Instructor: J. McGuire Class: Math 119

Homework 1

Each of the following assignments is worth 25 points. Each numbered problem will be worth 5 points. These will be turned in via Canvas, please do not cram your work onto this page. Each numbered problem will receive 2 points for a serious attempt and 3 points for work and correctness.

1. Find the functions $f \circ g, g \circ f, f \circ f, g \circ g$ and their domains.

   (a)
   $$f(x) = \frac{x}{x+1}, g(x) = \frac{1}{x}$$

   (b)
   $$f(x) = \frac{1}{\sqrt{x}}, g(x) = x^2 - 4x$$

2. Determine whether the functions below are one-to-one.

   (a) $-2x + 4$

   (b) $x^4 + 5, 0 \leq x \leq 2$

3. Assume that f is a one-to-one function. If $g(x) = x^2 + 4x$ with $x \geq -2$, find $g^{-1}(5)$.

4. Find **(a)** the reference number for each value of t and **(b)** the terminal point determined by t.

   (a) $t = \frac{13\pi}{4}$

   (b) $t = -\frac{11\pi}{3}$

   (c) $+\frac{13\pi}{6}$

5. The point P is on the unit circle. Find $P(x, y)$ from the given information. The $x-$coordinate of P is $\frac{5}{13}$, and the $y-$coordinate is negative.

Homework 2

1. Find the exact values of the trigonometric function at the given real number.
   (a) $\sin \frac{3\pi}{4}$
   (b) $\tan -\frac{7\pi}{6}$
   (c) $\sec \frac{11\pi}{6}$
2. Find the value of each of the six trigonometric functions (if it's defined) at the given real number t.
$$t = \frac{3\pi}{2}$$
3. Find the period, and graph the function.
$$\cot \frac{\pi}{2}x$$
4. Variable stars are ones whose brightness varies periodically. One of the most visibly is R Leonis; its brightness is modeled by the function
$$b(t) = 7.9 - 2.1\cos\left(\frac{\pi}{159}t\right)$$
   where t is measured in days.
   (a) Find the period of R Leonis.
   (b) Find the maximum and minimum brightness.
   (c) Using either desmos or any other calculator, graph the function b.
5. (a) Prove that if f is periodic with period p, then $\frac{1}{f}$ is also periodic with period p. (Using the definition of a periodic function.)
   (b) Using this fact, prove that cosecant and secant both have period 2π. (You can just explain why we can use the previous fact here.)

Homework 3

1. Find the exact value of the expression, if it's defined.
   (a) $\sin(\tan^{-1}(-1))$
   (b) $\sin(\tan^{-1}(-\sqrt{3}))$
2. Find the exact value of each expression, if it's defined.
   (a) $\cos^{-1}\left(-\frac{1}{2}\right)$
   (b) $\sin^{-1}\left(\frac{-\sqrt{2}}{2}\right)$
   (c) $\tan^{-1} 1$
3. In a predator/prey model, the predator populated is modeled by the function
$$y = 900\cos(2t) + 8000$$
   where t is measured in years.
   (a) What's the maximum population?
   (b) Find the length of time between successive periods of maximum populations.
4. Find the amplitude, period, phase, and horizontal shift.
$$y = 5\sin\left(2t - \frac{\pi}{2}\right).$$
5. A set of data is given.

t	y
0	2.1
2	1.1
4	-0.8
6	-2.1
8	-1.3
10	0.6
12	1.9
14	1.5

   (a) Make a scatter plot of the data. (Either by hand or with any online graphing).
   (b) Find a cosine functions of the form $y = a\cos(\omega(t - c)) + b$ that models the data as in the Modeling 5 Video.
   (c) Graph the function you found in part (b) together with the scatter plot. How well does the curve fit the data? (Also either by any online graphin)

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Homework 4

1. Find the distance along an arc on the surface of the earth that subtends a central angle of 1 minute (1 minute $= \frac{1}{60}$ degrees). This distance is called a *nautical mile*. (The radius of the earth is 3960 miles.)

2. A truck with 48−in. diameter wheels is traveling at 50 miles/hour.

   (a) Find the angular speed of the wheels in radians / min.

   (b) How many revolutions per minute do the wheels make?

3. Use the Law of Sines to solve for all possible triangles that satisfy the given conditions.

$$b = 73, c = 82, \angle B = 58^o$$

4. Solve triangle ABC.

   (a)
$$a = 20, b = 25, c = 22$$

   (b)
$$a = 73.5, \angle B = 61^o, \angle C = 83^o$$

   (c)
$$b = 125, c = 162, \angle B = 40^o$$

   (d)
$$a = 50, b = 65, \angle A = 55^o$$

5. Find the area of the whose sides have the given lengths.

   (a)
$$a = 11, b = 100, c = 101$$

   (b)
$$a = 9, b = 12, c = 15$$

Homework 5

1. Suppose that $x = R\cos(\theta)\sin(\phi), y = R\sin(\theta)\sin(\phi), z = R\cos(\phi)$. Verify the identity $x^2 + y^2 + z^2 = R^2$.

2. Use the Addition Formulas for Cosine and Sine to prove the Addition Formula for Tangent. [*Hint*: Use
$$\tan(s+t) = \frac{\sin(s+t)}{\cos(s+t)}$$
and divide the numerator and denominator by $\cos(s)\cos(t)$.]

