Nationalistic bias in collusion prosecution: The case for international antitrust agreements*

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Abstract

We study the incentives of competition authorities to prosecute collusive practices of domestic and foreign firms in a multi-market contact model between two firms operating in two countries. In equilibrium, the country of origin of the firms might prefer to delay prosecution to protect profits in foreign markets. This strategic delay is valuable because prosecution in the country of origin of the firms activates an information spillover that triggers prosecution in the foreign country. Prosecution delays, however, are not optimal from the point of view of global welfare, something that could be solved by integrating the competition authorities. With multiple industries, both countries can be better off under integration or signing an international antitrust agreement.

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1 Introduction

Globalization has brought new challenges to antitrust policy (see, for example, Connor (2004), Connor and Helmers (2007), Barnett (2007)). National competition authorities have reacted to the rise of globalization by devoting special attention to collusive practices involving multinational companies. The challenges associated with international antitrust enforcement have also led to proposals for increasing international cooperation among competition authorities (see, for example, Barnett (2007)). Nevertheless, most of the formal literature that studies anticompetitive behavior focuses on closed economies and disregards interactions among national competition authorities.[1] The aim of this article is to provide a formal

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[1]We discuss some interesting exceptions in Section 2.

analysis of the incentives of antitrust authorities to prosecute domestic and foreign firms involved in anticompetitive behavior when prosecution in one country generates informational spillovers to other countries.

We show that, in equilibrium, some countries may delay prosecution of national firms even in the presence of evidence of collusive behavior in the domestic market. This strategic delay is potentially damaging for the rest of the world. Prosecution delays are also not optimal from the point of view of global welfare. Moreover, we show that in a multi-industry world, countries can be better off under an integrated competition authority that maximizes the combined welfare or, equivalently, signing an international agreement in which they commit to have competition authorities that only target consumer welfare. Under integration, each country loses the collusion profits in the foreign market, but it gains the increase in consumer surpluses in domestic markets.

We start off our analysis in Section 3 by developing a simple two-country model of collusion and antitrust policy in which two multinational firms operate in both countries. In each country, there is a competition authority in charge of enforcing antitrust laws which targets total welfare. In every period, the firms decide whether to collude or compete. Afterwards, each competition authority receives information about the multinational firms' behavior and decides whether to prosecute the firms or not in a sequential manner. First, the competition authority in the country of origin of the firms receives a signal that directly informs about the collusion behavior of the firms in the domestic market as well as in the foreign market and, using this information, it decides whether to prosecute the firms or not. Second, the competition authority of the foreign country observes the prosecution decision of the country of origin, possibly receives an independent signal about the firms' behavior and, finally, decides whether to prosecute the firms or not.[2]

Our baseline model in Section 4 considers a scenario where the competition authority of the foreign country has no independent detection ability and must fully rely on the prosecution decision of the country of origin to obtain evidence about collusion in its own market before it can decide whether to prosecute the firms. The model generates three key results. First, the foreign country always prosecutes foreign-owned firms as soon as there is information of prosecution in the country of origin.

Second, the competition authority in the country of origin prosecutes a domestic firm that is colluding both in the domestic and the foreign market if the gain in consumer surplus exceeds the reduction in profits (including fines) from both domestic and foreign markets. The intuition is the following. Prosecuting collusion in the country of origin always increases domestic surplus because of the reduction in deadweight loss. However, due to the informational spillover across competition authorities, prosecution in the country of origin triggers prosecution in the foreign country, which activates fines and eliminates the profits from collusion that national firms are making in the foreign country. Thus, in equilibrium, it could be the case that the competition authority in the country of origin of the firms chooses not to prosecute collusion.

Third, even if the country of origin prosecutes the firms, the foreign country is affected by the antitrust policy of the country of origin. In particular, when the probability of detecting collusion in the country of origin is low, firms collude in both countries until they are detected and prosecuted in the country

[2] The idea is that it tends to be more difficult for a country to independently detect collusive behavior of foreign than domestic firms. As consequence, competition authorities often rely on the information that spills from firms being prosecuted in their country of origin. Indeed, the observation of the collusion cases brought about by foreign antitrust authorities has become a standard screening procedure (in addition to other forms of investigation) to detect collusion. See OECD (2013) for a report on best practices in cartel investigation.

of origin. Afterwards, the competition authority of the foreign country learns about the collusion and prosecutes the firms as well. On the contrary, when the probability of detecting collusion in the country of origin is high, firms only collude in the foreign country and they are never detected. This suggests that multinational companies might be competing in their home-country—usually a developed country with a professional and well-funded competition authority—but colluding in foreign markets—usually developing countries which do not have a competition authority capable of independently detecting collusion.

Having characterized the key trade-offs that drive the decisions of firms and competition authorities when the foreign country has no independent detection ability, in Section 5 we study how the solution of the model changes when the foreign country can also detect collusion independently. More specifically, we show that the equilibrium is characterized by finite delays in prosecution. To the best of our knowledge, this is the first paper that formally models and obtains prosecution delays as an equilibrium. The intuition for why delays in prosecution are finite in this extended model is as follows. The country of origin's decision to restrain from prosecuting is less valuable than in the baseline model, since now foreign collusion profits can no longer be indefinitely protected with a lenient prosecution policy. Therefore, the country of origin is more willing to prosecute the firms as soon as collusion is detected, ameliorating its nationalistic bias in the prosecution decision. Furthermore, even if the country of origin chooses not to prosecute the firms when they are colluding in both countries, eventually, the foreign country detects and prosecutes collusion on its own. When this happens, the country of origin will prosecute collusion too, since there are no more collusive profits to protect abroad.

In section 6, we discuss the implications of our results for international antitrust policy cooperation. To do so, we add to the model a second industry in which the country of origin is different than in the first industry. In this scenario, a country can lead investigations in one industry and be a follower in the other. More specifically, in industry X, the competition authority in country A receives a signal that directly informs about the collusion behavior of the firms in the domestic market and in the foreign market; afterwards, the competition authority of country B receives a signal about the collusion behavior of the firms in both markets and observes the decision of country A to prosecute the firms or not. In industry Y the situation is reversed: the competition authority in country B receives a signal that directly informs about the collusion behavior of the firms in the domestic market and in the foreign market; afterwards, the competition authority of country A receives a signal about the collusion behavior of the firms in both markets and observes the decision of country B to prosecute the firms or not. The multi-industry extension opens the door to mutual gains from international cooperation through an international agreement or the integration of the competition authorities.[3] Indeed, in a multi-industry world, both countries can be better off under an integrated competition authority that maximizes the combined welfare. The intuition is that, under integration, each country loses the collusion profits in the foreign market, but it gains the increase in consumer surpluses in both domestic markets. When countries are of different sizes, this might not be enough to avoid winners and losers from integration, but in the case of perfectly symmetric countries, we prove that both countries gain with an integrated competition authority. Finally, an equivalent outcome can be reached through an international agreement in which governments commit to have competition authorities that only target consumer welfare. This outcome can be sustained only in a repeated setting, as in the one shot game, if one of the countries is fully

[3]In the one-industry model, while the equilibrium does not maximize the combined welfare of the countries (as it involves a strategic prosecution delay), there is not much room for international cooperation to fix the problem as the country of origin of the firms has no incentive to change its antitrust policy.

committed to only protecting domestic consumers, then the other country could have no incentive to negotiate an international agreement since delaying the prosecution of national firms operating in foreign markets is a dominant strategy for a competition authority with a mandate to maximize national welfare.

The rest of the paper is organized as follows. Section 2 discusses the relationships between this paper and the literature on multi-market collusion; international trade agreements and export cartels; and the empirical evidence on antitrust enforcement by multiple jurisdictions. Section 3 develops the setup of the model. Section 4 characterizes the equilibrium under our baseline scenario, that is, when detection ability is fully aligned with ownership. Section 5 characterizes the equilibrium, when detection ability is only partially aligned with ownership. Section 6 extends the model to a multi-industry setting and explores the implications for international antitrust agreements and integration. Section 7 concludes. All proofs can be found in the Appendix.

2 Related Literature

There are two bodies of theoretical literature related to this paper: (i) multi-market collusion; and (ii) international trade agreements and export cartels. In addition, the paper is connected with a few empirical works that explore the evidence on antitrust enforcement by multiple jurisdictions.

2.1 Multi-market Collusion

This paper is naturally linked to the literature on multi-market collusion. Bernheim and Whinston (1990) were the first to formalize the idea of simultaneous collusion in multiple markets. They show that under asymmetric markets a larger set of collusive outcomes can be sustained in a multi-market setup. In the context of an open economy, Bond and Syropoulos (2008), Akinbosoye et al. (2012) and Agnosteva et al. (2018) study the relationship between international trade, collusion and multimarket contact. Bond and Syropoulos (2008) shows that if trade costs are not too high, then trade liberalization may facilitate collusion and reduce welfare. Akinbosoye et al. (2012) develop a segmented-markets duopoly model with differentiated goods and shows that when goods are close substitutes and trade costs are high, a reduction in trade costs makes collusion easier. Agnosteva et al. (2018) develop a model of multimarket collusion and empirically show that trade costs exert a negative and significant effect on cartel discipline. Our paper contributes to this literature focusing on the endogenous determination of antitrust enforcement in the context of an open economy and the strategic interactions that could emerge between national competition authorities.

Few papers study antitrust enforcement in a multi-market contact model. Choi and Gerlach (2012a,b, 2013), employ a multi-market model with symmetric countries to study the relationship between demand linkages and antitrust enforcement. They show that antitrust enforcement in one market spills over to other markets in a way that depends on whether the products are complements or substitutes. Choi and Gerlach (2013) serves as the basis for Choi and Gerlach (2012b), where they study the case of firms producing substitute products in different countries when there are trade frictions and local competition authorities have prosecution costs. They show that enforcement is non-monotonic with respect to trade integration. In particular, cartel enforcement is high if the economies are either closely integrated or trade costs are very high. Finally, they compare two regimes, one with local competition authorities and the other with a global competition authority and identify two sources of inefficiency associated with local prosecution, namely, decentralized information and cross-market externalities. Finally, Choi and Gerlach (2012a) study multi-market collusion with leniency and different information sharing policies between competition authorities. Our work departs from Choi and Gerlach (2012a) and Choi and Gerlach (2012b) in several important ways. First, we assume that competition authorities seek to maximize national welfare instead of the consumer surplus of domestic. Second, we employ a timing and information structure that allows us to model a leader and a follower in antitrust enforcement instead of having competition authorities deciding simultaneously. This is a relevant scenario to explore, given that the country of origin of the firms has more accessible information regarding the collusive behavior of the firms. It also captures the asymmetry in resources and capabilities between competition authorities in developed and developing countries.[4] More importantly, this information structure naturally pushes the leading country to internalize the potential information spillovers of its prosecution decisions. Third, in our model the nature of the relationships between products is not relevant for the sustainability of collusive agreements. The linkage in our setting is purely through an information channel. Finally, in our model, integrating antitrust policy has cross-country distributive effects. In particular, we show that in a one-industry model, one country is always worse off under integration.

2.2 International Trade Agreements and Export Cartels

The way we approach antitrust enforcement in an international setting has similarities with the literature on the terms of trade approach to international trade agreements (Bagwell and Staiger (1999)). In the context of trade policy, a country that seeks to maximize national welfare has an incentive to impose a tariff in order to improve its terms of trade at the expense of its trade partners. Analogously, the country of origin of the firms organizing collusion has an incentive to postpone prosecution in order to protect the market power of domestic firms in foreign markets. In the context of trade policy, trade agreements could make both countries better off inducing a mutual reduction in tariffs. In our two-industry model, the integration of antitrust policy in a single competition authority could make both countries better off eliminating prosecution delays in both industries.

Our paper is also related to the literature on export cartels. Consider a given industry and a country whose exporting firms sell at competitive prices in the domestic market, but they form a cartel to export and, hence, charge collusive prices to foreign customers. Then, everybody in this country is better off than if export cartels are banned. Producers collect collusive profits, while domestic consumers face the same or lower prices. The only losers are foreign consumers, who must pay higher prices.[5,6] The main innovation that our paper introduces to the literature on export cartels is the idea that the enforcement of antitrust policy can be informative for foreign competition authorities. Also, rather than assuming the existence of an export cartel, we endogenously derive it a possible equilibrium outcome.

[4]See, for example Levenstein and Suslow (2003).

[5]Indeed, export cartels are lawful in several countries. As Levenstein and Suslow (2004) explains, this can be either explicit (through statutes that specify that export cartels are exempt of antitrust laws) or implicit when antitrust laws only take into account domestic effects.

[6]Some authors do not agree with this logic. For example, Schultz (2002) develops a model in which export cartels can slacken the incentive constraint for collusion in the domestic market, with negative consequences for the domestic economy.

2.3 Antitrust Enforcement by Multiple Jurisdictions

A few works have explored the limited exiting empirical evidence on antitrust enforcement by multiple jurisdictions. Regarding the existence of a nationalistic bias on antitrust prosecution, Garrett (2014) documents that between 2001 and 2012, 45% of antitrust prosecutions carried out by the U.S. Department of Justice were to foreign companies (namely 78 out of 175 companies). Data on Sherman Act violations yielding fines over 10 million dollars also shows that, between 1995 and 2017, only 19 of the 139 firms fined were domestic companies (see DOJ 2017). In the same vein, Cremieux and Snyder (2016) find evidence that the U.S. Department of Justice and the European Commission tend to over-prosecute firms based in the rest of the world, i.e., those outside the U.S. and Europe. These finding are consistent with our model as, in equilibrium, competition authorities immediately prosecute foreign firms engaging in collusion but, in some cases, they delay the prosecution of collusion between domestic firms. There is also a growing consensus that in many Asian countries the enforcement of antitrust laws is rather weak (see, for example, Connor 2007b). This is also consistent with our model as many Asian companies heavily depend on export markets and, therefore, protecting collusive profits in foreign markets is more likely to outweigh the burden on domestic consumers.

It is more complicated to clearly document strategic delays on antitrust prosecution and link the delays with theoretical mechanisms. Prosecution data from U.S. and Europe suggest that the European Commission tends to follow the US Department of Justice prosecuting collusion of European firms[7]. This is consistent with our model. Feinberg and Husted (2013) empirically explore free riding on antitrust enforcement between U.S. states. They find that the number of states participating in the litigation process promotes the free-riding behavior, while the resources available to the state government ameliorates it. Additionally, they use the number of horizontal conspiracies as a measurement of case complexity and find that it is associated with delays in prosecution efforts. Our model generates equilibrium strategic delays in prosecution through a completely different channel. In our model, it is not the case that a jurisdiction prefers to wait until another jurisdiction pays the cost of dissolving the cartels. The problem is that the jurisdiction of origin of the firms might prefer to delay prosecution to protect the profits of its firms in other jurisdictions.

3 A Simple Model of Antitrust Policy

This section develops a simple model of collusion and antitrust policy in which the firms involved are multinational corporations operating in several countries. In particular, consider 2 multinational firms ($i \in \{1, 2\}$) which operate in 2 countries ($j \in \{A, B\}$) and must decide whether to collude or compete. In each country there is a competition authority which has the power to fine firms if evidence of collusion is detected. Time is infinite, discrete and indexed by $t \in \{0, 1, ...\}$. All agents have a common discount factor $\delta \in (0, 1)$. The timing of events within each period is as follows:

1. Both firms simultaneously choose to collude or compete in each country.

[7] Using information from DOJ/EC prosecution decisions from 1994 to 2014. Based on replication data in Cremieux and Snyder (2016).

2. Nature sends a signal to the competition authority of country A with evidence of collusion in country A. Upon observing the signal, the competition authority of country A decides whether to prosecute the firms or not.
3. Nature sends a signal to the competition authority of country B with evidence of collusion in country B. Upon observing the signal, the competition authority of country B decides whether to prosecute the firms or not.

This timing is compatible with different signalling structures and contexts. In particular, note that the signal with evidence of collusion received by the competition authority of country B could depend on the prosecution decision selected by the competition authority of country A.

3.1 Firms and Consumers

Let $(a_t^{i,A}, a_t^{i,B})$ denote the decision of firm i in period t, where $a_t^{i,j} = 1$ indicates that firm i chooses to collude in country j and $a_t^{i,j} = 0$ indicates that firm i chooses to compete in country j. Let $\pi_t^{i,j}$ denote the profits earned by firm i from its operations in country j in period t (before paying any fine). Specifically, profits are given by:

Firm 1 - Firm 2	$a_t^{2,j} = 1$	$a_t^{2,j} = 0$
$a_t^{1,j} = 1$	$\pi^{c,j}$, $\pi^{c,j}$	0, $\pi^{d,j}$
$a_t^{1,j} = 0$	$\pi^{d,j}$, 0	0, 0

where $\pi^{d,j} > \pi^{c,j} > 0$. Thus, if both firms collude in country j, each obtains collusion profits $\pi^{c,j}$. However, each firm has a short term incentive to deviate from collusion, which leaves no profits for the rival. Finally, competition fully dissipates profits.[8] Let f_t^j denotes the fine imposed to a firm in period t by the competition authority of country j. Then, the profits net of fines obtained by firm i in period t are $\pi_t^i = \pi_t^{i,A} - f_t^A + \pi_t^{i,B} - f_t^B$ and the expected discounted profits of firm i are $\Pi_t^i = \mathbf{E}_t \left[\sum_{\tau=t}^{\infty} \delta^{\tau-t}\pi_\tau^i\right]$, where the expectation operator is conditional on the information available to the firms when they make a decision in period t.

The consumer surplus, CS_t^j, obtained by consumers from country j in period t is $CS^{c,j}$ when firms collude, and $CS^{com,j}$ otherwise.

3.2 Competition Authorities, Collusion Detection and Antitrust Prosecution

In each period, immediately after firms make their decisions, the competition authority of each country receives a signal with evidence of the behavior of the firms. Let $s_t^j \in \{0, 1\}$ denote the signal received by the competition authority of country j. $s_t^j = 1$ ($s_t^j = 0$) indicates that the competition authority of country j detects (does not detect) evidence of collusion in period t. The signal conveys evidence of collusion only if firms are effectively colluding. Otherwise, the competition authority receives a no collusion signal. After observing the signal, the competition authorities decide whether or not to prosecute the firms. Let $p_t^j \in \{0, 1\}$ denote the prosecution decision of the competition authority j in period t, where $p_t^j = 1$

[8] The underlying assumption is that the firms sell homogeneous products and compete in prices. Hence, when reverting to competition upon either a choice of not colluding or a failure of collusion, the profits of the firms are zero. Alternatively, in the case that firms compete in quantities or in prices with differentiated products, we can assume that profits are normalized, i.e., all profits are relative to competitive profits. These alternative assumptions do not affect the analysis.

indicates prosecution and $p_t^j = 0$ indicates no prosecution. We assume that a competition authority cannot prosecute without detecting evidence of collusion. Thus, when $s_t^j = 0$, it must be the case that $p_t^j = 0$. If evidence of collusion is detected, i.e., $s_t^j = 1$, and the competition authority prosecutes the firms, i.e., $p_t^j = 1$, then, each firm must pay a fine $f_t^j = f^j$ and thereafter, the firms are induced to compete in all subsequent periods as in Harrington Jr. (2014).[9]

The national welfare of country j in period t is given by:

$$w_t^j = CS_t^j + 2f_t^j + \lambda\left(\sigma^{1,j}\pi_t^1 + \sigma^{2,j}\pi_t^2\right) \tag{1}$$

The first term is the consumer surplus in country j; the second is the fines collected by the competition authority of country j; the third term is the sum of the shares of the profits (net of fines) of firms 1 and 2 accruing to citizens of country j, where $\sigma^{i,j} \in [0,1]$ indicates the share of firm i owned by citizens from country j. This last term is multiplied by $\lambda \in [0,1]$ which represents the weight that country j assigns to firms profits (in other words, their producer surplus).

We assume that $CS^{com,j} > CS^{c,j} + 2f^j$ for all j, i.e., in each country, the aggregate surplus under competition ($CS^{com,j}$) is higher than the consumer surplus under collusion ($CS^{c,j}$) plus the fines of collusion ($2f^j$). This means that even for a country that does not value producer surplus ($\lambda = 0$), aggregate surplus under competition is higher than under collusion in a given period t.[10]

The objective of the competition authority of country j is to maximize the expected discounted welfare of the country, which is given by:[11]

$$W_t^j = \mathbf{E}_t\left[\sum\nolimits_{\tau=t}^{\infty} \delta^{\tau-t} w_\tau^j\right] \tag{2}$$

3.3 Definition of Equilibrium

The collusion and antitrust policy model is an infinite dynamic game with four players: the two firms and the two competition authorities. In the Appendix we formally define a strategy and payoff function for each player, a collusion signal function for each competition authority, and an equilibrium for the collusion and antitrust policy game. Specifically, an equilibrium of the collusion and antitrust game is a pair of collusion strategies for the firms $\left(\mathbf{a}^1, \mathbf{a}^2\right)$ and a pair of prosecution strategies for the competition authorities $\left(\mathbf{p}^A, \mathbf{p}^B\right)$ that, given the collusion signal functions $\left(\mathbf{s}^A, \mathbf{s}^B\right)$, form a subgame perfect Nash equilibrium.

4 Detection Ability Fully Aligned with Firm Ownership

This section studies the equilibrium of the model when both firms are owned by citizens of country A and the competition authority of country B is not able to detect evidence of collusion on its own, but it

[9] Note that we are implicitly assuming that the actions of the firms $a_t^{i,j}$ are public information, i.e., known by the firms and also by the competition authorities, but that knowing that there is collusion is not the same as having enough evidence to prosecute the firms. Only when $s_t^j = 1$, the competition authority of country j has the required information to successfully prosecute the firms.

[10] As $f_t^j - \pi^{c,j} > 0$, this assumption implies $CS^{com,j} > CS^{c,j} + 2\pi^{c,j}$.

[11] See Section 7 for a discussion on the several ways to justify (2) as the objective function of the competition authority. See Amir et al. (2009) for a similar approach in a merger control context.

can learn from the prosecution process in country A. Formally, regarding the ownership distribution of the firms, assume that $\sigma^{1,A} = \sigma^{1,B} = 1$, which implies that the profits of both firms obtained in country B are accounted for in the welfare of country A, while country B's welfare only includes the consumer surplus in country B and the fines collected there. With respect to collusion detection, consider the following assumption regarding the signals received by each competition authority.

Assumption 1. *The signals received by the competition authorities are:*

$$s_t^A = \begin{cases} 1 \text{ with probability } \alpha^A & \text{if } a_t^{1,A} = a_t^{2,A} = 1, \\ 0 \text{ with probability } \left(1 - \alpha^A\right) & \text{if } a_t^{1,A} = a_t^{2,A} = 1, \\ 0 & \text{otherwise.} \end{cases}$$

$$s_t^B = \begin{cases} 1 \quad \text{if } a_t^{1,B} = a_t^{2,B} = 1 \text{ and } p_\tau^A = 1 \text{ for some } \tau \leq t, \\ 0 \quad \text{otherwise.} \end{cases}$$

Assumption 1 states that the competition authority of country A can detect evidence of collusion on its own, while the competition authority of country B must rely on observing that country A has prosecuted the firms in order to detect collusion in country B.

The assumed ownership distribution of the firms together with Assumption 1 could capture the following situation. Consider two multinational firms whose shareholders are from a developed country A, which counts with a professional competition authority with the capacity to detect collusion. The firms also operate in a developing country B, whose competition authority does not have the resources and/or the expertise to detect and prosecute collusion on its own. However, if the competition authority of country B observes country A prosecuting the firms in country A, it will learn how to detect and prosecute collusion in country B.

In the next subsection we study the the antitrust policies selected by the competition authorities of both countries given the decisions of the firms to collude or not, and then, following backward induction, we proceed to study the equilibrium collusion decisions of the firms.

4.1 Prosecution Decisions

The following lemmas formally characterize the prosecution policies of both countries.

Lemma 1 *Suppose that Assumption 1 holds. Then, the competition authority of country B prosecutes the firms as soon as collusion is detected in country B.* ***Proof***: *See Appendix A.1.* ■

The intuition behind Lemma 1 is very simple. Since the competition authority of country B only benefits from the consumer surplus in country B and the fines (which are paid by foreign firms), it immediately prosecutes the firms as soon as collusion is detected.

Lemma 2 *Suppose that Assumption 1 holds. Assume that the competition authority of country A detects collusion, i.e., $s_t^A = 1$.*

1. *If the firms are not colluding in country B, then A always prosecutes the firms.*

2. *If the firms are also colluding in country B, then A prosecutes the firms if and only if*

$$\delta \geq \delta^*(\lambda) = \frac{2\big(\lambda f^B - (1-\lambda) f^A\big)}{CS^{com,A} - CS^{c,A} + 2\lambda\left(f^A - \pi^{c,A}\right) - 2f^A + 2\lambda\left(f^B - \pi^{c,B}\right)}, \tag{3}$$

*such that there exists $\lambda^* \in (0,1)$ where $\delta^*(\lambda) \geq 0$ for all $\lambda \geq \lambda^*$. Additionally, $\delta^*(\lambda)$ increases with λ. **Proof**: See Appendix A.1.* ■

The intuition behind Lemma 2 is as follows. When firms are colluding in both countries, if the competition authority of country A prosecutes the firms, this will trigger prosecution in country B. As a consequence, firms owned by shareholders from country A will have to pay fines in country B and, in the future, they will be forced to compete, which eliminates their profits from collusion in country B. Thus, prosecution in country A increases the total surplus in country A, but it reduces the profits of the firms in country B. Both effects will exists as long as country A sets a weight on the producer surplus above a certain threshold ($\lambda > \lambda^*$). Additionally, the second effect becomes stronger with increases in λ. When firms are only colluding in country A, the second effect disappears and, hence, the best policy for the competition authority of country A is to prosecute the firms when collusion is detected.

4.2 Collusion Decisions

We now proceed to study the equilibrium collusion decisions of the firms given the antitrust policies selected by the competition authorities of both countries. There are several sets of alternative assumptions that we could make regarding the scope of the collusive agreements (global versus market-specific collusion) and the punishment that firms can use to sustain collusion (global versus in each market separately). Proposition 1 explores the case in which firms can make market-specific collusion decisions and they employ the harshest possible punishment to deviators, i.e., they stop colluding in both countries even when the other firm deviates from collusion in only one market (see Appendix A.3 for variations of Proposition 1).[12] In any case, there might be multiple outcomes that can be sustained as a subgame perfect Nash equilibrium. To deal with this multiplicity, we assume that firms always coordinate in their most preferred equilibrium, i.e., the one that generates the highest expected profits for each firm. Note that this does not mean that we ignore other possible equilibria. On the contrary, for each set of parameters we deduce and compare all possible equilibria and select the equilibrium outcome with the highest expected profits.

