Analysis of Chip Heat Sink Design

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Description of the problem and objectives

In this project a chip that has an aluminum heat sink soldered onto itself is being analyzed using various heat transfer analysis methods. The first method being 1D heat approximation by making certain assumptions, equations that were arrived to empirically can be applied to this case. The Second method being Finite Difference Method to approximate the temperature gradient across the profile of the fin and from that being able to calculate the heat transfer. A coarse FDM with a smaller node/element grid is used and a fine FDM with a much larger node/element grid is used and they are contrasted with one another.

1D approximation method for an array of fins

For the 1-dimensional approximation of heat behavior across the fin, certain conditions were assumed to be present. One of them being, an adiabatic tip condition and a uniform cross-sectional area from base through to the tip. The adiabatic tip condition assumes that there is no heat transfer from the top surface of the fin, if the heat sink is seen from the bottom side, hence there is insulation at the tip. There are many different tip conditions, which will yield the heat transfer equation to have different boundary conditions and in effect change the overall equation. The general form of the energy equation for extended surfaces can be seen below. We will modify it according the boundary conditions that apply to our case.

$$\frac{d^2T}{dx^2} + \left(\frac{1}{A_c}\frac{dA_c}{dx}\right)\frac{dT}{dx} - \left(\frac{1}{A_c}\frac{h}{k}\frac{dA_s}{dx}\right)(T - T_s) = 0$$

The surface area exposed to the fluid is denoted as A_s and can be calculated by taking the perimeter of the fin and multiplying it by the depth of the fin.

$$\mathrm{As} = \mathrm{Px}$$

$$\frac{\mathrm{dAs}}{\mathrm{dx}} = \mathrm{P}$$

Since we assume the cross-section of the fin to be uniform the change across the profile of the x axis will equal zero.

$$\frac{\mathrm{dAc}}{\mathrm{dx}} = 0$$

Taking these values into consideration and plugging them into the general energy equation for extended surfaces, the following relationship is attained.

$$\frac{d^2T}{dx^2} - \left(\frac{hP}{kA_c}\right)(T - T_\infty) = 0$$

In order to simplify the solving of this differential equation, the change in temperature will be replaced with θ yielding the following expression.

$$\theta(x) \equiv (T(x) - T_\infty)$$

And because the Temperature of the fluid T_∞ is constant, the following derivatives of θ and $T(x) - T_\infty$ with respect to x are equal.

$$\frac{d\theta}{dx} = \frac{d(T(x) - T_\infty)}{dx}$$

$$\frac{d\theta}{dx} = \frac{dT(x)}{dx} - \frac{dT_\infty}{dx}$$

$$\frac{dT_\infty}{dx} = 0$$

$$\frac{d\theta}{dx} = \frac{dT(x)}{dx}$$

Adjusting the general energy equation for extended surfaces, with this expression, the following relationship is derived.

$$\frac{d^2\theta}{dx^2} - \left(\frac{hP}{kA_c}\right)\theta \; = \; 0$$

In order to solve this equation using the general solution for a linear, second order, homogenous the following m term must be introduced.

$$m^2 = \left(\frac{hP}{kA_c}\right)$$

Now that the equation meets the form required for the general solution, it can be solved using the boundary conditions that pertain to our project.

$$\frac{d^2\theta}{dx^2} - m^2\theta = \; 0$$

The general solution for this second order to differential equation can be seen below.

$$\theta(x) = \; C_1 e^{mx} + \; C_2 e^{-mx}$$

To solve for C_1 and C_2 the boundary conditions that apply to our project must be applied. Since we assume there is no significant convective heat loss from the tip of the fin, the condition below is valid.

$$\left.\frac{d\theta}{dx}\right|_{x=L} = 0$$

The result of plugging this condition into the general solution yields the following.

$$\frac{d\theta(x)}{dx} = \; mC_1 e^{mL} + \; -mC_2 e^{-mL}$$

$$0 = m(C_1 e^{mL} - C_2 e^{-mL})$$

$$0 = C_1 e^{mL} - C_2 e^{-mL}$$

After solving these constants, the expression below is attained.

$$\frac{\theta}{\theta_b} = \frac{\cosh(m(L-x))}{\cosh(mL)}$$

We also know that the Temperature at x=0 is the Temperature at the base.

$$\theta(0) = T_b - T_\infty$$

$$\theta_b = T_b - T_\infty$$

Since the θ represents the change in temperature at any location along the x direction of the fin and the temperature of the flow, the ratio above can then be solved to calculate any temperature across the profile of the fin.

$$\theta(x) = T_x - T_\infty$$

$$\frac{T_x - T_\infty}{T_b - T_\infty} = \frac{\cosh(m(L-x))}{\cosh(mL)}$$

$$T_x = T_\infty + (T_b - T_\infty)\frac{\cosh(m(L-x))}{\cosh(mL)}$$

The heat transfer for the fin under the same boundary conditions can be calculated using the expressions below.

$$M = \sqrt{hPkA_c}\theta_b$$

$$q_f = Mtanh(mL)$$

Using the 1D heat transfer approximation method described previously, the following relationships were attained. The results from plotting the number of fins versus the heat transfer from the chip can be seen below in figure 1. The heat transfer removed from the chip seems to taper off at 9 fins, so any additional fins added above 9 would be unnecessary.

