Circuit Theory and Electronics Fundamentals

T2 Laboratory Report

Group 34

José Bento, Nº 95815
Thomas Childs, Nº 95847
Luís Pacheco, Nº 96425

Aerospace Engineering (MEAer), Técnico, University of Lisbon

April 5, 2021

Contents

1 Introduction

The objective of this laboratory assignment is to study a circuit containing a capacitor and a sinusoidal voltage source v_s.The value of this sinusoidal voltage source varies in time according to the following equations:

$$v_s(t) = V_s u(-t) + sin(2\pi f t)u(t) \tag{1}$$

with

$$u(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1, & \text{if } t \geq 0 \end{cases} \tag{2}$$

In this circuit there is also a linearly dependent voltage and current source. The circuit also contains 7 resistors.

The nodes of the circuit were numbered arbitrarily (from V_0 to V_7), and it was considered that *node 0* was the ground node. The voltage-controlled current source I_b has a linear dependence on Voltage V_b, of constant K_b. The voltage V_b is the voltage drop at the ends of resistor R_3. The current-controlled voltage source V_d has a linear dependece on current I_d, of constant K_d. The control current I_d is the current that passes through the resistor R_6. The circuit can be seen in **Figure 1**.

These values for the capacitance, resistors and the constants for the dependent sources were obtained using the Python script provided by the Professor and using the number 95815 as the seed.The seed number can be altered in the top Makefile (line 9). By doing so, all figures and tables will be updated acording to the new values.

In Section 2, a theoretical analysis of the circuit is presented. Here the circuit is analised for $t < 0$ using the nodal analysis and the equivalent resistence R_{eq} as seen from the capacitor terminals is determined. In this section both the natural and forced solutions for V_6 are also determined as well as the frequency responses for V_c, V_s and V_6. In Section 3, the circuit is analysed by simulation using the program Ngspice. An operating point analysis is used to analyse the circuit when $t < 0$ and another one to determine the time constant.A transient analysis is used to determine the natural and forced responses on node 6. A frequency analysis is also performed on node 6. The conclusions of this study are outlined in Section 4, where the theoretical results obtained in Section 2 are compared to the simulation results obtained in Section 3.

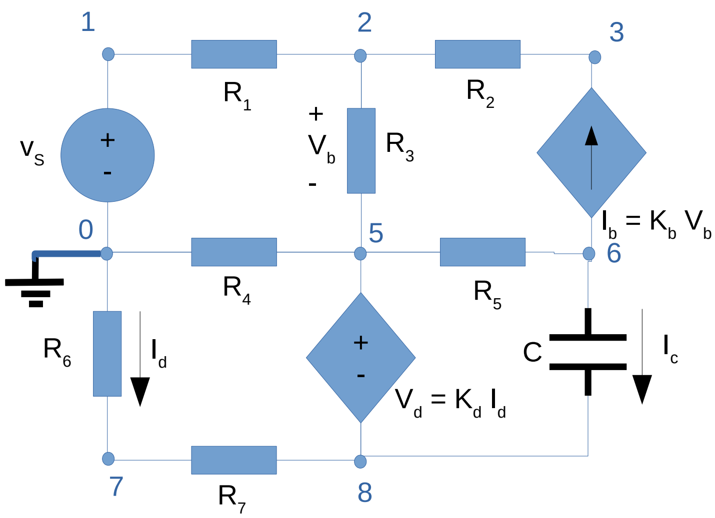

Figure 1: Circuit in study

2 Theoretical Analysis

In this section, the circuit shown in **Figure 2** is analysed theoretically.

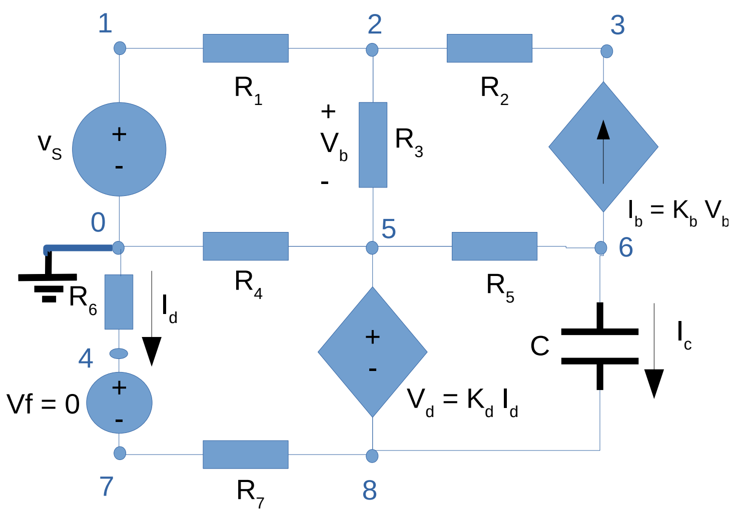

Figure 2: Diagram of the circuit considered for the computations and simulations

2.1 Analysis for $t < 0$

The nodal method is aplied to the circuit in order to determine the voltage in all nodes and the current on all branches . The nodal method aplies KVL and for $t < 0$ no current passes through the capacitor, and therefore this component behaves like an open circuit.

