Kellogg School of Management
FINC 585-3: Asset Pricing — Prof: Torben G. Andersen & Zhengyang Jiang
Spring 2024 — Jose Antunes Neto

Assignment 5

Problem 1.

Connor and Korajczyk (1986)[1] develop a method to measure the performance of a mutual fund portfolio. Using a large number of funds, they show that the Treynor and Black (1973) Appraisal Ratio completely captures investors ranking over funds. A simplified version of the model is described as follows:

Let fund's j excess return at time t be denoted as $r_{j,t}$ for $j = 1, \ldots, N$ and the riskless asset return be $r_{0,t}$. These returns are generated by a finite set of latent factors $\mathbf{f_t} = \left(f_t^{(1)}, \ldots, f_t^{(K)}\right)$ where K is known. The return of fund j is then given by:

$$r_{j,t} = \alpha_j + \sum_{k=1}^{K} \beta_j^{(k)} \left(\gamma^{(k)} + f_t^{(k)}\right) + \varepsilon_{j,t} \tag{1}$$

with $\mathbb{E}(\mathbf{f_t}) = 0$, $\mathbb{E}(\mathbf{f_t}\mathbf{f_t}') = I_K$ and $\mathbb{E}(\varepsilon_{j,t}) = 0$, $\mathbb{V}(\varepsilon_{j,t}) = \sigma_j^2$. We also assume that $\varepsilon_{\cdot,t}$ is uncorrelated between funds and mean-independent on the set of factors. The Appraisal Ratio is defined as $t_j = \alpha_j/\sigma_j$, and according to Theorem 4, this ratio is sufficient to rank funds[2].

Let $\mathbf{R}$ be the $N \times T$ matrix of fund returns. The authors show that equation (1) can be estimated by the following algorithm.

1) Compute a $k \times T$ principal component matrix of $\frac{1}{n}\mathbf{R}'\mathbf{R}$, denote it as $\widehat{\mathbf{f}}$;

2) Run a time series regression

$$r_j = \widehat{\alpha}_j + \sum_{k=1}^{K} \widehat{\beta}_j^{(k)} \left(\widehat{\gamma}^{(k)} + \widehat{f}^{(k)}\right) + \widehat{\varepsilon}_j \tag{2}$$

3) Calculate $\widehat{t_j} = \widehat{\alpha}_j/\widehat{\sigma}_j$.

4) For a large N and T, this estimator is consistent and asymptotically normal:

$$\operatorname*{plim}_{N\to\infty} T^{1/2}\left(\widehat{t_j} - t_j\right) \xrightarrow{d} N\left(0, 1 + \sum_{k=1}^{K} \gamma_k^2\right) \qquad \text{as } T \to \infty \tag{3}$$

[1] There is a typographical mistake in the paper, later corrected by this errata
[2] i.e, an investor would strictly prefer to switch from investing in fund j to fund l if, and only if, $t_j > t_l$

In this exercise, we will apply this model to a dataset of mutual funds. The dataset `fund_returns.csv` available on CANVAS contains a time series of monthly log-returns for a large set of mutual funds from 2000 to 2023 obtained from CRSP.[3]. We will also use the data on the Fama-French factors and risk-free rate available on the `ffdaily.csv` file.

A) For k from 1 to 6, calculate the principal components of the funds' excess returns. Plot the explained variance ratio for each of these components. How do these PC compare with the Fama-French factors?

B) Connor and Korajczyk (1993) provide a methodology to choose the optimal number of factors K in an approximate factor model. Their test is based on the idea that additional (non-informative) factors will not increase the explained variance of the model. Their algorithm is described as the following iteration:

   1) For a given number of factors k, estimate the model with k and $k+1$ factors. Let the residuals be $\widehat{\varepsilon}_{j,t}$ and $\widehat{\varepsilon}^*_{j,t}$, respectively;

   2) Calculate adjusted squared residuals

$$\begin{aligned} \widehat{\sigma}_{j,t} &= \widehat{\varepsilon}_{j,t} / \left[1 - (k+1)/T - k/N\right] \\ \widehat{\sigma}^*_{j,t} &= \widehat{\varepsilon}^*_{j,t} / \left[1 - (k+2)/T - k/N\right] \end{aligned} \tag{4}$$

