Advanced Programming in Artificial Intelligence

Local Search and SAT

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Local Search

- Introduction
- The Satisfiability Problem
- Random SAT Problem generator
- Local Search Algorithms for SAT

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Local Search and SAT

Introduction

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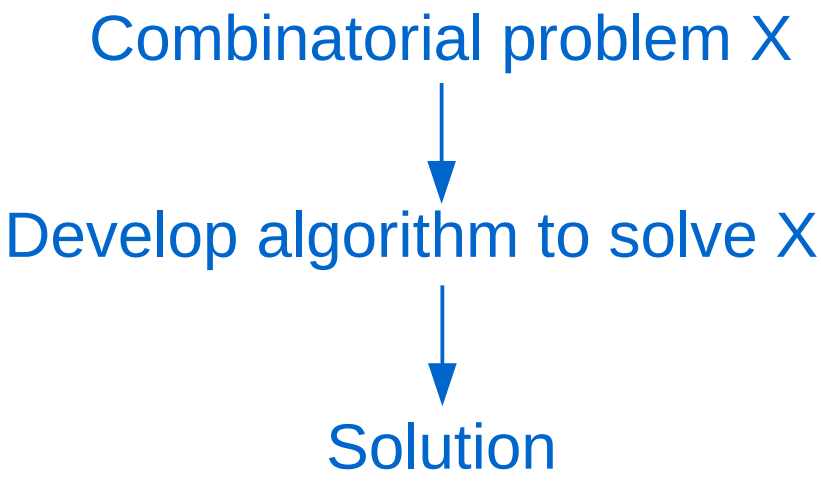
Introduction

- Real-world problems: Planning, Scheduling, Circuit design, Software verification, Automated theorem proving...



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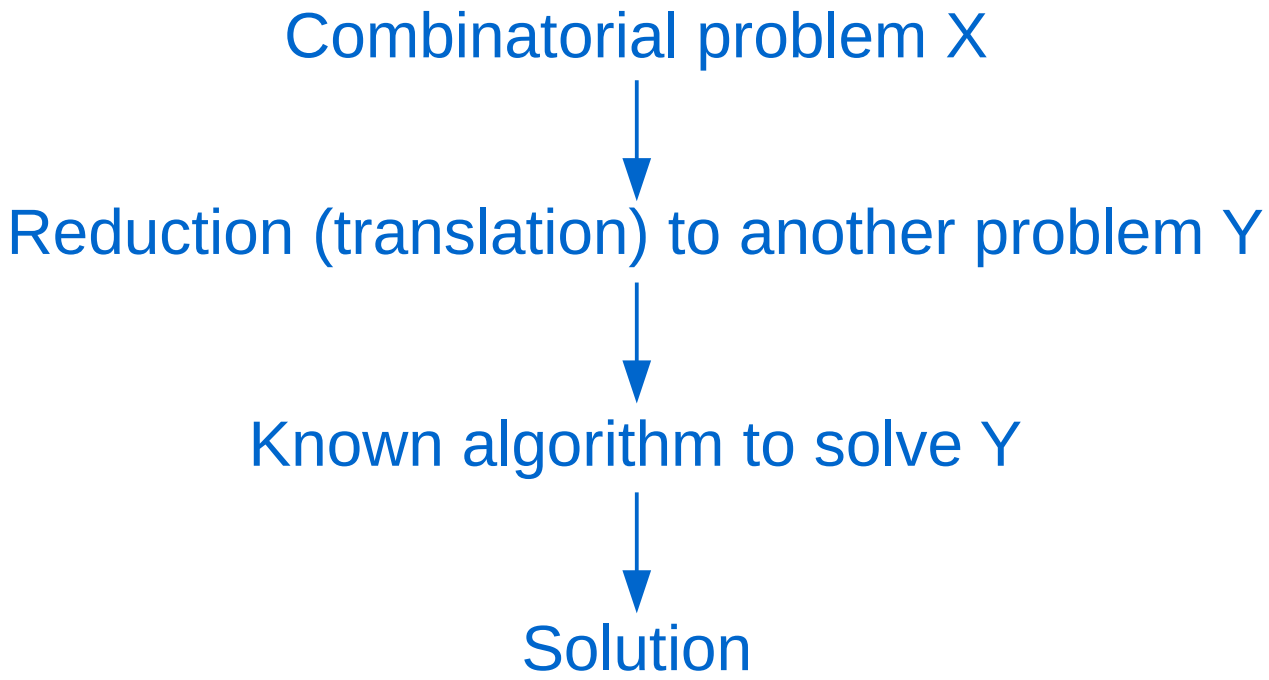
Introduction

