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Optimization of wind turbine placement in a wind farm using a new pseudo-random number generation method

Saïd Zergane [a], Arezki Smaili [a, *], Christian Masson [b]

[a] Mechanical Engineering and Development Laboratory, Ecole Nationale Polytechnique, B.P. 182, El Harrach, Algiers, 16200, Algeria
[b] Laboratoire de recherche sur l'aérodynamique des éoliennes en milieu nordique, Ecole de Technologie Supérieure, 1100 Rue Notre-Dame O, Montréal, QC H3C 1K3, Canada

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A B S T R A C T

In this paper, with the goal of maximizing the power production of a wind farm and reducing the wake effect resulting from front-end turbines, we present a new optimization method based on the generation of pseudo-random numbers as a mathematical approach; we have used this method along with the Jensen linear wake model in order to study optimal wind turbine positioning in a farm of given dimensions. For this purpose, a computer program has been developed to carry out numerical simulations based on the maximum total power produced. Using a typical wind turbine for uniform and unidirectional wind speed, the simulation results that we have obtained are presented and discussed. Compared to previous works based on genetic algorithms and viral basis methods, this optimization has yielded recorded enhancements of up to 6.5% on resulting wind farm power. Furthermore, we have found that an optimum number of wind turbines can be properly determined for any given wind farm.

1. Introduction

The overexploitation of fossil fuels such as oil and natural gas during recent decades and the consequences this has had on massive carbonic gas release in the atmosphere have been the main reasons for a distinct return to renewable energies. Wind energy is a renewable, clean and sustainable alternative; it does not produce greenhouse gas. To date, there has been considerable development in the installation of wind turbines in windy sites in order to produce electricity. However, wind farm yield essentially depends on where its turbines are located. Due to the wake effect, major reductions in wind speed can be seen. Consequently, the power produced undergoes a deficit in the area downstream of the turbine.

Several studies have been carried out, focussing on the optimal location of wind turbines in wind farms; such studies have employed a variety of analysis models and methods aimed at optimizing the results. It appears that the first analysis models used to describe the wake were presented in a study by Betz [1] and by Lanchester [2] during the early 1920s; their works laid the foundation for future research on setting up wind farms. Thus, in 1974, Templin [3] proposed a simple and efficient model for calculating the effect on wind farm performance of the relative positioning of the farm's wind turbines. In Templin's work, wind turbines are represented by a continuous roughness distribution and it is assumed that the vertical distribution of the velocities of the flow is logarithmic. The influence of wind turbines on flow is determined by equalizing the difference of momentum in the velocity field between the incident and the downstream flow of the machines along with their drag. Such an analysis, however, is restricted to farms composed of an infinite number of machines, so that the minimum spacing this theory predicts should be viewed as an upper limit. Later, Masson et al. [4] proposed a numerical wind farm method in order to optimize the arrangement of the machines. This numerical approach consists in describing flow using Navier-Stokes equations and modelling the rotor's presence in the flow using a distribution of external forces in the momentum equations. The resulting mathematical model was solved using a CFD (computational fluid dynamics) method based on control volume finite element formulation. These forces were evaluated based on the blade element theory and were also used to calculate the power generated by the machine. The formulation used prior knowledge to solve the problem exposed by a single wind turbine or set of wind turbines, in stable or unstable atmospheric boundary

* Corresponding author. Mechanical Engineering and Development Laboratory, Ecole Nationale Polytechnique, B.P. 182, El Harrach, Algiers, 16200, Algeria.
E-mail address: arezki.smaili@g.enp.edu.dz (A. Smaili).

