

Agricultural Index Insurance: An Optimization Approach

José I. Velarde Morales Linwei Xin

University of Chicago
Booth School of Business

October 14, 2023

The Problem of Agricultural Risk

- Farmers face a lot of risk, and the lack of risk management tools forces them to use coping strategies that hurt their long term welfare.
- Traditional insurance is prohibitively costly in most developing countries due to lack of data and high verification costs.
- Moral hazard, adverse selection, and the presence of large covariate shocks make the problem of agricultural insurance especially hard.

A Proposed Solution: Index Insurance

- In index insurance, an index (or statistic) is created using easily observable quantities (e.g. rainfall), and it is used to determine whether the insured party suffered an adverse event.
- If the index falls below a pre-determined threshold, the insurance company automatically issues out payments to the insured.
- This allows the insurance company to circumvent the issue of verification and moral hazard, since the actions of individual farmers cannot affect the index.

Index Insurance in Practice

- Since it was first proposed, index insurance programs have been implemented in many countries including India, Mexico, Tanzania, Malawi, Kenya, and many others (Jensen and Barrett (2017)).
- Today, tens of millions of farmers worldwide are covered by index insurance programs (Greatrex et al. (2015)).
- However, in most of these cases, the insurance has to be heavily subsidized by governments due to high cost and low demand (Greatrex et al. (2015)).

Project Overview

- Traditionally, the contract for each insured zone is designed independently of all other zones.
- The goal of this project is to make insurance less costly by improving the design of the insurance contracts.
- Our method simultaneously determines the contract parameters for different areas, while taking into account the correlation between the areas, reducing risk for the insurer.

Index Insurance Literature

- **Impact of Index Insurance:** Overall, there is evidence that index insurance reduces reliance on detrimental risk-coping strategies, increases investment, and leads to riskier, but more profitable production decisions (Jensen and Barrett (2017); Cole et al. (2013); Mobarak and Rosenzweig (2013); Karlan et al. (2014)).
- **Demand for Index Insurance:** Demand for index insurance tends to be low and highly price sensitive (Jensen and Barrett (2017); Cole et al. (2013); Cai, De Janvry, and Sadoulet (2020), Casaburi and Willis (2018)).
- **Design of Index Insurance:** There has been relatively little research studying the design of index insurance. The method developed by Chantarat et al. (2013) is the most commonly used in academic publications (Jensen, Stoeffler, et al. (2019); Flatnes, Carter, and Mercovich (2018)).

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Index Insurance: General Context

- Index insurance is a named-peril insurance.
- It is often tied to access to credit.
- Offered at different levels (micro level, meso level, and macro level)
- Often based on public-private partnerships. Research institutions usually design the product, and insurance companies (or governments) sell it.

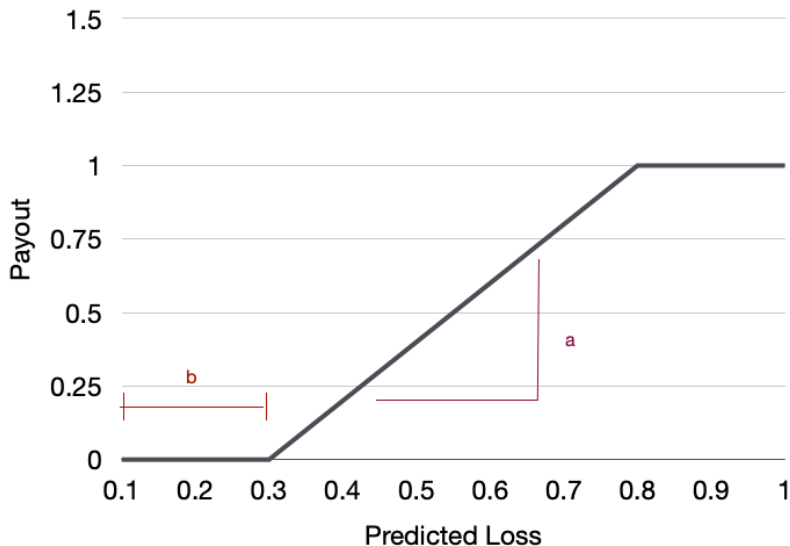
Index Insurance: Design

- ① Prediction: building a model to predict loss.
- ② Contract design: designing contracts specifying payouts based on model predictions.
- ③ Pricing: product pricing

