Contents

1 Hamiltonian

Hamiltonian of the atom

$$\mathcal{H} = \sum 4.0 V_{e-e} \tag{1}$$

Number of electrons in the d shell = 1

Lower case l, s denote single particle operators

Upper case L, S, J denote multi particle operators

2 Spectrum

There are 10 different states

There are 1 different manifolds

Manifold	Degeneracy	ΔE (eV)	Energy (eV)
GS	10	0.000	0.000

3 Eigenstates and expectation values

Energy (GHz)	Energy (meV)	L_x	L_y	L_z	S_x	S_y	S_z
0.000	0.000	0.000	0.000	-2.000	0.000	0.000	0.500
0.000	0.000	0.000	0.000	-1.000	0.000	0.000	0.500
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.500
0.000	0.000	0.000	0.000	1.000	0.000	0.000	0.500
0.000	0.000	0.000	0.000	2.000	0.000	0.000	0.500
0.000	0.000	0.000	0.000	-2.000	0.000	0.000	-0.500
0.000	0.000	0.000	0.000	-1.000	0.000	0.000	-0.500
0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.500
0.000	0.000	0.000	0.000	1.000	0.000	0.000	-0.500
0.000	0.000	0.000	0.000	2.000	0.000	0.000	-0.500

4 Proyection of operator on GS

$$S_x = \begin{pmatrix} 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \end{pmatrix} \tag{2}$$

$$S_y = \begin{pmatrix} 0 & \frac{1}{2}i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{2}i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2}i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{2}i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2}i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}i \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2}i & 0 \end{pmatrix} \tag{3}$$

$$S_z = \begin{pmatrix} -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \tag{4}$$

$$S^2 = \begin{pmatrix} 0.75 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.75 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.75 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.75 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.75 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.75 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.75 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.75 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.75 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.75 \end{pmatrix} \tag{5}$$

$$L_x = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & \sqrt{\frac{3}{2}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & \sqrt{\frac{3}{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{\frac{3}{2}} & 0 & 0 & 0 & \sqrt{\frac{3}{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{\frac{3}{2}} & 0 & 0 & 0 & \sqrt{\frac{3}{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{\frac{3}{2}} & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sqrt{\frac{3}{2}} & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \tag{6}$$

$$L_y = \begin{pmatrix} 0 & 0 & 1i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1i & 0 & 0 & 0 & 0 & 0 & 0 \\ -1i & 0 & 0 & 0 & \sqrt{\frac{3}{2}}i & 0 & 0 & 0 & 0 & 0 \\ 0 & -1i & 0 & 0 & 0 & \sqrt{\frac{3}{2}}i & 0 & 0 & 0 & 0 \\ 0 & 0 & -\sqrt{\frac{3}{2}}i & 0 & 0 & 0 & \sqrt{\frac{3}{2}}i & 0 & 0 & 0 \\ 0 & 0 & 0 & -\sqrt{\frac{3}{2}}i & 0 & 0 & 0 & \sqrt{\frac{3}{2}}i & 0 & 0 \\ 0 & 0 & 0 & 0 & -\sqrt{\frac{3}{2}}i & 0 & 0 & 0 & 1i & 0 \\ 0 & 0 & 0 & 0 & 0 & -\sqrt{\frac{3}{2}}i & 0 & 0 & 0 & 1i \\ 0 & 0 & 0 & 0 & 0 & 0 & -1i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1i & 0 & 0 \end{pmatrix} \tag{7}$$

$$L_z = \begin{pmatrix} -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix} \tag{8}$$

$$L^2 = \begin{pmatrix} 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 \end{pmatrix} \tag{9}$$

$$J_x = \begin{pmatrix} 0 & \frac{1}{2} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & \frac{1}{2} & \sqrt{\frac{3}{2}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & 0 & 0 & \sqrt{\frac{3}{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{\frac{3}{2}} & 0 & 0 & \frac{1}{2} & \sqrt{\frac{3}{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{\frac{3}{2}} & \frac{1}{2} & 0 & 0 & \sqrt{\frac{3}{2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{\frac{3}{2}} & 0 & 0 & \frac{1}{2} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sqrt{\frac{3}{2}} & \frac{1}{2} & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{1}{2} & 0 \end{pmatrix} \tag{10}$$

$$J_y = \begin{pmatrix} 0 & \frac{1}{2}i & 1i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{2}i & 0 & 0 & 1i & 0 & 0 & 0 & 0 & 0 & 0 \\ -1i & 0 & 0 & \frac{1}{2}i & \sqrt{\frac{3}{2}}i & 0 & 0 & 0 & 0 & 0 \\ 0 & -1i & -\frac{1}{2}i & 0 & 0 & \sqrt{\frac{3}{2}}i & 0 & 0 & 0 & 0 \\ 0 & 0 & -\sqrt{\frac{3}{2}}i & 0 & 0 & \frac{1}{2}i & \sqrt{\frac{3}{2}}i & 0 & 0 & 0 \\ 0 & 0 & 0 & -\sqrt{\frac{3}{2}}i & -\frac{1}{2}i & 0 & 0 & \sqrt{\frac{3}{2}}i & 0 & 0 \\ 0 & 0 & 0 & 0 & -\sqrt{\frac{3}{2}}i & 0 & 0 & \frac{1}{2}i & 1i & 0 \\ 0 & 0 & 0 & 0 & 0 & -\sqrt{\frac{3}{2}}i & -\frac{1}{2}i & 0 & 0 & 1i \\ 0 & 0 & 0 & 0 & 0 & 0 & -1i & 0 & 0 & \frac{1}{2}i \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1i & -\frac{1}{2}i & 0 \end{pmatrix} \tag{11}$$

