MATH 416: Last Homework

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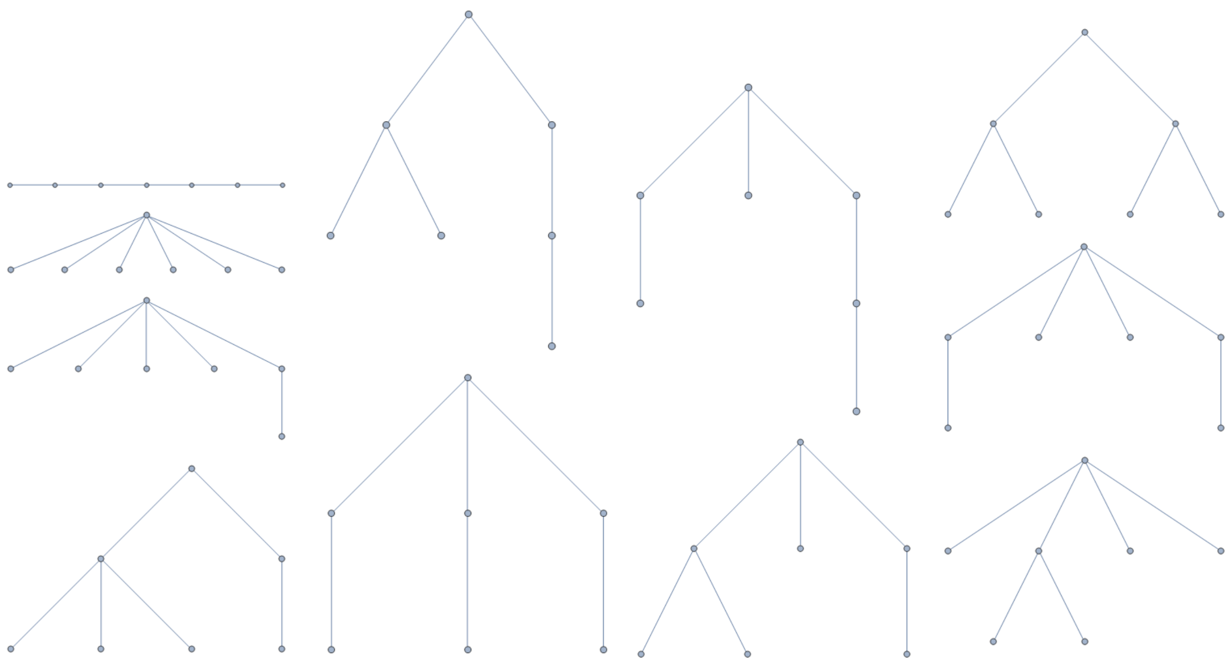
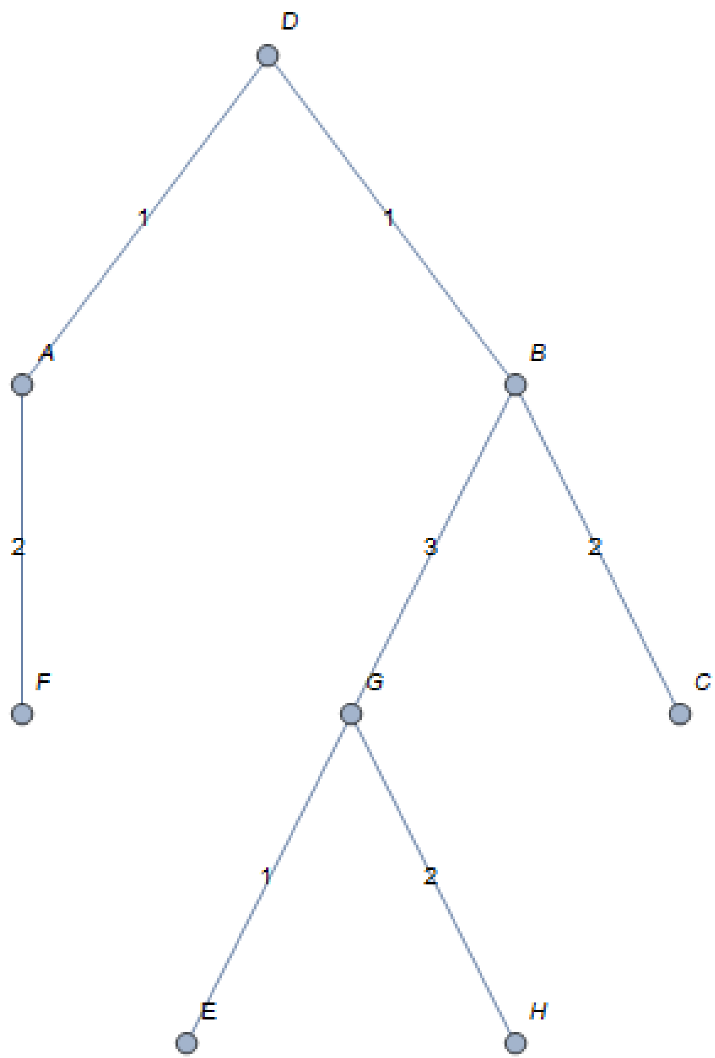
Problem (1.) Draw all non-isomorphic trees of order 7



