, Thailand, among others) to learn more about the context.

- **Objective:** minimize risk faced by farmers.

- **Constraints:** Budget constraints and payout frequency are very important from both demand and supply side.

Risk Measures

We are interested in minimizing the risk faced by farmers, so we need a measure of this risk.

Definition

For a random variable z, representing loss, the $(1-\epsilon)$ Value at Risk (VaR) is given by

$$VaR_{1-\epsilon}(z) := \inf \{t : P(z \leq t) \geq 1-\epsilon\}$$

Definition

For a random variable z, representing loss, the $(1-\epsilon)$ Conditional Value at Risk $(CVaR)$ is given by

$$CVaR_{1-\epsilon}(z) := \mathbb{E}\left[z | z \geq VaR_{1-\epsilon}(z)\right]$$

Idealized CVaR Model

- **Objective:** conditional value at risk of the farmers' loss net of insurance.
- **Constraint 1:** piecewise linear structure of the contract.
- **Constraint 2:** definition of premium
- **Constraint 3:** definition of required capital
- **Constraint 4:** payout frequency constraint

$$
\begin{aligned}
\min_{a,b,\pi,K} \quad & \mathrm{CVaR}_{1-\epsilon}\left(\ell + \pi - I(\theta)\right) \\
\text{s.t.} \quad & I(\theta) = \min\left\{\max\left\{0, a\hat{\ell}(\theta) - b\right\}, 1\right\} && (1) \\
& \pi = \mathbb{E}\left[I(\theta)\right] + c_\kappa K && (2) \\
& K = \mathrm{CVaR}_{1-\epsilon_K}\left(I(\theta)\right) - \mathbb{E}[I(\theta)] && (3) \\
& \underline{f} \leq \mathbb{P}\left(I(\theta) > 0\right) \leq \overline{f} && (4) \\
& \pi \leq \overline{\pi}. && (5)
\end{aligned}
$$

Multiple Zone

$$
\begin{aligned}
\min_{a,b,K} \max_{z} \quad & \mathrm{CVaR}_{1-\epsilon}\left(\ell_z + \pi_z - I_z(\theta_z)\right) && (6)\\
\text{s.t.} \quad & I_z(\theta_z) = \min\left\{\max\left\{a_z \hat{\ell}_z(\theta_z) + b_z, 0\right\}, 1\right\} && (7)\\
& \pi_z = \mathbb{E}\left[I_z(\theta_z)\right] + \frac{c_\kappa}{\sum_z s_z} K && (8)\\
& K = \mathrm{CVaR}_{1-\epsilon_K}\left(\sum_z s_z I_z(\theta_z)\right) - \mathbb{E}\left[\sum_z s_z I_z(\theta_z)\right] && (9)\\
& \underline{f} \leq \mathbb{P}\left(I_z(\theta_z) > 0\right) \leq \overline{f} && (10)\\
& \pi_z \leq \overline{\pi_z} && (11)
\end{aligned}
$$

Convex Approximation

$$\min_{a,b,K,\pi} \max_{z} \quad \mathrm{CVaR}_{1-\epsilon}\left(\ell_z + \pi_z - \underline{I_z(\theta_z)}\right) \tag{12}$$

$$\text{s.t.} \quad \pi_z = \mathbb{E}\left[\overline{I_z(\theta_z)}\right] + \frac{c_\kappa}{\sum_z s_z} K \tag{13}$$

$$K = \mathrm{CVaR}_{1-\epsilon_K}\left(\sum_z s_z \overline{I_z(\theta_z)}\right) - \mathbb{E}\left[\sum_z s_z \underline{I_z(\theta_z)}\right] \tag{14}$$

$$a\hat{\ell}_{\underline{f}} \leq b \leq a\hat{\ell}_{\overline{f}} \tag{15}$$

$$\overline{I_z(\theta_z)} = \max\left\{0, a_z\hat{\ell}_z(\theta_z) + b_z\right\} \tag{16}$$

$$\underline{I_z(\theta_z)} = \min\left\{a_z\hat{\ell}_z(\theta_z) + b_z, 1\right\} \tag{17}$$

$$\pi_z \leq \overline{\pi_z}.$$

LP Reformulation

Using the results from Rockafellar and Uryasev (2002), we get:

