TÉCNICO LISBOA

Unmanned Aerial Vehicles

MEAer

2022/2023 - First Semester

Modeling and Identification of the Parrot AR.Drone

Laboratory Guide

September 2022

1 Introduction

1.1 Objectives

The following topics are addressed in this laboratory:

1. Kinematic and dynamic modeling of a quadrotor vehicle.
2. Linearization of the quadrotor system dynamics about the hovering condition.
3. Analysis of inner and outer control loops for the height and vertical speed.
4. Identification of the height closed-loop control system.
5. Identification of the pitch angle closed-loop control system.

1.2 Organization and timeline

This guide consists of a set of questions that explore different aspects related to modeling and identification of the Parrot AR.Drone.

There are two kinds of questions: theoretical questions, marked as (T), and laboratory questions, marked as (L). As a guideline, the theoretical questions should be solved outside the laboratory sessions and the lab time should be used wisely to collect experimental data for subsequent analysis.

A single-column report in pdf format with **no more than 15 pages** together with the matlab script files (`.m`), containing the code developed to answer the questions, must be submitted through fenix in the designated dates (check the course's website). Please use the cover page available in the course's website (or similar with the same information) as front page.

1.3 Academic ethics code

All members of the academic community of the University of Lisbon (faculty, researchers, staff members, students, and visitors) are required to uphold high ethical standards. Hence, the report submitted by each group of students must be original and correspond to their actual work.

2 Modeling

Let $\{I\}$ denote a local inertial reference frame aligned with the North East Down (NED) orientation and let $\{B\}$ denote the body-fixed reference frame with origin at the vehicle's center of mass. Under these assumptions, the vehicle's kinematics can be written as

$$\begin{cases} \dot{\mathbf{p}} = R(\boldsymbol{\lambda})\mathbf{v} \\ \dot{\boldsymbol{\lambda}} = Q(\boldsymbol{\lambda})\boldsymbol{\omega} \end{cases},$$

where

$$\mathbf{p} = \begin{bmatrix} x & y & z \end{bmatrix}^T \in \mathbb{R}^3$$

is the position of $\{B\}$ with respect to $\{I\}$,

$$\mathbf{v} = \begin{bmatrix} u & v & w \end{bmatrix}^T \in \mathbb{R}^3$$

is the linear velocity of $\{B\}$ with respect to $\{I\}$, expressed in $\{B\}$,

$$\boldsymbol{\lambda} = \begin{bmatrix} \phi & \theta & \psi \end{bmatrix}^T$$

is the vector of Z-Y-X Euler angles, where $\phi \in \mathbb{R}$, $\theta \in]-\pi/2, \pi/2[$, and $\psi \in \mathbb{R}$ are the roll, pitch, and yaw Euler angles, respectively,

$$\boldsymbol{\omega} = \begin{bmatrix} p & q & r \end{bmatrix}^T \in \mathbb{R}^3$$

is the angular velocity of $\{B\}$ with respect to $\{I\}$, expressed in $\{B\}$,

$$\begin{aligned} R(\boldsymbol{\lambda}) &= R_Z(\psi)R_Y(\theta)R_X(\phi) = \\ &= \begin{bmatrix} \cos\theta\cos\psi & \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi \\ \cos\theta\sin\psi & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi \\ -\sin\theta & \sin\phi\cos\theta & \cos\phi\cos\theta \end{bmatrix} \in SO(3) \end{aligned}$$

is the rotation matrix from $\{B\}$ to $\{I\}$, and

$$Q(\boldsymbol{\lambda}) = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \frac{\sin\phi}{\cos\theta} & \frac{\cos\phi}{\cos\theta} \end{bmatrix} \in \mathbb{R}^{3\times3}$$

is the angular kinematics Jabobian.

