Oscillatory Patterns Arising Out of Stop-And-Go Traffic

Joseph C. McGuire, Keith Rhodewalt, and Jorge Ruiz Gonzalez
Sonoma State University, Rohnert Park, CA 94928

Introduction

If you've ever driven in California, then you've experienced stop-and-go traffic. This frequently experienced phenomenon is characterized by traffic patterns that abruptly change from free-flow to nearly stopped vehicles. The goal of our research was to understand this problem, primarily by modeling a particular road system and the stop-and-go traffic that arises out of it. The kind of system we analyzed in our research was one in which a strip of road diverges into two parallel routes and then comes back together at a merge.

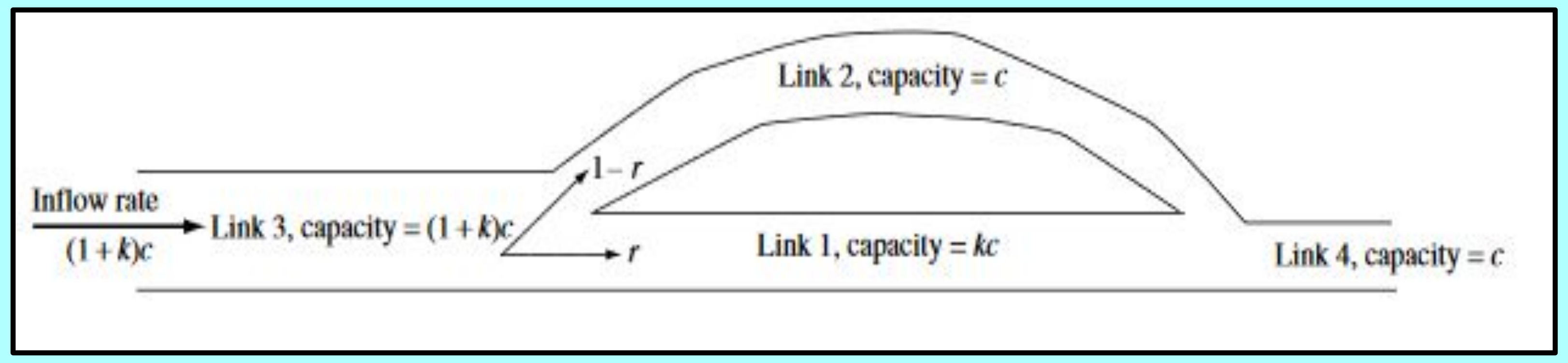

Figure 1: A homogenous road network w/Merge & Diverge

Background

The primary model of traffic flow that we use to investigate stop-and-go oscillations is the Lighthill, Whitham, and Richards (LWR model) [3].

$$\rho_t + f(\rho)_x = 0,$$

where f denotes traffic flow and ρ denotes traffic density.

This model is also referred to as the hydrodynamic model because it describes the flow of traffic as being similar to the flow of water through a channel.

The model can be formulated as an optimization problem on a discretized roadway[2]:

$$Q = \min\{C, S, D\},$$

where Q is the flow through the cell, C is the cell's flow capacity, S is the supply of the cell, and D is the cell's demand. Merges and Diverges are dictated by:

$$a_{i4} = \frac{D_i}{D_1 + D_2}$$

$$v_{3j} = r_{3j} \cdot \min\{D_3, \frac{S_1}{r}, \frac{S_2}{1-r}\}$$

$$v_{i4} = a_{i4} \cdot \min\{D_1 + D_2, S_4\}$$

a_{i4} is the merge ratio from link i to link 4, r_{3j} is the diverge ratio from link 3 to link j, and v_{ij} is flow from link i to link j.

