# MATHEMATICS 2, preparatory problems for the test T1

# Summer Semester, 2011/12

Structure of the test: 5 problems on the definite integrals and their applications, and solution of systems of linear equations by various methods.

**1.** Evaluate the definite integrals:

a) $\int_0^1 xe^x\,dx$; b) $\int_0^\pi x\sin x\,dx$; c) $\int_0^\pi x^2\sin x\,dx$; d) $\int_1^e \ln x\,dx$;

e) $\int_1^e x\ln x\,dx$; f) $\int_0^2 \frac{x}{x^2-9}\,dx$; g) $\int_3^4 \frac{1}{x^2-3x+2}\,dx$; h) $\int_1^e \frac{1}{x(1+\ln x)}\,dx$;

i) $\int_0^1 x\sqrt{1-x^2}\,dx$; j) $\int_1^3 \frac{1}{x+x^2}\,dx$; k) $\int_0^1 \frac{x+3}{9+x^2}\,dx$; l) $\int_0^1 \frac{1}{2x^2+3}\,dx$;

m) $\int_1^e \frac{\ln^2 x}{x}\,dx$; n) $\int_0^1 x\cdot e^{-x}\,dx$.

**2.** Find the area $A(O)$ of the plane region $O$ determined by graphs of the following curves (draw the region):

a) $y=6x-x^2,\ y=0$; b) $y=-x,\ y=x+2$ for $1\le x\le 3$;
c) $y=2x^2+10,\ y=4x+16$; d) $y=x^2, y=\sqrt{x}$;
e) $x=y^2+2,\ y=x-8$; f) $y=2x^2+10,\ y=4x+16,\ x=-2,\quad x=5$;
g) $y=0,\ y=\sin x$ for $0\le x\le\pi$; h) $y=\ln x,\ y=0$ for $1\le x\le e$;
i) $y=x^2-2x,\ y=x$; j) $y=x^2,\ y=x^2/4,\ y=1$;
k) $y=x^2,\ y^2=x$; l) $y=x^2-x-6,\ y=-x^2+5x+14$;
m) $y=x^3,\ y=4x$; n) $y=2x^3,\ y^2=4x$;

o) $xy = 10,\ \ x + y = 7$; p) $y = x^2,\ \ y = x^3$;
r) $y = x^n,\ \ y = 0,\ \ x = 1$; s) $y = x^2 - 3x + 2$ and $o_x$ for $x \in \langle 0,\ 3\rangle$;
t) $y = x^3,\ \ y = -x,\ \ y = 1$; u) $y = \sqrt{x},\ \ y = 1,\ \ x = 4$;
v) $x = y^2 + 2y,\ \ y = x - 8$; w) $y = x^2 - x,\ \ y = x - x^2$;
x) $f(x) = \sqrt{x},\ \ g(x) = \sqrt{4 - x},\ \ o_x$; y) $y = x^2 - x,\ \ y = 3x - x^2$.

**3.** Evaluate the Average Value $AV$ of a function on the interval $\langle a,\ b\rangle$, sketch the function, the interval and $AV$:

a) $f(x) = 1 + x,\ \langle 0,\ 1\rangle$; b) $f(x) = 1 + x,\ \langle -1,\ 1\rangle$; c) $f(x) = 1 - x^2,\ \langle -1,\ 1\rangle$;
d) $f(x) = x(1 - x^2),\ \langle -2,\ 2\rangle$; e) $f(x) = x \cdot \sqrt{1 - x^2},\ \langle 0,\ 1\rangle$; f) $f(x) = x \cdot \sqrt{4 - x^2},\ \langle -2,\ 2\rangle$;
g) $f(x) = x(2 - x),\ \ \langle 0,\ 2\rangle$; h) $f(x) = |x(2 - x)|,\ \ \langle 0,\ 4\rangle$; i) $f(x) = e^x,\ \ \langle 0,\ 1\rangle$;
j) $f(x) = e^x + e^{-2x},\ \ \langle -1,\ 1\rangle$; k) $f(x) = \sin x,\ \ \langle 0,\ \pi\rangle$; l) $f(x) = \sin x,\ \ \langle 0,\ 2\pi\rangle$;
m) $f(x) = |\sin x|,\ \ \langle 0,\ 2\pi\rangle$; n) $f(x) = \cos x,\ \ \langle 0,\ \frac{\pi}{2}\rangle$; o) $f(x) = x^3 + x^2,\ \ \langle -1,\ 1\rangle$;
p) $f(x) = \ln x,\ \ \langle 1,\ e\rangle$.