Index Insurance: Definition and Parameters

- Index insurance uses a signal, θ , to predict agricultural loss, $\hat{\ell}(\theta)$
- Contract form: $I(\theta) = \min \left\{ \max \left\{ a\hat{\ell}(\theta) - b, 0 \right\}, 1 \right\}$, a, b are the contract parameters.
- Expected cost for insurer: $C(I(\theta)) = \mathbb{E}[I(\theta)] + c_{\kappa}K(I(\theta))$, where c_{κ} is the cost of capital, and K is required capital.
- $K(I(\theta)) = CVaR_{1-\epsilon_P}(I(\theta)) - \mathbb{E}[I(\theta)]$.

Example of Index Insurance Contract



Practitioner Interviews

- We conducted interviews with researchers and practitioners that had implemented index insurance programs in several countries (Malawi, Kenya, Senegal, Thailand, among others) to learn more about the context.
- **Objective:** minimize risk faced by farmers
- **Constraints:** Price constraints and payout frequency

Definition

For a random variable z , representing loss, the $(1 - \epsilon)$ Value at Risk (VaR) is given by

$$\text{VaR}_{1-\epsilon}(z) := \inf \{t : P(z \leq t) \geq 1 - \epsilon\}$$

Definition

For a random variable z , representing loss, the $(1 - \epsilon)$ Conditional Value at Risk (CVaR) is given by

$$\text{CVaR}_{1-\epsilon}(z) := \mathbb{E}[z | z \geq \text{VaR}_{1-\epsilon}(z)]$$

Idealized CVaR Model

$$\begin{aligned} \min_{a,b,\pi,K} \quad & \text{CVaR}_{1-\epsilon}(\ell + \pi - I(\theta)) \\ \text{s.t.} \quad & I(\theta) = \min \left\{ \max \left\{ 0, a\hat{\ell}(\theta) - b \right\}, 1 \right\} \quad (1) \\ & \pi = \mathbb{E}[I(\theta)] + c_K K \quad (2) \\ & K = \text{CVaR}_{1-\epsilon_K}(I(\theta)) - \mathbb{E}[I(\theta)] \quad (3) \\ & \underline{f} \leq \mathbb{P}(I(\theta) > 0) \leq \bar{f} \quad (4) \\ & \pi \leq \bar{\pi}. \quad (5) \end{aligned}$$

Idealized CVaR Model

$$\begin{aligned} \min_{a,b,\pi,K} \quad & \text{CVaR}_{1-\epsilon}(\ell + \pi - I(\theta)) \\ \text{s.t.} \quad & I(\theta) = \min \left\{ \max \left\{ 0, a\hat{\ell}(\theta) - b \right\}, 1 \right\} \end{aligned} \tag{6}$$

$$\pi = \mathbb{E}[I(\theta)] + c_K K \tag{7}$$

$$K = \text{CVaR}_{1-\epsilon_K}(I(\theta)) - \mathbb{E}[I(\theta)] \tag{8}$$

$$\underline{f} \leq \mathbb{P}(I(\theta) > 0) \leq \bar{f} \tag{9}$$

$$\pi \leq \bar{\pi}. \tag{10}$$

Multiple Zone

$$\min_{a,b,K} \max_z \text{CVaR}_{1-\epsilon}(\ell_z + \pi_z - I_z(\theta_z)) \quad (11)$$

$$\text{s.t.} \quad I_z(\theta_z) = \min \left\{ \max \left\{ a_z \hat{\ell}_z(\theta_z) + b_z, 0 \right\}, 1 \right\} \quad (12)$$