Programmed in Mathematica, source code can be provided. Beginning with the top left and going to the top right, we have degree sequences of:
(1.) $(2, 2, 2, 2, 2, 1, 1)$,
(2.) $(6, 1, 1, 1, 1, 1, 1)$,
(3.) $(5, 2, 1, 1, 1, 1, 1)$,
(4.) $(4, 2, 2, 1, 1, 1, 1)$,
(5.) $(3, 2, 2, 2, 1, 1, 1)$,
(6.) $(2, 2, 2, 1, 1, 1, 1)$,
(7.) $(3, 2, 2, 2, 1, 1, 1)$,
(8.) $(3, 2, 2, 1, 1, 1, 1)$,
(9.) $(4, 3, 1, 1, 1, 1, 1)$,
(10.) $(4, 2, 2, 1, 1, 1, 1)$,
(11.)$(3, 3, 2, 1, 1, 1, 1)$.

Problem (2.) **Use Dijkstra's algorithm in order to construct a distance tree for vertex A for the weighted graph below, with vertices labeled as shown. (For full credit, please show the steps of your algorithm by listing the vertices in your table in the order they were added to your distance-tree.)**



Vertices	A	D	F	B	G	E	C	H
Distance to Origin	0	1	2	2	3	4	4	5
Edge	$\emptyset$	{A,D}	{A,F}	{D,B}	{D,G}	{G,E}	{B,C}	{G,H}

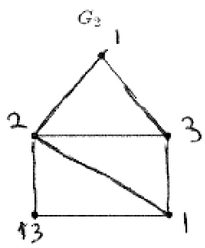
Problem (3.) Determine the chromatic numbers of the following graphs, and show that your answers are correct by specifying appropriate subgraphs or giving other appropriate arguments.

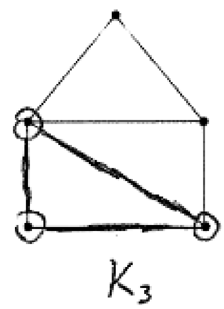
G_1

Note that G_1 has no odd cycles as subgraphs, since all the cycles of G_1 have an even order. Thus, by Theorem 11.4.1, G_1 is bipartite, hence by Theorem 12.1.4, $\chi(G_1) = 2$.

G_2

Note that G_2 has a sub graph of K_3, hence $\chi(G_2) \geq 3$. Moreover since we can come up with a 3-coloring,

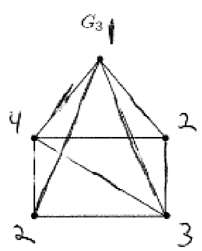


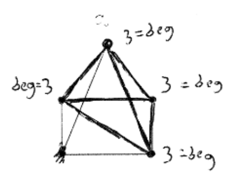


we also have $\chi(G_2) \leq 3$. Thus $\chi(G_2) = 3$.

G_3

Note that G_3 has a sub-graph of K_4, hence $\chi(G_3) \geq 4$. Moreover, since we can come up with a 4-coloring,





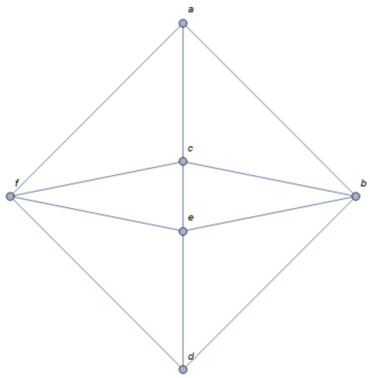
we have $\chi(G_3) \leq 4$. Thus $\chi(G_3) = 4$.

Problem (4.) Determine which of the following graphs are planar. If they are planar, draw a planar representation. If they are not planar, show that they are not planar using the Kurtowski's Theorem or some other method.

G_1

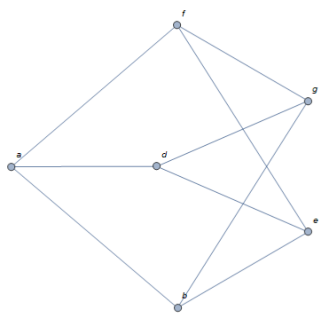
Note that we can contract e into f, making G_1 K_5, hence G_1 is not planar.

G_2



G_3

Since we have $K_{3,3}$ as a sub-graph, and since this isn't planar, we have G_3 isn't planar.