Proposition 1 *Suppose that Assumption 1 holds, firms punish deviations from collusion stopping collusion in both countries and they coordinate in their best equilibrium. Given $\lambda > \lambda^*$*

1. *Suppose that $\bar{\delta} < \delta < \delta^*(\lambda)$, where $\bar{\delta} = \max\left\{\frac{\pi^{d,A}-\pi^{c,A}}{\pi^{d,A}}, \frac{\pi^{d,B}-\pi^{c,B}}{\pi^{d,B}}\right\}$. Then, firms collude in both countries and they are never prosecuted.*

[12] In Appendix A.3 we prove 3 different versions of Proposition 1. First, we assume that firms can only collude in both countries at the same time. This simplifies Proposition 1 as it does not allow for multiple equilibria and therefore there is no equilibrium selection. Second, we assume that firms can collude in each country separately (and punishments work similarly). As more conditions are needed for sustainability when colluding in both countries analysis is simpler at equilibrium selection. Finally, we allow for short run information spillovers. This allows firms to collude in country A until detected and then switch to country B (and go undetected).

2. *Suppose that $\delta \geq \delta^*(\lambda)$. Then, there are thresholds $\bar{\alpha}_L(\delta)$ and $\bar{\alpha}_H(\delta)$ such that:*

   (a) *Suppose that $\alpha^A > \bar{\alpha}_H(\delta)$. Then, firms collude in country B and they are never prosecuted.*

   (b) *Suppose that $\bar{\alpha}_L(\delta) < \alpha^A \leq \bar{\alpha}_H(\delta)$. Then, there is a threshold $\hat{\alpha}_{M_1}(\delta)$ such that: If $\alpha^A \in (\bar{\alpha}_L(\delta), \hat{\alpha}_{M_1}(\delta)]$, then firms collude (only in country A or in both countries) until the first time $s_t^A = 1$, when they are prosecuted. Thereafter, there is competition. If $\alpha^A \in [\hat{\alpha}_{M_1}(\delta), \bar{\alpha}_H(\delta)]$, then firms only collude in country B and they are never prosecuted.*

   (c) *Suppose that $\alpha^A \leq \bar{\alpha}_L(\delta)$. Then, there is a threshold $\hat{\alpha}_{M_2}(\delta)$ such that: If $\alpha^A \in [0, \hat{\alpha}_{M_2}(\delta))$, then firms collude (only in country A or in both countries) until the first time $s_t^A = 1$, when they are prosecuted. Thereafter, there is competition. If $\alpha^A \in [\hat{\alpha}_{M_2}(\delta), \bar{\alpha}_L(\delta)]$, then firms only collude in country B and they are never prosecuted.* ***Proof***: *See Appendix A.2.* ■

In order to determine the equilibrium in collusion decisions of the firms, we must take in consideration two aspects: i) what types of collusion (single market A, single market B or multimarket A and B) are sustainable and ii) what type of collusion of the ones that are sustainable yields the highest profits for the firms. Also, given the sequential nature of the game, the firms take in consideration the reaction of the prosecution authorities in their decisions. Parts 1 and 2 of Proposition 1 correspond to the different prosecution policies that the competition authority of country A can follow, according to Lemma 2. Part 1 of Proposition 1 states that when the discount factor is below the threshold for prosecution established in Lemma 2, then the the competition authority of country A never prosecutes the firms as long as collusion is also occurring in the foreign country. In that case, if firms are patient enough to sustain collusion ($\delta > \bar{\delta}$), they can safely collude in both countries without facing any risk of prosecution, which ensures the highest possible profit.

Part 2 of Proposition 1 concerns the situation when the competition authority of country A chooses to prosecute collusion if it occurs in country A or both in country A and B.[13] This generates three possible equilibrium paths for the firms, which depend on the probability with which the competition authority of country A detects collusion. When the detection probability is high ($\alpha^A > \bar{\alpha}_H(\delta)$), the expected discounted profits from collusion in country A are not enough to sustain neither collusion in both countries nor collusion in country A as an equilibrium. Only collusion in country B can be sustained as an equilibrium. In that case, firms collude in country B and they are never detected.

When the detection probability adopts intermediate values or ($\bar{\alpha}_L(\delta) < \alpha^A \leq \bar{\alpha}_H(\delta)$) only two types of collusion can be sustained as an equilibrium. First, firms can always sustain collusion in country B because $\delta > \bar{\delta}$ implies $\pi^{c,B} > (1-\delta)\pi^{d,B}$. Second, either collusion in country A or collusion in both countries can be sustained as an equilibrium. In one case or the other, prosecution upon detection by the competition authority of country A always ensues. So either the firms collude in both countries (enjoy higher profits, but will also more likely be fined in the foreign country) or they will collude just in country A (lower profits, but lower expected fines as well). Comparing profits we find threshold $\alpha_{\hat{M}_1}$: if α^A is higher than $\alpha_{\hat{M}_1}$, then firms collude only in B and they are never prosecuted; whereas if α^A is lower than $\alpha_{\hat{M}_1}$, then firms collude also in A until they are caught and prosecuted.

Finally, when the detection probability is low ($\alpha^A \leq \bar{\alpha}_L(\delta)$), the expected discounted profits from collusion in country A and B are both high enough to sustain every type of collusion as an equilibrium.

[13]If collusion occurs only in the foreign country, neither country prosecutes. In other words, firms can be dissuaded to collude in country A, but they are not dissuaded to engage in collusion in country B.

Then, firms select the equilibrium with the highest expected discounted profits. If in such equilibrium firms are colluding in country A, eventually, the competition authority of country A will detect and prosecute the firms. Moreover, if firms are also colluding in country B, the competition authority of country B will follow the same course of action and, thereafter, firms will be forced to compete in both countries. The following example illustrates Proposition 1 for a simple Bertrand specification in which the demands reflect proportional market sizes across countries and fines are proportional to sales (or in this setup of zero costs, proportional to profits).[14]

Example 1. Suppose that the demand function in country j is given by $Q_d^j = k_c^j - P^j$ where P^j is the price in country j with $k_c^A = 1$ and $k_c^B = k_c > 0$. Thus, k_c measures the relative size of country B with respect to country A. Assume that the cost function of firm i in country j is $c^{i,j} = 0$. Then, profits under collusion are given by $\pi^{c,A} = 1/8$ and $\pi^{c,B} = k_c^2/8$; profits from deviation are $\pi^{d,A} = 1/4$ and $\pi^{d,B} = k_c^2/4$; and profits under competition are 0 in both countries. Finally, assume that fines are given by $f^A = \gamma\pi^{c,A}$ and $f^B = \gamma\pi^{c,B}$ with $\gamma \geq 1$.

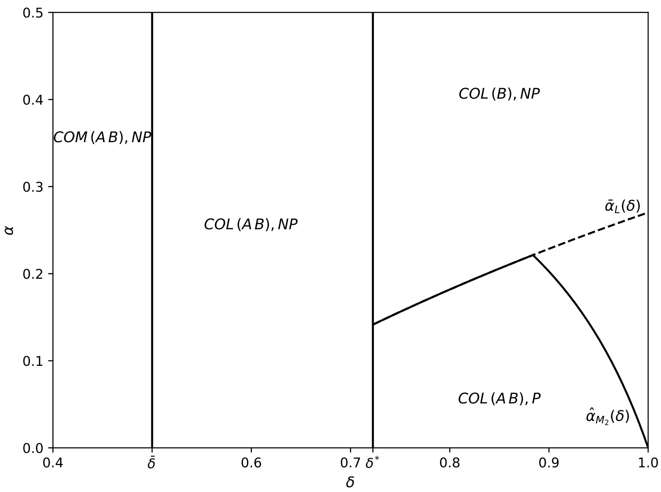

Figure 1: Equilibrium when detection ability is fully aligned with firm ownership. Note: $k_c^A = 1$ and $k_c^B = k_c = 0.55$, $\gamma = 1.2$ and $\lambda = 1$.

Figure 1 shows the equilibrium outcome for each value of δ and α^A for the specification in example 1 with $k_c = 0.55$ and $\gamma = 1.2$. In the figure $COM\,(A, B)$ indicates that firms compete in both countries; $COL\,(A, B)\,, NP$ that firms collude in both countries and they are never prosecuted; $COL\,(B)\,, NP$ that firms collude in country B and they are never prosecuted; and $COL\,(A, B)\,, P$, that firms collude in both countries until the first time $s_t^A = 1$, when they are prosecuted and forced to compete in both countries. For $\bar{\delta} < \delta < \delta^*(\lambda)$, firms collude in both countries and they are never prosecuted. For $\delta \geq \delta^*(\lambda)$, if $\alpha^A > \bar{\alpha}_L\,(\delta)$ or $\alpha^A \leq \bar{\alpha}_L\,(\delta)$ and $\alpha^A > \hat{\alpha}_{M_2}\,(\delta)$, firms collude in country B and they are never prosecuted; while if $\alpha^A \leq \bar{\alpha}_L\,(\delta)$ and $\alpha^A < \hat{\alpha}_{M_2}\,(\delta)$, firms collude in both countries until the first time $s_t^A = 1$, when they are prosecuted and forced to compete in both countries.

[14]In this example, $\bar{\alpha}_H(\delta)$ and $\bar{\alpha}_L(\delta)$ coincide.

Figure 1 also illustrates a very interesting implication of Proposition 1. Note that for $\delta \geq \delta^*(\lambda)$, an improvement in antitrust policy in country A (i.e., an increase in α^A) could hurt country B. Consider, for example, an increase in α^A such that, initially, the equilibrium is $COL\left(A,B\right),P$ and, after the change in α^A, it is $COL\left(B\right),NP$. Under $COL\left(A,B\right),P$, firms collude in both countries until the first time that $s_t^A = 1$, when the competition authorities of both countries prosecute the firms. In other words, eventually, firms compete in country B. On the contrary, under $COL\left(B\right),NP$, the competition authority of country B never learns how to prosecute the firms and, hence, collusion in country B lasts forever.

5 Detection Ability Partially Aligned with Firm Ownership

As in the previous section, assume that both firms are owned by citizens of country A (formally, $\sigma^{1,A} = \sigma^{2,A} = 1$), which implies that the profits of both firms obtained in country B are accounted for in the welfare of country A, while country B's welfare only includes the consumer surplus in country B and the fines collected there. Differently from the previous section, assume that the competition authority of country B has two channels to detect collusion: learn from the prosecution decisions of the competition authority of country A or detect collusion on its own. More specifically, consider the following assumption regarding the signals received by each competition authority. Additionally, we will also assume hereinafter that competition authorities assign the same weight to consumer and producer surplus ($\lambda = 1$).

Assumption 2. *The signal received by competition authority A is as in Assumption 1 and the signal received by competition authority B is given by:*

$$s_t^B = \begin{cases} 1 \textit{ with probability } \alpha^B & \textit{if } a_t^{1,B} = a_t^{2,B} = 1 \textit{ and } p_\tau^A = 0 \text{ for all } \tau \leq t, \\ 0 \textit{ with probability } \left(1-\alpha^B\right) & \textit{if } a_t^{1,B} = a_t^{2,B} = 1 \textit{ and } p_\tau^A = 0 \text{ for all } \tau \leq t, \\ 1 & \textit{if } a_t^{1,B} = a_t^{2,B} = 1 \textit{ and } p_\tau^A = 1 \text{ for some } \tau \leq t, \\ 0 & \textit{otherwise.} \end{cases}$$

Two remarks should be made about Assumption 2. First, as in the previous section, it is still the case that prosecution of the firms in country A is the most informative signal of collusion for competition authority B. Indeed, if competition authority A prosecutes the firms in country A, competition authority B immediately learns how to detect collusion in B in the present as well as in all future periods. Second, even if competition authority A does not prosecute the firms, it is possible that competition authority B detects collusion in B.

5.1 Prosecution Decisions

We begin characterizing the prosecution decisions in both countries. It is easy to verify that Lemma 1 also holds under Assumption 2. Regardless of how competition authority B detects collusion, the expected discounted welfare of country B if the firms are prosecuted is higher than if they are not prosecuted. Thus, as soon as a signal of collusion is received by the competition authority of country B, i.e. $s_t^B = 1$, it immediately prosecutes the firms.

Next, we turn to the prosecution decision of competition authority A when it detects collusion.

Lemma 3 *Suppose that Assumption 2 holds. Assume that the competition authority of country A detects collusion, i.e., $s_t^A = 1$.*

1. *If the firms are not colluding in country B, then A always prosecutes the firms.*

2. *If the firms are also colluding in country B, then A prosecutes the firms if and only if*

$$CS^{com,A} - CS^{c,A} - 2\pi^{c,A} \geq \frac{\left[1 - \left(1 - \alpha^A\right)\delta\right] 2\left(1 - \alpha^B\right)\left[\delta\pi^{c,B} + (1-\delta) f^B\right]}{\left[1 - \left(1 - \alpha^B\right)\left(1 - \alpha^A\right)\delta\right]\delta}, \tag{4}$$

Proof: *See Appendix B.1.* ■

The intuition behind Lemma 3 is very similar to the one behind Lemma 2. When firms are only colluding in country A, competition authority A prosecutes the firms as soon as collusion is detected because its prosecution decision does not have any impact on the profits of the firms in country B. When firms are colluding in both countries, prosecuting collusion in country A increases aggregate surplus in country A, but it reduces firms' profits in country B (forcing firms to compete in country B and making them pay fines). The main difference with Lemma 2 is that now competition authority B can detect collusion on its own and, hence, not prosecuting the firms is less valuable for country A. Formally, the right hand side of (4) is decreasing in α^B. Thus, as the probability that competition authority B detects collusion on its own increases, it is more likely that condition (4) holds. Finally, note that Lemma 3 is a generalization of Lemma 2. With $\alpha^B = 0$, the prosecution condition (4) becomes (3).

5.2 Collusion Decisions

We now proceed to study the equilibrium collusion decisions of the firms given the antitrust policies selected by the competition authorities of both countries. The following proposition focuses on the most interesting cases in which firms are willing to collude and the competition authority of country A does not prosecute the firms.

Proposition 2 *Suppose that Assumption 2 holds, firms punish deviations from collusion stopping collusion in both countries, they coordinate in their best equilibrium, and $\pi^{c,A} > (1-\delta)\pi^{d,A}$ and $\pi^{c,B} > (1-\delta)\pi^{d,B}$. Let*

$$\alpha^B > \frac{2\left[\delta\pi^{c,B} + (1-\delta) f^B\right] - \delta\left(\Delta S^A - 2\pi^{c,A}\right)}{2\left[\delta\pi^{c,B} + (1-\delta) f^B\right] + \frac{\delta(1-\alpha^A)}{1-(1-\alpha^A)\delta}\delta\left(\Delta S^A - 2\pi^{c,A}\right)}, \tag{5}$$

$$\alpha^A \leq \frac{\pi^{c,A} - (1-\delta)\pi^{d,A}}{\delta\pi^{d,A} + f^A}, \tag{6}$$

$$\alpha^B \leq \frac{\left(\pi^{c,A} + \pi^{c,B}\right) - (1-\delta)\left(\pi^{d,A} + \pi^{d,B}\right)}{\delta\left(\pi^{d,A} + \pi^{d,B}\right) - \frac{\delta(\pi^{c,A} - \alpha^A f^A)}{1-(1-\alpha^A)\delta} + f^B}, \tag{7}$$

$$\alpha^B < \frac{(1-\delta)\pi^{c,B} + \alpha^A\left[\delta\left(\pi^{c,A} + \pi^{c,B}\right) + (1-\delta) f^A\right]}{\left[1 - \left(1 - \alpha^A\right)\delta\right] f^B}. \tag{8}$$

1. *Suppose that (5)-(8) hold. Then, firms collude in both countries until the first time $s_t^B = 1$, when they are prosecuted in country B. Thereafter, they collude in country A until the first time $s_{t+\tau}^A = 1$ with $\tau \geq 1$, when they are prosecuted in country A. Thereafter, there is competition in both countries.*

2. *Suppose that (5)-(7) hold, but (8) does not hold. Then, firms only collude in country A until the first time $s_t^A = 1$, when they are prosecuted. Thereafter, there is competition.* ***Proof****: See Appendix B.2.* ■

In order to see the logic behind Proposition 2 it is useful to interpret conditions (5)-(8). (5) is the condition required for competition authority A not to prosecute the firms when they are colluding in both countries. (6) states that $\Pi^{c,A} = \frac{\pi^{c,A} - \alpha^A f^A}{1-(1-\alpha^A)\delta} \geq \pi^{d,A}$, which means that firms are willing to collude in country A even if they know that as soon as collusion is detected, they will be prosecuted. (7) states that $\Pi^{c,AB} = \frac{\pi^{c,A}+\pi^{c,B}-\alpha^B f^B}{[1-(1-\alpha^B)\delta]} + \frac{\delta\alpha^B\left(\pi^{c,A}-\alpha^A f^A\right)}{[1-(1-\alpha^B)\delta][1-(1-\alpha^A)\delta]} \geq \pi^{d,A} + \pi^{d,B}$, which implies that firms are willing to collude in both countries if they know that competition authority A will prosecute them, but only after competition authority B detects and prosecutes collusion in country B. Finally, (8) means that $\Pi^{c,AB} > \Pi^{c,A}$, i.e., firms prefer to collude in both countries rather than only in country A.

Part 1 of Proposition 2 describes an equilibrium in which firms start colluding in both countries, but they are not prosecuted by competition authority A, even when A detects collusion, i.e., even if $s_t^A = 1$.[15] However, competition authority B eventually detects collusion on its own (the first time that $s_t^B = 1$), prosecutes the firms and forces them to compete in country B. This, of course, does not mean that firms stop colluding in country A. Indeed, since $\Pi^{c,A} \geq \pi^{d,A}$, firms will keep their collusive agreement in country A until they are detected by competition authority A. This time, competition authority A will prosecute the firms, because now there are no profits to protect in country B. Firms can also sustain only colluding in country A, but this collusive agreement generate lower expected profits when $\Pi^{c,AB} > \Pi^{c,A}$. Finally, note that Proposition 2.1 is a generalization of Proposition 1.1. If $\alpha^B = 0$, competition authority B cannot detect collusion on its own and, hence, if competition authority A does not prosecute the firms, there is collusion in both countries forever.

Part 2 of Proposition 2 describes an equilibrium in which firms prefer to restrict collusion to country A because the fine that they will have to pay when collusion is detected in country B is too high. Note that this equilibrium is not very likely to occur. Indeed, if colluding in country B generates positive expected profits, firms will always prefer to start colluding in both countries. Formally, $\Pi^{c,B} = \frac{\pi^{c,B}-\alpha^B f^B}{1-(1-\alpha^B)\delta} > 0$ is sufficient for $\Pi^{c,AB} > \Pi^{c,A}$. However, Proposition 2.2 is useful to reveal a very interesting mechanism. Even when there is no intrinsic value in colluding in country B, firms might prefer to also collude in country B. Formally, even if $\Pi^{c,B} < 0$, it is possible that $\Pi^{c,AB} > \Pi^{c,A}$. The reason is that colluding in country B postpones prosecution in country A because when firms are colluding in both markets, competition authority A does not prosecute collusion in country A.

5.3 Prosecution Delays

The equilibrium in Proposition 2.1 involves a strategic prosecution delay. In order to see this, the following corollary compares the expected duration of collusion in equilibrium with an hypothetical situation in which both competition authorities prosecute collusion as soon as it is detected.

[15] This occurs because competition authority A accounts for the profits of the firms in country B and prosecution in country A will trigger prosecution in country B. Additionally, firms are able to collude in both countries because $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$.

Corollary 1 *Suppose that Assumption 2 holds, firms punish deviations from collusion stopping collusion in both countries, they coordinate in their best equilibrium, and* $\pi^{c,A} > (1-\delta)\,\pi^{d,A}$ *and* $\pi^{c,B} > (1-\delta)\,\pi^{d,B}$.

1. *Suppose that competition authority* $j \in \{A, B\}$ *prosecutes collusion the first time that* $s_t^j = 1$ *and, under such antitrust decisions, firms are still willing to collude in both countries. Then, the expected durations of collusion in countries A and B are given by* $\bar{d}^A = \frac{1-\alpha^A}{\alpha^A}$ *and* $\bar{d}^B = \frac{(1-\alpha^A)(1-\alpha^B)}{1-(1-\alpha^A)(1-\alpha^B)}$*, respectively.*

2. *Under the assumptions in Proposition 2.1. The expected durations of collusion in countries A and B are given by* $d^A = \frac{\alpha^A+\alpha^B-\alpha^A\alpha^B}{\alpha^B\alpha^A}$ *and* $d^B = \frac{1-\alpha^B}{\alpha^B}$*, respectively.*

3. *Equilibrium prosecution delays in countries A and B are given by:*

$$d^A - \bar{d}^A = \frac{1}{\alpha^B} \text{ and } d^B - \bar{d}^B = \frac{\alpha^A\left(1-\alpha^B\right)}{\alpha^B\left[1-\left(1-\alpha^A\right)\left(1-\alpha^B\right)\right]},$$

respectively. ***Proof****: See Appendix B.3.* ■

Two remarks apply to Corollary 1. First, note that the equilibrium in Proposition 2.1 involves a strategic delay in the prosecution of collusion. On average, collusion lasts $\left(\alpha^B\right)^{-1}$ extra periods in country A and $\frac{\alpha^A(1-\alpha^B)}{\alpha^B[1-(1-\alpha^A)(1-\alpha^B)]}$ extra periods in country B. Second, $\lim_{\alpha^B\to 0}\left(d^A - \bar{d}^A\right) = \infty$ and $\lim_{\alpha^B\to 0}\left(d^B - \bar{d}^B\right) = \infty$. Thus, the equilibrium in Proposition 1.1 generates an infinite prosecution delay.

6 Antitrust Policy, Integration and International Agreements

Up until this point we have studied the equilibrium prosecution decisions of two independent competition authorities. That is, the focus has been on the positive side of the problem. From a normative perspective, it is clear that in order to maximize the aggregate expected welfare of the world ($W_0^W = W_0^A + W_0^B$), both competition authorities should prosecute collusion as soon as it is detected. Indeed, this would be the policy selected by a globally integrated competition authority with a mandate to maximize W_0^W. Compared with the equilibrium prosecution decisions, full prosecution increases the world's welfare as well as the welfare of the country that does not own the firms, but it hurts the country of origin of the firms, that would prefer to delay prosecution. This misalignment between the welfare of the country of origin and global welfare implies that an international agreement to implement a global antitrust authority or to follow a different antitrust standard would be hard to achieve.[16] Note, however, that this logic might not apply if there are several industries and the firms of each country operate and try to organize collusion in different industries. In this section we argue that, by introducing multiple industries, with different countries of origin, there might be room for international agreements to be put in place. In order to formally explore this possibility, we incorporate a second industry to our model.

[16] If the country of origin of the firms is forced to participate in the international agreement (for instance because World Trade Organization or the European Commission impose it) it will have a strong incentive to withhold any information of collusion.

As in previous sections, assume there are two countries (A and B) with their respective competition authorities. In each country there are two industries, denoted by x and y. Only two companies operate in each industry: companies $1, x$ and $2, x$ in industry x and companies $1, y$ and $2, y$ in industry y. Let $\left(a_t^{i,z,A}, a_t^{i,z,B}\right)$ with $i \in \{1, 2\}$ and $z \in \{x, y\}$ denote the decision of firm i, z in period t, where $a_t^{i,z,j} = 1$ indicates that firm i, z chooses to collude in country j and $a_t^{i,z,j} = 0$ indicates that firm i, z chooses to compete in country j. Collusion can only occur within industry. Let $\left(\pi_t^{i,z,A}, \pi_t^{i,z,B}\right)$ denote the profits that company i, z obtains in period t from its operations in countries A and B. Assume that profits are given by:

Firm $1, z$ - Firm $2, z$	$a_t^{2,z,j} = 1$	$a_t^{2,z,j} = 0$
$a_t^{1,z,j} = 1$	$\pi^{c,z,j}$, $\pi^{c,z,j}$	0, $\pi^{d,z,j}$
$a_t^{1,z,j} = 0$	$\pi^{d,z,j}$, 0	0, 0

where $\pi^{d,z,j} > \pi^{c,z,j} > 0$. Let $CS_t^{z,j}$ be the consumer surplus in industry z and country j. Assume that $CS_t^{z,j} = CS^{c,z,j}$ when $a_t^{i,z,j} = a_t^{-i,z,j} = 1$ and $CS_t^{z,j} = CS^{com,z,j}$, otherwise, where $CS^{com,z,j} > CS^{c,z,j} + 2\pi^{c,z,j}$ for all z, j. Finally, denote by $f^{z,j} > \pi^{c,z,j}$ the fine charged by the competition authority of country j if firms are found organizing collusion in industry z.

The timing is as follows.

1. In each industry both firms simultaneously choose to collude or compete in each country.

2. Nature sends a signal to the competition authority of country j (k) with evidence of collusion in industry x (y) in country j (k). Upon observing the signal, the competition authority of country j (k) decides whether to prosecute the firms or not in industry x (y).

3. Nature sends a signal to the competition authority of country $-j$ ($-k$) with evidence of collusion in industry x (y) in country $-j$ ($-k$). Upon observing the signal, the competition authority of country $-j$ ($-k$) decides whether to prosecute the firms or not in industry x (y).[17]

A natural generalization of the ownership distribution studied in the case of one industry is to assume that each country owns the two firms of one of the industries. Regarding collusion detection, the following assumption is a generalization of Assumption 2 for an environment with two industries.

Assumption 3. *The signals received by the competition authorities are as in Assumption 2, with the roles of countries A and B reversed in the case of industry y (see Appendix C.1 for details).*

6.1 Independent Competition Authorities

Suppose that each country has an independent competition authority with a mandate to maximize the country's expected aggregate welfare. In such a context, it is straightforward to generalize Proposition 2.1 to the case of two industries (see Appendix C.2 for details). In equilibrium, competition authority A delays prosecuting firms in industry x because it does not want to trigger prosecution in country B, while the opposite happens in industry y, where competition authority B waits until firms are prosecuted in country A to prosecute collusion in country B.

