CHAPTER TWENTY-THREE

Elementary Algebra

Introduction

In arithmetic, we mostly use the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. These arithmatic symbols are also known as numerals (see the chapter 'Number system'). We also use signs '+' for 'addition', '–' for 'subtraction', '×' for 'multiplication' and '÷' for 'division' to do arithmetical calculations. In this chapter, we will make a beginning of the study of a new branch of mathematics known as *algebra*. Besides the arithmetic symbols (numerals) and signs for the operations of adition, subtraction, multiplication and division, algebra also uses letters from various alphabets like English, Greek etc. These letters are used either to generalise results or to represent unknown quantities. Algebra also enables us to find unknown quantities when all the values in a calculation are not known. Precisely, we can say that in algebra we mostly deal with letters and numerals and perform operations like addition, subtraction, multipliction and division.

In this chapter we shall be discussing various aspects of '*Algebraic Expressions*', '*Algebraic Equations*', '*Factors and Remainders Theorem*', '*LCM and HCF of Polynomials*' etc.

1. Algebraic Expressions
2. Algebraic Equations
3. Remainder and Factor Theorem
4. HCF and LCM of Polynomials
5. Problems Based on Inequality.

(1) Algebraic Expressions

In algebra, we generally come across two types of symbols, namely **constants** and **variables**.

A symbol having a fixed numerical value is called a constant and a symbol which takes various numerical values is called a Variable.

Following examples will illustrate our points.

Ex. 1: We know that the perimeter 'P' of a square of side 's' is given by P = 4 × s.
Here, 4 is a constant and 'P' and 's' are variables.

Ex. 2: The perimeter 'P' of a rectangle of sides 'l' and 'b' is given by P = 2 (l + b).
Here, 2 is a constant and 'l' and 'b' are variables.

Now see the definition of "algebraic expressions" given below.

"A combination of constant and variables connected by the signs of fundamental operations of addition, subtraction, multiplication and division is called an algebraic expression".

Various parts of an algebraic expression which are separated by the signs of '+' or '-' are called the '**terms**' of the expression. Consider the following examples.

Ex. 3: $2x^2 - 3xy + 5y^2$ is an algebraic expression consisting of three terms, namely, $2x^2$, $-3xy$ and $5y^2$.

Ex. 4: The expression $2x^3 - 3x^2 + 4x - 7$ is an algebraic expression consisting of four terms, namely, $2x^3$, $-3x^2$, $4x$ and -7.

Important Definitions

(i) Monomial: An algebraic expression containing only one term is called a monomial. As for example,

$-5, 3y, 7xy, \frac{2}{3}x^2yz, \frac{2}{3}x^2yz, \frac{5}{3}a^2b^2$ etc are all monomials.

(ii) Binomial: Two monomials containing unlike terms when added give a binomial as defined below.

"An algebraic expression containing two terms is called a binomial." As for example, the expressions 2x - 3, 3x + 2y, xyz - 5 etc are binomials.

Note that (3x + 7x) is not a binomial, because 3x + 7x = 10x, which is a monomial.

(iii) Trinomial: An algebraic expression containing three terms is called a trinomial.

In other words, if three monomials are such that no two contain like terms, then their sum is a trinomial. As for example, the expressions $a - b + 2$, $x^2 + y^2 - xy$, $x^3 - 2y^3 - 3x^2y^2z$ etc are trinomials.

(iv) Polynomials: An algebraic expression in which the variables involved have only non-negative integral powers, is called a polynomial. As for example,

$\frac{2}{3}x^2 - \frac{3}{2}x^2 + x - 5$ is a polynomial in variables x

whereas $\frac{1}{2}x^3 - 3x^2 + 5x^{1/2} + x - 1$ is not a polynomial, be-

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cause it contains a term $5x^{1/2}$ which contains $\frac{1}{2}$ as the power of variable x, which is not a non-negative integer. Consider another example,

$3-2x^2+4x^2y+8y-\frac{5}{3}xy^2$ is a polynomial in two variables x and y.

(a) Degree of a polynomial in one variable

In a polynomial in one variable, the highest power of the variable is called its degree. As for example, consider the following:

(i) 2x + 3 is a polynomial in x of degree 1.

(ii) $2x^2-3x+\frac{7}{5}$ is a polynomial in x of degree 2.

(iii) $3x^4-7x^2+\frac{5x}{2}+\frac{1}{3}$ is a polynomial in x of degree 4.

(iv) $\frac{2}{3}a^3-\frac{7}{2}a^2+4$ is a polynomial in a of degree 3.

(b) Degree of a polynomial in two variables

In a polynomial in more than one variable, the sum of the powers of the variables in each term is computed and the highest sum so obtained is called the degree of the polynomial. As for example,

$3x^4-2x^3y^2+7xy^3-9x+5y+4$ is a polynomial in x and y of degree 5, whereas $\frac{1}{2}-3x+7x^2y-\frac{3}{4}x^2y^2$ is a polynomial of degree 4 in x and y.

(c) Constant Polynomial

A polynomial consisting of a constant term only is called a constant polynomial. The degree of a constant polynomial is zero.

(d) Linear Polynomial

A polynomial of degree 1 is called a linear polynomial.

(e) Quadratic Polynomial

A polynomial of degree 2 is called a quadratic polynomial. Consider the following examples

$2x^2-3x+4$, $2-x+x^2$, $2y^2-\frac{3}{2}y+\frac{1}{4}$ are quadratic polynomials.

(f) Cubic Polynomial

A polynomial of degree 3 is called a cubic polynomial. As for example

x^3-7x^2+2x-3, $2+\frac{1}{2}y-\frac{3}{2}y^2+4y^3$ are cubic polynomials.

(g) Biquadratic Polynomial

A polynomial of degree 4 is called a biquadratic polynomial. As for example,

$3x^4-7x^3+x^2-x+9$, $4-\frac{2}{3}x^2+\frac{3}{5}x^4$ are biquadratic polynomials.

Note: A polynomial is said to be a monomial, a binomial or a trinomial according as it contains one term, 2 terms or 3 terms respectively.

(v) Factors: Each term in an algebraic expression is a product of one or more number(s) and/or literal(s). These number(s) and/or literal(s) are known as the factors of that term.

A constant factor is called a **numerical factor**, while a variable factor is known as a **literal factor**.

(vi) Coefficient: In a term of an algebraic expression any of the factors with the sign of the term is called the coefficient of the product of the other factors. Consider the following examples,

(a) In -5xy, the coefficient of x is -5y; the coefficient of y is -5x and the coefficient of xy is -5.

(b) In -x, the coefficient of x is -1.

(c) In $3a^2bc$, the coefficient of a^2 is 3bc, the coefficient of b is $3a^2c$ and the coefficient of c is $3a^2b$.

(vii) Constant Term: A term of the expression having no literal factors is called a constant term. As for example, in the algebraic expression $x^2 - xy + yz - 4$, the constant term is -4.

(viii) Like and Unlike Terms: The terms having the same literal factors are called like or similar terms, otherwise, they are called unlike terms. As for example, in the algebraic expression $2a^2b + 3ab^2 - 7ab - 4ba^2$, we have $2a^2b$ and $-4ba^2$ as like terms, whereas $3ab^2$ and -7ab are unlike terms.

Factorization of Algebraic Expressions

The process of writing an algebraic expression as the product of two or more algebraic expressions is called factorization and the algebraic expressions that may be multiplied to obtain the given algebraic expressions are called factors of the given expression. Consider the following examples,

Ex. 5: **(x + 3) and (x - 3) are the factors of $x^2 - 9$, because $(x + 3)(x - 3) = x^2 - 9$.**

Ex. 6: **x, x + 1, x (x + 1), $(x + 1)^2$ and $x(x + 1)^2$ are factors of $x^3 + 2x^2 + x$, because $x^3 + 2x^2 + x = x(x + 1)^2$.**

Now, let us have a look at the following types of problems that will illustrate our points.

Type-I: When Each Term of the Given Expression Contains a Common Monomial Factor

In this type of problems, we take out the common monomial factor and use the distributive property as illustrated in the following examples.

Ex. 7: **Factorize: $4x^2 - 6xy + 12x$**

Soln: The greatest common factor of the terms $4x^2$, 6xy and 12x is 2x.

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$\therefore 4x^2 - 6xy + 12x = 2x(2x - 3y + 6)$

Ex. 8: Factorize: $35a^2 - 21a^2b + 14a^2b^2$

Soln: The greatest common factor of the terms $35a^2$, $21a^2b$ and $14a^2b^2$ is $7a^2$.

$\therefore 35a^2 - 21a^2b + 14a^2b^2 = 7a^2(5 - 3b + 2b^2)$

Type-II: Factorization of Algebraic Expressions Having Binomial as a Common Factor

Ex. 9: Factorize:

(i) $3x(a + b) - 5y(a + b)$

(ii) $15(x - y)^2 - 5x(x - y) - x + y$

Soln: We have,

(i) $3x(a + b) - 5y(a + b) = (a + b)(3x - 5y)$

[Taking (a + b) common]

(ii) $15(x - y)^2 - 5x(x - y) - x + y$

$= 15(x - y)^2 - 5x(x - y) - (x - y)$

$= (x - y)\{15(x - y) - 5x - 1\}$

$= (x - y)(15x - 15y - 5x - 1)$

$= (x - y)(10x - 15y - 1)$

Ex. 10: Factorize:

(i) $x^3 + x^2 + x + 1$

(ii) $a^3 + 2a - 2a^2 - 4$

(iii) $x^2 - y + xy - x$

Soln: We have,

(i) $x^3 + x^2 + x + 1 = (x^3 + x^2) + (x + 1)$

$= x^2(x + 1) + 1(x + 1)$

$= (x + 1)(x^2 + 1)$

(ii) $a^3 + 2a - 2a^2 - 4 = (a^3 + 2a) + (-2a^2 - 4)$

$= a(a^2 + 2) - 2(a^2 + 2)$

$= (a^2 + 2)(a - 2)$

(iii) $x^2 - y + xy + x = x^2 + xy - x - y$

[Rearranging the terms]

$= (x^2 + xy) + (-x - y)$

$= x(x + y) - 1(x + y)$

$= (x + y)(x - 1)$

Type-III: Factorization of Expressions Expressible as the Square of a Binomial

Ex. 11: Factorize:

(i) $9a^2 + 12ab + 4b^2$

(ii) $x^4 + 2 + \frac{1}{x^4}$

(iii) $1 - 16x^2 + 64x^4$

Soln: We have,

(i) $9a^2 + 12ab + 4b^2 = (3a)^2 + 2 \times 3a \times 2b + (2b)^2$

$= (3a + 2b)^2$

(ii) $x^4 + 2 + \frac{1}{x^4}$

$$= (x^2)^2 + 2 \times x^2 \times \frac{1}{x^2} + \left(\frac{1}{x^2}\right)^2 = \left(x^2 + \frac{1}{x^2}\right)^2$$

(iii) $1 - 16x^2 + 64x^4 = 1^2 - 2 \times 8x^2 + (8x^2)^2 = (1 - 8x^2)^2$

Type-IV: Factorization of Expression Expressible as the Difference of two Squares

Ex. 12: Factorize:

a2 – b2= (a + b)(a – b)

(i) $16x^2 - 9y^2$

(ii) $-(a+b)^2 + \frac{4c^2}{49}$

(iii) $x^4 - x^4y^4$

Soln: We have,

(i) $16x^2 - 9y^2 = (4x)^2 - (3y)^2$

$= (4x - 3y)(4x + 3y)$

(ii) $-(a+b)^2 + \frac{4c^2}{49} = \left(\frac{2c}{7}\right)^2 - (a+b)^2$

$$= \left\{\frac{2c}{7} - (a+b)\right\}\left\{\frac{2c}{7} + (a+b)\right\}$$

$$= \left(\frac{2c}{7} - a - b\right)\left(\frac{2c}{7} + a + b\right)$$

(iii) $x^4 - x^4y^4 = x^4(1 - y^4)$

$= x^4\{(1)^2 - (y^2)^2\}$

$= x^4(1 + y^2)(1 - y^2)$

$= x^4(1 + y^2)(1 - y)(1 + y)$

Ex. 13: Factorize:

(i) $a^2b^2 - a^2 - b^2 + 1$

(ii) $x^2 - y^2 - 9z^2 + 6yz$

(iii) $50x^2 - 32y^2$

Soln: We have

(i) $a^2b^2 - a^2 - b^2 + 1 = (a^2b^2 - a^2) + (-b^2 + 1)$

$= a^2(b^2 - 1) - 1(b^2 - 1)$

$= (b^2 - 1)(a^2 - 1)$

$= (b - 1)(b + 1)(a - 1)(a + 1)$

$= (a - 1)(a + 1)(b - 1)(b + 1)$

(ii) $x^2 - y^2 - 9z^2 + 6yz = \cancel{x^2(y^2 + 9z^2 - 6yz)}$

$= x^2 - [y^2 + (3z)^2 - 2 \times y \times 3z]$

$= x^2 - (y - 3z)^2$

$= \{x + (y - 3z)\}\{x - (y - 3z)\}$

$= (x + y - 3z)(x - y + 3z)$

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(iii) $50x^2 - 32y^2 = 2(25x^2 - 16y^2)$
$= 2[(5x)^2 - (4y)^2]$
$= 2(5x - 4y)(5x + 4y)$

Type V: Factorization of Algebraic Expressions Expressible as the Cube of the Sum or Difference of Two Terms

To factorize algebraic expressions expressible as the cube of the sum or difference of two terms, we shall be using the following identities:

(i) $a^3 + 3a^2b + 3ab^2 + b^3 = (a + b)^3$
(ii) $a^3 - 3a^2b + 3ab^2 - b^3 = (a - b)^3$

Following examples will illustrate the use of these identities in factorizing polynomials consisting of four terms.