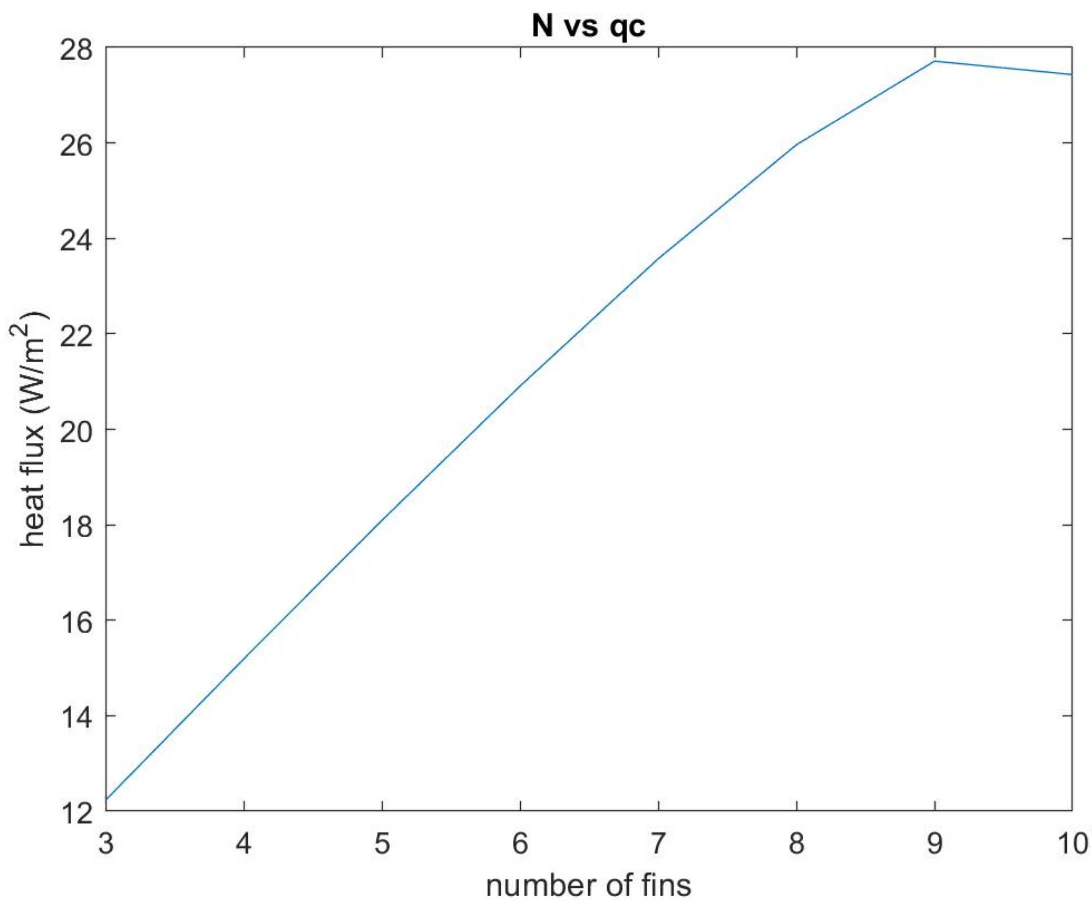

Figure 1 - Number of Fins vs. chip Heat Flux

Figure 2 shows how the temperature at the base of the fins stays relatively constant throughout adding more fins. This is mainly because the being supplied is not changing, nor is the geometry supplying the heat, so the base temperature remains the same even though more fins are added to the heat sink.

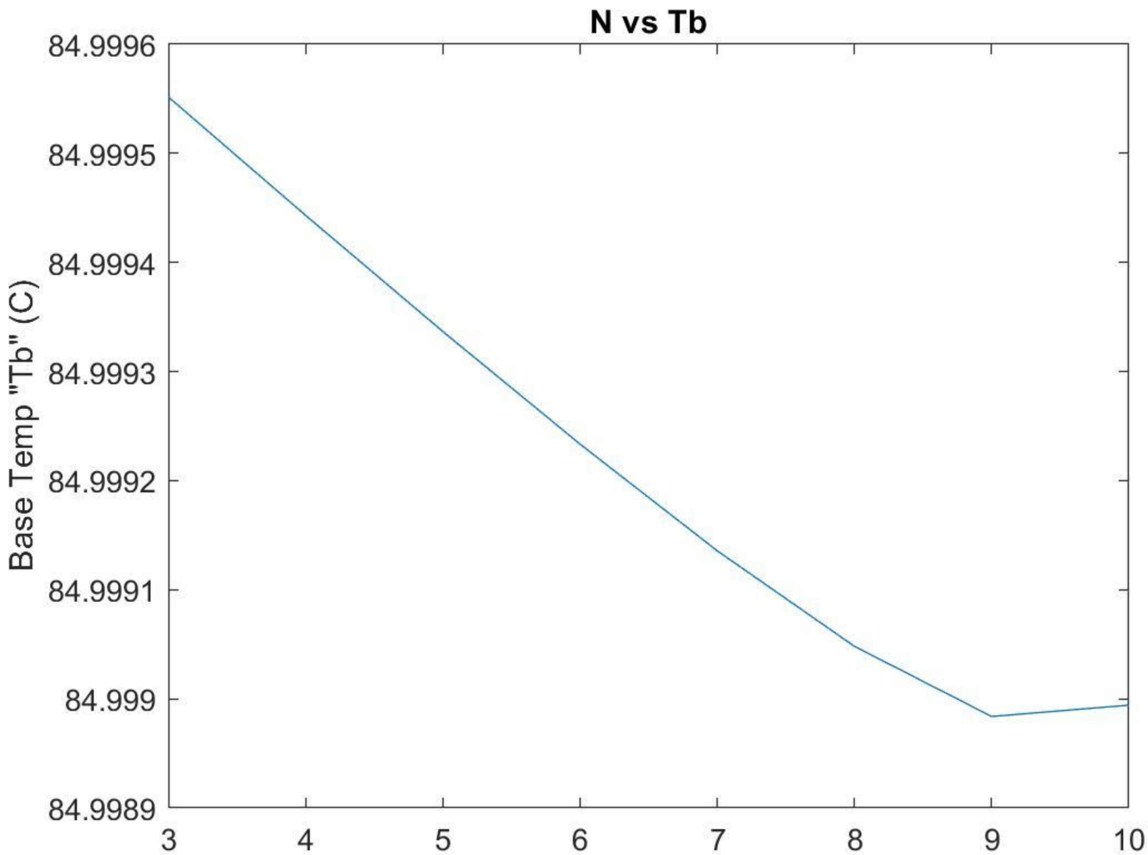

Figure 2 - Number of Fins vs. Temperature at Base of Heat Sink

The temperature at the tip of the fin however does not stay constant as more fins are added. As can be clearly seen in Figure 3 the tip temperature drastically decreases as more fins are added. When there are 3 fins in the heat sink the tip, temperature is close to 84℃ and when the fins are increased to 10 the tip temperature is decreased to approximately 65℃. This can be attributed to more fins dispersing the load of heat produced by the chip, therefore decreasing the temperature at the tips of the heat sink.

