

Assignment 5

Problem 1.

For this question, we are going to use the dataset `industry.csv` available on CANVAS. This dataset contains 38 Industry Portfolio daily returns available on Kenneth French's [web-site](#). Along with this, we will also use the Fama-French 5 + Momentum factors under the `ffdaily.csv` file.

- A) For a range of $k = 1, \dots, 8$, calculate the k^{th} principal components of these industry excess returns (use the risk-free data available on `ffdaily.csv`). Remember to standardize the returns to have zero mean and unity variance before extracting the principal components. Plot the total explained variance ratio for each of these models. Using this measure, how many factors would you choose? Assuming normality of the log-returns, calculate both the AIC and BIC for each model as well. How many factors would you choose based on these criteria? Compare the results.
- B) Pick up the model with 8 factors. Using rotation, how do the principal components correlate with the FF factors?
- C) Use the Canonical Correlation Analysis to analyze the relationship between the FF factors and the principal components. Using 1 to 5 CCA components, calculate the canonical correlation between these sets. How does it change with the number of components?
- D) Plot the loadings of the PCA for each industry. Give a brief explanation of the results.

Problem 2.

Connor and Korajczyk (1986) develop a method to measure the performance of a mutual fund portfolio. Using a large number of funds, they show that the Treynor and Black (1973) Appraisal Ratio completely captures investors ranking over funds. A simplified version of the model is described as follows:

Let fund's j excess return at time t be denoted as $r_{j,t}$ for $j = 1, \dots, N$ and the riskless asset return be $r_{0,t}$. These returns are generated by a finite set of latent factors $\mathbf{f}_t = (f_t^{(1)}, \dots, f_t^{(K)})$ where K is known. The return of fund j is then given by:

$$r_{j,t} = \alpha_j + \sum_{k=1}^K \beta_j^{(k)} \left(\gamma^{(k)} + f_t^{(k)} \right) + \varepsilon_{j,t} \quad (1)$$

with $\mathbb{E}(\mathbf{f}_t) = 0$, $\mathbb{E}(\mathbf{f}_t \mathbf{f}_t') = I_K$ and $\mathbb{E}(\varepsilon_{j,t}) = 0$, $\mathbb{E}(\varepsilon_{j,t} \varepsilon_{j',t}') = \sigma_j^2$, with σ_j bounded from below. We also assume that $\varepsilon_{j,t}$ is uncorrelated between funds and independent on the set of factors. The Appraisal Ratio is defined as $t_j = \alpha_j / \sigma_j$, and according to Theorem 4, this ratio is sufficient to rank funds¹.

Let \mathbf{R} be the $N \times T$ matrix of fund returns. The authors show that equation (1) can be estimated by the following algorithm.

- 1) Compute a $k \times T$ principal component matrix of $\frac{1}{n} \mathbf{R}' \mathbf{R}$, denote it as $\hat{\mathbf{f}}$;
- 2) Run a time series regression

$$r_j = \hat{\alpha}_j + \sum_{k=1}^K \hat{\beta}_j^{(k)} \left(\hat{\gamma}^{(k)} + \hat{f}^{(k)} \right) + \hat{\varepsilon}_j \quad (2)$$

- 3) Calculate $\hat{t}_j = \hat{\alpha}_j / \hat{\sigma}_j$.
- 4) For a large N and T , this estimator is consistent and asymptotically normal:

$$\text{dlim}_{T \rightarrow \infty} \text{plim}_{N \rightarrow \infty} T^{1/2} (\hat{t}_j - t_j) \sim N \left(0, 1 + \sum_{k=1}^K \gamma_k^2 \right) \quad (3)$$

In this exercise, we will apply this model to a dataset of mutual funds. The dataset `fund_returns.csv` available on CANVAS contains a time series of monthly log-returns for a large set of mutual funds from 2000 to 2023 obtained from CRSP.² We will also use the data on the Fama-French factors and risk-free rate available on the `ffdaily.csv` file.

- A) For k from 1 to 8, calculate the principal components of the funds' excess returns. Plot the explained variance ratio for each of these components. How do these PC compare with the Fama-French factors?

¹ i.e., an investor would strictly prefer to switch from investing in fund j to fund l if, and only if, $t_j > t_l$

² This data is available on WRDS under the MONTHLY_RETURNS dataset on the CRSP library.

B) Connor and Korajczyk (1993) provide a methodology to choose the optimal number of factors K in an approximate factor model. Their test is based on the idea that additional (non-informative) factors will not increase the explained variance of the model. Their algorithm is described as the following iteration:

- 1) For a given number of factors k , estimate the model with k and $k + 1$ factors. Let the residuals be $\hat{\varepsilon}_{j,t}$ and $\hat{\varepsilon}_{j,t}^*$, respectively;
- 2) Calculate adjusted squared residuals

$$\begin{aligned}\hat{\sigma}_{j,t} &= \hat{\varepsilon}_{j,t} / [1 - (k + 1)/T - k/N] \\ \hat{\sigma}_{j,t}^* &= \hat{\varepsilon}_{j,t}^* / [1 - (k + 2)/T - k/N]\end{aligned}\tag{4}$$

- 3) Calculate $\hat{\Delta}$ by subtracting the cross-sectional means of $\hat{\sigma}_{j,t}$ in odd periods from the cross-sectional means of $\hat{\sigma}_{j,t+1}^*$ in even periods:

$$\hat{\Delta}_s = \mu_{2s-1} - \mu_{2s}^* \quad s = 1, \dots, \lfloor T/2 \rfloor\tag{5}$$

where $\mu_t = N^{-1} \sum_{j=1}^N \hat{\sigma}_{j,t}^2$ and $\mu_t^* = N^{-1} \sum_{j=1}^N (\hat{\sigma}_{j,t}^*)^2$. Under the null,

$$\text{dlim}_{N \rightarrow \infty} \sqrt{N} \hat{\Delta} \sim N(0, \Gamma)\tag{6}$$

- 4) Using the time series of $\hat{\Delta}$, calculate the mean and covariance matrix $\hat{\Gamma}$ and perform a one-sided zero mean test:

$$\mathcal{H}_0 : \mathbb{E} [\hat{\Delta}] \leq 0$$

- 5) If the null is rejected, start over with $k + 1$ factors. Otherwise, set $K = k$ as the optimal number of factors.

Using their test, find the optimal K . Use this number for the rest of the exercise.

- C) Estimate (2) for each fund and calculate their Appraisal Ratio. Plot the distribution of this measure. According to this, how many funds do really outperform the market? Use the distribution on (3) to test the significance of their performance.
- D) Equation (1) assumes that the factor loadings $\beta_j^{(k)}$ are constant over time. Split the sample in half. Redo C) for each subsample and compare the results. How many funds do outperform in both periods? Interpret.

References

- G. Connor and R. A. Korajczyk. Performance measurement with the arbitrage pricing theory: A new framework for analysis. *Journal of financial economics*, 15(3):373–394, 1986.
- G. Connor and R. A. Korajczyk. A test for the number of factors in an approximate factor model. *the Journal of Finance*, 48(4):1263–1291, 1993.
- J. L. Treynor and F. Black. How to use security analysis to improve portfolio selection. *The journal of business*, 46(1):66–86, 1973.