RACIONALES (I)

$$\int \frac{2x+1}{x^2-4}\,dx$$

$$\begin{array}{r|rrr} & 1 & 0 & -4 \\ 2 & & 2 & 4 \\ \hline & 1 & 2 & \boxed{0} \\ -2 & & -2 & \\ \hline & 1 & \boxed{0} & \end{array}$$

$$\frac{2x+1}{\cancel{x^2-4}} = \frac{2x+1}{(x+2)(x-2)} = \frac{A_1}{(x+2)} + \frac{A_2}{(x-2)} = \frac{A_1(x-2)+A_2(x+2)}{\cancel{(x+2)(x-2)}}$$

$$2x+1 = A_1(x-2) + A_2(x+2)$$

$$2x+1 = \cancel{A_1x} - A_1 2 + \cancel{A_2x} + A_2 2$$

$$\underbrace{2x+1}_{} = \underbrace{x(A_1+A_2)}_{x} + \underbrace{(-2A_1+2A_2)}_{x}$$

$$\left.\begin{array}{l} 2 = A_1 + A_2 \\ 1 = -2A_1 + 2A_2 \end{array}\right\} \begin{array}{l} 4 = 2A_1 + 2A_2 \\ 1 = -2A_1 + 2A_2 \\ \hline 5 = 0 + 4A_2 \end{array}$$

$$\boxed{A_2 = \frac{5}{4}}$$

$$2 = A_1 + \frac{5}{4}$$

$$2 - \frac{5}{4} = A_1$$

$$\boxed{A_1 = \frac{3}{4}}$$

$$\int \frac{2x+1}{x^2-4}\,dx = \int \left(\frac{\frac{3}{4}}{(x+2)} + \frac{\frac{5}{4}}{(x-2)} \right)$$

$$= \frac{3}{4}\int \frac{1}{x+2}\,dx + \frac{5}{4}\int \frac{1}{x-2}\,dx =$$

$$= \boxed{\frac{3}{4} \ln(x+2) + \frac{5}{4} \ln(x-2) + C}$$

RACIONALES (II)

$$\int \frac{x^2+2}{x^3-9x^2+27x-27} \cdot dx$$

$$\begin{array}{r|rrrr} & 1 & -9 & 27 & -27 \\ 3 & & 3 & -18 & 27 \\ \hline & 1 & -6 & 9 & \boxed{0} \\ 3 & & 3 & 9 & \\ \hline & 1 & -3 & \boxed{0} & \\ 3 & & 3 & & \\ \hline & 1 & \boxed{0} & & \end{array} \quad (x-3)$$

$$\frac{x^2+2}{\cancel{(x+3)^3}} = \frac{A_1}{(x+3)} + \frac{A_2}{(x+3)^2} + \frac{A_3}{(x+3)^3} = \frac{A_1(x-3)^2 + A_2(x-3) + A_3}{\cancel{(x+3)^3}}$$

MCM

$$x^2+2 = A_1(x-3)^2 + A_2(x-3) + A_3 = A_1x^2 + 9A_1 + A_2x - 3A_2 + A_3$$

$$x^2+2 = A_1x^2 + 9A_1 - 6xA_1 + A_2x - 3A_2 + A_3$$

$$\underset{1}{x^2} + \underset{0}{0 \cdot x} + \underset{2}{2} = A_1x^2 + 9A_1 - 6xA_1 + A_2x - 3A_2 + A_3$$

$$x^2 + 0 \cdot x + 2 = \underset{1}{A_1x^2} + \underset{0}{x(-6A_1 + A_2)} + \underset{2}{(+9A_1 - 3A_2 + A_3)}$$

$$\boxed{A_1 = 1}$$

$$0 = -6A_1 + A_2$$

$$0 = -6 + A_2$$

$$\boxed{A_2 = 6}$$

$$2 = +9A_1 - 3A_2 + A_3$$

$$2 = +9(1) - 3(6) + A_3$$

$$2 = 9 - 18 + A_3 \;;\; 2 - 9 + 18 = A_3$$

$$\boxed{A_3 = 11}$$

$$\int \frac{x^2+2}{x^3-9x^2+27x-27}\,dx = \int \frac{1}{x-3}\,dx + \int \frac{6}{(x-3)^2}\,dx + \int \frac{11}{(x-3)^3}\,dx =$$

$$x-3=t$$
$$dt = d(x-3) = dx$$

$$= \int \frac{1}{t}\,dt + \int \frac{6}{t^2}\,dt + \int \frac{11}{t^3}\,dt = \ln t + 6\int \frac{1}{t^2}\,dt + 11\int \frac{1}{t^3}\,dt$$

$$= \ln t + 6\int t^{-2}\,dt + 11\int \frac{-2}{-2}\,t^{-3}\,dt = \ln t - 6\int -t^{-2}\,dt + 11\int \frac{-2}{-2}\,t^{-3}\,dt =$$

$$= \ln t - 6\cdot\frac{1}{t} - \frac{11}{2}\,t^{-2}$$

Cambio de variable

$$\ln(x-3) + \frac{6}{(x-3)} - \frac{11}{2}(x-3)^{-2} + C$$

* Orden de prioridad ⇒ ALPES

Para ver cual le corresponde la u. ⇒

A → arcsen / arctg
L → f. logarítmicas
P → f. potenciales
E → f. exponenciales
S → f. sen/cos