3. Prove the identities below.
   (a) $\cos^2(5x) - \sin^2(5x) = \cos(10x)$
   (b) $\dfrac{2(\tan(x) - \cot(x))}{\tan^2(x) - \cot^2(x)} = \sin(2x)$
   (c) $\dfrac{\sin(4x)}{\sin(x)} = 4\cos(x)\cos(2x)$

4. As the moon revolves around the earth, the side that faces the earth is usually just partially illuminated by the sun. The phases of the moon describe how much of the surface appears to be in sunlight. An astronomical measure of phase is given by the fraction F of the lunar disc that is lit. When the angle between the sun, earth, and moon is θ $(0 \leq \theta \leq 360^o)$, then
$$F = \frac{1}{2}(1 - \cos(\theta))$$
Determine the angles θ that correspond to the following phases:
   (a) $F = 0$ (new moon)
   (b) $F = 0.25$ (a crescent moon)
   (c) $F = 0.5$ (first or last quarter)
   (d) $F = 1$ (full moon)

5. A standing wave has maximum amplitude 7 and nodes at $0, \pi/2, \pi, 3\pi/2, 2\pi$. It attains a maximum at $x = \dfrac{\pi}{4}$ and a minimum at $x = \dfrac{3\pi}{4}$. Each point that is node a node moves up and down with period 4π. Find a function of the form
$$y(x,t) = A\sin(\alpha x)\cos(\beta t)$$
that models this wave.

Homework 6

1. Convert the polar equations to rectangular coordinates
   (a) $r = 7$
   (b) $\theta = \pi$
2. Convert the equation to polar form.
   (a) $x^2 - y^2 = 1$
   (b) $x = 4$
   (c) $y = x^2$
3. Consider the polar equation $r = a\cos(\theta) + b\sin(\theta)$
   (a) Express the equation in rectangular coordinates, and use this to show that the graph of the equations is a circle. What are the center and radius?
   (b) Use your answer in part (a) to graph the equation $r = 2\sin(\theta) + 2\cos(\theta)$. (Desmos or hand is fine.)
4. Write the complex number in polar form with arguments θ between 0 and 2π.
   (a) $1 + i$
   (b) $\sqrt{2}$
   (c) $2i(1 + i)$
5. Solve the equation $z^3 = 1$ for z. (Remember complex roots).

Homework 7

1. A pair of parametric equations is given. **(a)** Sketch the curve represented by the parametric equations. (Do this by hand, but it's okay to just trace the Desmos graph) Use arrows to indicate the direction of the curve as t increases. **(b)** Find a rectangular-coordinate equation for the curve.

   (a) $x = e^{-t}, y = e^{t}$

   (b) $x = \sec(t), y = \tan^2(t), 0 \leq t < \pi/2$

   (c) $x = \sin^2(t), y = \cos(t)$

2. Show that the maximum height reached by a projectile as a function of its initial speed v_0 and its firing angle θ is
$$y = \frac{v_0^2 \sin^2(\theta)}{2g}$$

3. Find an equation for the parabola that has its vertex at the origin and satisfies the given condition(s).

   (a) Focus: $F(0,6)$

   (b) Directrix $x = \dfrac{-1}{8}$

4. The **ancillary circle** of an ellipse is the circle with radius equal to half the length of the minor axis and center the same as the ellipse. The ancillary circle is thus the largest circle that can fit within an ellipse.

   (a) Find an equation for the ancillary circle of the ellipse $x^2 + 4y^2 = 16$.

   (b) For the ellipse and ancillary circle of part (a), show that if (s,t) is a point on the ancillary circle, then $(2s,t)$ is a point on the ellipse.

5. In the derivation of the equation of the hyperbola at the beginning of this section we said that the equation
$$\sqrt{(x+c)^2 + y^2} - \sqrt{(x-c)^2 + y^2} = \pm 2a$$
simplifies to
$$(c^2 - a^2)x^2 - a^2y^2 = a^2(c^2 - a^2)$$
Supply the steps needed to show this.

Homework 8

1. Determine what the value of F must be if the graph of the equation
$$4x^2 + y^2 + 4(x - 2y) + F = 0$$
is ...
   (a) An ellipse
   (b) A single point
   (c) Have no solutions (empty set)

2. Complete the square to determine whether the graph of the equation is an ellipse, a parabola, a hyperbola, or a degenerate conic. If the graph is an ellipse, find the center, foci, vertices and lengths of the minor and major axes. If it's a parabola, find the vertex, focus, and directrix. If it's a hyperbola, find the center, foci, vertices, and asymptotes. Then sketch the graph of the equations. If the equation has no graph, explain why.
   (a) $y^2 = 4(x + 2y)$
   (b) $9x^2 - 36x + 4y^2 = 0$

3. Solve the equations
$$x = X\cos(\phi) - Y\sin(\phi)$$
$$y = X\sin(\phi) + Y\cos(\phi)$$
for X and Y in terms of x and y. [*Hint*: To begin, multiply the first equation by $\cos(\phi)$ and the second by $\sin(\phi)$, and then add the equations to solve for X.]

4. **(a)** Use the discriminant to determine whether the graph of the equation is a parabola, an ellipse, or a hyperbola. **(b)** Use a rotation of axes to eliminate the $xy-$term. **(c)** Sketch the graph.
   (a) $xy = 8$
   (b) $11x^2 - 24xy + 4y^2 + 20 = 0$
   (c) $x^2 + 2xy + y^2 + x - y = 0$

5. The polar equation of an ellipse can be expressed in terms of its eccentricity e and the length a of its major axis. Show that the polar equation of an ellipse with directrix $x = -d$ can be written in the form
$$r = \frac{a(1 - e^2)}{1 - e\cos(\theta)}$$
[Hint: Use the relation $a^2 = \dfrac{e^2d^2}{(1 - e^2)^2}$.]