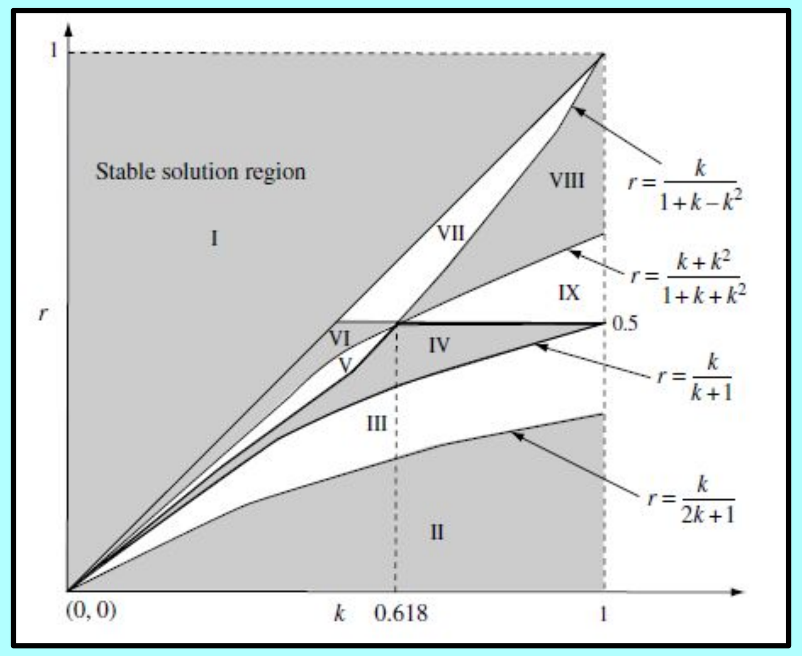

Figure 2: Relationship between diverge ratio (r) and capacity ratio (k) [1]. Each subspace represents a specific traffic pattern.

Methods and Results

Initially, we developed a spreadsheet implementation of the road network which kept track of flow and density over small segments of the road, at six-second intervals.

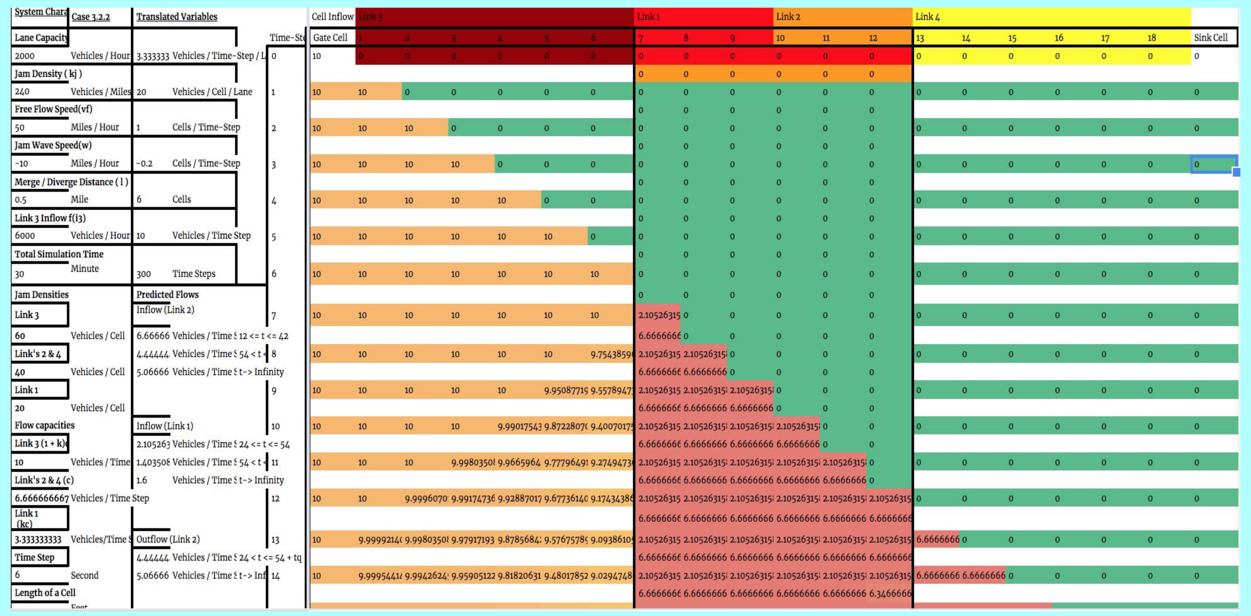

Figure 3: A portion of our spreadsheet

```
Density3[t_, x_] :=
 (Piecewise[
   {{DensityTable3[[t, x]] = Link3InflowCell, x == 1},
    (*The density function for the First Cell (feeder cell)*)
    {DensityTable3[[t, x]] = DensityTable3[[t - 1, x]] + FlowTable3[[t - 1, x]] - FlowTable1[[t - 1, 1]] - FlowTable2[[t - 1, 1]], x == LengthLink3}},
   (*The density function for the Last Cell (diverge*)
   DensityTable3[[t, x]] = DensityTable3[[t - 1, x]] + FlowTable3[[t - 1, x]] - FlowTable3[[t - 1, x + 1]]])
(*The density function for every cell in between first and last*)
```

```
Flow3[t_, x_] :=
 (Piecewise[
   {{FlowTable3[[t, x]] = Link3InflowCell, x == 1}},
   (*The flow function for the First Cell (feeder cell)*)
   FlowTable3[[t, x]] = Min[DensityTable3[[t, x - 1]], FlowCapLink3, Abs[wCell] (JamDensityLink3 - DensityTable3[[t, x]])]])
(*The flow function for every other cell*)
(*NOTE: in Link 3, the flows are pretty much all based on Daganzo.*)
```

Figure 4: A sample of our Mathematica code

We converted this model into a Mathematica program, which can run a similar simulation over smaller time intervals to provide output that resembles the continuous oscillations seen in reality.