$$
\begin{aligned}
\min_{a,b,\alpha,\omega,\gamma,t,m,K,\pi} \quad & m \\
\text{s.t.} \quad & t_z + \frac{1}{\epsilon}\sum_j p^j \gamma_z^j \leq m, \forall z \\
& \gamma_z^j \geq s_z \left(\ell^j + \pi_z - \omega_z^j\right) - t_z, \forall j, \forall z \\
& \gamma_z^j \geq 0, \forall j, \forall z \\
& \pi_z = \frac{1}{N}\sum_j \alpha_z^j + \frac{1}{\sum_z s_z} c_\kappa K \\
& t_K + \frac{1}{\epsilon_K}\sum_j p^j \gamma_K^j \leq K + \frac{1}{N}\sum_j \sum_z s_z \omega_z^j \\
& \gamma_K^j \geq \sum_z s_z \alpha_z^j - t_K, \forall j \\
& \gamma_K^j \geq 0, \forall j \\
& \alpha_z^j \geq a_z \hat{\ell}_z(\theta_z^j) + b_z, \forall j, \forall z \\
& \alpha_z^j \geq 0, \forall j, \forall z \\
& \omega_z^j \leq a_z \hat{\ell}_z(\theta_z^j) + b_z, \forall j, \forall z \\
& \omega_z^j \leq 1, \forall j, \forall z \\
& \pi_z \leq \overline{\pi_z}, \forall z.
\end{aligned}
$$

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Baseline Method

- We compare our method to the method developed in Chantarat et al. (2013). This method is the most commonly used in academic publications and is what is used in Kenya's index insurance program.
- Method uses a linear regression model to predict losses in each insured area. A different model is estimated for each area.
- Contracts are of the form: $I(\theta) = s \times \min\left\{\max\left\{0, \hat{\ell}(\theta) - \ell^*\right\}, 1\right\}$ where $\hat{\ell}(\theta)$ is the predicted loss, ℓ^* is the strike value, and s is the total insured amount. In other words, their contract pays farmers for the full predicted loss beyond a threshold, ℓ^*. This threshold, ℓ^* is the contract's strike value. The strike value is chosen to maximize the correlation of payouts and losses.

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Setup

We use the following data generating processes:

- Main DGP: $\ell_z = \frac{1}{1+e^{f(\theta_z)}}$
- Linear case: $f(\theta) = \beta\theta + \epsilon$
- Nonlinear case: $f(\theta) = \beta_0 + \beta_1\theta + \beta_2\theta^2 + \ldots + \beta_n\theta^n + \epsilon$

In both cases, $\theta \sim \mathcal{N}((0,0),\Sigma), \epsilon \sim \mathcal{N}(0,\rho I)$. In both cases, β_i is drawn randomly. With each data generating process, we test the following scenarios:

- **No correlation case:** $corr(\theta_1, \theta_2) = 0$
- **Positive correlation case:** $corr(\theta_1, \theta_2) = 0.8$
- **Negative correlation case:** $corr(\theta_1, \theta_2) = 0.8$

Simulation Details

In each simulation we:

1. Generate training and test samples, samples will be of the form $\left\{\ell_1^i, \theta_1^i, \ell_2^i, \theta_2^i\right\}_{i=1}^N$ where ℓ is loss and θ is the predictor.
2. Use training data and model predictions to design contracts using both methods.
3. Apply insurance contracts designed by the two methods to farmers in the test set and compare outcomes.

We run this 500 times for each scenario. We report the mean and 95% confidence intervals of each performance metric across the 500 simulations.

Performance Metrics

For each sample in the test set, we calculate the net loss, $\Delta\ell_z^i \triangleq \ell_z^i + \pi_z - I_z(\theta_z^i)$. Then we calculate the following metrics:

- **Farmer Welfare Measures:** Conditional Value at Risk (CVaR), Value at Risk (VaR), Semi-Variance
- **Insurer Cost/Risk Measures:** Required Capital, Average Cost

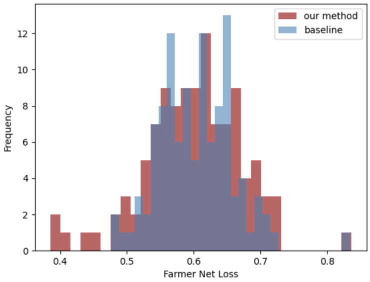
Summary of Results: (Almost) Correctly Specified Model

Our model offers slightly worse protection than the baseline, but at a much lower cost and risk for the insurer. This is achieved through both, better risk management, and smarter allocation of payouts.