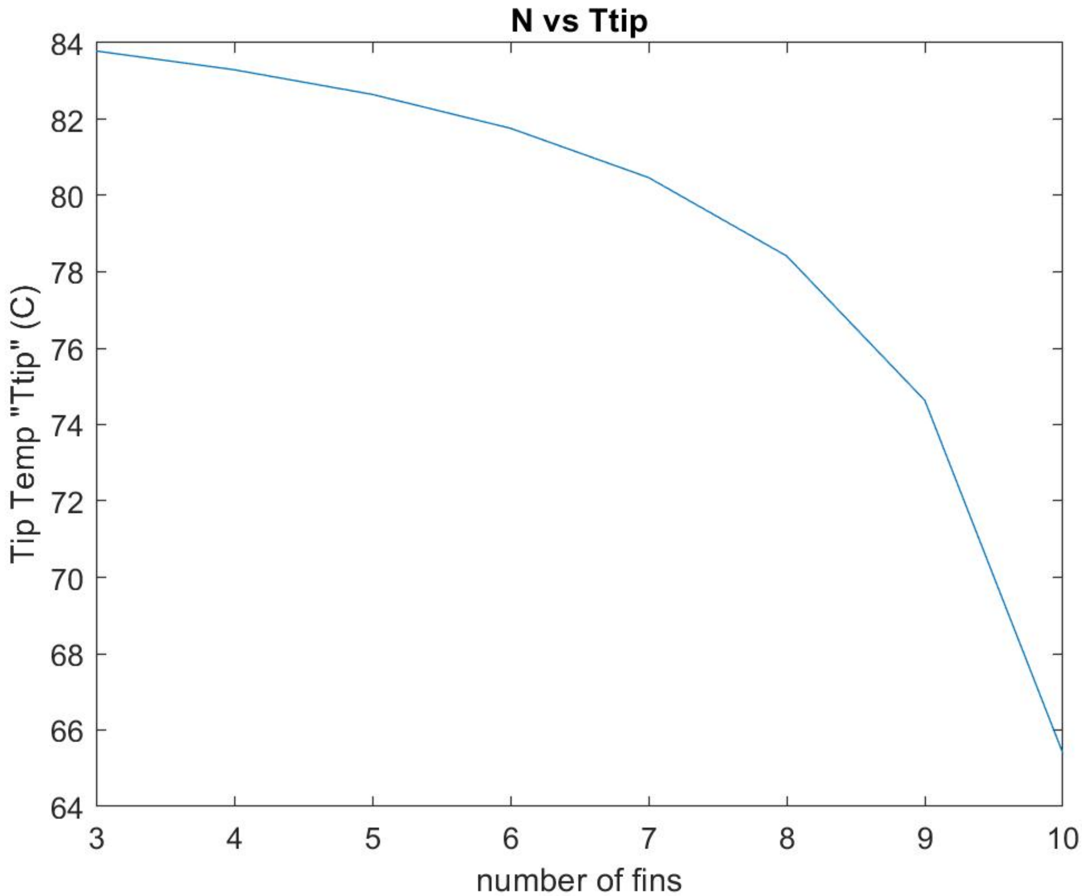

Figure 3 - Number of Fins vs. Tip Temperature of the Fins

In Figure 4 the heat transfer from the fins can be seen to decrease as number of fins are added to the heat sink. This can be attributed to the overall surface area of the fins increasing, therefore decreasing the amount of heat released per fin. Even the thought the net heat transfer across the entire heat sink is increasing as more fins are added, as we saw in Figure 1.

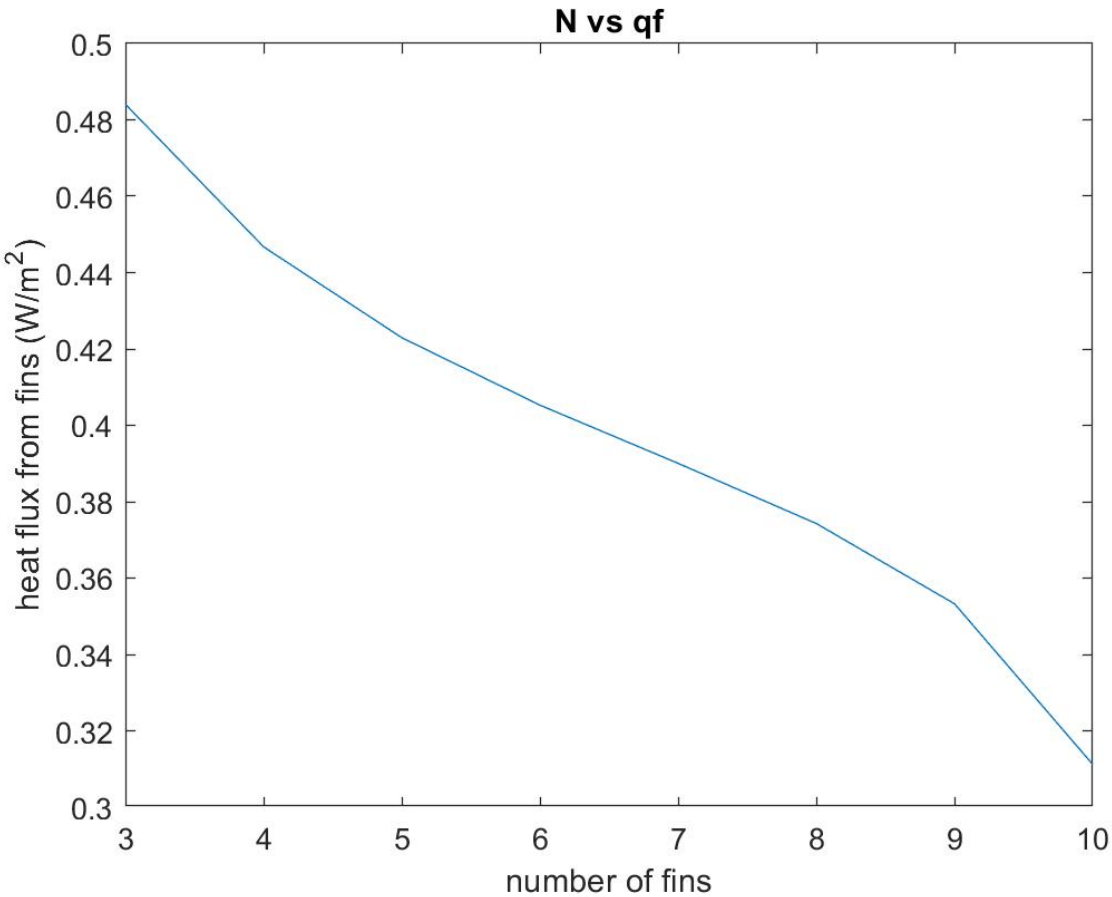

Figure 4 - Number of Fins vs Heat Transfer from Fins

Figure 5 shows how the total weight of the heat sink decreases as more fins are added. Since the number of fins affects the thickness of the fins, the total mass of the heat sink decreases as more fins are added. Thus, the total weight of the heat sink decreases as more fins are added.

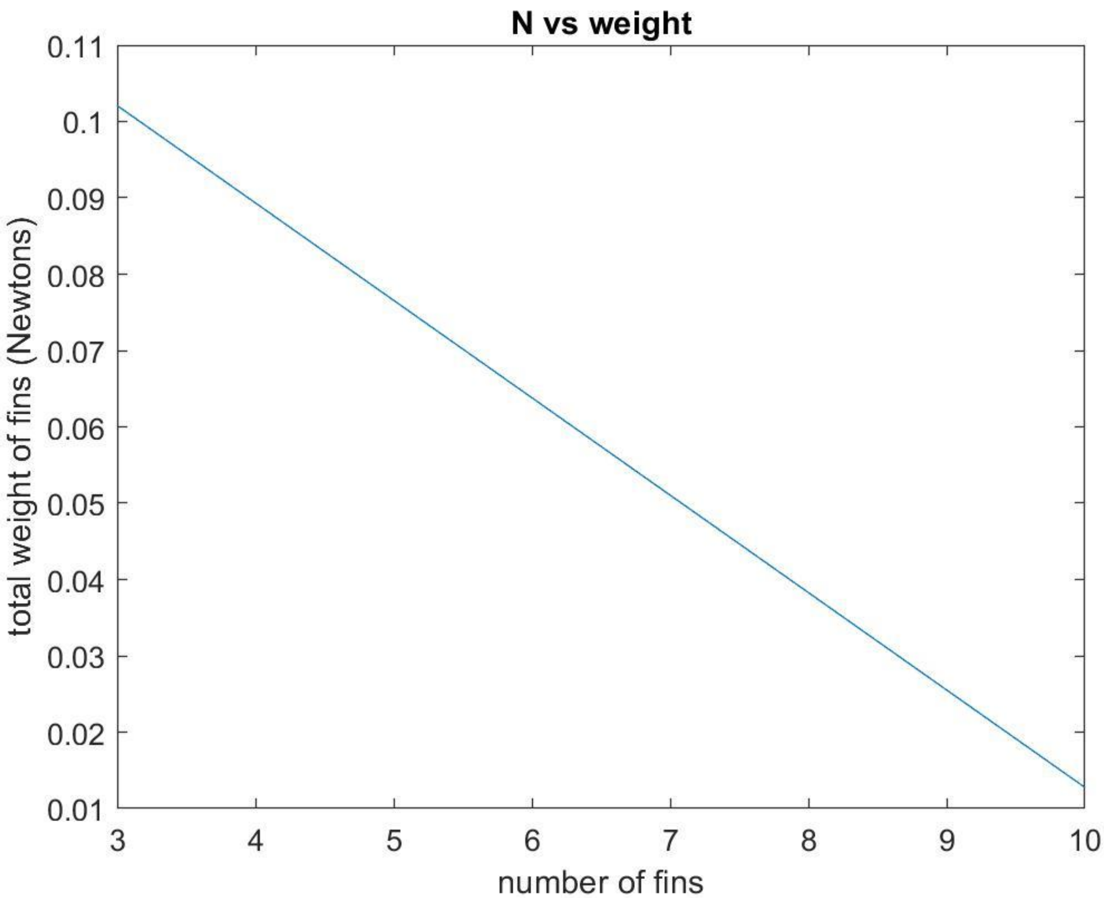

Figure 5 - Number of Fins vs Total Weight of Heat Sink

2D Finite difference method for a single fin.