$$V_0 = 0 \tag{3}$$

$$V_4 = V_7 \tag{4}$$

$$V_5 - V_8 = K_d \frac{V_0 - V_4}{R_6} \tag{5}$$

$$V_1 - V_0 = V_s \tag{6}$$

$$\frac{V_2 - V_1}{R_1} + \frac{V_2 - V_5}{R_3} + \frac{V_2 - V_3}{R_2} = 0 \tag{7}$$

$$\frac{V_3 - V_2}{R_2} - K_b(V_2 - V_5) = 0 \tag{8}$$

$$\frac{V_5 - V_2}{R_3} + \frac{V_5 - V_0}{R_4} + \frac{V_5 - V_6}{R_5} + \frac{V_8 - V_7}{R_7} = 0 \tag{9}$$

$$K_b(V_2 - V_5) + \frac{V_6 - V_5}{R_5} = 0 \tag{10}$$

$$\frac{V_4 - V_0}{R_6} + \frac{V_7 - V_8}{R_7} = 0 \tag{11}$$

Name	Node method
@c	0
@Gb	-2.915670e-04
@r1	2.780492123128611e-04
@r2	2.915670410357471e-04
@r3	-1.351782872288611e-05
@r4	-1.226887222704183e-03
@r5	-2.915670410357471e-04
@r6	9.488380103913221e-04
@r7	9.488380103913222e-04
v(1)	5.24359648479
v(2)	4.95273954039
v(3)	4.36746937599
v(4)	-1.96340246583
v(5)	4.99411230240
v(6)	5.90453662186
v(7)	-1.96340246583
v(8)	-2.92676735760

Table 1: A variable that starts with "@ " is of type *current* and expressed in milliampere (mA); all the other variables that start with a "V" are of the type *voltage* and expressed in Volt (V).

Name	Simulation
@c[i]	0.000000e+00
@gb[i]	-2.91567e-04
@r1[i]	2.780494e-04
@r2[i]	2.915672e-04
@r3[i]	-1.35178e-05
@r4[i]	-1.22689e-03
@r5[i]	-2.91567e-04
@r6[i]	9.488377e-04
@r7[i]	9.488377e-04
v(1)	5.243596e+00
v(2)	4.952739e+00
v(3)	4.367468e+00
v(4)	-1.96340e+00
v(5)	4.994112e+00
v(6)	5.904536e+00
v(7)	-1.96340e+00
v(8)	-2.92677e+00

Table 2: Step 1: Operating point for $t < 0$. A variable preceded by @ is of type *current* and expressed in miliAmpere; other variables are of type *voltage* and expressed in Volt.

2.2 Equivalent resistor as seen from the capacitor terminals

To compute the equivalent resistance as seen by C the independent source V_c needs to be switched off. We do this by replacing it with a short circuit ($V_s = 0$). In addition, due to the presence of dependent sources, we also needed to replace the capacitor with a voltage source $V_x = V_6 - V_8$. We use the V_6 and V_8 from the previous section beacause the voltage drop at the terminals of the capacitor needs to be a continuous function (there can not be an energy discontinuity in the capacitor). With this in mind, a nodal analysis is performed in order to determine the current I_x that is supplied by V_x. With these values we can determine R_{eq} ($R_{eq} = V_x/I_x$). All these procedures were required in order to determine the time constant τ ($\tau = R_{eq} * C$). The time constant is crucial to determine the natural and forced solutions for V_6, which will be done in the next subsections. The equations considered for these calculations were 3, 4, 5, 6, 7, 8 and the following:

$$\frac{V_1 - V_2}{R_1} + \frac{V_0 - V_4}{R_6} + \frac{V_0 - V_5}{R_4} = 0 \tag{12}$$

$$K_b(V_2 - V_5) + \frac{V_6 - V_5}{R_5} + I_x = 0 \tag{13}$$

$$\frac{V_4 - V_0}{R_6} + \frac{V_7 - V_8}{R_7} = 0 \tag{14}$$

$$V_x = V_6 - V_8 \tag{15}$$

Name	Theoretical values
@Gb	0.00000000000
@r1	0
@r2	0
@r3	0
@r4	0
@r5	-2.828260531660058e-03
@r6	0
@r7	0
v(1)	0.00000000000
v(2)	0.00000000000
v(3)	0.00000000000
v(4)	0.00000000000
v(5)	0.00000000000
v(6)	8.83130397945
v(7)	0.00000000000
v(8)	0.00000000000
Ix	-0.00282826053
Vx	8.83130397945
Req	-3.122521e+03
τ	-3.181599e-03

