Corrección de la Segunda Prueba

1. Resolver las siguientes EDP(Ecuaciones Diferenciales Parciales)

   (a) $u_x = u_y + u$

   (b) $a^2 \frac{d^2u}{dx^2} = \frac{d^2u}{dt^2}$

   Desarrollo

   (a) $u_x = u_y + u$

$$X^{'}Y = Y^{'}X + XY$$
$$\frac{X^{'}}{X} = \frac{Y^{'}+Y}{Y} = -\lambda$$

$$\lambda = 0$$

$$\begin{array}{cc} X^{'} = 0 & Y^{'} + Y = 0 \\ D = 0 & D + 1 = 0 \\ & D = -1 \\ X_{(x)} = C_1 & Y_{(y)} = C_2 e^{-y} \end{array}$$

$u = X_{(x)}Y_{(y)} = C_1(C_2e^{-y})$
$u = X_{(x)}Y_{(y)} = C_1C_2e^{-y}$

$$\lambda = \alpha^2$$

$$\begin{array}{cc} X^{'} + \alpha^2 X = 0 & Y^{'} + Y + \alpha^2 Y = 0 \\ & D + 1 + \alpha^2 = 0 \\ D + \alpha^2 = 0 & D = -1 - \alpha^2 \\ X_{(x)} = C_1 e^{-\alpha^2 x} & Y_{(y)} = C_2 e^{-y-\alpha^2 y} \end{array}$$

$u = X_{(x)}Y_{(y)} = C_1e^{-\alpha^2 x}C_2e^{-y-\alpha^2 y}$

$$\lambda = -\alpha^2$$

$$\begin{array}{cc} X^{'} - \alpha^2 X = 0 & Y^{'} + Y - \alpha^2 Y = 0 \\ & D + 1 - \alpha^2 = 0 \\ D - \alpha^2 = 0 & D = -1 + \alpha^2 \\ X_{(x)} = C_1 e^{\alpha^2 x} & Y_{(y)} = C_2 e^{-y+\alpha^2 y} \end{array}$$

$u = X_{(x)}Y_{(y)} = C_1e^{-\alpha^2 x}C_2e^{-y-\alpha^2 y}$

2. $a^2 \frac{d^2u}{dx^2} = \frac{d^2u}{dt^2}$

$$a^2X^{''}t = Xt^{''}$$
$$a^2\frac{X^{''}}{X} = \frac{t^{''}}{t} = -\lambda$$

$$a^2X^{''} + \lambda X = 0 \quad t^{''} + \lambda t = 0$$

$$\lambda = 0$$

$$\begin{array}{cc} a^2X^{''} = 0 & t^{''} = 0 \\ a^2D^2 = 0 & D^2 = 0 \\ D = 0 & D = 0 \\ X_{(x)} = C_1 + C_2x & Y_{(y)} = C_3 + C_4y \end{array}$$

$$u = X_{(x)}Y_{(y)} = (C_1 + C_2x)(C_3 + C_4y)$$

$$\lambda = \alpha^2$$

$$\begin{array}{cc} a^2X^{''} + \alpha^2X = 0 & t^{''} + \alpha^2t = 0 \\ a^2D^2 + \alpha^2 = 0 & D^2 + \alpha^2 = 0 \\ D^2 = -\frac{\alpha^2}{a^2} & D^2 = -\alpha^2 \\ D = \pm\frac{\alpha}{a}j & D = \pm\alpha \\ X_{(x)} = C_1\cos(\frac{\alpha}{a}x) + C_2\sin(\frac{\alpha}{a}x) & Y_{(y)} = C_3\cos(\alpha y) + C_4\sin(\alpha y) \end{array}$$

$$u = X_{(x)}Y_{(y)} = (C_1\cos(\tfrac{\alpha}{a}x) + C_2\sin(\tfrac{\alpha}{a}x))(C_3\cos(\alpha y) + C_4\sin(\alpha y))$$