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Nomenclature

A_F	Wind farm area (m)
a	Integer called the germ
b	Random integer
$Cost$	Cost of installation
C_T	Thrust coefficient
D	Turbine diameter (m)
M	Number of iterations
Mod	Modulo operator
N	Number of turbines
N_c	Number of cells within a given wind farm
Obj	Objective function
P	Turbine power (kW)
P_m	Total power of an arrangement m of turbines (kW)
P_{max}	Total power of an optimal arrangement of turbines (kW)
P_n	Power of a single wind turbine (kW)
R	Turbine radius (m)
r	Wake radius in x position (m)
x	Downstream position (m)
y	Longitudinal position (m)
$X(n)$	Elements of a random sequence
U	Wind speed (m/s)
U_{def}	Wind speed deficit (m/s)
Z	Hub height (m)
Z_0	Ground roughness (m)
α	Entrainment constant

layers, on flat or uneven ground. Elsewhere, Moseti et al. [5] were likely the first to tackle the optimal arrangement of wind turbines using genetic algorithms as an optimization procedure and a wake model. In their work focused on predominant wind direction, they conclude that the transverse and longitudinal distances between turbines are five times the diameter of a single wind turbine. In order to reduce the wake effect on total power production of the farm, Moseti et al. [5] represent the objective function as the cost of the investment on the power produced by the farm, including infrastructure: the lower the objective function value, the more optimal the results. Later, Grady et al. [6] also used the genetic algorithm approach along with a wake model in order to obtain optimal placement of wind turbines. They mainly investigated the effects of non-uniform wind speed with variable direction in a given farm. On the other hand, Marmidis et al. [7], proposed a novel procedure based on a statistical method, itself based on the Monte Carlo simulation method. The proposed Monte-Carlo-based method is iterative: using a computer, one generates several arrangements corresponding to the number of iterations. In each arrangement, the turbines are located at random in the wind farm. The optimization objective function inspired by the work of Moseti et al. [5], based on the average installation criteria for maximum energy production and minimum investment cost was also taken into account [8]. Lastly, Itauarte-Villareal et al. [9] suggested a simulation method using a new viral optimization algorithm, in order to find the optimal solution for the wind turbine placement problem, for a given unidirectional, uniform wind speed. They noted that enhancements can be attained in objective function values.

It is important to note that in all these studies, the simplified Jensen wake model was used to describe the resulting turbine wakes in a given farm.

In this paper, we propose a new pseudo-random number generation method as a mathematical approach for optimizing turbine location in a given farm. The Jensen model is also used to describe turbine wake. For this purpose, a MATLAB program was developed. Simulations were carried out using the characteristics of a typical wind turbine, as reported elsewhere [5], for a wind speed that is unidirectional and uniform. We then compared the simulation results obtained to those of previous studies.

2. Wake modelling

In 1983, Jensen [10] developed a simple and efficient model that treats the wake in a way that includes the effects of turbulence associated with flow; this model is old, however, and predicts wake characteristics that evolve analytically as a function of rotor diameter and ground roughness. The simplicity and ease of implementing Jensen's model, using methods such as Matlab, C ++, and Delphi, has been the object of several applications in the optimal micro-location of wind turbines within given wind farms and in a number of commercial codes [11]. Wake expansion along a horizontal plane through the hub of a turbine, idealized according to the Jensen model is shown in Fig. 1.

Wind speed in the wake is described by the following relation:

$$U_x = U\left(1 - \frac{R^2\left(1 - \sqrt{1 - C_T}\right)}{r^2}\right) \tag{1}$$

Therefore, the downstream wind speed deficit for a single wake is

$$U_{def} = U\left(\frac{R^2\left(1 - \sqrt{1 - C_T}\right)}{r^2}\right) \tag{2}$$

where C_T is the trust coefficient, R is the length of the rotor blade, and r is the wake radius at a distance x, according to the following relationship:

$$r = R + \alpha x \tag{3}$$

where α is an entrainment constant defined as follows [6]:

