Kellogg School of Management
FINC 585-3: Asset Pricing — Prof: Torben G. Andersen & Zhengyang Jiang
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Assignment 5

Problem 1.

Connor and Korajczyk (1986) develop a method to measure the performance of a mutual fund portfolio. Using a large number of funds, they show that the Treynor and Black (1973) Appraisal Ratio completely captures investors ranking over funds. A simplified version of the model is described as follows:

Let fund's j excess return at time t be denoted as $r_{j,t}$ for $j = 1, \dots, N$ and the riskless asset return be $r_{0,t}$. These returns are generated by a finite set of latent factors $\mathbf{f_t} = \left(f_t^{(1)}, \dots, f_t^{(K)}\right)$ where K is known. The return of fund j is then given by:

$$r_{j,t} = \alpha_j + \sum_{k=1}^{K} \beta_j^{(k)} \left(\gamma^{(k)} + f_t^{(k)}\right) + \varepsilon_{j,t} \tag{1}$$

with $\mathbb{E}(\mathbf{f_t}) = 0$, $\mathbb{E}(\mathbf{f_t}\mathbf{f_t}') = I_K$ and $\mathbb{E}(\varepsilon_{j,t}) = 0$, $\mathbb{V}(\varepsilon_{j,t}) = \sigma_j^2$. We also assume that $\varepsilon_{\cdot,t}$ is uncorrelated between funds and mean-independent on the set of factors. The Appraisal Ratio is defined as $t_j = \alpha_j/\sigma_j$, and according to Theorem 4, this ratio is sufficient to rank funds[1].

Let $\mathbf{R}$ be the $N \times T$ matrix of fund returns. The authors show that equation (1) can be estimated by the following algorithm.

1) Compute a $k \times T$ principal component matrix of $\frac{1}{n}\mathbf{R}'\mathbf{R}$, denote it as $\widehat{\mathbf{f}}$;

2) Run a time series regression

$$r_j = \widehat{\alpha}_j + \sum_{k=1}^{K} \widehat{\beta}_j^{(k)} \left(\widehat{\gamma}^{(k)} + \widehat{f}^{(k)}\right) + \widehat{\varepsilon}_j \tag{2}$$

3) Calculate $\widehat{t_j} = \widehat{\alpha}_j/\widehat{\sigma}_j$.

4) For a large N and T, this estimator is consistent and asymptotically normal:

$$\operatorname*{plim}_{N\to\infty} T^{1/2}\left(\widehat{t_j} - t_j\right) \xrightarrow{d} N\left(0, 1 + \sum_{k=1}^{K} \gamma_k^2\right) \qquad \text{as } T \to \infty \tag{3}$$

[1] i.e, an investor would strictly prefer to switch from investing in fund j to fund l if, and only if, $t_j > t_l$

In this exercise, we will apply this model to a dataset of mutual funds. The dataset `fund_returns.csv` available on CANVAS contains a time series of monthly log-returns for a large set of mutual funds from 2000 to 2023 obtained from CRSP.[2]. We will also use the data on the Fama-French factors and risk-free rate available on the `ffdaily.csv` file.

A) For k from 1 to 6, calculate the principal components of the funds' excess returns. Plot the explained variance ratio for each of these components. How do these PC compare with the Fama-French factors?

B) Connor and Korajczyk (1993) provide a methodology to choose the optimal number of factors K in an approximate factor model. Their test is based on the idea that additional (non-informative) factors will not increase the explained variance of the model. Their algorithm is described as the following iteration:

   1) For a given number of factors k, estimate the model with k and $k+1$ factors. Let the residuals be $\widehat{\varepsilon}_{j,t}$ and $\widehat{\varepsilon}^*_{j,t}$, respectively;

   2) Calculate adjusted squared residuals

$$\begin{aligned} \widehat{\sigma}_{j,t} &= \widehat{\varepsilon}_{j,t} / \left[1 - (k+1)/T - k/N\right] \\ \widehat{\sigma}^*_{j,t} &= \widehat{\varepsilon}^*_{j,t} / \left[1 - (k+2)/T - k/N\right] \end{aligned} \tag{4}$$

   3) Calculate $\widehat{\Delta}$ by subtracting the cross-sectional means of $\widehat{\sigma}_{j,t}$ in odd periods from the cross-sectional means of $\widehat{\sigma}^*_{j,t+1}$ in even periods:

$$\widehat{\Delta}_s = \mu_{2s-1} - \mu^*_{2s} \qquad s = 1, \ldots, \lfloor T/2 \rfloor \tag{5}$$

   where $\mu_t = N^{-1} \sum_{j=1}^{N} \widehat{\sigma}^2_{j,t}$ and $\mu^*_t = N^{-1} \sum_{j=1}^{N} \left(\widehat{\sigma}^*_{j,t}\right)^2$. Under the null, as $N \to \infty$

$$\sqrt{N}\widehat{\Delta} \xrightarrow{d} N\left(0, \Gamma\right) \tag{6}$$

   4) Using the time series of $\widehat{\Delta}$, calculate the mean and covariance matrix $\widehat{\Gamma}$ and perform a one-sided zero mean test:

$$\mathcal{H}_0 : \mathbb{E}\left[\widehat{\Delta}\right] \leq 0 \qquad \mathcal{H}_1 : \mathbb{E}\left[\widehat{\Delta}\right] > 0$$

$$\sqrt{\frac{T}{2} - 1}\left[\bar{\Delta}\widehat{\Gamma}^{-1/2}\right] \sim t(T/2 - 1) \tag{7}$$

   5) If the null is rejected, start over with $k+1$ factors. Otherwise, set $K = k$ as the optimal number of factors.

   Using their test, find the optimal K. Use this number for the rest of the exercise.

C) Estimate (2) for each fund and calculate their Appraisal Ratio. Plot the distribution of this measure. According to this, how many funds do really outperform the market? Use the distribution on (3) to test the significance of their performance.

[2] This data is available on WRDS under the `MONTHLY_RETURNS` dataset on the `CRSP` library.

D) Equation (1) assumes that the factor loadings $\beta_j^{(k)}$ are constant over time. Split the sample in half. Redo C) for each subsample and compare the results. How many funds do outperform in both periods? Interpret.

References

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