JudgeShopping

Can Wang

July 2019

1 Value function when $v = r$

When $v = r$, i.e., the defendant can observe all available choices when entering the system, the integrated value function is

$$V(\mathbf{h}, r) = -\log\left(\sum_{i=0}^{r-1} \exp(-id - u_d(h_i))\right).$$

In the following we first show that $V(\mathbf{h}, r)$ is bounded.

Since $u_d(h_i) \geq 0$,

$$V(\mathbf{h}, r) \leq -\log\left(\sum_{i=0}^{r-1} \exp(-id)\right) = -\log\left(\frac{1-\exp(-rd)}{1-\exp(-d)}\right),$$

then for any $\mathbf{h}$, $V(\mathbf{h}, r)$ is upper bounded by $-\log\left(\frac{1}{1-\exp(-d)}\right)$ as r goes to infinity. For a given sequence $\mathbf{h}$, the quantity $V(\mathbf{h}, r)$ increases in r. Thus $V(\mathbf{h}, r)$ converges when r goes to infinity but the limit depends on $\mathbf{h}$.

We consider the special cases when h is a repeating sequence.

Suppose the elements of $\mathbf{h}$ are all identical and have the value h, then we have

$$V(\mathbf{h}, r) = -\log\left(\sum_{i=0}^{r-1} \exp(-id - u_d(h))\right) = u_d(h) - \log\left(\frac{1-\exp(-rd)}{1-\exp(-d)}\right).$$

The probability that the defendant chooses the judge from the i^{th} week is

$$p_{i,v,r}(\mathbf{h}) = \frac{\exp(-id - u_d(h))}{\exp(-u_d(h))\left(\frac{1-\exp(-rd)}{1-\exp(-d)}\right)} = \frac{\exp(-id)(1-\exp(-d))}{1-\exp(-rd)},$$

which shows that when the judges in different weeks are equivalent, the probability of choosing a judge decreases exponentially with rate d (delay cost) as waiting time (the number of weeks) increases. Moreover, the probability of choosing the first week decreases with r since greater r means more available choices. When r goes to infinity, the probability of choosing the first week is $1 - \exp(-d)$, which increases with the delay cost, and becomes 1 when the delay cost is infinity.

Suppose $r = mn$ and $\mathbf{h} = (h_0, h_1, ..., h_{n-1})_m$, which represents that the sequence $h_0, h_1, ..., h_n$ is repeated for m times, then we have

$$\begin{aligned} V(\mathbf{h}, r) &= -\log\left(\sum_{j=0}^{m-1}\sum_{i=0}^{n-1} \exp(-(jn+i)d - u_d(h_i))\right) \\ &= -\log\left(\sum_{i=0}^{n-1} \exp(-id - u_d(h_i))\right) - \log\left(\frac{1-\exp(-md)}{1-\exp(-d)}\right). \end{aligned}$$

The probability that the defendant chooses the judge from the $(jn+i)^{th}$ week is

$$p_{jn+i,v,r}(\mathbf{h}) = \frac{\exp(-(jn+i)d - u_d(h_i))}{\left(\sum_{k=0}^{n-1}\exp(-kd - u_d(h_k))\right)\left(\frac{1-\exp(-md)}{1-\exp(-d)}\right)}$$
$$= \frac{\exp(-id - u_d(h_i))}{\sum_{k=0}^{n-1}\exp(-kd - u_d(h_k))}\frac{\exp(-jnd)(1-\exp(-d))}{1-\exp(-md)},$$

which has similar properties to the case of the identical sequence as m increases and goes to infinity.

When $v = r = 1$,

$$V(\mathbf{h}, r) = V(\mathbf{h}, 1) = -\log\left(\exp(-u_d(h_0))\right) = u_d(h_0)$$

and the defendant chooses his only choice with probability 1.

When $v = r = 2$,

$$V(\mathbf{h}, r) = V(\mathbf{h}, 2) = -\log\left(\exp(-u_d(h_0)) + \exp(-u_d(h_1))\right)$$

The percentage reduction in $V(\mathbf{h}, r)$ is

$$\frac{V(\mathbf{h},1) - V(\mathbf{h},2)}{V(\mathbf{h},1)} = \frac{\log\left(1 + \exp(u_d(h_0) - u_d(h_1))\right)}{u_d(h_0)}.$$

By Taylor expansion,

$$\log(1+\exp(x)) \approx \log(2) + \frac{1}{2}x + \frac{1}{8}x^2.$$

Then when $u_d(h_0)$ and $u_d(h_1)$ are not too much different, the percentage reduction is approximately

$$\frac{V(\mathbf{h},1) - V(\mathbf{h},2)}{V(\mathbf{h},1)} \approx \frac{\log(2) + \frac{1}{2}(u_d(h_0) - u_d(h_1)) + \frac{1}{8}(u_d(h_0) - u_d(h_1))^2}{u_d(h_0)}.$$

2 V(h,n) vs. % of time judge is at home

Suppose there are J counties and each county has a home judge, where the home judge stays in the home county with probability p_s and travel in the other $n-1$ counties with probability $p_t = \frac{1-p_s}{n-1}$.

Suppose the judges in the J counties have harshness level $\eta_1, ..., \eta_J$. Consider the cases in the first county. They would face η_1 with probability p_s and face any one of $\eta_2, ..., \eta_n$ with equal probability p_t.

Consider the case $v = r$, the probability that a defendant $(\theta\tau, c_d)$ facing a judge sequence $\mathbf{h}$ chooses the judge from the i^{th} week is

$$p_{i,v,r}(\mathbf{h}) = \frac{\exp(-id - u_d(h_i))}{\sum_{j=0}^{r-1}\exp(-jd - u_d(h_j))} = \frac{\exp(-id)}{\sum_{j=0}^{r-1}\exp(-jd + \Delta_u(i,j))},$$

where $\Delta_u(i,j) = u_d(h_i) - u_d(h_j)$.

If we choose $u_d(h) = \min(\theta\tau + c_d, u(h))$, $u(h) = h\theta\tau$ and $l(h) = (h-\delta)\theta\tau$, then

$$\Delta_u(i,j) = \min(\theta\tau + c_d, u(h_i)) - \min(\theta\tau + c_d, u(h_j)) = \theta\tau\left(\min(1 + \frac{c_d}{\theta\tau}, h_i) - \min(1 + \frac{c_d}{\theta\tau}, h_j)\right).$$

Denote the utility-severity ratio $1 + \frac{c_d}{\theta\tau}$ by γ. Then the defendant $(\theta\tau, \gamma)$ who chose judge h will go to trial if

$$l(h) = (h-\delta)\theta\tau > \theta\tau + c_d,$$

i.e.,

$$h > \delta + \gamma.$$