The dynamics can be described by

$$\begin{cases} m\dot{\mathbf{v}} = -m\,S(\boldsymbol{\omega})\mathbf{v} + \mathbf{f} \\ J\dot{\boldsymbol{\omega}} = -S(\boldsymbol{\omega})J\boldsymbol{\omega} + \mathbf{n} \end{cases}, \tag{1}$$

where $S : \mathbb{R}^3 \mapsto \mathbb{R}^{3\times3}$ is the skew-symmetric operator defined such that $S(\mathbf{a})\mathbf{b} = \mathbf{a} \times \mathbf{b}$, $m \in \mathbb{R}^+$ is the mass, $J \in \mathbb{R}^{3\times3}$ is the inertia tensor matrix given by

$$J = \begin{bmatrix} J_{xx} & J_{xy} & J_{xz} \\ J_{xy} & J_{yy} & J_{yz} \\ J_{xz} & J_{yz} & J_{zz} \end{bmatrix},$$

and

$$\mathbf{f} = \begin{bmatrix} f_x & f_y & f_z \end{bmatrix}^T \in \mathbb{R}^3 \quad \text{and} \quad \mathbf{n} = \begin{bmatrix} n_x & n_y & n_z \end{bmatrix}^T \in \mathbb{R}^3,$$

are the external forces and torques expressed in $\{B\}$, respectively.

The Parrot AR.Drone is based on a classic quadrotor design, with four rotors mounted symmetrically along two orthogonal axes, as depicted in Fig. 1, where the individual thrusts are denoted by $f_i \in \mathbb{R}$, $i \in \{1, \ldots, 4\}$. The corresponding individual torques are denoted by $n_i = cf_i \in \mathbb{R}$, $i \in \{1, \ldots, 4\}$, where $c \in \mathbb{R}$ is a scalar constant. In order to control all three angular degrees of freedom, two pairs of rotors are counter-rotating, as shown in the figure.

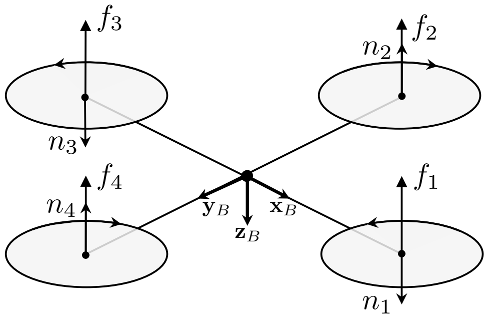

Figure 1: Simplified model of the Parrot AR.Drone

Define the vector of individual rotor thrusts as

$$\mathbf{f}_T := \begin{bmatrix} f_1 & f_2 & f_3 & f_4 \end{bmatrix}^T \in \mathbb{R}^4$$

and the gravitational force expressed in $\{I\}$ as $\mathbf{f}_g = mg\mathbf{e}_3 \in \mathbb{R}^3$, where $\mathbf{e}_3 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$.

2.1. **(T)** Show that $\mathbf{f}$ and $\mathbf{n}$ in (1) can be written as functions of $\mathbf{f}_T$ and $\mathbf{f}_g$, such that

$$\mathbf{f} = M\mathbf{f}_g + N\mathbf{f}_T \tag{2}$$

and

$$\mathbf{n} = P\mathbf{f}_T. \tag{3}$$

Determine expressions for the matrices M, N, and P.

2.2. **(T)** The body frame $\{B\}$ of the Parrot AR.Drone has an orientation that is different from the one shown in Fig. 1, with the $\mathbf{x}_B$ axis midway between the arms of the first and second rotors (45^o angular displacement in the horizontal plane). Do the expressions for $\mathbf{f}$ and $\mathbf{n}$ change because of this redefinition of $\{B\}$? If so, derive the new expression(s).

2.3. **(T)** Define a new control input

$$\mathbf{u} = \begin{bmatrix} T \\ \mathbf{n} \end{bmatrix} \in \mathbb{R}^4,$$

where the scalar T is given by $T = \sum_{i=1}^{4} f_i$ and $\mathbf{n} \in \mathbb{R}^3$ is given by (3), yielding

$$L\mathbf{f}_T = \mathbf{u},$$

where $L \in \mathbb{R}^{4\times 4}$. Discuss the usefulness of considering this linear input transformation and write an explicit expression for the inverse transformation from $\mathbf{u}$ to $\mathbf{f}_T$.