[17]Note that with this timing the signal received by the competition authority of country $-j$ ($-k$) about industry x (y) could depend on the prosecution decision implemented by the competition authority of country j (k).

6.2 A Globally Integrated Competition Authority

Suppose that the competition authorities of both countries merge and form a globally integrated competition authority with a mandate to maximize the world's aggregate expected welfare.[18] In such environment, it is not difficult to show that the global competition authority will always prosecute collusion as soon as it is detected (see Appendix C.3 for details). This, of course, will change the incentives of the firms to organize collusion. In order to keep the analysis as simple as possible and to avoid exaggerating the welfare impact of integration, we focus on a region of the parameter space in which firms are still willing to collude in both countries until detected and, if they are only detected in one country, keep colluding in the other country. Thus, in equilibrium, firms in industry x (y) collude in both countries until the first time $s_t^{x,A} = 1$ ($s_t^{y,B} = 1$), when they are prosecuted in both countries, or until the first time $s_t^{x,A} = 0$ and $s_t^{x,B} = 1$ ($s_t^{y,B} = 0$ and $s_t^{y,A} = 1$), when they are prosecuted in country B (A). In the later case, firms keep colluding in country A (B) until the first time $s_{t+\tau}^{x,A} = 1$ ($s_{t+\tau}^{y,B} = 1$) with $\tau \geq 1$ (see Appendix C.3 for details). In other words, a globally integrated competition authority might not be able to dissuade firms to collude, but, at least, it eliminates the prosecution delays associated with independent competition authorities with a mandate to maximize national welfare.

6.3 Welfare Comparison

The following proposition compares the expected welfare of each country when each competition authority maximizes the welfare of its country and under integration.

Proposition 3 *Suppose that Assumption 3 holds, firms punish deviations from collusion stopping collusion in both countries, they coordinate in their best equilibrium and* $\pi^{c,z,j} > (1-\delta)\,\pi^{d,z,j}$ *for* $z \in \{x, y\}$ *and* $j \in \{A, B\}$. *Assume that* $\left(\alpha^{x,A}, \alpha^{x,B}\right) \in R^{x,A} \cap \bar{R}^{x,A}$ *and* $\left(\alpha^{y,B}, \alpha^{y,A}\right) \in R^{y,B} \cap \bar{R}^{y,B}$.[19] *Then:*

1. *Country A benefits from integration if and only if* $\Delta W_0^{x,A} + \Delta W_0^{y,A} > 0$ *and (if* $j = B$*), where*

$$\Delta W_0^{x,A} = \frac{\alpha^{x,A}}{\left[1-\left(1-\alpha^{x,B}\right)\delta\right]}\left\{\frac{\delta\left(\Delta CS^{x,A} - 2\pi^{c,x,A}\right)}{\left[1-\left(1-\alpha^{x,A}\right)\delta\right]} - \frac{2\left(1-\alpha^{x,B}\right)\left[\delta\pi^{c,x,B} + (1-\delta)\,f^{x,B}\right]}{\left[1-\left(1-\alpha^{x,A}\right)\left(1-\alpha^{x,B}\right)\delta\right]}\right\},$$

$$\Delta W_0^{y,A} = \frac{\alpha^{y,B}\left(1-\alpha^{y,A}\right)\left[\delta\Delta CS^{y,A} + (1-\delta)\,2f^{y,A}\right]}{\left[1-\left(1-\alpha^{x,A}\right)\left(1-\alpha^{x,B}\right)\delta\right]\left[1-\left(1-\alpha^{y,A}\right)\delta\right]},$$

where $\Delta CS^{z,j} = CS^{com,z,j} - CS^{c,z,j}$.

[18] For example, this could capture the situation of the European Union if part of the integration process includes the consolidation of all the national competition authorities into one competition authority for the whole union. Alternatively, the situation under integration could better approximate the present antitrust policy in the United States, while independent competition authorities could capture an alternative institutional arrangement in which antitrust policy is fully delegated to the states.

[19] See Appendix C.2 and C.3 for the definitions of $R^{x,A}$, $R^{y,B}$, $\bar{R}^{x,A}$ and $\bar{R}^{y,B}$.

2. *Country B benefits from integration if and only if* $\Delta W_0^{x,B} + \Delta W_0^{y,B} > 0$*, where*

$$\Delta W_0^{x,B} = \frac{\alpha^{x,A}\left(1-\alpha^{x,B}\right)\left[\delta \Delta CS^{x,B} + (1-\delta)\, 2f^{x,B}\right]}{\left[1-\left(1-\alpha^{y,A}\right)\left(1-\alpha^{y,B}\right)\delta\right]\left[1-\left(1-\alpha^{x,B}\right)\delta\right]},$$

$$\Delta W_0^{y,B} = \frac{\alpha^{y,B}}{\left[1-\left(1-\alpha^{y,A}\right)\delta\right]}\left\{\frac{\delta\left(\Delta CS^{y,B} - 2\pi^{c,y,B}\right)}{\left[1-\left(1-\alpha^{y,B}\right)\delta\right]} - \frac{2\left(1-\alpha^{y,A}\right)\left[\delta\pi^{c,y,A} + (1-\delta)\, f^{y,A}\right]}{\left[1-\left(1-\alpha^{y,A}\right)\left(1-\alpha^{y,B}\right)\delta\right]}\right\}.$$

3. *Moreover, if* $\pi^{c,z,j} = \pi^c$*,* $\pi^{d,z,j} = \pi^d$*,* $\Delta CS^{z,j} = \Delta CS$*,* $\alpha^{z,j} = \alpha$*, and* $f^{z,j} = f$ *for all* $z \in \{x,y\}$ *and* $j \in \{A,B\}$*, it is always the case that both countries are better off under integration.* ***Proof****: See Appendix C.4.* ■

The intuition behind Proposition 3 is simple. On the one hand, country A obtains a lower aggregate welfare in industry x under integration ($\Delta W_0^{x,A} < 0$). The reason is that an independent competition authority will delay prosecution only when the profits from collusion in country B outweigh the benefit from stopping collusion in country A. On the other hand, country A obtains a higher aggregate welfare in industry y under integration ($\Delta W_0^{y,A} > 0$) because an integrated competition authority does not delay the prosecution of firms from country B organizing collusion in country A. Note that it is perfectly possible that $\Delta W_0^{x,A} + \Delta W_0^{y,A} > 0$ and, hence, country A is better off under integration. The welfare comparisons for country B follow the same logic, except that we must reverse the industries. In other words, under integration, country B obtains a higher aggregate welfare in industry x, but a lower aggregate welfare in industry x ($\Delta W_0^{x,B} > 0$ and $\Delta W_0^{y,B} < 0$). Again, if $\Delta W_0^{x,B} + \Delta W_0^{y,B} > 0$, country B is also better off under integration. Finally, Proposition 3.3 considers a particular case in which everything is symmetric (across industries and countries). In such a case, the extra profits that one country is obtaining from collusion in the other country in one industry are perfectly offset by the extra profits that foreign firms are obtaining from collusion in the domestic market in the other industry. Thus, once we aggregate both industries, integration is neutral with respect to collusive profits, but it eliminates the deadweight loss associated with collusion.

Proposition 3 shows that, given the incentives that independent national authorities have to delay prosecution, both countries might be better off if they integrate their competition authorities. The following example further explore when countries support integration for a simple Bertrand specification.

Example 2. Suppose that the demand function of good z in country j is given by $Q_d^{z,j} = k_c^j - P^{z,j}$ with $k_c^A = 1$ and $k_c^B = k_c$. Also suppose that $\alpha^{x,A} = \alpha^{y,B}$ and $\alpha^{x,B} = \alpha^{y,A}$.[20] Thus, k_c is a measure of the relative size of country B with respect to country A. Assume that the cost function of firm i,z in country j is $c^{i,z,j} = 0$ and that fines are given by $f^{z,j} = \gamma\pi^{c,z,j}$, with $\gamma > 1$. Then, $\Delta CS^{x,A} = \Delta CS^{y,A} = 3/8$, $\Delta CS^{x,B} = \Delta CS^{y,B} = 3\left(k_c\right)^2/8$, $\pi^{c,x,A} = \pi^{c,y,A} = 1/8$ and $\pi^{c,x,B} = \pi^{c,y,B} = \left(k_c\right)^2/8$. Employing Proposition 3 it is easy to prove that there are two thresholds $\bar{k}_L > 0$ and $\bar{k}_H > \bar{k}_L$ such that if $k_c > \bar{k}_H$, then country A does not support integration; if $k_c < \bar{k}_L$, then country B does not support integration; and if $\bar{k}_L \leq k_c \leq \bar{k}_H$, both countries support integration (see the details in Appendix C.5). The intuition behind this result is as follows. When k_c is high enough (formally, $k_c > \bar{k}_H$), country A does not support integration because country B is relatively big and, therefore, the profits from collusion in industry x in

[20] Remember that as Proposition 3 states, $(\alpha^{x,A}, \alpha^{x,B}) \in R^{x,A} \cap \bar{R}^{x,A}$ and $(\alpha^{y,B}, \alpha^{y,A}) \in R^{y,B} \cap \bar{R}^{y,B}$. These restrictions can be rewritten as $k \in R^k = [k_L^R, k_H^R]$.

country B outweigh the benefits from stopping collusion in both industries in country A. Analogously, when k_c is low enough (formally, $k_c < \bar{k}_L$), country B does not support integration because country A is relatively big and, therefore, the profits from collusion in industry y in country A outweigh the benefits from stopping collusion in both industries in country B. Thus, only when the countries are relatively similar (formally, $\bar{k}_L \leq k_c \leq \bar{k}_H$), they are both better off under integration.

6.4 International Agreements and the Goal of Competition Authorities

Is it possible to support the equilibrium under a globally integrated competition authority without actually integrating them? If only countries were willing to commit not to delay prosecution, then they could replicate the equilibrium under integration. At first sight, a simple way of inducing this is to assume that the goal of each competition authority is to maximize the consumer surplus of domestic consumers (rather than national welfare). In such environment, collusion in both industries will be prosecuted as soon as it is detected because competition authorities do not take into account the profits.

Unfortunately, the problem is not so straightforward. To see this, suppose that one of the competition authorities (say the competition authority of country A) has a mandate to maximize the consumer surplus of domestic consumers rather than national welfare. True to its mandate, competition authority A will prosecute collusion as soon as it is detected in each industry. But then, the optimal reaction of country B will be to strategically delay prosecution in industry y. In equilibrium, there will be no strategic delay in stopping collusion in industry x (where firms are owned by citizens of country A), but, on average, collusion will last more than necessary in industry y (where firms are owned by citizens of country B). In terms of national welfare, this is the worst possible outcome for country A and the best possible one for country B. In other words, the antitrust prosecution game has the structure of a prisoner's dilemma. Regardless of what competition authority A (B) does, the best reaction of country B (A) is to instruct its competition authority to delay prosecution in industry y (x). Thus, committing not to delay prosecution does not change the behavior of the other competition authority.[21]

The solution is to reach an international agreement in which each country agrees to instruct its competition authority to only consider the well-being of domestic consumers as long as the other country follows the same course of action. Thus, in a global economy the mandate of a competition authority should not be defined independently of the mandate of the other competition authority. Moreover, note that a unilaterally commitment to maximize the well-being of domestic consumers will not bring the other country to the bargaining table. What is required is a combination of carrots and sticks. The message to the other country should be: "There is a *quid pro quo* agreement through which we commit to maximize the well-being of domestic consumers, but only if you do exactly the same. Otherwise, we will instruct our competition authority to maximize national welfare".

7 Discussion and Conclusions

This paper pushes the frontier of the analysis of antitrust policy in open economies. We develop a political economy model of antitrust enforcement in an open economy and characterize the equilibrium prosecution policies selected by benevolent national competition authorities. In the several scenarios

[21] This follows the same logic of trade agreements (see Bagwell and Staiger (1999)). If a country is fully committed to free trade (that is zero tariff), the best response of the other country is to impose the unilateral optimum tariff.

studied in our model, we show that collusion reduces the world's aggregate welfare and, therefore, should be prevented. However, we also show that national competition authorities may have biased incentives towards the prosecution of collusive activities. In particular, the country of origin of the firms has weaker incentives to prosecute collusion because domestic prosecution spirals into foreign prosecution, which reduces the profits of domestic firms in foreign markets. This misalignment between the equilibrium prosecution policies and the global welfare maximizing solution could be solved by integrating the competition authorities. Integration would, nevertheless find resistance by the country of origin of the firms, undermining its efficacy. Such a solution is more likely to succeed in a multi-industry world where each country specializes in a different industry. Indeed, we have shown that in a multi-industry world each country could be better off if an internationally integrated competition authority decided on prosecution based on global welfare.

Our results have important implications for the design of antitrust enforcement institutions and agencies. First, although there might be benefits from decentralizing antitrust enforcement to sub-national entities, our model suggests that countries should centralize antitrust enforcement in a national competition authority.

Second, our results suggest that competition authorities should consider the origin of the firms and their foreign operations when they decide to initiate a collusion case. Political transparency could be a problem. How can the public distinguish a competition authority captured by the firms from one dedicated to maximize national welfare that does not prosecute some firms to protect their foreign profits? For some industries both could be observationally equivalent. There might also exist practical barriers to implement such policy. For example, firms can employ accounting tricks to assign profits to different countries in order to inflate their foreign operations and avoid prosecution.

Finally in this paper we focus on the case in which competition authorities set to maximize total welfare rather than consumer surplus. There are several ways to justify the total welfare standard as the objective function of the competition authority, both from a positive as well as a normative perspective. First, there is ample heterogeneity in the stated mission of competition authorities: while in some countries, the mission is narrowly defined as 'to protect consumers from anticompetitive practices', in other countries the mission includes other goals such as efficiency and the development of a national economy.[22] Second, even when the goal of the competition authority is to maximize consumers' surpluses, political

[22]Historically, the declared mission of antitrust authorities has been the maximization of consumer welfare but, recently, scholars and practitioners have started advocating for broader goals, like the pursuit of total welfare or even objectives beyond what welfare can measure, such as national security (e.g., Khan (2016), Khan and Vaheesan (2017) and Glick (2019)). In reality, national antitrust authorities are already quite heterogeneous in terms of the scope of their stated mission, some of them focusing on the pursuit of consumer welfare and others pursuing broader goals, that overlap with industrial policy, like the development of the national economy. In the US, the Department of Justice enforces antitrust laws and the Federal Trade Commission protects consumers from anticompetitive practices. The mission of the Antitrust Division of the Department of Justice is to "promote economic competition through enforcing and providing guidance on antitrust laws and principles"(see https://www.justice.gov/atr/mission), while the mission of the Federal Trade Commission is to "protect consumers by preventing anticompetitive, deceptive and unfair business practices "(see https://www.ftc.gov/about-ftc). In the European Union, the European Commission has a broader goal as it understands that competition encompasses efficiency, innovation and price reductions (see http://ec.europa.eu/competition/antitrust/overview en.html). The goal of Japan's Fair-Trade Commission seems to be even broader as it includes the development of the national economy. Specifically, it's mission is "to promote the democratic and sound development of the national economy as well as to assure the interest of general consumers by promoting fair and free competition through prohibition of private monopolization, unreasonable restraint of trade (such as cartels and bid riggings) and unfair trade practices"(see http://www.jftc.go.jp/en/about jftc/role).

economy considerations may dictate that firms' profits are not neglected.[23] Third, as we discussed in Section 2.3, the empirical evidence shows that foreign multinational firms are a common target of antitrust prosecution, possibly suggesting evidence of a nationalist bias in antitrust enforcement. Fourth, although in a closed economy employing consumer surplus or total surplus as the normative criteria to evaluate market performance usually leads to the same policy recommendation (i.e., prosecute collusion), we have shown that this is not necessarily the case in an open economy in which prosecution has international informational spillovers. Thus, in a global economy a competition authority committed to protect domestic consumers could end up reducing national welfare.[24]

Finally, in a multi-industry setting, we show that unilaterally committing to protect domestic consumers could have negative effects on international antitrust cooperation. Instead, what is required to maximize the total surplus of each country and, hence, global surplus, is an international agreement in which all countries simultaneously commit to prosecute collusion regardless of the origin of the firms or their participation in foreign markets.

There are several avenues to expand our analysis. For example, we have developed a two-country model, but the mechanism behind our results should also apply to a setting with multiple countries. More importantly, in such setting there will be room for the strategic formation of coalitions of countries that decide to integrate their competition authorities. Finally, the logic highlighted in this paper can be naturally generalized beyond antitrust enforcement to areas such as international regulation and abuse of dominant position.

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[23]See, for example Cremieux and Snyder (2016), who states: "enforcement behaviors toward foreign and domestic firms can shed light on the underlying objectives that guide antitrust enforcement [...] and reveal, for example, whether political interests known to affect international trade policies also affect antitrust enforcement".

[24]The differences between alternative objectives of antitrust policy can be also observed across the three main areas of antitrust law—collusion, mergers, and monopolization. The pursuit of total welfare rather than consumer surplus can be easily explained—on the basis of scale effects in production and innovation—in the areas of monopolization or merger control (See Amir et al. (2009)). In the case of collusion deterrence, the efficiency defense is hardly justifiable in closed economies as collusion is always detrimental to total welfare. However, as we have shown, in an open economy, a country can benefit from the collusive behavior of national firms as long as it extends to foreign markets as well.

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Online Appendix to “Nationalistic bias in collusion prosecution: The case for international antitrust agreements”

This Appendix formally define the collusion and antitrust policy game and presents the proofs of all lemmas and propositions.

The collusion and antitrust policy game: Define a t-history as a sequence $h_t = \left\{\left(a_\tau^{1,A}, a_\tau^{1,B}\right), \left(a_\tau^{2,A}, a_\tau^{2,B}\right), \left(p_\tau^A, p_\tau^B\right)\right\}_{\tau=0}^{\tau=t-1}$. That is, h_t contains the decisions of the firms and the competition authorities up to period $t-1$. Let H_t denote the set of all feasible t-histories. Let $A_t^j\left(h_t\right) \subseteq \{0,1\}$ denote the set of actions available to the firms in country j in period t when the history of the game is h_t. Since once firms are prosecuted, they must compete in all future periods, $A_t^j\left(h_t\right) = \{0\}$ whenever in history h_t there exists $\tau \leq t-1$ such that $p_\tau^j = 1$. Otherwise $A_t^j\left(h_t\right) = \{0,1\}$. Let $P_t^j\left(s_t^j\right) \subseteq \{0,1\}$ be the set of available prosecution decisions for the competition authority of country j in period t when the collusion signal received in period t is $s_t^j \in \{0,1\}$. Since only when the competition authority receives a positive signal of collusion, it can prosecute the firms, $P_t^j(0) = \{0\}$ and $P_t^j(1) = \{0,1\}$.

- A *strategy for firm* $i \in \{1,2\}$ is a sequence of functions $\mathbf{a}^i = \left\{\mathbf{a}_t^i\right\}_{t=0}^{\infty}$, where $\mathbf{a}_t^i : H_t \to \{0,1\}^2$ and $\mathbf{a}_t^i\left(h_t\right) = \left(\mathbf{a}_t^{i,A}\left(h_t\right), \mathbf{a}_t^{i,B}\left(h_t\right)\right) = \left(a_t^{i,A}, a_t^{i,B}\right) \in A_t^A\left(h_t\right) \times A_t^B\left(h_t\right)$. That is, for each t-history firm i must decide to collude (if such alternative is still available) or compete in each country in period t. Moreover, we assume that in order to sustain collusion firms employ the standard grim-trigger strategy. For example, if firms make market-specific collusion decisions and punish deviations from collusion in each market separately, then $\mathbf{a}_t^i\left(h_t\right)$ adopts the following form: $\mathbf{a}_t^{i,A}\left(h_t\right) = 1$ if $a_\tau^{i,A} = 1$ for all $\tau \leq t$ and $i \in \{1,2\}$, otherwise, $\mathbf{a}_t^{i,A}\left(h_t\right) = 0$; and $\mathbf{a}_t^{i,B}\left(h_t\right) = 1$ if $a_\tau^{i,B} = 1$ for all $\tau \leq t$ and $i \in \{1,2\}$; otherwise, $\mathbf{a}_t^{i,B}\left(h_t\right) = 0$.

- A *strategy for competition authority* $j \in \{A,B\}$ is a sequence of functions $\mathbf{p}^j = \left\{\mathbf{p}_t^j\right\}_{t=0}^{\infty}$, where $\mathbf{p}_t^j : H_t \times \{0,1\}^5 \to \{0,1\}$ with $\mathbf{p}^j\left(h_t, a_t^{1,A}, a_t^{1,B}, a_t^{2,A}, a_t^{2,B}, s_t^j\right) = p_t^j \in P_t^j\left(s_t^j\right)$. That is, given a t-history h_t, actions of the firms $\left(a_t^{1,A}, a_t^{1,B}\right) \in \{0,1\}^2$, $\left(a_t^{2,A}, a_t^{2,B}\right) \in \{0,1\}^2$ and the collusion signal $s_t^j \in \{0,1\}$ received in period t, competition authority j must decide whether to prosecute the firms or not in period t.

- The *collusion signal function of competition authority* j is a sequence of functions $\mathbf{s}^j = \left\{\mathbf{s}_t^j\right\}_{t=0}^{\infty}$, where $\mathbf{s}_t^j : H_t \times \{0,1\}^3 \to [0,1]$ and $\mathbf{s}_t^j\left(h_t, a_t^{1,j}, a_t^{2,j}, p_t^{-j}\right) = \Pr\left(s_t^j = 1\right)$. That is, for each t-history, actions of the firms in country j in period t, and prosecution decision of the other competition authority, nature selects the signal received by competition authority j. Since only when the firms are colluding, the competition authority can receive a positive collusion signal, if $\left(a_t^{1,j}, a_t^{2,j}\right) \neq (1,1)$, then $\mathbf{s}_t^j\left(h_t, a_t^{1,j}, a_t^{2,j}, p_t^{-j}\right) = 0$.

The combination of a pair of collusion strategies for the firms $\left(\mathbf{a}^1, \mathbf{a}^2\right)$, a pair of prosecution strategies for the competition authorities $\left(\mathbf{p}^A, \mathbf{p}^B\right)$, and a pair of collusion signal functions $\left(\mathbf{s}^A, \mathbf{s}^B\right)$ induce a probability distribution function over $\left\{\pi_\tau^i\right\}_{\tau=t}^{\infty}$, the sequence of profits net of fines obtained by firm $i \in \{1,2\}$, and a probability distribution over $\left\{w_\tau^j\right\}_{\tau=t}^{\infty}$, the sequence of welfare obtained by country $j \in \{A, B\}$. Then, the payoff of firm i induced by $\left(\mathbf{a}^1, \mathbf{a}^2, \mathbf{p}^A, \mathbf{p}^B, \mathbf{s}^A, \mathbf{s}^B\right)$ is given by $\Pi_t^i = \mathbf{E}_t\left[\sum_{\tau=t}^{\infty} \delta^{\tau-t}\pi_\tau^i \middle| h_t\right]$, where the expectation operator is conditional on h_t, the information available to the firms when they make a decision in period t. Similarly, the payoff of competition authority j induced by $\left(\mathbf{a}^1, \mathbf{a}^2, \mathbf{p}^A, \mathbf{p}^B, \mathbf{s}^A, \mathbf{s}^B\right)$ is given by $W_t^j = \mathbf{E}_t\left[\sum_{\tau=t}^{\infty} \delta^{\tau-t} w_\tau^j \middle| \left(h_t, a_t^{1,A}, a_t^{1,B}, a_t^{2,A}, a_t^{2,B}, s_t^j\right)\right]$, where the expectation operator is conditional on $\left(h_t, a_t^{1,A}, a_t^{1,B}, a_t^{2,A}, a_t^{2,B}, s_t^j\right)$, the information available to the competition authority j when it selects its prosecution decision in period t.

Definition 1 *An* ***equilibrium*** *of the collusion and antitrust game is a pair of collusion strategies for the firms* $\left(\mathbf{a}^1, \mathbf{a}^2\right)$ *and a pair of prosecution strategies for the competition authorities* $\left(\mathbf{p}^A, \mathbf{p}^B\right)$ *that, given the collusion signal functions* $\left(\mathbf{s}^A, \mathbf{s}^B\right)$*, form a subgame perfect Nash equilibrium.*

A Detection Ability Fully Aligned with Firm Ownership

Appendix A presents the proofs of all the lemmas and propositions in Section 4 as well as the extensions discussed at the end of this section.

A.1 Proofs of Lemmas 1 and 2

Lemma 1 *Suppose that Assumption 1 holds. Then, the competition authority of country B prosecutes the firms as soon as collusion is detected in country B.*

Proof. Suppose that the competition authority of country B detects collusion in period t, i.e., $s_t^B = 1$. Note that this can only occur if $a_t^{1,B} = a_t^{2,B} = 1$ and $p_\tau^A = 1$ for some $\tau \leq t$. If the competition authority of country B prosecutes the firms, then the expected discounted welfare of country B at period t is

$$W_t^B\left(p_t^B = 1|s_t^B = 1\right) = CS^{c,B} + 2f^B + \frac{\delta}{1-\delta}S^{com,B},$$

while if firms are not prosecuted, it is

$$W_t^B\left(p_t^B = 0|s_t^B = 1\right) = \frac{CS^{c,B}}{1-\delta}.$$

Since $CS^{com,B} > CS^{c,B}$, it must be the case that $W_t^B\left(p_t^B = 1|s_t^B = 1\right) > W_t^B\left(p_t^B = 0|s_t^B = 1\right)$. Thus, whenever B detects collusion, it immediately prosecutes the firms. ■

Lemma 2. *Suppose that Assumption 1 holds. Assume that the competition authority of country A detects collusion, i.e., $s_t^A = 1$.*

1. *If the firms are not colluding in country B, then A always prosecutes the firms.*

2. *If the firms are also colluding in country B, then A prosecutes the firms if and only if*

$$\delta \geq \delta^*(\lambda) = \frac{2\left[\lambda f^B - (1-\lambda)f^A\right]}{CS^{com,A} - CS^{c,A} + 2\lambda\left(f^A - \pi^{c,A}\right) - 2f^A + 2\lambda\left(f^B - \pi^{c,B}\right)}$$

such that there exists $\lambda^* \in (0,1)$ where $\delta^*(\lambda) \geq 0$ for all $\lambda \geq \lambda^*$. Additionally, $\delta^*(\lambda)$ increases with λ .