* $\int u \cdot dv = u \cdot v - \int v \cdot du$

$\int x \cdot \operatorname{sen} x \, dx = u \cdot v - \int v \cdot du$

se deriva

$u = x \quad du = 1dx$

$dv = \operatorname{sen} x \, dx \quad v = -\cos x$

se integra

se sustituye

$x \cdot (-\cos x) - \int -\cos x \, dx = -x \cos x + \operatorname{sen} x$

Un día vi un valiente soldadito vestido de uniforme

$\int u \cdot dv = \underbrace{\quad}_{u} \underbrace{\quad}_{v} - \underbrace{\quad}_{\int} \underbrace{\quad}_{v} \underbrace{\quad}_{du}$

* $\int x \cdot \ln x \, dx = u \cdot v - \int v \cdot du$

$u = \ln x \quad du = \frac{1}{x} dx$

$dv = x \cdot dx \quad v = \frac{x^2}{2}$

$\ln x \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot \frac{1}{\cancel{x}} \, dx$

$\ln x \cdot \frac{x^2}{2} - \frac{1}{2} \int x \, dx$

$\int x \cdot \ln x \, dx = \ln x \cdot \frac{x^2}{x} - \frac{1}{2} \cdot \frac{x^2}{2}$

* $\int \frac{1}{\cos^2 3x}\, dx = \frac{1}{3}\int \frac{3}{\cos^2 3x} = \frac{1}{3} \ln \cos^2 3x^{-1}$

* $\int e^{x^2+2x}\,(x+1) = \frac{1}{2}\int 2\, e^{x^2+2x}\,(x+1) = \frac{1}{2}\, e^{x^2+2x} = \frac{e^{x^2+2x}}{2}$

* $\int (x^2+x)^2\, dx = \int x^4 + x^2 + 2x^3 = \frac{x^5}{5} + \frac{x^3}{3} + 2\frac{x^4}{4}$

$\boxed{\text{EXAMEN}}$

* $f(x) = \ln \frac{x}{x-1}$

* $f(x) = \operatorname{sen}[\cos(e^{x^2})]$

* $f(x) = (x^2 + \sqrt{x} + \operatorname{sen} x)$

* $f(x) = e^{\frac{x}{x-1}} + \sqrt{3}$

* $\int e^x + x^3 + \sqrt{x}\, dx$

* $\int e^{x^2} \cdot x \quad dx$

* $\int \frac{3x^2+2x}{x^3+x^2+3}\, dx$

* $\int x \; e^x\, dx$

* Interpretación geométrica de la derivada.

DESCOMPOSICIÓN

$$A = \int \left(\frac{5}{\cos^2 x} - \frac{2}{x} + \frac{5}{\sqrt{x}} \right) dx$$

$$\int \left(\frac{5}{\cos^2 x} \right) dx - \int \frac{2}{x}\, dx + \int \frac{5}{\sqrt{x}}\, dx$$

$$A_1 = 5 \int \frac{1}{\sqrt{x}}\, dx = 5 \int \frac{1}{x^{\frac{1}{2}}}\, dx = 5 \int \frac{2}{2}\, x^{-\frac{1}{2}} dx = 5 \cdot 2 \int \frac{1}{2} \cdot x^{-\frac{1}{2}} dx = 10 \cdot x^{\frac{1}{2}}$$

$$A_2 = 2 \int \frac{1}{x}\, dx = 2 \cdot \ln x$$

$$A_3 = 5 \int \frac{1}{\cos^2 x}\, dx = 5\, \mathrm{tg}\, x$$

$$\boxed{5\, \mathrm{tg}\, x - \frac{2}{x} + 10 x^{\frac{1}{2}} + C}$$

CAMBIO DE VARIABLE

$$\int x \sqrt{x+2}\; dx$$

$x+2 = t$

$x = t-2$

$dx = dt \;(dx = d(t-2))$

$$\int (t-2)\sqrt{t}\, dt = \int (t\sqrt{t} - 2\sqrt{t})\, dt =$$

$$= \underbrace{\int t \cdot t^{\frac{1}{2}}\, dt}_{A_1} - \underbrace{\int 2 \cdot t^{\frac{1}{2}}\, dt}_{A_2}$$

$$A_1 = \int t^{\frac{3}{2}}\, dt = \int \frac{2}{2} \cdot \frac{5}{5} \cdot t^{\frac{3}{2}}\, dt = 2 \cdot \frac{1}{5} \int \frac{5}{2}\, t^{\frac{3}{2}}\, dt =$$

$$= \frac{2}{5}\, t^{\frac{5}{2}}$$

$$A_2 = 2 \int t^{\frac{1}{2}}\, dt = 2 \int \frac{2}{2} \cdot \frac{3}{3}\, t^{\frac{1}{2}}\, dt \quad 2 \cdot 2 \cdot \frac{1}{3} \int \frac{3}{2}\, t^{\frac{1}{2}}\, dt =$$

$$= \frac{4}{3}\, t^{3/2}$$

$$A_T = \frac{2}{5}\, t^{5/2} + \frac{4}{3}\, t^{\frac{3}{2}}$$

$t^{\frac{3}{2}} \quad \frac{3}{2}\, t^{\frac{1}{2}}$

$t^{\frac{1}{2}+1} = t^{3/2} = \frac{3}{2}\, t^{1/2}$

$t^{1/2+1} = t^{3/2} = \frac{3}{2} \cdot t^{1/2}$

POR PARTES

ALPES: A → Arctg, L → Log, P → Potencial, E → expo, S → [illegible]

solucionar un [illegible] soldadito vestido de uniforme.

