

unfold : User Manual

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Chapter 1

What is `unfolder`?

`unfolder` is an experimental toolkit that enables the user to unfold full fledged functional programs (i.e. programs that use higher order, function composition and lazy evaluation with partially undefined terms).

`unfolder` is written in Prolog (more specifically, Ciao Prolog). In its current state the programs to be unfolded are read from the Prolog code itself, where they are represented as Prolog facts. These facts represent guarded rules belonging to a functional program written in a generic functional language. That is, `unfolder` does not currently read functional programs (written in Haskell or any other existing functional language); instead, it reads the source to be unfolded from an internal representation.

1.1 Unfolding

Unfolding is the process of replacing a function invocation by the definition of that very same function. By repeating the process as many times as necessary, what we get is a program that has the same meaning as the original *folded* but that is simpler than the original one in terms of function composition (a completely unfolded program has no function composition at all but this is not possible in general). That simpler form of the program can be seen as a set of facts that plainly express the meaning of the original program (its semantics).

Unfolding generates valid functional code that can be run just as the original one to produce the same result (provided full unfolding has been possible).

Unfolding is an iterative process: Starting with an empty code, every iteration produces a better approximation to the meaning of the original code. These successive approximations draw closer to the real meaning of the program by having more rules that deal with more input values or by letting existing rules handle better approximations of large terms.

Every approximation to the final meaning of the functional program

under examination is nothing more than a set of functional rules. The set of functional rules contained within a given approximation is called **an interpretation**. The unfolding process begins with an empty interpretation (denoted as I_0) and every iteration of unfolding takes current interpretation along with the original program rules to generate the next interpretation (that is, jumping from I_n to I_{n+1}).

Chapter 2

Writing Programs for unfolder

As stated in chapter 1, `unfolder` reads the programs it must unfold from Prolog source code. Programs to be unfolded are made up of a number of rules. All rules must adhere to the following format:

```
rule(Function_Name,Patterns,Guard,Body,Where)
```

where each argument of `rule` describes a part of a given rule. For example, the two rules for the well known `append` function are written as follows:

```
rule(append,[nil,X],_,X,[]).  
rule(append,[cons(X,Xs),Ys],_,cons(X,append(Xs,Ys)),[]).
```

These two Prolog facts denote the following two Haskell rules:

```
append [] x = x  
append (x:xs) ys = (x:(append xs ys))
```

The example above shows how program rules must be written:

- Free functional variables are represented by free logic variables.
- Constructors are written as Prolog atoms (that is, they start by a lowercase letter). Lists are represented by constructors `nil` (for `[]`) and `cons` (for `(_:_)`).
- No type declaration is necessary. The `unfolder` does not currently know anything about types, so is up to the user to ensure that all programs are well typed.

In turn, each part of the rule must obey the following restrictions:

- The head must be an atom representing the name of the function. All rules defining the same function must bear the same atom at this position.
- The second argument for **rule** represents the pattern of every rule. A pattern is written as a list of terms. All functions defined are considered to be *fully curried*. If tuples have to be used, they can be written as $(term_1, \dots, term_n)$.
- The guard of the rule is next. An empty guard is represented by a free Prolog variable. If the guard is not empty, it must be written using normal applicative notations (that is, $f(1, 2)$ must be written as $f(1, 2)$). Boolean conjunction is written as **and**(_, _).
- The fourth argument of **rule** contains the body of the rule. It has to be written according to the same rules that govern the writing of guards.
- Finally, the last argument is an experimental one intended to contain local (*where*) declarations. It is currently unused and must contain an empty list (`[]`).

Chapter 3

Running `unfolder`

3.1 Execution

`unfolder` is currently included in one Prolog file that must be run from within Ciao Prolog. Therefore, Ciao must be started and then `unfolding.pl` must be loaded:

```
$ ciao
Ciao 1.15.0-14760: mi jun 27 18:36:00 CEST 2012 [install]
?- [unfolding].

yes
?-
```

The rules that represent the functional program to be unfolded must, at the time of writing this, be present inside the Prolog code.

As said before, unfolding is an iterative process. Basically, the process of unfolding a functional program involves running an iteration and examining its result until needed.

In `unfolder`, an iteration is run by invoking the Prolog predicate `unfolding_operator/0`. This takes the functional program rules (represented as Prolog facts) and current interpretation to generate the next interpretation.

Once `unfolding_operator/0` has been run, the content of current interpretation can be examined by means of `show_int/0`.

The unfolding of the `append` function is shown below:

```
$ ciao
Ciao 1.15.0-14760: mi jun 27 18:36:00 CEST 2012 [install]
?- [unfolding].

yes
```

```

?- unfolding_operator,show_int.
* append(Nil,b) = b
* append(Cons(b,c),d) = Cons(b,Bot)

yes
?- unfolding_operator,show_int.
* append(Nil,b) = b
* append(Cons(b,Nil),c) = Cons(b,c)
* append(Cons(b,Cons(c,d)),e) = Cons(b,Cons(c,Bot))

yes
?- unfolding_operator,show_int.
* append(Nil,b) = b
* append(Cons(b,Nil),c) = Cons(b,c)
* append(Cons(b,Cons(c,Nil)),d) = Cons(b,Cons(c,d))
* append(Cons(b,Cons(c,Cons(d,e))),f) = Cons(b,Cons(c,Cons(d,Bot)))

yes
?-

```

Three unfolding iterations are shown above (I_1 , I_2 and I_3). Note that every step involves invoking `unfolding_operator/0` to create a new interpretation and invoking `show_int/0` just after that to see what the new interpretation contains. Of course, there is no problem in invoking `unfolding_operator/0` any number of times between two invocations of `show_int/0`.