$$\pi_z = \mathbb{E}[I_z(\theta_z)] + \frac{c_K}{\sum_z s_z} K \quad (13)$$

$$K = \text{CVaR}_{1-\epsilon_K} \left(\sum_z s_z I_z(\theta_z) \right) - \mathbb{E} \left[\sum_z s_z I_z(\theta_z) \right] \quad (14)$$

$$\underline{f} \leq \mathbb{P}(I_z(\theta_z) > 0) \leq \bar{f} \quad (15)$$

$$\pi_z \leq \overline{\pi_z} \quad (16)$$

Multiple Zone

$$\min_{a,b,K} \max_z \text{CVaR}_{1-\epsilon}(\ell_z + \pi_z - I_z(\theta_z)) \quad (17)$$

$$\text{s.t.} \quad I_z(\theta_z) = \min \left\{ \max \left\{ a_z \hat{\ell}_z(\theta_z) + b_z, 0 \right\}, 1 \right\} \quad (18)$$

$$\pi_z = \mathbb{E}[I_z(\theta_z)] + \frac{c_K}{\sum_z s_z} K \quad (19)$$

$$K = \text{CVaR}_{1-\epsilon_K} \left(\sum_z s_z I_z(\theta_z) \right) - \mathbb{E} \left[\sum_z s_z I_z(\theta_z) \right] \quad (20)$$

$$\underline{f} \leq \mathbb{P}(I_z(\theta_z) > 0) \leq \bar{f} \quad (21)$$

$$\pi_z \leq \overline{\pi_z} \quad (22)$$

Convex Relaxation

$$\min_{a,b,K,\pi} \max_z \text{CVaR}_{1-\epsilon} \left(\ell_z + \pi_z - \underline{I_z(\theta_z)} \right) \quad (23)$$

$$\text{s.t.} \quad \pi_z = \mathbb{E} \left[\overline{I_z(\theta_z)} \right] + \frac{c_{\kappa}}{\sum_z s_z} K \quad (24)$$

$$K = \text{CVaR}_{1-\epsilon_K} \left(\sum_z s_z \overline{I_z(\theta_z)} \right) - \mathbb{E} \left[\sum_z s_z \underline{I_z(\theta_z)} \right] \quad (25)$$

$$a \hat{\ell}_{\underline{f}} \leq b \leq a \hat{\ell}_{\overline{f}} \quad (26)$$

$$\overline{I_z(\theta_z)} = \max \left\{ 0, a_z \hat{\ell}_z(\theta_z) + b_z \right\} \quad (27)$$

$$\underline{I_z(\theta_z)} = \min \left\{ a_z \hat{\ell}_z(\theta_z) + b_z, 1 \right\} \quad (28)$$

$$\pi_z \leq \overline{\pi_z}.$$

LP Reformulation

Using the results from Rockafellar and Uryasev (2002), we get:

$$\begin{aligned}
 & \min_{a,b,\alpha,\omega,\gamma,t,m,K,\pi} \quad m \\
 \text{s.t.} \quad & t_z + \frac{1}{\epsilon} \sum_j p^j \gamma_z^j \leq m, \forall z \\
 & \gamma_z^j \geq s_z (\ell^j + \pi_z - \omega_z^j) - t_z, \forall j, \forall z \\
 & \gamma_z^j \geq 0, \forall j, \forall z \\
 & \pi_z = \frac{1}{N} \sum_j \alpha_z^j + \frac{1}{\sum_z s_z} c_K K \\
 & t_K + \frac{1}{\epsilon_K} \sum_j p^j \gamma_K^j \leq K + \frac{1}{N} \sum_j \sum_z s_z \omega_z^j \\
 & \gamma_K^j \geq \sum_z s_z \alpha_z^j - t_K, \forall j \\
 & \gamma_K^j \geq 0, \forall j \\
 & \alpha_z^j \geq a_z \hat{\ell}_z(\theta_z^j) + b_z, \forall j, \forall z \\
 & \alpha_z^j \geq 0, \forall j, \forall z \\
 & \omega_z^j \leq a_z \hat{\ell}_z(\theta_z^j) + b_z, \forall j, \forall z \\
 & \omega_z^j \leq 1, \forall j, \forall z \\
 & \pi_z \leq \bar{\pi}_z, \forall z.
 \end{aligned}$$