$$\int x^2 \cos x \, dx \qquad x^2 \to u, \quad \cos x \, dx \to dv$$

$$\boxed{\int u \, dv = uv - \int v \, du}$$

$u = x^2$ $\quad v = \operatorname{sen} x$

$dv = \cos x \, dx$ $\quad du = 2x \, dx$

$$uv - \int v \, du$$

$$x^2 \operatorname{sen} x - \int \operatorname{sen} x \, 2x \, dx \quad = I$$

$$I = \int \operatorname{sen} x \, 2x \, dx = 2 \int x \operatorname{sen} x \, dx \qquad x \to u, \quad \operatorname{sen} x \, dx \to dv$$

$u = x$ $\quad V = -\cos x$

$dv = \operatorname{sen} x \, dx$ $\quad du = 1 \, dx$ $\quad uv - \int v \, d[illegible]$

$$2\left(x(-\cos x) - \int (-\cos x) \, du\right)$$

$$I = [x(-\cos x) - (-\operatorname{sen} x)] \, 2$$

$$\boxed{x^2 \operatorname{sen} x - 2x(-\cos x) + 2 \operatorname{sen} x + C}$$

$$\int 2x \cos x \, dx \qquad 2x \to u, \quad \cos x \, dx \to dv$$

$u = 2x$ $\quad V = \operatorname{sen} x$

$dv = \cos x \, dx$ $\quad du = 2 \, dx$

$$uv - \int v \, du$$

$$2x \operatorname{sen} x - \int \operatorname{sen} x \, 2 \, dx =$$

$$= 2x \operatorname{sen} x - 2 \int \operatorname{sen} x \, dx =$$

$$= 2x \operatorname{sen} x - 2(-\cos x) =$$

$$= \boxed{2x \operatorname{sen} x + 2 \cos x + C}$$

Integrales

- $\int f(x)\,dx = F(x) + k$

  $F'(x) = f(x)$
- $\int f \pm g\,dx = \int f\,dx \pm \int g\,dx$
- $\int k f\,dx = k \int f\,dx$

- $\int f' \cdot f^n = \dfrac{f^{n+1}}{n+1}$
- $\int \dfrac{f'}{f} = \ln f$
- $\int f' e^f = e^f$
- $\int f' \operatorname{sen} f = -\cos f$
- $\int f' \cos f = \operatorname{sen} f$
- $\int a^f \cdot f' = \dfrac{a^f}{\ln a}$
- $\int \dfrac{1}{\cos^2 f} = \operatorname{tg} f$
- $\int \dfrac{1}{\operatorname{sen}^2 f} = -\operatorname{cotg} f$
- $\int \dfrac{-f'}{\sqrt{1-f^2}} = \arccos f$
- $\int \dfrac{f'}{\sqrt{1+f^2}} = \operatorname{arcsen} f$
- $\int \dfrac{f'}{1+f^2} = \operatorname{arctg} f$

- Recta tg: $y - f(a) = f'(a)(x-a)$
- $\sqrt[m]{x^n} = x^{\frac{n}{m}}$
- $\operatorname{sen} 2x = 2 \operatorname{sen} x \cdot \cos x$
- $\operatorname{sen}^2 x + \cos^2 x = 1$

- Regla de Barrow: Si f es una función continua en $[a, b]$ y F es una primitiva de f, entonces $\int_a^b f(x)\,dx = F(b) - F(a)$

- Cálculo de áreas: Dadas dos funciones, f y g, tales que $f(x) \geq g(x)$ $[a, b]$, el área del recinto limitado por sus gráficas entre las abscisas a y b es:

$$A = \int_a^b [f(x) - g(x)]\,dx$$

* Inmediatas

$\sqrt{x} = x^{\frac{1}{2}}$

- $\int (2x^2 - \sqrt{x} + x^{-3})\,dx = \int 2x^2 dx - \int \sqrt{x}\,dx + \int x^{-3} dx = \frac{2}{3}x^3 - \frac{2}{3}x^{3/2} +$

- $\int \left(\frac{1}{x^2} - \frac{1}{\sqrt{x}} + x^{-1}\right) dx = \int x^{-2} dx - \int x^{-1/2} dx + \int x^{-1} dx = -\frac{1}{x} - 2x^{1/2} + \ln|x| +$