   3) Calculate $\widehat{\Delta}$ by subtracting the cross-sectional means of $\widehat{\sigma}_{j,t}$ in odd periods from the cross-sectional means of $\widehat{\sigma}^*_{j,t+1}$ in even periods:

$$\widehat{\Delta}_s = \mu_{2s-1} - \mu^*_{2s} \qquad s = 1, \ldots, \lfloor T/2 \rfloor \tag{5}$$

   where $\mu_t = N^{-1} \sum_{j=1}^{N} \widehat{\sigma}^2_{j,t}$ and $\mu^*_t = N^{-1} \sum_{j=1}^{N} \left(\widehat{\sigma}^*_{j,t}\right)^2$. Under the null, as $N \to \infty$

$$\sqrt{N}\widehat{\Delta} \xrightarrow{d} N\left(0, \Gamma\right) \tag{6}$$

   4) Using the time series of $\widehat{\Delta}$, calculate the mean and covariance matrix $\widehat{\Gamma}$ and perform a one-sided zero mean test:

$$\mathcal{H}_0 : \mathbb{E}\left[\widehat{\Delta}\right] \leq 0 \qquad \mathcal{H}_1 : \mathbb{E}\left[\widehat{\Delta}\right] > 0$$

$$\sqrt{\frac{T}{2} - 1}\left[\bar{\Delta}\widehat{\Gamma}^{-1/2}\right] \sim t(T/2 - 1) \tag{7}$$

   5) If the null is rejected, start over with $k+1$ factors. Otherwise, set $K = k$ as the optimal number of factors.

   Using their test, find the optimal K. Use this number for the rest of the exercise.

C) Estimate (2) for each fund and calculate their Appraisal Ratio. Plot the distribution of this measure. According to this, how many funds do really outperform the market? Use the distribution on (3) to test the significance of their performance.

[3] This data is available on WRDS under the `MONTHLY_RETURNS` dataset on the `CRSP` library.

D) Equation (1) assumes that the factor loadings $\beta_j^{(k)}$ are constant over time. Split the sample in half. Redo C) for each subsample and compare the results. How many funds do outperform in both periods? Interpret.

Solution.

A) After combining the industry portfolios dataset with the FF factors, the remaining dataset contains 288 monthly excess returns for 4674 mutual funds. Calculating the principal components for $k = 1, \ldots, 5$, Figure 1 plots the explained variance ratio for each of these portfolios. From this figure, we see that the amount of variance explained by the first factors is quite low compared to the previous exercise. The correlation

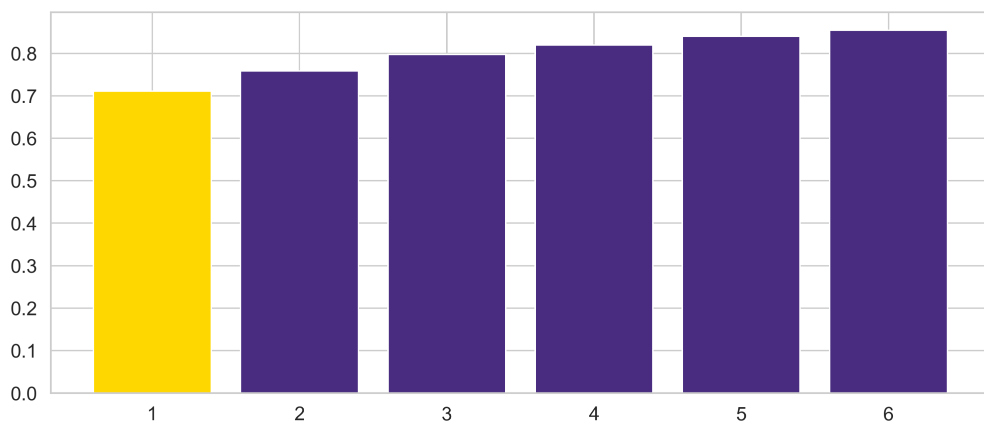

Figure 1: Explained Variance Ratio for the set of k components

between the 6 principal components and the FF factors is depicted in Figure 2. The first principal component still mostly captures the market portfolio. However, we see a greater influence of the momentum factors in each of the components. The second PC captures other important factors such as HML, RMW and CMA. Finally, SMB is mostly correlated with PC4, PC5 and PC6.

B) I use the Connor and Korajczyk (1993) to find the optimal number of factors. Using the entire sample, only 1 factor is required to explain a large portion of the variance of the 4674 mutual funds. Figure 1 shows the explained variance ratio for the first 6 components and the optimal number. We can see that there is not a significant increase in the explained variance ratio compared to the model with 6 components as the first factor already explains most of the variance.

C) Using the optimal number of factors, I estimate the Appraisal Ratio for each fund in the sample and calculate the critical values of (3). The distribution is shown in Figure 3. We see that there is a significant number of funds outperforming the market in the overall sample. A total of 1271 funds have a positive and significant appraisal ratio when using the asymptotic distribution.