Proof. Suppose that firms are only colluding in country A while there is competition in country B and the competition authority of country A detects collusion in period t, i.e., $s_t^A = 1$. If the competition authority of country A prosecutes the firms, then the expected discounted welfare of country A at period t is[25]

$$W_t^A \left(p_t^A = 1 | s_t^A = 1, A\right) = CS^{c,A} + 2\lambda \left(\pi^{c,A} - f^A\right) + 2f^A + \frac{\delta}{1-\delta} CS^{com,A},$$

while if firms are not prosecuted, it is

$$W_t^A \left(p_t^A = 0 | c_t^A = 1, A\right) = \frac{CS^{c,A} + 2\lambda\pi^{c,A}}{1-\delta}.$$

Since $\frac{\partial W_t^A\left(p_t^A=1|s_t^A=1,A\right)}{\partial\lambda} < 0$ and $\frac{\partial W_t^A\left(p_t^A=0|c_t^A=1,A\right)}{\partial\lambda} > 0$, then at $\lambda = 1$, $CS^{com,A} > CS^{c,A} + 2\pi^{c,A}$ always holds. It must be the case that $W_t^A \left(p_t^A = 1|s_t^A = 1, A\right) > W_t^A \left(p_t^A = 0|s_t^A = 1, A\right)$ for all possible values of λ.

Suppose that firms are colluding in both countries and the competition authority of country A detects collusion in period t, i.e., $s_t^A = 1$. If the competition authority of country A decides to prosecute the firms, then the competition authority of country B will detect collusion in country B as well, i.e., $s_t^B = 1$. Hence, firms will be also prosecuted in country B. Thus, if firms are prosecuted in country A, the expected discounted welfare of country A at period t will be

$$W_t^A \left(p_t^A = 1 | s_t^A = 1, AB\right) = CS^{c,A} + 2\lambda \left(\pi^{c,A} - f^A + \pi^{c,B} - f^B\right) + 2f^A + \frac{\delta}{1-\delta} CS^{com,A}.$$

If the competition authority of country A does not prosecute the firms, then there is no way that the competition authority of country B finds that firms are also colluding in country B. Then, firms will continue colluding in both countries and the expected discounted welfare of country A at period t will be

$$W_t^A \left(p_t^A = 0 | s_t^A = 1, AB\right) = \frac{CS^{c,A} + 2\lambda\left(\pi^{c,A} + \pi^{c,B}\right)}{1-\delta}.$$

$W_t^A \left(p_t^A = 1|s_t^A = 1, AB\right) > W_t^A \left(p_t^A = 0|s_t^A = 1, AB\right)$ if and only if $\delta \left(CS^{com,A} - CS^{c,A} - 2\lambda\pi^{c,A}\right) > 2\left[\lambda\delta\pi^{c,B} + (1-\delta)\left(\lambda f^B - (1-\lambda)f^A\right)\right]$ or equivalently, $\delta \geq \delta^*(\lambda) = \frac{2\left[\lambda f^B - (1-\lambda)f^A\right]}{CS^{com,A} - CS^{c,A} + 2\lambda(f^A - \pi^{c,A}) - 2f^A + 2\lambda(f^B - \pi^{c,B})}$. As $CS^{com,A} - CS^{c,A} - 2f^A > 0$, then $CS^{com,A} - CS^{c,A} + 2\lambda\left(f^A - \pi^{c,A}\right) - 2f^A + 2\lambda\left(f^B - \pi^{c,B}\right) > 0$ always holds. Moreover, $\lambda f^B - (1-\lambda)f^A > 0$ if and only if $\lambda > \lambda^* = \frac{f^A}{f^A + f^B} \in (0,1)$. Therefore, $\delta^*(\lambda) \geq 0$ for $\lambda \geq \lambda^*$. Finally, it is easy to verify that $\frac{\partial\delta^*(\lambda)}{\partial\lambda} > 0$ for all $\lambda \geq \lambda^*$. ■

[25] Note that we employ the following notation. $W_t^A(p_t^A = 1|s_t^A = 1, A)$ is the discounted expected welfare of country A when firms are only colluding in country A, $s_t^A = 1$ and competition authority A choses to prosecute the firms. $W_t^A(p_t^A = 1|s_t^A = 1, AB)$ is the discounted expected welfare of country A when firms are colluding in both countries, $s_t^A = 1$ and competition authority A choses to prosecute the firms.

A.2 Proof of Proposition 1

Proposition 1 (*Market-specific collusion and global punishments*). *Suppose that Assumption 1 holds, firms punish deviations from collusion stopping collusion in both countries and they coordinate in their best equilibrium. Given $\lambda > \lambda^*$,*

1. *Suppose that $\bar{\delta}^{SG} < \delta < \delta^*(\lambda)$, where $\bar{\delta}^{SG} = \max\left\{\frac{\pi^{d,A}-\pi^{c,A}}{\pi^{d,A}}, \frac{\pi^{d,B}-\pi^{c,B}}{\pi^{d,B}}\right\}$. Then, firms collude in both countries and they are never prosecuted.*

2. *Suppose that $\delta \geq \delta^*(\lambda)$. Then, there are thresholds $\bar{\alpha}_L(\delta)$ and $\bar{\alpha}_H(\delta)$ such that:*

   (a) *Suppose that $\alpha^A > \bar{\alpha}_H(\delta)$. Then, firms collude in country B and they are never prosecuted.*

   (b) *Suppose that $\bar{\alpha}_L(\delta) < \alpha^A \leq \bar{\alpha}_H(\delta)$. Then, there is a threshold $\hat{\alpha}_{M_1}(\delta)$ such that: If $\alpha^A \in (\bar{\alpha}_L(\delta), \hat{\alpha}_{M_1}(\delta)]$, then firms collude (only in country A or in both countries) until the first time $s_t^A = 1$, when they are prosecuted. Thereafter, there is competition. If $\alpha^A \in [\hat{\alpha}_{M_1}(\delta), \bar{\alpha}_H(\delta)]$, then firms only collude in country B and they are never prosecuted.*

   (c) *Suppose that $\alpha^A \leq \bar{\alpha}_L(\delta)$. Then, there is a threshold $\hat{\alpha}_{M_2}(\delta)$ such that: If $\alpha^A \in [0, \hat{\alpha}_{M_2}(\delta))$, then firms collude (only in country A or in both countries) until the first time $s_t^A = 1$, when they are prosecuted. Thereafter, there is competition. If $\alpha^A \in [\hat{\alpha}_{M_2}(\delta), \bar{\alpha}_L(\delta)]$, then firms only collude in country B and they are never prosecuted.*

Proof. To establish the optimal collusion strategy of the firms in each region of parameters we proceed in three steps. First, we compute the expected discounted profits under each type of collusion. Second, we deduce the conditions under which each type of collusion is sustainable. Finally, among the sustainable types of collusion, we dedude the type that maximizes the expected discounted profits of the firms.

1. Suppose that $\bar{\delta}^{SG} < \delta < \delta^*(\lambda)$.

Profits under each type of collusion: From Lemmas 1 and 2, the expected discounted profits of a firm under collusion in country A, collusion in country B and collusion in both countries are given by:

$$\Pi^{c,A} = \frac{\pi^{c,A} - \alpha^A f^A}{1-(1-\alpha^A)\,\delta}, \; \Pi^{c,B} = \frac{\pi^{c,B}}{1-\delta}, \; \Pi^{c,AB} = \frac{\pi^{c,A}+\pi^{c,B}}{1-\delta},$$

respectively.

Sustainability of each type of collusion: Let's deduce the conditions under which each type of collusion can be sustained as a subgame perfect Nash equilibrium.

- For collusion in country A to be an equilibrium it must be the case that $\Pi^{c,A} \geq \pi^{d,A}$, which holds if and only if $\alpha^A \leq \bar{\alpha}^A$, where
$$\bar{\alpha}^A = \frac{\pi^{c,A} - (1-\delta)\,\pi^{d,A}}{\delta\pi^{d,A} + f^A}.$$

- For collusion in country B to be an equilibrium it must be the case that $\Pi^{c,B} \geq \pi^{d,B}$, which holds which always holds because $\delta > \bar{\delta}^{SG}$ implies $\pi^{c,B} > (1-\delta)\,\pi^{d,B}$.

- For collusion in both countries to be an equilibrium it must be the case that $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$. Note that, since the punishment for deviation is competition in both countries, once a firm decides to violate the collusive agreement, it deviates in both countries. Since $\delta > \bar{\delta}^{SG}$ implies that $\pi^{c,j} > (1-\delta)\,\pi^{d,j}$ for $j = A, B$, $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$ always holds.

Therefore, if $\alpha^A \leq \bar{\alpha}^A$, the three types of collusion can be sustained, while if $\alpha^A > \bar{\alpha}^A$, only collusion in country B or collusion in both countries can be sustained.

Selection: Let's compare the expected discounted profits under each type of collusion. It is easy to verify that $\Pi^{c,AB} > \max\left\{\Pi^{c,A}, \Pi^{c,B}\right\}$, which implies that the optimal collusion strategy of the firms is to collude in both countries.

2. Suppose that $\delta \geq \delta^*(\lambda)$.

Profits under each type of collusion: From Lemmas 1 and 2, the expected profits of a firm under collusion in country A, collusion in country B and collusion in both countries are given by:

$$\Pi^{c,A} = \frac{\pi^{c,A} - \alpha^A f^A}{1-(1-\alpha^A)\,\delta},\ \Pi^{c,B} = \frac{\pi^{c,B}}{1-\delta},\ \Pi^{c,AB} = \frac{\pi^{c,A} + \pi^{c,B} - \alpha^A\left(f^A + f^B\right)}{1-(1-\alpha^A)\,\delta},$$

respectively.

Sustainability of each type of collusion: Let's deduce the conditions under which each type of collusion can be sustained as a subgame perfect Nash equilibrium.

- For collusion in country A to be an equilibrium, it must be the case that $\Pi^{c,A} \geq \pi^{d,A}$, which holds if and only if $\alpha^A \leq \bar{\alpha}^A$, where
$$\bar{\alpha}^A = \frac{\pi^{c,A} - (1-\delta)\,\pi^{d,A}}{\delta\pi^{d,A} + f^A}$$

- For collusion in country B to be an equilibrium, it must be the case that $\Pi^{c,B} \geq \pi^{d,B}$, which always holds because $\delta > \bar{\delta}^{SG}$ implies $\Pi^{c,B} > \pi^{d,B}$.

- For collusion in both countries to be an equilibrium, it must be the case that no firm has an incentive to deviate in both countries, which requires $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$, which holds if and only if $\alpha^A \leq \bar{\alpha}^{AB}$, where
$$\bar{\alpha}^{AB} = \frac{\left(\pi^{c,A} + \pi^{c,B}\right) - (1-\delta)\left(\pi^{d,A} + \pi^{d,B}\right)}{\delta\left(\pi^{d,A} + \pi^{d,B}\right) + f^A + f^B}$$

Selection: Let's compare the expected discounted profits under each type of collusion.

- $\Pi^{c,AB} > \Pi^{c,A}$ if and only if $\alpha^A < \hat{\alpha}^{A,AB}$, where
$$\hat{\alpha}^{A,AB} = \frac{\pi^{c,B}}{f^B}$$

- $\Pi^{c,AB} > \Pi^{c,B}$ if and only if $\alpha^A < \hat{\alpha}^{B,AB}$, where
$$\hat{\alpha}^{B,AB} = \frac{(1-\delta)\,\pi^{c,A}}{\delta\pi^{c,B} + (1-\delta)\left(f^A + f^B\right)}$$

- $\Pi^{c,A} > \Pi^{c,B}$ if and only if $\alpha^A < \hat{\alpha}^{A,B}$, where

$$\hat{\alpha}^{A,B} = \frac{(1-\delta)\left(\pi^{c,A} - \pi^{c,B}\right)}{\delta\pi^{c,B} + (1-\delta)\, f^A}$$

To determine the optimal collusion strategy of the firms let's consider the following three cases:

a. Suppose that $\alpha^A > \max\left\{\bar{\alpha}^A, \bar{\alpha}^{AB}\right\}$. Then, only collusion in country B can be sustained as an equilibrium. Then, the optimal collusion strategy of the firms is to collude in country B.

b. Suppose that $\min\left\{\bar{\alpha}^A, \bar{\alpha}^{AB}\right\} < \alpha^A \leq \max\left\{\bar{\alpha}^A, \bar{\alpha}^{AB}\right\}$. Then, there are two possible cases. First, suppose that $\bar{\alpha}^A < \bar{\alpha}^{AB}$ and $\bar{\alpha}^A < \alpha^A \leq \bar{\alpha}^{AB}$. Then, only collusion in country B or in both countries can be sustained. Moreover, firms prefer to collude in both countries when $\alpha^A < \hat{\alpha}^{B,AB}$ and to collude in B when $\alpha^A > \hat{\alpha}^{B,AB}$. Thus, the optimal collusion strategy of the firms is to collude in both countries if $\alpha^A < \hat{\alpha}^{B,AB}$ and collude in B if $\alpha^A > \hat{\alpha}^{B,AB}$. Second, suppose that $\bar{\alpha}^A > \bar{\alpha}^{AB}$ and $\bar{\alpha}^{AB} < \alpha^A \leq \bar{\alpha}^A$. Then, only collusion in country B or collusion in country A can be sustained. Moreover, firms prefer to collude in A when $\alpha^A < \hat{\alpha}^{A,B}$ and to collude in B when $\alpha^A > \hat{\alpha}^{A,B}$. Thus, the optimal collusion strategy of the firms is to collude in A if $\alpha^A < \hat{\alpha}^{A,B}$ and collude in B if $\alpha^A > \hat{\alpha}^{A,B}$.

c. Suppose that $\alpha^A \leq \min\left\{\bar{\alpha}^A, \bar{\alpha}^{AB}\right\}$. Then, the three types of collusion can be sustained. Firms prefer to collude in A when $\hat{\alpha}^{A,AB} < \alpha^A < \hat{\alpha}^{A,B}$, they prefer to collude in both countries when $\alpha^A < \min\left\{\hat{\alpha}^{A,AB}, \hat{\alpha}^{B,AB}\right\}$, and they prefer to collude in B when $\alpha^A > \max\left\{\hat{\alpha}^{A,B}, \hat{\alpha}^{B,AB}\right\}$. Thus, the optimal collusion strategy of the firms is to collude in A if $\hat{\alpha}^{A,AB} < \alpha^A < \hat{\alpha}^{A,B}$, collude in both countries if $\alpha^A < \min\left\{\hat{\alpha}^{A,AB}, \hat{\alpha}^{B,AB}\right\}$, and collude in B if $\alpha^A > \hat{\alpha}^{A,B}$.

To obtain Proposition 1 define

$$\bar{\alpha}_L(\delta) = \min\left\{\bar{\alpha}^A, \bar{\alpha}^{AB}\right\},\ \bar{\alpha}_H(\delta) = \max\left\{\bar{\alpha}^A, \bar{\alpha}^{AB}\right\}$$
$$\hat{\alpha}_{M_1}(\delta) = \begin{cases} \hat{\alpha}^{A,B} & \text{if } \bar{\alpha}^{AB} < \bar{\alpha}^A \\ \hat{\alpha}^{B,AB} & \text{if } \bar{\alpha}^{AB} \geq \bar{\alpha}^A \end{cases},\ \hat{\alpha}_{M_2}(\delta) = \max\left\{\hat{\alpha}^{A,B}, \hat{\alpha}^{B,AB}\right\}.$$

To obtain a detailed characterization of the equilibrium consider the following four possible situations.

1. Suppose that $\frac{\pi^{c,B}-(1-\delta)\pi^{d,B}}{\delta\pi^{d,B}+f^B} < \frac{\pi^{c,A}-(1-\delta)\pi^{d,A}}{\delta\pi^{d,A}+f^A}$ and $\frac{\pi^{c,B}}{f^B} < \frac{(1-\delta)\pi^{c,A}}{\delta\pi^{c,B}+(1-\delta)(f^A+f^B)}$. Then, $\bar{\alpha}^{AB} < \bar{\alpha}^A$ and $\hat{\alpha}^{B,AB} < \hat{\alpha}^{A,B}$. Therefore:
   (a) If $\alpha^A > \bar{\alpha}^A$, then firms only collude in country B.
   (b) If $\bar{\alpha}^{AB} < \alpha^A \leq \bar{\alpha}^A$, then firms collude in country A when $\alpha^A < \hat{\alpha}^{A,B}$ and they collude in country B when $\alpha^A > \hat{\alpha}^{A,B}$.
   (c) If $\alpha^A \leq \bar{\alpha}^{AB}$, then firms collude in both countries when $\alpha^A < \hat{\alpha}^{A,AB}$, collude only in country A when $\hat{\alpha}^{A,AB} < \alpha^A < \hat{\alpha}^{A,B}$, and collude only in B when $\alpha^A > \hat{\alpha}^{A,B}$.

2. Suppose that $\frac{\pi^{c,B}-(1-\delta)\pi^{d,B}}{\delta\pi^{d,B}+f^B} < \frac{\pi^{c,A}-(1-\delta)\pi^{d,A}}{\delta\pi^{d,A}+f^A}$ and $\frac{\pi^{c,B}}{f^B} \geq \frac{(1-\delta)\pi^{c,A}}{\delta\pi^{c,B}+(1-\delta)(f^A+f^B)}$. Then, $\bar{\alpha}^{AB} < \bar{\alpha}^A$ and $\hat{\alpha}^{B,AB} \geq \hat{\alpha}^{A,B}$. Therefore:
   (a) If $\alpha^A > \bar{\alpha}^A$, then firms only collude in country B.
   (b) If $\bar{\alpha}^{AB} < \alpha^A \leq \bar{\alpha}^A$, then firms collude in country A when $\alpha^A < \hat{\alpha}^{A,B}$ and they collude in country B when $\alpha^A > \hat{\alpha}^{A,B}$.

(c) If $\alpha^A \leq \bar{\alpha}^{AB}$, then firms collude in both countries when $\alpha^A < \hat{\alpha}^{B,AB}$ and collude only in B when $\alpha^A > \hat{\alpha}^{B,AB}$.

3. Suppose that $\frac{\pi^{c,B}-(1-\delta)\pi^{d,B}}{\delta\pi^{d,B}+f^B} \geq \frac{\pi^{c,A}-(1-\delta)\pi^{d,A}}{\delta\pi^{d,A}+f^A}$ and $\frac{\pi^{c,B}}{f^B} < \frac{(1-\delta)\pi^{c,A}}{\delta\pi^{c,B}+(1-\delta)(f^A+f^B)}$. Then, $\bar{\alpha}^{AB} \geq \bar{\alpha}^A$ and $\hat{\alpha}^{B,AB} < \hat{\alpha}^{A,B}$. Therefore:

   (a) If $\alpha^A > \bar{\alpha}^{AB}$, then firms only collude in country B.

   (b) If $\bar{\alpha}^A < \alpha^A \leq \bar{\alpha}^{AB}$, then firms collude in both countries when $\alpha^A < \hat{\alpha}^{B,AB}$ and they collude in country B when $\alpha^A > \hat{\alpha}^{B,AB}$.

   (c) If $\alpha^A \leq \bar{\alpha}^A$, then firms collude in both countries when $\alpha^A < \hat{\alpha}^{A,AB}$, collude only in country A when $\hat{\alpha}^{A,AB} < \alpha^A < \hat{\alpha}^{A,B}$, and collude only in B when $\alpha^A > \hat{\alpha}^{A,B}$.

4. Suppose that $\frac{\pi^{c,B}-(1-\delta)\pi^{d,B}}{\delta\pi^{d,B}+f^B} \geq \frac{\pi^{c,A}-(1-\delta)\pi^{d,A}}{\delta\pi^{d,A}+f^A}$ and $\frac{\pi^{c,B}}{f^B} \geq \frac{(1-\delta)\pi^{c,A}}{\delta\pi^{c,B}+(1-\delta)(f^A+f^B)}$. Then, $\bar{\alpha}^{AB} \geq \bar{\alpha}^A$ and $\hat{\alpha}^{B,AB} \geq \hat{\alpha}^{A,B}$. Therefore:

   (a) If $\alpha^A > \bar{\alpha}^{AB}$, then firms only collude in country B.

   (b) If $\bar{\alpha}^A < \alpha^A \leq \bar{\alpha}^{AB}$, then firms collude in both countries when $\alpha^A < \hat{\alpha}^{B,AB}$ and they collude in country B when $\alpha^A > \hat{\alpha}^{B,AB}$.

   (c) If $\alpha^A \leq \bar{\alpha}^A$, then firms collude in both countries when $\alpha^A < \hat{\alpha}^{B,AB}$ and collude only in B when $\alpha^A > \hat{\alpha}^{B,AB}$.

This completes the proof of Proposition 1. ■

A.3 Extensions

Table A.1 summarizes the 4 possible cases that we study depending on the scope of collusion (global or market-specific), the punishment strategy employed by the firms to sustain collusion (global or market-specific), and the duration of the information spillover.

Scope of Collusion	Punishment Strategy	Information Spillover	Section
Market-specific	Global	Permanent	A.2
Global	Global	Permanent	A.3.1
Market-specific	Market-specific	Permanent	A.3.2
Market-specific	Global	Short-run	A.3.3

Table A.1: Extensions

A.3.1 Global Collusion

Proposition 1 (*Global collusion*). *Suppose that Assumption 1 holds, firms can either collude in both countries or in none of them and they coordinate in their best equilibrium. Given $\lambda > \lambda^*$,*

1. *Suppose that $\bar{\delta}^G < \delta < \delta^*(\lambda)$, where $\bar{\delta}^G = \frac{\pi^{d,A}+\pi^{d,B}-\pi^{c,A}-\pi^{c,B}}{\pi^{d,A}+\pi^{d,B}}$. Then, firms collude in both countries and they are never prosecuted.*

2. *Suppose that $\delta \geq \delta^*(\lambda)$. Then, there is a threshold $\bar{\alpha}^{AB}(\delta)$ such that:*

   (a) *If $\alpha^A \leq \bar{\alpha}^{AB}(\delta)$, then there is global collusion until the first time $s_t^A = 1$, when firms are prosecuted in both countries. Thereafter, there is competition.*

   (b) *If $\alpha^A > \bar{\alpha}^{AB}(\delta)$, then there is always competition.*

Proof.

1. Suppose that $\bar{\delta}^G < \delta < \delta^*(\lambda)$. Then, from Lemmas 1 and 2, if firms collude in both countries they will not be prosecuted. Therefore, the expected profits of a firm under collusion are given by:

$$\Pi^{c,AB} = \frac{\pi^{c,A} + \pi^{c,B}}{1-\delta}.$$

(Recall that firms can either collude in both countries or in none of them). Collusion can be sustained as a subgame perfect Nash equilibrium whenever $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$, which always holds since $\bar{\delta}^G < \delta$ implies $\pi^{c,A} + \pi^{c,B} > (1-\delta)\left(\pi^{d,A} + \pi^{d,B}\right)$.

2. Suppose $\delta \geq \delta^*(\lambda)$. Then, from Lemmas 1 and 2, if firms collude in both countries they will be prosecuted the first time $s_t^A = 1$. Therefore, the expected profits of a firm under collusion are given by $\Pi^{c,AB} = \alpha^A\left(\pi^{c,A} + \pi^{c,B} - f^A - f^B\right) + \left(1-\alpha^A\right)\left(\pi^{c,A} + \pi^{c,B} + \delta\Pi^{c,AB}\right)$, which implies

$$\Pi^{c,AB} = \frac{\pi^{c,A} + \pi^{c,B} - \alpha^A\left(f^A + f^B\right)}{1-\left(1-\alpha^A\right)\delta}.$$

Collusion can be sustained as a subgame perfect Nash equilibrium if and only if $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$. (Since firms are not allowed to collude in each market separately, when a firm violates the collusive agreement, it deviates in both countries). $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$ if and only if $\alpha^A \leq \bar{\alpha}^{AB}(\delta)$, where

$$\bar{\alpha}^{AB}(\delta) = \frac{\left(\pi^{c,A} + \pi^{c,B}\right) - (1-\delta)\left(\pi^{d,A} + \pi^{d,B}\right)}{f^A + f^B + \delta\left(\pi^{d,A} + \pi^{d,B}\right)}$$

Therefore, if $\alpha^A \leq \bar{\alpha}^{AB}(\delta)$, firms collude until they are prosecuted, while if $\alpha^A > \bar{\alpha}^{AB}(\delta)$ firms do not collude at all. ■

Proposition 1.1 states that when the discount factor is bellow some threshold ($\delta < \delta^*(\lambda)$), the expected discounted welfare of country A is higher if collusion in both countries is allowed, and, hence, the competition authority of country A never prosecutes the firms. Then, firms can safely collude in both countries without facing any risk of prosecution. Since firms also need to sustain collusion, we require $\pi^{c,A} + \pi^{c,B} > (1-\delta)\left(\pi^{d,A} + \pi^{d,B}\right)$ or, which is equivalent, $\delta > \bar{\delta}^G$. Proposition 1.2 studies the situation in which $\delta \geq \delta^*(\lambda)$ and, hence, the expected discounted welfare of country A is higher if collusion is stopped. In such a situation the competition authority of country A prosecutes the firms as soon as collusion is detected, which also triggers prosecution in country B. This prosecution policy might not be enough to dissuade firms to collude when the detection probability is low. Indeed, for $\alpha^A \leq \bar{\alpha}^{AB}(\delta)$, the expected discounted profits from collusion are high enough to sustain collusion. If this is the case, in equilibrium, there will be collusion until the competition authority of country A detects it and prosecutes the firms. Thereafter, firms will be forced to compete. On the contrary, when $\alpha^A > \bar{\alpha}^{AB}$, the expected discounted profits from collusion are not enough to sustain collusion. The antitrust policy effectively dissuades firms from colluding.