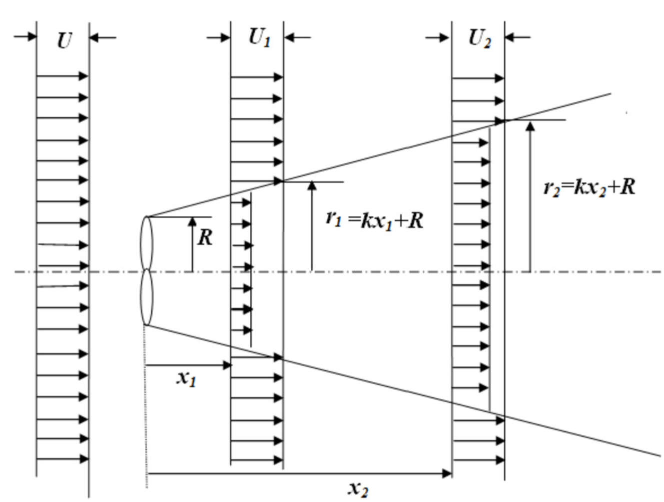

Fig. 1. Jensen wake model.

$$\alpha = \frac{0.5}{ln\left(\frac{Z}{Z_0}\right)} \tag{4}$$

In the case of a wind turbine striking several wakes of a number of different upstream turbines, the principle of the superposition of wind speed deficit is adopted. The downstream wind speed deficit is then given as [7,12].

$$U_{def} = U\sqrt{\sum_{i=1}^{N}\left(\frac{R^2\left(1-\sqrt{1-C_T}\right)}{(R+\alpha x_i)^2}\right)^2} \tag{5}$$

where N is the total number of turbines, Z is the hub height, Z_0 is the ground roughness and x_i is the turbine's position in a given farm; Eq. (1) is then rewritten in the form:

$$U_x = U\left(1-\frac{R^2\left(1-\sqrt{1-C_T}\right)}{\sqrt{\sum_i^N (R+\alpha x_i)^4}}\right) \tag{6}$$

3. The pseudo-random number generation method

Several methods, such as genetic algorithms [11], the Monte Carlo method [13], and the viral basis [9] have been developed and utilized by many researchers and scientists worldwide in order to determine the optimal location of wind turbines and maximize power output in wind farms.

In this paper, we present a new mathematical approach based on the generation of pseudo-random numbers [14] for placing wind turbines in a given wind farm. It is an iterative method that relies on deterministic functions which are often used to produce pseudo-random numbers according to recurring sequences in the form:

$$X(N+1) = (aX(N)+b)mod[N_C+1] \tag{7}$$

where N_C is the number of cells in a given farm, *mod* is the modulo operator which represents the remainder of the entire division, N is the total number of turbines that do not exceed N_C and a and b are positive integer numbers.

This means that the elements of this sequence are the remainders of the division on the divisor N_C+1, while at the same time choosing the integers a and b so that they are non-zero; the first element $X(1)$ is less than N_C+1. In all cases, the pseudo-generated numbers are between 0 and N_C and depend on the constants a, b and c as well as on the initial term $X(1)$. It is instructive to note that this method relies up on deterministic functions; if we keep the same parameters, the sequence generated is reproducible and periodic.

Also, it is important to note that the pseudo-random number generation method turns out to be promising and more convenient for practical and engineering problems, due to its ease of programming and its modest use of computer resources (i.e. it does not require lengthy computational time). Thus, such an optimization method might play a prominent role in the micro-localization of wind turbines in a given site, and hence in the development and design of efficient wind farms.

4. Optimization procedure

The wind farm used for the optimization procedure is square in shape, and subdivided into N_C identical cells. As reported above, the dimension of a given cell is $5D$ x $5D$ (D refers to the diameter of the turbine). As shown in Fig. 2, the centre of each cell represents the possible location of a single wind turbine at maximum. The area of the farm A_F is given as functions of turbine diameter D, and the number of cells N_C by the following relationship:

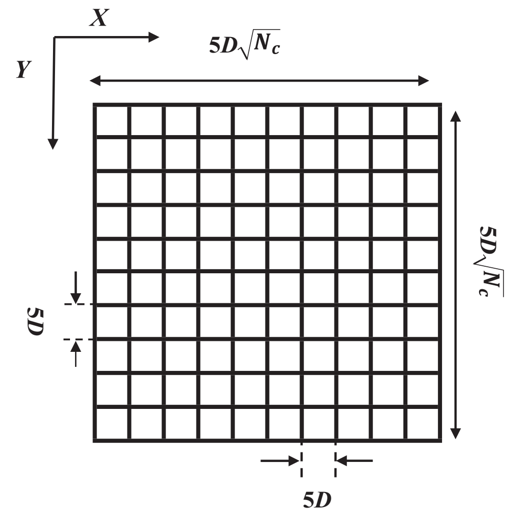

Fig. 2. Subdivided wind farm area

Fig. 2. Proposed wind farm area.