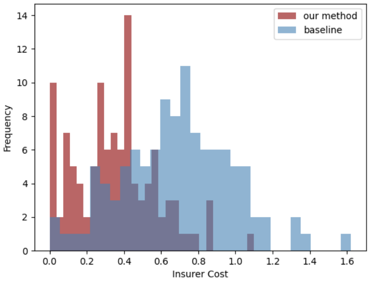
- **Metrics on which our model does better:** Maximum Semi-Variance, Required Capital, Average Cost
- **Metrics on which baseline model does better:** Maximum CVaR
- **Metrics on which we tie:** Maximum VaR

Histograms: Model Correctly Specified

- Distribution of farmer loss is similar under both
- Cost for insurer is radically different



(a) Farmer net loss



(b) Insurer cost

Figure: Distribution of outcomes

Results: Correctly Specified Prediction Model

Model	Max CVaR	Max VaR	Max SemiVar	$\lvert VaR_2 - VaR_1 \rvert$	Required Capital	Average Cost
Baseline	0.60	0.62	0.02	0.01	0.64	0.81
	[0.58, 0.63]	[0.6, 0.65]	[0.01, 0.04]	[0.0, 0.04]	[0.55, 0.79]	[0.65, 0.92]
Opt	0.64	0.63	0.01	0.01	0.55	0.48
	[0.61, 0.67]	[0.61, 0.66]	[0.01, 0.03]	[0.0, 0.03]	[0.43, 0.69]	[0.43, 0.54]
Diff	-0.04	-0.01	0.01	0.00	0.10	0.33
	[-0.07, -0.01]	[-0.02, 0.0]	[0.0, 0.02]	[-0.02, 0.03]	[0.05, 0.16]	[0.15, 0.43]

(a) No correlation

Model	Max CVaR	Max VaR	Max SemiVar	$\lvert VaR_2 - VaR_1 \rvert$	Required Capital	Average Cost
Baseline	0.61	0.64	0.02	0.01	0.88	0.82
	[0.59, 0.65]	[0.62, 0.66]	[0.01, 0.03]	[0.0, 0.03]	[0.72, 1.1]	[0.68, 0.96]
Opt	0.65	0.65	0.01	0.01	0.74	0.52
	[0.63, 0.68]	[0.62, 0.67]	[0.0, 0.02]	[0.0, 0.03]	[0.6, 0.98]	[0.44, 0.59]
Diff	-0.03	-0.01	0.01	0.00	0.13	0.31
	[-0.06, -0.02]	[-0.02, 0.0]	[0.0, 0.01]	[-0.02, 0.02]	[0.09, 0.17]	[0.18, 0.43]

(b) Positive correlation

Results: Correctly Specified Prediction Model

Model	Max CVaR	Max VaR	Max SemiVar	$\|VaR_2 - VaR_1\|$	Required Capital	Average Cost
Baseline	6.19	6.3	19.72	0.85	3.33	5.56
	[6.13, 6.26]	[6.24, 6.36]	[19.27, 20.17]	[0.79, 0.9]	[3.28, 3.38]	[5.48, 5.64]
Opt	6.11	5.98	15.09	0.22	2.91	5.53
	[6.06, 6.17]	[5.93, 6.02]	[14.68, 15.5]	[0.2, 0.23]	[2.86, 2.95]	[5.45, 5.61]
Diff	0.08	0.32	4.63	0.63	0.42	0.04
	[0.05, 0.11]	[0.29, 0.35]	[4.35, 4.9]	[0.57, 0.69]	[0.39, 0.45]	[0.03, 0.04]

Table: Negative correlation

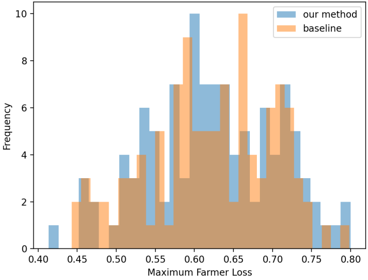
Summary of Results: Misspecified Model

In this case, our results are even stronger. We match the performance of the basline method on most metrics, and the cost is significantly lower.

