Agricultural Index Insurance: An Optimization Approach

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The Problem of Agricultural Risk

- Farmers face a lot of risk, and the lack of risk management tools forces them to use coping strategies that hurt their long term welfare.
- Traditional insurance is prohibitively costly in most developing countries due to lack of data and high verification costs.
- Moral hazard, adverse selection, and the presence of large covariate shocks make the problem of agricultural insurance especially hard.

A Proposed Solution: Index Insurance

- Researchers developed index insurance as a less costly way to offer insurance in developing countries.
- In index insurance, an index (or statistic) is created using easily observable quantities (e.g. rainfall), and it is used to determine whether the insured party suffered an adverse event.
- If the index falls below a pre-determined threshold, the insurance company automatically issues out payments to the insured.
- This allows the insurance company to circumvent the issue of verification and moral hazard, since the actions of individual farmers cannot affect the index.

Index Insurance in Practice

- Since it was first proposed, index insurance programs have been implemented in many countries including India, Mexico, Tanzania, Malawi, Kenya, and many others (Jensen and Barrett (2017)).
- Today, tens of millions of farmers worldwide are covered by index insurance programs (Greatrex et al. (2015)).
- However, in most of these cases, the insurance has to be heavily subsidized by governments due to high cost and low demand (Greatrex et al. (2015)).

Project Overview

- The goal of this project is to make insurance less costly by improving the design of the insurance contracts.
- Traditionally, the contract for each insured zone is designed independently of all other zones.
- Our method simultaneously determines the contract parameters for different areas, while taking into account the correlation between the areas.
- This allows it to make better trade offs between coverage and the costs associated with capital requirements.

What I would like feedback on

- Evaluation (what metrics to report, evaluation with observational and synthetic data, etc.)
- Data imputation

Index Insurance Literature

- **Impact of Index Insurance:** Overall, there is evidence that index insurance reduces reliance on detrimental risk-coping strategies, increases investment, and leads to riskier, but more profitable production decisions (Jensen and Barrett (2017); Cole et al. (2013); Mobarak and Rosenzweig (2013); Karlan et al. (2014)).
- **Demand for Index Insurance:** Demand for index insurance tends to be low and highly price sensitive (Jensen and Barrett (2017); Cole et al. (2013); Cai, De Janvry, and Sadoulet (2020),Casaburi and Willis (2018)).
- **Design of Index Insurance:** There has been relatively little research studying the design of index insurance. The method developed by Chantarat et al. (2013) is the most commonly used in academic publications (Jensen, Stoeffler, et al. (2019); Flatnes, Carter, and Mercovich (2018)).

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Index Insurance: General Context

- Index insurance is a named-peril insurance.
- It is often tied to access to credit.
- Offered at different levels (micro level, meso level, and macro level)
- Often based on public-private partnerships. Research institutions usually design the product, and insurance companies (or governments) sell it.

Index Insurance: Design

Index insurance design generally consists of three steps:

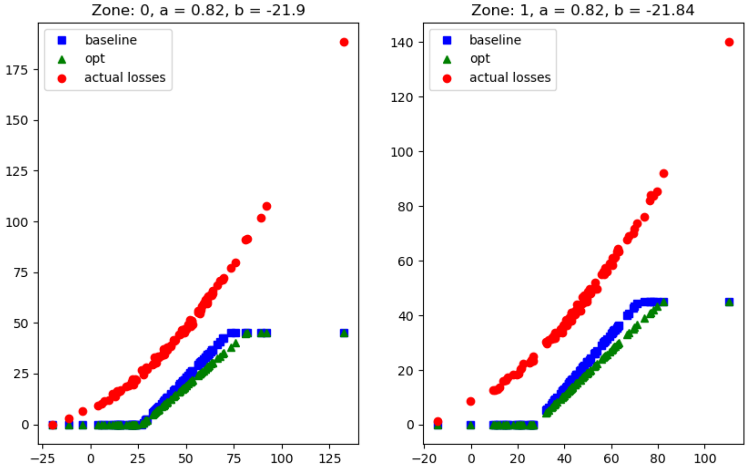
1. Prediction: building a model to predict loss.
2. Contract design: designing contracts specifying payouts based on model predictions.
3. Pricing: product pricing

This project focuses on the second step. Currently, a research team usually designs the insurance product, and the insurance company then prices the product.