$$J_z = \begin{pmatrix} -\frac{5}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{3}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{3}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{5}{2} \end{pmatrix} \tag{12}$$

$$J^2 = \begin{pmatrix} 8.75 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4.75 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 7.75 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5.75 & \sqrt{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{6} & 6.75 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6.75 & \sqrt{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sqrt{6} & 5.75 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7.75 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 4.75 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8.75 \end{pmatrix} \tag{13}$$

$$\sum l_x^2 = \begin{pmatrix} 1 & 0 & 0 & 0 & \sqrt{\frac{3}{2}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & \sqrt{\frac{3}{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{5}{2} & 0 & 0 & 0 & \frac{3}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{5}{2} & 0 & 0 & 0 & \frac{3}{2} & 0 & 0 \\ \sqrt{\frac{3}{2}} & 0 & 0 & 0 & 3 & 0 & 0 & 0 & \sqrt{\frac{3}{2}} & 0 \\ 0 & \sqrt{\frac{3}{2}} & 0 & 0 & 0 & 3 & 0 & 0 & 0 & \sqrt{\frac{3}{2}} \\ 0 & 0 & \frac{3}{2} & 0 & 0 & 0 & \frac{5}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{3}{2} & 0 & 0 & 0 & \frac{5}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{\frac{3}{2}} & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sqrt{\frac{3}{2}} & 0 & 0 & 0 & 1 \end{pmatrix} \tag{14}$$

$$\sum l_y^2 = \begin{pmatrix} 1 & 0 & 0 & 0 & -\sqrt{\frac{3}{2}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -\sqrt{\frac{3}{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{5}{2} & 0 & 0 & 0 & -\frac{3}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{5}{2} & 0 & 0 & 0 & -\frac{3}{2} & 0 & 0 \\ -\sqrt{\frac{3}{2}} & 0 & 0 & 0 & 3 & 0 & 0 & 0 & -\sqrt{\frac{3}{2}} & 0 \\ 0 & -\sqrt{\frac{3}{2}} & 0 & 0 & 0 & 3 & 0 & 0 & 0 & -\sqrt{\frac{3}{2}} \\ 0 & 0 & -\frac{3}{2} & 0 & 0 & 0 & \frac{5}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{3}{2} & 0 & 0 & 0 & \frac{5}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\sqrt{\frac{3}{2}} & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\sqrt{\frac{3}{2}} & 0 & 0 & 0 & 1 \end{pmatrix} \tag{15}$$

$$\sum l_z^2 = \begin{pmatrix} 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \end{pmatrix} \tag{16}$$

$$\sum l_x^4 + l_y^4 + l_z^4 = \begin{pmatrix} 21 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 21 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 18 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 18 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 24 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 24 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 18 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 18 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 21 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 21 \end{pmatrix} \tag{17}$$

$$V_{e-e} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{18}$$

$$\sum \vec{l} \cdot \vec{s} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2} & \sqrt{\frac{3}{2}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{\frac{3}{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{\frac{3}{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sqrt{\frac{3}{2}} & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{19}$$

5 Wavefunctions

5.1 Ground state manifold

$$\Psi_1 = \begin{pmatrix} & |-2\downarrow\rangle \end{pmatrix} \tag{20}$$

$$\Psi_2 = \begin{pmatrix} & |-2\uparrow\rangle \end{pmatrix} \tag{21}$$

$$\Psi_3 = \begin{pmatrix} & |-1\downarrow\rangle \end{pmatrix} \tag{22}$$

$$\Psi_4 = \begin{pmatrix} & |-1\uparrow\rangle \end{pmatrix} \tag{23}$$

$$\Psi_5 = \begin{pmatrix} & |0\downarrow\rangle \end{pmatrix} \tag{24}$$

$$\Psi_6 = \begin{pmatrix} & |0\uparrow\rangle \end{pmatrix} \tag{25}$$

$$\Psi_7 = \begin{pmatrix} & |+1\downarrow\rangle \end{pmatrix} \tag{26}$$

$$\Psi_8 = \big(\quad |+1\uparrow\rangle \big) \tag{27}$$

$$\Psi_9 = \big(\quad |+2\downarrow\rangle \big) \tag{28}$$

$$\Psi_{10} = \big(\quad |+2\uparrow\rangle \big) \tag{29}$$