- Real-world problems: Planning, Scheduling, Circuit design, Software verification, Automated theorem proving → SAT



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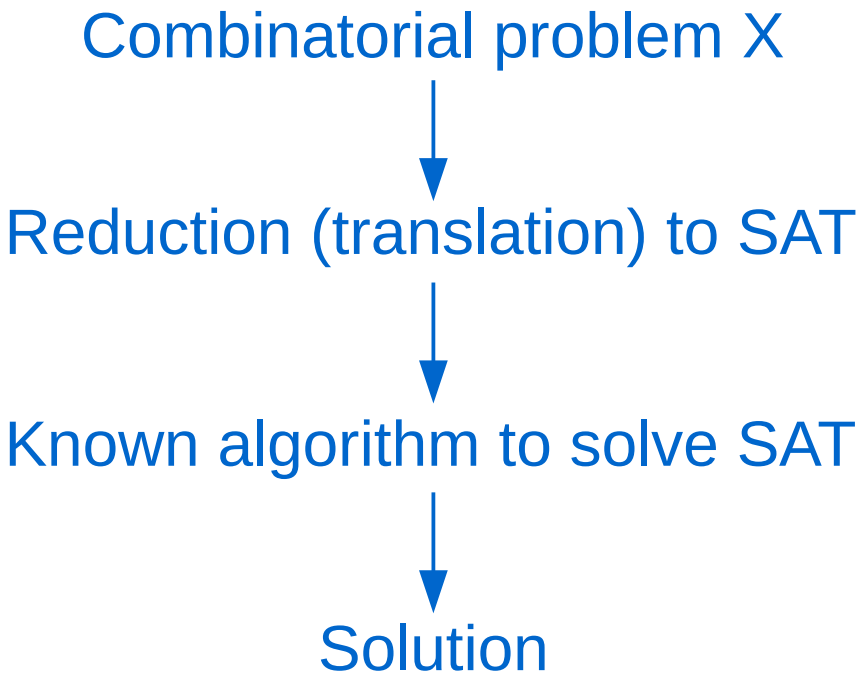
Introduction

- Real-world problems: Planning, Scheduling, Circuit design, Software verification, Automated theorem proving → SAT



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The Satisfiability Problem

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The Satisfiability Problem

The Boolean Satisfiability problem (SAT)

- SAT is the problem of determining if a propositional formula in Conjunctive Normal Form (CNF) is satisfiable
  - In other words, if there exists an interpretation which satisfies the formula

The CNF Formulas

Set of variables $X = \{x_1, x_2, ..., x_n\}$

- Literal: a positive or negative variable
  - Ex: x_i, $\neg x_i$
- Clause: a disjunction of literals
  - Ex: $x_1 \vee \neg x_2 \vee x_3$, x_2, $\neg x_2 \vee \neg x_3$...
- CNF Formula: a conjunction of clauses
  - Ex: $(x_1 \vee \neg x_2 \vee x_3) \wedge (x_2) \wedge (\neg x_2 \vee \neg x_3)$

  Note: $\vee$ = logical or, $\wedge$ = logical and

The CNF Formulas

- CNF Formula: a conjunction of clauses
  - Ex: $(x_1 \vee \neg x_2 \vee x_3) \wedge (x_2) \wedge (\neg x_2 \vee \neg x_3)$

    Note: v = logical or, ^ = logical and
  - Alternative notation: sets notation
    - Ex: $\{ \{ x_1 \vee \neg x_2 \vee x_3 \}, \{ x_2 \}, \{ \neg x_2 \vee \neg x_3 \} \}$
    - $\square$ denotes an empty clause
    - $\emptyset$ denotes an empty formula

The CNF Formulas

- Interpretation (*I*): is an assignment of truth values {0, 1} (false, true) to variables
- We say that *I* satisfies a:
  - Literal x_i iff $I(x_i) = 1$
  - Literal $\neg x_i$ iff $I(x_i) = 0$
  - Clause *c* iff *I* satisfies at least on literal of *c*
  - CNF Formula *F* iff *I* satisfies all the clauses of *F*
- A CNF Formula *F* is satisfiable iff exist an interpretation that satisfies *F*. Otherwise *F* is unsatisfiable.