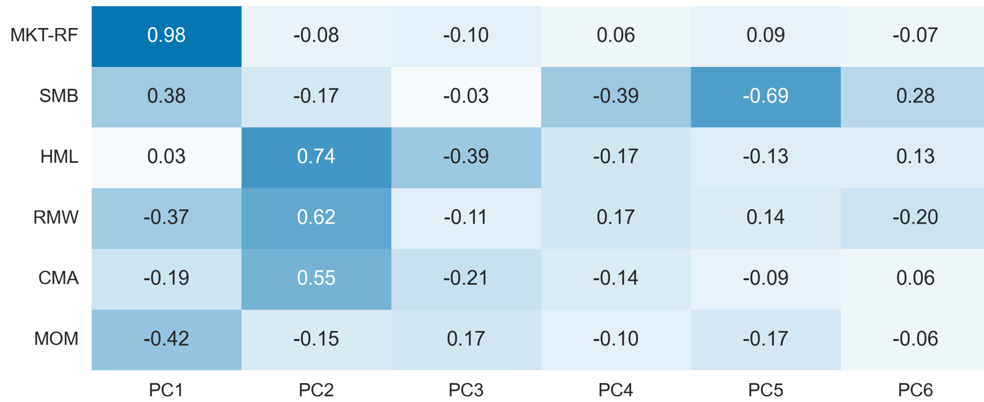

Figure 2: Correlation between Principal Components and FF Factors

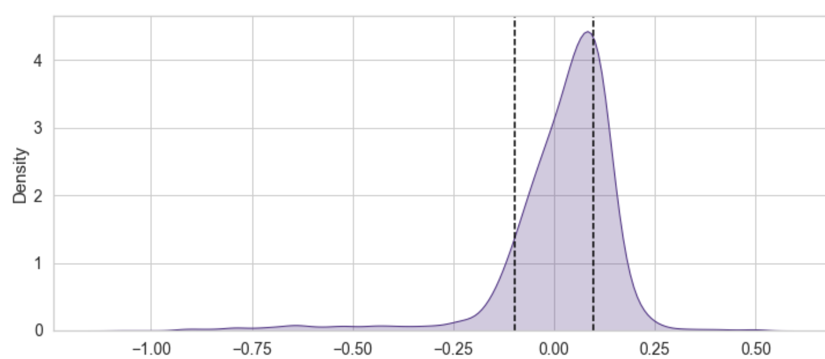

Figure 3: Distribution of Appraisal Ratio

D) Using data for over 20 years of returns, it is hard to expect that the sensitivity of funds to each factor would have stayed the same. Different macroeconomic conditions, for example, can affect the strategies and performances of each fund. To account for this, I estimate the same model using 2 halves of the sample. The first subsample contains data from 2000 to 2011, while the second spans from 2012 to 2023. I run the model for each subsample and calculate the optimal value of factors. Unlike in the overall sample, the subsamples indicated an optimal number of 1 and 6 factors respectively, smaller than the full sample. Figure 4 shows the distribution of the Appraisal Ratio for both subsamples. As we see, these ratios tend to be higher in the first subsample, compared to the second. A total of 1638 ($\approx 35\%$s) of funds outperformed in the first half and 980 ($\approx 21\%$) in the second half and only a 498 ($\approx 10\%$) of funds did well in both samples. This exhibits the difficulty in maintaining a good strategy throughout different market conditions.

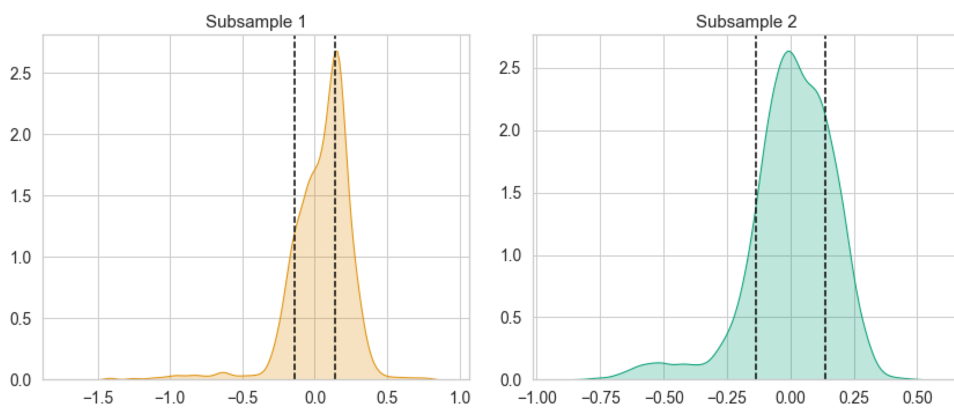

Figure 4: Distribution of Appraisal Ratio for Subsamples

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