Ex. 14: Factorize: $8x^3 + 12x^2 + 6x + 1$.

Soln: We have,
$8x^3 + 12x^2 + 6x + 1$
$= (2x)^3 + 3 \times (2x)^2 \times 1 + 3 \times (1)^2 \times 2x + (1)^3$
$= (2x + 1)^3$
[Using $a^3 + 3a^2b + 3ab^2 + b^3 = (a + b)^3$]
$= (2x + 1)(2x + 1)(2x + 1)$

Ex. 15: Factorize: $a^3 - 18a^2 + 108a - 216$

Soln: We have,
$a^3 - 18a^2 + 108a - 216$
$= (a)^3 - 3 \times (a)^2 \times 6 + 3 \times a \times (6)^2 - (6)^3$
$= (a - 6)^3$ [Using: $a^3 - 3a^2b + 3ab^2 - b^3 = (a - b)^3$]
$(a - 6)(a - 6)(a - 6)$

Ex. 16: Resolve the following into factors:

(i) $x^3 + x^2y + \frac{xy^2}{3} + \frac{y^3}{27}$

(ii) $\frac{8x^3}{27} - \frac{4x^2}{3y} + \frac{2x}{y^2} - \frac{1}{y^3}$

Soln: (i) We have,

$$x^3 + x^2y + \frac{xy^2}{3} + \frac{y^3}{27}$$

$$= (x)^3 + 3 \times (x)^2 \times \frac{y}{3} + 3 \times x \times \left(\frac{y}{3}\right)^2 + \left(\frac{y}{3}\right)^3$$

$$= \left(x + \frac{y}{3}\right)^3 = \left(x + \frac{y}{3}\right)\left(x + \frac{y}{3}\right)\left(x + \frac{y}{3}\right)$$

(ii) We have,

$$\frac{8x^3}{27} - \frac{4x^2}{3y} + \frac{2x}{y^2} - \frac{1}{y^3}$$

$$= \left(\frac{2x}{3}\right)^3 - 3 \times \left(\frac{2x}{3}\right)^2 \times \frac{1}{y} + \left(\frac{2x}{3}\right) \times \left(\frac{1}{y}\right)^2 - \left(\frac{1}{y}\right)^3$$

$$= \left(\frac{2x}{3} - \frac{1}{y}\right)^3 = \left(\frac{2x}{3} - \frac{1}{y}\right)\left(\frac{2x}{3} - \frac{1}{y}\right)\left(\frac{2x}{3} - \frac{1}{y}\right)$$

Ex. 17: Factorize

(i) $x^6 - 3x^4 + 3x^2 - 1$ (ii) $x^6 - 3x^2 + \frac{3}{x^2} - \frac{1}{x^6}$

Soln: (i) We have,
$x^6 - 3x^4 + 3x^2 - 1$
$= (x^2)^3 - 3 \times (x^2)^2 \times 1 + 3 \times (x^2) \times (1)^2 - (1)^3$
$= (x^2 - 1)^3 = \{(x - 1)(x + 1)\}^3 = (x - 1)^3 (x + 1)^3$

(ii) We have,

$$x^6 - 3x^2 + \frac{3}{x^2} - \frac{1}{x^6}$$

$$= (x^2)^3 - 3 \times (x^2)^2 \times \frac{1}{x^2}$$

$$+ 3 \times (x^2) \times \left(\frac{1}{x^2}\right)^2 - \left(\frac{1}{x^2}\right)^3$$

$$= \left(x^2 - \frac{1}{x^2}\right)^3 = \left\{\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)\right\}^3$$

$$= \left(x + \frac{1}{x}\right)^3 \left(x - \frac{1}{x}\right)^3$$

Type VI: Factorization of Algebraic Expressions Expressible as the Sum and Difference of Two Cubes

In order to factorize the algebraic expressions expressible as the sum or differnce of two cubes, we use the following identities:

(i) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
(ii) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Following examples will illustrate the use of these identities in the factorization of algebraic expressions.

Ex. 18: Factorize:

(i) $8x^3 + 125y^3$ (ii) $x^3 - 8y^3$

Soln:

(i) We have,
$8x^3 + 125y^3 = (2x)^3 + (5y)^3$
$= (2x + 5y)\{(2x)^2 - 2x \times 5y + (5y)^2\}$
[Using: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$]
$= (2x + 5y)(4x^2 - 10xy + 25y^2)$

(ii) We have, $x^3 - 8y^3 = (x)^3 - (2y)^3$
$= (x - 2y)\{(x)^2 - x \times 2y + (2y)^2\}$
$= (x - 2y)(x^2 + 2xy + 4y^2)$
[Using $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$]

Ex. 19: Factorize

(i) $a^3 - 0.216$ (ii) $p^6 - 64q^6$

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Soln: (i) We have,

$$a^3 - 0.216 = a^3 - (0.6)^3$$
$$= (a - 0.6)\{a^2 + 0.6a + (0.6)^2\}$$
$$= (a - 0.6)(a^2 + 0.6a + 0.36)$$

(ii) $p^6 - 64q^6 = (p^2)^3 - (8q^2)^3$

$$= (p^2 - 8q^2)\{(p^2)^2 - p^2 \times 8q^2 + (8q^2)^2\}$$
$$= (p^2 - 8q^2)(p^4 - 8p^2q^2 + 64q^4)$$

Ex. 20: Factorize:

(i) $(x + 1)^3 - (x - 1)^3$

(ii) $(x + 1)^3 + (x - 1)^3$

(iii) $8(x + y)^3 - 27(x - y)^3$

Soln:

(i) We have,

$(x + 1)^3 - (x - 1)^3$

$= \{(x + 1) - (x - 1)\}\{(x + 1)^2 + (x + 1)(x - 1) + (x - 1)^2\}$

$= (x + 1 + x + 1)\{(x^2 + 2x + 1) + (x^2 - 1) + (x^2 - 2x + 1)\}$

$= 2(x^2 + 2x + 1 + x^2 - 1 + x^2 - 2x + 1) = 2(3x^2 + 1)$

(ii) We have,

$(x + 1)^3 - (x - 1)^3 = \{(x + 1) + (x - 1)\}\{(x + 1)^2 - (x + 1)(x - 1) + (x - 1)^2\}$

$= (x + 1 + x - 1)\{(x^2 + 2x + 1) + (x^2 - 1) + (x^2 - 2x + 1)\}$

$= 2x(x^2 + 2x + 1 - x^2 + 1 + x^2 - 2x + 1) = 2x(x^2 + 3)$

(iii) We have,

$8(x + y)^3 - 27(x - y)^3$

$= \{2(x + y)\}^3 - \{3(x - y)\}^3$

$= \{2(x + y) - 3(x - y)\}[\{2(x + y)\}^2 - 2(x + y) \times 3(x - y) + \{3(x - y)\}^2]$

$= (2x + 2y - 3x + 3y)\{2(x^2 + 2xy + y^2) - 6(x^2 - y^2) + 3(x^2 - 2xy + y^2)\}$

$= (-x + 5y)(2x^2 + 4xy + 2y^2 - 6x^2 + 6y^2 + 3x^2 - 6xy + 3y^2)$

$= (-x + 5y)(-x^2 - 2xy + 11y^2)$

Ex. 21: Factorize

(i) $x^6 - y^6$ (ii) $x^{12} - y^{12}$

Soln:

(i) We have,

$$x^6 - y^6 = (x^2)^3 - (y^2)^3$$
$$= (x^2 - y^2)\{(x^2)^2 + x^2 \times y^2 + (y^2)^2\}$$
$$= (x^2 - y^2)(x^4 + x^2y^2 + y^4)$$
$$= (x - y)(x + y)\{(x^4 + 2x^2y^2 + y^4) - x^2y^2\}$$

[Adding and subtracting x^2y^2 in the second bracket]

$$= (x - y)(x + y)\{(x^2 + y^2)^2 - (xy)^2\}$$
$$= (x - y)(x + y)\{(x^2 + y^2 - xy)(x^2 + y^2 + xy)\}$$
$$= (x + y)(x - y)(x^2 - xy + y^2)(x^2 + xy + y^2)$$

(ii) $x^{12} - y^{12} = (x^4)^3 - (y^4)^3$

$$= (x^4 - y^4)\{(x^4)^2 + x^4 \times y^4 + (y^4)^2\}$$
$$= \{(x^2)^2 - (y^2)^2\}(x^8 + x^4y^4 + y^8)$$
$$= (x^2 - y^2)(x^2 + y^2)(x^8 + 2x^4y^4 + y^8 - x^4y^4)$$
$$= (x - y)(x + y)(x^2 + y^2)\{(x^4 + y^4)^2 - (x^2y^2)^2\}$$
$$= (x - y)(x + y)(x^2 + y^2)(x^4 + y^4 - x^2y^2)(x^4 + y^4 + x^2y^2)$$
$$= (x - y)(x + y)(x^2 + y^2)(x^4 + y^4 - x^2y^2)\{(x^4 + y^4 + 2x^2y^2) - x^2y^2\}$$
$$= (x - y)(x + y)(x^2 + y^2)(x^4 + y^4 - x^2y^2)\{(x^2 + y^2)^2 - (xy)^2\}$$
$$= (x - y)(x + y)(x^2 + y^2)(x^4 + y^4 - x^2y^2)(x^2 + y^2 - xy)(x^2 + y^2 + xy)$$
$$= (x - y)(x + y)(x^2 + y^2)(x^4 + y^4 - x^2y^2)(x^2 - xy + y^2)(x^2 + xy + y^2)$$

Ex. 22: Factorize:

(i) $x^3 + 3x^2 + 3x - 7$

(ii) $x^3 - 3x^2 + 3x + 7$

Soln: (i) We have,

$x^3 + 3x^2 + 3x - 7 = (x^3 + 3x^2 + 3x + 1) - 8$ [Adding and subtracting 1]

$= (x + 1)^3 - 2^3 = \{(x + 1) - 2\}\{(x + 1)^2 + 2(x + 1) + 2^2\}$

$= (x + 1 - 2)(x^2 + 2x + 1 + 2x + 2 + 4)$

$= (x - 1)(x^2 + 4x + 7)$

(ii) We have,

$x^3 - 3x^2 + 3x + 7 = (x^3 - 3x^2 + 3x - 1) + 8$ [Adding and subtracting 1]

$= (x - 1)^3 + 2^3$

$= \{(x - 1) + 2)\}\{(x - 1)^2 - (x - 1) \times 2 + 2^2\}$

$= (x - 1 + 2)\{(x - 1)^2 - 2(x - 1) + 4\}$

$= (x + 1)(x^2 - 2x + 1 - 2x + 2 + 4)$

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$= (x + 1)(x^2 - 4x + 7)$

Ex. 23: Prove that:

$$\frac{0.87\times0.87\times0.87+0.13\times0.13\times0.13}{0.87\times0.87-0.87\times0.13+0.13\times0.13}=1$$

Soln: We have

$$\frac{0.87\times0.87\times0.87+0.13\times0.13\times0.13}{0.87\times0.87-0.87\times0.13+0.13\times0.13}$$

$$=\frac{(0.87)^3+(0.13)^3}{(0.87)^2-0.87\times0.13+(0.13)^2}$$

$$=\frac{a^3+b^3}{a^2-ab+b^2}\text{, where } a = 0.87 \text{ and } b = 0.13$$

$$=\frac{(a+b)(a^2-ab+b^2)}{(a^2-ab+b^2)}$$

$= a + b = (0.87 + 0.13) = 1$

Type VII: Factorization of Algebraic Expressions of the form $a^3 + b^3 + c^3 - 3abc$

In order to factorize the algebraic expressions of the form $a^3 + b^3 + c^3 - 3abc$, we use the following identity:

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

Following examples will illustrate the use of the above identity in factorizing algebraic expressions.

Ex. 24: Factorize:

(i) $8x^3 + 27^3 + z^3 - 18xyz$

(ii) $a^3 - 8b^3 - 64c^3 - 24abc$

Soln: (i) We have,

$8x^3 + 27y^3 + z^3 - 18xyz$

$= (2x)^3 + (3y)^3 + z^3 - 3 \times 2x \times 3y \times z$

$= (2x + 3y + z)\{(2x)^2 + (3y)^2 + z^2 - 2x \times 3y - 3y \times z - z \times 2x\}$

$= (2x + 3y + z)(4x^2 + 9y^2 + z^2 - 6xy - 3yz - 2zx)$

(ii) We have,

$a^3 - 8b^3 - 64c^3 - 24abc$

$= a^3 + (-2b)^3 + (-4c)^3 - 3 \times a \times (-2b) \times -4c$

$= \{a + (-2b) + (-4c)\}\{a^2 + (-2b)^2 + (-4c)^2 - a \times -2b - (-2b) \times (-4c) - (-4c) \times a\}$

$= (a - 2b - 4c)(a^2 + 4b^2 + 16c^2 + 2ab - 8bc + 4ca)$

Ex. 25: Factorize:

$(a + b)^3 + (b + c)^3 + (c + a)^3 - 3(a + b)(b + c)(c + a)$

Soln: We have,

$(a + b)^3 + (b + c)^3 + (c + a)^3 - 3(a + b)(b + c)(c + a)$

$= \{(a + b) + (b + c) + (c + a)\}\{(a + b)^2 + (b + c)^2 + (c + a)^2 - (a + b)(b + c) - (b + c)(c + a) - (c + a)(a + b)\}$

$= (2a + 2b + 2c)\{(a^2 + 2ab + b^2) + (b^2 + 2bc + c^2) + (c^2 + 2ca + a^2) - (ab + ac + b^2 + bc) - (bc + ba + c^2 + ca) - (ca + cb + a^2 + ab)\}$

$= 2(a + b + c)(2a^2 + 2b^2 + 2c^2 + 2ab + 2bc + 2ca - ab - ac - b^2 - bc - bc - ba - c^2 - ca - ca - cb - a^2 - ab)$

$= 2(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

Type VIII: Factorization of Algebraic Expressions of the form $a^3 + b^3 + c^3$, when $a + b + c = 0$

In the previous section, we have learnt that

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

If $a + b + c = 0$, then

$a^3 + b^3 + c^3 - 3abc = 0 \times (a^2 + b^2 + c^2 - ab - bc - ca) = 0$

$\Rightarrow a^3 + b^3 + c^3 = 3abc$

Thus, if $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

We shall now use the above conditional identity to factorize algebraic expressions in the following examples.