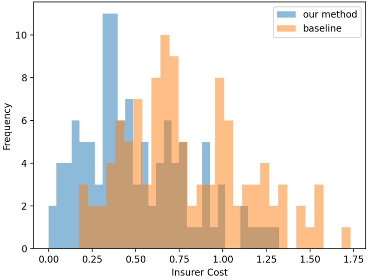
- **Metrics on which our model does better:** Maximum Semi-Variance, Required Capital
- **Metrics on which baseline model does better:** Maximum CVaR
- **Metrics on which we tie:** Maximum VaR, Average Cost

Histograms: Model Misspecified

- Distribution of farmer loss is similar under both
- Cost for insurer is radically different



(a) Farmer net loss



(b) Insurer cost

Figure: Distribution of outcomes

Results: Misspecified Prediction Model

Model	Max CVaR	Max VaR	Max SemiVar	$\lvert VaR_2 - VaR_1 \rvert$	Required Capital	Average Cost
Baseline	0.65	0.67	0.02	0.02	0.70	0.91
	[0.6, 0.84]	[0.61, 0.77]	[0.01, 0.08]	[0.0, 0.07]	[0.21, 0.93]	[0.7, 0.97]
Opt	0.71	0.67	0.03	0.02	0.61	0.56
	[0.64, 0.86]	[0.61, 0.77]	[0.01, 0.08]	[0.0, 0.08]	[0.0, 0.86]	[0.0, 0.74]
Diff	-0.04	-0.00	-0.00	-0.00	0.10	0.36
	[-0.1, 0.02]	[-0.02, 0.06]	[-0.03, 0.02]	[-0.03, 0.03]	[-0.03, 0.26]	[0.18, 0.81]

(a) No correlation

Model	Max CVaR	Max VaR	Max SemiVar	$\lvert VaR_2 - VaR_1 \rvert$	Required Capital	Average Cost
Baseline	0.68	0.69	0.02	0.01	0.98	0.95
	[0.62, 0.85]	[0.63, 0.79]	[0.01, 0.05]	[0.0, 0.04]	[0.26, 1.21]	[0.59, 1.03]
Opt	0.73	0.69	0.03	0.02	0.83	0.57
	[0.66, 0.86]	[0.62, 0.76]	[0.01, 0.08]	[0.0, 0.05]	[0.0, 1.19]	[0.0, 0.8]
Diff	-0.04	0.00	-0.00	-0.00	0.13	0.36
	[-0.09, 0.01]	[-0.02, 0.07]	[-0.04, 0.01]	[-0.03, 0.02]	[-0.07, 0.38]	[0.14, 0.81]

(b) Positive correlation

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Data Sources

- **NDVI Data:** The Normalized Difference Vegetation Index (NDVI) is a satellite-based indicator of the amount and health of vegetation. We use NDVI data for Kenya between 2000-2015.

- **Kenya Household Survey Data:** This survey was conducted as part of the development of the Index based livestock insurance (IBLI) program in northern Kenya. This dataset has information on household location, livestock levels and changes for each month in 2010-2013. There are 900 households in this dataset.

Data Creation

1. Calculate NDVI metrics for each village in each season. These metrics will be the features for our predictive model.
2. Use household data to calculate average livestock mortality in each village in each season.
3. Merge the two resulting datasets to create a dataset for regression.
4. Train prediction model for each cluster and use this model to predict herd mortality at the village level.
5. Add model predictions to household data.

Evaluation Procedure

We use leave one out cross validation to evaluate our method. In each iteration, we leave out one year of data for testing.

1. Split data into training and test sets
2. Train prediction model on training data, create predictions for the test set
3. Use training data and model predictions to design contracts using both methods.
4. Apply insurance contracts designed by the two methods to farmers in the test set and compare outcomes.

Results

The insurance contracts developed by our model provide slightly better protection at a much lower cost. The cost of our contracts is 9% lower, and the cost of capital is 13% lower.

Model	Max CVaR	Max VaR	$\lvert VaR_2 - VaR_1 \rvert$	Average Cost
Baseline	0.69	0.52	0.13	5263.64
Opt	0.65	0.53	0.10	3794.80

Table: Results using Kenya household data

Conclusions

- The contracts designed by our model are able to offer better protection at a similar costs, or comparable protection at lower costs than the baseline method.
- It outperforms the baseline when the prediction model is incorrectly specified and on the Kenyan pastoralist data.
- Our method is more cost effective because it takes into account spatial correlations between areas and the costs of capital requirements. Thus, the model makes better trade offs between costs and coverage than the baseline method.