A.3.2 Market-specific Collusion and Punishments

Proposition 1 (*Market-specific collusion and punishments*). *Suppose that Assumption 1 holds, firms punish deviations from collusion in each market separately and they coordinate in their best equilibrium. Given* $\lambda > \lambda^*$,

1. *Suppose that* $\bar{\delta}^S < \delta < \delta^*(\lambda)$, *where* $\bar{\delta}^S = \max\left\{\frac{\pi^{d,A}-\pi^{c,A}}{\pi^{d,A}}, \frac{\pi^{d,B}-\pi^{c,B}}{\pi^{d,B}}\right\}$. *Then firms collude in both countries and they are never prosecuted.*

2. *Suppose that* $\delta \geq \delta^*(\lambda)$. *Then, there are thresholds* $\bar{\alpha}_L(\delta)$ *and* $\bar{\alpha}_H(\delta)$ *such that:*

   (a) *Suppose that* $\alpha^A > \bar{\alpha}_H(\delta)$. *Then, firms collude in country* B *and they are never prosecuted.*

   (b) *Suppose that* $\bar{\alpha}_L(\delta) < \alpha^A \leq \bar{\alpha}_H(\delta)$. *Then, there is a threshold* $\hat{\alpha}_M(\delta)$ *such that: If* $\alpha^A \in (\bar{\alpha}_L(\delta), \hat{\alpha}_M(\delta)]$, *firms only collude in country* A *until the first time* $s_t^A = 1$, *when they are prosecuted. Thereafter, there is competition. If* $\alpha^A \in [\hat{\alpha}_M(\delta), \bar{\alpha}_H(\delta)]$, *firms only collude in country* B *and they are never prosecuted.*

   (c) *If* $\alpha^A \leq \bar{\alpha}_L(\delta)$, *then firms collude in both countries until the first time* $s_t^A = 1$, *when they are prosecuted. Thereafter, there is competition in both countries.*

Proof. To establish the optimal collusion strategy of the firms in each region of parameters we proceed in three steps. First, we compute the expected discounted profits under each type of collusion. Second, we deduce the conditions under which each type of collusion is sustainable. Finally, among the sustainable types of collusion, we dedude the type that maximizes the expected discounted profits of the firms.

1. Suppose that $\bar{\delta}^S < \delta < \delta^*(\lambda)$.

Profits under each type of collusion: From Lemmas 1 and 2, if firms collude in both countries or they only collude in country B, they will never be prosecuted, while if they only collude in country A, they will be prosecuted the first time $s_t^A = 1$. Therefore, the expected profits of a firm under collusion in country A, collusion in country B and collusion in both countries are given by:

$$\Pi^{c,A} = \frac{\pi^{c,A} - \alpha^A f^A}{1 - (1-\alpha^A)\delta}, \; \Pi^{c,B} = \frac{\pi^{c,B}}{1-\delta}, \; \Pi^{c,AB} = \frac{\pi^{c,A} + \pi^{c,B}}{1-\delta},$$

respectively. In order to deduce $\Pi^{c,A}$ note that $\Pi^{c,A} = \alpha^A\left(\pi^{c,A} - f^A\right) + \left(1-\alpha^A\right)\left(\pi^{c,A} + \delta\Pi^{c,A}\right)$.

Sustainability of each type of collusion: Let's deduce the conditions under which each type of collusion can be sustained as a subgame perfect Nash equilibrium.

- For collusion in country A to be an equilibrium it must be the case that $\Pi^{c,A} \geq \pi^{d,A}$, which holds if and only if $\alpha^A \leq \bar{\alpha}^A$, where
$$\bar{\alpha}^A = \frac{\pi^{c,A} - (1-\delta)\,\pi^{d,A}}{\delta\pi^{d,A} + f^A}.$$

- For collusion in country B to be an equilibrium it must be the case that $\Pi^{c,B} \geq \pi^{d,B}$, which holds which always holds because $\delta > \bar{\delta}^S$ implies $\Pi^{c,B} > \pi^{d,B}$.

- For collusion in both countries to be an equilibrium, it must be the case that no firm has an incentive to deviate in country A, in country B, or in both countries, which requires $\Pi^{c,AB} \geq \pi^{d,A} + \Pi^{c,B}$, $\Pi^{c,AB} \geq \Pi^{c,A} + \pi^{d,B}$ and $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$, respectively. Since $\pi^{c,j} > (1-\delta)\,\pi^{d,j}$ for $j = A, B$, all these conditions hold.

Therefore, if $\alpha^A \leq \bar{\alpha}^A$, the three types of collusion can be sustained, while if $\alpha^A > \bar{\alpha}^A$, only collusion in country B or collusion in both countries can be sustained.

Selection: Let's compare the expected discounted profits under each type of collusion. It is easy to verify that $\Pi^{c,AB} > \max\left\{\Pi^{c,A}, \Pi^{c,B}\right\}$, which implies that the optimal collusion strategy of the firms is to collude in both countries.

2. Suppose that $\delta \geq \delta^*(\lambda)$.

Profits under each type of collusion: From Lemmas 1 and 2, if firms only collude in country B, they will never be prosecuted, while if they collude in country A or in both countries, they will be prosecuted the first time that $s_t^A = 1$. Therefore, the expected profits of a firm under collusion in country A, collusion in country B and collusion in both countries are given by:

$$\Pi^{c,A} = \frac{\pi^{c,A} - \alpha^A f^A}{1-(1-\alpha^A)\,\delta},\ \Pi^{c,B} = \frac{\pi^{c,B}}{1-\delta},\ \Pi^{c,AB} = \frac{\pi^{c,A} + \pi^{c,B} - \alpha^A\left(f^A + f^B\right)}{1-(1-\alpha^A)\,\delta},$$

respectively. In order to deduce $\Pi^{c,AB}$, note that $\Pi^{c,AB} = \alpha^A\left(\pi^{c,A} + \pi^{c,B} - f^A - f^B\right) + \left(1-\alpha^A\right)\left(\pi^{c,A} + \pi^{c,B} + \delta\Pi^{c,AB}\right)$.

Sustainability of each type of collusion: Let's deduce the conditions under which each type of collusion can be sustained as a subgame perfect Nash equilibrium.

- For collusion in country A to be an equilibrium, it must be the case that $\Pi^{c,A} \geq \pi^{d,A}$, which holds if and only if $\alpha^A \leq \bar{\alpha}^A$, where
$$\bar{\alpha}^A = \frac{\pi^{c,A} - (1-\delta)\,\pi^{d,A}}{\delta\pi^{d,A} + f^A}.$$

- For collusion in country B to be an equilibrium, it must be the case that $\Pi^{c,B} \geq \pi^{d,B}$, which always holds.

- For collusion in both countries to be an equilibrium, it must be the case that no firm has an incentive to deviate in country A, in country B, or in both countries. No firm has an incentive to stop colluding in country A if $\Pi^{c,AB} \geq \pi^{d,A} + \Pi^{c,B}$, which holds if and only if $\alpha^A \leq \frac{\pi^{c,A}-(1-\delta)\pi^{d,A}}{\delta\pi^{d,A}+f^A+\frac{\delta\pi^{c,B}}{1-\delta}+f^B}$. No firm has an incentive to stop colluding in country B if $\Pi^{c,AB} \geq \pi^{d,B} + \Pi^{c,A}$, which holds if and only if $\alpha^A \leq \frac{\pi^{c,B}-(1-\delta)\pi^{d,B}}{\delta\pi^{d,B}+f^B}$. No firm has an incentive to deviate in both countries if $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$, which holds if and only if $\alpha^A \leq \frac{\pi^{c,A}-(1-\delta)\left(\pi^{d,A}+\pi^{d,B}\right)}{\delta\pi^{d,A}+\delta\pi^{d,B}+f^A}$. Therefore, for collusion in both countries to be an equilibrium it must be the case that $\alpha^A \leq \bar{\alpha}^{AB}$, where
$$\bar{\alpha}^{AB} = \min\left\{\frac{\pi^{c,A} - (1-\delta)\,\pi^{d,A}}{\delta\pi^{d,A} + f^A + \frac{\delta\pi^{c,B}}{1-\delta} + f^B}, \frac{\pi^{c,B} - (1-\delta)\,\pi^{d,B}}{\delta\pi^{d,B} + f^B}, \frac{\pi^{c,A} - (1-\delta)\left(\pi^{d,A} + \pi^{d,B}\right)}{\delta\left(\pi^{d,A} + \pi^{d,B}\right) + f^A + f^B}\right\}$$

Selection: Let's compare expected discounted profits under each type of collusion.

- $\Pi^{c,AB} > \Pi^{c,A}$ if and only if $\alpha^A < \hat{\alpha}^{A,AB}$, where

$$\hat{\alpha}^{A,AB} = \frac{\pi^{c,B}}{f^B}$$

- $\Pi^{c,AB} > \Pi^{c,B}$ if and only if $\alpha^A < \hat{\alpha}^{B,AB}$, where

$$\hat{\alpha}^{B,AB} = \frac{(1-\delta)\,\pi^{c,A}}{\delta\pi^{c,B} + (1-\delta)\left(f^A + f^B\right)}$$

- $\Pi^{c,A} > \Pi^{c,B}$ if and only if $\alpha^A < \hat{\alpha}^{A,B}$, where

$$\hat{\alpha}^{A,B} = \frac{(1-\delta)\left(\pi^{c,A} - \pi^{c,B}\right)}{\delta\pi^{c,B} + (1-\delta)\,f^A}$$

To determine the optimal collusion strategy of the firms, note that $\bar{\alpha}^A > \frac{\pi^{c,A}-(1-\delta)\pi^{d,A}}{\delta\pi^{d,A}+f^A+\frac{\delta\pi^{c,B}}{1-\delta}+f^B}$, which implies that $\bar{\alpha}^A > \bar{\alpha}^{AB}$. Therefore, we must consider the following three cases:

1. Suppose that $\alpha^A > \bar{\alpha}^A$. Then, only collusion in country B can be sustained as an equilibrium. Thus, if $\alpha^A > \bar{\alpha}^A$, the optimal collusion strategy of the firms is to collude in B.

2. Suppose that $\bar{\alpha}^{AB} < \alpha^A \leq \bar{\alpha}^A$. Then, only collusion in country B or collusion in country A can be sustained as an equilibrium. Moreover, firms prefer to collude in A when $\alpha^A < \hat{\alpha}^{A,B}$ and to collude in B when $\alpha^A > \hat{\alpha}^{A,B}$. Thus, the optimal collusion strategy of the firms is to collude in A if $\alpha^A < \hat{\alpha}^{A,B}$ and collude in B if $\alpha^A > \hat{\alpha}^{A,B}$.

3. Suppose that $\alpha^A \leq \bar{\alpha}^{AB}$. Then, the three types of collusion can be sustained as an equilibrium. Note that $\frac{\pi^{c,B}}{f^B} > \frac{\pi^{c,B}-(1-\delta)\pi^{d,B}}{\delta\pi^{d,B}+f^B}$, which implies that $\bar{\alpha}^{AB} < \hat{\alpha}^{A,AB}$. Also note that $\frac{(1-\delta)\pi^{c,A}}{\delta\pi^{c,B}+(1-\delta)(f^A+f^B)} > \frac{\pi^{c,A}-(1-\delta)\pi^{d,A}}{\delta\pi^{d,A}+f^A+\frac{\delta\pi^{c,B}}{1-\delta}+f^B}$, which implies that $\bar{\alpha}^{AB} < \hat{\alpha}^{B,AB}$. Therefore, the optimal collusion strategy of the firms is to collude in both countries.

To obtain Proposition 1 define $\bar{\alpha}_L(\delta) = \bar{\alpha}^A$, $\bar{\alpha}_H(\delta) = \bar{\alpha}^{AB}$, $\hat{\alpha}_M(\delta) = \hat{\alpha}^{A,B}$. ■

A.3.3 Short-run Information Spillover

Assumption 1 states that when competition authority A prosecutes the firms in country A, competition authority B observes the process and learns how to detect evidence of collusion and prosecute it in country B in all future periods. A weaker version of this assumption is to assume that prosecution in country A in period t only allows competition authority B to detect and prosecute collusion in period t, but it is not informative in future periods. Formally:

Assumption 1.bis. *The signal received by competition authority A is as in Assumption 1 and the signal received by competition authority B is given by:*

$$s_t^B = \begin{cases} 1 & \textit{if } a_t^{1,B} = a_t^{2,B} = 1 \textit{ and } p_t^A = 1, \\ 0 & \textit{otherwise.} \end{cases}$$

Note that while under Assumption 1, $s_t^B = 1$ when $a_t^{1,A} = a_t^{2,A} = 1$ and $p_\tau^A = 1$ for at least one $\tau \leq t$, under Assumption 1.bis, $s_t^B = 1$ only when $a_t^{1,B} = a_t^{2,B} = 1$ and $p_t^A = 1$. Thus, under Assumption 1, prosecution in country A in one period, allows competition authority B to detect and prosecute collusion in every future period, while under Assumption 1', prosecution in country A is only temporarily informative for competition authority B. The main implication of replacing Assumption 1 by Assumption 1.bis is that now firms have access to new ways of organizing collusion. In particular, firms can collude only in country A until they are detected and, thereafter, start colluding in country B. Collusion in country B will never be detected because it begins after $p_t^A = 1$ and, hence, there is no way that competition authority B detects it.

Proposition 1 (Short-run information spillover). *Suppose that Assumption 1.bis holds, firms punish deviations from collusion stopping collusion in both countries and they coordinate in their best equilibrium. Given $\lambda > \lambda^*$,*

1. *Suppose that $\bar{\delta}^{SR} < \delta < \delta^*(\lambda)$, where $\bar{\delta}^{SR} = \max\left\{\frac{\pi^{d,A}-\pi^{c,A}}{\pi^{d,A}}, \frac{\pi^{d,B}-\pi^{c,B}}{\pi^{d,B}}\right\}$. Then firms collude in both countries and they are never prosecuted.*

2. *Suppose that $\delta \geq \delta^*(\lambda)$ and $f^A > \left(\pi^{c,A} - \pi^{d,A}\right) + \delta\frac{\pi^{c,B}}{1-\delta}$. Then, there are thresholds $\bar{\alpha}_L(\delta)$ and $\bar{\alpha}_H(\delta)$ such that:*

   (a) *Suppose that $\alpha^A > \bar{\alpha}_H(\delta)$. Then, firms collude in country B and they are never prosecuted.*

   (b) *Suppose that $\bar{\alpha}_L(\delta) < \alpha^A \leq \bar{\alpha}_H(\delta)$. Then, there is a threshold $\hat{\alpha}_{M_1}(\delta)$ such that: If $\alpha^A \in (\bar{\alpha}_L(\delta), \hat{\alpha}_{M_1}(\delta)]$, then one of the following two paths will occur. (1) Firms collude in both countries until the first time $s_t^A = 1$, when they are prosecuted. Thereafter, there is competition in both countries. (2) Firms collude in country A until the first time $s_t^A = 1$, when they are prosecuted. Thereafter, they collude in country B, where they will never be prosecuted. If $\alpha^A \in [\hat{\alpha}_{M_1}(\delta), \bar{\alpha}_H(\delta)]$, then firms only collude in country B and they are never prosecuted.*

   (c) *Suppose* that $\alpha^A \leq \bar{\alpha}_L(\delta)$. *Then, there is a threshold $\hat{\alpha}_{M_2}(\delta)$ such that: If $\alpha^A \in [0, \hat{\alpha}_{M_2}(\delta))$, then one of the following two paths will occur. (1) Firms collude in both countries until the first time $s_t^A = 1$, when they are prosecuted. Thereafter, there is competition in both countries. (2) Firms collude in country A until the first time $s_t^A = 1$, when they are prosecuted. Thereafter, they collude in country B, where they will never be prosecuted. If $\alpha^A \in [\hat{\alpha}_{M_2}(\delta), \bar{\alpha}_L(\delta)]$, then firms only collude in country B and they are never prosecuted.*

Proof. Let's first prove that Lemmas 1 and 2 hold when Assumption 1 is replaced by Assumption 1.bis.

Lemma 1: It is easy to verify that competition authority B will always prosecute the firms as soon as collusion is detected because country B does not own the firms. Thus, Lemma 1 holds when Assumption 1 is replaced by Assumption 1.bis.

Lemma 2. Suppose that firms are only colluding in country A while there is and will always be competition in country B. Assume that $s_t^A = 1$. If the competition authority of country A prosecutes the firms, then the expected discounted welfare of country A at period t is $W_t^A\left(p_t^A = 1|s_t^A = 1, A\right) = CS^{c,A} + 2\lambda\left(\pi^{c,A} - f^A\right) + 2f^A + \frac{\delta}{1-\delta}CS^{com,A}$, while if firms are not prosecuted, it is $W_t^A\left(p_t^A = 0|s_t^A = 1, A\right) = \frac{CS^{c,A}+2\lambda\pi^{c,A}}{1-\delta}$. Since $CS^{com,A} > CS^{c,A} + 2\pi^{c,A}$, it must be the case that $W_t^A\left(p_t^A = 1|s_t^A = 1, A\right) > W_t^A\left(p_t^A = 0|s_t^A = 1, A\right)$.

Suppose that firms are only colluding in country A, there is competition in country B, but as soon as they are prosecuted in country A, firms will start colluding in country B. Assume that $s_t^A = 1$. If the competition authority of country A prosecutes the firms, then the expected discounted welfare of country A at period t is $W_t^A\left(p_t^A = 1|s_t^A = 1, A\right) = CS^{c,A} + 2\lambda\left(\pi^{c,A} - 2f^A\right) + 2f^A + \frac{\delta}{1-\delta}\left(CS^{com,A} + 2\lambda\pi^{c,B}\right)$, while if the firms are not prosecuted, it is $W_t^A\left(p_t^A = 0|s_t^A = 1, A\right) = \frac{CS^{c,A}+2\lambda\pi^{c,A}}{1-\delta}$. This condition will hold for all possible values of δ as $CS^{com,A} - CS^{c,A} - 2f^A > 0$.

Suppose that firms are colluding in both countries and $s_t^A = 1$. If the competition authority of country A decides to prosecute the firms, the expected discounted welfare of country A at period t will be $W_t^A\left(p_t^A = 1|s_t^A = 1, AB\right) = CS^{c,A} + 2\lambda\left(\pi^{c,A} - f^A + \pi^{c,B} - f^B\right) + 2f^A + \frac{\delta}{1-\delta}CS^{com,A}$. If the competition authority of country A does not prosecute the firms, then the the expected discounted welfare of country A at period t will be $W_t^A\left(p_t^A = 0|s_t^A = 1, AB\right) = \frac{CS^{c,A}+2\lambda\left(\pi^{c,A}+\pi^{c,B}\right)}{1-\delta}$. $W_t^A\left(p_t^A = 1|s_t^A = 1, AB\right) > W_t^A\left(p_t^A = 0|s_t^A = 1, AB\right)$ if and only if $\delta\left(CS^{com,A} - CS^{c,A} - 2\lambda\pi^{c,A}\right) > 2\delta\lambda\pi^{c,B} + (1-\delta)(2\lambda(f^A + f^B) - f^A)$. Then, as $CS^{com,A} - CS^{c,A} - 2f^A > 0$ there exists $\lambda^* \in (0,1)$ where $\delta^*(\lambda) \geq 0$ for all $\lambda \geq \lambda^*$ with $\lambda^* = \frac{f^A}{f^A+f^B}$. Thus, Lemma 2 holds when Assumption 1 is replaced by Assumption 1.bis.

Optimal collusion strategy:

1. Suppose that $\bar{\delta}^{SR} < \delta < \delta^*(\lambda)$.

From Lemmas 1 and 2, if firms are colluding in both countries they will never be prosecuted. Therefore, if firms always collude in both countries, the expected discounted profits of a firm are $\Pi^{c,AB} = \frac{\pi^{c,A}+\pi^{c,B}}{1-\delta}$. Note that there is no other form of collusion that will induce higher discounted expected profits. Moreover, for collusion in both countries to be sustained as a subgame perfect Nash equilibrium, it must be the case that $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$, which always holds because $\delta > \bar{\delta}^{SR}$ implies that $\pi^{c,j} > (1-\delta)\,\pi^{d,j}$ for $j = A, B$. Thus, the optimal collusion strategy of the firms is to collude in both countries.

2. Suppose that $\delta \geq \delta^*(\lambda)$.

Profits under each type of collusion: From Lemmas 1 and 2, if firms are only colluding in country B, they will never be prosecuted, while if they are colluding in country A or in both countries, they will be prosecuted the first time that $s_t^A = 1$. Therefore, there are three types of collusion that firms must consider: 1) collude only in country A until detected, then start colluding in country B; 2) always collude only in country B; and 3) always collude in both countries. The expected discounted profits of a firm associated with each type of collusion are given by:

$$\Pi^{c,A} = \frac{\pi^{c,A} - \alpha^A f^A + \alpha^A\delta\frac{\pi^{c,B}}{1-\delta}}{1-(1-\alpha^A)\,\delta},\ \Pi^{c,B} = \frac{\pi^{c,B}}{1-\delta},\ \Pi^{c,AB} = \frac{\pi^{c,A} + \pi^{c,B} - \alpha^A\left(f^A + f^B\right)}{1-(1-\alpha^A)\,\delta},$$

respectively. In order to deduce $\Pi^{c,A}$ and $\Pi^{c,AB}$ note that $\Pi^{c,A} = \alpha^A\left(\pi^{c,A} - f^A + \delta\Pi^{c,B}\right) + \left(1-\alpha^A\right)\left(\pi^{c,A} + \delta\Pi^{c,A}\right)$ and $\Pi^{c,AB} = \alpha^A\left(\pi^{c,A} + \pi^{c,B} - f^A - f^B\right) + \left(1-\alpha^A\right)\left(\pi^{c,A} + \pi^{c,B} + \delta\Pi^{c,AB}\right)$.

Sustainability of each type of collusion: Let's deduce the conditions under which each type of collusion can be sustained as a subgame perfect Nash equilibrium.

- For collusion in country A until detected, then collusion in country B to be an equilibrium, it must be the case that $\Pi^{c,A} \geq \pi^{d,A}$, which holds if and only if $\alpha^A \leq \bar{\alpha}^A$, where

$$\bar{\alpha}^A = \begin{cases} \frac{\pi^{c,A}-(1-\delta)\pi^{d,A}}{\delta\pi^{d,A}+f^A-\delta\frac{\pi^{c,B}}{1-\delta}} & \text{if } f^A > \left(\pi^{c,A} - \pi^{d,A}\right) + \delta\frac{\pi^{c,B}}{1-\delta} \\ 1 & \text{if } f^A \leq \left(\pi^{c,A} - \pi^{d,A}\right) + \delta\frac{\pi^{c,B}}{1-\delta} \end{cases}$$

- For collusion in country B to be an equilibrium it must be the case that $\Pi^{c,B} \geq \pi^{d,B}$, which always holds because $\delta > \bar{\delta}^{SR}$.

- For collusion in both countries to be an equilibrium, it must be the case that $\Pi^{c,AB} \geq \pi^{d,A} + \pi^{d,B}$, which holds if and only if $\alpha^A \leq \bar{\alpha}^{AB}$, where

$$\bar{\alpha}^{AB} = \frac{\left(\pi^{c,A} + \pi^{c,B}\right) - (1-\delta)\left(\pi^{d,A} + \pi^{d,B}\right)}{\delta\left(\pi^{d,A} + \pi^{d,B}\right) + f^A + f^B}$$

Selection: Let's compare expected discounted profits under each type of collusion.

- $\Pi^{c,AB} > \Pi^{c,A}$ if and only if $\alpha^A < \hat{\alpha}^{A,AB}$, where

$$\hat{\alpha}^{A,AB} = \frac{(1-\delta)\,\pi^{c,B}}{\delta\pi^{c,B} + (1-\delta)\,f^B}$$

- $\Pi^{c,AB} > \Pi^{c,B}$ if and only if $\alpha^A < \hat{\alpha}^{B,AB}$, where

$$\hat{\alpha}^{B,AB} = \frac{(1-\delta)\,\pi^{c,A}}{\delta\pi^{c,B} + (1-\delta)\left(f^A + f^B\right)}$$

- $\Pi^{c,A} > \Pi^{c,B}$ if and ony if $\alpha^A < \hat{\alpha}^{A,B}$, where

$$\hat{\alpha}^{A,B} = \frac{\pi^{c,A} - \pi^{c,B}}{f^A}$$

To determine the optimal collusion strategy of the firms, we must consider the following three cases:

1. Suppose that $\max\left\{\bar{\alpha}^A, \bar{\alpha}^{AB}\right\} < \alpha^A \leq 1$ (Note that if $f^A \leq \left(\pi^{c,A} - \pi^{d,A}\right) + \delta\frac{\pi^{c,B}}{1-\delta}$, then $\bar{\alpha}^A = 1$ and, hence, this region is empty). Then, only collusion in country B can be sustained as an equilibrium. Therefore, the optimal collusion strategy of the firms is to collude in B.

2. Suppose that $\min\left\{\bar{\alpha}^A, \bar{\alpha}^{AB}\right\} < \alpha^A \leq \max\left\{\bar{\alpha}^A, \bar{\alpha}^{AB}\right\}$. Then, there are two possible cases. First, suppose that $\bar{\alpha}^A < \bar{\alpha}^{AB}$ and $\bar{\alpha}^A < \alpha^A \leq \bar{\alpha}^{AB}$. Then, only collusion in country B or in both countries can be sustained. Moreover, firms prefer to collude in both countries when $\alpha^A < \hat{\alpha}^{B,AB}$ and to collude in B when $\alpha^A > \hat{\alpha}^{B,AB}$. Thus, the optimal collusion strategy of the firms is to collude in both countries if $\alpha^A < \hat{\alpha}^{B,AB}$ and collude in B if $\alpha^A > \hat{\alpha}^{B,AB}$. Second, suppose that $\bar{\alpha}^A > \bar{\alpha}^{AB}$ and $\bar{\alpha}^{AB} < \alpha^A \leq \bar{\alpha}^A$. Then, only collusion in country B or collusion in country A can be sustained. Moreover, firms prefer to collude in A when $\alpha^A < \hat{\alpha}^{A,B}$ and to collude in B when $\alpha^A > \hat{\alpha}^{A,B}$. Thus, the optimal collusion strategy of the firms is to collude in A if $\alpha^A < \hat{\alpha}^{A,B}$ and collude in B if $\alpha^A > \hat{\alpha}^{A,B}$.