Ex. 26: Factorize:

$(x - y)^3 + (y - z)^3 + (z - x)^3$

Soln: Let $x - y = a$, $y - z = b$ and $z - x = c$. Then,

$a + b + c = x - y + y - z + x - x = 0$

$\therefore a^3 + b^3 + c^3 = 3abc$

$\Rightarrow (x - y)^3 + (y - z)^3 + (z - x)^3 = 3(x - y)(y - z)(z - x)$

Ex. 27: Factorize:

$p^3(q - r)^3 + q^3(r - p)^3 + r^3(p - q)^3$

Soln: We have,

$p^3(q - r)^3 + q^3(r - p)^3 + r^3(p - q)^3$

$= \{p(q - r)\}^3 + \{q(r - p)\}^3 + \{r(p - q)\}^3$

Let $p(q - r) = a$, $q(r - p) = b$ and $r(p - q) = c$. Then,

$a + b + c = p(q - r) + q(r - p) + r(p - q)$

$= pq - pr + qr - qp + rp - rq$

$= pq - pr + qr - pq + pr - qr = 0$

$\therefore a^3 + b^3 + c^3 = 3abc$

$\Rightarrow \{p(q - r)^3 + \{q(r - p)\}^3 + \{r(p - q)\}^3 = 3p(q - r) \times q(r - p) \times r(p - q)$

$\Rightarrow p^3(q - r)^3 + q^3(r - p)^3 + r^3(p - q)^3 = 3pqr(p - q)(q - r)(r - p)$

Ex. 28: Factorize: $(a^2 - b^2)^3 + (b^2 - c^2)^3 + (c^2 - a^2)^3$

Soln: Let $x = a^2 - b^2$, $y = b^2 - c^2$ and $z = c^2 - a^2$. Then,

$x + y + z = a^2 - b^2 + b^2 - c^2 + c^2 - a^2 = 0.$

$\therefore x^3 + y^3 + z^3 = 3xyz$

$\Rightarrow (a^2 - b^2)^3 + (b^2 - c^2) + (c^2 - a^2)^3$

$= 3(a^2 - b^2)(b^2 - c^2)(c^2 - a^2)$

$= 3(a + b)(a - b)(b + c)(b - c)(c + a)(c - a)$

$= 3(a + b)(b + c)(c + a)(a - b)(b - c)(c - a)$

Ex. 29: Factorize: $(x - 2y)^3 + (2y - 3z)^3 + (3z - x)^3$

Soln: Let $a = x - 2y$, $b = 2y - 3x$ and $c = 3x - x$. Then,

$a + b + c = x - 2y + 2y - 3z + 3z - x = 0$

$\therefore a^3 + b^3 + c^3 = 3abc$

$\Rightarrow (x - 2y)^3 + (2y - 3z)^3 + (3z - x)^3 = 3(x - 2y)(2y - 3z)(3z - x)$

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Type IX: Factorization of Quadratic Trinomials

In this section, we shall learn to factorize quadratic trinomials of the form $ax^2 + bx + c$. The following stepwise procedure will be useful to factorize a quadratic trinomial $ax^2 + bx + c$.

Procedure

Step I: Obtain the quadratic trinomial $ax^2 + bx + c$.

Step II: Obtain a =coefficient of x^2, b = coefficient of x and c = constant term.

Step III: Find the product of the coefficient of x^2 and the constant term i.e. ac.

Step IV: Split up the coefficient of x i.e. b into two parts whose sum is b and product ac and write the middle term as the sum of two terms.

Step V: Factorize the expression obtained in step IV by grouping the terms. Factors so obtained will be the required factor of the given quadratic trinomial.

Following examples will illustrate the above procedure.

Ex. 30: Factorize:

(i) $x^2 + 5x + 6$ (ii) $x^2 - 5x + 6$ (iii) $x^2 - x - 6$

Soln: (i) Given expression is $x^2 + 5x + 6$.

We have, coefficient of $x^2 = 1$

Coefficient of x = 5 and constant term = 6.

We try to split up the coefficient of x i.e. 5 into two parts whose sum is 5 and product equal to the coefficient of x^2 and constant term i.e. $1 \times 6 = 6$.

Clearly, $2 + 3 = 5$ and $2 \times 3 = 6$.

So, we write the middle term 5x as 2x + 3x.

$$\therefore\ x^2 + 5x + 6 = x^2 + 2x + 3x + 6$$
$$= (x^2 + 2x) + (3x + 6)$$
$$= x(x + 2) + 3(x + 2)$$
$$= (x + 2)(x + 3)$$

(ii) Given expression is $x^2 - 5x + 6$.

We have, coefficient of $x^2 = 1$, coefficient of x = -5 and constant term = 6.

We now try to split the coefficient of x i.e. -5 into two parts whose sum is -5 and product equal to the product of the coefficient of x^2 and constant term i.e.

$1 \times 6 = 6$. Clearly, $(-2) + (-3) = -5$ and $-2 \times -3 = 6$.

So, we write the middle term -5x as -2x - 3x.

$$\therefore\ x^2 - 5x + 6 = x^2 - 2x - 3x + 6$$
$$= (x^2 - 2x) - (3x - 6)$$
$$= x(x - 2) - 3(x - 2) = (x - 2)(x - 3)$$

(iii) Given expression is $x^2 - x - 6$.

We have, coefficient of $x^2 = 1$, coefficient of x = -1, and constant term = -6.

We now split up the coefficient of x i.e. -1 as the sum of two numbers whose sum is -1 and product equal to the product of the coefficient of x^2 and constant term i.e.

$1 \times -6 = -6$. Clearly, $-3 + 2 = -1$ and $-3 \times 2 = -6$

So, we write the middle term -x as -3x + 2x.

$$\therefore\ x^2 - x + 6 = x^2 - 3x + 2x - 6$$
$$= (x^2 - 3x) + (2x - 6)$$
$$= x(x - 3) + 2(x - 3)$$
$$= (x - 3)(x + 2)$$

Ex. 31: Factorize:

(i) $2x^2 + 5x + 3$

(ii) $6x^2 + 5x - 6$

(iii) $-2x^2 - 3x + 2$

Soln:

(i) The given expression is $2x^2 + 5x + 3$

Here, coefficient of $x^2 = 2$, coefficient of x = 5, and constant term = 3.

We shall now split up the coefficient of the middle term i.e. 5 into two parts such that their sum is 5 and product equal to the product of coefficient of x^2 and constant term, i.e. $2 \times 3 = 6$. Clearly, $2 + 3 = 5$ and $2 \times 3 = 6$. So, we replace the middle term by 2x + 3x. Thus, we have

$$2x^2 + 5x + 3 = 2x^2 + 2x + 3x + 3$$
$$= (2x^2 + 2x) + (3x + 3)$$
$$= 2x(x + 1) + 3(x + 1)$$
$$= (x + 1)(2x + 3)$$

(ii) The given expression is $6x^2 + 5x - 6$

Here, coefficient of $x^2 = 6$. Coefficient of x = 5, constant term = -6.

We shall now split up the coefficient of x i.e. 5 into two parts such that their sum is equal to coefficient of x i.e. 5 and product equal to the product of coefficient of x^2 and constant term i.e. $6 \times -6 = -36$

Clearly, $9 + (-4) = 5$ and $9 \times -4 = -36$. So, we replace the middle term by 9x - 4x. Thus, we have

$$6x^2 + 5x - 6 = 6x^2 + 9x - 4x - 6$$
$$= 3x(2x + 3) - 2(2x + 3) = (2x + 3)(3x - 2)$$

(iii) The given expression is $-2x^2 - 3x + 2$.

Here, coefficient of $x^2 = -2$, coefficient of x = -3 and constant term = 2.

We shall now split up the coefficient of the middle term i.e. -3 into two parts such that their sum is -2 and the product is equal to the product of the coefficient of x^2 and constant term i.e. $-2 \times 2 = -4$.

Clearly, $-4 + 1 = -3$ and $-4 \times 1 = -4$

So, we write the middle term -3x as -4x + x. Thus, we have

$$-2x^2 - 3x + 2 = -2x^2 - 4x + x + 2$$
$$= -2x(x + 2) + 1(x + 2) = (x + 2)(-2x + 1)$$

Ex. 32: Factorize:

(i) $12x^2 - 23xy + 10y^2$

(ii) $12x^2 + 7xy - 10y^2$

(iii) $6x^2 + 35xy - 6y^2$

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Soln:

(i) The given expression is $12x^2 - 23xy + 10y^2$

Here, coefficient of $x^2 = 12$, coefficient of $x = -23y$, and constant term $= 10y^2$.

Now, we split up the coefficient of the middle term i.e. $-23y$ into two parts whose sum is $-23y$ and product equal to the product of the coefficient of x^2 and constant term

i.e. $12 \times 10y^2 = 120y^2$.

Clearly, $-15y - 8y = -23y$ and $-15y \times -8y = 120y^2$

So, we replace the middle term by $-15xy - 8xy$.

Thus, we have

$$12x^2 - 23xy + 10y^2 = 12x^2 - 15xy - 8xy + 10y^2$$
$$= 3x(4x - 5y) - 2y(4x - 5y)$$
$$= (4x - 5y)(3x - 2y)$$

(ii) The given expression is $12x^2 + 7xy - 10y^2$

Here coefficient of $x^2 = 12$, coefficient of $x = 7y$ and constant term $= -10y^2$.

We shall now split up the coefficient of the middle term i.e. $7y$ into two parts whose sum is $7y$ and product equal to the product of the coefficient of x^2 and constant term

i.e. $12 \times -10y^2 = -120y^2$.

Clearly, $15y - 8y = 7y$ and $15y \times (-8y) = -120y^2$.

So, we replace the middle term

$7xy$ by $15xy - 8xy$.

Thus, we have

$$12x^2 + 7xy - 10y^2 = 12x^2 + 15xy - 8xy - 10y^2$$
$$= 3x(4x + 5y) - 2y(4x + 5y)$$
$$= (4x + 5y)(3x - 2y)$$

(iii) The given expression is $6x^2 + 35xy - 6y^2$.

Here, coefficient of $x^2 = 6$, coefficient of $x = 35y$ and constant term $= -6y^2$.

We shall now split up the coefficient of x i.e. $35y$ into two parts whose sum is $35y$ and product equal to the product of coefficient of x^2 and constant term i.e.

$6 \times (-6y^2) = -36y^2$.

Clearly, $36y - y = 35y$ and $36y \times (-y) = -36y^2$.

So, we replace the middle term by $36xy - xy$.

Thus, we have

$$6x^2 + 35xy - 6y^2 = 6x^2 + 36xy - xy - 6y^2$$
$$= 6x(x + 6y) - y(x + 6y)$$
$$= (6x - y)(x + 6y)$$

Ex. 33: Factorize:

(i) $x^2 + 3\sqrt{2}x + 4$ (ii) $x^2 - 3\sqrt{3}x + 6$

Soln:

(i) The given expression is $x^2 + 3\sqrt{2}x + 4$.

Here, coefficient of $x^2 = 1$, coefficient of $x = 3\sqrt{2}$, and constant term $= 4$.

We shall now split up the coefficient of the middle term i.e. $3\sqrt{2}$ into two parts whose sum is $3\sqrt{2}$ and product equal to the product of the coefficient of x^2 and the constant term i.e. $1 \times 4 = 4$.

Clearly, $2\sqrt{2} + \sqrt{2} = 3\sqrt{2}$ and $2\sqrt{2} \times \sqrt{2} = 4$.

So, we replace the middle term $3\sqrt{2x}$ by $2\sqrt{2x} + \sqrt{2x}$. Thus, we have

$$x^2 + 3\sqrt{2}x + 4 = x^2 + 2\sqrt{2}x \times \sqrt{2}x + 4$$
$$= (x^2 + 2\sqrt{2}x) + (\sqrt{2}x + 4)$$
$$= x(x + 2\sqrt{2}) + \sqrt{2}(x + 2\sqrt{2})$$
$$= (x + 2\sqrt{2})(x + \sqrt{2})$$

(ii) The given expression is $x^2 - 3\sqrt{3}x + 6$.

Here, coefficient of $x^2 = 1$, coefficient of $x = -3\sqrt{3}$, and constant term $= 6$.