Table 3: A variable that starts with a "V" is of type *voltage* and expressed in Volt (V). The variable R_{eq} is expressed in Ohm and the variable τ is expressed in seconds

Name	Simulation values
@gb[i]	4.151983e-18
@r1[i]	-3.95949e-18
@r2[i]	-4.15198e-18
@r3[i]	1.924970e-19
@r4[i]	-8.72784e-19
@r5[i]	-2.82826e-03
@r6[i]	4.336809e-19
@r7[i]	-8.83871e-19
v(1)	0.000000e+00
v(2)	4.141871e-15
v(3)	1.247625e-14
v(4)	-8.97403e-16
v(5)	3.552714e-15
v(6)	8.831302e+00
v(7)	-8.97403e-16
v(8)	0.000000e+00
Ix	-2.82826e-03
Vx	8.831302e+00
Req	-3.12252e+03

Table 4: Step 2: Operating point for $v_s(0) = 0$. A variable preceded by @ is of type *current* and expressed in miliAmpere; variables are of type *voltage* and expressed in Volt. The equivalent resistance is in Ohms

2.3 Natural solution for V_6

The natural solution depends on the initial charge (voltage), on R_{eq} and C and it is computed by removing all independent sources and applying KVL. In Octave, to compute the Natural solution the general formula derived in the theoretical classes was used: $V_{6n}(t) = Ae^{\frac{-t}{\tau}}$. In this formula τ is the time constant determined in the previous section and A is a constant that can be determined through the boundary conditions (when $t = 0$, $A = V_x$).

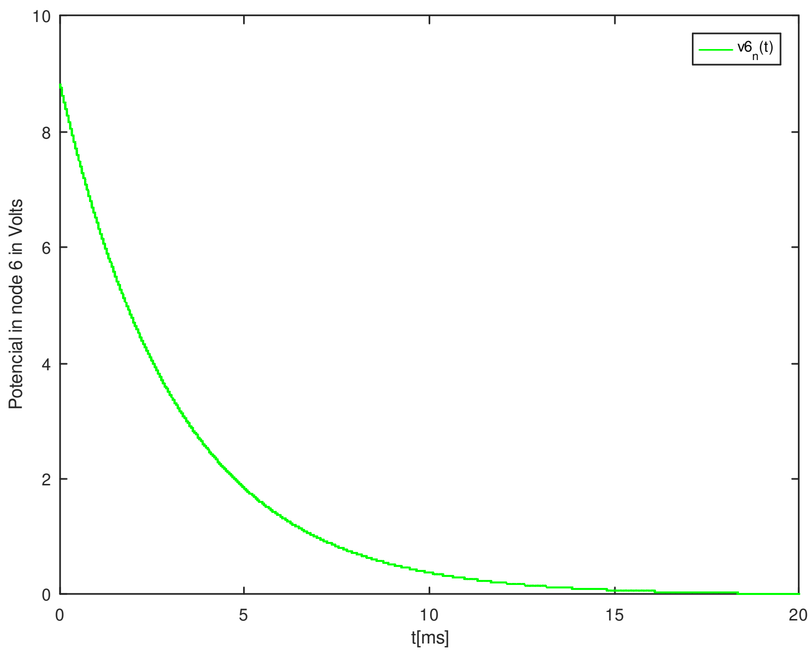

Figure 3: Natural response of V_6 as a function os time in the interval from [0,20] ms

2.4 Forced solution for V_6 with $f = 1000Hz$

In this section the forced solution V_{6f} is determined for the same time interval and for a frequency of 1KHz. To do this a nodal analysis was used, but insted of resistances and capacitances, impedances were used. It was also considered that the magnitude of the phasor representing the voltage sorce $\tilde{V_s}$ was 1 ($V_s = 1$), a result of expression 1. By taking all these steps the phasor voltages in all nodes were determined in accordance to the following equations:

$$Z = \frac{1}{wCj} \tag{16}$$

$$\tilde{V_s} = -j \tag{17}$$

$$\tilde{V_0} = 0 \tag{18}$$

$$\tilde{V_4} = \tilde{V_7} \tag{19}$$

$$\tilde{V_5} - \tilde{V_8} = K_d \frac{\tilde{V_0} - \tilde{V_4}}{R_6} \tag{20}$$