2.4. **(T)** Define a new position vector ${}^B\mathbf{p} = R^T\mathbf{p}$ and let $\mathbf{x} = \begin{bmatrix} {}^B\mathbf{p}^T & \mathbf{v}^T & \boldsymbol{\lambda}^T & \boldsymbol{\omega}^T \end{bmatrix}^T \in \mathbb{R}^{12}$ and $\mathbf{x}_1 = \begin{bmatrix} \mathbf{p}^T & \dot{\mathbf{p}}^T & \boldsymbol{\lambda}^T & \boldsymbol{\omega}^T \end{bmatrix}^T \in \mathbb{R}^{12}$ denote two alternative state vectors for the system that describes a quadrotor. The nonlinear dynamics take the form $\dot{\mathbf{x}} = \mathbf{g}(\mathbf{x}, \mathbf{u})$ and $\dot{\mathbf{x}}_1 = \mathbf{g}_1(\mathbf{x}_1, \mathbf{u})$, respectively. Determine the expressions for $\mathbf{g}(\mathbf{x}, \mathbf{u})$ and $\mathbf{g}_1(\mathbf{x}_1, \mathbf{u})$. *Suggestion:* to derive the kinematics equation for ${}^B\dot{\mathbf{p}}$, notice that $S(.)^T = -S(.)$ and $R(\mathbf{a} \times \mathbf{b}) = (R\mathbf{a}) \times (R\mathbf{b})$.

2.5. **(T)** Consider that, at equilibrium, the quadrotor is hovering at a given position $\mathbf{p}_0$ with an arbitrary but constant yaw angle ψ_0, which together fully describe the equilibrium point for the system $\dot{\mathbf{x}} = \mathbf{g}(\mathbf{x}, \mathbf{u})$. Let $\mathbf{x}_0 = \begin{bmatrix} {}^B\mathbf{p}_0^T & \mathbf{v}_0^T & \boldsymbol{\lambda}_0^T & \boldsymbol{\omega}_0^T \end{bmatrix}^T$ denote the equilibrium state and $\mathbf{u}_0$ the corresponding equilibrium input. Determine $\mathbf{x}_0$ and $\mathbf{u}_0$ and give a physical interpretation for the result. *Suggestion:* Start by determining the equilibrium values for T_0, ϕ_0, and θ_0.

2.6. **(T)** Define the error variables $\tilde{\mathbf{x}} = \begin{bmatrix} \tilde{\mathbf{p}}^T & \tilde{\mathbf{v}}^T & \tilde{\boldsymbol{\lambda}}^T & \tilde{\boldsymbol{\omega}}^T \end{bmatrix}^T$, where

$$\begin{aligned}
\tilde{\mathbf{p}} &= {}^B\mathbf{p} - R^T\mathbf{p}_0 = R^T(\mathbf{p} - \mathbf{p}_0), \\
\tilde{\mathbf{v}} &= \mathbf{v} - \mathbf{v}_0, \\
\tilde{\boldsymbol{\lambda}} &= \boldsymbol{\lambda} - \boldsymbol{\lambda}_0, \\
\tilde{\boldsymbol{\omega}} &= \boldsymbol{\omega} - \boldsymbol{\omega}_0,
\end{aligned}$$

and

$$\tilde{\mathbf{u}} = \mathbf{u} - \mathbf{u}_0.$$

Show that the linearized system dynamics can be written as

$$\dot{\tilde{\mathbf{x}}} = \mathbf{A}\tilde{\mathbf{x}} + \mathbf{B}\tilde{\mathbf{u}}$$

and determine expressions for $\mathbf{A} \in \mathbb{R}^{12\times 12}$ and $\mathbf{B} \in \mathbb{R}^{12\times 4}$. *Suggestion:* define $\mathbf{A}$ and $\mathbf{B}$ using block matrices. Note that $\tilde{\mathbf{p}}$ is not simply given by the diference between $\mathbf{p}$ and $\mathbf{p}_0$.