We shall now split up the coefficient of the middle term i.e. $-3\sqrt{3}$ into two parts whose sum is $-3\sqrt{3}$ and product equal to the coefficient of x^2 and constant term i.e. $1 \times 6 = 6$.

Clearly, $-2\sqrt{3} - \sqrt{3} = -3\sqrt{3}$ and

$$-2\sqrt{3} \times -\sqrt{3} = 6.$$

So, we replace the middle term

$$-3\sqrt{3}x \text{ by } -2\sqrt{3}x - \sqrt{3}x.$$

Thus, we have

$$x^2 - 3\sqrt{3}x + 6 = x^2 - 2\sqrt{3}x - \sqrt{3}x + 6$$
$$= x^2 - 2\sqrt{3}x - \sqrt{3}x + 2\sqrt{3} \times \sqrt{3}$$
$$= x(x - 2\sqrt{3}) - \sqrt{3}(x - 2\sqrt{3})$$
$$[\because 6 = 2\sqrt{3} \times \sqrt{3}]$$
$$= (x - 2\sqrt{3})(x - \sqrt{3})$$

Ex. 34: Factorize: $(2x + 3y)^2 - 5(2x + 3y) - 14$

Soln: The given expression is

$(2x + 3y)^2 - 5(2x + 3y) - 14$

Let $2x + 3y = a$. Then,

$$(2x + 3y)^2 - 5(2x + 3y) - 14 = a^2 - 5a - 14$$
$$= a^2 - 7a + 2a - 14$$
$$= a(a - 7) + 2(a - 7)$$
$$= (a - 7)(a + 2)$$
$$= (2x + 3y - 7)(2x + 3y + 2)$$

2. Algebraic Equations

In this section we will discuss two types of equations — *Linear Equations* and *Quadratic Equations*.

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(i) Linear Equations

There are two types of linear equations.

(a) Linear equations in one variable

(b) Linear equations in two variables

(a) Linear Equations in One Variable

In this section we shall study solution of linear equations containing one variable only and their applications to solve simple practical problems.

An *equation* is a statement of equality of two algebraic expressions which involve one or more unknown quantities called the "variables". As for example,

(i) $2x + 3 = 0$ (ii) $\frac{2}{3}y + 1 = \frac{y}{3}$

(iii) $2x + 5 = 3x + 9$ (iv) $x^2 + 4 = 2x$

(v) $t - \frac{1}{2}t = 3t + 1$ (vi) $t - \frac{1}{t} = 4$

(vii) $x^3 + 4x^2 = 3x + 2$

are equations.

In the above examples equations in (i), (ii) and (iii) involve only linear polynomials, whereas in (iv), (v), (vi) and (vii) quadratic and cubic polynomials are involved. So, we define a linear equation as follows:

An equation involving linear polynomials is called a linear equation. Consider the following example,

Ex.35: Which of the following equations are linear equations?

(i) $\frac{3}{2}x + 4 = 2x - 3$ **(ii) $y - 2 = 4y + 3$**

(iii) $y + \frac{1}{y} = 5$ **(iv) $(x - 1)^2 = x^2 + 5$**

(v) $(x - 1)(x - 2) = 4$

(vi) $(x - 1)(x - 3) = (x + 2)(x + 3)$

Soln: Clearly, polynomials involved in equations (i) and (ii) are linear. So, equations (i) and (ii) are linear equations.

(iii) We have, $y + \frac{1}{y} = 5 \Rightarrow \frac{y^2 + 1}{y} = 5$ [Taking LCM]

$\Rightarrow y^2 + 1 = 5y$ [By cross-multiplication]

Clearly, this equation involves a quadratic polynomial. So, it is not a linear equation.

(iv) We have, $(x - 1)^2 = x^2 + 5$

$\Rightarrow x^2 - 2x + 1 = x^2 + 5 \Rightarrow -2x + 1 = 5.$

Clearly, it is a linear equation.

(v) We have $(x - 1)(x - 2) = 4 \Rightarrow x^2 - 3x + 2 = 4.$

Clearly, it involves a quadratic polynomial. So, it is not a linear equation.

(vi) We have, $(x - 1)(x - 3) = (x + 2)(x + 3)$

$\Rightarrow x^2 - 4x + 3 = x^2 + 5x + 6 \Rightarrow -4x + 3 = 5x + 6$

Clearly, it is a linear equation.

Solution of a Linear Equation

A value of the variable, which when substituted for the variable in the equation makes the two sides of the equation equal, is called the solution of the equation.

Consider the equation $3x + 4 = -5$.

If we substitute the value -3 for x, we get

LHS $= 3 \times -3 + 4 = -9 + 4 = -5 =$ RHS

Therefore, $x = -3$ is a solution of the equation $3x + 4 = -5$.

If we substitute $x = 2$ in the given equation, we get

LHS $= 3 \times 2 + 4 = 6 + 4 = 10 \neq$ RHS

So, $x = 2$ is not a solution of the equation $3x + 4 = 5$.

Now consider the following examples that will further illustrate our points.

Worked Out Examples

Ex. 36: Verify that 4 is a solution of $5x - 2 = 18$.

Soln: If we substitute $x = 4$, we get

LHS $= 5 \times 4 - 2 = 18 =$ RHS

So, $x = 4$ is a solution of the given equation.

Ex. 37: Verify that -3 is a solution of the equation $3x + 5 = -4$

Soln: If we substitute $x = -3$ in the given equation, we get

LHS $= 3 \times -3 + 5 = -9 + 5 = -4 =$ RHS

So, $x = -3$ is a solution of the given equation.

Ex. 38: Verify that -2 is a solution of the equation $3x + 5 = 2x + 3$

Soln: If we substitute $x = -2$, we have

LHS $= 3 \times -2 + 5 = -6 + 5 = -1$

and, RHS $= 2 \times -2 + 3 = -4 + 3 = -1$

So, LHS = RHS.

Thus, $x = -2$ is a solution of the given equation.

Ex. 39: Examine whether $\frac{1}{2}$ is a solution of the equation $5x + \frac{3}{2} = 7x + \frac{1}{2}$.

Soln: If we substitute $x = \frac{1}{2}$, we get

LHS $= 5 \times \frac{1}{2} + \frac{3}{2} = \frac{5}{2} + \frac{3}{2} = \frac{5+3}{2} = \frac{8}{2} = 4$ and,

RHS $= 7 \times \frac{1}{2} + \frac{1}{2} = \frac{7}{2} + \frac{1}{2} = \frac{7+1}{2} = \frac{8}{2} = 4.$

So, LHS = RHS.

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Thus, $x = \frac{1}{2}$ is a solution of the given equation.

Ex. 40: Examine whether 3 is a solution of the equation $2x - 5 = 3x - 2$

Soln: If we substitute $x = 3$, we get

LHS $= 2 \times 3 - 5 = 6 - 5 = 1$

and RHS $= 3 \times 3 - 2 = 9 - 2 = 7$.

So, LHS $\neq$ RHS

Thus, $x = 3$ is not a solution of the given equation.

Note: Solving a linear equation means finding a value of the variable which satisfies the equation.

Rules for Solving an Equation

As discussed in the previous section that solving an equation means finding a value of the vriable which satisfies the equation. To solve an equation we use the following properties of equality:

(a) Same quantity can be added to both sides of an equation without changing the equality.

(b) Same quantity can be subtracted from both the sides of an equation without changing the equality.

(c) Both the sides of an equation can be multiplied by the same non-zero number without changing the equality.

(d) Both the sides of an equation may be divided by the same non-zero number without changing the equality.

The following examples illustrate the use of the above properties in solving linear equations.

Worked Out Examples

Ex. 41: Solve: $3x + 2 = 11$.

Soln: We have: $3x + 2 = 11$

$\Rightarrow 3x + 2 + (-2) = 11 + (-2)$

[Adding -2 to both sides]

$\Rightarrow 3x = 9$

$\Rightarrow \frac{1}{3}(3x) = \frac{1}{3}(9)$

[Dividing both sides by 3]

$\Rightarrow x = 3$.

Thus, $x = 2$ is the solution of the given equation.

Ex. 42: Solve: $5x + 9 = 2x + 21$

Soln: We have: $5x + 9 = 2x + 21$

$\Rightarrow 5x + 9 + (-9) = 2x + 21 + (-9)$

[Adding -9 to both sides]

$\Rightarrow 5x = 2x + 12$

$\Rightarrow 5x - 2x = 2x + 12 - 2x$

[Subtracting 2x from both sides]

$\Rightarrow 3x = 12 \Rightarrow x = 4$.

Thus, $x = 4$ is the solution of the given equation.

Ex. 43: Solve: $2x + 6 = x - 8$

Soln: We have: $2x + 6 = x - 8$

$\Rightarrow 2x + 6 - 6 = x - 8 - 6$

[Subtracting 6 from both sides]

$\Rightarrow 2x = x - 14$

$\Rightarrow 2x - x = x - 14 - x$

[Subtracting x from both sides]

$\Rightarrow x = -14$

Thus, $x = -14$ is the solution of the given equation.

Note: In Ex. 41 above, we have added -2 to both sides of the equation $3x + 2 = 11$ to obtain $3x = 9$. Now, instead of adding -2 to both sides, if we transpose 2 to the RHS after changing its sign, then also we get the same equation. Thus, whether we add or subtract a quantity on both the sides of an equation or we transpose the term on the other side after changing its sign the effect remains same. In other words, addition or subtraction by the same quantity on the two sides of an equation is equivalent to transposition of the quantity on the other sides after changing its sign.

Transposition Method of Solving Linear Equations

Following step-wise procedure will illustrate the "Transposition Method" of solving linear equations.

Step I: Obtain the linear equation.

Step II: Transpose the terms involving the variable to the left hand side and those terms not involving the variable to the right hand side. It must be remembered that while transposing a term from one side to the other side, the sign of the term is changed.

Step III: Simplify the two sides in their simplest forms to obtain the equation of the form $ax = b$.

Step IV: Solve the equation obtained in step III by dividing both sides by the coefficient of x.

The following examples will illustrate the above method.

Worked Out Examples

Ex. 44: Solve the following equations:

(i) $3x + 2 = 23$

(ii) $8x + \frac{21}{4} = 3x + 7$

(iii) $2x - (3x - 4) = 3x - 5$

(iv) $2x + \sqrt{2} = 3x - 4 - 3\sqrt{2}$

Soln: (i) The given equation is

$3x + 2 = 23$

$\Rightarrow 3x = 23 - 2$ [By transposition]

$\Rightarrow 3x = 21 \Rightarrow x = \frac{21}{3} = 7$

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Thus, 7 is the solution of the given equation.

(ii) The given equation is

$$8x + \frac{21}{4} = 3x + 7 \Rightarrow 8x - 3x = 7 - \frac{21}{4}$$

[By transposition]

$$\Rightarrow 5x = \frac{28-21}{4}$$ [On taking LCM on RHS]

$$\Rightarrow 5x = \frac{7}{4} \Rightarrow x = \frac{1}{5} \times \frac{7}{4}$$

[Dividing both sides by 5]

$$\Rightarrow x = \frac{7}{20}$$

Thus, 7/20 is the solution of the given equation.

(iii) The given equation is

$2x - (3x - 4) = 3x - 5$

$\Rightarrow 2x - 3x + 4 = 3x - 5$

[By simplifying the two sides]

$\Rightarrow -x + 4 = 3x - 5$

$\Rightarrow -x - 3x = -5 - 4$

[By transposition]

$\Rightarrow -4x = -9$

$\Rightarrow 4x = 9$ [Multiplying both sides by -1]

$$\Rightarrow x = \frac{9}{4}$$

Thus, 9/4 is the solution of the given equation.

(iv) The given equation is

$$2x + \sqrt{2} = 3x - 4 - 3\sqrt{2}$$

$$\Rightarrow 2x - 3x = -4 - 3\sqrt{2} - \sqrt{2}$$

[By transposition]

$$\Rightarrow -x = -4 - 4\sqrt{2}$$

[By simplifying both sides]

$$\Rightarrow -x = -4(1 + \sqrt{2})$$

$$\Rightarrow x = 4(1 + \sqrt{2})$$

[Multiplying both sides by -1]

Thus, $4(1 + \sqrt{2})$ is the solution of the given equation.

Ex. 45: Solve the following equations:

(i) $\frac{4x}{5} = \frac{7}{9}$ **(ii)** $7x : 63 = 1 : 9$ **(iii)** $\frac{1.5x}{3} = \frac{7}{2}$ **(iv)** $-\frac{3.4x}{3} = \frac{1.7}{9}$

Soln: (i) The given equation is $\frac{4x}{5} = \frac{7}{9}$

$\Rightarrow 9 \times 4x = 5 \times 7$

[By cross-multiplication $\frac{4x}{5} \times \frac{7}{9}$]

$$\Rightarrow 36x = 35 \Rightarrow x = \frac{35}{36}$$

Thus, $x = \frac{35}{36}$ is the solution of the given equation.

(ii) The given equation is

$7x : 63 = 1 : 9$

$$\Rightarrow \frac{7x}{63} = \frac{1}{9}$$

$\Rightarrow 9 \times 7x = 63 \times 1$

[By cross-multiplication $\frac{7x}{63} \times \frac{1}{9}$]

$$\Rightarrow 63x = 63 \Rightarrow x = \frac{63}{63} = 1$$

Thus, x = 1 is the solution of the given equation.

(iii) The given equation is

$$\frac{1.5x}{3} = \frac{7}{2} \Rightarrow 2 \times 1.5x = 3 \times 7$$

[By cross-multiplication]

$$\Rightarrow 3x = 21 \Rightarrow x = \frac{21}{3} = 7.$$

Thus, x = 7 is the solution of the given equation.