$$\tilde{V_1} - \tilde{V_0} = \tilde{V_s} \tag{21}$$

$$\frac{\tilde{V_2} - \tilde{V_1}}{R_1} + \frac{\tilde{V_2} - \tilde{V_5}}{R_3} + \frac{\tilde{V_2} - \tilde{V_3}}{R_2} = 0 \tag{22}$$

$$\frac{\tilde{V_3} - \tilde{V_2}}{R_2} - K_b(\tilde{V_2} - \tilde{V_5}) = 0 \tag{23}$$

$$\frac{\tilde{V_1} - \tilde{V_2}}{R_1} + \frac{\tilde{V_0} - \tilde{V_4}}{R_6} + \frac{\tilde{V_0} - \tilde{V_5}}{R_4} = 0 \tag{24}$$

$$K_b(\tilde{V_2} - \tilde{V_5}) + \frac{\tilde{V_6} - \tilde{V_5}}{R_5} + \frac{\tilde{V_6} - \tilde{V_8}}{Z} = 0 \tag{25}$$

$$\frac{\tilde{V_4} - \tilde{V_0}}{R_6} + \frac{\tilde{V_7} - \tilde{V_8}}{R_7} = 0 \tag{26}$$

$$\tilde{V_x} = \tilde{V_6} - \tilde{V_8} \tag{27}$$

The complex amplitudes of the phasors are presented in **Table 5**

Name	Complex amplitude voltage
V0	0
V1	1
V2	9.445310207903378e-01
V3	8.329148493139050e-01
V4	3.744381306846604e-01
V5	9.524211706387480e-01
V6	5.602948262043124e-01
V7	3.744381306846604e-01
V8	5.581602943867834e-01

Table 5: Complex amplitudes in all nodes in Volts

2.5 Final total solution $v_6(t)$

In this section the final total solution V_6 for a frequency of 1KHz is determined by superimposing the natural and forced solutions determined in previous sections (V_6=V_{n6}+V_{6f}) In **Figure: 4** the voltage of the independent source V_s and the voltage of V_6 were plotted for the time interval of [-5,20] ms.

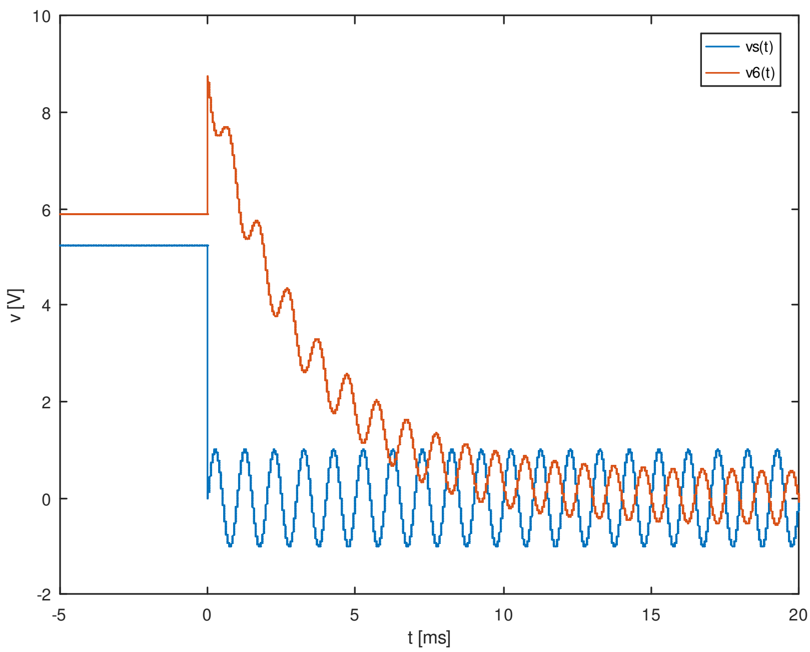

Figure 4: Voltage of $V_6(t)$ and $V_s(t)$ as functions of time from [-5,20] ms

2.6 Frequency responses $v_c(f)$, $v_s(f)$ and $v_6(f)$ for frequency range 0.1 Hz to 1 MHz

For this section, we considered $v_s(t) = sin(2\pi ft)$. As we can see, the magnitude and the phase do not depend on the frequency f. Therefore, we are to expect these values to remain constant for both these variables in the figures 5 and 6.