The CNF Formulas

- Examples
  - $\{ \square , \{ x_2 \}, \{ \neg x_2 \vee \neg x_3 \} \}$ is unsatisfiable
    - Every formula with an empty clause is trivially unsatisfiable
  - $\{ \varnothing \}$ is satisfiable
    - The empty formula is satisfiable
  - $\{ \{ x_1 \vee \neg x_2 \vee x_3 \}, \{ x_2 \}, \{ \neg x_2 \vee \neg x_3 \} \}$?

The CNF Formulas

- Examples
  - $\{ \square , \{ x_2 \}, \{ \neg x_2 \vee \neg x_3 \} \}$ is unsatisfiable
    - Every formula with an empty clause is trivially unsatisfiable
  - $\{ \varnothing \}$ is satisfiable
    - The empty formula is satisfiable
  - $\{ \{ x_1 \vee \neg x_2 \vee x_3 \}, \{ x_2 \}, \{ \neg x_2 \vee \neg x_3 \} \}$ is satisfiable
    - $x_1 = 1, x_2 = 1, x_3 = 0$ is an interpretation that satisfies the formula
    - There are more interpretations ?

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Random SAT Problem generator

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CNF format

- Comments: Line starting with character 'c'
  - Ex: c Random CNF formula
- Problem Line: p cnf *num_variables num_clauses*
  - Ex: p cnf 4 10
- Clauses: List of clauses, one per line
  - Clause: sequence of integers between *num_variables* and *-num_variables* ending with 0 on the same line
    - Positive number → positive literal of the variable
    - Negative number → negative literal of the variable
    - Ex: -3 5 24 -18 0

CNF format

SAT input format example

```
c
c A random CNF formula
c
p cnf 3 4
1 -2 0
-1 2 -3 0
-3 2 0
1 3 0
```

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CNF format

SAT input format example

```
c
c A random CNF formula
c
p cnf 3 4
1 -2 0
-1 2 -3 0
-3 2 0
1 3 0
```

Is there a solution?
1 = true
2 = true
3 = false

Solver output solution:
SATISFIABLE
1 2 -3 0

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Random CNF generator

- Example of a Random k-SAT generator
  - Random generator of CNF problems
  - *K* is the length of the clauses
  - Parameters of the generator
    - Number of variables
    - Number of clauses
    - Length of the clauses (K)
    - Random seed