- $\int e^{4x+1}\,dx = \frac{1}{4} e^{4x+1} + C$

$\text{sen}\, 2x = 2\,\text{sen}\, x \cdot \cos x$

- $\int \cos(2x+1)\,dx = \frac{1}{2}\int 2\cos(2x+1)\,dx = \frac{1}{2}\,\text{sen}\,(2x+1) + C$

- $\int x^2\,\text{sen}\,(x^3+1)\,dx = \frac{1}{3}\int 3x^2\,\text{sen}\,(x^3+1)\,dx = \frac{1}{3}\,(-\cos(x^3+1)) + C$

- $\int \frac{\text{sen}\, 2x}{1+\text{sen}^2 x}\,dx = \int \frac{2\,\text{sen}\, x \cdot \cos x}{1+\text{sen}^2 x}\,dx = \ln(1+\text{sen}^2 x)$

- $\int e^{\text{sen}^2 x}\,\text{sen}\, 2x\,dx = \int e^{\text{sen}^2 x}\, 2\,\text{sen}\, x \cos x\,dx = e^{\text{sen}^2 x} + C$

- $\int e^x \cos e^x\,dx = \text{sen}\, e^x + C$

- $\int \frac{\text{sen}\,(\ln x)}{x}\,dx = -\cos(\ln x) + C$

- $\int \frac{x^2}{1+x^6}\,dx = \frac{1}{3}\int \frac{3x^2}{1+(x^3)^2}\,dx = \frac{1}{3}\,\text{arctg}\, x^3$

* Por partes

- $\int (x+2)\cos x\,dx \quad \begin{cases} u = x+2 & du = dx \\ dv = \cos x\,dx & v = \text{sen}\, x \end{cases}$

$$\int (x+2)\cos x\,dx = (x+2)\,\text{sen}\, x - \int \text{sen}\, x\,dx$$

$$= (x+2)\,\text{sen}\, x + \cos x + C$$

- $\int x \operatorname{sen} x \, dx = \begin{cases} u = x & du = dx \\ dv = \operatorname{sen} x \, dx & v = -\cos x \end{cases}$

$$\int x \operatorname{sen} x \, dx = x(-\cos x) - \int -\cos x \, dx$$

$$= x(-\cos x) + \operatorname{sen} x + C$$

- $\int x^2 \operatorname{sen} x \, dx = \begin{cases} u = x^2 & du = 2x \, dx \\ dv = \operatorname{sen} x \, dx & v = -\cos x \end{cases}$

$$\int x^2 \operatorname{sen} x \, dx = x^2(-\cos x) - \int -\cos x \; 2x \, dx$$

$$+\int -\cos x \; 2x \, dx = 2 \int x \cos x \, dx = \begin{cases} u = x & du = dx \\ dv = \cos x \, dx & v = \operatorname{sen} x \end{cases}$$

$$\int x \cos x \, dx = 2\left(x \operatorname{sen} x - \int \operatorname{sen} x \, dx\right)$$

$$\int x \cos x \, dx = (x \operatorname{sen} x + \operatorname{sen} x)\, 2 \quad \Big\} \; x^2(-\cos x) + 2(x \operatorname{sen} x + \operatorname{sen} x \, d$$

- $\int \operatorname{arcotg} x \, dx = \begin{cases} u = \operatorname{arctg} x & du = \dfrac{1}{1+x^2} \, dx \\ dv = dx & v = x \end{cases}$

$$\int \operatorname{arctg} x \, dx = x \operatorname{arctg} x - \int \frac{x}{1+x^2} \, dx$$

$$= x \operatorname{arctg} x - \frac{1}{2} \int \frac{2x}{1+x^2} \, dx$$

$$= x \operatorname{arctg} x - \frac{1}{2} \ln(1+x^2)$$

*Cambio de variable

- $\int x\, e^{x^2}\, dx = \begin{cases} x^2 = t \\ 2x\, dx = dt \end{cases} \quad dx = \frac{dt}{2x}$

$$\int \cancel{x}\, e^t \frac{dt}{2\cancel{x}} = \frac{1}{2}\int e^t\, dt = \frac{1}{2} e^t + C_1 = \frac{1}{2} e^{x^2} + C_1$$

- $\int$

* Racionales (I)

- $\int \frac{2x+1}{x^2-4}\, dx$

$$\begin{array}{r|rrr} & 1 & 0 & -4 \\ 2 & & 2 & 4 \\ \hline & 1 & 2 & \llcorner 0 \\ -2 & & -2 & \\ \hline & 1 & \llcorner 0 & \end{array}$$

$(x-2)(x+2)$

$$\frac{2x+1}{\cancel{(x-2)(x+2)}} = \frac{A_1}{(x+2)} + \frac{A_2}{(x-2)} = \frac{A_1(x-2) + A_2(x+2)}{\cancel{(x+2)(x-2)}}$$

$2x+1 = A_1(x-2) + A_2(x+2)$

$x=2 \left\{ 2\cdot 2+1 = A_1\cdot 0 + A_2\cdot 4 \, ; \; 5 = 4A_2 \, ; \; A_2 = \frac{5}{4} \right.$