2.7. **(T)** Consider the individual elements of $\tilde{\mathbf{x}}$ and $\tilde{\mathbf{u}}$ as specified in

$$\tilde{\mathbf{x}} = \begin{bmatrix} \tilde{x} & \tilde{y} & \tilde{z} & \tilde{\phi} & \tilde{\theta} & \tilde{\psi} & \tilde{u} & \tilde{v} & \tilde{w} & \tilde{p} & \tilde{q} & \tilde{r} \end{bmatrix}^T$$

and

$$\tilde{\mathbf{u}} = \begin{bmatrix} \tilde{T} & \tilde{n}_x & \tilde{n}_y & \tilde{n}_z \end{bmatrix}^T,$$

respectively. Denote by

$$\mathbf{X}(s) = \begin{bmatrix} X(s) & Y(s) & Z(s) & \Phi(s) & \Theta(s) & \Psi(s) & U(s) & V(s) & W(s) & P(s) & Q(s) & R(s) \end{bmatrix}^T$$

and

$$\mathbf{U}(s) = \begin{bmatrix} T(s) & N_x(s) & N_y(s) & N_z(s) \end{bmatrix}^T$$

the Laplace transforms of $\tilde{\mathbf{x}}$ and $\tilde{\mathbf{u}}$, respectively. Determine the transfer functions

$$G_\phi(s) = \frac{\Phi(s)}{N_x(s)}, \quad G_\theta(s) = \frac{\Theta(s)}{N_y(s)}, \quad G_\psi(s) = \frac{\Psi(s)}{N_z(s)},$$

$$G_x(s) = \frac{X(s)}{N_y(s)}, \quad G_y(s) = \frac{Y(s)}{N_x(s)}, \quad \text{and} \quad G_z(s) = \frac{Z(s)}{T(s)}.$$

Note: to obtain the transfer functions, assume that the moment of inertia J is diagonal.

2.8. **(T)** Relate $G_\phi(s)$ with $G_y(s)$ and $G_\theta(s)$ with $G_x(s)$. Give a physical interpretation for the obtained result and discuss the effect of considering different equilibria ψ_0 for the yaw angle.

2.9. **(T)** For the sake of simplicity, let $h = -z$ denote the height of the quadrotor above the ground. Consider the inner-outer loop control scheme for the height depicted in Fig. 2 and the alternative control scheme shown in Fig. 3. In both diagrams, h_{ref} denotes the reference height.

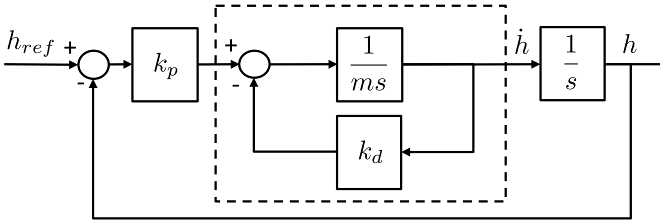

Figure 2: Height control scheme 1

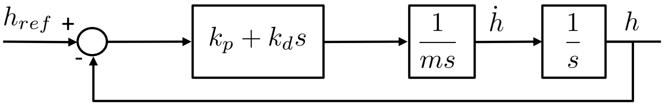

Figure 3: Height control scheme 2

Determine the closed-loop transfer functions for both control schemes and discuss the differences and possible advantages and disadvantages of each scheme.

2.10. **(T)** Considering the controller from Fig. 2, sketch the root-locus with respect to $k_p > 0$, assuming that $k_d > 0$ is fixed but arbitrary. Discuss how changing the control gains k_p and k_d affects the step response of the closed-loop system.

3 Identification of the closed-loop height dynamics

The main goal of this section is to identify the closed-loop height dynamics first in simulation and then experimentally.