3. Suppose that $0 \leq \alpha^A \leq \min\left\{\bar{\alpha}^A, \bar{\alpha}^{AB}\right\}$. Then, the three types of collusion can be sustained as an equilibrium. Moreover, firms prefer to collude in A when $\hat{\alpha}^{A,AB} < \alpha^A < \hat{\alpha}^{A,B}$, they prefer to collude in B when $\alpha^A > \max\left\{\hat{\alpha}^{A,B}, \hat{\alpha}^{B,AB}\right\}$, and they prefer to collude in both countries when $\alpha^A < \min\left\{\hat{\alpha}^{B,AB}, \hat{\alpha}^{A,AB}\right\}$. Thus, the optimal collusion strategy of the firms is to collude in A when $\hat{\alpha}^{A,AB} < \alpha^A < \hat{\alpha}^{A,B}$, collude in B when $\alpha^A > \max\left\{\hat{\alpha}^{A,B}, \hat{\alpha}^{B,AB}\right\}$, and collude in both countries when $\alpha^A < \min\left\{\hat{\alpha}^{B,AB}, \hat{\alpha}^{A,AB}\right\}$.

To obtain Proposition 1, assume that $f^A > \left(\pi^{c,A} - \pi^{d,A}\right) + \delta\frac{\pi^{c,B}}{1-\delta}$ and define

$$\bar{\alpha}_L(\delta) = \min\left\{\bar{\alpha}^A, \bar{\alpha}^{AB}\right\}, \; \bar{\alpha}_H(\delta) = \max\left\{\bar{\alpha}^A, \bar{\alpha}^{AB}\right\}$$
$$\hat{\alpha}_{M_1}(\delta) = \begin{cases} \hat{\alpha}^{B,AB} & \text{if } \bar{\alpha}^A < \bar{\alpha}^{AB} \\ \hat{\alpha}^{A,B} & \text{if } \bar{\alpha}^{AB} < \bar{\alpha}^A \end{cases}, \; \hat{\alpha}_{M_2}(\delta) = \max\left\{\hat{\alpha}^{B,AB}, \hat{\alpha}^{A,B}\right\}$$

To obtain a detailed characterization of the equilibrium note that:

- If $\hat{\alpha}^{A,B} > \hat{\alpha}^{A,AB}$, then $\hat{\alpha}^{A,B} > \hat{\alpha}^{B,AB} > \hat{\alpha}^{A,AB}$; while if $\hat{\alpha}^{A,AB} > \hat{\alpha}^{A,B}$, then $\hat{\alpha}^{A,AB} > \hat{\alpha}^{B,AB} > \hat{\alpha}^{A,B}$. To prove this, note that (i) $\hat{\alpha}^{A,B} > \hat{\alpha}^{A,AB}$ if and only if $\pi^{c,A} > \pi^{c,B} + \frac{(1-\delta)\pi^{c,B} f^A}{\delta\pi^{c,B} + (1-\delta) f^B}$; (ii) $\hat{\alpha}^{A,B} > \hat{\alpha}^{B,AB}$ if and only if $\pi^{c,A} > \pi^{c,B} + \frac{(1-\delta)\pi^{c,A} f^A}{\delta\pi^{c,B} + (1-\delta)(f^A + f^B)}$; and (iii) $\hat{\alpha}^{B,AB} > \hat{\alpha}^{A,AB}$ if and only if $\pi^{c,A} > \pi^{c,B} + \frac{(1-\delta)\pi^{c,B} f^A}{\delta\pi^{c,B} + (1-\delta) f^B}$.
- If $\hat{\alpha}^{A,AB} > \hat{\alpha}^{A,B}$, then $\hat{\alpha}^{A,AB} > \hat{\alpha}^{B,AB} > \hat{\alpha}^{A,B}$. To prove this, note that (i) $\hat{\alpha}^{A,AB} > \hat{\alpha}^{A,B}$ if and only if $\pi^{c,A} < \pi^{c,B} + \frac{(1-\delta)\pi^{c,B} f^A}{\delta\pi^{c,B} + (1-\delta) f^B}$; (ii) $\hat{\alpha}^{B,AB} > \hat{\alpha}^{A,B}$ if and only if $\pi^{c,A} < \pi^{c,B} + \frac{(1-\delta)\pi^{c,A} f^A}{\delta\pi^{c,B} + (1-\delta)(f^A + f^B)}$; and (iii) $\hat{\alpha}^{A,AB} > \hat{\alpha}^{B,AB}$ if and only if $\pi^{c,A} < \pi^{c,B} + \frac{(1-\delta)\pi^{c,B} f^A}{\delta\pi^{c,B} + (1-\delta) f^B}$.

Therefore, we must consider the following six possible situations.

1. Suppose that $f^A \leq \left(\pi^{c,A} - \pi^{d,A}\right) + \delta\frac{\pi^{c,B}}{1-\delta}$ and $\pi^{c,A} > \pi^{c,B} + \frac{(1-\delta)\pi^{c,B} f^A}{\delta\pi^{c,B} + (1-\delta) f^B}$. Then, $\bar{\alpha}^A = 1$ and $\hat{\alpha}^{A,B} > \hat{\alpha}^{B,AB} > \hat{\alpha}^{A,AB}$.
   (a) If $\alpha^A > \bar{\alpha}^{AB}$, then firms collude in A when $\alpha^A < \hat{\alpha}^{A,B}$ and collude in B when $\alpha^A > \hat{\alpha}^{A,B}$.
   (b) If $\alpha^A \leq \bar{\alpha}^{AB}$, then firms in collude in B when $\alpha^A > \hat{\alpha}^{A,B}$, in A when $\hat{\alpha}^{A,AB} < \alpha^A < \hat{\alpha}^{A,B}$, and in both countries when $\alpha^A < \hat{\alpha}^{A,AB}$.
2. Suppose that $f^A \leq \left(\pi^{c,A} - \pi^{d,A}\right) + \delta\frac{\pi^{c,B}}{1-\delta}$ and $\pi^{c,A} < \pi^{c,B} + \frac{(1-\delta)\pi^{c,B} f^A}{\delta\pi^{c,B} + (1-\delta) f^B}$. Then, $\bar{\alpha}^A = 1$ and $\hat{\alpha}^{A,AB} > \hat{\alpha}^{B,AB} > \hat{\alpha}^{A,B}$.
   (a) If $\alpha^A > \bar{\alpha}^{AB}$, then firms collude in A when $\alpha^A < \hat{\alpha}^{A,B}$ and collude in B when $\alpha^A > \hat{\alpha}^{A,B}$.
   (b) If $\alpha^A \leq \bar{\alpha}^{AB}$, then firms collude in B when $\alpha^A > \hat{\alpha}^{B,AB}$ and in both countries when $\alpha^A < \hat{\alpha}^{B,AB}$.
3. Suppose that $f^A > \left(\pi^{c,A} - \pi^{d,A}\right) + \delta\frac{\pi^{c,B}}{1-\delta}$, $\frac{\left(\pi^{c,A} + \pi^{c,B}\right) - (1-\delta)\left(\pi^{d,A} + \pi^{d,B}\right)}{\delta\left(\pi^{d,A} + \pi^{d,B}\right) + f^A + f^B} < \frac{\pi^{c,A} - (1-\delta)\pi^{d,A}}{\delta\pi^{d,A} + f^A - \delta\frac{\pi^{c,B}}{1-\delta}}$ and $\pi^{c,A} > \pi^{c,B} + \frac{(1-\delta)\pi^{c,B} f^A}{\delta\pi^{c,B} + (1-\delta) f^B}$. Then, $\bar{\alpha}^{AB} < \bar{\alpha}^A < 1$ and $\hat{\alpha}^{A,B} > \hat{\alpha}^{B,AB} > \hat{\alpha}^{A,AB}$.
   (a) If $\alpha^A > \bar{\alpha}^A$, then firms only collude in country B.
   (b) If $\bar{\alpha}^{AB} < \alpha^A \leq \bar{\alpha}^A$, then firms collude in A when $\alpha^A < \hat{\alpha}^{A,B}$ and collude in B when $\alpha^A > \hat{\alpha}^{A,B}$.
   (c) If $\alpha^A \leq \bar{\alpha}^{AB}$, then firms in collude in B when $\alpha^A > \hat{\alpha}^{A,B}$, in A when $\hat{\alpha}^{A,AB} < \alpha^A < \hat{\alpha}^{A,B}$, and in both countries when $\alpha^A < \hat{\alpha}^{A,AB}$.

4. Suppose that $f^A > \left(\pi^{c,A} - \pi^{d,A}\right) + \delta \frac{\pi^{c,B}}{1-\delta}$, $\frac{\left(\pi^{c,A}+\pi^{c,B}\right)-(1-\delta)\left(\pi^{d,A}+\pi^{d,B}\right)}{\delta\left(\pi^{d,A}+\pi^{d,B}\right)+f^A+f^B} < \frac{\pi^{c,A}-(1-\delta)\pi^{d,A}}{\delta\pi^{d,A}+f^A-\delta\frac{\pi^{c,B}}{1-\delta}}$ and $\pi^{c,A} < \pi^{c,B} + \frac{(1-\delta)\pi^{c,B}f^A}{\delta\pi^{c,B}+(1-\delta)f^B}$. Then, $\bar{\alpha}^{AB} < \bar{\alpha}^A < 1$ and $\hat{\alpha}^{A,AB} > \hat{\alpha}^{B,AB} > \hat{\alpha}^{A,B}$.

   (a) If $\alpha^A > \bar{\alpha}^A$, then firms only collude in country B.

   (b) If $\bar{\alpha}^{AB} < \alpha^A \leq \bar{\alpha}^A$, then firms collude in A when $\alpha^A < \hat{\alpha}^{A,B}$ and collude in B when $\alpha^A > \hat{\alpha}^{A,B}$.

   (c) If $\alpha^A \leq \bar{\alpha}^{AB}$, then firms collude in B when $\alpha^A > \hat{\alpha}^{B,AB}$ and in both countries when $\alpha^A < \hat{\alpha}^{B,AB}$.

5. Suppose that $f^A > \left(\pi^{c,A} - \pi^{d,A}\right) + \delta \frac{\pi^{c,B}}{1-\delta}$, $\frac{\left(\pi^{c,A}+\pi^{c,B}\right)-(1-\delta)\left(\pi^{d,A}+\pi^{d,B}\right)}{\delta\left(\pi^{d,A}+\pi^{d,B}\right)+f^A+f^B} > \frac{\pi^{c,A}-(1-\delta)\pi^{d,A}}{\delta\pi^{d,A}+f^A-\delta\frac{\pi^{c,B}}{1-\delta}}$ and $\pi^{c,A} > \pi^{c,B} + \frac{(1-\delta)\pi^{c,B}f^A}{\delta\pi^{c,B}+(1-\delta)f^B}$. Then, $\bar{\alpha}^A < \bar{\alpha}^{AB}$ and $\hat{\alpha}^{A,B} > \hat{\alpha}^{B,AB} > \hat{\alpha}^{A,AB}$.

   (a) If $\alpha^A > \bar{\alpha}^{AB}$, then firms only collude in country B.

   (b) If $\bar{\alpha}^A < \alpha^A \leq \bar{\alpha}^{AB}$, then firms collude in B when $\alpha^A > \hat{\alpha}^{B,AB}$ and in both countries when $\alpha^A < \hat{\alpha}^{B,AB}$.

   (c) If $\alpha^A \leq \bar{\alpha}^A$, then firms collude in B when $\alpha^A > \hat{\alpha}^{A,B}$, in A when $\hat{\alpha}^{A,AB} < \alpha^A < \hat{\alpha}^{A,B}$, and collude in both countries when $\alpha^A < \hat{\alpha}^{A,AB}$.

6. Suppose that $f^A > \left(\pi^{c,A} - \pi^{d,A}\right) + \delta \frac{\pi^{c,B}}{1-\delta}$, $\frac{\pi^{c,A}-(1-\delta)\pi^{d,A}}{\delta\pi^{d,A}+f^A-\delta\frac{\pi^{c,B}}{1-\delta}} > \frac{\left(\pi^{c,A}+\pi^{c,B}\right)-(1-\delta)\left(\pi^{d,A}+\pi^{d,B}\right)}{\delta\left(\pi^{d,A}+\pi^{d,B}\right)+f^A+f^B}$ and $\pi^{c,A} < \pi^{c,B} + \frac{(1-\delta)\pi^{c,B}f^A}{\delta\pi^{c,B}+(1-\delta)f^B}$. Then, $\bar{\alpha}^A < \bar{\alpha}^{AB}$ and $\hat{\alpha}^{A,AB} > \hat{\alpha}^{B,AB} > \hat{\alpha}^{A,B}$.

   (a) If $\alpha^A > \bar{\alpha}^{AB}$, then firms only collude in country B.

   (b) If $\bar{\alpha}^A < \alpha^A \leq \bar{\alpha}^{AB}$, then firms collude in B when $\alpha^A > \hat{\alpha}^{B,AB}$ and in both countries when $\alpha^A < \hat{\alpha}^{B,AB}$.

   (c) If $\alpha^A \leq \bar{\alpha}^A$, firms collude in B when $\alpha^A > \hat{\alpha}^{B,AB}$, and collude in both countries when $\alpha^A < \hat{\alpha}^{B,AB}$.

This completes the proof of Proposition 1. ■

The main novelty of this proposition is the firms strategy of colluding in country A until detected and then switching to country B, which generates higher expected profits than colluding solely in country A. The reason is that collusion in country B cannot be detected once firms have been prosecuted in country A. This result points toward the idea that if firms are detected in their own market, they will start collusion in foreign markets. The following example illustrates the results for a simple Bertrand specification.

Example 1.bis. Consider the specification in Example 1 when Assumption 1 is replaced by Assumption 1.bis. Figure A,1 shows the equilibrium outcome for each value of δ and α^A when $k_c = 0.55$ and $\gamma = 1.2$. In the figure $COM\,(A, B)$ indicates that firms compete in both countries; $COL\,(A, B)\,, NP$ that firms collude in both countries and they are never prosecuted; $COL\,(B)\,, NP$ that firms collude in country B and they are never prosecuted; $COL\,(A^*, B)\,, P$, that firms collude only country A until the first time

$s_t^A = 1$, and thereafter they collude in country B; and $COL\,(A, B)\,, P$, that firms collude in both countries until the first time $s_t^A = 1$, when they are prosecuted and forced to compete in both countries. Figure A.1 also shows how an improvement in antitrust policy in country A (i.e., an increase in α^A) affects country B. Consider, for example, an increase in α^A such that, initially, the equilibrium is $COL\,(A, B)\,, P$ and, after the change in α^A, it is $COL\,(A^*, B)\,, P$. Under $COL\,(A, B)\,, P$, firms collude in both countries until the first time that $s_t^A = 1$, when the competition authority of country A prosecutes the firms, the competition authority of country B learns about this and also prosecuted the firms in country B. On the contrary, under $COL\,(A^*, B)\,, P$, firms initially are not colluding in country B and they only begin colluding in B when they are detected and prosecuted in country A. Thus, a change in α^A induces a temporal switch in the pattern of collusion in country B. Naturally, depending on the discount factor, country B could be better or worse off due to the change in α^A.

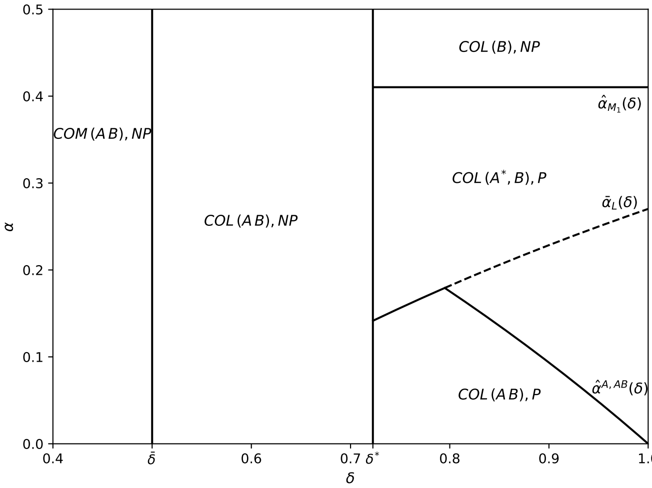

Figure A.1 Equilibrium under short-run information spillovers. Note: $k_c^A = 1$ and $k_c^B = k_c = 0.55$, $\gamma = 1.2$ and $\lambda = 1$.

B Detection Ability Partially Aligned with Firm Ownership

Appendix B presents the proofs of all the lemmas and propositions in Section 5.

B.1 Proof of Lemma 3

Lemma 3 *Suppose that Assumption 2 holds. Assume that the competition authority of country A detects collusion, i.e.,* $c_t^A = 1$.

1. *If the firms are not colluding in country B, then A always prosecutes the firms.*

2. *If the firms are also colluding in country B, then A prosecutes the firms if and only if*

$$CS^{com,A} - CS^{c,A} - 2\pi^{c,A} \geq \frac{\left[1 - \left(1 - \alpha^A\right)\delta\right] 2\left(1 - \alpha^B\right)\left[\delta\pi^{c,B} + (1 - \delta)\, f^B\right]}{\left[1 - \left(1 - \alpha^B\right)\left(1 - \alpha^A\right)\delta\right]\delta},$$

or, which is equivalent, $\alpha^B > \alpha^* = \frac{2\left[\delta\pi^{c,B}+(1-\delta)f^B\right]-\delta\left(CS^{com,A}-CS^{c,A}-2\pi^{c,A}\right)}{2[\delta\pi^{c,B}+(1-\delta)f^B]+\frac{\delta(1-\alpha^A)}{1-(1-\alpha^A)\delta}\delta(CS^{com,A}-CS^{c,A}-2\pi^{c,A})}$.

Proof. As in the proof of Lemma 2 we must distinguish two possible cases: when firms are only colluding in country A and when they are colluding in both countries.

1. Suppose that firms are only colluding in country A and $s_t^A = 1$. If competition authority A prosecutes the firms, the expected discounted welfare of country A at period t is

$$W_t^A(p_t^A = 1|s_t^A = 1, A) = CS^{c,A} + 2\left(\pi^{c,A} - f^A\right) + 2f^A + \frac{\delta}{1-\delta}CS^{com,A},$$

while if firms are not prosecuted, it is

$$W_t^A(p_t^A = 0|s_t^A = 1, A) = \frac{CS^{c,A} + 2\pi^{c,A}}{1-\delta}$$

Since $CS^{com,A} > CS^{c,A}+2\pi^{c,A}$, it must be the case that $W_t^A(p_t^A = 1|s_t^A = 1, A) > W_t^A(p_t^A = 0|s_t^A = 1, A)$. Thus, when firms are only colluding in country A, as soon as $c_t^A = 1$, competition authority A immediately prosecutes the firms.

2. Suppose that firms collude in both countries and $s_t^A = 1$. If competition authority A decides to prosecute the firms, then, from Lemma 1, competition authority B will detect collusion in country B as well and, hence, firms will also be prosecuted in country B. Therefore, from period $t+1$ there will be competition in both countries. Then, the expected discounted welfare of country A at period t is:

$$W_t^A\left(p_t^A = 1|s_t^A = 1, AB\right) = CS^{c,A} + 2\left(\pi^{c,A} - f^A + \pi^{c,B} - f^B\right) + 2f^A + \frac{\delta}{1-\delta}CS^{com,A}.$$

If competition authority A does not prosecute the firms, then with probability α^B competition authority B receives $s_t^B = 1$ and with probability $\left(1-\alpha^B\right)$ it receives $s_t^B = 0$. If $s_t^B = 1$, then competition authority B prosecutes the firms in period t and in period $t+1$ with probability α^A competition authority A will receive $s_{t+1}^A = 1$ and with probability $\left(1-\alpha^A\right)$ it will receive $s_{t+1}^A = 0$. Then, the expected discounted welfare of country A at period t is

$$\begin{aligned}W_t^A\left(p_t^A = 0|s_t^A = 1, AB\right) &= CS^{c,A} + 2\pi^{c,A} + 2\pi^{c,B} - \alpha^B 2f^B \\ &+ \alpha^B\delta\left[\alpha^A W_t^A\left(p_t^A = 1|s_t^A = 1, A\right) + \left(1-\alpha^A\right)W_t^A\left(s_t^A = 0, A\right)\right] \\ &+ \left(1-\alpha^B\right)\delta W_t^A\left(p_t^A = 0|s_t^A = 1, AB\right)\end{aligned}$$

where $W_t^A\left(p_t^A = 1|s_t^A = 1, A\right)$ and $W_t^A\left(s_t^A = 0, A\right)$ are given by

$$W_t^A(p_t^A = 1|s_t^A = 1, A) = CS^{c,A} + 2\pi^{c,A} + \frac{\delta}{1-\delta}CS^{com,A},$$
$$W_t^A\left(s_t^A = 0, A\right) = CS^{c,A} + 2\pi^{c,A} + \delta\left[\alpha^A W_t^A(p_t^A = 1|s_t^A = 1, A) + \left(1-\alpha^A\right)W_t^A\left(s_t^A = 0, A\right)\right].$$

Solving for $W_t^A\left(p_t^A = 0|s_t^A = 1, AB\right)$ we obtain:

$$W_t^A\left(p_t^A = 0|s_t^A = 1, AB\right) = \frac{\frac{\left[1-\left(1-\alpha^A\right)\delta+\alpha^B\delta\right]\left(CS^{c,A}+2\pi^{c,A}\right)}{1-(1-\alpha^A)\delta} + 2\left(\pi^{c,B} - \alpha^B f^B\right) + \frac{\alpha^A\alpha^B\delta^2 CS^{com,A}}{(1-\delta)[1-(1-\alpha^A)\delta]}}{1-\left(1-\alpha^B\right)\delta}.$$

Finally, $W_t^A\left(p_t^A=1|s_t^A=1,AB\right) > W_t^A\left(p_t^A=0|s_t^A=1,AB\right)$ if and only $CS^{com,A} - CS^{c,A} - 2\pi^{c,A} \geq \frac{\left[1-\left(1-\alpha^A\right)\delta\right]2\left(1-\alpha^B\right)\left[\delta\pi^{c,B}+(1-\delta)f^B\right]}{\left[1-(1-\alpha^B)(1-\alpha^A)\delta\right]\delta}$, or, which is equivalent, $\alpha^B > \alpha^* = \frac{2\left[\delta\pi^{c,B}+(1-\delta)f^B\right]-\delta\left(CS^{com,A}-CS^{c,A}-2\pi^{c,A}\right)}{2\left[\delta\pi^{c,B}+(1-\delta)f^B\right]+\frac{\delta(1-\alpha^A)}{1-(1-\alpha^A)\delta}\delta\left(CS^{com,A}-CS^{c,A}-2\pi^{c,A}\right)}$. ■

B.2 Proof of Proposition 2

Proposition 2 *Suppose that Assumption 2 holds, firms punish deviations from collusion stopping collusion in both countries, they coordinate in their best equilibrium, and* $\pi^{c,A} > (1-\delta)\pi^{d,A}$ *and* $\pi^{c,B} > (1-\delta)\pi^{d,B}$*. Let*

$$\alpha^B > \alpha^* = \frac{2\left[\delta\pi^{c,B}+(1-\delta)f^B\right]-\delta\left(CS^{com,A}-CS^{c,A}-2\pi^{c,A}\right)}{2\left[\delta\pi^{c,B}+(1-\delta)f^B\right]+\frac{\delta(1-\alpha^A)}{1-(1-\alpha^A)\delta}\delta\left(CS^{com,A}-CS^{c,A}-2\pi^{c,A}\right)},$$

$$\alpha^A \leq \bar{\alpha}^A = \frac{\pi^{c,A}-(1-\delta)\pi^{d,A}}{\delta\pi^{d,A}+f^A},$$

$$\alpha^B \leq \bar{\alpha}^{AB} = \frac{\left(\pi^{c,A}+\pi^{c,B}\right)-(1-\delta)\left(\pi^{d,A}+\pi^{d,B}\right)}{\delta\left(\pi^{d,A}+\pi^{d,B}\right)-\frac{\delta\left(\pi^{c,A}-\alpha^A f^A\right)}{1-(1-\alpha^A)\delta}+f^B},$$

$$\alpha^B < \hat{\alpha}^{A,AB} = \frac{(1-\delta)\pi^{c,B}+\alpha^A\left[\delta\left(\pi^{c,A}+\pi^{c,B}\right)+(1-\delta)f^A\right]}{\left[1-\left(1-\alpha^A\right)\delta\right]f^B}.$$

1. *Suppose that the four inequalities above hold. Then, firms collude in both countries until the first time* $s_t^B=1$*, when they are prosecuted in country* B*. Thereafter, they collude in country* A *until the first time* $s_{t+\tau}^A=1$ *with* $\tau\geq 1$*, when they are prosecuted in country* A*. Thereafter, there is competition in both countries.*

2. *Suppose that the first three inequalities above hold, but the fourth does not hold. Then, firms only collude in country* A *until the first time* $s_t^A=1$*, when they are prosecuted. Thereafter, there is competition.*

Proof. We proceed in three steps. First, we compute the expected discounted profits under each type of collusion. Second, we deduce the conditions under which each type of collusion is sustainable. Finally, among the sustainable types of collusion, we deduce the type that maximizes the expected discounted profits of the firms.

Profits under each type of collusion: Suppose that $CS^{com,A} - CS^{c,A} - 2\pi^{c,A} \geq \frac{\left[1-\left(1-\alpha^A\right)\delta\right]2\left(1-\alpha^B\right)\left[\delta\pi^{c,B}+(1-\delta)f^B\right]}{\left[1-(1-\alpha^B)(1-\alpha^A)\delta\right]\delta}$ or, which is equivalent, $\alpha^B > \alpha^* = \frac{2\left[\delta\pi^{c,B}+(1-\delta)f^B\right]-\delta\left(CS^{com,A}-CS^{c,A}-2\pi^{c,A}\right)}{2\left[\delta\pi^{c,B}+(1-\delta)f^B\right]+\frac{\delta(1-\alpha^A)}{1-(1-\alpha^A)\delta}\delta\left(CS^{com,A}-CS^{c,A}-2\pi^{c,A}\right)}$. Then, from Lemmas 1 and 3, if firms collude in both countries or they only collude in country B, they will be prosecuted in country B the first time that $s_t^B=1$, while if they only collude in country A, they will be prosecuted the first time $s_t^A=1$. Therefore, the expected profits of a firm under collusion in country A, collusion in country B and collusion in both countries are given by:

$$\Pi^{c,A}=\frac{\pi^{c,A}-\alpha^{A}f^{A}}{1-\left(1-\alpha^{A}\right)\delta},\ \Pi^{c,B}=\frac{\pi^{c,B}-\alpha^{B}f^{B}}{1-\left(1-\alpha^{B}\right)\delta},\ \Pi^{c,AB}=\frac{\pi^{c,A}+\pi^{c,B}-\alpha^{B}f^{B}+\frac{\delta\alpha^{B}\left(\pi^{c,A}-\alpha^{A}f^{A}\right)}{1-(1-\alpha^{A})\delta}}{1-\left(1-\alpha^{B}\right)\delta},$$

respectively. In order to deduce $\Pi^{c,AB}$ note that $\Pi^{c,AB}=\pi^{c,A}+\pi^{c,B}+\alpha^{B}\left(-f^{B}+\delta\Pi^{c,A}\right)+\left(1-\alpha^{B}\right)\left(\pi^{c,A}+\pi^{c,B}+\delta\Pi^{c,AB}\right)$.