(iv) The given equation is

$$\frac{-3.4x}{3} = \frac{1.7}{9}$$

$\Rightarrow 9 \times (-3.4x) = 3 \times 1.7$

[By cross-multiplication]

$\Rightarrow -30.6x = 5.1$

$$\Rightarrow x = \frac{5.1}{-30.6} = -\frac{5.1}{30.6} = -\frac{1}{6}$$

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Thus, $x = -\frac{1}{6}$ is the solution of the given equation.

Ex. 46: Solve the following equations:

(i) $3 : 5x = 9 : 5, x \neq 0$

(ii) $\frac{5}{x} + 11 = \frac{2}{x} + 9$, where $x \neq 0$

(iii) $\frac{5}{x-5} = \frac{7}{x-7}$, where $x \neq 0, x \neq 7$

Soln: (i) The given equation is

$3 : 5x = 9 : 5$

$\Rightarrow \frac{3}{5x} = \frac{9}{5}$ [By cross multiplication]

$\Rightarrow 3 \times 5 = 5x \times 9$

$\Rightarrow 15 = 45x$

$\Rightarrow x = \frac{15}{45} = \frac{1}{3}$

Thus, $x = \frac{1}{3}$ is the solution of the given equation.

(ii) The given equation is

$\frac{5}{x} + 11 = \frac{2}{x} + 9 \Rightarrow \frac{5}{x} - \frac{2}{x} = 9 - 11$

$\Rightarrow \frac{5-2}{x} = 9 - 11 \Rightarrow \frac{3}{x} = -\frac{2}{1}$

$\Rightarrow 3 = -2x$ [By cross-multiplication]

$\Rightarrow -2x = 3 \Rightarrow x = \frac{3}{-2} = \frac{-3}{2}$

Thus, $x = -3/2$ is the solution of the given equation.

(iii) The given equation is

$\frac{5}{x-5} = \frac{7}{x-7}$

$\Rightarrow 5(x - 7) = 7(x - 5)$

[By cross-multiplication]

$\Rightarrow 5x - 35 = 7x - 35 \Rightarrow 5x - 7x = -35 + 35$

$\Rightarrow -2x = 0 \Rightarrow x = \frac{0}{-2} = 0$

Thus, $x = 0$ is the solution of the given equation.

Ex. 47: Solve the following equations:

(i) $\frac{5}{3x-2} - \frac{1}{8} = 0$, $x \neq \frac{2}{3}$

(ii) $\frac{x}{x-2} = 3, x \neq 2$

(iii) $\frac{2x+3}{x+7} = \frac{5}{8}$, $x \neq -7$

(iv) $\frac{3x-5}{7x-5} = \frac{1}{9}$, $x \neq \frac{5}{7}$

Soln: (i) The given equation is

$\frac{5}{3x-2} - \frac{1}{8} = 0 \Rightarrow \frac{5}{3x-2} = \frac{1}{8}$

$\Rightarrow 5 \times 8 = 1 \times (3x - 2)$

[By cross-multiplication]

$\Rightarrow 40 = 3x - 2 \Rightarrow 40 + 2 = 3x$

$\Rightarrow 3x = 40 + 2 \Rightarrow 3x = 42 \Rightarrow x = \frac{42}{3} = 14$

Thus, $x = 14$ is the solution of the given equation.

(ii) the given equation is

$\frac{x}{x-2} = 3 \Rightarrow \frac{x}{x-2} = \frac{3}{1}$

$\Rightarrow x \times 1 = 3 \times (x - 2)$

[By cross-multiplication]

$\Rightarrow x = 3x - 6 \Rightarrow x - 3x = -6 \Rightarrow -2x = -6$

$\Rightarrow 2x = 6$ [Multiplying both sides by -1]

$\Rightarrow x = \frac{6}{2} = 3$.

Thus, $x = 3$ is the solution of the given equation.

(iii) The given equation is

$\frac{2x+3}{x+7} = \frac{5}{8} \Rightarrow 8 \times (2x + 3) = 5 \times (x + 7)$

$\Rightarrow 16x + 24 = 5x + 35 \Rightarrow 16x - 5x = 35 - 24$

$\Rightarrow 11x = 11 \Rightarrow x = \frac{11}{11} = 1$

Thus, $x = 1$ is the solution of the given equation.

(iv) The given equation is

$\frac{3x-5}{7x-5} = \frac{1}{9}$

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$\Rightarrow 9 \times (3x - 5) = 1 \times (7x - 5)$

[By cross-multiplication]

$\Rightarrow 27x - 45 = 7x - 5 \Rightarrow 27x - 7x = 45 - 5$

$\Rightarrow 20x = 40 \Rightarrow x = \frac{40}{20} = 2$

Thus, x = 2 is the solution of the given equation.

Ex. 48: Solve the following equations:

(i) $15(x - 1) + 4(x + 3) = 2(7 + x)$

(ii) $8(x - 3) - (6 - 2x) = 2(x + 2) - 5(5 - x)$

(iii) $14x - (5x - 9) - [4 - 3x - (2x - 3)] = 30$

(iv) $25x - 19 - \{3 - (4x - 5)\} = 3x - (6x - 5)$

Soln: (i) The given equation is

$15(x - 1) + 4(x + 3) = 2(7 + x)$

$\Rightarrow (15x - 15) + (4x + 12) = 14 + 2x$

$\Rightarrow (15x + 4x) - 15 + 12 = 14 + 2x$

$\Rightarrow 19x - 3 = 14 + 2x$

$\Rightarrow 19x - 2x = 14 + 3$ [By transposition]

$\Rightarrow 17x = 17 \Rightarrow x = \frac{17}{17} = 1.$

Thus, x = 1 is the solution of the given equation.

(ii) The given equation is

$8(x - 3) - (6 - 2x) = 2(x + 2) - 5(5 - x)$

$\Rightarrow (8x - 24) - (6 - 2x) = (2x + 4) - (25 - 5x)$

$\Rightarrow 8x - 24 - 6 + 2x = 2x + 4 - 25 + 5x$

$\Rightarrow 8x + 2x - 24 - 6 = 2x + 5x + 4 - 25$

$\Rightarrow 10x - 30 = 7x - 21$

$\Rightarrow 10x - 7x = 30 - 21$ [By transposition]

$\Rightarrow 3x = 9 \Rightarrow x = \frac{9}{3} = 3$

Thus, x = 3 is the solution of the given equation.

(iii) The given equation is

$14x - (5x - 9) = \{4 - 3x - (2x - 3)\} = 30$

$\Rightarrow 14x - (5x - 9) - (4 - 3x - 2x + 3) = 30$

$\Rightarrow 14x - 5x + 9 - (7 - 5x) = 30$

$\Rightarrow 14x - 5x + 9 - 7 + 5x = 30$

$\Rightarrow 14x + 2 = 30 \Rightarrow 14x = 30 - 2$

$\Rightarrow 14x = 28 \Rightarrow x = \frac{28}{14} = 2$

Thus, x = 2 is the solution of the given equation.

(iv) The given equation is

$25x - 19 - \{3 - (4x - 5)\} = 3x - (6x - 5)$

$\Rightarrow 25x - 19 - (3 - 4x + 5) = 3x - (6x - 5)$

$\Rightarrow 25x - 19 - (8 - 4x) = 3x - (6x - 5)$

$\Rightarrow 25x - 19 - 8 + 4x = 3x - 6x + 5$

$\Rightarrow 29x - 27 = -3x + 5 \Rightarrow 29x + 3x = 27 + 5$

$\Rightarrow 32x = 32 \Rightarrow x = \frac{32}{32} = 1.$

Thus, x = 1 is the solution of the given equation.

Ex. 49: Solve each of the following equations:

(i) $\frac{x-1}{3} - \frac{4x+1}{4} = \frac{1}{12}$

(ii) $\frac{2x-3}{5} + \frac{x+3}{4} = \frac{4x+1}{7}$

(iii) $\frac{5x-4}{8} - \frac{x-3}{5} = \frac{x+6}{4}$

(iv) $\frac{x-2}{4} + \frac{1}{3} = x - \frac{2x-1}{3}$

Soln: (i) The given equation is

$\frac{x-1}{3} - \frac{4x+1}{4} = \frac{1}{12}$

$\Rightarrow \frac{4(x-1) - 3(4x+1)}{12} = \frac{1}{12}$

[Taking LCM on LHS]

$\Rightarrow \frac{4x - 4 - 12x - 3}{12} = \frac{1}{12}$

$\Rightarrow \frac{-8x - 7}{12} = \frac{1}{12}$

$\Rightarrow 12 \times (-8x - 7) = 12 \times 1$

[By cross-multiplication]

$\Rightarrow -8x - 7 = 1$ [Dividing both sides by 12]

$\Rightarrow -8x = 1 + 7$

$\Rightarrow -8x = 8 \Rightarrow x = \frac{8}{-8} = -1$

Thus, x = -1 is the solution of the given equation.

(ii) The given equation is

$\frac{2x-3}{5} + \frac{x+3}{4} = \frac{4x+1}{7}$

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$$\Rightarrow \frac{4(2x-3)+5(x+3)}{20} = \frac{4x+1}{7}$$

[Taking LCM on LHS]

$$\Rightarrow \frac{8x-12+5x+15}{20} = \frac{4x+1}{7}$$

$$\Rightarrow \frac{13x+3}{20} = \frac{4x+1}{7}$$

$\Rightarrow 7 \times (13x + 3) = 20 \times (4x + 1)$

[By cross-multiplication]

$\Rightarrow 91x+21=80x+20 \Rightarrow 91x - 80x = 20 - 21$

$$\Rightarrow 11x = -1 \Rightarrow x = -\frac{1}{11}$$

Thus, x = -1/11 is the solution of the given equation.

(iii) The given equation is

$$\frac{5x-4}{8} - \frac{x-3}{5} = \frac{x+6}{4}$$

The LCM of 8, 5 and 4 is 40. Multiplying both sides by 40 the given equation reduces to

$5(5x - 4) - 8(x - 3) = 10(x + 6)$

$\Rightarrow 25x - 20 - 8x + 24 = 10x + 60$

$\Rightarrow 17x + 4 = 10x + 60 \Rightarrow 17x - 10x = 60 - 4$

$$\Rightarrow 7x = 56 \Rightarrow x = \frac{56}{7} = 8.$$

Thus, x = 8 is the solution of the given equation.

(iv) The given equation is

$$\frac{x-2}{4} + \frac{1}{3} = \frac{x}{1} - \frac{2x-1}{3}$$

Multiplying both sides by LCM of 4, 3, 1 and 3 i.e., 12, the given equation reduces to

$3(x - 2) + 4 = 12x - 4 (2x - 1)$

$\Rightarrow 3x - 6 + 4 = 12x - 8x + 4$

$\Rightarrow 3x - 2 = 4x + 4 \Rightarrow 3x - 4x = 4 + 2$

$\Rightarrow -x = 6 \Rightarrow x = -6.$

Thus, x = -6 is the solution of the given equation.

Ex. 50: Solve the following equations:

(i) $\frac{2}{x} - \frac{5}{3x} = \frac{1}{3}; x \neq 0$

(ii) $\frac{6}{x} + 11 = \frac{2}{x} + 9; x \neq 0$

(iii) $\frac{2x+3}{5x} - \frac{7}{x} + 4 = \frac{2}{3x}; \quad x \neq 0$

(iv) $\frac{2}{x-5} = \frac{x+3}{x-5}; \quad x \neq 5$

Soln: (i) The given equation is

$$\frac{2}{x} - \frac{5}{3x} = \frac{1}{3}$$

$$\Rightarrow \frac{2 \times 3 - 5}{3x} = \frac{1}{3}$$ [Taking LCM on LHS]

$$\Rightarrow \frac{6-5}{3x} = \frac{1}{3} \Rightarrow \frac{1}{3x} = \frac{1}{3}$$

$\Rightarrow 3 = 3x$ [By cross-multiplication]

$$\Rightarrow 3x = 3 \Rightarrow x = \frac{3}{3} = 1$$

Thus, x = 1 is the solution of the given equation.

(ii) The given equation is

$$\frac{6}{x} + 11 = \frac{2}{x} + 9 \Rightarrow \frac{6}{x} - \frac{2}{x} = 9 - 11$$

$$\Rightarrow \frac{6-2}{x} = -2 \Rightarrow \frac{4}{x} = -2$$

$\Rightarrow 4 = -2x$ [By cross-multiplication]

$$\Rightarrow -2x = 4 \Rightarrow x = \frac{4}{-2} = -2$$

Thus, x = -2 is the solution of the given equation.

(iii) The given equation is

$$\frac{2x+3}{5x} - \frac{7}{x} + 4 = \frac{2}{3x}$$

$$\Rightarrow \frac{2x+3}{5x} - \frac{7}{x} + \frac{4}{1} = \frac{2}{3x}$$

$\Rightarrow 3(2x + 3) - 15 \times 7 + 4 \times 15x = 2 \times 5$

[Multiplying both sides by the LCM 5x, x and 3x i.e. 15x]

$\Rightarrow 6x + 9 - 105 + 60x = 10$

$\Rightarrow 66x = 10 - 9 + 105$

$\Rightarrow 66x = 106$

$$\Rightarrow x = \frac{106}{66} = \frac{53}{33}$$

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Thus, $x = \frac{53}{33}$ is the solution of the given equation.