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Baseline Method

Baseline method: Chantarat et al. (2013). Common in academic publications, used to design Kenya's IBLI program. Method consists of:

- 1 Use linear model to predict loss.
- 2 Contracts are of the form: $I(\theta) = \min \left\{ \max \left\{ \hat{\ell}(\theta) - b, 0 \right\}, 1 \right\}$. b is chosen to maximize the correlation of payouts and losses.

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What we wanted to test

- How is our method's performance affected by errors in the prediction model?
- How are costs affected by correlation between zones?

Setup

We use a linear prediction model and the following data generating process

$\ell_z = \frac{1}{1+e^{f(\theta_z)}}$ with $f(\theta_z)$:

- Linear case: $f(\theta) = \beta\theta + \epsilon$
- Nonlinear case: $f(\theta) = \beta_0 + \beta_1\theta + \beta_2\theta^2 + \dots + \beta_n\theta^n + \epsilon$

In both cases, $\theta \sim \mathcal{N}((0, 0), \Sigma)$, $\epsilon \sim \mathcal{N}(0, \rho I)$. In both cases, β_i is drawn randomly. Vary ρ to test effect of correlation.

Simulation Details

- 1 Generate training and test samples.
- 2 Use training data and model predictions to design contracts using both methods.
- 3 Apply insurance contracts designed by the two methods to farmers in the test set and compare outcomes.

We run this 200 times for every scenario we test.

Performance Metrics

For each sample in the test set, we calculate the net loss. Then we calculate the following metrics:

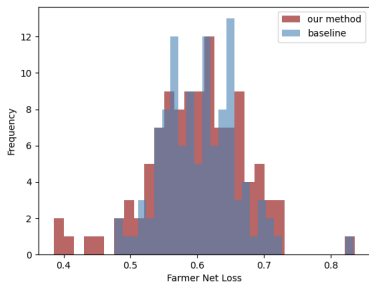
Farmer Welfare Measures	Insurer Cost/Risk Measures
Conditional Value at Risk (CVaR) Value at Risk (VaR) Semi-Variance	Average Cost Required Capital

Summary of Results

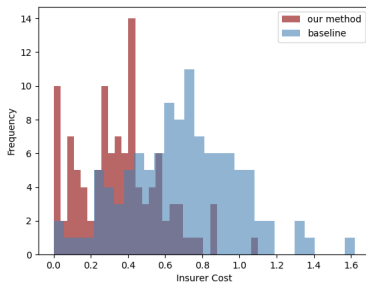
- Our model offers comparable protection at much lower costs
- More robust to prediction errors and correlated losses.
- Adjusts payout strategy depending on correlation.

Farmer Risk and Insurer Costs

- Distribution of farmer loss is similar under both
- Cost for insurer is radically different