$$A_F = 25N_C D^2 \tag{8}$$

On a wind farm of N_C cells, the proposed method consists in generating M pseudo-random distributions (M refers to the number of iterations) of N wind turbines; then the total power is evaluated at each distribution. The resulting powers are designated as follows; P_m, where $m = 1 \ldots M$, and where m denotes a given distribution (i.e. iterations). Therefore, the optimal distribution can be readily obtained: it corresponds to the maximal power P_{max} among the different arrangements.

Fig. 3 shows the proposed flowchart for determining the optimum arrangement of pseudo-randomly distributed turbines at a given site, that is, for seeking an arrangement that provides maximum power. In order to do this, the calculation is initialized by the generation of pseudo-random numbers by introducing the required parameters, such as the number of turbines N; the number of cells N_C, and the number of iterations M. According to Eq. (7), a matrix of M columns and N rows is obtained, so that each column corresponds to a single possible arrangement, and each turbine within the site may take a pseudo-random position with which a number varying between 1 and 100 is associated. Then, wind speed in the wake can be calculated in order to determine the power of a single wind turbine P_n. Hence, the total power of each arrangement P_m can be computed as follows:

$$P_m = \sum_{n=1}^{N} P_n \tag{9}$$

Once the total power P_m of all possible arrangements is obtained, the optimal arrangement required to produce maximum total power can be readily determined. Lastly, it is important to note that optimization can also be carried out via an objective function *(Obj)* by introducing the cost effect *(Cost)* of wind turbine installation, as has been reported elsewhere [15].

The cost of N wind turbines installed is defined as follows [5]:

$$Cost = N\left(\frac{2}{3}+\frac{1}{3}exp^{-0.0017N^2}\right) \tag{10}$$

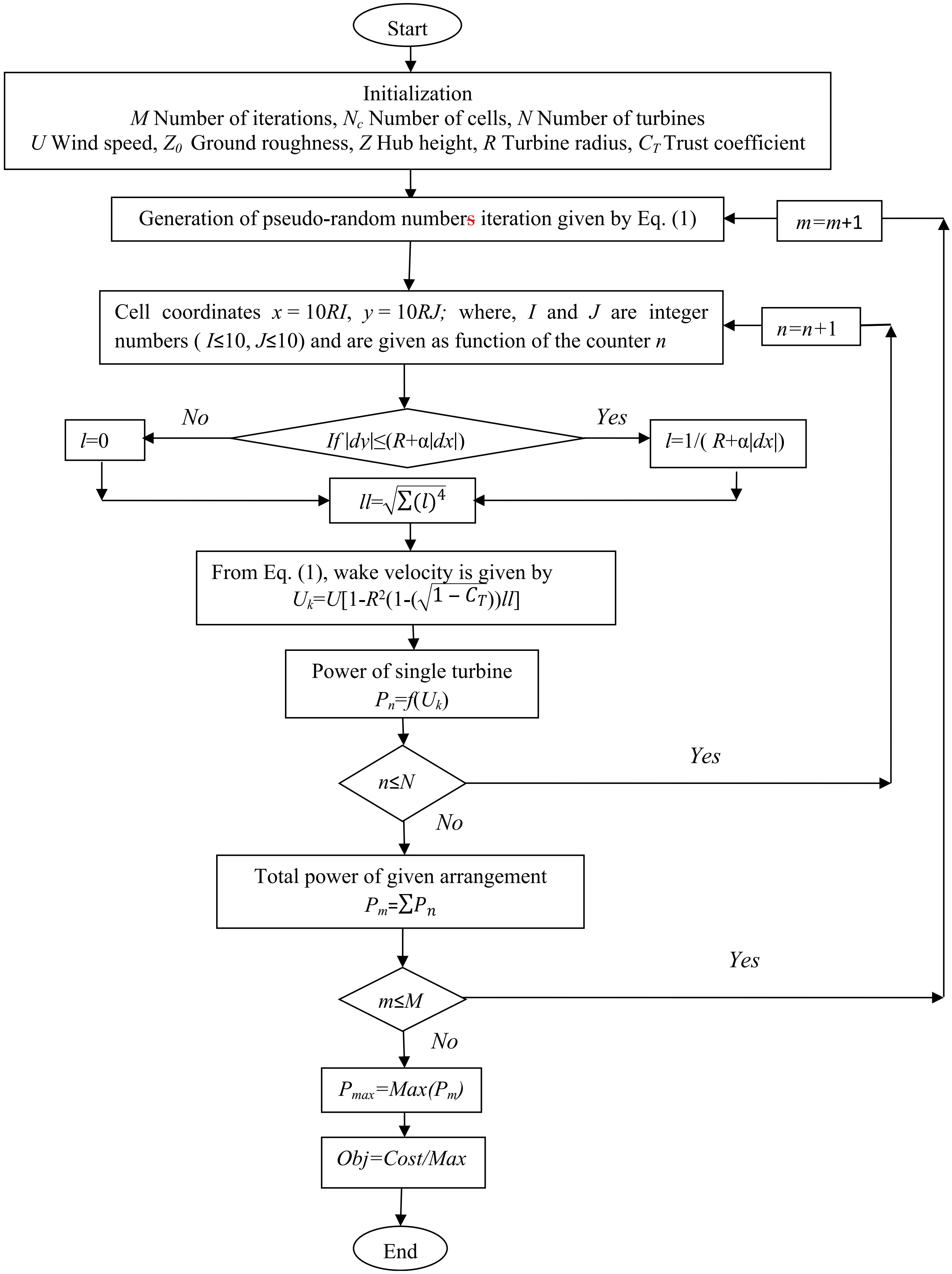

Fig. 3. Flowchart of the proposed optimization method.

Table 1
Wind turbine characteristics and operating conditions.

Z (m)	R(m)	C_T	U (m/s)	Z_0 (m)
60	40	0.88	12	0.3

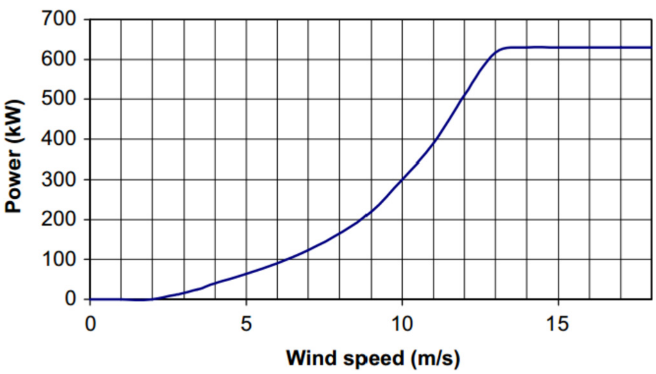

Fig. 4. Wind turbine power curve.

where N is the number of wind turbines in a given wind farm.

Then the objective function is given by:

$$Obj = \frac{Cost}{P} \tag{11}$$

where P refers to the total power of the optimal arrangement of a given wind farm.

5. Results and discussion

First of all, and in order to investigate the validity of the proposed optimization method, previous studies by Moseti et al. [5], Grady et al. [6] and Ituarte-villarreal et al. [9] are considered in this paper. For this purpose, to perform the numerical simulations, the same typical wind turbine and operating conditions are used. Table 1 presents the characteristics of wind turbine and operating conditions, and Fig. 4 shows the corresponding power curve.

