

Introduction

If you’ve ever driven in California, then you’ve experienced stop-and-go traffic. This frequently experienced phenomenon is characterized by traffic patterns that abruptly change from free-flow to nearly stopped vehicles. The goal of our research was to understand this problem, primarily by modeling a particular road system and the stop-and-go traffic that arises out of it. The kind of system we analyzed in our research was one in which a strip of road diverges into two parallel routes and then comes back together at a merge.

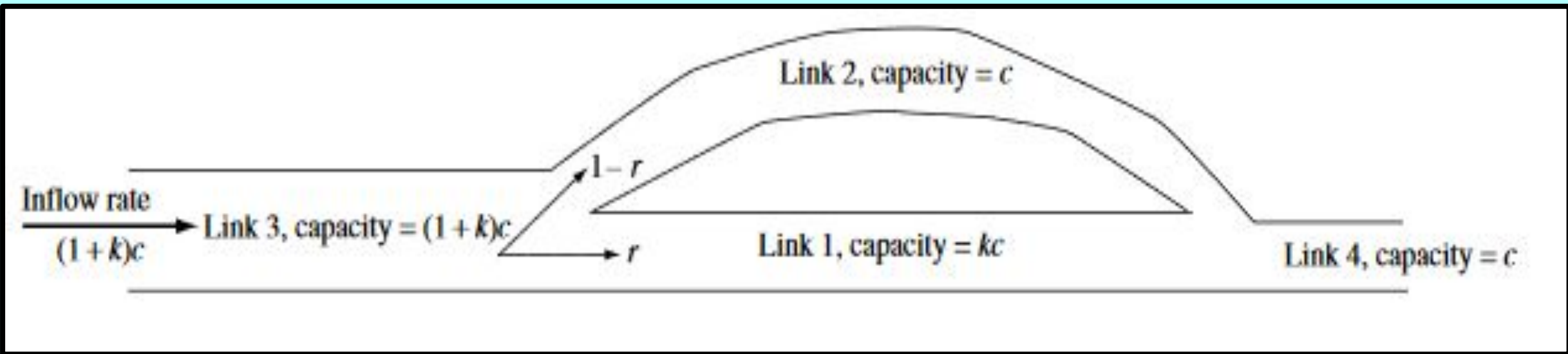


Figure 1: A homogenous road network w/Merge & Diverge

Background

The primary model of traffic flow that we use to investigate stop-and-go oscillations is the Lighthill, Whitham, and Richards (LWR model) [3].

$$\rho_t + f(\rho)_x = 0,$$

where f denotes traffic flow and ρ denotes traffic density.

This model is also referred to as the hydrodynamic model because it describes the flow of traffic as being similar to the flow of water through a channel.

The model can be formulated as an optimization problem on a discretized roadway[2]:

$$Q = \min\{C, S, D\},$$

where Q is the flow through the cell, C is the cell’s flow capacity, S is the supply of the cell, and D is the cell’s demand. Merges and Diverges are dictated by:

$$a_{i4} = \frac{D_i}{D_1 + D_2}$$

$$v_{3j} = r_{3j} \cdot \min\{D_3, \frac{S_1}{r}, \frac{S_2}{1-r}\}$$

$$v_{i4} = a_{i4} \cdot \min\{D_1 + D_2, S_4\}$$

a_{i4} is the merge ratio from link i to link 4,
 r_{3j} is the diverge ratio from link 3 to link j ,
and v_{ij} is flow from link i to link j .