$x=-2 \left\{ 2(-2)+1 = A_1(-4) + A_2\cdot 0 \, ; \; -3 = -4A_1 \, ; \; A_1 = \frac{3}{4} \right.$

$$\int \frac{2x+1}{x^2-4}\, dx = \int \frac{\frac{3}{4}}{(x+2)}\, dx + \int \frac{\frac{5}{4}}{(x-2)}\, dx$$

$$= \frac{3}{4} \ln(x+2) + \frac{5}{4} \ln(x-2)$$

• $\int \frac{3x}{x^3-3x^2+4}\,dx \qquad (x+1)(x-2)^2$

$\frac{3x}{x^3-3x^2+4} = \frac{A_1}{(x+1)} + \frac{A_2}{(x-2)} + \frac{A_3}{(x-2)^2} = \frac{A_1(x-2)^2 + A_2(x+1)(x-2)\, A_3(x+1)}{x^3-3x^2+4}$

$3x = A_1(x-2)^2 + A_2(x+1)(x-2) + A_3(x+1)$

$x=2 \left\{ 3\cdot 2 = 3A_3 \;;\; A_3 = \frac{6}{3} \;;\; A_3 = 2 \right.$

$x=-1 \left\{ 3\cdot(-1) = 6A_1 \;;\; A_1 = \frac{-3}{6} \;;\; A_1 = \frac{-1}{3} \right.$

$x=0 \left\{ 3\cdot 0 = \frac{-1}{3}(4) + A_2(-2) + 2 \;;\; \frac{-4}{3} - 2A_2 + 2 = 0 \;;\; \frac{2}{3} - 2A_2 = 0 \right.$

$+2A_2 = \frac{2}{3} \;;\; A_2 = \frac{\frac{2}{3}}{+2} \;;\; A_2 = \frac{2}{+6} \;;\; A_2 = \frac{1}{3}$

$\int \frac{3x}{x^3-3x^2+4}\,dx = \int \frac{-1}{3(x+1)}\,dx + \int \frac{1}{3(x-2)}\,dx + \int \frac{2}{(x-2)^2}\,dx$

$= \frac{-1}{3}\ln(x+1) + \frac{1}{3}\ln(x-2) - 2\,\frac{1}{(x-2)^2} + C$

• $\int \frac{3x-2}{x^2-8x+15}\,dx \qquad (x-3)(x-5)$

$\frac{3x-2}{x^2-8x+15} = \frac{A_1}{(x-3)} + \frac{A_2}{(x-5)} = \frac{A_1(x-5) + A_2(x-3)}{x^2-8x+15}$

$3x-2 = A_1(x-5) + A_2(x-3)$

$x=5 \left\{ 3\cdot 5 - 2 = A_1\cdot 0 + 2A_2 \;;\; A_2 = \frac{13}{2} \right.$

$x=3 \left\{ 3\cdot 3 - 2 = -2A_1 + A_2\cdot 0 \;;\; A_1 = \frac{7}{-2} \right.$

$\frac{3x-2}{x^2-8x+15} = \frac{7}{-2(x-3)} + \frac{13}{2(x-5)} \;;\; \int \frac{3x-2}{x^2-8x+15}\,dx = \int \frac{7}{-2(x-3)}\,dx + \int \frac{13}{2(x-5)}\,dx$

$\int \frac{3x-2}{x^2-8x+15}\,dx = \frac{7}{-2}\ln(x-3) + \frac{13}{2}\ln(x-5) + C$

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Tema 13: Integrales Indefinidas

Definición: Una función F es **primitiva** de f si y solo si F'=f.

Si F es una primitiva de f, también son primitivas de f todas las funciones de la forma F+C, siendo C∈R.

Definición: el conjunto formado por todas las primitivas de una función f se llama integral indefinida de f y se representa por $\int f(x)\,dx$= F(x)+C donde C es la constante de integración.

Propiedades de la integral indefinida

$$\int \frac{-f'}{\sqrt{1-f^2}}\,dx = \arccos f$$

$$\int \frac{f'}{\sqrt{1+f^2}}\,dx = \text{arctan}\, f$$

1. $\int k \cdot f(x)\,dx = k \cdot \int f(x)dx$
2. $\int (f(x) + g(x))\,dx = \int f(x)dx + \int g(x)dx$
3. $\int f(g(x)) \cdot g'(x)dx = F\big(g(x)\big) + C$