Index Insurance: Definition and Parameters

- Index insurance uses a signal, θ, that is used to predict agricultural loss, $\hat{\ell}(\theta)$
- Index insurance contracts normally have the form: $I(\theta) = \min\left\{\left(a\hat{\ell}(\theta) + b\right)^+, P\right\}$, where P is the maximum payout, and a, b are the contract parameters.
- The expected cost, $C(I(\theta))$, of an insurance contract, $I(\theta)$ for an insurer is: $C(I(\theta)) = \mathbb{E}[I(\theta)] + c_\kappa K(I(\theta))$, where c_κ is the cost of holding capital, and K is the amount of capital required to insure the contract.
- The formula for K is $K(I(\theta)) = CVaR_{1-\epsilon_P}(I(\theta)) - \mathbb{E}[I(\theta)]$.

Example of Payout Functions



Practitioner Interviews

- We conducted interviews with researchers and practitioners that had implemented index insurance programs in several countries (Malawi, Kenya, Senegal, Thailand, among others) to learn more about the context.
- **Objective:** minimize risk faced by farmers, maybe minimize probability that wealth drops below a certain threshold.
- **Constraints:** Budget constraints are very important from both demand and supply side.

Risk Measures

We are interested in minimizing the risk faced by farmers, so we need a measure of this risk.

Definition

For a random variable z, representing loss, the $(1-\epsilon)$ Value at Risk (VaR) is given by

$$VaR_{1-\epsilon}(z) := \inf\{t : P(z \leq t) \geq 1-\epsilon\}$$

Definition

For a random variable z, representing loss, the $(1-\epsilon)$ Conditional Value at Risk $(CVaR)$ is given by

$$CVaR_{1-\epsilon}(z) := \mathbb{E}\left[z | z \geq VaR_{1-\epsilon}(z)\right]$$

Idealized CVaR Model

- **Objective:** conditional value at risk of the farmers' loss net of insurance.
- **Constraint 1:** piecewise linear structure of the contract.
- **Constraint 2:** budget constraint.
- **Constraint 3:** definition of required capital.

$$
\begin{aligned}
\min_{a,b,\pi,K} \quad & CVaR_{1-\epsilon}\left(\ell + \mathbb{E}[I(\theta)] - I(\theta)\right) \\
\text{s.t.} \quad & I(\theta) = \min\{(a\hat{\ell}(\theta) + b)^{+}, P\} && (1) \\
& \mathbb{E}\left[I(\theta)\right] + c_{\kappa} K \leq B && (2) \\
& K = \left(CVaR_{1-\epsilon}\left(I(\theta)\right) - \mathbb{E}[I(\theta)]\right) && (3)
\end{aligned}
$$

Multiple Zone

$$\min_{a,b,K} \max_{z} \quad CVaR_{1-\epsilon}\left(\ell_z + \mathbb{E}[I_z(\theta_z)] - I_z(\theta_z)\right) \tag{4}$$

$$\text{s.t.} \quad I_z(\theta_z) = \min\{(a_z \hat{\ell}_z(\theta_z) + b_z)^+, P_z\} \tag{5}$$

$$\mathbb{E}\left[\sum_z I_z(\theta_z)\right] + c_\kappa K \leq B \tag{6}$$

$$K = CVaR_{1-\epsilon_K}\left(\sum_z I_z(\theta_z)\right) - \mathbb{E}\left[\sum_z I_z(\theta_z)\right] \tag{7}$$

Convex Approximation

$$\min_{a,b,K} \max_{z} \quad CVaR_{1-\epsilon}\left(\ell_z + \mathbb{E}\left[\overline{I_z}(\theta_z)\right] - \underline{I_z}(\theta_z)\right) \tag{8}$$

$$\text{s.t.} \quad \mathbb{E}\left[\sum_z \overline{I_z}(\theta_z)\right] + c_\kappa K \leq B \tag{9}$$

$$K = CVaR_{1-\epsilon_K}\left(\sum_z \overline{I_z}(\theta_z)\right) - \mathbb{E}\left[\sum_z \underline{I_z}(\theta_z)\right] \tag{10}$$

$$\overline{I_z}(\theta_z) = \max\left\{0, a_z\hat{\ell}_z(\theta_z) + b_z\right\} \tag{11}$$

$$\underline{I_z}(\theta_z) = \min\{a_z\hat{\ell}_z(\theta_z) + b_z, P_z\} \tag{12}$$