(iv) The given equation is

$$\frac{2}{x-5} = \frac{x+3}{x-5}$$

$\Rightarrow 2 = x + 3$ [Multiplying both sides by x - 5]

$\Rightarrow x + 3 = 2 \Rightarrow x = 2 - 3 \Rightarrow x = -1.$

Thus, x = -1 is the solution of the given equation.

Ex. 51: Solve the following equations

(i) $(x + 1)(2x + 1) = (x + 3)(2x + 3) - 14$

(ii) $2(x + 1)(x + 3) + 8 = (2x + 1)(x + 5)$

(iii) $6(x^2 - 3x + 2) - 2(x^2 - 1) = 4(x + 1)(x + 2) - 24$

Soln: (i) The given equation is

$(x + 1)(2x + 1) = (x + 3)(2x + 3) - 14$

$\Rightarrow 2x^2 + 2x + x + 1 = 2x^2 + 6x + 3x + 9 - 14$

$\Rightarrow 2x^2 + 3x + 1 = 2x^2 + 9x - 5$

$\Rightarrow 2x^2 + 3x - 2x^2 - 9x = -5 - 1$

[By transposition]

$\Rightarrow -6x = -6$

$\Rightarrow 6x = 6$ [Multiplying both sides by -1]

$\Rightarrow x = \frac{6}{6} = 1$

(ii) The given equation is

$2(x + 1)(x + 3) + 8 = (2x + 1)(x + 5)$

$\Rightarrow 2(x^2 + x + 3x + 3) + 8 = 2x^2 + 10x + x + 5$

$\Rightarrow 2(x^2 + 4x + 3) + 8 = 2x^2 + 11x + 5$

$\Rightarrow 2x^2 + 8x + 14 = 2x^2 + 11x + 5$

$\Rightarrow 2x^2 + 8x - 2x^2 - 11x = 5 - 14$

$\Rightarrow -3x = -9 \Rightarrow x = \frac{-9}{-3} = 3.$

Thus, x = 3 is the solution of the given equation.

(iii) The given equation is

$6(x^2 - 3x + 2) - 2(x^2 - 1) = 4(x + 1)(x + 2) - 24$

$\Rightarrow 6(x^2 - 3x + 2) - 2(x^2 - 1) = 4(x^2 + 3x + 2) - 24$

$\Rightarrow 6x^2 - 18x + 12 - 2x^2 + 2 = 4x^2 + 12x + 8 - 24$

$\Rightarrow 4x^2 - 18x + 14 = 4x^2 + 12x - 16$

$\Rightarrow 4x^2 - 18x - 4x^2 - 12x = -16 - 14$

$\Rightarrow -30x = -30 \Rightarrow x = \frac{-30}{-30} = 1.$

Thus, x = 1 is the solution of the given equation.

Ex. 52: Solve the following equations:

(i) $\frac{3x+2}{x-1} = \frac{3x+4}{x+1}$; $x \neq -1, 1$

(ii) $\frac{5x-5}{4x+7} = \frac{5x-31}{4x-23}$; $x \neq \frac{7}{4}$; $x \neq \frac{23}{4}$

Soln: (i) The given equation is

$$\frac{3x+2}{x-1} = \frac{3x+4}{x+1}$$

$\Rightarrow (3x + 2)(x + 1) = (3x + 4)(x - 1)$

[By cross-multiplication]

$\Rightarrow 3x^2 + 3x + 2x + 2 = 3x^2 - 3x + 4x - 4$

$\Rightarrow 3x^2 + 5x + 2 = 3x^2 + x - 4$

$\Rightarrow 3x^2 + 5x - 3x^2 - x = -4 - 2$

[By transposition]

$\Rightarrow 4x = -6 \Rightarrow x = -\frac{6}{4} = -\frac{3}{2}$

Thus, $x = -\frac{3}{2}$ is the solution of the given equation.

(ii) The given equation is

$$\frac{5x-5}{4x+7} = \frac{5x-31}{4x-23}$$

$\Rightarrow (5x - 5)(4x - 23) = (4x + 7)(5x - 31)$

[By cross multiplication]

$\Rightarrow 20x^2 - 115x - 20x + 115$

$= 20x^2 - 124x + 35x - 127$

$\Rightarrow 20x^2 - 135x + 115 = 20x^2 - 89x - 217$

$\Rightarrow -46x = -332$

$\Rightarrow x = \frac{-332}{-46} = \frac{166}{23}.$

Thus, $x = \frac{166}{23}$ is the solution of the given equation.

Ex. 53: Solve the following equations:

(i) $\frac{x^2-9}{x+3} = 5$, $x \neq -3$

(ii) $\frac{x^2+5x+4}{x^2+3x+2} = \frac{3}{2}$, $x \neq -1, -2$

(iii) $\frac{x^2+5x+6}{x^2+4x+4} = \frac{3x+1}{5x-2}$, $x \neq -2, \frac{2}{5}$

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(iv) $\frac{3}{x-1}+\frac{4}{x-2}=\frac{7}{x-3}$, $x \neq 1,2,3$

Soln: (i) The given equation is

$$\frac{x^2-9}{x+3}=5 \Rightarrow \frac{(x-3)(x+3)}{x+3}=5$$

$\Rightarrow x-3=5$

$\left[\because x \neq -3 \therefore x+3 \neq 0 \text{ So, we can cancel } x+3 \text{ the } N^r \text{ and } D^r\right]$

$\Rightarrow x=5+3 \Rightarrow x=8.$

Thus, x = 8 is the solution of the given equation.

(ii) The given equation is

$$\frac{x^2+5x+4}{x^2+3x+2}=\frac{3}{2}$$

$$\Rightarrow \frac{x^2+4x+x+4}{x^2+2x+x+2}=\frac{3}{2}$$

$$\Rightarrow \frac{x(x+4)+(x+4)}{x(x+2)+(x+2)}=\frac{3}{2}$$

$$\Rightarrow \frac{(x+4)(x+1)}{(x+2)(x+1)}=\frac{3}{2}$$

$$\Rightarrow \frac{x+4}{x+2}=\frac{3}{2}$$

[Cancelling (x + 1) from N^r and D^r]

$\Rightarrow 2(x+4)=3(x+2)$

[By cross multiplication]

$\Rightarrow 2x+8=3x+6 \Rightarrow 2x-3x=6-8$

$\Rightarrow -x=-2$

$\Rightarrow x=2$ [Multiplying both sides by -1]

Thus, x = 2 is the solution of the given equation.

(iii) The given equation is

$$\frac{x^2+5x+6}{x^2+4x+4}=\frac{3x+1}{5x-2}$$

$$\Rightarrow \frac{x^2+3x+2x+6}{x^2+4x+4}=\frac{3x+1}{5x-2}$$

$$\Rightarrow \frac{x(x+3)+2(x+3)}{(x+2)^2}=\frac{3x+1}{5x-2}$$

$$\Rightarrow \frac{(x+3)(x+2)}{(x+2)^2}=\frac{3x+1}{5x-2}$$

$$\Rightarrow \frac{x+3}{x+2}=\frac{3x+1}{5x-2}$$

[Cancelling (x + 2) from N^r and D^r]

$\Rightarrow (x+3)(5x-2)=(x+2)(3x+1)$

[By cross-multiplication]

$\Rightarrow 5x^2+15x-2x-6=3x^2+6x+x+2$

$\Rightarrow 5x^2+13x-6=3x^2+7x+2$

$\Rightarrow 5x^2+13x-3x^2-7x=2+6$

$\Rightarrow 2x^2+6x=8$

$\Rightarrow x^2+3x=4$ [Dividing both sides by 2]

$\Rightarrow x^2+4x-x-4=0$

$\Rightarrow x(x+4)-1(x+4)=0$

$\Rightarrow (x+4)(x-1)=0$

$\Rightarrow x+4=0$ or $x-1=0$

$\Rightarrow x=-4$ or $x=1$.

(iv) The given equation is

$$\frac{3}{x-1}+\frac{4}{x-2}=\frac{7}{x-3}$$

$\Rightarrow 3(x-2)(x-3)+4(x-1)(x-3)$
$=7(x-1)(x-2)$

[Multiplying both sides by (x - 1) (x - 2) (x - 3)]

$\Rightarrow 3(x^2-3x-2x+6)+4(x^2-3x-x+3)$
$=7(x^2-x-2x+2)$

$\Rightarrow 3(x^2-5x+6)+4(x^2-4x+3)$
$=7(x^2-3x+2)$

$\Rightarrow 3x^2-15x+18+4x^2-16x+12$
$=7x^2-21x+14$

$\Rightarrow 7x^2-31x+30=7x^2-21x+14$

$\Rightarrow 7x^2-31x-7x^2+21x=14-30$

$$\Rightarrow -10x=-16 \Rightarrow x=\frac{-16}{-10}=\frac{16}{10}=\frac{8}{5}$$

Hence, $x=\frac{8}{5}$ is the solution of the given equation.

Application of Linear Equations

In this section we shall use linear equations to solve simple practical problems which frequently occur in our day to day life. Here, we will be discussing in short various types of questions in which linear equations are applied. For details, refer to the respective chapters discussed earlier in this book. Following types of questions are given

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below.

Type-I: Problems Based on Numbers
Type-II: Problems Based on Mensuration
Type-III: Problems Based on Ages
Type-IV: Problems Based on Time and Distance
Type-V: Problems Based on Time and Work
Type-VI: Miscellaneous Problems

The following procedure will be helpful to do so.

Step I: Read the problem carefully and denote the unknown quantity (or the quantity to be determined) by some variable x (say). If there are more than one unknown quantities, then denote one of them by x (say) and write the others in terms of x.

Step II: From the information given in the problem, formulate a linear equation for x.

Step III: Solve the linear equation obtained in step II to find x.

Following examples illustrate the above procedure.

Worked Out Examples

Type I: Problems Based on Numbers

Ex. 54: Find a number which, when added to its half, gives 33.

Soln: Let the required number be x. Then, half of the number x is $x/2$.

It is given that the number added to its half gives 33.

$$\therefore\ x+\frac{x}{2}=33$$

$$\Rightarrow \frac{2x+x}{2}=33 \Rightarrow \frac{3x}{2}=33$$

$$\Rightarrow 3x=66 \Rightarrow x=\frac{66}{3}=22.$$

So, the required number = 22.

Ex. 55: A number added to its two-third is equal to 35. Find the number.

Soln: Let the required number be x. Then two third of this number is $\frac{2}{3}x$. We are given that

Number + Two-third of the number = 35

$$\Rightarrow x+\frac{2}{3}x=35 \Rightarrow \frac{3x+2x}{3}=35$$

$$\Rightarrow \frac{5x}{3}=35 \Rightarrow 5x=3\times 35$$

$$\Rightarrow x=\frac{3\times 35}{5}=3\times 7=21.$$

So, the required number = 21.

Ex. 56: Divide 64 into two parts such that one part is three times the other.

Soln: Let one part be x. Then, the second part is 3x.

$\therefore\ x+3x=64$ (Given)

$$\Rightarrow 4x=64 \Rightarrow x=\frac{64}{4}=16$$

So, one part = 16, and second part = 3x = 3 × 16 = 48.

Thus, the two parts are 16 and 48.

Ex. 57: Find the two numbers whose sum and differences are 25 and 5, respectively.

Soln: Let, one number be x. Then,

second number = 25 - x

[$\because$ Sum of the numbers = 25]

Now, difference of the numbers = 5

$$\Rightarrow x-(25-x)=5 \Rightarrow x-25+x=5$$

$$\Rightarrow 2x=30 \Rightarrow x=\frac{30}{2}=15$$

$\therefore$ One number = 15

Second number = 25 - x = 25 - 15 = 10.

Ex. 58: Divide 300 into two parts so that half of one part may be less than the other by 48.

Soln: Let one part be x. Then, other part = 300 - x. It is given that half of one part is 48 less than the other.

$$\therefore\ \frac{1}{2}x=(300-x)-48$$

$$\Rightarrow \frac{x}{2}=300-x-48$$

$$\Rightarrow \frac{x}{2}+x=252 \Rightarrow \frac{3}{2}x=252$$

$$\Rightarrow 3x=252\times 2 \Rightarrow 3x=252\times 2$$

$$\Rightarrow 3x=504 \Rightarrow x=\frac{504}{3}=168$$

$\therefore$ One part = 168,

another part = 300 - x = 300 - 168 = 132.

Ex. 59: The sum of three consecutive natural numbers is 225. Find the numbers.

Soln: Let the three consecutive natural numbers be x, x + 1 and x + 2. Then,

sum = 225

$$\Rightarrow x+x+1+x+2=225 \Rightarrow 3x+3=225$$

$$\Rightarrow 3x=225-3 \Rightarrow 3x=222 \Rightarrow x=74$$

So, the three consecutive natural numbers are 74, 75 and 76.

Ex. 60: The sum of three consecutive odd natural num-

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bers is 153. Find the numbers.

Soln: Let three consecutive odd natural numbers be 2x - 1, 2x + 1 and 2x + 3. Then,

sum = 153

$\Rightarrow 2x - 1 + 2x + 1 + 2x + 3 = 153 \Rightarrow 6x + 3 = 153$

$\Rightarrow 6x = 153 - 3 = 150 \Rightarrow x = \frac{150}{6} = 25$

Thus, the consecutive odd natural numbers are: 2x - 1 = 2 × 25 - 1 = 49, 2x + 1 = 2 × 25 + 1 = 51 and 2x + 3 = 2 × 50 + 3 = 53.