Sustainability of each type of collusion: We deduce conditions under which each type of collusion can be sustained as a subgame perfect Nash equilibrium.

- For collusion in country A to be an equilibrium it must be the case that $\Pi^{c,A}\geq\pi^{d,A}$, which holds if and only if $\alpha^{A}\leq\bar{\alpha}^{A}=\frac{\pi^{c,A}-(1-\delta)\pi^{d,A}}{\delta\pi^{d,A}+f^{A}}$.
- For collusion in country B to be an equilibrium it must be the case that $\Pi^{c,B}\geq\pi^{d,B}$, which holds if and only if $\alpha^{B}\leq\bar{\alpha}^{B}=\frac{\pi^{c,B}-(1-\delta)\pi^{d,B}}{\left(\delta\pi^{d,B}+f^{B}\right)}$.
- For collusion in both countries to be an equilibrium it must be the case that $\Pi^{c,AB}\geq\pi^{d,A}+\pi^{d,B}$, which holds if and only if $\alpha^{B}\leq\bar{\alpha}^{AB}=\frac{\left(\pi^{c,A}+\pi^{c,B}\right)-(1-\delta)\left(\pi^{d,A}+\pi^{d,B}\right)}{\delta\left(\pi^{d,A}+\pi^{d,B}\right)-\frac{\delta\left(\pi^{c,A}-\alpha^{A}f^{A}\right)}{1-(1-\alpha^{A})\delta}+f^{B}}$.

Selection: Note that if $\Pi^{c,A}\geq\pi^{d,A}$, then $\Pi^{c,AB}>\Pi^{c,B}$. Hence, when collusion in country A and collusion in both countries can be sustained as an equilibrium, firms always prefer to collude in both countries rather than only in country B. Finally, $\Pi^{c,AB}>\Pi^{c,A}$ if and only if $\alpha^{B}<\hat{\alpha}^{A,AB}=\frac{(1-\delta)\pi^{c,B}+\alpha^{A}\left[\delta\left(\pi^{c,A}+\pi^{c,B}\right)+(1-\delta)f^{A}\right]}{[1-(1-\alpha^{A})\delta]f^{B}}$.

Therefore, if $\alpha^{B}>\alpha^{*}$, $\alpha^{A}\leq\bar{\alpha}^{A}$, $\alpha^{B}\leq\bar{\alpha}^{AB}$, and $\alpha^{B}<\hat{\alpha}^{A,AB}$, firms collude in both countries, while if $\alpha^{B}>\alpha^{*}$, $\alpha^{A}\leq\bar{\alpha}^{A}$, $\alpha^{B}\leq\bar{\alpha}^{AB}$, and $\alpha^{B}>\hat{\alpha}^{A,AB}$ firms collude in country A. ■

B.3 Proof of Corollary 1

Corollary 1 *Suppose that Assumption 2 holds, firms punish deviations from collusion stopping collusion in both countries, they coordinate in their best equilibrium, and* $\pi^{c,A}>(1-\delta)\,\pi^{d,A}$ *and* $\pi^{c,B}>(1-\delta)\,\pi^{d,B}$.

1. *Suppose that competition authority* $j\in\{A,B\}$ *prosecutes collusion the first time that* $s_t^j=1$ *and, under such anti-trust decisions, firms are still willing to collude in both countries. Then, the expected durations of collusion in countries A and B are given by* $\bar{d}^{A}=\frac{1-\alpha^{A}}{\alpha^{A}}$ *and* $\bar{d}^{B}=\frac{\left(1-\alpha^{A}\right)\left(1-\alpha^{B}\right)}{1-(1-\alpha^{A})(1-\alpha^{B})}$, *respectively.*
2. *Under the assumptions in Proposition 2.1. The expected durations of collusion in countries A and B are given by* $d^{A}=\frac{\alpha^{A}+\alpha^{B}-\alpha^{A}\alpha^{B}}{\alpha^{B}\alpha^{A}}$ *and* $d^{B}=\frac{1-\alpha^{B}}{\alpha^{B}}$, *respectively.*
3. *Equilibrium prosecution delays in countries A and B are given by:*

$$d^{A}-\bar{d}^{A}=\frac{1}{\alpha^{B}}\text{ and }d^{B}-\bar{d}^{B}=\frac{\alpha^{A}\left(1-\alpha^{B}\right)}{\alpha^{B}\left[1-\left(1-\alpha^{A}\right)\left(1-\alpha^{B}\right)\right]},$$

respectively.

Proof. We compute the expected duration of collusion in each country under two different situations: First, when firms are prosecuted the first time that $s_t^j = 1$. Second, under the equilibrium prosecution strategies in Proposition 2.1.

1. Suppose that both competition authorities always prosecute collusion as soon as they detect it and firms are still willing to collude in both countries. In such environment, the expected duration of collusion will be given by:

$$\bar{d}^A = \sum_{t=0}^{\infty} t \left(1-\alpha^A\right)^t \alpha^A = \frac{1-\alpha^A}{\alpha^A},$$

for country A and by:

$$\bar{d}^B = \sum_{k=0}^{\infty} k \left[\left(1-\alpha^A\right)\left(1-\alpha^B\right)\right]^k \left[1-\left(1-\alpha^A\right)\left(1-\alpha^B\right)\right] = \frac{\left(1-\alpha^A\right)\left(1-\alpha^B\right)}{1-\left(1-\alpha^A\right)\left(1-\alpha^B\right)}$$

for country B. The logic behind these formulas is straightforward. When competition authority A prosecutes the firms as soon as collusion in country A is detected, the probability that the firms are detected in period k is given by $\left(1-\alpha^A\right)^k \alpha^A$, i.e., the probability that firms are not detected from $t=0$ to $t=k-1$ times the probability that firms are detected in period $t=k$. Analogously, the probability that collusion in country B is detected in period k is given by $\left[\left(1-\alpha^A\right)\left(1-\alpha^B\right)\right]^k \left[1-\left(1-\alpha^A\right)\left(1-\alpha^B\right)\right]$, i.e., the probability that collusion is not detected neither in country A nor in country B from $t=0$ to $t=k-1$ times the probability that collusion is detected either in country A or in country B in period $t=k$. Finally, using the properties of the geometric distribution, it is easy to compute $\bar{d}^A$ and $\bar{d}^B$.

2. The expected duration of collusion associated with the equilibrium in Proposition 2.1 is given by:

$$d^A = \sum_{k=0}^{\infty} k \left[\sum_{j=0}^{k-1} \left(1-\alpha^B\right)^j \alpha^B \left(1-\alpha^A\right)^{k-1-j} \alpha^A\right] = \frac{\alpha^A+\alpha^B-\alpha^A\alpha^B}{\alpha^B\alpha^A},$$

for country A and by:

$$d^B = \sum_{k=0}^{\infty} k \left(1-\alpha^B\right)^k \alpha^B = \frac{1-\alpha^B}{\alpha^B},$$

for country B. The probability that competition authority A prosecutes collusion in period k is given by $\sum_{j=0}^{k-1} \left(1-\alpha^B\right)^j \alpha^B \left(1-\alpha^A\right)^{k-1-j} \alpha^A$, where $\left(1-\alpha^B\right)^j \alpha^B$ is the probability that competition authority B detects collusion in period $j \leq k-1$ and $\left(1-\alpha^A\right)^{k-1-j} \alpha^A$ is the probability that competition authority A detects collusion $k-j$ periods after collusion was detected and prosecuted in country B. Thus, $\left(1-\alpha^B\right)^j \alpha^B \left(1-\alpha^A\right)^{k-1-j} \alpha^A$ is the probability that B prosecutes in period j and A prosecutes $k-j$ periods after. (Recall that in the equilibrium described in Proposition 2.1, firms start colluding in both countries, but competition authority A only prosecutes the firms after competition authority B detects and prosecutes collusion in country B). Regarding country B, in equilibrium, the probability that competition authority B detects collusion in period k is $\left(1-\alpha^B\right)^k \alpha^B$. Again, using the properties of the geometric distribution, it is simple to compute d^A and d^B.

3. Employing $\bar{d}^A$, $\bar{d}^B$, d^A, and d^B, we obtain $d^A - \bar{d}^A = \frac{1}{\alpha^B}$ and $d^B - \bar{d}^B = \frac{\alpha^A\left(1-\alpha^B\right)}{\alpha^B\left[1-\left(1-\alpha^A\right)\left(1-\alpha^B\right)\right]}$, the prosecution delays in countries A and B, respectively. ■

C Antitrust Policy, Integration and International Agreements

Appendix C develops the details and presents the proofs of all the lemmas and propositions in Section 6.

C.1 Assumption 3

The following assumption is a generalization of Assumption 2 for an environment with two industries.
Assumption 3 *The signals received by the competition authorities are:*

1. *Industry x:*

$$s_t^{x,A} = \begin{cases} 1 \textit{ with probability } \alpha^{x,A} & \textit{if } a_t^{1,x,A} = a_t^{2,x,A} = 1, \\ 0 \textit{ with probability } \left(1 - \alpha^{x,A}\right) & \textit{if } a_t^{1,x,A} = a_t^{2,x,A} = 1, \\ 0 & \textit{otherwise.} \end{cases}$$

$$s_t^{x,B} = \begin{cases} 1 \textit{ with probability } \alpha^{x,B} \textit{ if } a_t^{1,x,B} = a_t^{2,x,B} = 1 \textit{ and } p_\tau^{x,A} = 0 \textit{ for all } \tau \leq t, \\ 1 \textit{ if } a_t^{1,x,B} = a_t^{2,x,B} = 1 \textit{ and } p_\tau^{x,A} = 1 \textit{ for some } \tau \leq t, \\ 0 \textit{ with probability } \left(1 - \alpha^{x,B}\right) \textit{ if } a_t^{1,x,B} = a_t^{2,x,B} = 1 \textit{ and } p_\tau^{x,A} = 0 \textit{ for all } \tau \leq t, \\ 0 \textit{ otherwise.} \end{cases}$$

2. *Industry y:*

$$s_t^{y,B} = \begin{cases} 1 \textit{ with probability } \alpha^{y,B} & \textit{if } a_t^{1,y,B} = a_t^{2,y,B} = 1, \\ 0 \textit{ with probability } \left(1 - \alpha^{y,B}\right) & \textit{if } a_t^{1,y,B} = a_t^{2,y,B} = 1, \\ 0 & \textit{otherwise.} \end{cases}$$

$$s_t^{y,A} = \begin{cases} 1 \textit{ with probability } \alpha^{y,A} \textit{ if } a_t^{1,y,A} = a_t^{2,y,A} = 1 \textit{ and } p_\tau^{y,B} = 0 \textit{ for all } \tau \leq t \\ 1 \textit{ if } a_t^{1,y,A} = a_t^{2,y,A} = 1 \textit{ and } p_\tau^{y,B} = 1 \textit{ for some } \tau \leq t, \\ 0 \textit{ with probability } \left(1 - \alpha^{y,A}\right) \textit{ if } a_t^{1,y,A} = a_t^{2,y,A} = 1 \textit{ and } p_\tau^{y,B} = 0 \textit{ for all } \tau \leq t \\ 0 \textit{ otherwise.} \end{cases}$$

C.2 Independent Competition Authorities

Proposition 3 (Equilibrium with two-industries and independent competition authorities). *Suppose that Assumption 3 holds, firms punish deviations from collusion stopping collusion in both countries, they coordinate in their best equilibrium, and $\pi^{c,z,j} > (1-\delta)\,\pi^{d,z,j}$ for $z \in \{x, y\}$ and $j \in \{A, B\}$.*

1. *Industry x. Suppose that $\left(\alpha^{x,A}, \alpha^{x,B}\right) \in R^{x,A}$. Then, in industry x firms collude in both countries until the first time $s_t^{x,B} = 1$, when they are prosecuted in country B. Thereafter, they collude in country A until the first time $s_{t+\tau}^{x,A} = 1$ with $\tau \geq 1$, when they are prosecuted in country A. Thereafter, there is competition in both countries.*

2. *Industry y. Suppose that $\left(\alpha^{y,B}, \alpha^{y,A}\right) \in R^{y,B}$. Then, in industry y firms collude in both countries until the first time $s_t^{y,A} = 1$, when they are prosecuted in country A. Thereafter, they collude in country B until the first time $s_{t+\tau}^{y,B} = 1$ with $\tau \geq 1$, when they are prosecuted in country B. Thereafter, there is competition in both countries.*

Proof. The proof is identical to the proof of Proposition 2.1. The only required change in notation is to add the industry superindex $z \in \{x, y\}$ and define $R^{z,j} = \left\{ \left(\alpha^{z,j}, \alpha^{z,-j} \right) \in [0,1] \times [0,1] : \textit{(9)-(12) hold} \right\}$, where $z \in \{x, y\}$, $j \in \{A, B\}$ and

$$\alpha^{z,-j} > \frac{2\left[\delta \pi^{c,z,-j} + (1-\delta) f^{z,-j}\right] - \delta \left(\Delta S^{z,j} - 2\pi^{c,z,j}\right)}{2\left[\delta \pi^{c,z,-j} + (1-\delta) f^{z,-j}\right] + \frac{\delta(1-\alpha^{z,j})}{1-(1-\alpha^{z,j})\delta} \delta \left(\Delta S^{z,j} - 2\pi^{c,z,j}\right)}, \tag{9}$$

$$\alpha^{z,j} \leq \frac{\pi^{c,z,j} - (1-\delta) \pi^{d,z,j}}{\delta \pi^{d,z,j} + f^{z,j}}, \tag{10}$$

$$\alpha^{z,-j} \leq \frac{\left(\pi^{c,z,A} + \pi^{c,z,B}\right) - (1-\delta) \left(\pi^{d,z,A} + \pi^{d,z,B}\right)}{\delta \left(\pi^{d,z,A} + \pi^{d,z,B}\right) - \frac{\delta(\pi^{c,z,j} - \alpha^{z,j} f^{z,j})}{1-(1-\alpha^{z,j})\delta} + f^{z,-j}}, \tag{11}$$

$$\alpha^{z,-j} < \frac{(1-\delta) \pi^{c,z,-j} + \alpha^{z,j} \left[\delta \left(\pi^{c,z,j} + \pi^{c,z,-j}\right) + (1-\delta) f^{z,j}\right]}{\left[1 - (1-\alpha^{z,j}) \delta\right] f^{z,-j}}. \tag{12}$$

This completes the proof of Proposition 3 (Two-industries and Independent Competition Authorities). ■

Welfare under no integration

1. Consider industry x under no integration. Suppose that $\left(\alpha^{x,A}, \alpha^{x,B}\right) \in R^{x,A}$. Then, in equilibrium, firms collude in both countries until the first time $s_t^{x,B} = 1$, when they are prosecuted in country B. Thereafter, they collude in country A until the first time $s_{t+\tau}^{x,A} = 1$ with $\tau \geq 1$, when they are prosecuted in country A. Thereafter, there is competition in both countries. Then, the expected discounted welfare of country A in industry x is:

$$W_0^{x,A} = \left(CS^{c,x,A} + 2\pi^{c,x,A} + 2\pi^{c,x,B}\right) + \alpha^{x,B} \left(-2f^{x,B} + \delta \tilde{W}_0^{x,A}\right) + \left(1 - \alpha^{x,B}\right) \delta W_0^{x,A},$$

where

$$\tilde{W}_0^{x,A} = \frac{CS^{c,x,A} + 2\pi^{c,x,A} + \frac{\alpha^{x,A}\delta}{1-\delta} CS^{com,x,A}}{1 - (1-\alpha^{x,A}) \delta}.$$

Hence:

$$W_0^{x,A} = \frac{\left[\frac{1-\delta+\alpha^{x,A}\delta+\alpha^{x,B}\delta}{1-(1-\alpha^{x,A})\delta}\right] \left(CS^{c,x,A} + 2\pi^{c,x,A}\right) + 2\left(\pi^{c,x,B} - \alpha^{x,B} f^{x,B}\right) + \frac{\alpha^{x,A}\alpha^{x,B}\delta^2 CS^{com,x,A}}{[1-(1-\alpha^{x,A})\delta](1-\delta)}}{\left[1 - (1-\alpha^{x,B}) \delta\right]}$$

The expected discounted welfare of country B in industry x is

$$W_0^{x,B} = \frac{CS^{c,x,B} + 2\alpha^{x,B} f^{x,B} + \frac{\alpha^{x,B}\delta}{1-\delta} CS^{com,x,B}}{1 - (1-\alpha^{x,B}) \delta}.$$

2. Consider industry y under no integration. Suppose that $\left(\alpha^{y,B}, \alpha^{y,A}\right) \in R^{y,B}$. Then, in equilibrium, firms collude in both countries until the first time $s_t^{y,A} = 1$, when they are prosecuted in country A. Thereafter, they collude in country B until the first time $s_{t+\tau}^{y,B} = 1$ with $\tau \geq 1$, when they are prosecuted

in country B. Thereafter, there is competition in both countries. Then, the expected discounted welfare of country B in industry y is:

$$W_0^{y,B} = \frac{\left[\frac{1-\delta+\alpha^{y,B}\delta+\alpha^{y,A}\delta}{1-(1-\alpha^{y,B})\delta}\right]\left(CS^{c,y,B}+2\pi^{c,y,B}\right)+2\left(\pi^{c,y,A}-\alpha^{y,A}f^{y,A}\right)+\frac{\alpha^{y,B}\alpha^{y,A}\delta^2 CS^{com,y,B}}{[1-(1-\alpha^{y,B})\delta](1-\delta)}}{\left[1-\left(1-\alpha^{y,A}\right)\delta\right]},$$

while the expected discounted welfare of country A in industry y is

$$W_0^{y,A} = \frac{CS^{c,y,A}+2\alpha^{y,A}f^{y,A}+\frac{\alpha^{y,A}\delta}{1-\delta}CS^{com,y,A}}{1-\left(1-\alpha^{y,A}\right)\delta}.$$

C.3 A Globally Integrated Competition Authority

Proposition 3 (Equilibrium with two-industries and a globally integrated competition authority). *Suppose that Assumption 3 holds, firms punish deviations from collusion stopping collusion in both countries, they coordinate in their best equilibrium and* $\pi^{c,z,j} > (1-\delta)\,\pi^{d,z,j}$ *for* $z \in \{x,y\}$ *and* $j \in \{A,B\}$.

1. *Industry x. Suppose that* $\left(\alpha^{x,A},\alpha^{x,B}\right) \in \bar{R}^{x,A}$. *Then, firms collude in both countries until the first time* $s_t^{x,A}=1$, *when they are prosecuted in both countries, or until the first time* $s_t^{x,A}=0$ *and* $s_t^{x,B}=1$, *when they are prosecuted in country B. In the later case, firms keep colluding in country A until the first time* $s_{t+\tau}^{x,A}=1$ *with* $\tau \geq 1$.

2. *Industry y. Suppose that* $\left(\alpha^{y,B},\alpha^{y,A}\right) \in \bar{R}^{y,B}$. *Then, firms collude in both countries until the first time* $s_t^{y,B}=1$, *when they are prosecuted in both countries, or until the first time* $s_t^{y,B}=0$ *and* $s_t^{y,A}=1$, *when they are prosecuted in country A. In the later case, firms keep colluding in country B until the first time* $s_{t+\tau}^{y,B}=1$ *with* $\tau \geq 1$.

Proof. The world's aggregate welfare in period t is $w_t = (CS_t^x + PS_t^x) + (CS_t^y + PS_t^y)$, where CS_t^z and PS_t^z are the consumer and producer surpluses in period t in industry $z \in \{x,y\}$, respectively. The world's aggregate expected discounted welfare is $W_t^W = \mathbf{E}_t\left[\sum_{\tau=t}^{\infty}\delta^{\tau-t}w_\tau\right]$. Since $CS^{com,z,j} > CS^{c,z,j}+2\pi^{c,z,j}$, $CS_t^z + PS_t^z$ adopts a maximum when firms operating in industry z compete. Thus, in order to maximize W_t^W collusion must be prosecuted as soon as it is detected.

1. Consider industry x. We proceed in three steps. First, we compute the expected discounted profits under each type of collusion. Second, we deduce the conditions under which each type of collusion is sustainable. Finally, among the sustainable types of collusion, we deduce the type that maximizes the expected discounted profits of the firms.

Profits under each type of collusion: The expected discounted profits of a firm under collusion in country A, collusion in country B and collusion in both countries are given by:

$$\Pi^{c,x,A} = \frac{\pi^{c,x,A}-\alpha^{x,A}f^{x,A}}{1-\left(1-\alpha^{x,A}\right)\delta},\ \Pi^{c,x,B} = \frac{\pi^{c,x,B}-\alpha^{x,B}f^{x,B}}{1-\left(1-\alpha^{x,B}\right)\delta},$$
$$\Pi^{c,x,AB} = \Pi^{c,x,A} + \frac{\pi^{c,x,B}-\left[1-\left(1-\alpha^{x,A}\right)\left(1-\alpha^{x,B}\right)\right]f^{x,B}}{1-\left(1-\alpha^{x,A}\right)\left(1-\alpha^{x,B}\right)\delta}.$$

respectively.

Sustainability of each type of collusion: We deduce conditions under which each type of collusion can be sustained as a subgame perfect Nash equilibrium.

- For collusion in country A to be an equilibrium, it must be the case that $\Pi^{c,x,A} \geq \pi^{d,x,A}$, which holds if and only if $\alpha^{x,A} \leq \frac{\pi^{c,x,A}-(1-\delta)\pi^{d,x,A}}{f^{x,A}+\delta\pi^{d,x,A}}$.
- For collusion in country B to be an equilibrium it must be the case that $\Pi^{c,x,B} \geq \pi^{d,x,B}$, which holds if and only if $\alpha^{x,B} \leq \frac{\pi^{c,x,B}-(1-\delta)\pi^{d,x,B}}{f^{x,B}+\delta\pi^{d,x,B}}$.
- For collusion in both countries to be an equilibrium it must be the case that $\Pi^{c,x,AB} \geq \pi^{d,x,A}+\pi^{d,x,B}$, which holds if and only if

  $$\sum_{j=A,B} \pi^{c,x,j}-\alpha^{x,A}f^{x,j}-\left[1-\left(1-\alpha^{x,A}\right)\delta\right]\pi^{d,x,j} \geq \alpha^{x,B}\left(1-\alpha^{x,A}\right)\left[\delta\pi^{d,x,B}+f^{x,B}-\delta\left(\Pi^{c,x,A}-\pi^{d,x,A}\right)\right]$$

  The left hand side of this inequality is positive if and only if $\alpha^{x,A} < \alpha_H = \frac{\pi^{c,x,A}+\pi^{c,x,B}-(1-\delta)\left(\pi^{d,x,A}+\pi^{d,x,B}\right)}{f^{x,A}+f^{x,B}-\delta\left(\pi^{d,x,A}+\pi^{d,x,B}\right)}$, while the right hand side is positive if and only if $\alpha^{x,A} > \alpha_L = \frac{\pi^{c,x,A}-\left(\frac{1-\delta}{\delta}\right)f^{x,B}-(1-\delta)\left(\pi^{d,x,A}+\pi^{d,x,B}\right)}{f^{x,A}+f^{x,B}+\delta\left(\pi^{d,x,A}+\pi^{d,x,B}\right)}$. Moreover, note that $\alpha_L < \alpha_H$. Hence, the inequality holds when $\alpha^{x,A} \leq \alpha_L$ or $\alpha_L < \alpha^{x,A} < \alpha_H$ and $\alpha^{x,B} \leq \frac{\sum_{j=A,B}\pi^{c,x,j}-\alpha^{x,A}f^{x,j}-\left[1-\left(1-\alpha^{x,A}\right)\delta\right]\pi^{d,x,j}}{\left(1-\alpha^{x,A}\right)\left[\delta\pi^{d,x,B}+f^{x,B}-\delta\left(\Pi^{c,x,A}-\pi^{d,x,A}\right)\right]}$.

Selection: $\Pi^{c,x,AB} > \Pi^{c,x,A}$ if and only if $\alpha^{x,B} < \frac{\pi^{c,x,B}-\alpha^{x,A}f^{x,B}}{\left(1-\alpha^{x,A}\right)f^{x,B}}$, while $\Pi^{c,x,AB} > \Pi^{c,x,B}$ if and only if $\frac{\pi^{c,x,A}-\alpha^{x,A}f^{x,A}}{1-\left(1-\alpha^{x,A}\right)\delta} > \frac{\alpha^{x,A}\left(1-\alpha^{x,B}\right)\left[\delta\pi^{c,x,B}+(1-\delta)f^{x,B}\right]}{\left[1-\left(1-\alpha^{x,B}\right)\delta\right]\left[1-\left(1-\alpha^{x,A}\right)\left(1-\alpha^{x,B}\right)\delta\right]}$.