Convex approximations

LP Reformulation

Using the results from Rockafellar and Uryasev (2002), we get:

$$\min_{a,b,\alpha,\gamma,t,m,K^P} \quad m \tag{13}$$

$$\text{s.t.} \quad t_z + \frac{1}{\epsilon}\sum_j p^j \gamma_z^j \leq m, \forall z \tag{14}$$

$$\gamma_z^j \geq \ell^j - \min\left\{(a_z \hat{\ell}_z(\theta_z^j) + b_z), P_z\right\} - t_z, \forall j, \forall z \tag{15}$$

$$\gamma_z^j \geq 0, \forall j, \forall z \tag{16}$$

$$B \geq \frac{1}{N}\sum_j \sum_z \alpha_z^j + c_\kappa K \tag{17}$$

$$t_K + \frac{1}{\epsilon_K}\sum_j p^j \gamma_K^j \leq K + Z\pi_{SQ} \tag{18}$$

$$\gamma_K^j \geq \sum_z \alpha_z^j - t_K, \forall j \tag{19}$$

$$\gamma_K^j \geq 0, \forall j \tag{20}$$

$$\alpha_z^j \geq a_z \hat{\ell}_z(\theta_z^j) + b_z, \forall j, \forall z \tag{21}$$

$$\alpha_z^j \geq 0, \forall j, \forall z \tag{22}$$

Content

Baseline Method

- We compare our method to the method developed in Chantarat et al. (2013). This method is the most commonly used in academic publications and is what is used in Kenya's index insurance program.
- A linear regression model is used to predict losses in each insured area. A different model is estimated for each area. Each area normally has multiple villages in it.
- Contracts are of the form: $I(\theta) = \min \left\{ (\hat{\ell}(\theta) - \ell^*)^+, P \right\}$ where $\hat{\ell}(\theta)$ is the predicted loss, ℓ^* is the strike value, and P is the maximum payout. In other words, their contract pays farmers for the full predicted loss beyond a threshold, ℓ^*. This threshold, ℓ^* is the contract's strike value. The strike value is chosen to maximize the correlation of payouts and losses.

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Setup

We use the following data generating processes:

- Linear DGP: $\ell = \beta\theta + \epsilon$
- Nonlinear DGP: $\ell = \beta\theta^2 + \epsilon$

In both cases we have: $\theta \sim \mathcal{N}((5,5), \Sigma), \beta = diag(1.5, 1.5), \epsilon \sim \mathcal{N}(0, I)$.

With each data generating process, we test the following scenarios:

- **No correlation case:** $corr(\theta_1, \theta_2) = 0$
- **Positive correlation case:** $corr(\theta_1, \theta_2) = 0.8$
- **Negative correlation case:** $corr(\theta_1, \theta_2) = 0.8$

Simulation Details

In each simulation we:

1. Generate training and test samples, samples will be of the form $\left\{\ell_1^i, \theta_1^i, \ell_2^i, \theta_2^i\right\}_{i=1}^N$ where ℓ is loss and θ is the predictor.
2. Train linear prediction model. We run $\ell = \beta_0 + \beta_1\theta + \epsilon$. Use model to generate predictions $\hat{\ell}(\theta)$ for training and test data.
3. Use training data and model predictions to design contracts using both methods.
4. Given test data and predicted losses, calculate payouts from both contracts.
5. Calculate performance metrics on test data.

We run this 500 times for each scenario. We report the mean and 95% confidence intervals of each performance metric across the 500 simulations.

Performance Metrics

For each sample in the test set, we calculate the net loss, $\Delta\ell_z^i \triangleq \ell_z^i + \pi_z - I_z(\theta_z^i)$.

