Agricultural Index Insurance: An Optimization Approach

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The Problem of Agricultural Risk

- Farmers face a lot of risk, and the lack of risk management tools forces them to use coping strategies that hurt their long term welfare.
- Traditional insurance is prohibitively costly in most developing countries due to lack of data and high verification costs.
- Moral hazard, adverse selection, and the presence of large covariate shocks make the problem of agricultural insurance especially hard.

A Proposed Solution: Index Insurance

- In index insurance, an index (or statistic) is created using easily observable quantities (e.g. rainfall), and it is used to determine whether the insured party suffered an adverse event.
- If the index falls below a pre-determined threshold, the insurance company automatically issues out payments to the insured.
- This allows the insurance company to circumvent the issue of verification and moral hazard, since the actions of individual farmers cannot affect the index.

Index Insurance in Practice

- Since it was first proposed, index insurance programs have been implemented in many countries including India, Mexico, Tanzania, Malawi, Kenya, and many others (Jensen and Barrett (2017)).
- Today, tens of millions of farmers worldwide are covered by index insurance programs (Greatrex et al. (2015)).
- However, in most of these cases, the insurance has to be heavily subsidized by governments due to high cost and low demand (Greatrex et al. (2015)).

Project Overview

- Traditionally, contracts are designed to maximize correlation between payouts and losses.
- The goal of this project is to develop an optimization-based approach to contract design, that will allow us to include constraints.
- Our method simultaneously determines the contract parameters for different areas, while taking into account the correlation between the areas, reducing risk for the insurer.

Index Insurance Literature

- **Impact of Index Insurance:** Overall, there is evidence that index insurance reduces reliance on detrimental risk-coping strategies, increases investment, and leads to riskier, but more profitable production decisions (Jensen and Barrett (2017); Cole et al. (2013); Mobarak and Rosenzweig (2013); Karlan et al. (2014)).
- **Demand for Index Insurance:** Demand for index insurance tends to be low and highly price sensitive (Jensen and Barrett (2017); Cole et al. (2013); Cai, De Janvry, and Sadoulet (2020),Casaburi and Willis (2018)).
- **Design of Index Insurance:** There has been relatively little research studying the design of index insurance. The method developed by Chantarat et al. (2013) is the most commonly used in academic publications (Jensen, Stoeffler, et al. (2019); Flatnes, Carter, and Mercovich (2018)). Recently, Chen et al 2023 developed a NN based method to design index insurance.

Content

Three methods to design index insurance

- Predict then optimize, choose contract that maximizes correlation between payouts and losses. No constraints.
- End to end, use NN to design contracts, can handle few constraints.

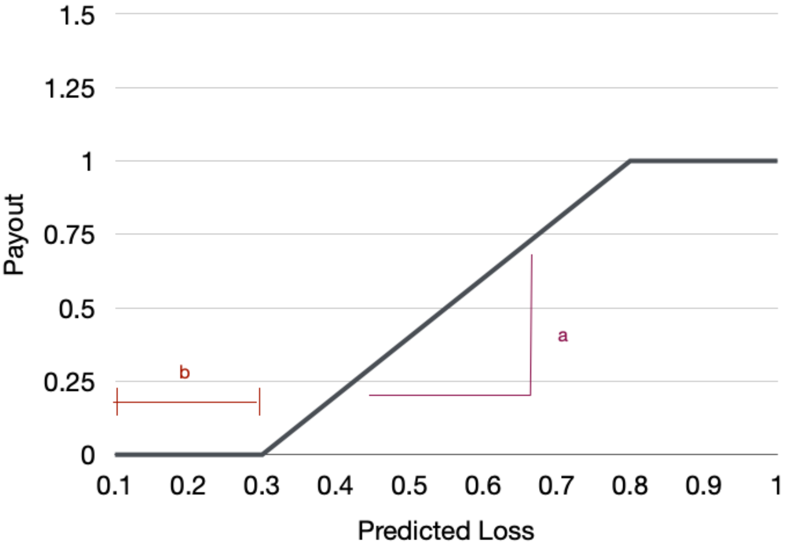
Index Insurance: Design

1. Prediction: building a model to predict loss.
2. Contract design: designing contracts specifying payouts based on model predictions.
3. Pricing: product pricing

Index Insurance: Definition and Parameters

- Index insurance uses a signal, θ, to predict agricultural loss, $\hat{\ell}(\theta)$
- Contract form: $I(\theta) = \min\left\{\max\left\{a\hat{\ell}(\theta) - b, 0\right\}, 1\right\}$, a, b are the contract parameters.