**4.** The velocity of an object $v(x)$ in meter per minute varies during the first 20 minutes of its motion as follows:

– from the start of the motion ($x = 0$) to 4th minute it was $v(x) = 0,5x$ m/min,
– from the 4th minute to the 10th minute it was constant $v(x) = 2$ m/min, and then
– from 10th to 20th minute it was $v(x) = 0,8x - 6$ m/min.

Find the Average Value $AV$ of the object velocity within 20 minutes.

**5.** Find the average value $AV$ of the function $f(x) = \left(x - \dfrac{1}{x}\right)^2$ on the interval $\langle 1,\ 3\rangle$.

**6.** Find all solutions of the system of linear equations, write the solution in a structured form::

a) $\begin{array}{rcl} x_1 + 2x_2 \phantom{{}+3x_3} - 6x_4 &=& 0 \\ 2x_1 + x_2 + 3x_3 \phantom{{}-18x_4} &=& 0 \\ 7x_1 + 8x_2 + 6x_3 - 18x_4 &=& 0 \end{array}$ b) $\begin{array}{rcl} 3x_1 + 2x_2 + x_3 &=& 10 \\ 2x_1 + 3x_2 + x_3 &=& 2 \\ 2x_1 + x_2 + 3x_3 &=& 22 \end{array}$ c) $\begin{array}{rcl} 3x_1 + 2x_2 + x_3 &=& 0 \\ 2x_1 + 3x_2 + x_3 &=& 0 \\ 2x_1 + x_2 + 3x_3 &=& 0 \end{array}$

**7.** Find all solutions of the homogeneous system of linear equations with the matrix $A$ of the system, write the solution in a structured form:

a) $A = \begin{pmatrix} 3 & 4 & 6 & -10 \\ 2 & 3 & 4 & 0 \\ 4 & 5 & 8 & -20 \end{pmatrix}$ b) $A = \begin{pmatrix} 3 & 0 & 0 & 1 \\ 0 & 1 & -1 & 1 \\ 2 & 1 & 0 & 1 \end{pmatrix}$ c) $A = \begin{pmatrix} -5 & 10 & 20 & 13 \\ -15 & -20 & 1 & 11 \\ 1 & 0 & 9 & 0 \\ 12 & 30 & 37 & 2 \end{pmatrix}$

**8.** Solve the linear systems, using Cramer's rule; the system is written in the form of the table:

a)

| $x_1$ | $x_2$ | $x_3$ | |
|---|---|---|---|
| 4 | 3 | −1 | −24 |
| 5 | 1 | 2 | −25 |
| 5 | −4 | 5 | −13 |

b)

| $x_1$ | $x_2$ | $x_3$ | $x_4$ | |
|---|---|---|---|---|
| 4 | 0 | 3 | −1 | 1 |
| 5 | 1 | 0 | 2 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 5 | 4 | 0 | 1 | 0 |

**9.** Evaluate determinants $det\,(A \cdot A^T)$, $det\,(A^T \cdot A)$ for the matrix $A = \begin{pmatrix} 2 & -3 & 1 & 0 \\ 0 & 1 & 0 & -2 \end{pmatrix}$ ($A^T$ is the matrix transposed to the matrix $A$).

**10.** Evaluate the determinant $det\,(A \cdot A^T)$ for the matrix $A = \begin{pmatrix} 2 & -3 & 0 \\ 3 & -1 & 2 \\ 1 & 1 & 0 \end{pmatrix}$.

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