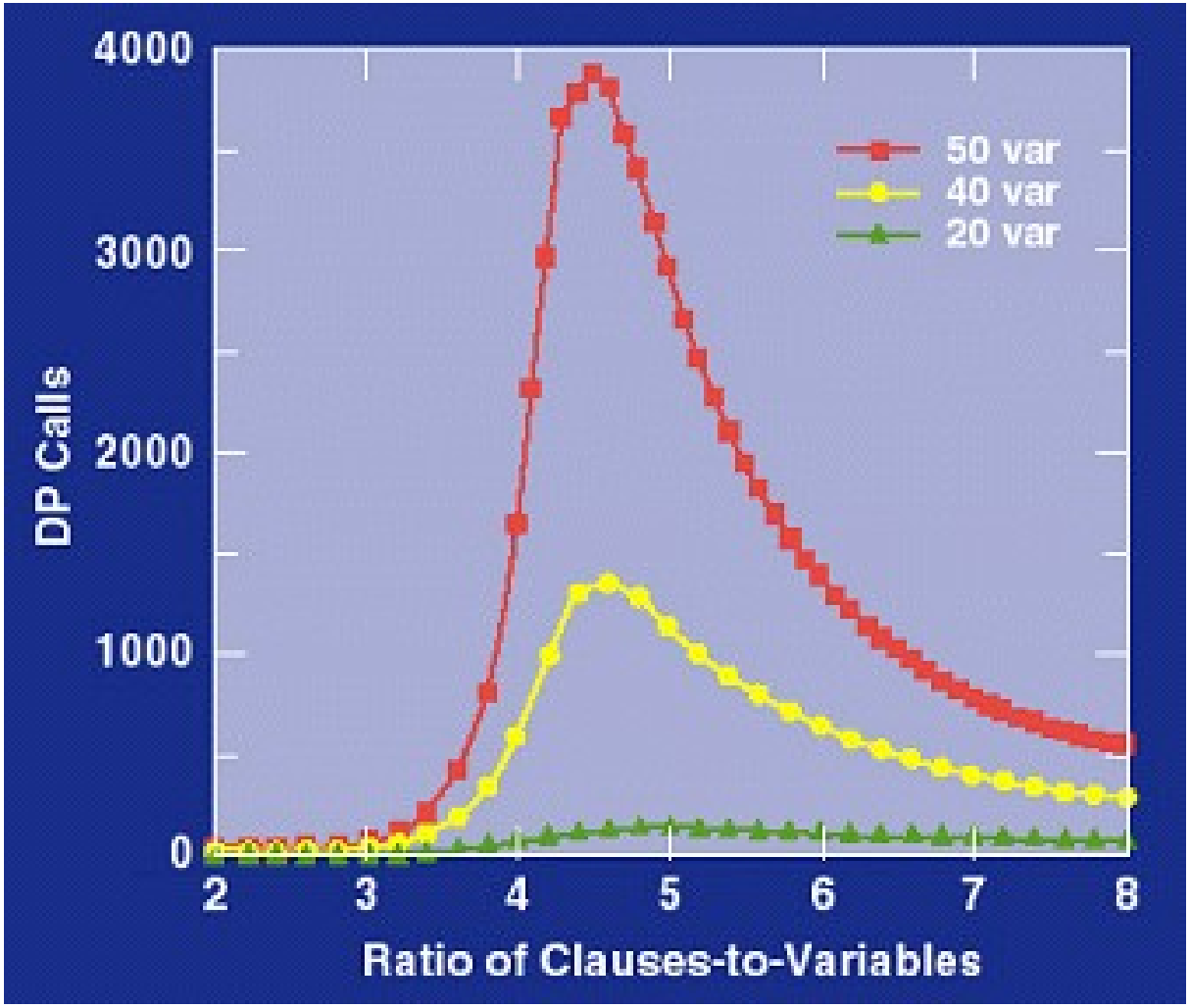
Random 3-SAT Hardness

Easy-hard-easy pattern

The hardness depends on:

$$\frac{\#\text{clauses}}{\#\text{variables}}$$

(peak around 4.25)



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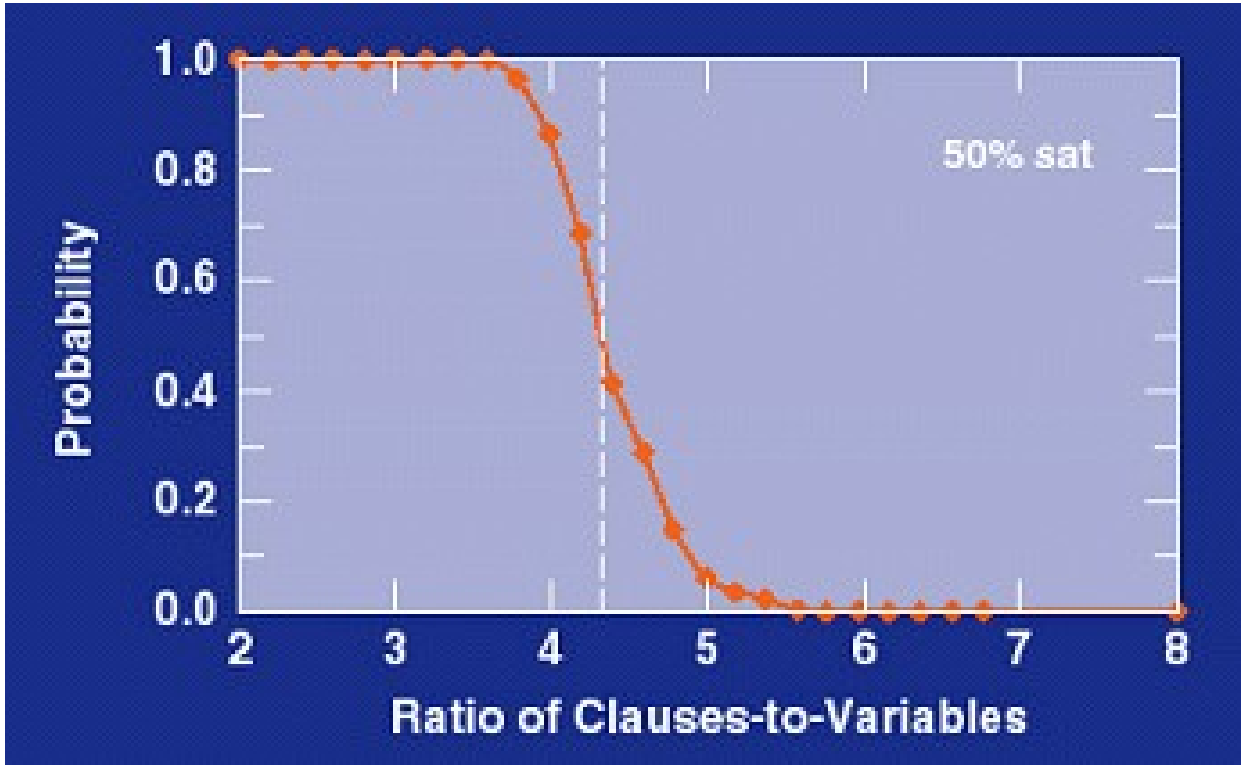
Random 3-SAT Probability