Ex. 61: What number increased by 6% of itself gives 2544?

Soln: Let the required number be x. Then,

6% of $x = \frac{6x}{100}$

We are given that, $x + \frac{6x}{100} = 2544$

$\Rightarrow x + \frac{3}{50}x = 2544 \Rightarrow \frac{50x + 3x}{50} = 2544$

$\Rightarrow \frac{53x}{50} = 2544 \Rightarrow 53x = 2544 \times 50$

$\Rightarrow x = \frac{2544 \times 50}{53} \Rightarrow x = 48 \times 50 = 2400$

Thus, the required number is 2400.

Ex. 62: The sum of two numbers is 99. If one number is 20% more than the other, find the two numbers.

Soln: Let one number be x. Then the other number is (99 - x).

We are given that x is 20% more than (99 - x)

i.e. x = (99 - x) + 20% of (99 - x)

$\Rightarrow x = (99 - x) + \frac{20}{100} \times (99 - x)$

$\Rightarrow x = (99 - x) + \frac{1}{5}(99 - x)$

$\Rightarrow x = \frac{5(99 - x) + (99 - x)}{5}$

$\Rightarrow x = \frac{495 - 5x + 99 - x}{5}$

$\Rightarrow x = \frac{594 - 6x}{5} \Rightarrow 5x = 594 - 6x$

$\Rightarrow 5x + 6x = 594$

$\Rightarrow 11x = 594 \Rightarrow x = \frac{594}{11} = 54$

Thus, the two numbers are: x = 54 and 99 - x = 99 - 54 = 45.

Ex. 63: Divide 243 into three parts such that half of the first part, one third of the second part and one fourth of the third part, shall be equal.

Soln: Let First part = A , Second part = B and Third part = C

Then, $\frac{A}{2} = \frac{B}{3} = \frac{C}{4} = x(\text{say})$

$\Rightarrow A = 2x, B = 3x$ and $C = 4x$.

Now, A + B + C = 243

$\Rightarrow 2x + 3x + 4x = 243$

$\Rightarrow 9x = 243$

$\Rightarrow x = \frac{243}{9} = 27.$

$\therefore A = 2 \times 27 = 54, B = 3 \times 27 = 81$ and $C = 4 \times 27 = 108$.

Hence, three parts are 54, 81 and 108.

Ex. 64: A number consists of two digits. The digit in ten's place exceeds the digit in unit's place by 4. The sum of the digits is 1/7 of the number. Find the number.

Soln: Let the digit at units's place be x. Then, the digits at ten's place = x + 4

$\therefore$ Number = 10 (x + 4) + x = 10x + 40 + x = 11x + 40

Sum of the digits = $\frac{1}{7}$ (Number) (Given)

$\Rightarrow 2x + 4 = \frac{1}{7}(11x + 40)$

$\Rightarrow 14x + 28 = 11x + 40$

$\Rightarrow 14x - 11x = 40 - 28 \Rightarrow 3x = 12$

$\Rightarrow x = \frac{12}{3} = 4.$

$\therefore$ Number = 11x + 40 = 11 × 4 + 40 = 44 + 40 = 84

Ex. 65: A number consists of two digits. The digit at the ten's place is two times the digit at the unit's place. The number, formed by reversing the digits, is 27 less than the original number. Find the original number.

Soln: Let the digit at unit's place be x. Then,

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the digit at ten's place = 2x

$\therefore$ Number = $2x \times 10 + x = 20x + x = 21x$.

Number formed by reversing the digits = $10 \times x + 2x = 10x + 2x = 12x$.

Now, $12x = 21x - 27$ (Given)

$\Rightarrow 12x - 21x = -27 \Rightarrow -9x = -27 \Rightarrow x = \frac{-27}{-9} = 3$.

$\therefore$ Number = $21 \times 3 = 63$.

Ex. 66: A number consists of two digits of which the ten's digit exceeds the unit's digit by 6. The number itself is equal to ten times the sum of the digits. Find the number.

Soln: Let the unit's digit be x. Then,

ten's digit = $x + 6$.

$\therefore$ Number = $10(x + 6) + x = 10x + 60 + x$
$= 11x + 60$.

and, sum of the digits = $x + x + 6 = 2x + 6$

Now, Number = 10 (Sum of the digits)

$\Rightarrow 11x + 60 = 10(2x + 6) \Rightarrow 11x + 60 = 20x + 60$

$\Rightarrow 11x - 20x = 60 - 60 \Rightarrow -9x = 0 \Rightarrow x = 0$.

$\therefore$ Number = $11 \times 0 + 60 = 60$.

Ex. 67: The denominator of a fraction is 1 more than its numerator. If 1 is subtracted from both the numerator and denominator, the fraction becomes $\frac{1}{2}$. Find the fraction.

Soln: Let the numerator of the fraction be x. Then, Denominator = $x + 1$.

$\therefore$ Fraction = $\frac{x}{x+1}$.

New fraction obtained by subtracting 1 from both the numerator and denominator is

$$\frac{x-1}{x+1-1} = \frac{x-1}{x}$$

$\therefore \frac{x-1}{x} = \frac{1}{2}$ (Given)

$\Rightarrow 2(x - 1) = x \Rightarrow 2x - 2 = x$

$\Rightarrow 2x - x = 2 \Rightarrow x = 2$.

$\therefore$ Fraction = $\frac{x}{x+1} = \frac{2}{2+1} = \frac{2}{3}$

Ex. 68: The difference between two numbers is 642. When the greater is divided by the smaller, the quotient is 8 and the remainder is 19. Find the numbers.

Soln: Let one number be x. Then,

another number = $642 + x$

We know that

Dividend = Quotient × Divisor + Remainder.

Here,

Dividend = $642 + x$, Divisor = x, Quotient = 8 and Remainder = 19.

$\therefore 642 + x = 8x + 19$

$\Rightarrow x - 8x = 19 - 642 \Rightarrow -7x = -623$

$\Rightarrow x = \frac{-623}{-7} = 89$

Thus, the numbers are 89 and $642 + x$
$= 642 + 89 = 731$.

Type II. Problems Based on Mensuration

Ex. 69: The length of a rectangle is 5 cm more than the breadth. If the perimeter of the rectangle is 40 cm find the length and breadth of the rectangle.

Soln: Let the breadth of the rectangle be x cm. Then,

Length = $(x + 5)$ cm.

$\therefore$ Perimeter = 2(length + breadth)
$= 2(x + 5 + x) = 2(2x + 5)$
$= 2 \times 2x + 2 \times 5 = 4x + 10$.

Now, Perimeter = 40

$\Rightarrow 4x + 10 = 40 \Rightarrow 4x = 40 - 10$

$\Rightarrow 4x = 30 \Rightarrow x = 7.5$ cm.

$\therefore$ Length = $(x + 5) = (7.5 + 5)$ cm = 12.5 cm

and, Breadth = $x = 7.5$ cm

Ex. 70: The length of a rectangle is 4 cm more than the breadth, and the perimeter is 11 cm more than the breadth. Find the length and breadth of the rectangle.

Soln: Let the breadth of the rectangle be x cm. Then, length = $(x + 4)$ cm.

$\therefore$ Perimeter = 2(length + breadth)
$= 2(x + 4 + x) = 2(2x + 4) = 4x + 8$

Now, Perimeter = Breadth + 11 (Given)

$\Rightarrow 4x + 8 = x + 11 \Rightarrow 4x - x = 11 - 8$

$\Rightarrow 3x = 3 \Rightarrow x = \frac{3}{3} = 1$

$\therefore$ Length = $(x + 4)$ cm = $(1 + 4)$ cm = 5 cm.

Breadth = x cm = 1 cm.

Ex. 71: The perimeter of a rectangular field is 80 m. If the length of the field is decreased by 2 m and its breadth is increased by 2 m, the area is increased by 36 m². Find the length and breadth of the rectangular field.

Soln: Let the length and breadth of the rectangular field be x and y metres respectively. Then,

Perimeter = $2(x + y)$

$\Rightarrow 2(x + y) = 80$ [$\because$ Perimeter = 80 m (Given)]

$\Rightarrow x + y = 40$ [Dividing both sides by 2]

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$\Rightarrow y = 40 - x$ ..... (i)

Area = Length × Breadth = $xy = x(40 - x)$ [Using (i)]

We are given that the area of the field is increased by 36 m² if the length of the field is decreased by 2 m and breadth is increased by 2 m.

$\therefore$ New length = $(x - 2)$ m.

New breadth = $(y + 2)$ m = $(40 - x + 2)$ m [Using (i)]

= $(42 - x)$ m

New area = $(x - 2)(42 - x)$ m²

Now,

New Area = Old Area + 36

$\Rightarrow (x - 2)(42 - x) = x(40 - x) + 36$

$\Rightarrow 42x - x^2 - 84 + 2x = 40x - x^2 + 36$

$\Rightarrow 44x - x^2 - 84 = 40x - x^2 + 36$

$\Rightarrow 44x - x^2 - 40x + x^2 = 84 + 36$

$\Rightarrow 4x = 120 \Rightarrow x = 30.$

$\therefore$ Length = $x = 30$ m

And, Breadth = $40 - x = (40 - 30)$ m = 10 m.

Ex. 72: The perimeter of a rectangular field is 140 m. If the length of the field is increased by 2 m and its breadth decreased by 3 m, the area is decreased by 66 m². Find the length and breadth of the field.

Soln: Let, length = x metres, and breadth = y metres.

Then, perimeter = $2(x + y)$ metres.

$\therefore 2(x + y) = 140 \Rightarrow x + y = 70$

$\Rightarrow y = 70 - x$ .... (i)

Thus, length = x metres, and breadth = $(70 - x)$ metres

$\therefore$ Area = $[x(70 - x)]$ m²

New length = $(x + 2)$ metres

New breadth = $(70 - x - 3)$ metres = $(67 - x)$ metres

$\therefore$ New area = $(x + 2)(67 - x)$ m²

Now,

New area = Old area - 66

$\Rightarrow (x + 2)(67 - x) = x(70 - x) - 66$

$\Rightarrow 67x - x^2 + 134 - 2x = 70x - x^2 - 66$

$\Rightarrow 65x - x^2 + 134 = 70x - x^2 - 66$

$\Rightarrow 65x - x^2 - 70x + x^2 = -66 - 134$

$\Rightarrow -5x = -200 \Rightarrow 5x = 200$

$\Rightarrow x = \frac{200}{5} \Rightarrow x = 40.$

$\therefore$ Length of the field = 40 m

And, breadth of the field = $(70 - 40)$ m = 30 m.

Ex. 73: In an isosceles triangle, each of the two equal sides is 4 cm more than the base. If the perimeter of the triangle is 29 cm, find the sides of the triangle.

Soln: Let the base of the triangle be x cm. Then the remaining two equal sides are $(x + 4)$ cm.

$\therefore$ Perimeter = $(x + 4) + x + (x + 4) = (3x + 8)$ cm.

Now, perimeter = 29 cm

$\Rightarrow 3x + 8 = 29 \Rightarrow 3x = 29 - 8 \Rightarrow 3x = 21$

$\Rightarrow x = \frac{21}{3} = 7 \quad \therefore x + 4 = 7 + 4 = 11.$

Thus the sides of the triangle are 7 cm, 11 cm and 7 cm.

Type III: Problems Based on Ages

Ex. 74: A boy is now one-third as old as his father. Twelve years hence he will be half as old as his father. Determine the present age of the boy and that of his father.

Soln: Let the present age of the father be x years. Then,

present age of the son = $\frac{x}{3}$ years.

After twelve years:

Father's age = $(x + 12)$ years

Son's age = $\left(\frac{x}{3} + 12\right)$ years.

We are given that after 12 years the son will be half as old as his father.

$\therefore \frac{x}{3} + 12 = \frac{1}{2}(x + 12) \Rightarrow \frac{x + 36}{3} = \frac{x + 12}{2}$

$\Rightarrow 2(x + 36) = 3(x + 12) \Rightarrow 2x + 72 = 3x + 36$

$\Rightarrow 2x - 3x = 36 - 72 \Rightarrow -x = -36$

$\Rightarrow x = 36.$

$\therefore$ present age of father = 36 years,

and, present age of son = $\frac{x}{3} = \frac{36}{3} = 12$ years.

Ex. 75: Kareem is three times as old as his son. After ten years, the sum of their ages will be 76 years. Find their present ages.

Soln: Let the present age of Kareem's son be x years. Then,

Kareem's age = 3x years.

After 10 years;

Kareem's age = $(3x + 10)$ years.

Kareem's son age = $(x + 10)$ years

$\therefore (3x + 10) + (x + 10) = 76$

$\Rightarrow 4x + 20 = 76 \Rightarrow 4x = 76 - 20$

$\Rightarrow 4x = 56 \Rightarrow x = \frac{56}{4} = 14$

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$\therefore$ Kareem's present age $= 3x = 3 \times 14$
$= 42$ years

Kareem's son age $= x = 14$ years.

Ex. 76: The present ages of Ravish and Shikha are in the ratio 15 : 8. After ten years, their ages will be in the ratio 5 : 3. Find their present ages.

Soln: Let the present ages of Ravish and Shikha be 15x years and 8x years.

After ten years:

Ravish's age $= (8x + 10)$ years

$\therefore \dfrac{15x+10}{8x+10} = \dfrac{5}{3}$ [Given]

$\Rightarrow 3(15x + 10) = 5(8x + 10)$

$\Rightarrow 45x + 30 = 40x + 50$

$\Rightarrow 45x - 40x = 50 - 30 \Rightarrow 5x = 20$

$\Rightarrow x = \dfrac{20}{5} = 4$

$\therefore$ Present age of Ravish $= 15x = 15 \times 4$
$= 60$ years

Present age of Shikha $= 8x = 8 \times 4 = 32$ years.