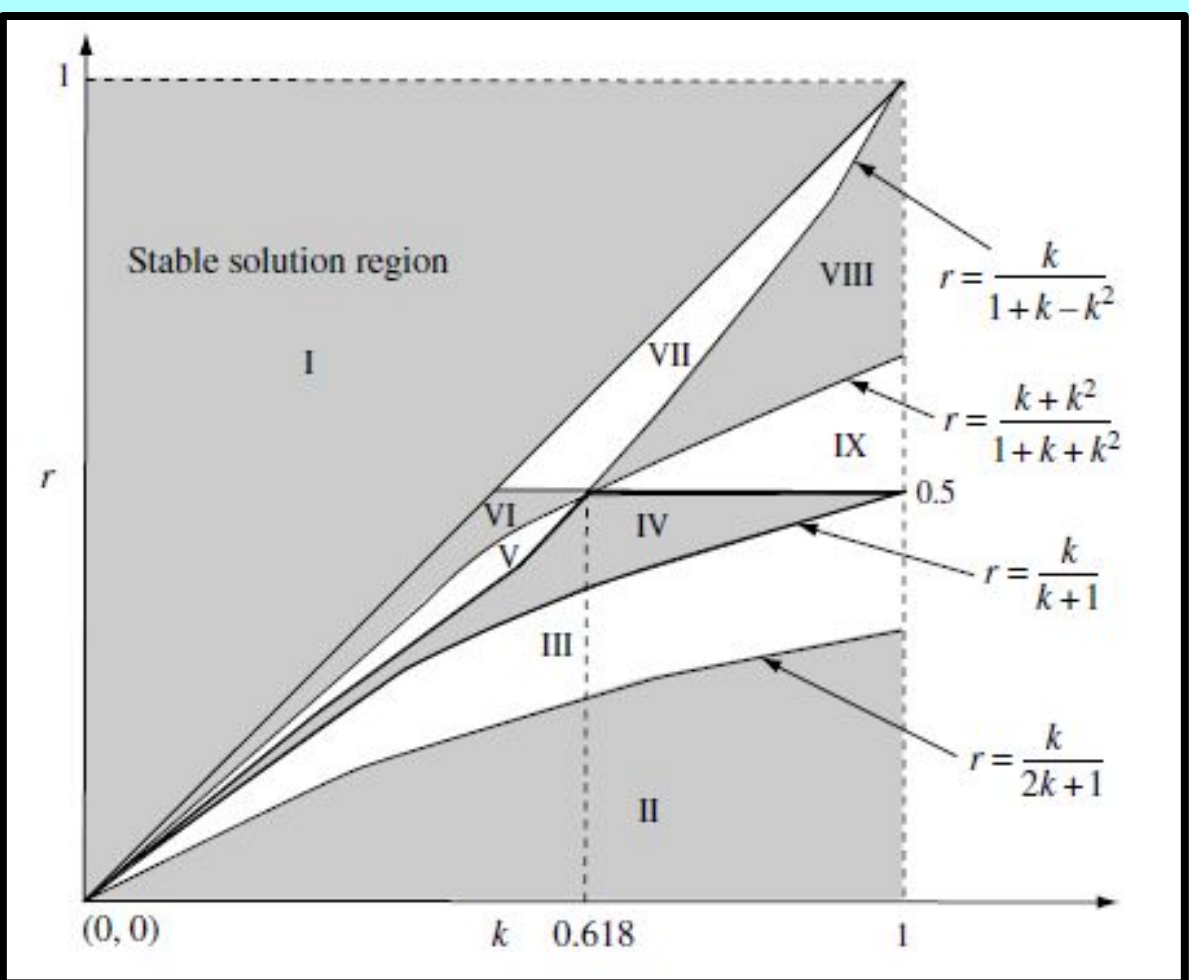


Figure 2: Relationship between diverge ratio (r) and capacity ratio (k) [1]. Each subspace represents a specific traffic pattern.

Methods and Results

Initially, we developed a spreadsheet implementation of the road network which kept track of flow and density over small segments of the road, at six-second intervals.

System Output	Time Step	Variable	Value	Link 1	Link 2	Link 3	Link 4
Link Capacity	0	Link 1 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 2 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 3 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 4 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 5 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 6 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 7 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 8 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 9 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 10 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 11 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 12 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 13 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 14 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 15 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 16 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 17 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 18 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 19 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 20 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 21 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 22 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 23 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 24 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 25 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 26 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 27 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 28 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 29 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 30 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 31 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 32 Capacity	1000	1000	1000	1000	1000
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Link Capacity	0	Link 36 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 37 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 38 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 39 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 40 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 41 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 42 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 43 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 44 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 45 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 46 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 47 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 48 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 49 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 50 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 51 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 52 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 53 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 54 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 55 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 56 Capacity	1000	1000	1000	1000	1000
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Link Capacity	0	Link 58 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 59 Capacity	1000	1000	1000	1000	1000
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Link Capacity	0	Link 62 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 63 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 64 Capacity	1000	1000	1000	1000	1000
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Link Capacity	0	Link 66 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 67 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 68 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 69 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 70 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 71 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 72 Capacity	1000	1000	1000	1000	1000
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Link Capacity	0	Link 74 Capacity	1000	1000	1000	1000	1000
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Link Capacity	0	Link 76 Capacity	1000	1000	1000	1000	1000
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Link Capacity	0	Link 81 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 82 Capacity	1000	1000	1000	1000	1000
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Link Capacity	0	Link 87 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 88 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 89 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 90 Capacity	1000	1000	1000	1000	1000
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Link Capacity	0	Link 92 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 93 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 94 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 95 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 96 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 97 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 98 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 99 Capacity	1000	1000	1000	1000	1000
Link Capacity	0	Link 100 Capacity	1000	1000	1000	1000	1000

