Lattice Boltzmann Method in Acoustics - Exercise List I

Prof. Andrey Ricardo da Silva

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A Benchmarking wave propagation in the LBGK model

The objective of this exercise is to compare numerical and analytical results for viscous two-dimensional (cylindrical) wave propagation. An initial implementation of the numerical scheme is provided in the reference code *lbkg1.m*. The comparisons should take into account different wave discretizations (cells per wavelength) to be implemented in the code. The viscous wave equation is given by:

$$(1 - \frac{2\nu}{c_0^2}\partial_t)\nabla^2 p = \frac{1}{c_p^2}\partial_t^2 p, \tag{1}$$

where ν and c_0 are the fluid's kinematic viscosity and the speed of sound, respectively. For an outgoing cylindrical wave, the exact stationary solution of Eq. (1) is given by

$$p = AH_0^{(2)}kr\exp(j\omega t), \tag{2}$$

where A is the wave amplitude, $H_0^{(2)}$ is the Hankel function of the second kind and order zero, and r is the radial distance from the sound source. $k = 2\pi f/c_p - j\alpha$ is the complex wavenumber and α is the spatial absorption coefficient, expressed as

$$\alpha = \frac{\omega}{c_p\sqrt{(2)}}\sqrt{\frac{\sqrt{1+(\omega\tau_s)^2}-1}{1+(\omega\tau_s)^2}}. \tag{3}$$

The solution in Eq.(2) is implemented in the provided Matlab function *cylinwave.m*.

B Procedures

1. Define the number of necessary time steps NTS for an acoustic disturbance to propagate from the center of the lattice until its boundaries. Remember that the lattice - as default in the provided Matlab code

lbgk1.m - is 300 $\times$ 300 cells.

2. Create an harmonic acoustic source in the center of the lattice. One way to accomplish that is to impose an harmonic density fluctuation such that $\rho(\mathbf{x}, ta\Delta t) = \rho_0 + A\sin(2\pi freq(ta-1)\Delta t)$, where $freq$ is the source frequency, ta is the step counter, Δt is the time step, and $A = 0.001$ is the density disturbance amplitude. **Tip:** The harmonic density fluctuation can be implemented inside the for-loop structure, right after the re-calculation of ρ. Test it.

3. Obtain the numerical result for the acoustic pressure field p_{an} at $ta = NTS$ along the lattice coordinate vector $\mathbf{x} = [150 : 300, 150]$. The numerical results should be obtained for three different wavelength discretizations (d = 8, 16, and 25 cells per wavelength). One way to change the wavelength discretization is to keep the lattice pitch Δx constant and vary the source frequency $freq$. Calculate also the phase of the source when $ta = NTS$ at $\mathbf{x} = [150, 150]$. Save the results for each discretization along with the distance vector x.

$$\phi = 2\pi f(n-5)\Delta t, \quad \text{gives the phase} \tag{4}$$

4. Calculate, in physical unities, the fluid kinematic viscosity, as well as the pressure amplitude at the acoustic source corresponding to the default lattice variables used in the simulations. Use these values to obtain the analytical pressure field p_a as a function of the distance vector x based on Eq. (2). You may use the Matalab function *cylin wave.m* for the calculations of p_a. Save the resulting vectors of p_a for different source frequencies $freq$, and repeat this step using, at this time, the kinematic viscosity of air in STP ($\nu_p = 1.5 \times 10^{-5}$ m^2/s) to determine the analytical acoustic field p_{air} as a function of the distance vector x. Save it.

5. In a single figure, plot $p_{an}(x)$ for each discretization scheme, as well as their respective analytical solution $p_a(x)$

6. In one figure, compare the different results obtained for $p_a(x)$ and $p_{air}(x)$.

C Questions

1. What is the observed effect of a low discretization scheme? Do these results qualitatively agree with the analysis conducted by Wilde (2006) with respect to wave dissipation (see slides from the last class)? Please, justify.

2. For high discretization schemes (16, 25 cells per wavelength) a slight disagreement between $p_{an}(x)$ and $p_a(x)$ should be noticeable when the wave approaches the lattice boundary. Can you explain why?

3. Due to the limitations of the LBGK model, the physical viscosity used in the simulations is $O(2)$ higher than the kinematic viscosity of air in normal conditions. Even so, the error between $p_{an}(x)$, $p_a(x)$, $p_{air}(x)$ reasonably small. Can you draw any conclusions over this fact?