Ex. 77: Ten years ago a father was six times as old as his daughter. After 10 years, he will be twice as old as his daughter. Determine their present ages.

Soln: Suppose ten years ago the age of the daughter was x years.

Then, ten years ago the age of her father was 6x years.

$\therefore$ Present age of father $= (6x + 10)$ years

Present age of daughter $= (x + 10)$ years

Ten years after:

Father's age $= (6x + 10 + 10)$ years
$= (6x + 20)$ years

Daughter's age $= (x + 10 + 10)$ years
$= (x + 20)$ years.

$\therefore 6x + 20 = 2(x + 20) \Rightarrow 6x + 20 = 2x + 40$

$\Rightarrow 6x - 2x = 40 - 20 \Rightarrow 4x = 20$

$\Rightarrow x = \dfrac{20}{4} = 5$

$\therefore$ Present age of father $= (6 \times 5 + 10)$
$= 40$ years.

Present age of daughter $= (5 + 10) = 15$ years.

Type IV: Problems Based on Time and Distance

Formulae to be used:

(i) Time $= \dfrac{\text{Distance}}{\text{Speed}}$

(ii) Distance = Speed × Time

Ex. 78: A man travels some distance at a speed of 12 km/hr and returns at 9 km/hr. If the total time taken by him is 2 hours 20 minutes, find the distance.

Soln: Let the distance be x km. Then,

Time taken at 12 km/hr $= \dfrac{x}{12}$ hr

$= \left(\dfrac{x}{12} \times 60\right)$ minutes $= 5x$ minutes.

Time taken at 9 km/hr $= \dfrac{x}{9}$ hr

$= \left(\dfrac{x}{9} \times 60\right)$ minutes $= \dfrac{20x}{3}$ minutes.

Since the total time taken is 2 hours 20 minutes i.e. 140 minutes.

$\therefore 5x + \dfrac{20}{3}x = 140 \Rightarrow \dfrac{15x + 20x}{3} = 140$

$\Rightarrow \dfrac{35x}{3} = 140 \Rightarrow 35x = 3 \times 140$

$\Rightarrow x = \dfrac{3 \times 140}{35} \Rightarrow x = 12$

Hence, the distance is 12 km.

Ex. 79: Walking at 4 km/hr, a person reaches his office 5 minutes late. If he walks at 5 km/hr, he will be 4 minutes too early. Find the distance of his office from his residence.

Soln: Let the required distance be x km.

Time taken at 4 km/hr $= \dfrac{x}{4}$ hr

$= \left(\dfrac{x}{4} \times 60\right)$ minutes $= 15x$ minutes

Time taken at 5 km/hr $= \dfrac{x}{5}$ hr

$= \left(\dfrac{x}{5} \times 60\right)$ minutes $= 12x$ minutes

Since the difference between the two times taken is $(5 + 4) = 9$ minutes.

$\therefore 15x - 12x = 9 \Rightarrow 3x = 9 \Rightarrow x = \dfrac{9}{3} = 3$.

Hence, the required distance is 3 km.

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Ex. 80: Two planes start from a city and fly in opposite directions, one averaging a speed of 40 km/hr greater than that of the other. If they are 3400 km apart after 5 hours, find their average speeds.

Soln: Let the average speed of one aeroplane be x km/hr. Then the average speed of the other is (x + 40) km/hr.

Distance travelled by the first aeroplane in 5 hours = 5x km.

Distance travelled by the second plane in 5 hours = 5 (x + 40) km

$\therefore$ 5x + 5(x + 40) = 3400

$\Rightarrow$ 5x + 5x + 200 = 3400 $\Rightarrow$ 10x + 200 = 3400

$\Rightarrow$ 10x = 3400 - 200 $\Rightarrow$ $x = \frac{3200}{10} = 320.$

For x = 320, we have x + 40 = 320 + 40 = 360.

Hence, the average speeds of the two aeroplanes are 320 km/hr and 360 km/hr.

Ex. 81: A steamer goes downstream from one port to another in 4 hours. It covers the same distance upstream in 5 hours. If the speed of the stream be 2 km/hr, find the distance between two ports.

Soln: Let the speed of the steamer be x km/hr.

Speed of the stream = 2 km/hr.

$\therefore$ Speed of the steamer downstream = (x + 2) km/hr

$\Rightarrow$ Distance covered during downstream in 4 hours = 4 (x + 2) km

Speed of the steamer upstream = (x - 2) km/hr

$\Rightarrow$ Distance covered during upstream in 5 hours = 5 (x - 2) km.

Now distance covered during upstream = Distance covered during downstream

$\Rightarrow$ 5 (x - 2) = 4(x + 2) $\Rightarrow$ 5x - 10 = 4x + 8

$\Rightarrow$ 5x - 4x = 10 + 8 $\Rightarrow$ x = 18.

Hence, the distance between two ports = 4 (x + 2) km = 4 (18 + 2) km = 80 km.

Ex. 82: The speed of a boat in still water is 8 km/hr. If it takes the same time in going 20 km downstream as it takes in going 12 km upstream. Find the speed of the water in the river.

Soln: Let the speed of the water in the river be x km/hr. Then,

Speed of the boat upstream = (8 - x) km/hr

Speed of the boat downstream = (8 + x) km/hr

Time taken in going 20 km downstream

$$= \frac{20}{8+x} \text{ hr}$$

Since the time taken in going upstream is same as in down stream.

$$\therefore \frac{20}{8+x} = \frac{12}{8-x}$$

$\Rightarrow$ 20(8 - x) = 12 (8 + x) $\Rightarrow$ 160 - 20x = 96 + 12x

$\Rightarrow$ -20x - 12x = 96 - 160 $\Rightarrow$ -32x = -64

$$\Rightarrow x = \frac{-64}{-32} = 2$$

Hence, the speed of the water in the river is 2 km/hr.

Ex. 83: A police car is ordered to chase a speeding car which is 5 km ahead and is travelling at an average speed of 80 km/hr. The police car is running at an average speed of 100 km/hr. How long does it take for the police car to overtake the speeding car?

Soln: Suppose the police car takes x hours to overtake the speeding car.

Distance travelled by the police car in x hours = (100 × x) km.

Distance travelled by the speeding car in x hours = (80 × x) km = 80x km.

Since the police car travels 5 km more than the speeding car in x hours.

$\therefore$ 100x = 80x + 5 $\Rightarrow$ 100x - 80x = 5

$$\Rightarrow 20x = 5 \Rightarrow x = \frac{5}{20} = \frac{1}{4} \text{ hr}$$

$$= \left(\frac{1}{4} \times 60\right) \text{ minutes} = 15 \text{ minutes.}$$

Thus, the police car takes 15 minutes to overtake the speeding car.

Type V Problems Based on Time and Work

Ex. 84: A and B together can do a piece of work in 4 days, but A alone can do it in 12 days. How many days would B alone take to do the same piece of work?

Soln: Suppose B alone takes x days to do the work.

Since, A alone can complete the work in 12 days.

Therefore, in one day A completes $\frac{1}{12}$ th part of the work.

Similarly, in one day B completes $\left(\frac{1}{x}\right)$ th part of work.

A and B together complete $\left(\frac{1}{4}\right)$ th part of the

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work in one day.

Now, A's one day's work + B's one day's work = (A + B)'s one day's work

$$\Rightarrow \frac{1}{12}+\frac{1}{x}=\frac{1}{4} \Rightarrow \frac{1}{x}=\frac{1}{4}-\frac{1}{12}$$

$$\Rightarrow \frac{1}{x}=\frac{3-1}{12} \Rightarrow \frac{1}{x}=\frac{1}{6} \Rightarrow 6=x.$$

Thus, B alone can do the work in 6 days.

Type VI Miscellaneous Problems

Ex. 85: A pharmacist needs to strengthen a 15% alcohol solution to one of 32% alcohol. How much pure alcohol should be added to 400 ml of the 15% solution?

Soln: Let x ml of the pure alcohol be added to 400 ml of 15% solution to convert it into 32% alcohol.

Now, total volume of the solution = (400 + x) ml.

Let us now find alcohol content in this solution. To find this we must find alcohol in 400 ml of the 15% solution.

Note that 15% solution means that in 100 ml, alcohol content is 15%.

Thus,

Alcohol in 100 ml = 15 ml

$$\Rightarrow \text{Alcohol in 400 ml} = \left(\frac{15}{100}\times 400\right)\text{ml} = 60 \text{ ml}$$

$\Rightarrow$ Total alcohol content in 400 ml solution = (60 + x) ml.

So, strength of this solution

$$= \left\{\left(\frac{60+x}{400+x}\right)\times 100\right\}\%$$

$$\Rightarrow \frac{60+x}{400+x}\times 100 = 32 \Rightarrow \frac{60+x}{400+x}=\frac{32}{100}$$

$$\Rightarrow \frac{60+x}{400+x}=\frac{8}{25} \Rightarrow 25(60+x)=8(400+x)$$

$\Rightarrow 1500 + 25x = 3200 + 8x$

$\Rightarrow 25 - 8x = 3200 - 1500 \Rightarrow 17x = 1700$

$$\Rightarrow x = \frac{1700}{17} = 100.$$

Thus, 100 ml of alcohol is added to 400 ml of the 15% solution to get 32% of the solution.

Ex. 86: There are benches in a class room. If 4 students sit on each bench, three benches are left vacant; and if 3 students sit on each bench, 3 students are left standing. What is the total number of students in the class?

Soln: Let there be x benches in the class.

When 4 students sit on each bench, three benches are left vacant.

Therefore, all students sit on (x - 3) benches.

So, number of students = 4(x - 3) ..... (i)

If 3 students sit on each bench, then 3 students are left standing.

So, number of students = 3x + 3 ..... (ii)

$\therefore$ 4 (x - 3) = 3x + 3 [From (i) and (ii)]

$\Rightarrow 4x - 12 = 3x + 3 \Rightarrow 4x - 3x = 12 + 3$

$\Rightarrow x = 15.$

Putting x = 15 in (i), we have

Number of students = 4 (15 - 3) = 48.

Ex. 87: A man invested Rs 35,000; a part of it at an annual rate of 12% and the rest at 14%. If he received a total annual interest of Rs 4460, how much did he invest at each rate?

Soln: Suppose he invests Rs x at an annual rate of 12% and the remaining Rs (35000 - x) at an annual rate of 14%. Then,

Interest earned on Rs x at the rate of 12%

$$= \frac{12x}{100}$$

Interest earned on Rs (35000 - x) at the rate of

$$14\% = \frac{14}{100}\times(35000-x)$$

$\therefore$ Total annual interest

$$= \frac{12x}{100}+\frac{14}{100}(35000-x)$$

$$= \frac{3x}{25}+\frac{7}{50}(35000-x)$$

$$= \frac{3x}{25}+\frac{7}{50}\times 35000-\frac{7}{50}x$$

$$= \frac{3x}{25}-\frac{7}{50}x+4900$$

It is given that total annual interest = Rs 4460.

$$\therefore \frac{3x}{25}-\frac{7x}{50}+4900 = 4460$$

$$\Rightarrow \frac{6x-7x}{50} = 4460 - 4900$$

$$\Rightarrow -\frac{x}{50} = -440 \Rightarrow x = 440\times 50 \Rightarrow x = 22000$$

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So, he invests Rs 22000 at the rate of 12% per annum and Rs 13000 at the rate of 14% per annum.

Ex. 88: A 700 g dry fruit pack costs Rs 72. It contains some cashew kernel, and the rest as dry grapes. If cashew kernel cost Rs 96 per kg and dry grapes cost Rs 112 per kg, what were the quantities of the two dry fruits separately?

Soln: Suppose the pack contains x gm of cashew kernel and (700 - x) gm of dry grapes.

Cost of cashew kernel at the rate of Rs 96 per kg =

$$\text{Rs}\left(\frac{96}{1000}x\right)$$

Cost of dry grapes at the rate of Rs 112 per kg

$$= \text{Rs}\left(\frac{112}{1000}\times(700-x)\right)$$

$$\therefore \frac{96x}{1000}+\frac{112}{1000}\times(700-x)=72$$

$$\Rightarrow \frac{96x+112\times(700-x)}{1000}=72$$

$$\Rightarrow 96x + 78400 - 112x = 72000$$

$$\Rightarrow 96x - 112x = 72000 - 78400$$

$$\Rightarrow -16x = -6400 \Rightarrow x = \frac{-6400}{-16} = 400$$

$\therefore$ Quantity of cashew kernel = 400 gm

Quantity of dry grapes = (700 - 400) gm

= 300 gm.

Ex. 89: By selling a car for Rs 72000, a person made a profit of 20%. What was the cost price of the car?

Soln: Let the cost price of the car be Rs x. Then,

$$\text{profit at the rate of 20\%} = \text{Rs}\left(\frac{20x}{100}\right) = \text{Rs}\left(\frac{x}{5}\right)$$

Now, Cost Price + Profit = Selling Price

$$\Rightarrow x+\frac{x}{5}=7200$$

$$\Rightarrow \frac{5x+x}{5}=7200$$

$$\Rightarrow \frac{6x}{5}=7200$$

$$\Rightarrow x = 7200\times\frac{5}{6} = 60{,}000.$$

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