Figure 3: A portion of our spreadsheet

```

Density3[t_, x_] :=
(Piecewise[
  {
    {DensityTable3[[t, x]] = LinkInflowCell, x == 1},
    {(*The density function for the first cell (feeder cell)*)
     DensityTable3[[t, x]] = DensityTable3[[t - 1, x]] + FlowTable3[[t - 1, 1]] - FlowTable2[[t - 1, 1]], x == LengthLink3}},
    {(*The density function for the last cell (diverge*)
     DensityTable3[[t, x]] = DensityTable3[[t - 1, x]] + FlowTable3[[t - 1, x]] - FlowTable3[[t - 1, x + 1]]}],
    {(*The density function for every cell in between first and last*)
     DensityTable3[[t, x]] = DensityTable3[[t - 1, x]] + FlowTable3[[t - 1, x]] - FlowTable3[[t - 1, x + 1]]}],
  ]
)

Flow3[t_, x_] :=
(Piecewise[
  {
    {FlowTable3[[t, x]] = LinkInflowCell, x == 1},
    {(*The flow function for the first cell (feeder cell)*)
     FlowTable3[[t, x]] = Min[DensityTable3[[t, x - 1]], FlowCapLink3, Abs[uCell]] (JamDensityLink3 - DensityTable3[[t, x]])}],
    {(*The flow function for every other cell*)
     FlowTable3[[t, x]] = Min[DensityTable3[[t, x - 1]], FlowCapLink3, Abs[uCell]] (JamDensityLink3 - DensityTable3[[t, x]])}],
  ]
)
(*NOTE: in Link 3, the flows are pretty much all based on Daganzo.*)

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Figure 4: A sample of our Mathematica code

We converted this model into a Mathematica program, which can run a similar simulation over smaller time intervals to provide output that resembles the continuous oscillations seen in reality.

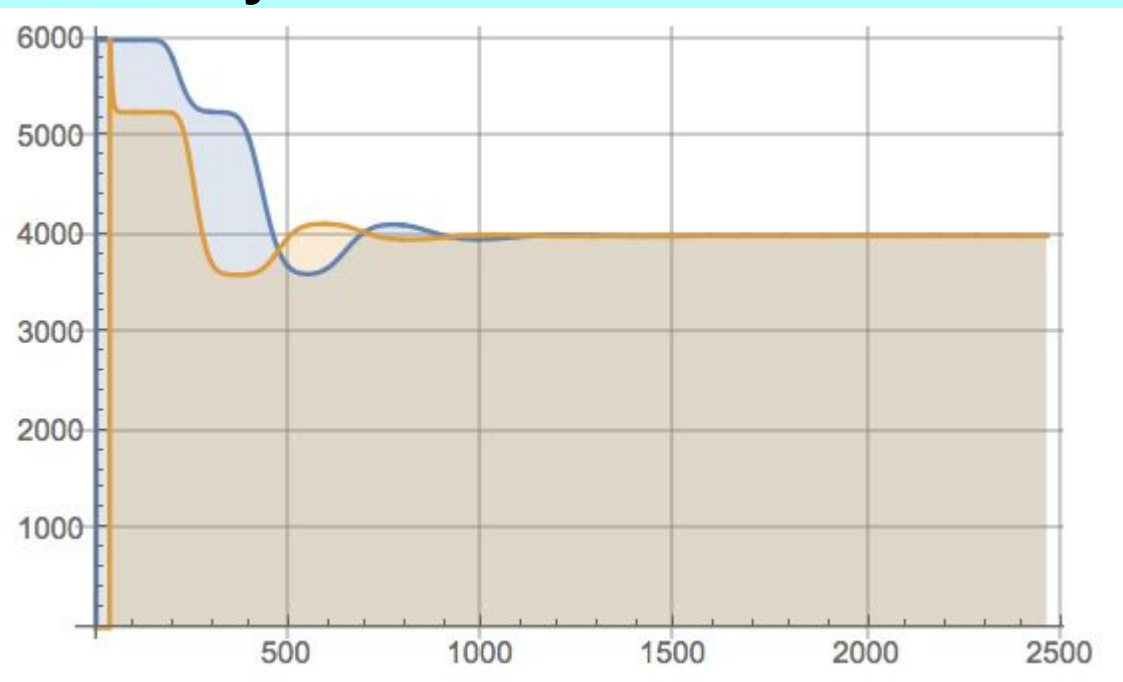


Figure 5: Inflow and outflow of Link 3 with a diverge ratio of $r=0.24$

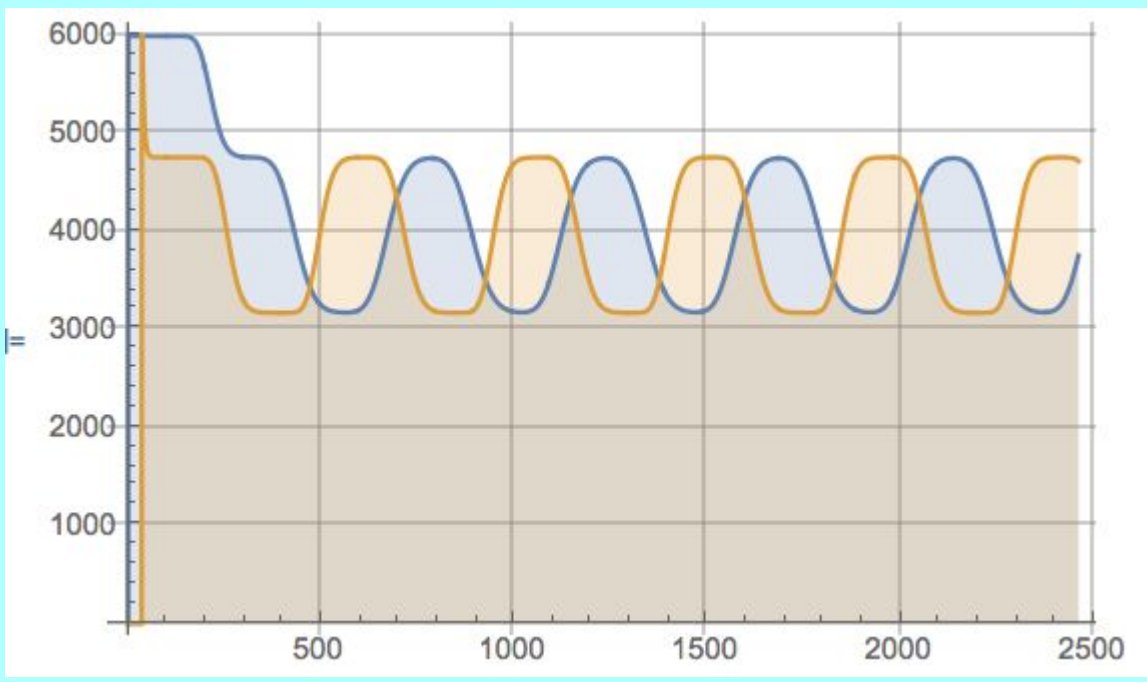