For the 2D Finite Difference method, a single fin's behavior is being analyzed. A grid of nodes and elements is set up across the fin. Relationships at the left side, top side, bottom side, right top corner, right bottom corner and the right side are inputted into excel in order to define the temperatures at each node. The boundary condition in this case is the base temperature, which is given in the problem statement as 85°C. Once the relationships are set the base temperature then feeds into the relationships mentioned and a temperature gradient across the fin is developed. The fin requires a large amount of iterations in order to stabilize, this was done by recalculating the temperatures until the temperatures stopped fluctuating across the fin. A coarse and fine FEM mesh were used to analyze the behavior of the fin. The results from both meshes were analyzed and contrasted with one another along with the 1D method as well.

Assuming that there is no internal heat generation and uniform thermal conductivity across the section being analyzed a 2D grid can be used to determine the temperature across the grid to a very high level of precision, depending on how far apart the nodes are in the mesh.

The following equations were used to establish the relationships between the nodes:

Top boundary,

$$T_{i,j} = \frac{\left[(T_{i-1,j})+(T_{i+1,j})+(2*T_{i,j-1})+\frac{2h\Delta x}{kT\infty}\right]}{4+\frac{2h\Delta x}{k}}$$

Bottom boundary,

$$T_{i,j} = \frac{\left[(T_{i-1,j})+(T_{i+1,j})+(2*T_{i,j+1})+\frac{2h\Delta x}{kT\infty}\right]}{4+\frac{2h\Delta x}{k}}$$

Right bottom corner,

$$T_{i,j} = \frac{\left[(2*T_{i-1,j})+(2*T_{i,j+1})+\frac{2h\Delta x}{kT\infty}\right]}{4+\frac{2h\Delta x}{k}}$$

Right top corner,

$$T_{i,j} = \frac{\left[(2*T_{i-1,j})+(2*T_{i,j-1})+\frac{2h\Delta x}{kT\infty}\right]}{4+\frac{2h\Delta x}{k}}$$

Internal nodes,

$$T_{i,j} = \frac{[(T_{i-1,j})+(T_{i+1,j})+(T_{i,j-1})+(T_{i,j+1})]}{4}$$

Right boundary,

$$T_{i,j} = \frac{[(2*T_{i-1,j})+(T_{i,j-1})+(T_{i,j+1})]}{4}$$

In figure 6 the conduction heat transfer across the fin is compared when analyzed using the 1D, FDM coarse and FDM fine. The FDM coarse used a 13 x 5 grid of nodes across the fin and the fine grid used a 17 x 49 node grid. Since it is known that the fine is much more accurate, from the figure below we can see that in this case the 1D approximation is closer the fine FDM.

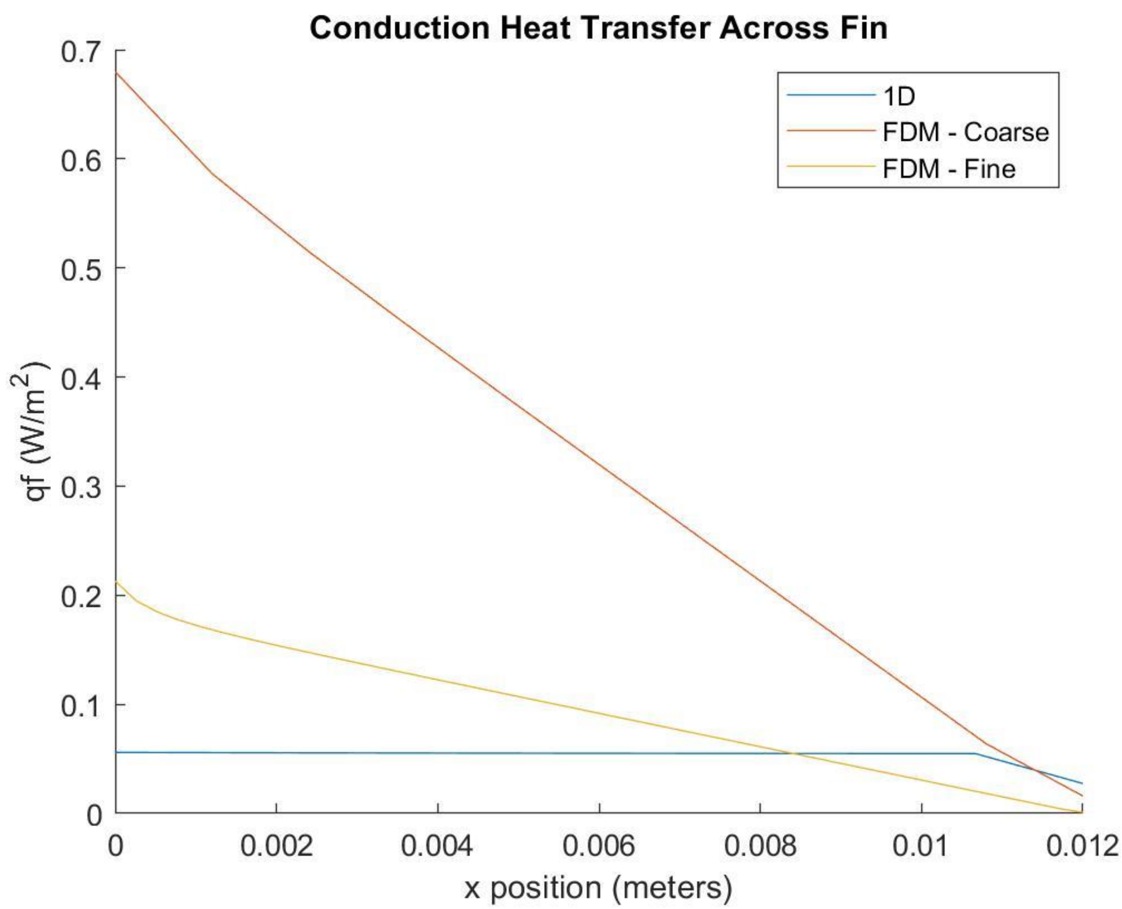

Figure 6 - Conduction Heat Transfer Across Fin

The fin temperature gradient across the fin when using all three methods exhibit the same trend. However, the 1D and fine FDM are closer to each other. The temperature at the base decreases and tapers off at the tip, the range for all three models is between 84 and 83.5℃.

