Kellogg School of Management
FINC 585-3: Asset Pricing
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Assignment 1

The objective of this assignment is to gain familiarity with basic properties of univariate equityindex return series and some standard test statistics gauging the existence of return dependence. Both the daily S&P 500 and CRSP return series are available under `data/` on the Canvas site.

Problem 1.

Provide a sketch of a proof for the Hausman Principle, given on page 11.

Proposition. *Consider the following estimators:*

- $\widehat{\sigma}_a^2$ *is Efficient (MLE) Estimator under the Null Hypothesis, i.e, it has Smallest Asymptotic Variance amongst consistent estimators*
- $\widehat{\sigma}_b^2$ *is Consistent, albeit NOT Efficient Estimator under the Null*

$$\textbf{\textit{Hausman Principle:}}\ \mathbb{C}(\widehat{\sigma}_a^2, \widehat{\sigma}_b^2) = \mathbb{V}(\widehat{\sigma}_a^2)$$

Problem 2.

Derive the asymptotic distribution for the VD statistic in page 12.

VD Statistic:

$$VD = \widehat{\sigma}_a^2 - \widehat{\sigma}_b^2$$

Problem 3.

Do the exercise on page 13 (on the VR statistic with overlapping returns). Hint: If it seems tricky, you can check the literature for inspiration.

Exercise: (Overlapping returns)

Under the Null Hypothesis,

$$\sqrt{nq}\left(\widehat{\rho}_1, \ldots, \widehat{\rho}_{q-1}\right) \xrightarrow{d} N\left(0, I_{(q-1)\times(q-1)}\right)$$

We also saw,

$$\widehat{VR}(q) = 1 + 2\sum_{k=1}^{(q-1)}\left(1 - \frac{k}{q}\right)\widehat{\rho}_k$$

Use the Delta method to show that, if we use overlapping returns to estimate the higher order return auto-covariances then,

$$\sqrt{nq}\left(\widehat{VR}(q) - 1\right) \xrightarrow{d} N\left(0, \frac{2(q-1)(2q-1)}{3q}\right)$$

Problem 4.

Hong, Linton, and Zhang (2017) (HLZ) provide a correction and extension to the Variance Ratio statistic explored by Lo and MacKinlay (1988) (no formal proofs needed below).

a) Identify the changes in assumptions they introduce relative to Lo & MacKinlay.

b) What are the advantages of their proposed new assumptions?

c) State the limiting distribution result for the univariate VR statistic in their paper.

Problem 5.

Apply the Spearman Corerlation Test on page 4 of the notes to explore dependences for the daily S&P 500 index returns.

a) First, do the test as suggested for January 1991-December 2006 period

b) Second, do the same test for the sample period 2007-2022

c) Repeat the above, but using the absolute returns in lieu of the returns

Problem 6.

Implement the Lo and MacKinlay (1988) ratio test for the CRSP value-weighted and equal-weighted indices over their sample period, and for the periods 1991-2006 and 2007-2022. Try to assess significance according to the Lo & Mackinlay and HLZ robust limiting distributions.

Problem 7.

Provide a proof of the Roll estimator for the bid-ask spread based on the return auto-correlation. Using the original paper (Roll, 1984), it should be straightforward to pin down assumptions that will allow for a proof of the results.

Problem 8.

Prove the initial expressions for β and λ, and then all the formulas displayed under implications 1)-4) in the Kyle Model (Market Microstructure Notes).

References

S. Y. Hong, O. Linton, and H. J. Zhang. An investigation into multivariate variance ratio statistics and their application to stock market predictability. *Journal of Financial Econometrics*, 15(2):173–222, 2017.

A. W. Lo and A. C. MacKinlay. Stock market prices do not follow random walks: Evidence from a simple specification test. *The review of financial studies*, 1(1):41–66, 1988.

R. Roll. A simple implicit measure of the effective bid-ask spread in an efficient market. *The Journal of finance*, 39(4):1127–1139, 1984.