$$\lambda = -\alpha^2$$

$$\begin{array}{cc} a^2X^{''} - \alpha^2X = 0 & t^{''} - \alpha^2t = 0 \\ a^2D^2 - \alpha^2 = 0 & D^2 - \alpha^2 = 0 \\ D^2 = \frac{\alpha^2}{a^2} & D^2 = \alpha^2 \\ D = \pm\frac{\alpha}{a}j & D = \pm\alpha \\ X_{(x)} = C_1e^{\frac{\alpha}{a}x} + C_2e^{-\frac{\alpha}{a}x} & Y_{(y)} = C_3e^{\alpha x} + C_4e^{-\alpha x} \end{array}$$

$$u = X_{(x)}Y_{(y)} = (C_1e^{\frac{\alpha}{a}x} + C_2e^{-\frac{\alpha}{a}x})(C_3e^{\alpha x} + C_4e^{-\alpha x})$$

Determinar :

$$f(x) = \begin{cases} -1 & \text{si} \quad -2 < t < 0 \\ \\ 1 & \text{si} \quad 0 < t < 2 \end{cases}$$

Resolución

Serie de Fourier

Término a_0

$$p = 2$$

$$a_0 = \frac{1}{p}\int_{-p}^{p} f(t)dt = \frac{1}{2}\int_{-2}^{2} f(t)dt = \frac{1}{2}(\int_{-2}^{0} -1dt + \int_{0}^{2} 1dt)$$

$$a_0 = \frac{1}{2}(-t\mid_{-2}^{0} + t\mid_{0}^{2}) = \frac{1}{2}(2+2) = \frac{4}{2} = 2$$

Término a_n

$$a_n = \frac{1}{p}\int_{-p}^{p} f(t)cos(\frac{n\pi t}{p})dt = \frac{1}{2}\int_{-2}^{2} f(t)cos(\frac{n\pi t}{2})dt = \frac{1}{2}(\int_{-2}^{0} -1cos(\frac{n\pi t}{2})dt + \int_{0}^{2} 1cos(\frac{n\pi t}{2})dt)$$

$$an = 0$$

Término b_n

$$b_n = \frac{1}{p}\int_{-p}^{p} f(t)(\frac{n\pi t}{p})dt = \frac{1}{2}\int_{-2}^{2} f(t)sen(\frac{n\pi t}{2})dt = \frac{1}{2}(\int_{-2}^{0} -1sen(\frac{n\pi t}{2})dt + \int_{0}^{2} 1sen(\frac{n\pi t}{2})dt)$$

$$b_n = \frac{1}{2}[(\frac{2cos(\frac{n\pi t}{2})t}{n\pi}) |_{-2}^{0} -(\frac{2cos(\frac{n\pi t}{2})t}{n\pi}) |_{0}^{2}$$
$$b_n = \frac{2}{n\pi} - \frac{2}{n\pi}cos(n\pi)$$
$$b_n = \frac{2}{n\pi}(1 - cos(n\pi)$$

Serie de Fourier

$$f(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}(a_n cos(\frac{n\pi t}{p})) + b_n sen(\frac{n\pi t}{p}))$$
$$f(t) = 1 + \sum_{n=1}^{\infty}\frac{2}{n\pi}(1 - cos(n\pi))sen(\frac{n\pi t}{2})$$

Transformada de Fourier

$$F(j\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$
$$F(j\omega) = \int_{-2}^{2} f(t)e^{-j\omega t}dt$$
$$F(j\omega) = \int_{-2}^{0} -1e^{-j\omega t}dt + \int_{0}^{2} 1e^{-j\omega t}dt$$
$$F(j\omega) = (-\frac{e^{-j\omega t}}{-j\omega} |_{-2}^{0} - \frac{e^{-j\omega t}}{j\omega} |_{0}^{2})$$
$$F(j\omega) = (\frac{1 - e^{-j2\omega}}{j\omega} - \frac{e^{-j2\omega} - 1}{jw})$$
$$F(j\omega) = \frac{2}{j\omega}(1 - e^{-j2\omega})$$