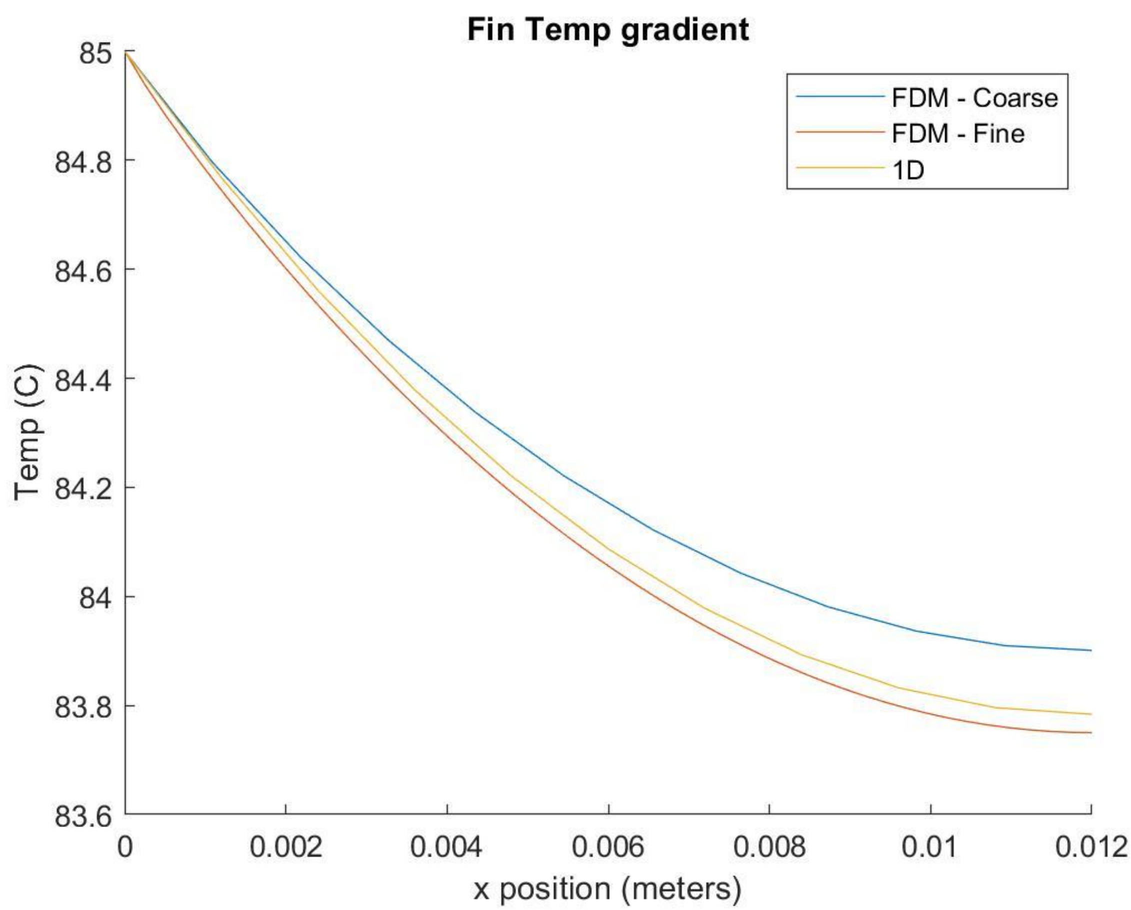

Figure 7 - Fin Temp Gradient

The fin heat transfer measured at the base of the fin can be seen in figure 8. The two FDM methods used display the same trend and the 1D displays a similar trend.

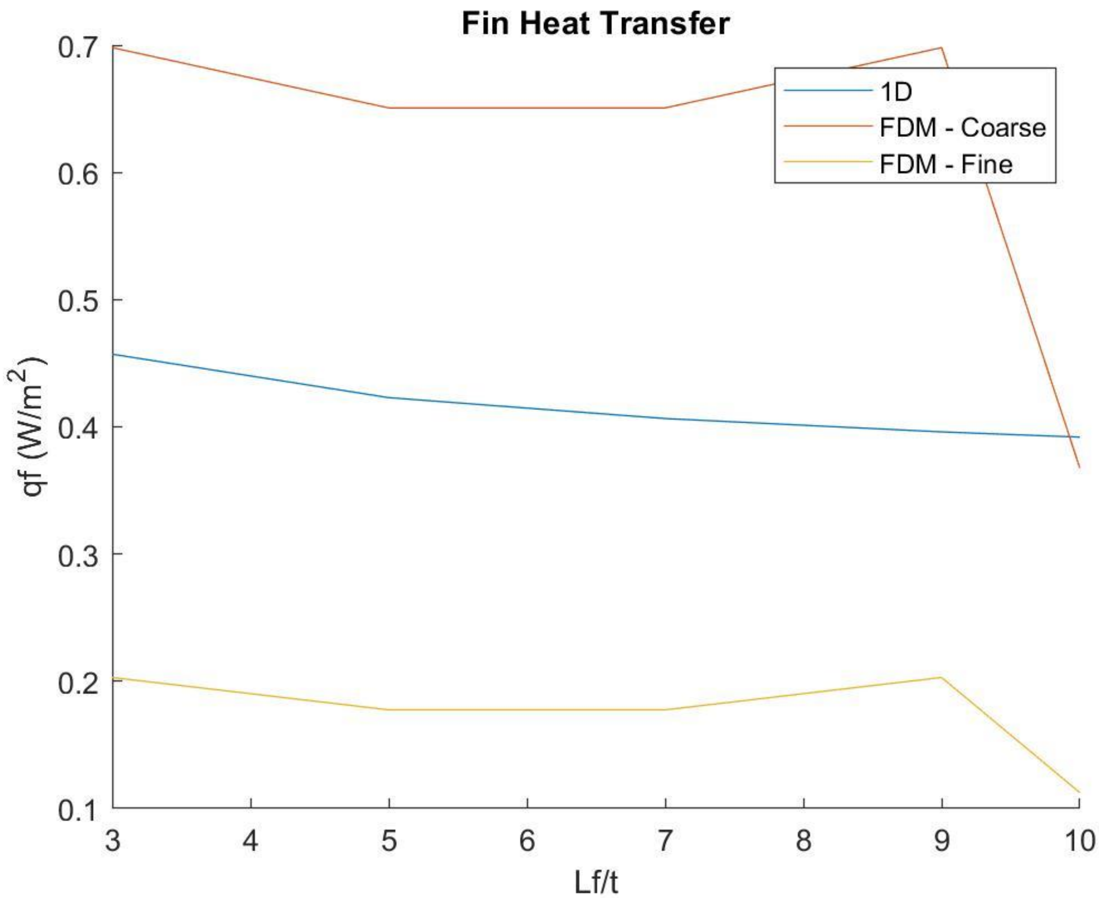

Figure 8 - Fin Heat Transfer

Conclusions

In conclusion when using the finite difference method to analyze the heat transfer and temperature behavior of a specimen, using a fine grid will yield more accurate values since the spacing between nodes is far less. This allows for the temperature at more points to be accounted, this in turn gives a more accurate and precise view of what is occurring through out the specimen's profile. The 1D and the coarse FDM method give accurate information, but to a certain degree. When using these two methods, it must be kept in mind that these do carry uncertainty along with them. Depending on how accurate the readings need to be, is what determines which method should be used. If a preliminary calculation needs to be done in order to quickly gauge the behavior of a specimen and accuracy/precision is not of high importance, then the 1D approximation is a viable option. However, as the importance of accuracy/precision in these measurements increases then the FDM coarse and then fine should be used.