This circuit can serve the purpouse of a low-pass filter. In other words, when the frequencies are low, there is plenty of time for the capacitor to charge up to practically the same voltage as the input, which means it will act aproximately as an open circuit, thus allowing a considerable potential drop from nodes 6 to 8.This means that for low frequencies the voltage in the capacitor is in phase with the voltage source. However when the frequencies are high, the capacitor only has a small time to charge up before the input changes direction, which means it will act aproximately as a short-circuit. Therefore, there will be close to none potential drop between nodes 6 and 8 and the capacitor and source will start to fall out of phase, for frequencies greater that the cutoff frequecy (f_c). This frequency can be calculated with the following formula. $f_c = \frac{1}{2.\pi.\tau}$. For the values provided this cutoff frequency is around 50Hz. This explains the steep drop in potential difference that we can see in the graph around the first and second decades. The phase difference between the capacitor voltage and the voltage source also begins to show at around this frequency as can be seen in **Figure: 6**.

Simplifying the circuit to a voltage source, capacitor and equivalent resistor yields the following equations, which help understand the phase and magnitude declination with the increase in frequency:

$$V_c = \frac{V_s}{\sqrt{1 + (R_{eq}.C.2\pi.f)^2}} \tag{28}$$

$$\phi_{V_c} = -\frac{\pi}{2} + arctan(R_{eq}.C.2\pi.f) \tag{29}$$

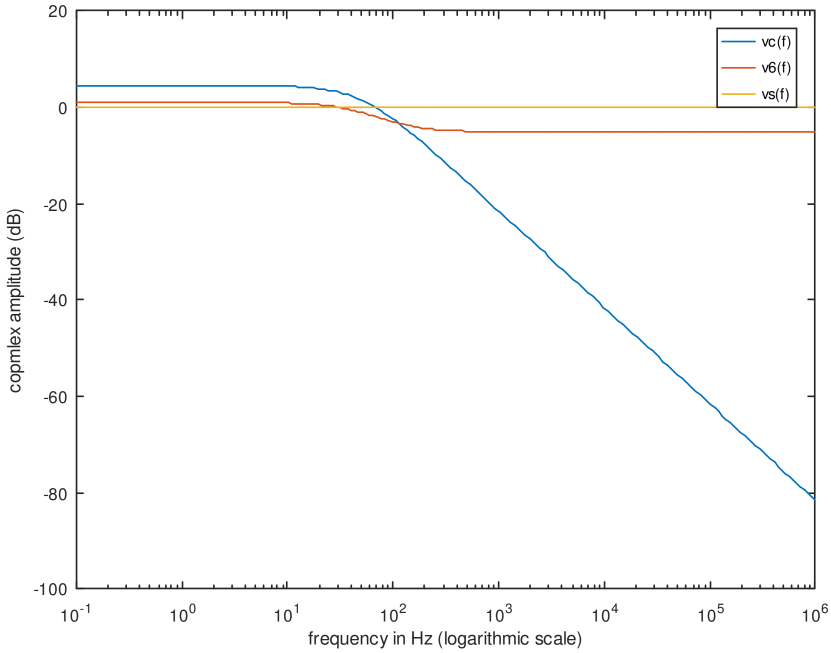

Figure 5: Graph for amplitude frequency response, in dB, of V_c, V_6 and V_s for frequencies ranging from 0.1Hz to 1MHz (logarithmic scale).