Figure 6: Inflow and outflow of Link 3 with a diverge ratio of $r=0.42$

Discussion and Future Research

Though having a diverge ratio similar to the capacity ratio can help alleviate traffic, our results show that once the rate of vehicles entering the system severely exceeds the rate at which they can leave, there is little that can be done to prevent a traffic jam situation.

Now that we are satisfied that our model can replicate the results of Nie/Zhang [1], we hope to apply it to other traffic patterns, by:

- changing the capacities
- implementing a time-variable diverge ratio
- artificially limiting the inflow rate (analogous to a metered onramp)
- altering the road configuration.

We would like to find a comparable freeway system in real-life so that we may compare our results to empirical data.

Lastly, we could use a “microscopic” model, which tracks each vehicle individually, as a comparison against the “macroscopic” LWR model.

References

[1] Nie (Marco), Yu, and H. Michael Zhang. “Oscillatory Traffic Flow Patterns Induced by Queue Spillback in a Simple Road Network.” *Transportation Science*, vol. 42, no. 2, 2008, pp. 236–248., doi:10.1287/trsc.1070.0229.

[2] Daganzo, Carlos F. “The Cell Transmission Model. Part I: A Simple Dynamic Representation of Highway Traffic..” *California Path Program*, July 1993, ideas.repec.org/p/cdl/itsrrp/qt0b6612tk.html.

[3] Lighthill, M. J., and G. B. Whitham. “On Kinematic Waves .” *Proceedings of the Royal Society*, 10 May 1955, royalsocietypublishing.org/doi/abs/10.1098/rspa.1955.0088.

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