Appendix

MatLab Code:

```
%% Inputs
clear
clc

Tc = 85; %Celsius

Tinf = 20; %Celsius

h = 100; %W/m^2 K
k = 180; %W/m K

Lf = .012; % meter
Lb = .003; % meter

L = Lf;

W = 20;%in millimeters
N=3:1:10;
t = ((W - 2.*N + 2)./N)./1000;

%re-assigning W value
W = 0.02;%in meters

P = 2.*(W+t);
Ac = t*W;
Af = 2.*((Lf.*W)+(Lf.*t));
At = W.^2 - N.*(W.*t) + N.*Af;
%% N vs qc

figure(1)
m = sqrt((h.*P)./(k.*Ac));

nf = (tanh(m.*L))./(m.*L);
no = 1-((N.*(Af./At)).*(1-nf));

Rs = (2*10^-5);
Rc = Lb/k;
Rf = 1./(no.*h.*At);

qc = (Tc-Tinf)./(Rs+Rc+Rf);
plot(N,qc)
xlabel('number of fins')
ylabel('heat flux (W/m^2)')
title('N vs qc')
%% N,Tb
figure(2)
Tb = qc.*Rf + Tinf;
plot(N,Tb)
```

```
xlabel('number of fins')
ylabel('Base Temp "Tb" (C)')
title('N vs Tb')
%% N,qf
figure(3)

theta_b = Tb - Tinf;
M = sqrt(h.*P.*k.*Ac.*theta_b);
qf = M.*tanh(m.*L);
%qf = nf.*h.*Af.*(Tb-Tinf);
plot(N,qf)
xlabel('number of fins')
ylabel('heat flux from fins (W/m^2)')
title('N vs qf')
%% N,Ttip
figure(4)
L = Lf;
x = Lf;
Ttip = Tinf + (Tb-Tinf).*cosh(m.*(L-x))./(cosh(m.*L));
plot(N,Ttip)
xlabel('number of fins')
ylabel('Tip Temp "Ttip" (C)')
title('N vs Ttip')
%% wt,N
figure(5)
d = 2710;%kg/m^3 density of alluminum
Vf = N.*t.*(Lf*W); %m^3Volume of fins
mf = d.*Vf; %kg mass of fins
wt = mf.*9.81;%Newtons weight of fins

plot(N,wt)
xlabel('number of fins')
ylabel('total weight of fins (Newtons)')
title('N vs weight')
%% Adiabatic Tip Condition

%theta = T - T_inf;
%theta_b = Tb - T_inf;
%m = sqrt(h*P/k*Ac);
%M = sqrt(h*P*k*Ac*theta_b);

% Temperature Distribution
%theta_L = cosh(m*(L-x))/cosh(m*L)

%Fin Heat Transfer Rate
%qf = M*tanh(m*L)
%% Plotting temps for fdm c/f and 1D

% T1 = Tc - qc(1,1)*Rs;
% Ttip = Ttip(1,1);

fdmCt = xlsread('fdmc.xlsx','B5:M5');
```

```
fdmFt = xlsread('fdmf.xlsx','B5:AW5');

figure(6)
hold on
%FDM Coarse temp gradient
xC = linspace(0,Lf,12);
plot(xC,fdmCt)

%FDM Fine temp gradient
xF = linspace(0,Lf,48);
plot(xF,fdmFt)

%1D temp gradient
m = m(1,1);
L = Lf;
x1D = linspace(0,Lf,11);
x = x1D;
Tb = Tb(1,1);
Tx = Tinf + (Tb-Tinf).*cosh(m.*(L-x))./cosh(m.*L);

plot(x,Tx)


title('Fin Temp gradient')
ylabel('Temp (C)')
xlabel('x position (meters)')

legend('FDM - Coarse','FDM - Fine','1D')

% -----------------------------------------------------
% q_conduction heat Transfer plots
figure(7)
hold on

% 1D heat Transfer
% qc1d = ones(1,10);
% n=0;
% for i = 1:length(Tx)-1
%     n = n+1;
%     qc1d(n) = (k/.001).*(Tx(1,n) - Tx(1,n+1));
% end
%
% qc1 = ones(1,10);
% a = 0;
% for i = 1:length(qc1d)
% a = a +1;
% qc1(a) = (0.5*.02*.001).*(qc1d(1,a) - qc1d(1,a+1));
% end
%
onedq = xlsread('1d.xlsx','C22:L22');
xd = linspace(0,Lf,10);
```

```
plot(xd,onedq)

%FDM Coarse heat transfer
fdmCq = xlsread('fdmc.xlsx','B11:L11');
xC = linspace(0,Lf,11);
plot(xC,fdmCq)

%FDM Fine heat transfer
fdmFq = xlsread('fdmf.xlsx','B11:AV11');
xF = linspace(0,Lf,47);
plot(xF,fdmFq)

title('Conduction Heat Transfer Across Fin')
ylabel('qc (W/m^2)')
xlabel('x position (meters)')

legend('1D','FDM - Coarse','FDM - Fine')


%% Plotting qf,Ttip and nf along Lf/t
% 1D qf for plot along L/t
Tinf = 20; %Celsius
Tb = 85;%Celsius

h = 100; %W/m^2 K
k = 180; %W/m K

Lf = .012; % meter
L = Lf;
Lb = .003; % meter
W = 0.02;%in meters

N = 1;

t = Lf./[3  5  7  9 10];

P = 2.*(W+t);
Ac = t*W;
Af = 2.*((Lf.*W)+(Lf.*t));
At = W.^2 - N.*(W.*t) + N.*Af;

m = sqrt((h.*P)./(k.*Ac));

theta_b = Tb - Tinf;
M = sqrt(h.*P.*k.*Ac.*theta_b);
qf = M.*tanh(m.*L);

figure(8)
hold on

plot(Lf./t,qf)
```

```
% FDM coarse qf for plot along L/t
fdmCqf = xlsread('fdmc.xlsx','B18:F18');
plot(Lf./t,fdmCqf)

% FDM Fine qf for plot along L/t
fdmFqf = xlsread('fdmf.xlsx','B16:F16');
plot(Lf./t,fdmFqf)


title('Fin Heat Transfer')
ylabel('qf (W/m^2)')
xlabel('Lf/t')

legend('1D','FDM - Coarse','FDM - Fine')
hold off



%Ttip------------------------------------------------------
figure(9)

hold on

% FDM coarse qf for plot along L/t
fdmCTtip = xlsread('fdmc.xlsx','B23:F23');
plot(Lf./t,fdmCTtip)

% FDM Fine qf for plot along L/t
fdmFqTtip = xlsread('fdmf.xlsx','B21:F21');
plot(Lf./t,fdmFqTtip)



%nf------------------------------------------------------
figure(10)
```

Excel Files:

fdm - coarse												
	temp											
	85	84.79578	84.62233	84.46994	84.33624	84.22061	84.12286	84.04293	83.98079	83.93642	83.9098	83.90093
	heat transfer											
	0.679812	0.586513	0.514954	0.448796	0.384082	0.319814	0.255722	0.191724	0.127785	0.063883	0.01597	
	qf"avg											
	0.698083	0.650694	0.650694	0.698083	0.367595							
	Ttip											
	83.90093	83.92756	83.93644	83.92756	83.90093							

k	180					
h	100					
Tinf	20	293.15				
Tb	85	358.15				
dx_1	0.001					
dy	0.001					
N	3		qf=	36759.49	qf"avg	0.698083
W	0.02			33048.79		0.650694
				32020.56		0.650694
dx_2	0.00025			33048.79		0.698083
dy_2	0.00025			36759.49		0.367595

