Patrol Strategies for Illegal Logging in the Brazilian Rainforest

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Introduction

We model illegal logging events as the result of a game between loggers and patrols. Patrols set a patrol strategy first. Then loggers maximize their profits P at each spot based on the patrol strategy and choose spots to cut down trees according to the optimal profit.

Different patrol strategies can be compared in our model by comparing some metrics based on the corresponding optimal profit to loggers. Besides, we can also get the optimal logging strategies, namely, the optimal moving-in path, moving-out path and logging time from our model.

Logging Model Overview

Given a spot, the logging time and paths, the profit

$$\tilde{P}(x, t, X_{\text{in}}, X_{\text{out}}) = \Phi B - C,$$

where B is the benefit, C is the cost and Φ is the probability of not being caught.

Benefit:

$$B = B(x)\frac{t}{T}.$$

Cost:

$$C = \alpha(\tau_{\text{in}} + \tau_{\text{out}}).$$

Probability of not being caught:

$$\Phi = e^{-\psi(x)t} e^{-\int_0^{\tau_{\text{out}}} \psi(X_{\text{out}}(s))ds}$$

Notations in formulas above:

- $B(x)$: Initial benefit at position x
- T: Maximal logging time
- t: Actual logging time
- ψ: Patrol density function
- $\mathbf{X}_{\text{out}}(s), \mathbf{X}_{\text{in}}(s)$: Time parameterizations of the moving-in and moving-out paths
- τ_{out}, τ_{in}: Travel times along the moving-in path $\mathbf{X}_{\text{out}}(s)$ and the moving-out path $\mathbf{X}_{\text{in}}(s)$
- α: Constant

Optimal profit function P:

$$P(x) = \max_{t, X_{\text{in}}, X_{\text{out}}} \tilde{P}(x, t, X_{\text{in}}, X_{\text{out}})$$

Numerical Solver

We use level-set method to solve two optimal control problems over the moving-in path X_{in} and moving-out path X_{out} by evolving corresponding two Hamilton-Jacobi equations respectively [1]. For the optimization problem over t, we choose finite values of $t \in [0, t_{\max}]$ and use the maximum over these finite values of t as an approximation of the maximum over t.

Experiments and results

A **patrol strategy** is the way to arrange the patrol density function $\psi(x)$.

Patrol budget E: A constraint on the patrol density function ψ.

$$\int_\Omega \psi(x)(1 + \mu d(x))^2 dx \le E$$

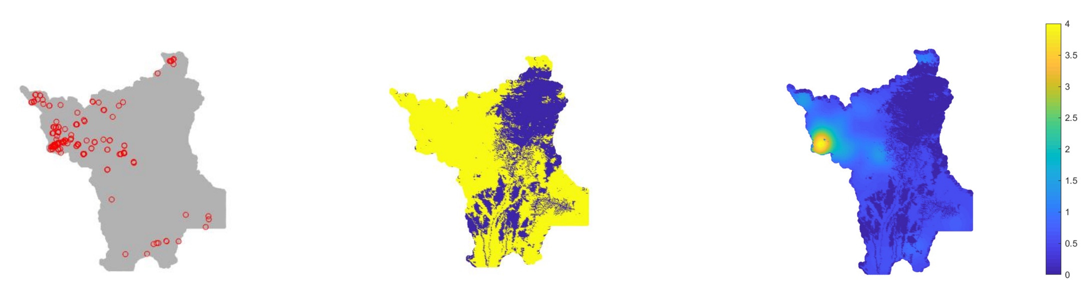
where μ is a constant and $d(x)$ is the distance function to major highways.

Metrics to evaluate patrol efficiency: P is the corresponding profit function to the patrol ψ.

- Pristine area ratio $PR = \frac{\int_\Omega \mathbb{1}_{\{P(x)\le 0\}}dx}{\int_\Omega 1 dx}$.
- Pristine benefit ratio $PB = \frac{\int_\Omega B(x)\mathbb{1}_{\{P(x)\le 0\}}dx}{\int_\Omega B(x)dx}$
- Weighted profit $WP = \frac{\int_\Omega P_+(x)^2 dx}{\int_\Omega P_+(x)dx}$.

Better patrol: higher PR and PB, lower WP.

