Problem Diagnosis

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1 Problem Description

In our last meeting, I brought up the problem that the optimization method would set $a = b = 0$ whenever both a and b were decision variables. This didn't make sense because it would seem that the objective could be improved upon by paying out sometimes. The program we were running was:

$$\min_{a,b,\pi} \quad t + \frac{1}{\epsilon}\sum_k p_k \gamma_k \tag{1}$$

$$\text{s.t. } I^k \geq a\hat{l}(\theta^k) + b, \forall k \tag{2}$$

$$0 \leq I^k \leq y, \forall k \tag{3}$$

$$E[I] \leq \bar{\pi} y \tag{4}$$

$$\gamma_k \geq l^k + E[I] - I^k - t, \forall k \tag{5}$$

$$\gamma_k \geq 0, \forall k \tag{6}$$

I found that the problem was caused by constraint (2). The program was issuing payouts, (ie $I^k > 0$ for some k), but it wasn't changing the values of a and b. However, the resulting payouts ended up being piecewise linear with a slope of one (see Figure 1 for an example). The intercept varied based on the premium constraint. Changing constraint (2) to $I^k = a\hat{l}(\theta^k) + b$ did give non-zero values for a, b, but the payout function ends up being strictly linear, and seemed to perform worse than the baseline (see Figure 2 for an example).

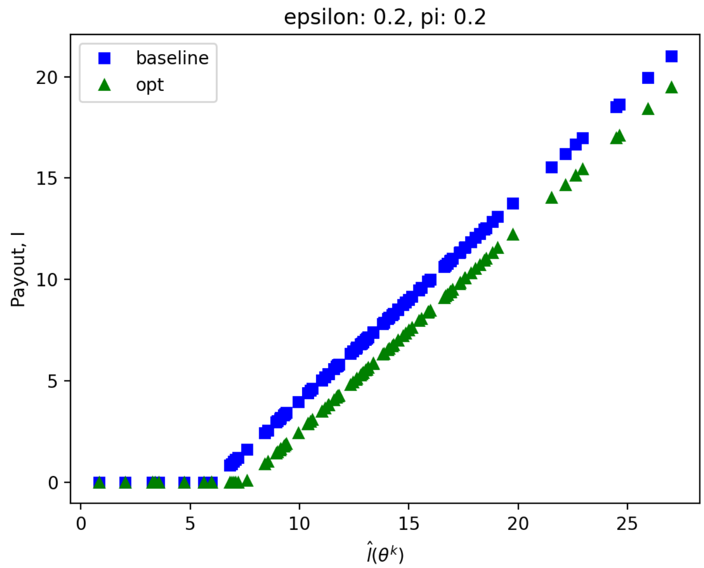

Figure 1: Payout function when $\epsilon = 0.2, \bar{\pi} = 0.2$. This plots I^k when we use the constraint $I^k \geq a\hat{l}(\theta^k) + b$

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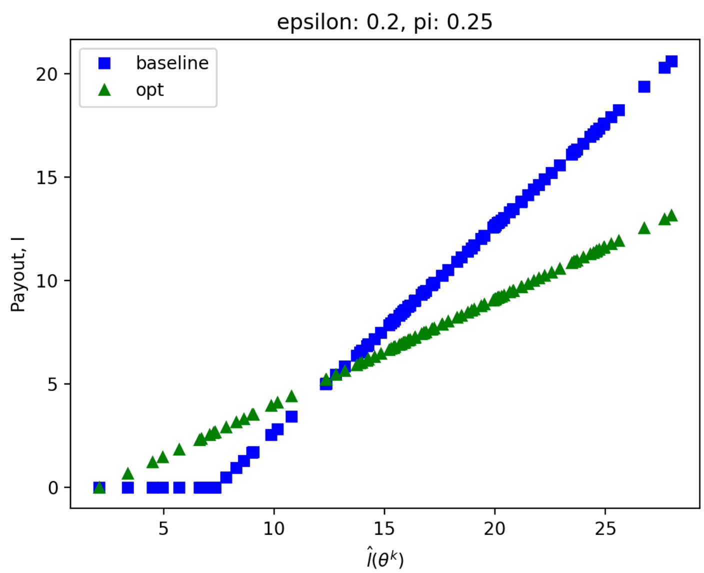

Figure 2: Payout function when $\epsilon = 0.2, \bar{\pi} = 0.25$. This plots I^k when we use the constraint $I^k = a\hat{l}(\theta^k) + b$

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Idea for how to proceed

- One option would be to try to fit a piecewise linear function to the values of I^k the program gives. We could set $b = \max\{\theta^k | I^k = 0\}$, and we could calculate the slope of the function by running the following regression: $I^k = \beta \hat{l}(\theta^k) + \epsilon$, where we would only include observations where $I^k > 0$. However, I'm not sure if this is sketchy.