								Celsius				
	1	2	3	4	5	6	7	8	9	10	11	12
5	85	84.79578	84.62233	84.46994	84.33624	84.22061	84.12286	84.04293	83.98079	83.93642	83.9098	83.90093
4	85	84.8164	84.64769	84.49641	84.36295	84.24734	84.14958	84.06962	84.00746	83.96306	83.93643	83.92756
3	85	84.82211	84.65564	84.50507	84.3718	84.25624	84.15848	84.07852	84.01634	83.97195	83.94531	83.93644
2	85	84.8164	84.64769	84.49641	84.36295	84.24734	84.14958	84.06962	84.00746	83.96306	83.93643	83.92756
1	85	84.79578	84.62233	84.46994	84.33624	84.22061	84.12286	84.04293	83.98079	83.93642	83.9098	83.90093
q"	36759.4946	31221.75	27429.57	24065.8	20813.81	17594.44	14386.92	11185.23	7987.124	4791.366	1596.972	
q"avg	0.67981243	0.586513	0.514954	0.448796	0.384082	0.319814	0.255722	0.191724	0.127785	0.063883	0.01597	
qcond	3.5890544											

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
17	85	84.93753	84.88169	84.82925	84.77908	84.73067	84.68379	84.63828	84.59405	84.55105	84.50925	84.46861	84.42912	84.39078	84.35356	84.31747
16	85	84.94323	84.889	84.83712	84.78719	84.73891	84.69209	84.64661	84.60241	84.55942	84.51762	84.47699	84.4375	84.39915	84.36194	84.32584
15	85	84.94639	84.89397	84.84305	84.79365	84.74567	84.69904	84.65368	84.60955	84.56661	84.52484	84.48422	84.44474	84.4064	84.36918	84.33309
14	85	84.94838	84.89742	84.84747	84.79868	84.7511	84.70472	84.65952	84.61549	84.57262	84.5309	84.49031	84.45085	84.41252	84.37531	84.33922
13	85	84.94971	84.89985	84.85072	84.80251	84.75532	84.70921	84.66419	84.62028	84.57749	84.53582	84.49526	84.45583	84.41751	84.38031	84.34423
12	85	84.95062	84.90155	84.85305	84.80532	84.75847	84.7126	84.66775	84.62396	84.58124	84.53962	84.4991	84.45969	84.42139	84.3842	84.34812
11	85	84.95121	84.90268	84.85463	84.80724	84.76065	84.71497	84.67025	84.62655	84.5839	84.54232	84.50183	84.46244	84.42415	84.38697	84.3509
10	85	84.95155	84.90332	84.85554	84.80836	84.76193	84.71637	84.67173	84.6281	84.58549	84.54394	84.50347	84.46409	84.42581	84.38863	84.35256
9	85	84.95166	84.90354	84.85583	84.80873	84.76236	84.71683	84.67223	84.62861	84.58601	84.54448	84.50401	84.46464	84.42636	84.38919	84.35312
8	85	84.95155	84.90332	84.85554	84.80836	84.76193	84.71637	84.67173	84.6281	84.58549	84.54394	84.50347	84.46409	84.42581	84.38863	84.35256
7	85	84.95121	84.90268	84.85463	84.80724	84.76065	84.71497	84.67025	84.62655	84.5839	84.54232	84.50183	84.46244	84.42415	84.38697	84.3509
6	85	84.95062	84.90155	84.85305	84.80532	84.75847	84.7126	84.66775	84.62396	84.58124	84.53962	84.4991	84.45969	84.42139	84.3842	84.34812
5	85	84.94971	84.89985	84.85072	84.80251	84.75532	84.70921	84.66419	84.62028	84.57749	84.53582	84.49526	84.45583	84.41751	84.38031	84.34423
4	85	84.94838	84.89742	84.84747	84.79868	84.7511	84.70472	84.65952	84.61549	84.57262	84.5309	84.49031	84.45085	84.41252	84.37531	84.33922
3	85	84.94639	84.89397	84.84305	84.79365	84.74567	84.69904	84.65368	84.60955	84.56661	84.52484	84.48422	84.44474	84.4064	84.36918	84.33309
2	85	84.94323	84.889	84.83712	84.78719	84.73891	84.69209	84.64661	84.60241	84.55942	84.51762	84.47699	84.4375	84.39915	84.36194	84.32584
1	85	84.93753	84.88169	84.82925	84.77908	84.73067	84.68379	84.63828	84.59405	84.55105	84.50925	84.46861	84.42912	84.39078	84.35356	84.31747
q"	44978.885	40203.16	37756.14	36125.7	34850.24	33757.09	32768.23	31843.08	30958.44	30100.23	29259.58	28430.75	27609.98	26794.79	25983.49	25174.95
q"avg	0.21295512	0.194898	0.184705	0.17744	0.171518	0.166313	0.161528	0.157004	0.152647	0.1484	0.144226	0.140102	0.136012	0.131946	0.127896	0.123858
qcond	4.38710814															

	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
17	84.28251	84.24866	84.21594	84.18433	84.15383	84.12445	84.09618	84.06903	84.04299	84.01806	83.99424	83.97153	83.94993	83.92944	83.91006	83.89179
16	84.29088	84.25703	84.2243	84.19268	84.16218	84.1328	84.10453	84.07737	84.05132	84.02639	84.00257	83.97986	83.95826	83.93777	83.91838	83.90011
15	84.29812	84.26427	84.23154	84.19992	84.16942	84.14003	84.11176	84.0846	84.05855	84.03362	84.00979	83.98708	83.96547	83.94498	83.9256	83.90732
14	84.30425	84.2704	84.23767	84.20605	84.17555	84.14616	84.11788	84.09072	84.06467	84.03973	84.0159	83.99319	83.97158	83.95108	83.9317	83.91342
13	84.30926	84.27541	84.24268	84.21106	84.18056	84.15117	84.12289	84.09572	84.06967	84.04473	84.0209	83.99818	83.97658	83.95608	83.93669	83.91842
12	84.31316	84.27931	84.24658	84.21496	84.18445	84.15506	84.12678	84.09962	84.07356	84.04862	84.02479	84.00207	83.98046	83.95997	83.94058	83.9223
11	84.31594	84.28209	84.24936	84.21774	84.18724	84.15784	84.12956	84.1024	84.07634	84.0514	84.02757	84.00485	83.98324	83.96274	83.94335	83.92507
10	84.31761	84.28376	84.25103	84.21941	84.18891	84.15951	84.13123	84.10407	84.07801	84.05307	84.02924	84.00651	83.9849	83.96441	83.94502	83.92674
9	84.31816	84.28432	84.25159	84.21997	84.18946	84.16007	84.13179	84.10462	84.07857	84.05362	84.02979	84.00707	83.98546	83.96496	83.94557	83.92729
8	84.31761	84.28376	84.25103	84.21941	84.18891	84.15951	84.13123	84.10407	84.07801	84.05307	84.02924	84.00651	83.9849	83.96441	83.94502	83.92674
7	84.31594	84.28209	84.24936	84.21774	84.18724	84.15784	84.12956	84.1024	84.07634	84.0514	84.02757	84.00485	83.98324	83.96274	83.94335	83.92507
6	84.31316	84.27931	84.24658	84.21496	84.18445	84.15506	84.12678	84.09962	84.07356	84.04862	84.02479	84.00207	83.98046	83.95997	83.94058	83.9223
5	84.30926	84.27541	84.24268	84.21106	84.18056	84.15117	84.12289	84.09572	84.06967	84.04473	84.0209	83.99818	83.97658	83.95608	83.93669	83.91842
4	84.30425	84.2704	84.23767	84.20605	84.17555	84.14616	84.11788	84.09072	84.06467	84.03973	84.0159	83.99319	83.97158	83.95108	83.9317	83.91342
3	84.29812	84.26427	84.23154	84.19992	84.16942	84.14003	84.11176	84.0846	84.05855	84.03362	84.00979	83.98708	83.96547	83.94498	83.9256	83.90732
2	84.29088	84.25703	84.2243	84.19268	84.16218	84.1328	84.10453	84.07737	84.05132	84.02639	84.00257	83.97986	83.95826	83.93777	83.91838	83.90011
1	84.28251	84.24866	84.21594	84.18433	84.15383	84.12445	84.09618	84.06903	84.04299	84.01806	83.99424	83.97153	83.94993	83.92944	83.91006	83.89179
q"	24368.42	23563.37	22759.44	21956.4	21154.07	20352.33	19551.09	18750.3	17949.9	17149.86	16350.15	15550.75	14751.63	13952.77	13154.17	12355.8
q"avg	0.119829	0.115807	0.11179	0.107776	0.103766	0.099759	0.095753	0.091751	0.087749	0.08375	0.079752	0.075756	0.071761	0.067767	0.063775	0.059784