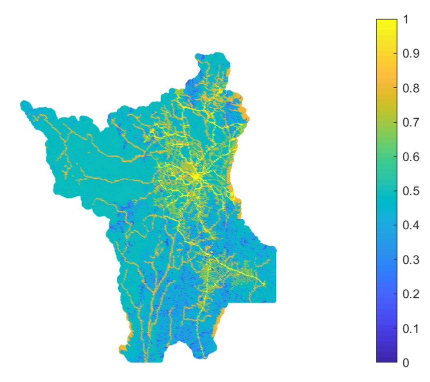
Benefit function $B(x)$: Generated from real events and indicator function of trees with kernel density estimation (KDE).



(a) Real events (b) Indicator (c) Benefit

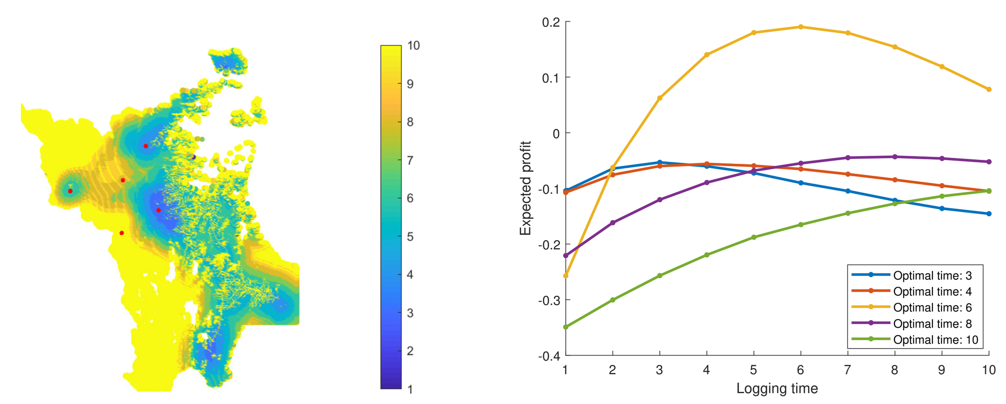
Generate benefit from real events.

Velocity field: Higher on roads or rivers, slower otherwise, modified by elevation. Velocity will influence the choice of optimal path.



Velocity field

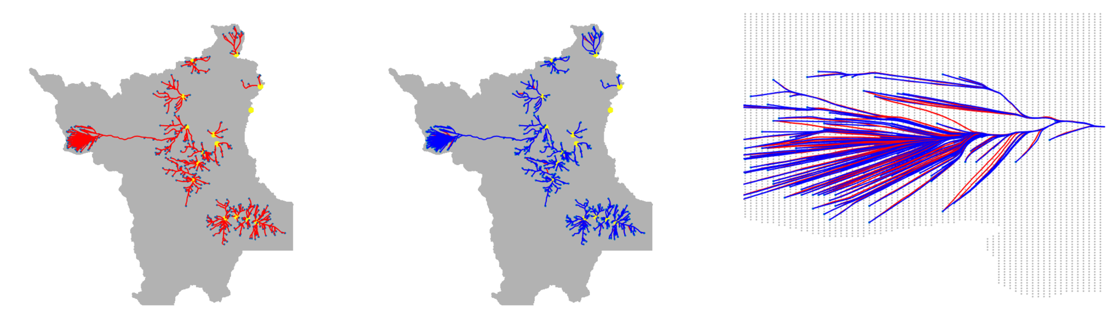
Optimal logging time: Patrol will influence the optimal logging time.



(a) Optimal time map (b) Profit changes with logging time at 5 spots

Optimal logging time varies with spots.

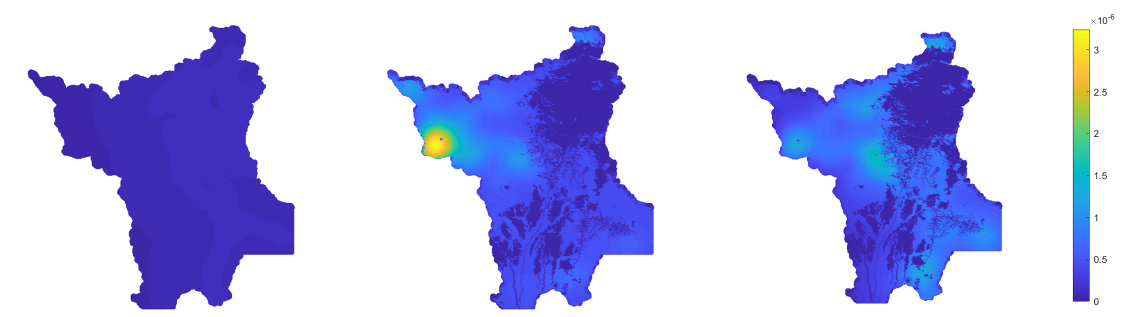
Optimal path We sample 500 points according to benefit function in our region and draw the optimal paths for logging. (Points cluster at high-benefit spots.)