Example of Index Insurance Contract



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Model

Farmers start with wealth, w_0 and experience loss ℓ. There is an index insurance contract I, that is determined by a p dimensional vector of indices $\theta = (\theta_1, ..., \theta_p)$. Premium for contract I is $\pi(I)$. Farmer wealth is:

$$
\begin{aligned}
w &= w_0 - \ell + I(\theta) - \pi(I) \\
\pi(I) &= \lambda \mathbb{E}[I(\theta)]
\end{aligned}
$$

Optimization Problem

They solve the following optimization problem:

$$\begin{aligned}
\max_{I \in \mathcal{I}} \quad & \mathbb{E}\left[U\left(w_0 - \ell - \pi + I(\theta)\right)\right] \\
\text{s.t.} \quad & \underline{\pi} \leq \pi(I) \leq \overline{\pi} && (1) \\
& \pi = \lambda \mathbb{E}\left[I(\theta)\right] && (2) \\
& U(w) = -(1/\alpha)e^{-\alpha w} && (3)
\end{aligned}$$

Solution and results

- They use a neural network to solve the optimization problem.
- Network they use has $\sim 10,000$ parameters.
- They test it on Illinois corn yield data and find that the contracts are utility improving on average.
- Their method is end-to-end, it goes directly from weather variables to payouts.

Drawbacks

- Not interpretable, can't set constraints on variables that policy makers might care about (e.g. deductible).
- Model requires a lot of data to train, the data they tested the model on went back to 1925, unrealistic for developing countries.
- Definition of premium used depends only on expected value of the payout and not on variance of the payouts, in practice the price would depend on riskiness of contract.

Overview

- We opt for a "predict-then-optimize" approach.
- We use specialized time-series feature extraction algorithms for feature extraction and traditional ML algorithms (e.g. Random Forest, Gradient Boosting, Support Vector Machines)
- These algorithms require less data to train.

Model

We use the same model, just a different definition of the premium.

- Premium: $\pi(I(\theta)) = \mathbb{E}[I(\theta)] + c_\kappa K(I(\theta))$, where c_κ is the cost of capital, and K is required capital.

- $K(I(\theta)) = CVaR_{1-\epsilon_P}(I(\theta)) - \mathbb{E}[I(\theta)]$.

I got this definition from *Risk Modeling for Appraising Named Peril Index Insurance Products: A Guide for Practitioners* (Mapfumo, Groenendaal, and Dugger (2017)). The EU's Solvency requirements are similar.

Idealized Model

$$\max_{a,b,\pi,K} \quad \mathbb{E}\left[U\left(w_0 - \ell - \pi + I(\theta)\right)\right] \tag{4}$$

$$\text{s.t.} \quad I(\theta) = \min\left\{\max\left\{0, a\hat{\ell}(\theta) + b\right\}, 1\right\} \tag{5}$$

$$\pi = \mathbb{E}\left[I(\theta)\right] + c_\kappa\left[\text{CVaR}_{1-\epsilon_K}\left(I(\theta)\right) - \mathbb{E}[I(\theta)]\right] \tag{6}$$

$$\pi \leq \overline{\pi}. \tag{7}$$

Convex Relaxation

We use the following convex and concave relaxations of $I(\theta)$:

$$\overline{I(\theta)} \triangleq \max\left\{0, a\hat{\ell}(\theta) + b\right\}, \quad \underline{I(\theta)} \triangleq \min\{a\hat{\ell}(\theta) + b, 1\}.$$

Note that $\overline{I(\theta)}$ is convex in $\hat{\ell}(\theta)$, $\underline{I(\theta)}$ is concave in $\hat{\ell}(\theta)$, and

$$\underline{I(\theta)} \leq I(\theta) \leq \overline{I(\theta)}.$$

We replace $I(\theta)$ in Problem (4) with either $\overline{I(\theta)}$ or $\underline{I(\theta)}$ where necessary to obtain a conservative and convex relaxation.