	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47
17	83.87463	83.85858	83.84364	83.8298	83.81707	83.80545	83.79494	83.78553	83.77723	83.77004	83.76395	83.75897	83.7551	83.75234	83.75068	83.75012
16	83.88295	83.86689	83.85195	83.83811	83.82538	83.81376	83.80324	83.79384	83.78554	83.77834	83.77226	83.76728	83.7634	83.76064	83.75898	83.75842
15	83.89016	83.8741	83.85915	83.84531	83.83258	83.82096	83.81044	83.80103	83.79273	83.78554	83.77945	83.77447	83.7706	83.76783	83.76617	83.76562
14	83.89626	83.8802	83.86525	83.85141	83.83868	83.82705	83.81654	83.80713	83.79882	83.79163	83.78554	83.78056	83.77669	83.77392	83.77226	83.77171
13	83.90125	83.88519	83.87024	83.8564	83.84366	83.83204	83.82152	83.81211	83.80381	83.79661	83.79052	83.78554	83.78167	83.7789	83.77724	83.77669
12	83.90513	83.88907	83.87412	83.86028	83.84754	83.83592	83.8254	83.81599	83.80768	83.80049	83.7944	83.78942	83.78554	83.78278	83.78112	83.78056
11	83.9079	83.89184	83.87689	83.86305	83.85031	83.83869	83.82817	83.81876	83.81045	83.80326	83.79717	83.79219	83.78831	83.78554	83.78388	83.78333
10	83.90957	83.89351	83.87855	83.86471	83.85197	83.84035	83.82983	83.82042	83.81211	83.80492	83.79883	83.79385	83.78997	83.7872	83.78554	83.78499
9	83.91012	83.89406	83.87911	83.86526	83.85253	83.8409	83.83038	83.82097	83.81267	83.80547	83.79938	83.7944	83.79053	83.78776	83.7861	83.78554
8	83.90957	83.89351	83.87855	83.86471	83.85197	83.84035	83.82983	83.82042	83.81211	83.80492	83.79883	83.79385	83.78997	83.7872	83.78554	83.78499
7	83.9079	83.89184	83.87689	83.86305	83.85031	83.83869	83.82817	83.81876	83.81045	83.80326	83.79717	83.79219	83.78831	83.78554	83.78388	83.78333
6	83.90513	83.88907	83.87412	83.86028	83.84754	83.83592	83.8254	83.81599	83.80768	83.80049	83.7944	83.78942	83.78554	83.78278	83.78112	83.78056
5	83.90125	83.88519	83.87024	83.8564	83.84366	83.83204	83.82152	83.81211	83.80381	83.79661	83.79052	83.78554	83.78167	83.7789	83.77724	83.77669
4	83.89626	83.8802	83.86525	83.85141	83.83868	83.82705	83.81654	83.80713	83.79882	83.79163	83.78554	83.78056	83.77669	83.77392	83.77226	83.77171
3	83.89016	83.8741	83.85915	83.84531	83.83258	83.82096	83.81044	83.80103	83.79273	83.78554	83.77945	83.77447	83.7706	83.76783	83.76617	83.76562
2	83.88295	83.86689	83.85195	83.83811	83.82538	83.81376	83.80324	83.79384	83.78554	83.77834	83.77226	83.76728	83.7634	83.76064	83.75898	83.75842
1	83.87463	83.85858	83.84364	83.8298	83.81707	83.80545	83.79494	83.78553	83.77723	83.77004	83.76395	83.75897	83.7551	83.75234	83.75068	83.75012
q"	11557.64	10759.69	9961.923	9164.332	8366.9	7569.614	6772.46	5975.423	5178.49	4381.646	3584.879	2788.174	1991.518	1194.896	398.2943	
q"avg	0.055795	0.051804	0.047816	0.043828	0.039841	0.035855	0.03187	0.027885	0.0239	0.019916	0.015933	0.011949	0.007966	0.003983	0.000996	

qf=	44978.89	qf"avg	0.202965
	36207.14		0.177533
	34806.24		0.177533
	36207.14		0.202965
	44978.88		0.112447

fdm - fine

temp

85	84.93753	84.88169	84.82925	84.77908	84.73067	84.68379	84.63828	84.59405	84.55105	84.50925	84.46861	84.42912	84.39077	84.35356	84.31747	84.28251	84.24866	84.21593	84.18432	84.15383	84.12445	84.09618	8

heat transfer

0.212955	0.194899	0.184705	0.17744	0.171519	0.166314	0.161529	0.157004	0.152647	0.1484	0.144226	0.140102	0.136012	0.131946	0.127896	0.123859	0.11983	0.115807	0.11179	0.107776	0.103766	0.099759	0.095754	0

qf"avg

0.202965	0.177533	0.177533	0.202965	0.112447

Ttip

83.75012	83.77669	83.78554	83.77669	83.75012

1D

k	180	
h	100	
Tinf	20	293.15
Tb	85	358.15
dx_1	0.001	
dy	0.001	
N	3	
W	0.02	
dx_2	0.00025	
dy_2	0.00025	

	85	84.70847	84.44801	84.21848	84.01978	83.85181	83.71449	83.60775	83.53154	83.48584	83.4706
	heat transfer										
q"	52474.67	46883.64	41315.11	35766.41	30234.88	24717.86	19212.71	13716.78	8227.435	2742.04	
q"avg	0.05591	0.055685	0.055487	0.055315	0.05517	0.055052	0.054959	0.054893	0.054854	0.02742	

References

Incropera, Frank P., and David P. DeWitt. *Introduction to Heat Transfer*. Wiley, 2009.