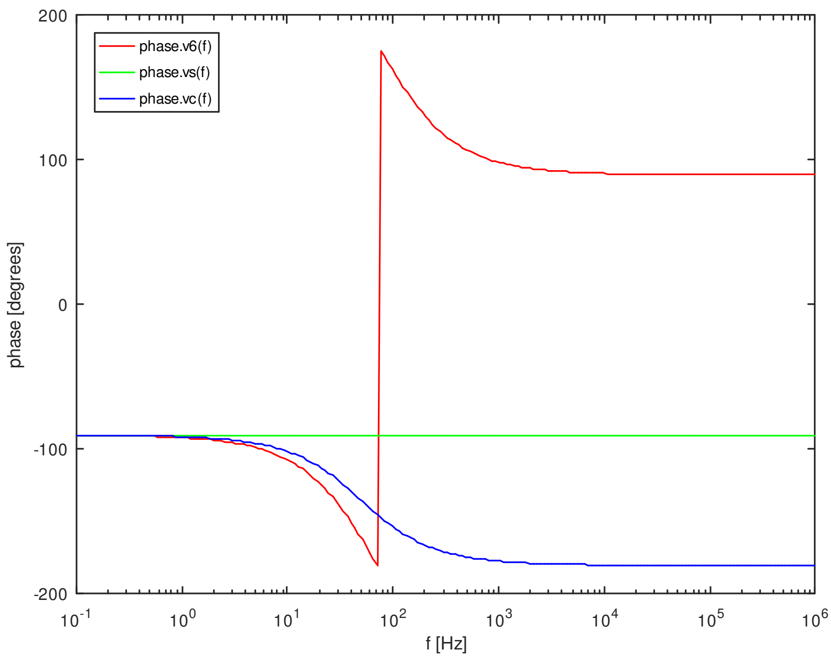

Figure 6: Graph for the phase response, in degrees of V_c, V_6 and V_s for frequencies ranging from 0.1Hz to 1MHz, displayed in a logarithmic scale. Note that the apparent peak discontinuity in the phase of V_6 is only due to the domain of the arctan function that gives the phase (angle of the phasor), and so the phase is in fact continuous.

3 Simulation Analysis

In this section we will describe the steps needed to simulate this circuit using the software Ngspice. Three types of analysis will be performed: Operating Point analysis, Transient analysis ans Frequency analysis.

The following steps in the simulations are to be conducted:

- for $t < 0$ (operating point only, in order to obtain the voltages in all nodes and the currents in all branches);

- operating point for $V_s(0) = 0$, replacing the capacitor with a a voltage source $V_x = V_6 - V_8$, where V_6 and V_8 are the voltages in nodes 6 and 8 as obtained in the previous step (this step is necessary given that we must the compute the boundary conditions that guarantee continuity in the capacitor's discharge - such may imply that the boundary conditions differ from those computed for $t < 0$);

- simulate the natural response of the circuit (using the boundary conditions V(6) and V(8) as obtained previously) using a transient analysis;

- repeating the third step, using V_s as given in **Figure 7** and f = 1kHz in order to simulate for the total response on node 6

- simulate the frequency response in node 6 for a frequency range 0.1 Hz to 1MHz.

$$v_s(t) = V_s u(-t) + \sin(2\pi f t) u(t)$$

$$u(t) = \begin{cases} 0, t < 0 \\ 1, t \geq 0 \end{cases}$$

Figure 7: Time step conditions

3.1 Operating Point Analysis for $t < 0$

There was a need to create a "fictional" voltage source, between node 7 and resistor 6 (providing 0V to the circuit in order not to alter the behaviour of the rest of the circuit) so as to be able to define the dependecy for the current-controlled voltage source V_d. This has no specific reason to be, other than the particularities of the Ngspice software. The circuit and nodes used for the simulation can be seen in **Figure 2**.

Table 6 shows the simulated operating point results for the circuit under analysis for $t < 0$.

Name	Value [mA or V]
@c[i]	0.000000e+00
@gb[i]	-2.91567e-04
@r1[i]	2.780494e-04
@r2[i]	2.915672e-04
@r3[i]	-1.35178e-05
@r4[i]	-1.22689e-03
@r5[i]	-2.91567e-04
@r6[i]	9.488377e-04
@r7[i]	9.488377e-04
v(1)	5.243596e+00
v(2)	4.952739e+00
v(3)	4.367468e+00
v(4)	-1.96340e+00
v(5)	4.994112e+00
v(6)	5.904536e+00
v(7)	-1.96340e+00
v(8)	-2.92677e+00

Table 6: Operating point for $t < 0$. A variable preceded by @ is of type *current* and expressed in miliAmpere; other variables are of type *voltage* and expressed in Volt.

3.2 Operating Point Analysis for $t = 0$

In this section the circuit was simulated using an operating point analysis with $V_s(0) = 0$ and with the capacitor replaced by a voltage source $V_x = V(6) - V(8)$ with these as obtained in the last step. This step was taken because we must compute the boundary conditions that guarantee continuity in the capacitor's discharge (such may imply that the boundary conditions differ from those computed for $t < 0$). In other words $V(6) - V(8)$ needs to be a continuos function in time (in this case particularly from $t < 0$ to $t = 0$), as there can not be a energy discontinuity in the capacitor ($E_C = \frac{1}{2}CV^2$). However, that does not imply that that $V(6)$ and $V(8)$ are continuos functions in time. In **Table 7** the simulation results are presented.