G_4

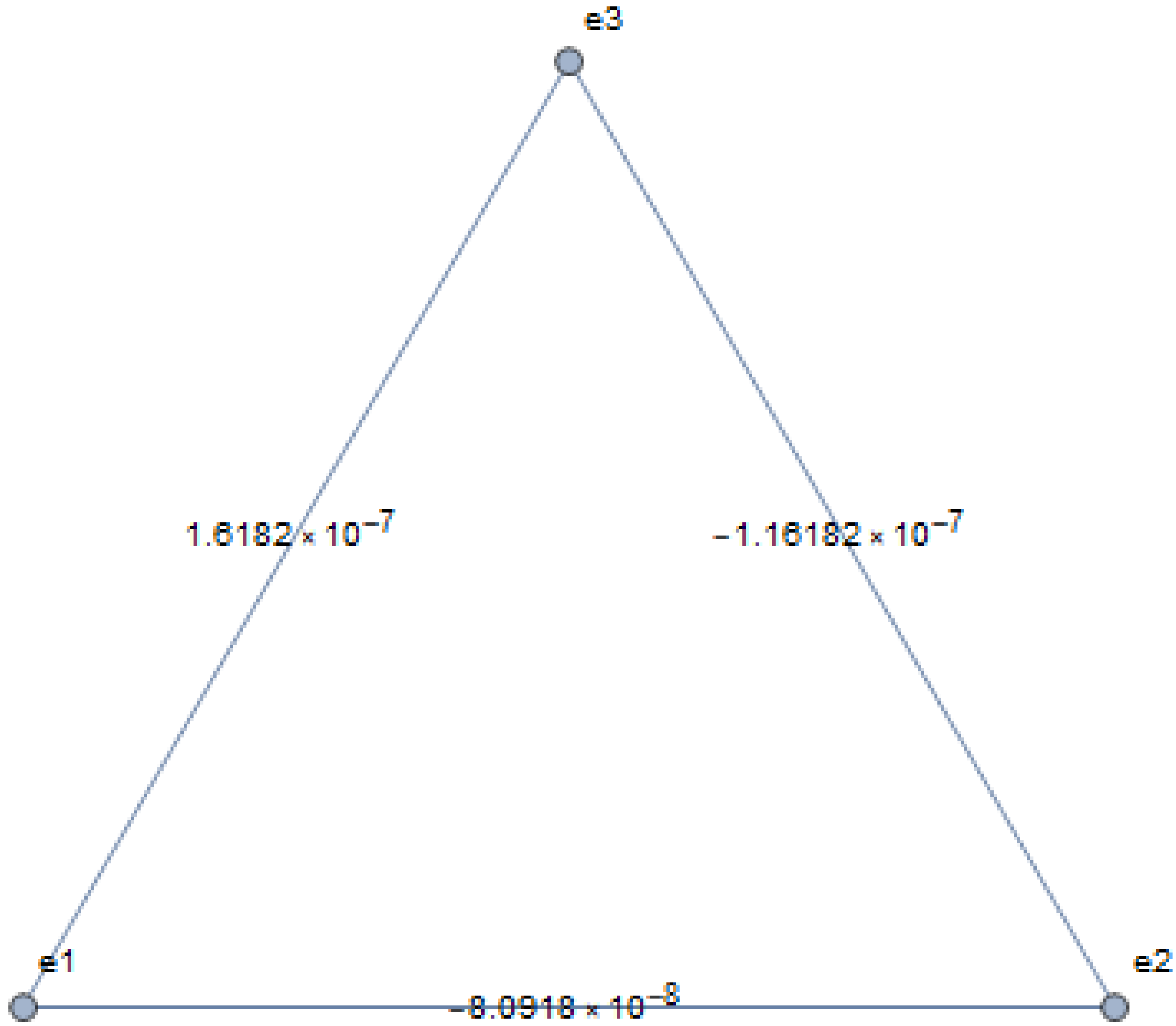
If we take g and contract it into h, then take h and contract it into i, then we have $K_{3,3}$ as a sub-graph.

Problem (5.) Give at least one example of the use of graphs and graph theory outside mathematics. Please specify what are the vertices and edges, and any other details we could understand. Include bibliography. (The more unexpected applications the better :)

Consider a Physics example:
In electrostatics, students deal with electric charges and their interactions with each other. Any kind of electric charge in the universe is caused by some combination of electrons and protons, electrons having a negative, attractive charge, while a proton has a positive, repellent charge. Note that the attraction is based on the polarity of a charge. We also have in a system of electrons or protons, each charge in that system will interact with every other charge in that system. So that we can view the system as a complete graph, where the edges represent the interaction between individual point charges and the vertices are the

point charges themselves. Furthermore, if we add weight's to the edges we can represent the interaction between two individual charges. Assuming that distance is the same between every charge.

So consider the circumstance where we have 3 charges of charge $3[C]$, $-3[C]$, and $6[C]$, where $[C]$ is a Coulomb, the unit of measurement for electric charges. Given that that each charge is 1[m] apart, then the following graph represents the electrostatic-force interaction of the point charges, where the edge labeling is the force in Newtons on the pair of charges:



See (http://hyperphysics.phy-astr.gsu.edu/hbase/electric/elefor.html) for information.