Tabla de integrales

Funciones simples	***Funciones compuestas***
$\int 0\,dx = C$	Para simplificar la notación, u denotará una función de x
$\int 1\,dx = x + C$	
$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, \quad \forall n \neq -1$	$\int u^n \cdot u'\,dx = \frac{u^{n+1}}{n+1} + C, \quad \forall n \neq -1$
$\int \frac{1}{x}\,dx = Ln\lvert x\rvert + C$	$\int \frac{1}{u} u'\,dx = Ln\lvert u\rvert + C$
$\int e^x\,dx = e^x + C$	$\int e^u \cdot u'\,dx = e^u + C$
$\int a^x dx = \frac{a^x}{\text{Ln a}} + C, \forall a \in R$	$\int a^u \cdot u'\,dx = \frac{a^u}{\text{Ln a}} + C, \forall a \in R$
$\int sen\,x\,dx = -cos x + C$	$\int sen\,u \cdot u'\,dx = -\cos u + C$
$\int cos\,x\,dx = sen\,x + C$	$\int cos\,u \cdot u'\,dx = sen\,x + C$
$\int \frac{1}{cos^2 x}\,dx = \int sec^2 x\,dx = tgx + C$	$\int \frac{1}{cos^2 u} \cdot u'dx = \int sec^2 u \cdot u'\,dx = tg\,u + C$
$\int \frac{1}{sen^2 x}\,dx = \int cosec^2 x\,dx = -cotgx + C$	$\int \frac{1}{sen^2 u} \cdot u'dx = \int cosec^2 u \cdot u'\,dx = -cotg\,u + C$
$\int tg\,x \cdot \sec x\;dx = sec\,x + C$	$\int tg\,u \cdot \sec u \cdot u'\;dx = sec\,u + C$
$\int cotg\,x \cdot \text{cosec}\,x\;dx = -cosec\,x + C$	$\int cotg\,u \cdot \text{cosec}\,u \cdot u'\;dx = -cosec\,u + C$

$$\int \frac{f'}{1+f^2}\,dx = \text{arcotg}\, f$$

Integrales

• $\int f(x)\,dx = F(x) + k$ Siempre se pone

$F'(x) = f(x)$

• $\int f' f^n = \dfrac{f^{n+1}}{n+1}$ $\int x^5\,dx = \dfrac{x^6}{6} + k$

• $\int f \pm g = \int f + \int g$

• $\int k f = k \int f$

• $\int f' e^f = e^f$

• $\sqrt[m]{x^n} = x^{\frac{n}{m}}$

• $\cancel{\int f' e^f = e^f}$

• $\int \dfrac{f'}{f} = \ln f$

• $\int f' \operatorname{sen} f = -\cos f$

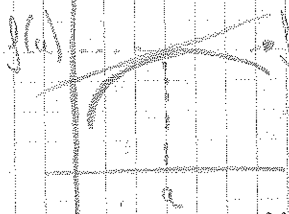
• $\int f' \cos f = \operatorname{sen} f$

• $y - f(a) = f'(a)\cdot(x - a)$

$f(x) \Rightarrow x = a$

Para hallar la recta tangente en un punto a de una función.

Ej:



$f(x) = x^2 + 3x$

$x = 1$

$f'(a) = 2x + 3$

$y - f(a) = f'(a)(x - a)$

$y - f(1) = f'(1)(x - 1)$

$y - 4 = 5(x - 1)$

$y - 4 = 5x - 5$

$y = 5x - 5 + 4$

$\boxed{y = 5x - 1}$

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* $\int \left(\dfrac{1}{x^2} + x^5\right)dx = \int (x^{-2} + x^5)\,dx = \left(\dfrac{x^{-2+1}}{-2+1} + \dfrac{x^6}{6}\right) + k$

* $\int \left(3x^5 + \sqrt{x^3}\right)dx = \int \left(3x^5 + x^{\frac{3}{2}}\right)dx = \left(\dfrac{3x^6}{6} + \dfrac{x^{\frac{5}{2}}}{\frac{5}{2}}\right) + k$

$= \left(\dfrac{x^6}{2} + \dfrac{2\sqrt{x^5}}{5}\right) + k$

* $\int \left(\sqrt[3]{x^2} + \dfrac{1}{x^3}\right)dx = \int \left(x^{\frac{2}{3}} + x^{-3}\right) = \left(\dfrac{x^{\frac{5}{3}}}{\frac{5}{3}} + \dfrac{x^{-2}}{-2}\right) + k$

* $\int \dfrac{1}{x^2} = \int x^{-2} = \left(\dfrac{x^{-1}}{-1}\right) + k$ Para que se anule (pág 263 ej 5)

$\Downarrow$

$\dfrac{3^{-1}}{-1} + k = 0$

$k = 0'333$

$*\ \int (e^x+\sqrt{x})\,dx = \int (e^x + x^{\frac{1}{2}}) = e^x + \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} = e^x + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} =$

$= e^x + \frac{x^{\frac{3}{2}}\cdot 2}{3} = \left(e^x + \frac{2x^{\frac{3}{2}}}{3}\right) + K$

$*\ \int \left(\frac{x}{3} + \cos x\right) dx = \frac{\frac{x^2}{2}}{3} + \operatorname{sen} x = \left(\frac{3x^2}{2} + \operatorname{sen} x\right) + K$

$*\ \int (3e^x - \operatorname{sen} x)\,dx = (3e^x - \cos x) + K$

$*\ \int (3\operatorname{sen} x + 5\cos x)\,dx = (3\cos x + 5\operatorname{sen} x) + K$

$*\ \int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) dx = \int \left(x^{\frac{1}{2}} - \frac{1}{x^{\frac{1}{2}}}\right) = \int (x^{\frac{1}{2}} - x^2) = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^3}{3} =$

$= \frac{2x^{\frac{2}{3}}}{3} - \frac{x^3}{3} = \left(\frac{2x^{\frac{2}{3}} - x^3}{3}\right) + K$