Convex Relaxation

$$\max_{a,b,K,\pi} \quad \mathbb{E}\left[U\left(w_0 - \ell - \pi + \underline{I(\theta)}\right)\right] \tag{8}$$

$$\text{s.t.} \quad \overline{I(\theta)} = \max\left\{0, a\hat{\ell}(\theta) + b\right\}$$

$$\underline{I(\theta)} = \min\left\{a\hat{\ell}(\theta) + b, 1\right\}$$

$$\pi = (1 - c_\kappa)\,\mathbb{E}\left[\overline{I(\theta)}\right] + c_\kappa \mathrm{CVaR}_{1-\epsilon_K}\left(\overline{I(\theta)}\right) \tag{9}$$

$$\pi \leq \overline{\pi}.$$

CP Reformulation

Using the results from Rockafellar and Uryasev (2002), we get:

$$\max_{a,b,\alpha,\omega,\gamma,t_K,K,\pi} \quad \frac{1}{N}\sum_j \sum_z U\left(w_{0,z} - \ell_z^j - \pi_z + I_z(\theta_z^j)\right)$$

$$\text{s.t.}\pi_z = \left(1 - \frac{c_\kappa}{\sum_z s_z}\right)\frac{1}{N}\sum_j \alpha_z^j + \frac{c_\kappa}{\sum_z s_z}\left(K - \frac{1}{N}\sum_j \sum_{z' \neq z} s_{z'}\omega_{z'}^j\right)$$

$$t_K + \frac{1}{\epsilon_K}\sum_j p^j \gamma_K^j \leq K$$

$$\gamma_K^j \geq \sum_z s_z \alpha_z^j - t_K, \forall j$$

$$\gamma_K^j \geq 0, \forall j$$

$$\alpha_z^j \geq a_z \hat{\ell}_z(\theta_z^j) + b_z, \forall j, \forall z$$

$$\alpha_z^j \geq 0, \forall j, \forall z$$

$$\omega_z^j \leq a_z \hat{\ell}_z(\theta_z^j) + b_z, \forall j, \forall z$$

$$\omega_z^j \leq 1, \forall j, \forall z$$

$$\pi_z \leq \overline{\pi_z}, \forall z.$$

Data

Used two main data sources

- Illinois annual corn yield data from the National Agricultural Statistics Service (NASS). Data is available at the county level from 1925-2022. 84 counties.
- Weather data from the PRISM climate group. Has monthly data on several weather variables (temperature, precipitation, etc). Available 1895-present.

- We use a 70/15/15 train/val/test split. Data is kept in chronological order. Training data has older years and test data has the newest years.

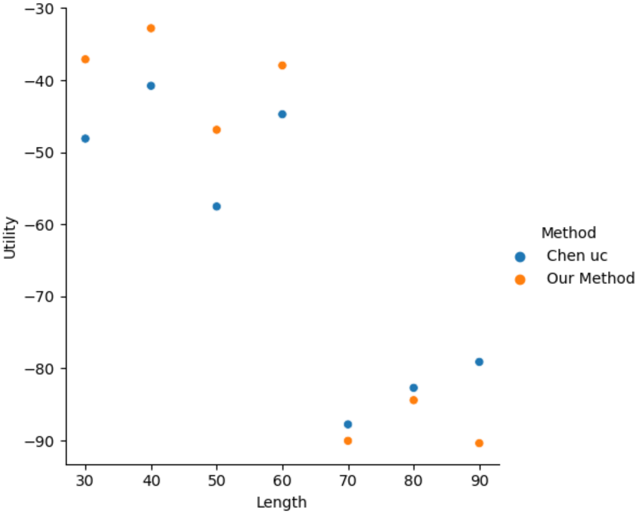
- We modified Chen's method to use the same definition of the premium as our method.