Fig. 5 shows the results obtained for a given number of turbines $N = 30$ and number of cells $N_c = 100$; the optimal arrangement for our study (case c) is compared with those used in previous works [6] and [9] (cases a and b). Table 2 presents the resulting values for total power and objective function. As it can be seen, the studies by Ituarte-Villareal et al. and by Grady et al., based on the viral optimization algorithm method and on genetic algorithms, respectively, led to the same optimal configuration of wind turbine placement in a given site. The study by Ituarte-Villareal suggests a slight enhancement in total power production and in objective function of about 3% as compared to the study by Grady et al. In our study, which is based on the pseudo-random number generation method, a different arrangement of wind turbine placement optimization is obtained with a total power production value that is higher than those obtained in Itaurte-Villareal et al. and in Grady et al. Using the same wake model, the optimization method we propose appears to be more promising, because we recorded enhancements of up to 6% in total power as well as in objective function.

Also, the validity of the proposed optimization method has been investigated through the previous work due to Moseti et al. [5]. Numerical simulations have been performed for the same wind turbine (Fig. 4) and under the same operating conditions as shown in Table 1. Fig. 6 shows the results obtained for the number of turbines $N = 26$ and the number of cells $N_C = 100$, where the optimal arrangement of our study (case b) is compared with those of the study by Moseti et al. In Table 3, using the same wake model and the same number of turbines, the proposed optimization method seems to be more promising, because we were able to increase the wind farm's power output from 12352 kW to 13201 kW, and to decrease the objective function from 0.0016167 to 0.0015154, as compared to the values obtained in Moseti et al. This means that approximately 6.5% in improvement can be recorded. These performance enhancements may be attributed to the optimization method (i.e. the pseudo-random number generation approach), since the same wake model (i.e. the Jensen model) has been used in our study and in all previous works.

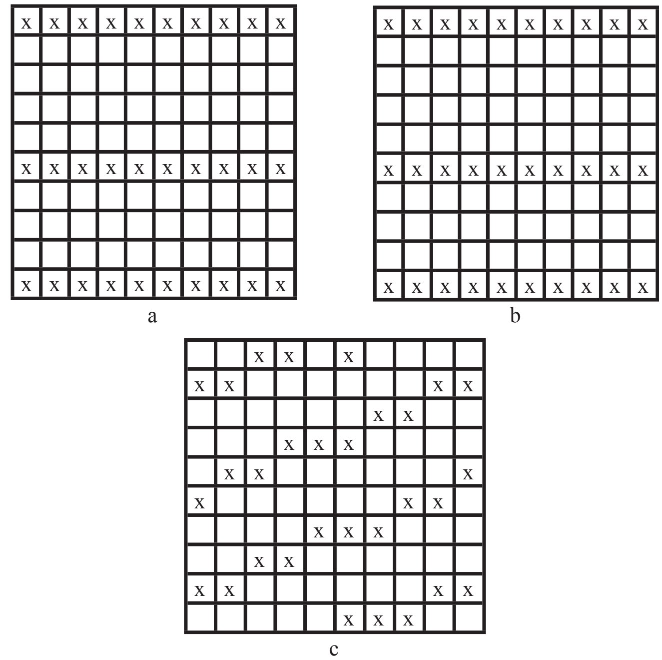

Fig. 5. Optimal configuration for 30 turbines: (a) Ituarte-Villarreal et al. [9], (b) Grady et al. [6], (c) Present study.

Table 2
Comparison of the results of the present study with those obtained previously for 30 turbines.

Case	turbine number	P(kW)	Objective function
Ituarte-Villareal et al. [9]	30	14814	0.0014910
Grady et al. [6]		14310	0.0015436
Present work		15149	0.0014580

Optimizing the production capacity of a wind farm of dimensions $5D\sqrt{N_c}x5D\sqrt{N_c}$, means seeking to increase the number of turbines in an optimal arrangement while at the same time minimizing the total amount of all costs associated with the wind power plant per unit of power. From the definition of the objective function, Eq. (11), the appropriate number of turbines for an optimal arrangement in a wind farm corresponds to the minimum value of the objective function. By varying the number of turbines in the given wind farm, the simulation results shown in Fig. 7 indicate that the optimum number of turbines to be installed can be reached and that N will be equals to 38.