$*\ \int -\frac{3}{x}\,dx = -1\int \frac{3}{x} = -1\left(\frac{3}{\frac{x^2}{2}}\right) = -1\left(\frac{6}{x^2}\right) = -\frac{6}{x^2} + K$

$*\ \int \left(\frac{1}{x^2} + \frac{1}{x^3}\right) dx = \frac{1}{\frac{x^3}{3}} + \frac{1}{\frac{x^4}{4}} = \left(\frac{3}{x^3} + \frac{4}{x^4}\right) + K$

$*\ \int \left(\frac{1}{x} + \frac{1}{2}x^5\right) dx = \int \frac{1}{x} + \frac{1}{2}\int x^5 = \frac{1}{\frac{x^2}{2}} + \frac{1}{2}\cdot\frac{x^6}{6} = \left(\frac{2}{x^2} + \frac{x^6}{12}\right) + K$

$*\ \int \left(e^x - \frac{3}{2}x\right) dx = \int \left(e^x - \frac{3x}{2}\right) = e^x - \frac{3\cdot\frac{x^2}{2}}{2} = \left(e^x - \frac{6x^2}{2}\right) + K$

$*\ \int (x+1)^3\,dx = \left(\frac{x^2}{2} + 1\right)^3 = \left(\frac{x^6}{6} + 1\right) + K$

$*\ \int \frac{1}{\sqrt{x^3}}\,dx = \int \frac{1}{x^{\frac{3}{2}}}\,dx = \frac{1}{\frac{x^{\frac{5}{2}}}{\frac{5}{2}}} = \frac{2}{5x^{\frac{3}{2}}} + K$

$*\ \int \sqrt[4]{x^3}\,dx = \int x^{\frac{3}{4}} = \frac{x^{\frac{7}{4}}}{\frac{7}{4}} = \frac{4x^{\frac{7}{4}}}{7} + K$

$*\ \int \frac{1}{x^9}\,dx = \frac{1}{\frac{x^{10}}{10}} = \frac{10}{x^{10}} + K$

* $f(x) = \ln \frac{x}{x-1}$

$$f'(x) = \frac{\frac{1(x-1)-1x}{(x-1)^2}}{\frac{x}{x-1}} = \frac{\frac{x-1-x}{(x-1)^2}}{\frac{x}{x-1}} = \frac{\frac{-1}{(x-1)^2}}{\frac{x}{x-1}} = \frac{-1(\cancel{x-1})}{x(x-1)^{\cancel{2}}} =$$

$$= \frac{-1}{x(x-1)} = \frac{-1}{x^2-x}$$

* $f(x) = \text{sen}\,[\cos(x^3)]$

$$f'(x) = -(2x\, e^{x^2})\,(\text{sen}\, -e^{x^2})\,(\cos(\cos(e^{x^2})))$$

* $f(x) = (x^2 + \sqrt{x} + \text{sen}\, x)$

$$f'(x) = 2x + \frac{1}{2\sqrt{x}} + \cos x$$

* $f(x) = e^{\frac{x}{x-1} + \sqrt{3}}$

$$f'(x) = \frac{1(x-1)-x}{(x-1)^2} \cdot e^{\frac{x}{x-1}}\, \sqrt{3}$$

* $\int e^x + x^3 + \sqrt{x}\; dx = \left(e^x + \frac{x^4}{4} + \frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right) + k$

* $\int e^{x^2} \frac{2x}{2}\; dx = \frac{1}{2} \int (2x\; e^{x^2})$ $\quad \int f'\, e^f = e$

$$= \frac{1}{2} e^{x^2} + k$$

* $\int \frac{3x^2 + 2x}{x^3 + x^2 + 3} = (\ln\; x^3 + x^2 + 3) + k$

* $\int x \cdot e^x \, dx$

$u = x \quad du = 1 \qquad u \cdot v - \int v \cdot du$

$dv = e^x dx \quad v = e^x \qquad x \cdot e^x - \int e^x \cdot 1$

$$\int x \cdot e^x \, dx = e^x (x - 1) + K$$

* $f(x) = \operatorname{tg} x = \dfrac{\operatorname{sen} x}{\cos x}$

$f'(x) = \dfrac{(\cos x \cdot \cos x) + \operatorname{sen} x \cdot \cos x}{(\cos x)^2} = \dfrac{(\cos x)^2 + \operatorname{sen} x \cdot \cos x}{(\cos x)^2} =$

$= \dfrac{\cos x (\cos x + \operatorname{sen} x)}{\cos^2 x} = \dfrac{\cos x + \operatorname{sen} x}{\cos x}$

* $\int \dfrac{\ln x}{x} dx \Rightarrow \int \dfrac{1}{x} \ln x = \dfrac{(\ln x)^2}{2} + K$

* $\int x \ln x \, dx = u \cdot v - \int v \cdot du$

$u = \ln x \quad du = \dfrac{1}{x} \qquad \ln x \cdot \dfrac{x^2}{2} - \int \dfrac{x^2}{2} \cdot \dfrac{1}{x}$

