Alternative Idleness Models

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1 Overview

This week I worked on implementing the non-linear model we discussed last time. Let g be whatever group we decide to calculate idleness for (Judge or County) and let i be our current observation, which is a judge-county combination. The non-linear model is:

$$\mu_g \text{Days}_i = \beta_P \text{Plea}_i + \beta_T \text{Trial}_i + \epsilon_i$$
$$\log(\text{Days}_i) = \log(\frac{1}{\mu_g}) + \log(\beta_P \text{Plea}_i + \beta_T \text{Trial}_i + \epsilon_i)$$

1.1 Basic Linear Model

I asked another student for help with the implementation, and they pointed out that the model we specified is a linear mixed effects model:

$$\mu_g \text{Days}_i = \beta_P \text{Plea}_i + \beta_T \text{Trial}_i + \epsilon_i$$
$$\text{Days}_i = \frac{\beta_P}{\mu_g} \text{Plea}_i + \frac{\beta_T}{\mu_g} \text{Trial}_i + \frac{\epsilon_i}{\mu_g}$$
$$\text{Days}_i = \beta_{P,g} \text{Plea}_i + \beta_{T,g} \text{Trial}_i + u_i$$

In other words, we specified a model which could be interpreted as fitting a different trial and plea processing rate for each judge or county.

1.2 Intercept Model

Alternatively, we could also add "fixed costs" or "startup costs" for working by using the model:

$$\mu_g \text{Days}_i = \beta_0 + \beta_P \text{Plea}_i + \beta_T \text{Trial}_i + \epsilon_i \quad (1)$$
$$\text{Days}_i = \beta_{0,g} + \beta_{P,g} \text{Plea}_i + \beta_{T,g} \text{Trial}_i + u_i \quad (2)$$

Here, β_0 is the fixed cost of working, and by setting $\beta_{0,g} = \frac{\beta_0}{\mu_g}$ we allow this fixed cost of working to vary by judge/county. The advantage of this model is that it more explicitly accounts for idleness through the $\beta_{0,g}$ term and that it allows that idleness to vary by judge/county. The disadvantage is that it would be estimating about 150 parameters. Another disadvantage is that it would take us back to using a fixed intercept.

1.3 Pros and Cons

Pros: these models are more well studied and easier to estimate. They are commonly used in biostatistics and other social sciences. I think this model would require less justification than our non-linear model. We could use this approach to construct measures of idleness for each judge/county by comparing their $\beta_{P,g}$ to that of the most productive judge/county **Cons:** It increases the number of parameters we are estimating, the basic model would have roughly 100 parameters and the intercept model would have around 150. I don't think the overall β_T/β_P would be identified. The intercept model would bring us back to an intercept model, which is what we were trying to avoid. The biggest con from this approach is that the estimates we got were wonky: several judges and counties end up with negative service rates.

2 County Models

Here, we estimate both the basic linear model and the intercept model described above using the county as the grouping variable. In other words, we estimate a different trial/plea service rate for each county.

2.1 Basic Linear Model

The model we estimate here is $\text{Days}_i = \beta_{P,c}\text{Plea}_i + \beta_{T,c}\text{Trial}_i + u_i$.

Table 1: County Basic Model

	Plea	Trial
Abbeville	0.299	2.205
Aiken	0.150	4.105
Allendale	0.072	0
Anderson	0.095	5.098
Bamberg	0.261	0
Barnwell	0.188	1.035
Beaufort	0.495	2.303
Berkeley	0.163	1.891
Calhoun	0.163	0
Charleston	0.173	3.612
Cherokee	0.213	3.675
Chester	0.509	3.952
Chesterfield	0.311	3.395
Clarendon	0.332	0.621
Colleton	0.320	-0.043
Darlington	0.287	1.402
Dillon	0.304	0
Dorchester	0.270	-0.585
Edgefield	0.265	0
Fairfield	0.339	3.583
Florence	0.128	8.542
Georgetown	0.317	-0.224
Greenville	0.093	7.562
Greenwood	0.223	0.987
Hampton	0.181	0
Horry	0.141	3.975
Jasper	0.230	1.565
Kershaw	0.217	0
Lancaster	0.363	1.117
Laurens	0.284	-3.115
Lee	0.247	3.792
Lexington	0.177	2.559
Marion	0.184	2.507
Marlboro	0.261	0
McCormick	0.295	0
Newberry	0.284	0.002
Oconee	0.171	0.733
Orangeburg	0.220	3.689
Pickens	0.249	4.422
Richland	0.149	5.787
Saluda	0.229	0.378
Spartanburg	0.020	8.094
Sumter	0.205	2.139
Union	0.324	0.916
Williamsburg	0.351	0
York	0.198	2.122