Summing up, suppose that $\left(\alpha^{x,A},\alpha^{x,B}\right) \in \bar{R}^{x,A}$, where $\bar{R}^{z,j} = \left\{\left(\alpha^{z,j},\alpha^{z,-j}\right) \in [0,1]\times[0,1] : \textit{(13)-(15) hold}\right\}$, where

$$\alpha^{z,j} \leq \frac{\pi^{c,z,j}-(1-\delta)\pi^{d,z,j}}{f^{z,j}+\delta\pi^{d,z,j}}, \tag{13}$$

$$\begin{gathered}\left[\alpha^{z,j} \leq \frac{\pi^{c,z,j}-\left(\frac{1-\delta}{\delta}\right)f^{z,j}-(1-\delta)\left(\pi^{d,z,A}+\pi^{d,z,B}\right)}{f^{z,A}+f^{z,B}+\delta\left(\pi^{d,z,A}+\pi^{d,z,B}\right)}\right] \;\; \textit{or} \\ \left[\frac{\pi^{c,z,j}-\left(\frac{1-\delta}{\delta}\right)f^{z,-j}-(1-\delta)\left(\pi^{d,z,A}+\pi^{d,z,B}\right)}{f^{z,A}+f^{z,B}+\delta\left(\pi^{d,z,A}+\pi^{d,z,B}\right)} < \alpha^{z,j} < \frac{\pi^{c,z,A}+\pi^{c,z,B}-(1-\delta)\left(\pi^{d,z,A}+\pi^{d,z,B}\right)}{f^{z,A}+f^{z,B}-\delta\left(\pi^{d,z,A}+\pi^{d,z,B}\right)} \;\; \textit{and} \right. \\ \left. \alpha^{z,-j} \leq \frac{\pi^{c,z,A}+\pi^{c,z,B}-\alpha^{z,j}\left(f^{z,A}+f^{z,B}\right)-\left[1-\left(1-\alpha^{z,j}\right)\delta\right]\left(\pi^{d,z,A}+\pi^{d,z,A}\right)}{\left(1-\alpha^{z,j}\right)\left[\delta\pi^{d,z,-j}+f^{z,-j}-\frac{\delta\left(\pi^{c,z,j}-\alpha^{z,j}f^{z,j}\right)}{1-\left(1-\alpha^{z,j}\right)\delta}+\delta\pi^{d,z,j}\right]}\right],\end{gathered} \tag{14}$$

$$\begin{gathered}\left[\frac{\pi^{c,z,-j}-(1-\delta)\pi^{d,z,-j}}{f^{z,-j}+\delta\pi^{d,z,-j}} < \alpha^{z,-j} < \frac{\pi^{c,z,-j}-\alpha^{z,j}f^{z,-j}}{\left(1-\alpha^{z,j}\right)f^{z,-j}}\right] \;\; \textit{or} \\ \left[\alpha^{z,-j} < \min\left\{\frac{\pi^{c,z,-j}-(1-\delta)\pi^{d,z,-j}}{f^{z,-j}+\delta\pi^{d,z,-j}}, \frac{\pi^{c,z,-j}-\alpha^{z,j}f^{z,-j}}{\left(1-\alpha^{z,j}\right)f^{z,-j}}\right\} \;\; \textit{and} \right. \\ \left. \frac{\pi^{c,z,j}-\alpha^{z,j}f^{z,j}}{1-\left(1-\alpha^{z,j}\right)\delta} > \frac{\alpha^{z,j}\left(1-\alpha^{z,-j}\right)\left[\delta\pi^{c,z,-j}+(1-\delta)f^{z,-j}\right]}{\left[1-\left(1-\alpha^{z,-j}\right)\delta\right]\left[1-\left(1-\alpha^{z,j}\right)\left(1-\alpha^{z,-j}\right)\delta\right]}\right]\end{gathered} \tag{15}$$

Then, when competition authorities are integrated, firms in industry x collude in both countries until they are detected and prosecuted.

2. Consider industry y. Following exactly the same steps, but reversing the roles of A and B and replacing x for y, we obtain that if $\left(\alpha^{y,B},\alpha^{y,A}\right)\in\bar{R}^{y,B}$, when competition authorities are integrated, firms in industry y collude in both countries until they are detected and prosecuted. ■

Welfare under integration

1. Consider industry x under integration. Suppose that $\left(\alpha^{x,A},\alpha^{x,B}\right)\in\bar{R}^{x,A}$. Then, firms collude in both countries until the first time $s_t^{x,A}=1$, when they are prosecuted in both countries, or until the first time $s_t^{x,A}=0$ and $s_t^{x,B}=1$, when they are prosecuted in country B. In the later case, firms keep colluding in country A until the first time $s_{t+\tau}^{x,A}=1$. Then, the expected discounted welfare of country A in industry x is:

$$\bar{W}_0^{x,A}=\left(CS^{c,x,A}+2\pi^{c,x,A}+2\pi^{c,x,B}\right)+\alpha^{x,A}\left(-2f^{x,B}+\delta\frac{CS^{com,x,A}}{1-\delta}\right)+$$
$$+\left(1-\alpha^{x,A}\right)\alpha^{x,B}\left(-2f^{x,B}+\delta\tilde{W}_0^{x,A}\right)+\left(1-\alpha^{x,A}\right)\left(1-\alpha^{x,B}\right)\delta\bar{W}_0^{x,A},$$

where

$$\tilde{W}_0^{x,A}=\left(CS^{c,x,A}+2\pi^{c,x,A}\right)+\alpha^{x,A}\delta\frac{CS^{com,x,A}}{1-\delta}+\left(1-\alpha^{x,A}\right)\delta\tilde{W}_0^{x,A}.$$

Hence,

$$\bar{W}_0^{x,A}=\frac{CS^{c,x,A}+2\pi^{c,x,A}}{1-\left(1-\alpha^{x,A}\right)\delta}+\frac{2\pi^{c,x,B}-\left[1-\left(1-\alpha^{x,A}\right)\left(1-\alpha^{x,B}\right)\right]2f^{x,B}}{1-\left(1-\alpha^{x,A}\right)\left(1-\alpha^{x,B}\right)\delta}+\frac{\alpha^{x,A}\delta CS^{com,x,A}}{\left[1-\left(1-\alpha^{x,A}\right)\delta\right]\left(1-\delta\right)}.$$

The expected discounted welfare of country B in industry x is:

$$\bar{W}_0^{x,B}=\frac{CS^{c,x,B}+\left[1-\left(1-\alpha^{x,A}\right)\left(1-\alpha^{x,B}\right)\right]\left(2f^{x,B}+\frac{\delta CS^{com,x,B}}{1-\delta}\right)}{1-\left(1-\alpha^{x,A}\right)\left(1-\alpha^{x,B}\right)\delta}.$$

2. Consider industry y under integration. $\left(\alpha^{y,B},\alpha^{y,A}\right)\in\bar{R}^{y,B}$. Then, firms collude in both countries until the first time $s_t^{y,B}=1$, when they are prosecuted in both countries, or until the first time $s_t^{y,B}=0$ and $s_t^{y,A}=1$, when they are prosecuted in country A. In the later case, firms keep colluding in country B until the first time $s_{t+\tau}^{y,B}=1$. Then, the expected discounted welfare of country B in industry y is:

$$\bar{W}_0^{y,B}=\frac{CS^{c,y,B}+2\pi^{c,y,B}}{1-\left(1-\alpha^{y,B}\right)\delta}+\frac{2\pi^{c,y,A}-\left[1-\left(1-\alpha^{y,A}\right)\left(1-\alpha^{y,B}\right)\right]2f^{y,A}}{1-\left(1-\alpha^{y,A}\right)\left(1-\alpha^{y,B}\right)\delta}+\frac{\alpha^{y,B}\delta CS^{com,y,B}}{\left[1-\left(1-\alpha^{y,B}\right)\delta\right]\left(1-\delta\right)},$$

while the expected discounted welfare of country A in industry y is:

$$\bar{W}_0^{y,A}=\frac{CS^{c,y,A}+\left[1-\left(1-\alpha^{y,A}\right)\left(1-\alpha^{y,B}\right)\right]\left(2f^{y,A}+\frac{\delta CS^{com,y,A}}{1-\delta}\right)}{1-\left(1-\alpha^{y,A}\right)\left(1-\alpha^{y,B}\right)\delta}.$$

C.4 Welfare Comparison (Proof of Proposition 3)

Proposition 3 (Welfare comparison). *Suppose that Assumption 3 holds, firms punish deviations from collusion stopping collusion in both countries, they coordinate in their best equilibrium and* $\pi^{c,z,j} > (1-\delta)\,\pi^{d,z,j}$ *for* $z \in \{x, y\}$ *and* $j \in \{A, B\}$. *Assume that* $\left(\alpha^{x,A}, \alpha^{x,B}\right) \in R^{x,A} \cap \bar{R}^{x,A}$ *and* $\left(\alpha^{y,B}, \alpha^{y,A}\right) \in R^{y,B} \cap \bar{R}^{y,B}$. *Then:*

1. *Country A benefits from integration if and only if* $\Delta W_0^{x,A} + \Delta W_0^{y,A} > 0$ *and (if* $j = B$*), where*

$$\Delta W_0^{x,A} = \frac{\alpha^{x,A}}{\left[1-\left(1-\alpha^{x,B}\right)\delta\right]}\left\{\frac{\delta\left(\Delta CS^{x,A}-2\pi^{c,x,A}\right)}{\left[1-\left(1-\alpha^{x,A}\right)\delta\right]}-\frac{2\left(1-\alpha^{x,B}\right)\left[\delta\pi^{c,x,B}+(1-\delta)\,f^{x,B}\right]}{\left[1-\left(1-\alpha^{x,A}\right)\left(1-\alpha^{x,B}\right)\delta\right]}\right\},$$

$$\Delta W_0^{y,A} = \frac{\alpha^{y,B}\left(1-\alpha^{y,A}\right)\left[\delta\Delta CS^{y,A}+(1-\delta)\,2f^{y,A}\right]}{\left[1-\left(1-\alpha^{y,A}\right)\left(1-\alpha^{y,B}\right)\delta\right]\left[1-\left(1-\alpha^{y,A}\right)\delta\right]},$$

2. *Country B benefits from integration if and only if* $\Delta W_0^{x,B} + \Delta W_0^{y,B} > 0$, *where*

$$\Delta W_0^{x,B} = \frac{\alpha^{x,A}\left(1-\alpha^{x,B}\right)\left[\delta\Delta CS^{x,B}+(1-\delta)\,2f^{x,B}\right]}{\left[1-\left(1-\alpha^{x,A}\right)\left(1-\alpha^{x,B}\right)\delta\right]\left[1-\left(1-\alpha^{x,B}\right)\delta\right]},$$

$$\Delta W_0^{y,B} = \frac{\alpha^{y,B}}{\left[1-\left(1-\alpha^{y,A}\right)\delta\right]}\left\{\frac{\delta\left(\Delta CS^{y,B}-2\pi^{c,y,B}\right)}{\left[1-\left(1-\alpha^{y,B}\right)\delta\right]}-\frac{2\left(1-\alpha^{y,A}\right)\left[\delta\pi^{c,y,A}+(1-\delta)\,f^{y,A}\right]}{\left[1-\left(1-\alpha^{y,A}\right)\left(1-\alpha^{y,B}\right)\delta\right]}\right\}.$$

3. *Moreover, if* $\pi^{c,z,j} = \pi^c$, $\pi^{d,z,j} = \pi^d$, $\Delta CS^{z,j} = \Delta CS$, $\alpha^{z,j} = \alpha$, *and* $f^{z,j} = f$ *for all* $z \in \{x, y\}$ *and* $j \in \{A, B\}$, *it is always the case that both countries are better off under integration.*

Proof. Suppose that $\left(\alpha^{x,A}, \alpha^{x,B}\right) \in R^{x,A} \cap \bar{R}^{x,A}$ and $\left(\alpha^{y,B}, \alpha^{y,A}\right) \in R^{y,B} \cap \bar{R}^{y,B}$. Then, the aggregate expected welfare of country $j \in \{A, B\}$ under no integration is $W_0^{x,j} + W_0^{y,j}$, while under integration it is $\bar{W}_0^{x,j} + \bar{W}_0^{y,j}$. Thus, country j is better off under integration than under no integration if and only if $\bar{W}_0^{x,j} - W_0^{x,j} + \bar{W}_0^{y,j} - W_0^{y,j} > 0$.

1. Define

$$\Delta W_0^{x,A} = \bar{W}_0^{x,A} - W_0^{x,A} = \frac{\alpha^{x,A}}{\left[1-\left(1-\alpha^{x,B}\right)\delta\right]}\left\{\begin{array}{c}\frac{\delta\left(\Delta CS^{x,A}-2\pi^{c,x,A}\right)}{\left[1-\left(1-\alpha^{x,A}\right)\delta\right]}\\ -\frac{2\left(1-\alpha^{x,B}\right)\left[\delta\pi^{c,x,B}+(1-\delta)f^{x,B}\right]}{\left[1-\left(1-\alpha^{x,A}\right)\left(1-\alpha^{x,B}\right)\delta\right]}\end{array}\right\},$$

$$\Delta W_0^{y,A} = \bar{W}_0^{y,A} - W_0^{y,A} = \frac{\alpha^{y,B}\left(1-\alpha^{y,A}\right)\left[\delta\Delta CS^{y,A}+(1-\delta)\,2f^{y,A}\right]}{\left[1-\left(1-\alpha^{y,A}\right)\left(1-\alpha^{y,B}\right)\delta\right]\left[1-\left(1-\alpha^{y,A}\right)\delta\right]}.$$

Then, country A is better off under integration if and only if $\Delta W_0^{x,A} + \Delta W_0^{y,A} > 0$. Moreover, note that $\left(\alpha^{x,A}, \alpha^{x,B}\right) \in R^{x,A}$ implies $\Delta W_0^{x,A} < 0$, while it is clear that $\Delta W_0^{y,A} > 0$.

2. Define

$$\Delta W_0^{x,B} = \frac{\alpha^{x,A}\left(1-\alpha^{x,B}\right)\left[\delta\Delta CS^{x,B}+(1-\delta)\,2f^{x,B}\right]}{\left[1-\left(1-\alpha^{x,A}\right)\left(1-\alpha^{x,B}\right)\delta\right]\left[1-\left(1-\alpha^{x,B}\right)\delta\right]},$$

$$\Delta W_0^{y,B} = \frac{\alpha^{y,B}}{\left[1-\left(1-\alpha^{y,A}\right)\delta\right]}\left\{\begin{array}{c}\frac{\delta\left(\Delta CS^{y,B}-2\pi^{c,y,B}\right)}{\left[1-\left(1-\alpha^{y,B}\right)\delta\right]}-\\ \frac{2\left(1-\alpha^{y,A}\right)\left[\delta\pi^{c,y,A}+(1-\delta)f^{y,A}\right]}{\left[1-\left(1-\alpha^{y,A}\right)\left(1-\alpha^{y,B}\right)\delta\right]}\end{array}\right\}.$$

Then, country B is better off under integration if and only if $\Delta W_0^{x,B} + \Delta W_0^{y,B} > 0$. Moreover, note that $\Delta W_0^{x,B} > 0$, while $\left(\alpha^{y,B}, \alpha^{y,A}\right) \in R^{y,B}$ implies $\Delta W_0^{y,B} < 0$.

3. Finally, assume that $\pi^{c,z,j} = \pi^c$, $\pi^{d,z,j} = \pi^d$, $\Delta CS^{z,j} = \Delta CS$, $\alpha^{z,j} = \alpha$, and $f^{z,j} = f$ for all $z \in \{x, y\}$ and $j \in \{A, B\}$. Then

$$\Delta W_0^{x,A} = \Delta W_0^{y,B} = \frac{\alpha}{\left[1 - (1-\alpha)\delta\right]} \left\{ \frac{\delta\left(\Delta CS - 2\pi^c\right)}{\left[1-(1-\alpha)\delta\right]} - \frac{2(1-\alpha)\left[\delta\pi^c + (1-\delta) f\right]}{\left[1 - (1-\alpha)^2\delta\right]} \right\},$$

$$\Delta W_0^{y,A} = \Delta W_0^{x,B} = \frac{\alpha(1-\alpha)\left[\delta\Delta CS + (1-\delta)2f\right]}{\left[1-(1-\alpha)^2\delta\right]\left[1-(1-\alpha)\delta\right]}.$$

Therefore:

$$\Delta W_0^{x,A} + \Delta W_0^{y,A} = \Delta W_0^{x,B} + \Delta W_0^{y,B} = \frac{\left[2 - \alpha - (1-\alpha)^2\delta\right]\alpha\delta\left(\Delta CS - 2\pi^c\right)}{\left[1-(1-\alpha)\delta\right]^2\left[1-(1-\alpha)^2\delta\right]} > 0.$$

Hence, it is always the case that both countries are better off under integration. ■

C.5 Example 2

Suppose that the demand function of good z in country j is given by $Q_d^{z,j} = k_c^j - P^{z,j}$ with $k_c^A = 1$ and $k_c^B = k_c$. The cost function of firm i in industry z in country j is $c^{i,z,j} = 0$. Assume that fines are given by $f^{z,j} = \gamma\pi^{c,z,j}$ with $\gamma > 1$. Then $\Delta CS^{x,A} = CS^{y,A} = 3/8$, $\Delta CS^{x,B} = \Delta CS^{y,B} = 3\left(k_c\right)^2/8$, $\pi^{c,x,A} = \pi^{c,y,A} = 1/8$, $\pi^{c,x,B} = \pi^{c,y,B} = \left(k_c\right)^2/8$. Moreover, there are two thresholds $\bar{k}_L > 0$ and $\bar{k}_H > \bar{k}_L$ such that: if $k_c > \bar{k}_H$, then country A does not support integration; if $k_c < \bar{k}_L$, then country B does not support integration; and if $\bar{k}_L \leq k_c \leq \bar{k}_H$, both countries support integration. To prove this, note that, from Proposition 3.1 we have:

$$\Delta W_0^{x,A} = \frac{\alpha^{x,A}}{\left[1-\left(1-\alpha^{x,B}\right)\delta\right]} \left\{ \frac{\delta\left(\Delta CS^{x,A} - 2\pi^{c,x,A}\right)}{\left[1-\left(1-\alpha^{x,A}\right)\delta\right]} - \frac{2\left(1-\alpha^{x,B}\right)\left[\delta\pi^{c,x,B} + (1-\delta) f^{x,B}\right]}{\left[1-\left(1-\alpha^{x,A}\right)\left(1-\alpha^{x,B}\right)\delta\right]} \right\},$$

$$\Delta W_0^{y,A} = \frac{\alpha^{y,B}\left(1-\alpha^{y,A}\right)\left[\delta\Delta CS^{y,A} + (1-\delta)2f^{y,A}\right]}{\left[1-\left(1-\alpha^{y,A}\right)\left(1-\alpha^{y,B}\right)\delta\right]\left[1-\left(1-\alpha^{y,A}\right)\delta\right]}.$$

Since $\Delta CS^{x,A} = \Delta CS^{y,A} = \Delta CS^A = 3/8$, $\Delta CS^{x,B} = \Delta CS^{y,B} = \left(k_c\right)^2 \Delta CS^A = 3\left(k_c\right)^2/8$, $\pi^{c,x,A} = \pi^{c,y,A} = \pi^{c,A} = 1/8$, $\pi^{c,x,B} = \pi^{c,y,B} = \left(k_c\right)^2 \pi^{c,A} = \left(k_c\right)^2/8$, and $f^{z,j} = \gamma\pi^{c,z,j}$ with $\gamma > 1$, these expressions become:

$$\Delta W_0^{x,A} = \frac{\alpha^{x,A}}{8\left[1-\left(1-\alpha^{x,B}\right)\delta\right]} \left\{ \frac{\delta}{\left[1-\left(1-\alpha^{x,A}\right)\delta\right]} - \frac{2\left(1-\alpha^{x,B}\right)\left(k_c\right)^2\left[\delta + (1-\delta)\gamma\right]}{\left[1-\left(1-\alpha^{x,A}\right)\left(1-\alpha^{x,B}\right)\delta\right]} \right\},$$

$$\Delta W_0^{y,A} = \frac{\alpha^{y,B}\left(1-\alpha^{y,A}\right)\left[3\delta + 2(1-\delta)\gamma\right]}{8\left[1-\left(1-\alpha^{y,A}\right)\left(1-\alpha^{y,B}\right)\delta\right]\left[1-\left(1-\alpha^{y,A}\right)\delta\right]}.$$

Therefore, $\Delta W_0^{x,A} + \Delta W_0^{y,A} > 0$ if and only if

$$k_c < \bar{k}_H = \sqrt{\frac{\frac{\alpha^{x,A}\delta}{[1-(1-\alpha^{x,A})\delta][1-(1-\alpha^{x,B})\delta]} + \frac{\alpha^{y,B}(1-\alpha^{y,A})[3\delta+2(1-\delta)\gamma]}{[1-(1-\alpha^{y,A})(1-\alpha^{y,B})\delta][1-(1-\alpha^{y,A})\delta]}}{\frac{2\alpha^{x,A}(1-\alpha^{x,B})[\delta+(1-\delta)\gamma]}{[1-(1-\alpha^{x,B})\delta][1-(1-\alpha^{x,A})(1-\alpha^{x,B})\delta]}}}.$$

From Proposition 3.2 we have:

$$\Delta W_0^{x,B} = \frac{\alpha^{x,A}\left(1-\alpha^{x,B}\right)\left[\delta \Delta CS^{x,B} + (1-\delta)\, 2 f^{x,B}\right]}{\left[1-\left(1-\alpha^{x,A}\right)\left(1-\alpha^{x,B}\right)\delta\right]\left[1-\left(1-\alpha^{x,B}\right)\delta\right]},$$

$$\Delta W_0^{y,B} = \frac{\alpha^{y,B}}{\left[1-\left(1-\alpha^{y,A}\right)\delta\right]}\left\{\frac{\delta\left(\Delta CS^{y,B} - 2\pi^{c,y,B}\right)}{\left[1-\left(1-\alpha^{y,B}\right)\delta\right]} - \frac{2\left(1-\alpha^{y,A}\right)\left[\delta\pi^{c,y,A} + (1-\delta)\, f^{y,A}\right]}{\left[1-\left(1-\alpha^{y,A}\right)\left(1-\alpha^{y,B}\right)\delta\right]}\right\}.$$

Since $\Delta CS^{x,A} = \Delta CS^{y,A} = 3/8$, $\Delta CS^{x,B} = \Delta CS^{y,B} = 3\,(k_c)^2/8$, $\pi^{c,x,A} = \pi^{c,y,A} = 1/8$, $\pi^{c,x,B} = \pi^{c,y,B} = (k_c)^2/8$, and $f^{z,j} = \gamma\pi^{c,z,j}$ with $\gamma > 1$, these expressions become:

$$\Delta W_0^{x,B} = \frac{\alpha^{x,A}\left(1-\alpha^{x,B}\right)(k_c)^2\left[3\delta + (1-\delta)\, 2\gamma\right]}{8\left[1-\left(1-\alpha^{y,A}\right)\left(1-\alpha^{y,B}\right)\delta\right]\left[1-\left(1-\alpha^{x,B}\right)\delta\right]},$$

$$\Delta W_0^{y,B} = \frac{\alpha^{y,B}}{8\left[1-\left(1-\alpha^{y,A}\right)\delta\right]}\left\{\frac{\delta\,(k_c)^2}{\left[1-\left(1-\alpha^{y,B}\right)\delta\right]} - \frac{2\left(1-\alpha^{y,A}\right)\left[\delta + (1-\delta)\,\gamma\right]}{\left[1-\left(1-\alpha^{y,A}\right)\left(1-\alpha^{y,B}\right)\delta\right]}\right\}.$$

Therefore, $\Delta W_0^{x,B} + \Delta W_0^{y,B} > 0$ if and only if

$$k_c > \bar{k}_L = \sqrt{\frac{\frac{2\alpha^{y,B}(1-\alpha^{y,A})[\delta+(1-\delta)\gamma]}{[1-(1-\alpha^{y,A})\delta][1-(1-\alpha^{y,A})(1-\alpha^{y,B})\delta]}}{\frac{\alpha^{y,B}\delta}{[1-(1-\alpha^{y,A})\delta][1-(1-\alpha^{y,B})\delta]} + \frac{\alpha^{x,A}(1-\alpha^{x,B})[3\delta+(1-\delta)2\gamma]}{[1-(1-\alpha^{y,A})(1-\alpha^{y,B})\delta][1-(1-\alpha^{x,B})\delta]}}}.$$

Suppose that $\alpha^{x,A} = \alpha^{y,B} = \alpha^D$ and $\alpha^{y,A} = \alpha^{x,B} = \alpha^F$. Then, $\bar{k}_L$ and $\bar{k}_H$ and are given by

$$\bar{k}_L = \sqrt{\frac{\frac{2(1-\alpha^F)[\delta+(1-\delta)\gamma]}{[1-(1-\alpha^D)(1-\alpha^F)\delta]}}{\frac{\delta}{[1-(1-\alpha^D)\delta]} + \frac{(1-\alpha^F)[3\delta+2(1-\delta)\gamma]}{[1-(1-\alpha^D)(1-\alpha^F)\delta]}}},\ \bar{k}_H = \sqrt{\frac{\frac{\delta}{[1-(1-\alpha^D)\delta]} + \frac{(1-\alpha^F)[3\delta+2(1-\delta)\gamma]}{[1-(1-\alpha^D)(1-\alpha^F)\delta]}}{\frac{2(1-\alpha^F)[\delta+(1-\delta)\gamma]}{[1-(1-\alpha^D)(1-\alpha^F)\delta]}}}.$$

Since $\frac{\delta}{[1-(1-\alpha^D)\delta]} > \frac{2(1-\alpha^F)(\delta+(1-\delta)\gamma)}{1-(1-\alpha^D)(1-\alpha^F)\delta} - \frac{\left(1-\alpha^F\right)[3\delta+2(1-\delta)\gamma]}{[1-(1-\alpha^D)(1-\alpha^F)\delta]}$, it is easy to verify that $\bar{k}_L < \bar{k}_H$. Hence, if $k_c > \bar{k}_H$, then country A does not support integration; if $k_c < \bar{k}_L$, then country B does not support integration; and if $\bar{k}_L \leq k_c \leq \bar{k}_H$, both countries support integration.

Finally, note that $(\alpha^{x,A}, \alpha^{x,B}) \in R^{x,A} \cap \bar{R}^{x,A}$ and $(\alpha^{y,B}, \alpha^{y,A}) \in R^{y,B} \cap \bar{R}^{y,A}$ can be rewritten as $k_c \in [\hat{k}_L, \hat{k}_H]$. Therefore, for a given (α^D, α^F), we need $[\hat{k}_L, \hat{k}_H] \cap [\bar{k}_L, \bar{k}_H] \neq \emptyset$, so there is an interval of values for which both countries support integration. Consider, the following numerical example. Let $\gamma = 1.2$, $\alpha^D = 0.0383$, $\alpha^F = 0.1169$. Then, $\bar{k}_L = 0.7474$, $\bar{k}_H = 1.3378$, $\hat{k}_L = 0.7922$ and $\hat{k}_H = 1.1887$. Thus, $\bar{k}_L < \hat{k}_L < \hat{k}_H < \bar{k}_H$ and, therefore, integration is always beneficial for both countries. ■