$dv = x \, dx \quad v = \dfrac{x^2}{2} \qquad \ln x \cdot \dfrac{x^2}{2} - \int \dfrac{x^2}{2x}$

$\ln x \cdot \dfrac{x^2}{2} - \dfrac{1}{2} \int x$

$\ln x \cdot \dfrac{x^2}{2} - \dfrac{1}{2} \cdot \dfrac{x^2}{2}$

$\ln x \cdot \dfrac{x^2}{2} - \dfrac{x^2}{4}$

* $\int$ Pag 161

1 (1) a) Calculo área bajo la gráfica de una función

b) Calcula las primitivas que cumplen $F(0)=($

1 (0'75) $F(x)=\int \frac{(\sqrt{x}-2x)^2}{\sqrt{x}}\,dx$

1 (0'75) $F(x)=\int \sqrt{9-x^2}\,dx$

(2) Integración por partes

$\int x e^x\,dx$ (1)

$\int x^2 \cos x\,dx$ (1'5)

(3) F racionales (método)

1 $\int \frac{1}{1-x^2}\,dx$

(4) Representa y calcula (2'5)

$f(x)=x+1$

$g(x)=x^2+1$

$(3+x)(3-x)$

$3+x=($

$x=(-)$

$(a+b)^2=a^2+b^2+2ab$

$(a-b)^2=a^2+b^2-2ab$

$a^2-b^2=(a+b)(a-b)$

① Calcula la integral por el método racional

$$\int \frac{1}{1-x^2}\,dx$$

$$\frac{1}{1-x^2} = \frac{A_1}{(x+1)} + \frac{A_2}{(x-1)} = \frac{A_1(x-1)+A_2(x+1)}{(x+1)(x-1)} \Rightarrow 1 = A_1(x-1)+A_2(x+1) =$$

$$= 1 = A_1x - A_1 + A_2x + A_2$$

$$0\cdot x + 1 = \underbrace{x(A_1+A_2)}_{0} + \underbrace{(A_2-A_1)}_{1}$$

$$\left.\begin{array}{l} A_1 + A_2 = 0 \\ -A_1 + A_2 = 1 \end{array}\right\}$$

$$2A_2 = 1$$

$$\boxed{A_2 = \tfrac{1}{2}} \quad \boxed{A_1 = -\tfrac{1}{2}}$$

$$\int \left(\frac{-\frac{1}{2}}{(x+1)} + \frac{\frac{1}{2}}{(x-1)}\right)dx = \int \frac{-\frac{1}{2}}{(x+1)}\,dx + \int \frac{\frac{1}{2}}{(x-1)}\,d$$

$$= -\frac{1}{2}\int \frac{1}{x+1}\,dx + \frac{1}{2}\int \frac{1}{x-1}\,dx =$$

$$= \underline{-\frac{1}{2}\ln(x+1) + \frac{1}{2}\ln(x-1) + C}$$

② Calcula la primitiva que cumpla $F(0)=0$

$$f(x) = \int \frac{(\sqrt{x}-2x)^2}{\sqrt{x}}\,dx = \int \frac{(\sqrt{x})^2+(2x)^2-2\sqrt{x}\,2x}{\sqrt{x}}\,dx = \int \frac{x+4x^2-2\ldots}{\sqrt{x}}$$

$$= \int \overbrace{\frac{x}{\sqrt{x}}}^{A_1}\,dx + \int \overbrace{\frac{4x}{\sqrt{x}}}^{A_2} - \int \overbrace{\frac{2\sqrt{x}\,2x}{\sqrt{x}}}^{A_3}\,dx \qquad \int \frac{x}{x^{1/2}}\cdot dx = \int x^{1-\frac{1}{2}}\cdot dx = \int x^{1/2}\,dx$$

$$A_1 = \int \frac{x}{\sqrt{x}}\,dx = \int \frac{x}{x^{1/2}}\,dx = \cancel{x\int \frac{1}{x^{1/2}}\,dx} = x\int \frac{2}{2}x^{-1/2}\,dx = 2x\int \frac{1}{2}x^{-1/2}\,dx = \underline{2x\cdot x^{1/2}}$$

$$A_2 = \int \frac{4x}{\sqrt{x}}\,dx = \int \frac{4x}{x^{1/2}}\,dx = \cancel{4x\int \frac{1}{x^{1/2}}\,dx} = 4x\int \frac{2}{2}x^{-1/2}\,dx = 2\cdot 4x\int \frac{1}{2}x^{-1/2}\,dx = \underline{8x\cdot x^{1/2}}$$

$$A_3 = \int \frac{2\sqrt{x}\,2x}{\sqrt{x}}\,dx = \int \frac{2\cancel{x^{1/2}}\,2x}{\cancel{x^{1/2}}}\,dx = \int 4x\,dx = 4\int x\,dx = 4\int \frac{2}{2}x\,dx = 4\cdot\frac{1}{2}\int 2$$

$$= \underline{2x^2}$$

$$2x\cdot x^{1/2} + 8x\cdot x^{1/2} - 2x^2 = \boxed{10x^{3/2} - 2x^2 + C}$$