Next Steps

- We are working with practitioners to improve the model and possibly test it in practice.
- We are working with the Bank of Thailand on the implementation of their satellite-based index insurance program.
- We are also talking to the International Research Institute for Climate and Society at Columbia, they have worked on the implementation of numerous index insurance programs in Africa.

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Idealized CVaR Model

- **Objective:** conditional value at risk of the farmers' loss net of insurance.
- **Constraint 1:** piecewise linear structure of the contract.
- **Constraint 2:** budget constraint.
- **Constraint 3:** definition of required capital.

$$\min_{a,b,\pi,K} \quad CVaR_{1-\epsilon}\left(\ell - I(\theta)\right)$$

$$\text{s.t. } I(\theta) = \min\{(a\hat{\ell}(\theta) + b)^+, P\} \tag{18}$$

$$\mathbb{E}\left[I(\theta)\right] + c_\kappa K \leq B \tag{19}$$

$$K = \left(CVaR_{1-\epsilon}\left(I(\theta)\right) - \mathbb{E}[I(\theta)]\right) \tag{20}$$

The problem is non-convex, so we need convex approximations

We use the following approximations of $I(\theta)$ to make the problem convex:

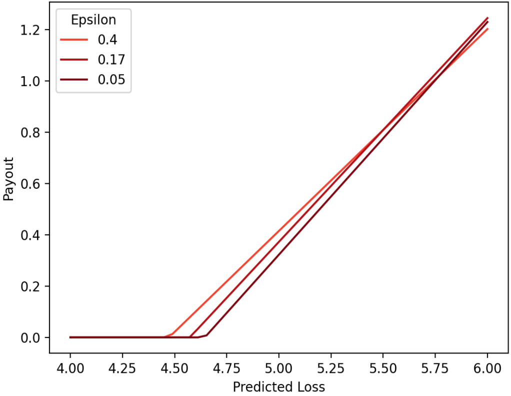
$$\overline{I(\theta)} \triangleq \max\left\{0, a\hat{\ell}(\theta) + b\right\}$$

$$\underline{I(\theta)} \triangleq \min\{a\hat{\ell}(\theta) + b, K\}$$

- Note that $\overline{I(\theta)} \geq I(\theta)$ and $\overline{I(\theta)}$ is convex. Conversely, $\underline{I(\theta)} \leq I(\theta)$ and $\underline{I(\theta)}$ is concave.
- We replace $I(\theta)$ with either $\overline{I(\theta)}$ or $\underline{I(\theta)}$ where necessary to obtain conservative and convex approximations.
- We also need approximations or proxies for $E[I(\theta)]$ in constraint . We use $\pi_{SQ} = E[I_{SQ}(\theta)]$, where I_{SQ} is the contract designed using the status quo method, as a proxy for $E[I(\theta)]$ in constraint .

Insights: Relationships between parameters and epsilon

As ϵ gets smaller, the slope increases and the function shifts to the right.



Results: Misspecified Prediction Model

Model	Max VaR	$\lvert VaR_2 - VaR_1 \rvert$	Required Capital	Average Cost
Baseline	27.42	1.65	53.85	40.73
	[25.13, 29.57]	[0.16, 5.56]	[44.81, 59.88]	[36.06, 45.83]
Opt	27.41	1.96	49.97	40.36
	[24.53, 29.83]	[0.15, 5.6]	[42.87, 58.53]	[35.52, 45.5]

(a) No correlation

Model	Max VaR	$\lvert VaR_2 - VaR_1 \rvert$	Required Capital	Average Cost
Baseline	27.16	1.23	57.1	41.36
	[24.7, 29.62]	[0.1, 5.05]	[51.16, 62.9]	[36.24, 46.52]
Opt	27.71	1.1	56.51	41.31
	[24.91, 30.31]	[0.09, 3.62]	[50.92, 62.26]	[35.98, 46.82]

(b) Positive correlation

Model	Max VaR	$\lvert VaR_2 - VaR_1 \rvert$	Required Capital	Average Cost
Baseline	27.52	1.9	25.2	36.42
	[25.09, 29.69]	[0.17, 6.0]	[17.99, 36.35]	[33.13, 41.88]
Opt	27.39	1.84	26.77	36.67
	[24.33, 29.71]	[0.16, 5.49]	[18.17, 37.5]	[33.21, 42.03]

(c) Negative correlation