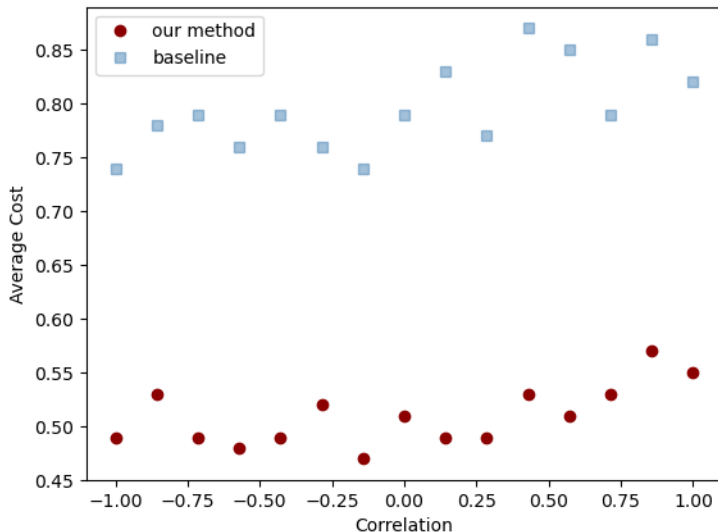
(a) Farmer net loss



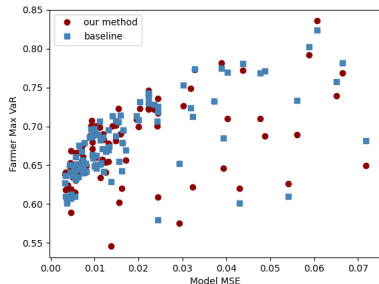
(b) Insurer cost

Figure: Distribution of outcomes

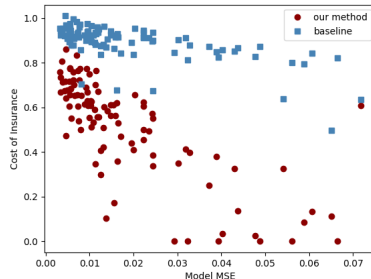
More robust to correlation of losses



More robust to errors in prediction model



(a) Farmer VaR

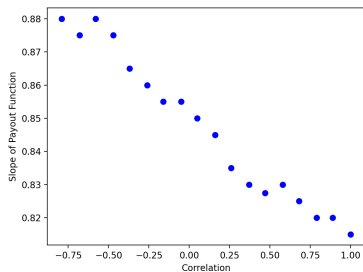


(b) Insurer cost

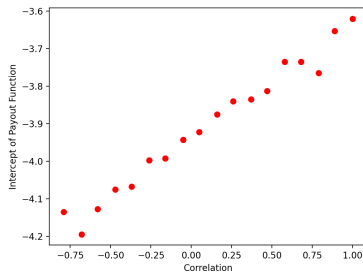
Figure: Distribution of outcomes

Adjusts payout strategy based on correlation of losses

- Positive correlation leads to smaller but more frequent payouts
- Negative correlation leads to less frequent but more aggressive payouts



(a) Slope of payout function vs correlation



(b) Intercept of payout function vs correlation

Figure: Relationship between parameters and correlation

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- **NDVI Data:** The Normalized Difference Vegetation Index (NDVI) is a satellite-based indicator of the amount and health of vegetation. We use NDVI data for Kenya between 2000-2015.
- **Kenya Household Survey Data:** This survey was conducted as part of the development of the Index based livestock insurance (IBLI) program in northern Kenya. This dataset has information on household location, livestock levels and changes for each month in 2010-2013. There are 900 households in this dataset.

Evaluation Procedure

We use leave one out cross validation to evaluate our method. In each iteration, we leave out one year of data for testing.

- 1 Split data into training and test sets
- 2 Use training data and model predictions to design contracts using both methods.
- 3 Apply insurance contracts designed by the two methods to farmers in the test set and compare outcomes.

Results

The insurance contracts developed by our model provide slightly better protection at a much lower cost. The cost of our contracts is 9% lower, and the cost of capital is 13% lower.

Model	Max CVaR	Max VaR	$ VaR_2 - VaR_1 $	Average Cost
Baseline	0.69	0.52	0.13	5263.64
Opt	0.65	0.53	0.10	3794.80

Table: Results using Kenya household data

Conclusions

- The contracts designed by our model are able to offer comparable protection at lower costs than the baseline method.
- Our method is able to outperform the baseline because it is more robust to errors in the prediction model.
- Our method is more cost effective because it takes into account spatial correlations between areas to better manage risk. Thus, the model makes better trade offs between costs and coverage than the baseline method.