- We use the training and validation data to design the contracts using both methods, apply the contracts to farmers in the test set, and compute performance metrics.

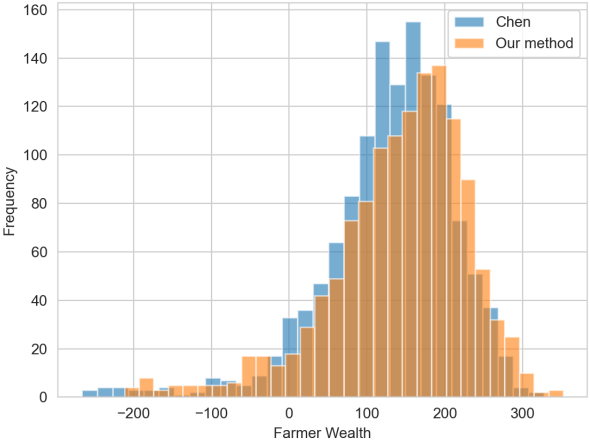
Overview

- No method really dominates the other.
- Generally, whichever method costs more tends to have higher utility for farmers.
- In terms of farmer utility, our method tends to work better with realistic data lengths.

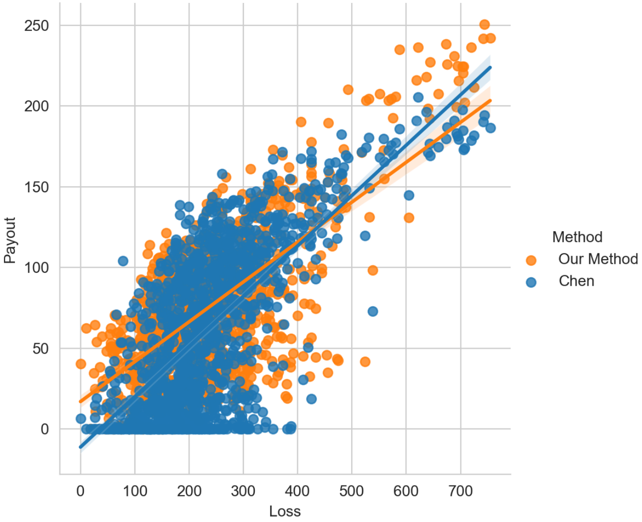
Data Shortening



Farmer Wealth distribution: 45 years of data



Payouts vs Losses



Multiple Zone

$$
\begin{aligned}
\max_{a,b,K,\pi} \quad & \mathbb{E}\left[\sum_z U\left(w_{0,z} - \ell_z^j - \pi_z + I_z(\theta_z^j)\right)\right] && (10)\\
\text{s.t.} \quad & \pi_z = \mathbb{E}\left[\overline{I_z(\theta_z)}\right] + \frac{c_\kappa}{\sum_z s_z} K && (11)\\
& K = \mathrm{CVaR}_{1-\epsilon_K}\left(\sum_z s_z \overline{I_z(\theta_z)}\right) - \mathbb{E}\left[\sum_z s_z \underline{I_z(\theta_{z'})}\right] && (12)\\
& \overline{I_z(\theta_z)} = \max\left\{0, a_z \hat{\ell}_z(\theta_z) + b_z\right\}\\
& \underline{I_z(\theta_z)} = \min\left\{a_z \hat{\ell}_z(\theta_z) + b_z, 1\right\}\\
& \pi_z \leq \overline{\pi_z}.
\end{aligned}
$$

Next steps

- Compare the cost of insuring the US midwest

Conclusions

- The contracts designed by our model are able to offer better protection at a similar costs, or comparable protection at lower costs than the baseline method.
- It outperforms the baseline when the prediction model is incorrectly specified and on the Kenyan pastoralist data.
- Our method is more cost effective because it takes into account spatial correlations between areas and the costs of capital requirements. Thus, the model makes better trade offs between costs and coverage than the baseline method.

