Assignment 4

This assignment studies conditions under which the Fama-MacBeth (FM) estimator is T-consistent (i.e., $\hat{\beta}_{FM} \to \beta$, as $T \to \infty$ for fixed N). We also look at cases, where the FM standard errors may be biased. You may find it useful to review the following papers:

- **Petersen, M.A. (2009), "Estimating Standard Errors in Finance Panel Data sets: Comparing Approaches," *Review of Financial Studies* 22(1):435-480.**
- **Skoulakis, G. (2008), "Panel Data Inference in Finance: Least-squares vs Fama-MacBeth, Working Paper, University of Maryland.**

Consider the following model:

$$y_{it} = \beta x_{it} + \varepsilon_{it} \qquad i = 1, ..., N, \quad t = 1, ..., T \tag{1}$$

where y_{it} is the dependent variable and x_{it} represents the $(K \times 1)$ vector of regressors. Suppose

$$\begin{aligned} x_{it} &= \mu_i + \eta_{it} \\ \varepsilon_{it} &= \gamma_i + \nu_{it} \end{aligned} \tag{2}$$

where μ_i and γ_i are the time-invariant random firm effects.

For a given sample size T, the Fama-Macbeth estimator is given by

$$\hat{\beta}_{FM}^{(T)} = \frac{1}{T} \sum_t \hat{\beta}_t \tag{3}$$

with

$$\hat{\beta}_t = \left(\sum_{i=1}^{N} x_{it} x_{it}' \right)^{-1} \left(\sum_{i=1}^{N} x_{it} y_{it} \right) \tag{4}$$

We want to check under what conditions the FM estimator is T-consistent (i.e. $\hat{\beta}_{FM} \to \beta$ as $T \to \infty$ for a fixed N).

We will start with some simulations: Set $K = 1$, $T = 5,000$ and $N = 20$. From a standard Normal distribution, independently generate:

1. $\mu_i, \quad i = 1, ..., 20$
2. $\gamma_i, \quad i = 1, ..., 20$
3. $\eta_{it}, \quad i = 1, ..., 20, \quad t = 1, ..., 5000$
4. $\nu_{it}, \quad i = 1, ..., 20, \quad t = 1, ..., 5000$

(a) Compute $\{\varepsilon_{it}, x_{it}\}$ using equation (2) and then generate the dependent variable using the equation

$$y_{it} = 2x_{it} + \varepsilon_{it} \text{ for } i = 1, ..., 20 \text{ and } t = 1, ..., 5000$$

Consider the following estimation methods:

1. **Traditional FM: For each period t, compute $\hat{\beta}^{(T)}_{FM}$ using equations (3) and (4). Include an intercept term even though the true intercept is zero. For a given T, call this estimator $\beta^{(T)}_{FM}$.**
2. **Demeaned FM: For each firm i, demean both the dependent variable and the regressor by subtracting the time-series averages to get $\tilde{y}_{it} = y_{it} - (1/T)\sum_t y_{it}$ and $\tilde{x}_{it} = x_{it} - (1/T)\sum_t x_{it}$. For a given T, call this estimator $\hat{\beta}^{(T)}_{DFM}$**

Compute $\hat{\beta}^{(T)}_{FM}$ and $\hat{\beta}^{(T)}_{DFM}$ using the first T periods of your generated data, where $T = 100, 200, ..., 4900, 5000$ (increments of 100). For each estimation method, plot both the estimates and the estimation errors $(\hat{\beta}^T - 2)$ as a function of the sample size T. What pattern do you see in each case? Why should you expect to see these patterns? Explain.

(b) Using the same sample for ε_{it} and η_{it}, set $\mu_i = 0$ and generate the data as

$$x_{it} = \eta_{it}$$
$$y_{it} = 2x_{it} + \varepsilon_{it}$$

Repeat (a) above using the new sample. Discuss any differences in the results.

(c) **Using the same sample for x_{it} and ν_{it}, set $\gamma_i = 0$ and generate the data as**

$$\varepsilon_{it} = \nu_{it}$$
$$y_{it} = 2x_{it} + \varepsilon_{it}$$

Repeat (a) above using the new sample. Discuss any differences in the results.

(d) **For (a), (b) and (c), show whether or not the FM estimator and the demeaned FM estimator are T-consistent (Hint: write the expression $\hat{\beta}_{FM} - \beta$ as a function of the x_{it}'s and the ε_{it}'s and substitute the definitions in (2). Then explain what happens when you let $T \to \infty$).**

(e) **So far we have been using only firm effects. How would the results in (d) change if we included time effects? Specifically, suppose that**

$$x_{it} = \delta_t + \eta_{it}$$
$$\varepsilon_{it} = \psi_t + \nu_{it}$$

Show (using an argument similar to the one in (d)) whether or not the traditional FM is consistent. What about the Demeaned FM? What are your overall conclusions about the consistent of the FM estimators?

(f) **For each case above, show whether or not the estimated variance of the FM estimator calculated as**

$$S^2(\hat{\beta}_{FM}^{(T)}) = \frac{1}{T}\sum_t \frac{(\hat{\beta}_t - \hat{\beta}_{FM}^{(T)})^2}{T-1}$$

is an unbiased estimator of the true variance $Var(\hat{\beta}_{FM}^{(T)})$. Also, show whether or not the true variance $Var(\hat{\beta}_{FM}^{(T)})$ goes to 0 as $T \to \infty$.