- **Maximum CVaR:** This is the expected value of the net loss given the loss is above a certain threshold. $CVaR_z = \mathbb{E}\left[\Delta\ell_z | \ell_z \geq VaR_{0.8}(\ell_z)\right]$.
- **Maximum VaR:** This the 80^{th} quantile of the distribution of the net loss.
- **Maximum Semi-variance**: This is the variance conditional on the loss being beyond a certain threshold. $\frac{1}{N-1}\sum_i (\Delta\ell_z^i - \overline{\ell})^2 \mathbb{1}\{\ell_z > \overline{\ell_z}\}$
- $|VaR_2 - VaR_1|$**:** Absolute value of the difference betweent the VaR of the two zones.
- **Required Capital:** $K(I(\theta)) = CVaR_{1-\epsilon_P}(\sum_z I_z(\theta)) - \mathbb{E}[\sum_z I_z(\theta)]$.
- **Average Cost:** We define this to be $\frac{1}{N}\sum_{i=1}^N \sum_z I_z(\theta_z^i) + c_\kappa K$. This is the empirical average of the cost of the insurance in every scenario in the test set plus the cost of capital

Summary of Results: Correctly Specified Model

Our model outperforms or matches the performance of the baseline contracts in the following areas:

- **Maximum Value at Risk:** the value at risk of the worst off zone is lower under our contracts, usually by about $5-6\%$.
- **Equity:** the difference in the value at risk of the two zones is lower under our contracts.
- **Maximum Semi-Variance** the variance conditional on loss exceeding a certain threshold is lower in our model, usually by about 20%.
- **Required Capital:** The contracts designed by our model consistently require less capital to fund than the baseline, $12-15\%$ less capital.

Results: Correctly Specified Prediction Model

Model	Max CVaR	Max VaR	Max SemiVar	$\lvert VaR_2 - VaR_1 \rvert$	Required Capital	Average Cost
Baseline	6.23	6.35	19.72	0.87	7.04	6.11
	[6.17, 6.3]	[6.29, 6.41]	[19.26, 20.17]	[0.81, 0.93]	[6.94, 7.14]	[6.02, 6.2]
Opt	6.18	5.98	14.78	0.21	6.12	6.08
	[6.13, 6.24]	[5.93, 6.02]	[14.38, 15.19]	[0.2, 0.23]	[6.03, 6.21]	[5.99, 6.17]
Diff	0.05	0.37	4.93	0.66	0.92	0.03
	[0.02, 0.08]	[0.34, 0.4]	[4.66, 5.21]	[0.6, 0.72]	[0.88, 0.95]	[0.02, 0.03]

(a) No correlation

Model	Max CVaR	Max VaR	Max SemiVar	$\lvert VaR_2 - VaR_1 \rvert$	Required Capital	Average Cost
Baseline	6.19	6.28	19.44	0.77	9.18	6.45
	[6.12, 6.26]	[6.22, 6.34]	[18.98, 19.91]	[0.72, 0.83]	[9.09, 9.28]	[6.36, 6.54]
Opt	6.21	5.94	14.24	0.19	7.81	6.41
	[6.15, 6.26]	[5.89, 5.99]	[13.84, 14.65]	[0.18, 0.21]	[7.7, 7.91]	[6.32, 6.5]
Diff	-0.02	0.34	5.2	0.58	1.38	0.04
	[-0.05, 0.01]	[0.31, 0.37]	[4.95, 5.46]	[0.52, 0.63]	[1.33, 1.43]	[0.03, 0.05]

(b) Positive correlation

Results: Correctly Specified Prediction Model

Model	Max CVaR	Max VaR	Max SemiVar	$\lvert VaR_2 - VaR_1 \rvert$	Required Capital	Average Cost
Baseline	6.19	6.3	19.72	0.85	3.33	5.56
	[6.13, 6.26]	[6.24, 6.36]	[19.27, 20.17]	[0.79, 0.9]	[3.28, 3.38]	[5.48, 5.64]
Opt	6.11	5.98	15.09	0.22	2.91	5.53
	[6.06, 6.17]	[5.93, 6.02]	[14.68, 15.5]	[0.2, 0.23]	[2.86, 2.95]	[5.45, 5.61]
Diff	0.08	0.32	4.63	0.63	0.42	0.04
	[0.05, 0.11]	[0.29, 0.35]	[4.35, 4.9]	[0.57, 0.69]	[0.39, 0.45]	[0.03, 0.04]

Table: Negative correlation

Summary of Results: Misspecified Model

A fair comparison is harder in this case because it becomes harder to enforce the budget constraint, and the models end up having different costs. Our model generally does better on the following metrics:

- **Maximum Semi-Variance** the variance conditional on loss exceeding a certain threshold is lower in our model, usually by about 20%.

- **Required Capital:** The contracts designed by our model generally require less capital to fund than the baseline, $12-15\%$ less capital.

- **Average Cost**: The contracts designed by our model tend to be $6-10\%$ less costly.