2.2 Intercept Model

The model we estimate here is: $\text{Days}_i = \beta_{0,c} + \beta_{P,c}\text{Plea}_i + \beta_{T,c}\text{Trial}_i + u_i$

Table 2: County Intercept Model

	(Intercept)	Plea	Trial
Abbeville	4.698	0.183	3.103
Aiken	3.664	0.143	3.232
Allendale	2.737	0.107	1.647
Anderson	2.086	0.081	4.699
Bamberg	4.248	0.166	0.855
Barnwell	3.292	0.128	1.292
Beaufort	8.502	0.332	1.321
Berkeley	3.607	0.141	1.266
Calhoun	2.826	0.110	1.236
Charleston	3.820	0.149	3.107
Cherokee	3.836	0.150	3.025
Chester	7.635	0.298	4.426
Chesterfield	4.358	0.170	3.155
Clarendon	6.382	0.249	0.198
Colleton	4.933	0.192	0.732
Darlington	5.338	0.208	1.384
Dillon	5.161	0.201	2.280
Dorchester	4.945	0.193	0.674
Edgefield	4.941	0.193	0.771
Fairfield	5.371	0.210	2.757
Florence	2.752	0.107	6.996
Georgetown	5.666	0.221	0.806
Greenville	1.899	0.074	6.825
Greenwood	4.941	0.193	0.494
Hampton	3.015	0.118	1.496
Horry	2.959	0.115	3.820
Jasper	3.099	0.121	1.985
Kershaw	4.346	0.170	0.938
Lancaster	7.155	0.279	0.636
Laurens	6.451	0.252	−2.108
Lee	2.890	0.113	3.425
Lexington	3.576	0.139	2.203
Marion	3.388	0.132	2.230
Marlboro	5.546	0.216	1.772
McCormick	4.605	0.180	1.393
Newberry	5.169	0.202	0.962
Oconee	3.587	0.140	0.305
Orangeburg	5.296	0.207	1.919
Pickens	4.605	0.180	3.924
Richland	3.272	0.128	5.446
Saluda	3.936	0.154	1.220
Spartanburg	0.370	0.014	7.518
Sumter	4.634	0.181	1.341
Union	5.233	0.204	1.972
Williamsburg	5.665	0.221	0.459
York	4.405	0.172	2.257

3 Judge Models

Here, we estimate both the basic linear model and the intercept model described above using the judge as the grouping variable. In other words, we estimate a different trial/plea service rate for each judge.

3.1 Basic Linear Model

The model we estimate here is $\mathrm{Days}_i = \beta_0 + \beta_{P,j}\mathrm{Plea}_i + \beta_{T,j}\mathrm{Trial}_i + u_i$. We add the β_0 term because the model without the intercept term failed to converge.