Simulations

The ARDrone Simulink Development Kit provides the means for both simulation of the Parrot Ar.Drone and experiments. To begin simulations, run the script `start_here`. Choose the option `(1) Simulation` and then the option `(2) Hover: Vehicle is held at constant position`. This will open the simulink model `ARDroneHoverSim.slx`.

First get familiarized with the several blocks, options, and scopes in `ARDroneHoverSim.slx`. In particular, note that the implemented model is based on a linearization about the hovering condition, similar to the one derived in Section 2.

To obtain answers for the following questions, it is advisable to save `ARDroneHoverSim.slx` with a different name, e.g. `ARDroneHoverSimHeight.slx`. This new model should be modified in order to perform the required simulations for the identification of the height dynamics.

3.1. **(L)** Modify the Simulink model `ARDroneHoverSimHeight.slx` to obtain step responses using four different values for the proportional height controller gain, as specified in Table 1. To run a simulation, and more importantly an experiment, bear in mind the following:

- Before pressing the run button, always check that the `Fly` switch is in the `land` mode and the `Commands` switch is in the `commands disabled` mode.
- Press the run button, select `take off` and allow the vehicle to stabilize at the reference height of 0.75 m.
- Select `commands enabled` (keeping the same reference height).
- After 2 seconds or more, apply the cumulative step (`height ref` = 0.75 + cumulative step).

Gain	Cumulative step
0.5	1.0 m
1.0	0.8 m
2.0	0.3 m
3.0	0.2 m

Table 1: Parameters for height step response experiments

For each value of the gain, plot the simulated responses. Comment on the reason for considering steps of decreasing magnitude as the gain increases.

Experiments

Before beginning the experiments, connect the computer via Wi-Fi to the Parrot Ar.Drone. To begin experiments, run the script `start_here`. Choose the option `(2) Wi-fi Control` and then the option `(1) Hover: Vehicle is held at constant position`. This will open the simulink model `ARDroneHover.slx`. Save the model with a different name, e.g. `ARDroneHoverHeight.slx` and work with it.

3.2. **(L)** As in 3.1, modify `ARDroneHoverHeight.slx` to obtain step responses, considering the height references described in Table 1. For each value of the gain, plot the real responses together with the simulated responses.

3.3. **(L)** Using the system identification toolbox of Matlab, obtain the closed-loop transfer functions that best approximate the experimental results considering a transfer function corresponding to the control scheme depicted in Fig. 2. Discuss the results.

3.4. **(L)** To further analyse the system, for each value of k_p, plot the pole and zero map for the closed-loop system and compare with the root-locus plot from Question 2.10. Discuss whether or not the theoretical, simulation, and experimental results are consistent and indicate possible reasons for the observed differences.

Suggestion: Consider the effect of having a delay in the feedback loop and how it can be incorporated in a root-locus plot by means of a Padé approximation.

4 Identification of the closed-loop pitch dynamics

The main goal of this section is to experimentally identify the closed-loop pitch dynamics. The procedure used to conduct the experiments is similar to the previous one. Again, it is advisable to save the simulink model with a different name, e.g. `ARDroneHoverPitch.slx`.

4.1. **(L)** Modify the Simulink model `ARDroneHoverPitch.slx` to obtain the experimental amplitude Bode diagram. Use sinusoidal input signals with frequency $\omega \in [1, 20]$ rad/s and maximum amplitude 0.2 for the lower frequencies.

4.2. **(L)** Based on the amplitudes of the responses obtained for each of the tested frequencies, determine the approximate parameters for a second order linear time invariant system with a minimum-phase zero that describes the closed-loop pitch dynamics. Discuss the results.

Suggestion: Use the Matlab function `nlinfit` to find the coefficients of the transfer function that best describes the acquired data.

4.3. **(L)** Obtain the response of the system to a pitch reference given by

$$\theta_{ref}(t) = \begin{cases} 0, & 0 \leq t < 15s \\ 0.2, & 15 \leq t < 18s \\ 0, & t \geq 18s \end{cases}.$$

Discuss the results, taking into account the step and frequency responses of the system.