Phase transition

From satisfiable formulas
to unsatisfiable formulas

(peak around 4.25)



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Experimentation

Reproduce the two previous plots

- Generate the benchmarks (random generator)
- Run the solver with all the benchmarks
  - Send the output to a file (for all the benchmarks)
- Parse the output needed from the results
- Summarize the results
  - Table
  - Plot
  - ...

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Local Search and SAT

Local Search algorithms for SAT

Local Search algorithms for SAT

- Do not follow a systematic method to explore all the search space
  - Not conscious of the explored space
- When they finish...
  - they have found a solution, or
  - they reached a limit of resources
    - In this case, they can not say anything about the solution
- Examples
  - GSAT
  - WalkSat

GSAT

```
F ← input CNF formula
for i = 1 to max_tries do
   I ← random interpretation for F
   for j = 1 to max_flips do
      if I satisfies F then
         return I
      endif
      S ← set of variables s.t. flipping them
         increments more the total number of satisfied
         clauses
      p ← a variable of S
      I ← I with the value of p flipped
   endfor
endfor
return "No solution found"
```

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GSAT

- GSAT can not be used to proof that a formula is unsatisfiable
  - Incomplete procedure
    - If GSAT gets an interpretation, then the formula is SAT
    - If GSAT can not find an interpretation, then we do not know if the formula is SAT or UNSAT
- GSAT is able to solve hard instances with more than 2.000 variables
- Different runs with the same parameters → different solutions
- Problems with local minimums

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GSAT: Escaping from local minimums

- Restarts
  - Restarts the search with a new random interpretation after a given number of flips
- Random walk (GWSAT)
  - with probability ω:
    - Flips a variable from a random unsatisfied clause
  - with probability 1 – ω:
    - Does like GSAT
- Tabu search
  - Some recent flips that take to visited solutions are forbidden
  - GSAT forces to explore new regions in flat areas

Walksat

```
F ← input CNF formula
for i = 1 to max_tries do
    I ← random interpretation for F
    for j = 1 to max_flips do
        if I satisfies F then
            return I
        endif
        C ← a clause of F not satisfied by I
        S ← set of variables that appear in C
        b ← min({broken(p,F,I) | p in S})
        if b > 0 and with probability ω then
            p ← a variable of S
        else
            p ← a variable of S s.t. broken(p,F,I) = b
        endif
        I ← I with the value of p flipped
    endfor
endfor
return “No solution found”
```

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GSAT and Walksat implementations

- There are several implementations of GSAT and Walksat
  - The original: Walksat/GSAT Home Page
    - GSAT is no longer maintained
  - UBCSAT: UBCSAT SATLib
    - Implements a very large sample of solvers
    - Current version 1.1.0

UBCSAT

- ./ubcsat -alg gsat -solve -i test.cnf
  - -alg <name>: name of the algorithm (-ha for the list)
  - -solve: stops when it finds a solution (see -target)
  - -i <file_name>: input CNF instance
  - -runs INT: number of independent tries (default 1)
  - -cutoff INT: number of flips per run (default 100.000)
  - -target INT: target solution quality (default 0)
  - -seed INT: initial random seed
  - -wp <[0,1]>: random walk probability (default 0.5)

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Bibliography

- “Artificial Intelligence: A Modern Approach”. S. Russell and P. Norvig. Prentice Hall.
- Wikipedia and several Internet sources.

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