However, it does worse on all other metrics. We can make it competitive using a heuristic reallocation policy.

Results: Misspecified Prediction Model

Model	Max CVaR	Max VaR	Max SemiVar	$\lvert VaR_2 - VaR_1 \rvert$	Required Capital	Average Cost
Baseline	35.54	27.43	820.02	2.25	53.1	40.94
	[35.28, 35.79]	[27.3, 27.56]	[806.9, 833.15]	[2.07, 2.43]	[52.67, 53.52]	[40.69, 41.2]
Opt	38.67	30.61	628.58	2.41	50.87	38.31
	[38.44, 38.9]	[30.45, 30.76]	[616.93, 640.23]	[2.25, 2.58]	[50.33, 51.41]	[38.06, 38.56]
Diff	-3.13	-3.18	191.44	-0.16	2.23	2.63
	[-3.25, -3.01]	[-3.32, -3.04]	[184.69, 198.19]	[-0.39, 0.06]	[1.95, 2.5]	[2.55, 2.72]

(a) No correlation

Model	Max CVaR	Max VaR	Max SemiVar	$\lvert VaR_2 - VaR_1 \rvert$	Required Capital	Average Cost
Baseline	34.97	27.15	817.11	1.73	56.97	41.51
	[34.69, 35.25]	[27.02, 27.29]	[803.66, 830.55]	[1.58, 1.87]	[56.67, 57.27]	[41.26, 41.77]
Opt	39.9	32.2	521.22	1.5	60.61	37.17
	[39.63, 40.16]	[32.03, 32.37]	[510.86, 531.58]	[1.39, 1.6]	[60.3, 60.91]	[36.94, 37.41]
Diff	-4.93	-5.04	295.88	0.23	-3.64	4.34
	[-5.05, -4.8]	[-5.19, -4.9]	[288.47, 303.29]	[0.06, 0.4]	[-3.88, -3.39]	[4.23, 4.45]

(b) Positive correlation

Content

Data Sources

- **NDVI Data:** The Normalized Difference Vegetation Index (NDVI) is a satellite-based indicator of the amount and health of vegetation. We use NDVI data for Kenya between 2000-2015. There are $\sim 23,000,000$ observations per time period, and 365 time periods.
- **Kenya Household Survey Data:** This survey was conducted as part of the development of the Index based livestock insurance (IBLI) program in northern Kenya. This dataset has information on household location, livestock levels and changes for each month in 2010-2013. There are 900 households in this dataset.
- **Kenya Geospatial Data:** This dataset contains the geospatial boundaries of larger villages in Kenya.

Data Creation

1. Data for prediction model
   1. NDVI Data
      1. Use geospatial information to merge NDVI data with village geospatial data.
      2. For each village, you now have many NDVI values. Use this to calculate village level features for relevant time periods.
      3. **Outcome:** data set with village level NDVI features across time.
   2. Survey Data
      1. Merge survey data with Kenya geospatial data.
      2. Calculate herd mortality for each village in each season.
      3. **Outcome:** data set with village level herd mortality across time
   3. Merge two datasets at the village by season level.
2. Train prediction model using data from previous step.
3. Add model predictions to household survey data

Evaluation Procedure

We use leave one out cross validation to evaluate our method. In each iteration, we leave out one year of data for testing.

1. Split regression data and household survey data into training/test set
2. Train prediction model on training data
3. Use model to calculate village level predicted losses, add this to household training data
4. Calculate insurance contracts based on the training data using both methods.
5. Use prediction model to calculate village level predicted losses on the test set.
6. Calculate insurance payouts on test data.

Results

The insurance contracts developed by our model provide slightly better protection at a much lower cost. The cost of our contracts is 9% lower, and the cost of capital is 13% lower.

Model	Max VaR	$\lvert VaR_2 - VaR_1 \rvert$	Required Capital	Average Cost
Baseline	1303	1316	3014405	6449
Opt	1286	1361	2626260	5864

Table: Results using Kenya household data

Conclusions

- In some cases, the contracts designed by our model are able to offer better protection at a similar costs, or comparable protection at lower costs than the baseline method.
- It outperforms the baseline when the prediction model is correclty specified and on the Kenyan pastoralist data.
- Our method is more cost effective because it takes into account spatial correlations between areas and the costs of capital requirements. Thus, the model makes better trade offs between costs and coverage than the baseline method.