Name	Value [mA or V and Ohm]
@gb[i]	4.151983e-18
@r1[i]	-3.95949e-18
@r2[i]	-4.15198e-18
@r3[i]	1.924970e-19
@r4[i]	-8.72784e-19
@r5[i]	-2.82826e-03
@r6[i]	4.336809e-19
@r7[i]	-8.83871e-19
v(1)	0.000000e+00
v(2)	4.141871e-15
v(3)	1.247625e-14
v(4)	-8.97403e-16
v(5)	3.552714e-15
v(6)	8.831302e+00
v(7)	-8.97403e-16
v(8)	0.000000e+00
Ix	-2.82826e-03
Vx	8.831302e+00
Req	-3.12252e+03

Table 7: Operating point for $v_s(0) = 0$. A variable preceded by @ is of type *current* and expressed in miliAmpere; variables are of type *voltage* and expressed in Volt. The equivalent resistance is in Ohms

3.3 Natural solution for V_6 using transient analysis

In this section the natural response of the circuit in the interval [0,20] ms was studied using a transient analysis simulation. To do so the boundary conditions V(6) and V(8) obtained in the previous section were used, as well as the NgSpice directive *.ic*. These values are being obtained from the previous simulations run, and not from the theoretical previsions.

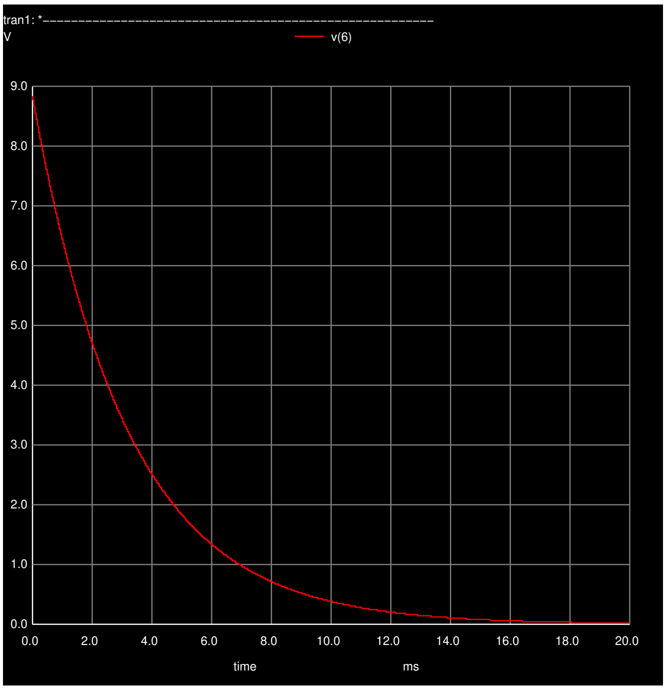

Figure 8: Simulated natural response of $V_6(t)$ in the interval [0,20] ms. The *x axis* represents the time in miliseconds and the *y axis* the Potencial in node 6 in Volts.

3.4 Total solution for V_6 using transient analysis

In this section the total response of V_6 (natural + forced) is simulated using transient analysis. This is done by repeating the previous section, but using V_s as given in **Figure 7** and f = 1kHz.

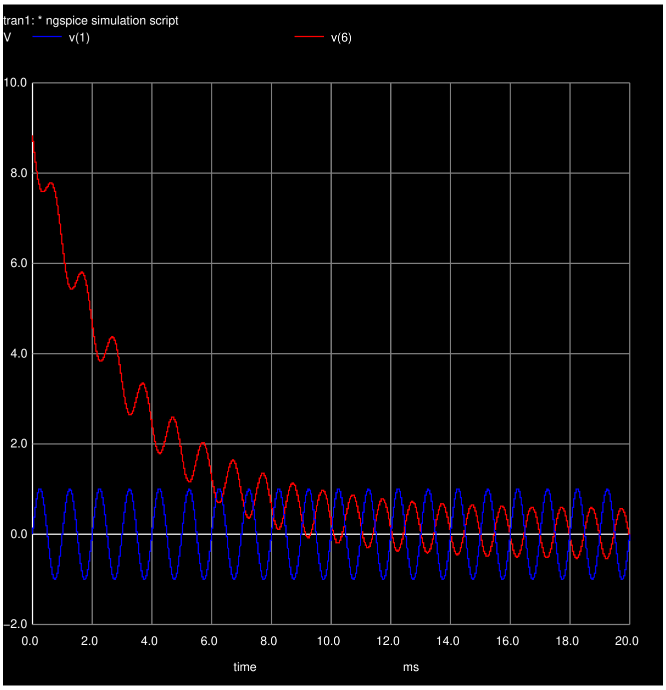

Figure 9: Simulated response of $V_6(t)$ and of the stimulus $V_s(t)$ as functions of time from [0,20] ms. The *x axis* represents the time in miliseconds and the *y axis* the Voltage in Volts.

3.5 Frequency response in node 6

In this section the frequency response in node 6 is simulated for the frequency range from 0.1 Hz to 1 MHz. The reasons of how and why $V_6(t)$ and $V_s(t)$ differ have been coverd in **subsection 2.6**.

