Joseph C. McGuire
Dr. H. Shi

Homework #6

PHYS 314
October 19, 2024

#21 p.187

A classical particle confined to the positive x-axis experiences a force whose potential energy is

$$U(x) = \frac{1}{x^2} - \frac{2}{x} + 1 \qquad \text{(SI units)}$$

a.

By finding its minimum value and determining its behaviors at $x = 0$ and $x = +\infty$, sketch this potential energy.

Solution:

First take note that if we take the limit as x approaches zero we get:

$$\lim_{x \to 0^+} \frac{1}{x^2} - \frac{2}{x} + 1 = \infty + \infty + 1 = \infty$$

and that as we take the limit as x approaches infinity:

$$\lim_{x \to +\infty} \frac{1}{x^2} - \frac{2}{x} + 1 = 0 + 0 + 1 = 1.$$

Additionally we can find the extrema by differentiating $U(x)$ and setting its derivative to zero:

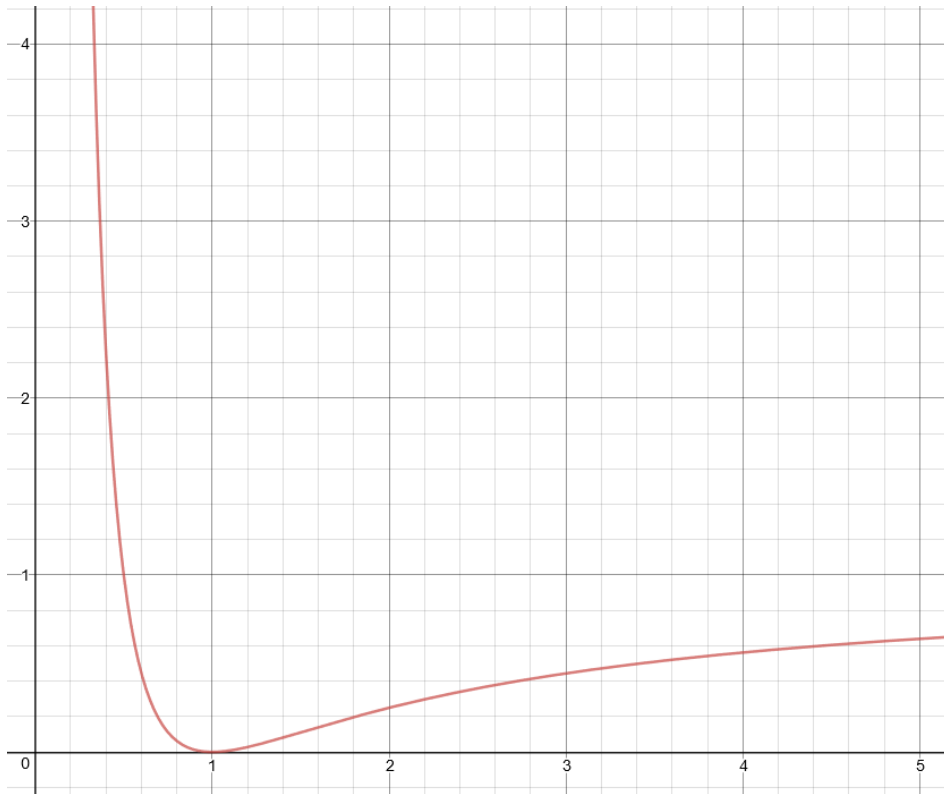
$$U'(x) = \frac{-2}{x^3} + \frac{2}{x^2} \iff 0 = \frac{-2}{x^3} + \frac{2}{x^2} \iff x = 1$$

Furthermore, if we look at U' we can determine the slope of U:

$$\frac{2}{x^2} - \frac{2}{x^3} < 0 \iff x^3 < x^2 \iff 0 < x < 1$$

$$\frac{2}{x^2} - \frac{2}{x^3} > 0 \iff x^3 > x^2 \iff x > 1$$

Thus we have a positive slope on the interval $(1, +\infty)$ and negative slope on the interval $(0, 1)$ and zero at $x = 1$. Thus the graph looks like:



b.

Suppose the particle has an energy of 0.5 J. Find any turning points. Would the particle be bound?

Solution: Setting the equation $U(x)$ equal to 0.5, we can solve for our turning points:

$$\frac{1}{x^2} - \frac{2}{x} + 1 = 0.5 \iff \frac{1-2x}{x^2} = -0.5 \iff 1 - 2x + 0.5x^2 = 0 \iff x = 2 \pm \sqrt{2}.$$

Since we have two positive intercepts (note that $\sqrt{2} < 2$), we can say that the particle is bound.

c.

Suppose the particle has an energy of 2.0 J. Find any turning points. Would the particle be bound?

Solution: Following the same process as in part(b), except with a value of 2.0, we can solve for the turning points:

$$\frac{1}{x^2} - \frac{2}{x} + 1 = 2.0 \iff \frac{1-2x}{x^2} = -0.5 \iff 1 - 2x + 2.0x^2 = 0 \iff x = \frac{-2 \pm \sqrt{8}}{2}.$$

This means we only have one positive intercept, thus the particle is unbound.

24.

An electron in the $n = 4$ state of a 5 nm wide infinite well makes a transition to the ground state, giving off energy in the form of a photon. What is the photon's wavelength?

Solution:

We have the formula for the energy states: $E_n = \frac{\pi^2\hbar^2}{2mL^2}$. Then the difference of the third excited state and the ground state is:

$$E_4 - E_1 = \frac{\pi^2\hbar^2}{2m_eL^2}(4^2 - 1^2) = \frac{\pi^2\hbar^2}{2m_e(5 \times 10^{-9})^2}(15) = 3.6 \times 10^{-20}\ J$$

This energy difference will be carried off by a photon, now we can find the wavelength:

$$\lambda = \frac{hc}{E_4 - E_1} = 5.5 \times 10^{-6}\ m = 550\ nm$$

25.

An electron is trapped in a quantum well (practically infinite). If the lowest-energy transition is to produce a photon of 450 nm wavelength, what should be the well's width?

Solution:

Using the same reasoning as (24), we can use the expression along with the fact $E = \frac{hc}{\lambda}$:
$E_1 = \frac{1^2\pi^2\hbar^2}{2m_eL^2} \implies \frac{hc}{\lambda} = \frac{\pi^2\hbar^2}{2mL^2} \implies L^2 = \frac{\pi^2\hbar^2(450\times10^{-9})}{2m_ehc} \implies L = 6.4\times10^{-10}\ m = 0.64\ nm.$
The well should be of width 0.64 nm.

28.

What is the probability that a particle in the first excited (n = 2) state of an infinite well would be found in the middle third of the well? How does this compare with the classical expectation? Why?

Solution:

We use the normalized probability distribution to find the probability:

$$\int_{\frac{L}{3}}^{\frac{2L}{3}} \frac{2}{L}\sin^2\frac{2\pi x}{L}$$

Using Mathematica to evaluate this the $L's$ cancel out and we get:

$$=$$