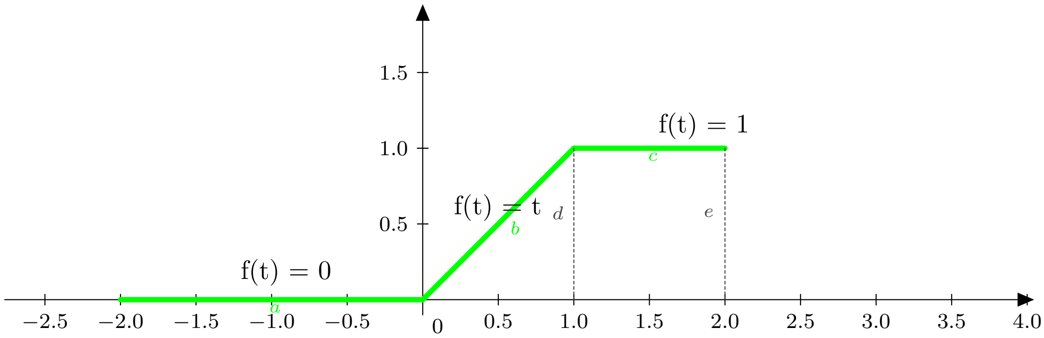
UNIVERSIDAD DE LAS FUERZAS ARMADAS ESPE
EXTENSIÓN LATACUNGA

MATEMÁTICA SUPERIOR

2) Halle la Serie de Fourier, la Transformada de Fourier, la transformada Inversa de Fourier y el Espectro de Frecuencias de la Función dada:

$$f(t) = \begin{cases} 0 & \text{si} \quad -2 < t < 0 \\ t & \text{si} \quad 0 < t < 1 \\ 1 & \text{si} \quad 1 < t < 2 \end{cases}$$



Resolución
Serie de Fourier
Término a_0
$T = 4$ y $\delta = -2$

$$a_0 = \frac{2}{T}\int_{\delta}^{\delta+T} f(t)dt = \frac{2}{4}\int_{-2}^{2} f(t)dt = \frac{1}{2}(0 + \int_0^1 tdt + \int_1^2 1dt)$$

$$a_0 = \frac{1}{2}(0 + \frac{t^2}{2}\ |_0^1 + t\ |_1^2) = \frac{1}{2}(\frac{1^2-0}{2} + 2 - 1) = \frac{3}{4}$$

Término a_n

$$a_n = \frac{2}{T}\int_{\delta}^{\delta+T} f(t)cos(n\omega t)dt = \frac{2}{4}\int_{-2}^{2} f(t)cos(n\omega t)dt = \frac{1}{2}(0 + \int_0^1 tcos(n\omega t)dt + \int_1^2 cos(n\omega t)dt)$$

$$a_n = \frac{1}{2}[\frac{sen(n\omega t)t}{n\omega} - \int_0^1 \frac{sen(n\omega t)}{n\omega}dt + |_0^1\ \frac{sen(n\omega)t}{n\omega}\ |_1^2]$$

$$a_n = \frac{1}{2}[(\frac{sen(n\omega t)}{n\omega} - \frac{cos(n\omega t)}{(n\omega)^2})\ |_0^1 + \frac{sen(n\omega t)t}{n\omega}\ |_1^2]$$

$$a_n = \frac{1}{2}[\frac{2sen(n\omega) - 0}{n\omega} - \frac{cos(n\omega) - cos(0)}{(n\omega)^2} + \frac{sen(2n\omega) - sen(n\omega)}{n\omega}]$$

$$a_n = \frac{1}{2}[\frac{sen(2n\omega)}{n\omega} - \frac{cos(n\omega) - 1}{(n\omega)^2}]$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$a_n = \frac{1}{2}[\frac{sen(n\pi)}{n\omega} - \frac{cos(n\frac{\pi}{2}) - 1}{(n\omega)^2}]$$

$$sen(n\pi) = 0$$
$$a_n = \frac{1}{2}[\frac{1 - cos(n\omega)}{(n\omega)^2}]$$
Término b_0
$$b_n = \frac{2}{T}\int_{\delta}^{\delta+T} f(t)sen(n\omega t)dt = \frac{2}{4}\int_{-2}^{2} f(t)sen(n\omega t)dt = \frac{1}{2}(+\int_{0}^{1} tsen(n\omega t)dt + \int_{1}^{2} sen(n\omega t)dt)$$
$$b_n = \frac{1}{2}[(\frac{-cos(n\omega t)t}{n\omega} + \int_{0}^{1} \frac{cos(n\omega t)}{n\omega}dt) \mid_0^1 - \frac{cos(n\omega t)}{n\omega} \mid_1^2]$$
$$b_n = \frac{1}{2}[(\frac{-cos(n\omega t)t}{n\omega} + \frac{sen(n\omega t)}{(n\omega)^2}) \mid_0^1 - \frac{cos(n\omega t)}{n\omega} \mid_1^2]$$
$$b_n = \frac{1}{2}[(-\frac{cos(n\omega) - 0cos(0)}{n\omega} + \frac{sen(n\omega) - sen(0)}{(n\omega)^2}) - \frac{cos(2n\omega) - cos(n\omega)}{n\omega}]$$
$$b_n = \frac{1}{2}[\frac{sen(n\omega)}{(n\omega)^2} - \frac{cos(2n\omega)}{n\omega}]$$
$$b_n = \frac{1}{2}[\frac{sen(n\omega)}{(n\omega)^2} - \frac{(-1)^n}{n\omega}]$$
Serie de Fourier
$$f(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}(a_n cos(nt) + b_n sen(nt))$$
$$f(t) = \frac{3}{8}a_0 + \frac{1}{2}\sum_{n=1}^{\infty}((\frac{1 - cos(n\omega)}{(n\omega)^2})cos(nt) + (\frac{sen(n\omega)}{(n\omega)^2} - \frac{(-1)^n}{n\omega})sen(nt))$$

Transformada de Fourier
$$F(j\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$
$$F(j\omega) = \int_{-2}^{2} f(t)e^{-j\omega t}dt$$
$$F(j\omega) = 0 + \int_{0}^{1} te^{-j\omega t}dt + \int_{1}^{2} e^{-j\omega t}dt$$
$$F(j\omega) = (-\frac{te^{-j\omega t}}{j\omega} - \frac{e^{-j\omega t}}{(j\omega)^2}) \mid_0^1 - \frac{e^{-j\omega t}}{j\omega} \mid_1^2$$
$$F(j\omega) = (-\frac{e^{-j\omega} - 0}{j\omega} - \frac{e^{-j\omega} - 1}{(j\omega)^2}) - \frac{e^{-2j\omega} - e^{-j\omega}}{j\omega}$$
$$F(j\omega) = \frac{1 - e^{-j\omega}}{(j\omega)^2} - \frac{e^{-2j\omega}}{j\omega}$$
Espectro de Frecuencias

4) Defina y explique que es una Serie de Bessel Es un caso particular de la series Generalizadas de Fourier

$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - \alpha^2)y = 0$