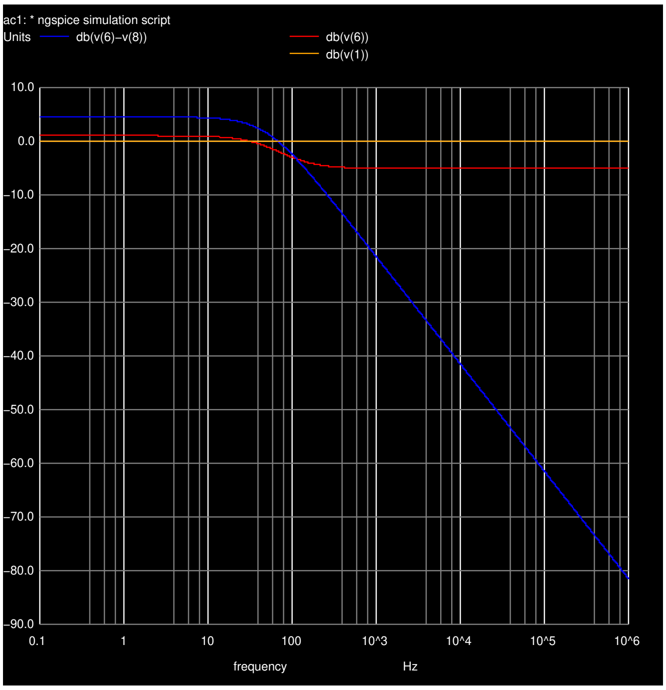

Figure 10: Magnitude of $V_s(f)$, $V_c(f)$ and of $V_6(f)$. The *x axis* represents the frequency in Hz, using a logarithmic scale and the *y axis* the magnitude in dB.

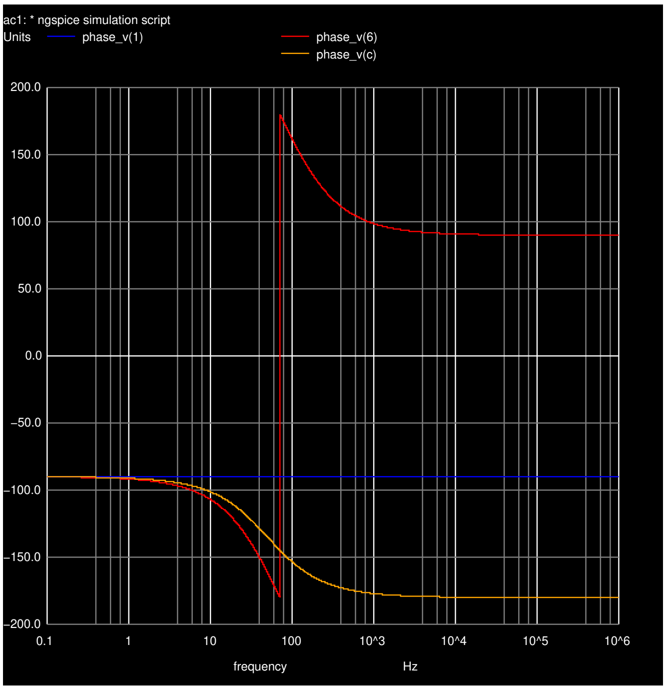

Figure 11: Phase of $V_s(f)$, $V_c(f)$ and of $V_6(f)$. The *x axis* represents the frequency in HZ, using a logarithmic scale and the *y axis* the phase in degrees.

4 Conclusion

In this laboratory assignment the objective of analysing a circuit containing multiple resistances, a capacitor and a sinusoidal voltage source v_s that varies in time has been achieved.

Static, time and frequency analyses have been performed both theoretically, using the Octave maths tool, and by circuit simulation, using the Ngspice tool. The theoretical results obtained match the simulation results quite precisely - one must call attention to the fact that certain values obtained theoretically were precisely zero, whilst im some cases (the boundary conditions simulated for t=0) the corresponding simulated results were of the order of 1e-15 - we considered these results to be effectively zero, given they were multiple orders of magnitude inferior to the remaining nominal values and aproximately of the magnitude of the floating point precision for the numerical representation types used.

The reason for this overall satisfactory match is the fact that this is a relatively straightforward circuit, containing only one capacitor, besides linear components - as such, any differences will be related to the model used in ngspice for the capacitor, and in particular for the transient boundary condition analysis. For more complex components and circuits, the theoretical and simulation models could differ more (given the greater complexity of the models implemented by the simulator Ngspice, as well as the interactions between these, when compared to those studied in the theoretical classes).