3.2 Understanding the Output of `unfolder`

The output of `unfolder` is the one generated by `show_int/0`. This predicate dumps current interpretation to the screen. Every functional rule that belongs to current interpretation is written in a somewhat beautified form. Although the result of unfolding is valid functional code, `unfolder` does not generate valid Haskell code yet (in particular the rules are still uncurried). This is work for the future.

The *beautification* of every interpretation element (every generated functional rule) comprises the following steps:

- Every interpretation element is preceded by an asterisk (`'*'`) to ease the reading of rules.
- Constructors are shown with uppercase initials. Lists are shown as `Nil` (for `[]`) and `Cons` (for `(_:_)`). Note that `Bot` (*bottom*) is a special constructor denoting the absence of information (usually written as \perp in books).

- Variables are shown with lowercase initials.
- Partial or higher order applications are represented by $\langle function \rangle @ \langle arguments_list \rangle$ where $\langle function \rangle$ can be a free variable or the name of a function and $\langle arguments_list \rangle$ is a list of arguments already applied to the function.

The three iterations run above show how the unfolding process moves closer and closer to the final meaning of the program until the final full meaning is eventually found (which does not happen in this case since `append` has an infinite semantics). Every iteration includes the previous one and adds more elements to it. Those new elements are able to deal with successively longer lists.

The next example shows that unfolding does not always add more rules to an interpretation even if the final result has not been found. Consider the following code:

```
data Nat = Zero | Suc Nat

ones :: [Int]
ones = (1:ones)

take_n :: Nat -> [Int] -> Int
take_n Zero _ = []
take_n Suc(n) (x:xs) = (x:(take_n n xs))

main5 :: [Int]
main5 = take_n Suc(Suc(Zero)) ones
```

Its first unfolding iterations are as follows:

```
$ ciao
Ciao 1.15.0-14760: mi jun 27 18:36:00 CEST 2012 [install]
?- [unfolding].

yes
?- unfolding_operator,show_int.
* ones = Cons(1,Bot)
* take_n(Zero,b) = Nil
* take_n(Suc(b),Cons(c,d)) = Cons(c,Bot)

yes
?- unfolding_operator,show_int.
* take_n(Zero,b) = Nil
* ones = Cons(1,Cons(1,Bot))
```

```

* take_n(Suc(Zero),Cons(b,c)) = Cons(b,Nil)
* take_n(Suc(Suc(b)),Cons(c,Cons(d,e))) = Cons(c,Cons(d,Bot))
* main5 = Cons(1,Bot)

yes
?- unfolding_operator,show_int.
* take_n(Zero,b) = Nil
* take_n(Suc(Zero),Cons(b,c)) = Cons(b,Nil)
* ones = Cons(1,Cons(1,Cons(1,Bot)))
* take_n(Suc(Suc(Zero)),Cons(b,Cons(c,d))) = Cons(b,Cons(c,Nil))
* take_n(Suc(Suc(Suc(b))),Cons(c,Cons(d,Cons(e,f)))) =
    Cons(c,Cons(d,Cons(e,Bot)))
* main5 = Cons(1,Cons(1,Bot))

yes
?- unfolding_operator,show_int.
* take_n(Zero,b) = Nil
* take_n(Suc(Zero),Cons(b,c)) = Cons(b,Nil)
* take_n(Suc(Suc(Zero)),Cons(b,Cons(c,d))) = Cons(b,Cons(c,Nil))
* ones = Cons(1,Cons(1,Cons(1,Cons(1,Bot))))
* take_n(Suc(Suc(Suc(Zero))),Cons(b,Cons(c,Cons(d,e)))) =
    Cons(b,Cons(c,Cons(d,Nil)))
* take_n(Suc(Suc(Suc(Suc(b)))),Cons(c,Cons(d,Cons(e,Cons(f,g)))))) =
    Cons(c,Cons(d,Cons(e,Cons(f,Bot))))
* main5 = Cons(1,Cons(1,Nil))

```

Take a look at function `ones`: It has just one rule in every iteration. By contrast, `take_n` gathers more rules as iterations go by. This is due to the fact that `ones` has just one pattern (the empty one) so all the rules are covered by a single case. Observe how the value returned by that rule grows bigger with every iteration (keep in mind that `Bot` is the infimum – least value – of the domain inside which interpretations are generated).

3.3 Other Examples

The code of `unfolder` contains many additional examples covering the main features of functional programming. They all are written as Prolog rules belonging to the `rule/5` predicate. All rules must be commented out except for those that are about to be tested. The steps for a test are the following ones:

1. Uncomment the rules to be tested. Comment all the others.
2. Load `unfolding.pl` into Ciao Prolog.

3. Run the iterations as described above.

The examples included in `unfolding.pl` usually contain a `mainN` function where `N` is a number. This function can be seen as the one that generates the answer that is being sought with the particular test case.

Topics covered by the examples include:

- Functions that need the `match` operator (see the relevant paper).
- Guarded rules.
- Experimenting with constraints within guards.
- Dealing with infinite structures and lazy evaluation.
- Higher order and partial applications.
- Treatment of predefined functions (i.e. those with no rules).
- Function composition.