(a) Moving-in (b) Moving-out (c) Zooming

Optimal paths.

Comparison among patrol strategies



(a) Distance only, $\psi(x)=C/(1+\mu d(x))$ (b) Benefit only, $\psi(x)=CB(x)$ (c) Both, $\psi(x)=CB(x)/(1+\mu d(x))^{11}$

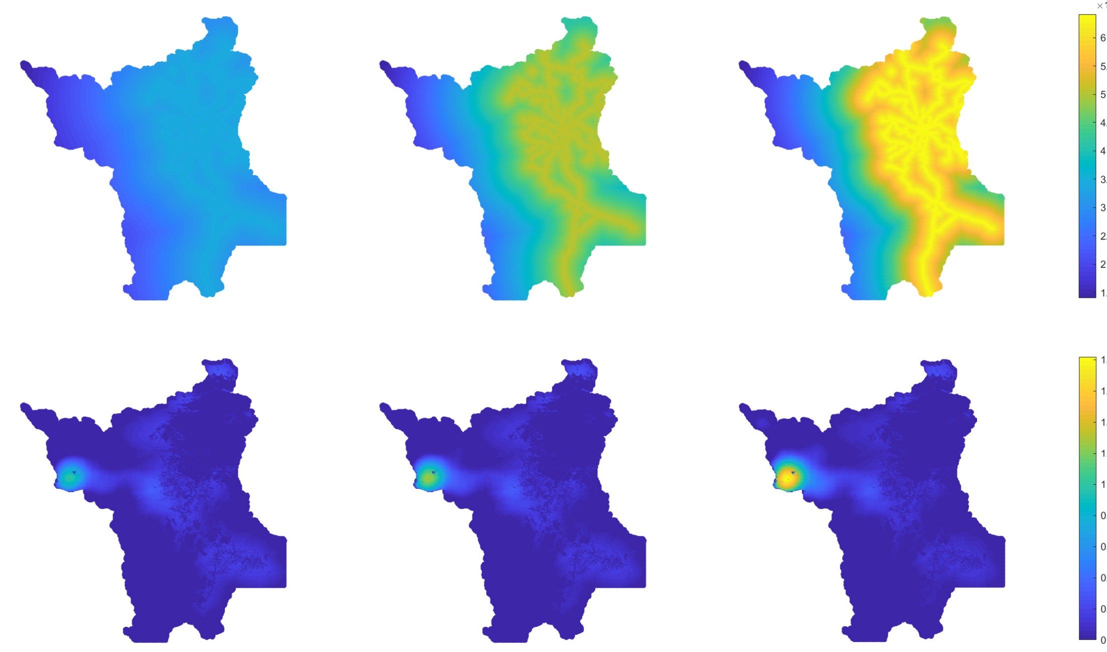
Different patrol strategies

(a) Distance (b) Benefit (c) Both

Profit map of different patrol strategies.

	WP	PR	PB
Distance only	0.2793	0.5674	0.3485
Benefit only	0.1696	0.6820	0.5105
Both	0.0661	0.7737	0.7883

Distributed patrol is better. Let patrol density $\psi(x) = C/(1 + \mu d(x))^r$



(a) $r = 1$ (b) $r = 3$ (c) $r = 7$

Different patrol strategies (first row) and corresponding profit function (second row) based on r.

The more distributed patrol, the better performance. Verify this by the table below:

	WP	PR	PB
$r = 1$	0.2793	0.5674	0.3485
$r = 3$	0.3813	0.5630	0.3302
$r = 7$	0.6192	0.5237	0.2459

References

[1] Arnold D J, Fernandez D, Jia R, et al.,*Modeling Environmental Crime in Protected Areas Using the Level Set Method.* SIAM Journal on Applied Mathematics, 2019

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