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Idealized CVaR Model

- **Objective:** conditional value at risk of the farmers' loss net of insurance.
- **Constraint 1:** piecewise linear structure of the contract.
- **Constraint 2:** budget constraint.
- **Constraint 3:** definition of required capital.

$$\min_{a,b,\pi,K} \quad CVaR_{1-\epsilon}\left(\ell - I(\theta)\right)$$

$$\text{s.t. } I(\theta) = \min\{(a\hat{\ell}(\theta) + b)^{+}, P\} \tag{13}$$

$$\mathbb{E}\left[I(\theta)\right] + c_{\kappa} K \leq B \tag{14}$$

$$K = \left(CVaR_{1-\epsilon}\left(I(\theta)\right) - \mathbb{E}[I(\theta)]\right) \tag{15}$$

The problem is non-convex, so we need convex approximations

We use the following approximations of $I(\theta)$ to make the problem convex:

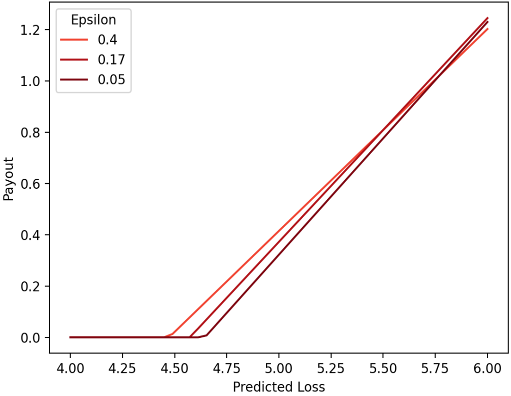
$$\overline{I(\theta)} \triangleq \max\left\{0, a\hat{\ell}(\theta) + b\right\}$$

$$\underline{I(\theta)} \triangleq \min\{a\hat{\ell}(\theta) + b, K\}$$

- Note that $\overline{I(\theta)} \geq I(\theta)$ and $\overline{I(\theta)}$ is convex. Conversely, $\underline{I(\theta)} \leq I(\theta)$ and $\underline{I(\theta)}$ is concave.
- We replace $I(\theta)$ with either $\overline{I(\theta)}$ or $\underline{I(\theta)}$ where necessary to obtain conservative and convex approximations.
- We also need approximations or proxies for $E[I(\theta)]$ in constraint . We use $\pi_{SQ} = E[I_{SQ}(\theta)]$, where I_{SQ} is the contract designed using the status quo method, as a proxy for $E[I(\theta)]$ in constraint .

Insights: Relationships between parameters and epsilon

As ϵ gets smaller, the slope increases and the function shifts to the right.



Results: Misspecified Prediction Model

Model	Max VaR	$\lvert VaR_2 - VaR_1 \rvert$	Required Capital	Average Cost
Baseline	27.42	1.65	53.85	40.73
	[25.13, 29.57]	[0.16, 5.56]	[44.81, 59.88]	[36.06, 45.83]
Opt	27.41	1.96	49.97	40.36
	[24.53, 29.83]	[0.15, 5.6]	[42.87, 58.53]	[35.52, 45.5]

(a) No correlation

Model	Max VaR	$\lvert VaR_2 - VaR_1 \rvert$	Required Capital	Average Cost
Baseline	27.16	1.23	57.1	41.36
	[24.7, 29.62]	[0.1, 5.05]	[51.16, 62.9]	[36.24, 46.52]
Opt	27.71	1.1	56.51	41.31
	[24.91, 30.31]	[0.09, 3.62]	[50.92, 62.26]	[35.98, 46.82]

(b) Positive correlation

Model	Max VaR	$\lvert VaR_2 - VaR_1 \rvert$	Required Capital	Average Cost
Baseline	27.52	1.9	25.2	36.42
	[25.09, 29.69]	[0.17, 6.0]	[17.99, 36.35]	[33.13, 41.88]
Opt	27.39	1.84	26.77	36.67
	[24.33, 29.71]	[0.16, 5.49]	[18.17, 37.5]	[33.21, 42.03]

(c) Negative correlation