Table 3: Judge Basic Model

	Trial	Plea	(Intercept)
Judge 1	4.836	0.230	5.155
Judge 10	0.807	0.173	5.155
Judge 11	5.855	0.080	5.155
Judge 12	6.593	0.139	5.155
Judge 13	3.623	0.157	5.155
Judge 14	0.515	0.179	5.155
Judge 15	−0.802	0.131	5.155
Judge 16	−0.691	0.051	5.155
Judge 17	13.078	0.048	5.155
Judge 18	−0.197	0.329	5.155
Judge 19	1.424	0.069	5.155
Judge 2	3.198	0.166	5.155
Judge 20	0	0.031	5.155
Judge 21	4.899	0.117	5.155
Judge 22	4.693	0.112	5.155
Judge 23	6.595	0.132	5.155
Judge 24	2.905	0.111	5.155
Judge 25	−1.568	0.115	5.155
Judge 26	−5.423	0.226	5.155
Judge 27	3.004	0.149	5.155
Judge 28	−2.644	0.133	5.155
Judge 29	3.840	0.171	5.155
Judge 3	2.783	0.117	5.155
Judge 30	6.012	0.039	5.155
Judge 31	8.579	0.185	5.155
Judge 32	2.318	0.118	5.155
Judge 33	3.090	0.136	5.155
Judge 34	5.987	0.122	5.155
Judge 35	−2.043	0.218	5.155
Judge 36	3.338	0.094	5.155
Judge 37	3.043	0.051	5.155
Judge 38	−0.623	0.161	5.155
Judge 39	0.963	0.120	5.155
Judge 4	2.913	0.236	5.155
Judge 40	4.536	0.102	5.155
Judge 41	−1.436	0.300	5.155
Judge 42	−1.599	0.108	5.155
Judge 43	0	0.117	5.155
Judge 44	3.804	0.091	5.155
Judge 45	3.064	0.285	5.155
Judge 46	0	0.054	5.155
Judge 47	0.491	0.186	5.155
Judge 48	4.237	0.157	5.155
Judge 49	11.850	0.119	5.155
Judge 5	4.278	0.078	5.155
Judge 50	2.308	0.161	5.155
Judge 6	4.598	0.083	5.155
Judge 7	0.668	0.212	5.155
Judge 8	0.805	0.087	5.155
Judge 9	1.428	0.155	5.155

3.2 Intercept Model

The model we estimate here is: $\text{Days}_i = \beta_{0,j} + \beta_{P,j}\text{Plea}_i + \beta_{T,j}\text{Trial}_i + u_i$

Table 4: Judge Intercept Model

	Plea	Trial	(Intercept)
Judge 1	0.234	6.033	−0.239
Judge 10	0.166	1.225	2.071
Judge 11	0.119	5.153	3.652
Judge 12	0.150	6.316	2.611
Judge 13	0.142	4.039	2.862
Judge 14	0.193	0.830	1.141
Judge 15	0.141	−0.784	2.904
Judge 16	0.049	−1.021	6.010
Judge 17	0.050	12.809	5.966
Judge 18	0.181	2.203	1.556
Judge 19	0.048	0.601	6.039
Judge 2	0.175	3.053	1.760
Judge 20	0.021	1.072	6.962
Judge 21	0.176	4.072	1.730
Judge 22	0.112	4.583	3.875
Judge 23	0.141	6.572	2.883
Judge 24	0.079	3.384	4.984
Judge 25	0.111	−1.552	3.908
Judge 26	0.237	−5.830	−0.357
Judge 27	0.168	2.833	1.997
Judge 28	0.116	−2.566	3.728
Judge 29	0.183	3.578	1.479
Judge 3	0.122	2.569	3.549
Judge 30	0.020	5.904	6.977
Judge 31	0.238	6.917	−0.391
Judge 32	0.118	2.113	3.659
Judge 33	0.138	3.301	3.008
Judge 34	0.126	5.810	3.407
Judge 35	0.227	−1.413	−0.002
Judge 36	0.064	2.895	5.502
Judge 37	0.020	2.366	6.968
Judge 38	0.166	−0.582	2.070
Judge 39	0.127	0.501	3.379
Judge 4	0.261	3.414	−1.158
Judge 40	0.100	4.518	4.271
Judge 41	0.328	−0.134	−3.427
Judge 42	0.107	−2.087	4.050
Judge 43	0.104	−0.957	4.143
Judge 44	0.089	3.747	4.660
Judge 45	0.322	4.019	−3.206
Judge 46	0.050	2.791	5.975
Judge 47	0.190	0.622	1.252
Judge 48	0.178	4.338	1.659
Judge 49	0.136	11.575	3.053
Judge 5	0.074	4.350	5.151
Judge 50	0.168	2.223	1.977
Judge 6	0.081	4.553	4.914
Judge 7	0.218	0.706	0.292
Judge 8	0.083	0.481	4.845
Judge 9	0.157	1.507	2.362