6. Conclusion

The results of the simulation obtained in this study demonstrate that the proposed method of optimization could more properly

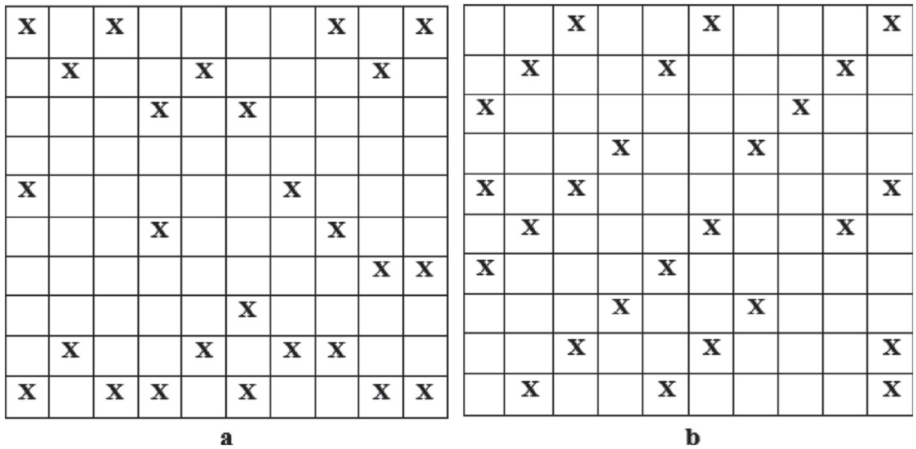

Fig. 6. Optimal configuration for 26 turbines: (a) Moseti et al. [5], (b) Present study.

Table 3
Comparison between the results of the present study with those obtained previously for 26 turbines.

Case	turbine number	*P*(kW)	Objective function
Moseti et al. [5]	26	12352	0.0016167
Present work		13201	0.0015154

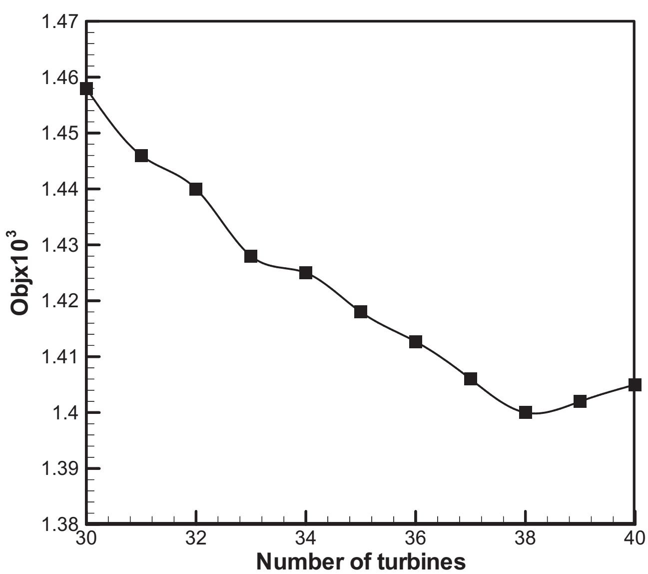

Fig. 7. Optimal number of turbines for a given farm.

predict the maximum power corresponding to the optimum placement of wind turbines in a wind farm. This method is based on the mathematical approach of generating pseudo-random numbers and adopting the Jensen linear wake model for uniform and unidirectional wind speed. A computer program has been developed to perform these calculations. We conclude that the proposed optimization method appears to be more suitable, because noticeable enhancements on total power production can be obtained in comparison with previous studies. Under the same conditions, we recorded improvements of between 3% and 6.5% over those obtained in previous works based on genetic algorithms and the viral methods. We have also shown that, for a given wind farm size, the optimal number of wind turbines can be properly determined. Lastly, with regard to future work, first, a comparative study of various optimization methods will be undertaken; in order to thoroughly investigate the reasons why by using such a method one can see performance improvements as compared to other methods. Also, more detailed wake models will be considered to properly investigate more complex problems such as the atmospheric boundary layer; ground conditions; turbulence associated with flow, and the wind turbine rotor.

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