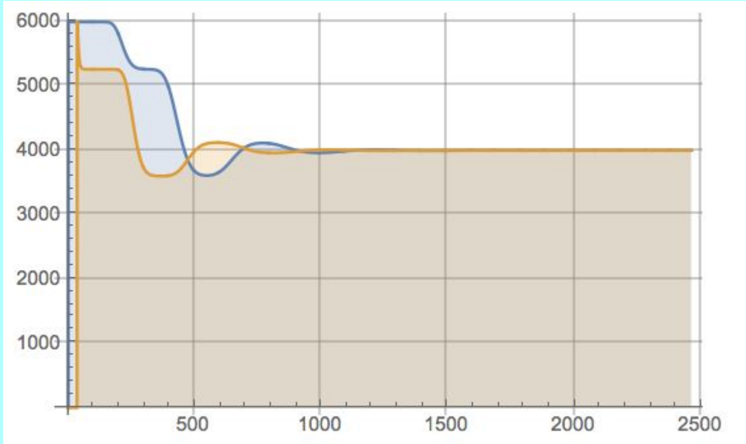

Figure 5: Inflow and outflow of Link 3 with a diverge ratio of r=0.24

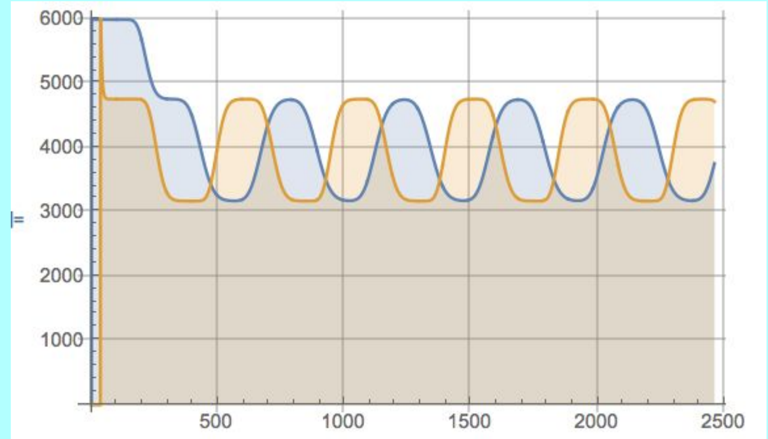

Figure 6: Inflow and outflow of Link 3 with a diverge ratio of r=0.42

Discussion and Future Research

Though having a diverge ratio similar to the capacity ratio can help alleviate traffic, our results show that once the rate of vehicles entering the system severely exceeds the rate at which they can leave, there is little that can be done to prevent a traffic jam situation.

Now that we are satisfied that our model can replicate the results of Nie/Zhang [1], we hope to apply it to other traffic patterns, by:

- changing the capacities
- implementing a time-variable diverge ratio
- artificially limiting the inflow rate (analogous to a metered onramp)
- altering the road configuration.

We would like to find a comparable freeway system in real-life so that we may compare our results to empirical data.

Lastly, we could use a "microscopic" model, which tracks each vehicle individually, as a comparison against the "macroscopic" LWR model.

References

[1] Nie (Marco), Yu, and H. Michael Zhang. "Oscillatory Traffic Flow Patterns Induced by Queue Spillback in a Simple Road Network." *Transportation Science*, vol. 42, no. 2, 2008, pp. 236–248., doi:10.1287/trsc.1070.0229.

[2] Daganzo, Carlos F. "The Cell Transmission Model. Part I: A Simple Dynamic Representation of Highway Traffic.." *California Path Program*, July 1993, ideas.repec.org/p/cdl/itsrrp/qt0b6612tk.html.

[3] Lighthill, M. J., and G. B. Whitham. "On Kinematic Waves ." *Proceedings of the Royal Society*, 10 May 1955, royalsocietypublishing.org/doi/abs/10.1098/rspa.1955.0088.

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