References

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Idealized CVaR Model

- **Objective:** conditional value at risk of the farmers' loss net of insurance.
- **Constraint 1:** piecewise linear structure of the contract.
- **Constraint 2:** budget constraint.
- **Constraint 3:** definition of required capital.

$$\min_{a,b,\pi,K} \text{CVaR}_{1-\epsilon}(\ell - I(\theta))$$

$$\text{s.t. } I(\theta) = \min\{(a\hat{\ell}(\theta) + b)^+, P\} \quad (29)$$

$$\mathbb{E}[I(\theta)] + c_\kappa K \leq B \quad (30)$$

$$K = (\text{CVaR}_{1-\epsilon}(I(\theta)) - \mathbb{E}[I(\theta)]) \quad (31)$$

The problem is non-convex, so we need convex approximations

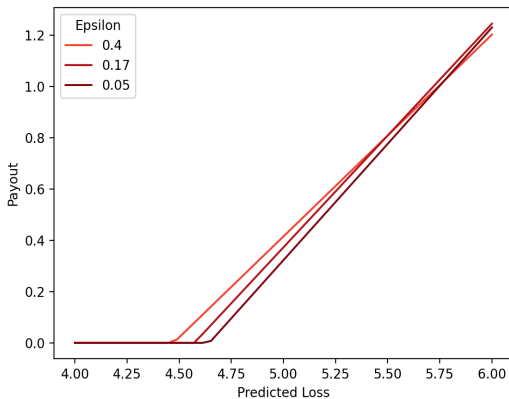
We use the following approximations of $I(\theta)$ to make the problem convex:

$$\begin{aligned}\overline{I(\theta)} &\triangleq \max \left\{ 0, a\hat{\ell}(\theta) + b \right\} \\ \underline{I(\theta)} &\triangleq \min \{ a\hat{\ell}(\theta) + b, K \}\end{aligned}$$

- Note that $\overline{I(\theta)} \geq I(\theta)$ and $\overline{I(\theta)}$ is convex. Conversely, $\underline{I(\theta)} \leq I(\theta)$ and $\underline{I(\theta)}$ is concave.
- We replace $I(\theta)$ with either $\overline{I(\theta)}$ or $\underline{I(\theta)}$ where necessary to obtain conservative and convex approximations.
- We also need approximations or proxies for $E[I(\theta)]$ in constraint . We use $\pi_{SQ} = E[I_{SQ}(\theta)]$, where I_{SQ} is the contract designed using the status quo method, as a proxy for $E[I(\theta)]$ in constraint .

Insights: Relationships between parameters and epsilon

As ϵ gets smaller, the slope increases and the function shifts to the right.



Results: Misspecified Prediction Model

Model	Max VaR	$ VaR_2 - VaR_1 $	Required Capital	Average Cost
Baseline	27.42 [25.13, 29.57]	1.65 [0.16, 5.56]	53.85 [44.81, 59.88]	40.73 [36.06, 45.83]
Opt	27.41 [24.53, 29.83]	1.96 [0.15, 5.6]	49.97 [42.87, 58.53]	40.36 [35.52, 45.5]

(a) No correlation

Model	Max VaR	$ VaR_2 - VaR_1 $	Required Capital	Average Cost
Baseline	27.16 [24.7, 29.62]	1.23 [0.1, 5.05]	57.1 [51.16, 62.9]	41.36 [36.24, 46.52]
Opt	27.71 [24.91, 30.31]	1.1 [0.09, 3.62]	56.51 [50.92, 62.26]	41.31 [35.98, 46.82]

(b) Positive correlation

Model	Max VaR	$ VaR_2 - VaR_1 $	Required Capital	Average Cost
Baseline	27.52 [25.09, 29.69]	1.9 [0.17, 6.0]	25.2 [17.99, 36.35]	36.42 [33.13, 41.88]
Opt	27.39 [24.33, 29.71]	1.84 [0.16, 5.49]	26.77 [18.17, 37.5]	36.67 [33.21, 42.03]

(c) Negative correlation