Next Steps

- We are working with practitioners to improve the model and possibly test it in practice.
- We are working with the Bank of Thailand on the implementation of their satellite-based index insurance program.
- We are also talking to the International Research Institute for Climate and Society at Columbia, they have worked on the implementation of numerous index insurance programs in Africa.

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Idealized CVaR Model

- **Objective:** conditional value at risk of the farmers' loss net of insurance.
- **Constraint 1:** piecewise linear structure of the contract.
- **Constraint 2:** budget constraint.
- **Constraint 3:** definition of required capital.

$$\min_{a,b,\pi,K} \quad CVaR_{1-\epsilon}\left(\ell - I(\theta)\right)$$

$$\text{s.t. } I(\theta) = \min\{(a\hat{\ell}(\theta) + b)^{+}, P\} \tag{23}$$

$$\mathbb{E}\left[I(\theta)\right] + c_{\kappa} K \leq B \tag{24}$$

$$K = \left(CVaR_{1-\epsilon}\left(I(\theta)\right) - \mathbb{E}[I(\theta)]\right) \tag{25}$$

The problem is non-convex, so we need convex approximations

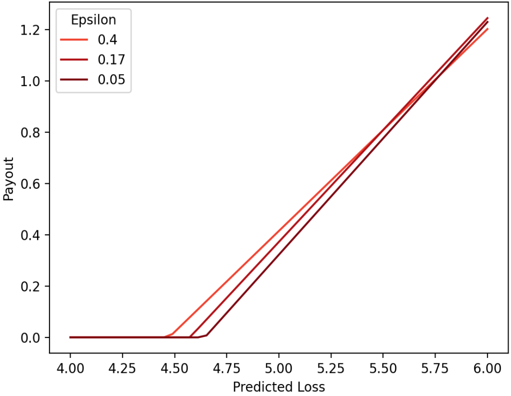
We use the following approximations of $I(\theta)$ to make the problem convex:

$$\overline{I(\theta)} \triangleq \max\left\{0, a\hat{\ell}(\theta) + b\right\}$$
$$\underline{I(\theta)} \triangleq \min\{a\hat{\ell}(\theta) + b, K\}$$

- Note that $\overline{I(\theta)} \geq I(\theta)$ and $\overline{I(\theta)}$ is convex. Conversely, $\underline{I(\theta)} \leq I(\theta)$ and $\underline{I(\theta)}$ is concave.
- We replace $I(\theta)$ with either $\overline{I(\theta)}$ or $\underline{I(\theta)}$ where necessary to obtain conservative and convex approximations.
- We also need approximations or proxies for $E[I(\theta)]$ in constraint 25. We use $\pi_{SQ} = E[I_{SQ}(\theta)]$, where I_{SQ} is the contract designed using the status quo method, as a proxy for $E[I(\theta)]$ in constraint 25.

Insights: Relationships between parameters and epsilon

As ϵ gets smaller, the slope increases and the function shifts to the right.



Results: Misspecified Prediction Model

Model	Max VaR	$\lvert VaR_2 - VaR_1 \rvert$	Required Capital	Average Cost
Baseline	27.42	1.65	53.85	40.73
	[25.13, 29.57]	[0.16, 5.56]	[44.81, 59.88]	[36.06, 45.83]
Opt	27.41	1.96	49.97	40.36
	[24.53, 29.83]	[0.15, 5.6]	[42.87, 58.53]	[35.52, 45.5]

(a) No correlation

Model	Max VaR	$\lvert VaR_2 - VaR_1 \rvert$	Required Capital	Average Cost
Baseline	27.16	1.23	57.1	41.36
	[24.7, 29.62]	[0.1, 5.05]	[51.16, 62.9]	[36.24, 46.52]
Opt	27.71	1.1	56.51	41.31
	[24.91, 30.31]	[0.09, 3.62]	[50.92, 62.26]	[35.98, 46.82]

(b) Positive correlation

Model	Max VaR	$\lvert VaR_2 - VaR_1 \rvert$	Required Capital	Average Cost
Baseline	27.52	1.9	25.2	36.42
	[25.09, 29.69]	[0.17, 6.0]	[17.99, 36.35]	[33.13, 41.88]
Opt	27.39	1.84	26.77	36.67
	[24.33, 29.71]	[0.16, 5.49]	[18.17, 37.5]	[33.21, 42.03]

(c) Negative correlation