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Homework #4

1 #12 p.134

Analyzing crystal diffraction is intimately tied to the various difference geometries in which the atoms can arranged in three dimensions and upon their differing effectiveness in reflecting waves. To grasp some of the considerations without too much trouble, consider the simple square arrangement of identical atoms shown in the figure. In diagram (a), waves are incident at angle θ with the crystal face and are detected at the same angle with the atomic plane. In diagram (b), the crystal has been rotated 45^o counterclockwise, and waves are now incident upon planes comprising different sets of atoms. If in the orientation of diagram (b), constructive interference is noted only at an angle $\theta = 40^o$, at what angle(s) will constructive interference be found in the orientation of diagram (a)? (Note: The spacing between atoms is the same in each diagram.)

Solution:

First note that when we rotate the square structure so that the wave is now incident upon a new atomic plane, but we still have distance of d between the atomic planes. So that first we must measur Our rotation is 45^o hence so that for the new distance d' we have: $\cos 45^o = \frac{d}{d'} \iff \frac{\sqrt{2}}{2} = \frac{d}{d'} \iff \sqrt{2}d' = 2d$. Furthermore, assuming that the bright fringe we are seeing is the first one; i.e m = 1. And that our angle of constructive interference is $\theta = 40^o$. Hence introducing that into the Bragg's Law relation, we can solve for our wavelength in terms of d':

$$2d \sin\theta = m\lambda \iff \sqrt{2}d' \sin 40^o = \lambda \iff \lambda = 0.909d'$$

Now we can use this to find where we have bright fringes in our new rotated structure:

$$2d' \sin\theta = m\lambda \iff 2d' \sin\theta = m(0.909d') \iff \sin\theta = m(0.4545)$$

The only real solutions to this are the angles:

$$\theta = 27.03^o, 65.37^o$$

Hence these are the angles where we have constructive interference.

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The setup depicted in Figure 4.6 is used in a diffraction experiment using X-rays of 0.26 nm wavelength. Constructive interference is noticed at angles of 23.0^o and 51.4^o, but none between. What is the spacing d of atomic planes?

Solution:

First we have a wavelength of $\lambda = 0.26$ nm and angles of constructive interference at $\theta = 23.0^o, 51.4^o$, then we can plug these in to Bragg's Law relation:

$$\begin{aligned} 2d\sin 23^o &= m(0.26 \times 10^{-9}) \quad 2d\sin 51.4^o = n(0.26 \times 10^{-9}) \\ \iff 2d &= m(6.54 \times 10^{-10}) \qquad 2d = n(3.327 \times 10^{-10}) \\ &\iff n = 2m \end{aligned}$$

So the 23^o is our first bright fringe, meaning we can now solve for the distance from the above equation:

$$2d = 6.54 \times 10^{-10} \iff d = 3.327 \times 10^{-10}\ m = 0.3327\ nm$$

So the spacing between the atomic planes is 0.3327 nm.

3 #16 p.134

A Bragg diffraction experiment is conducted using a beam of electrons accelerated through a 1.0 kV potential difference.

a.

If the spacing between atomic planes in the crystal is 0.1 nm, at what angles with respect to the panes will diffraction maximum be observed?

Solution:

Note that we have $d = 0.1\ nm$, and that $V = 1.0 \times 10^3\ V$. So from combining Conservation of Energy, the Wavelength of a Matter Wave, and the definition of momentum: $qV = \frac{1}{2}m_e v^2$, $\lambda = \frac{h}{p}$, $p = m_e v$, we get:

$$eV = \frac{1}{2}\left(\frac{h}{m_e\lambda}\right)^2 m_e \iff V = \frac{h^2}{2\lambda^2 m_e e}$$

We can use this to solve for our wavelength λ:

$$\lambda^2 = \frac{h^2}{2Vm_e e} \iff \lambda = 3.881 \times 10^{-11}\ m = 0.03881\ nm$$

Introducing this into Bragg's Law:

$$2d\sin\theta = m\lambda \iff \sin\theta = m(0.194)$$

Solving for θ we get:

$$\theta = 11.19^o, 22.84^, 35.6^o, 50.91^o, 75.98^o$$

Corresponding to the 1st, 2nd, 3rd, 4th, and 5th bright fringes respectively. These are our diffraction maximum's.

b.

If a beam of X-rays produces diffraction maxima at the same angles as the electron beam, what is the X-ray photon energy?

Solution: From the photoelectric effect, we have the formula for the energy of a photon:$E = hf = \frac{hc}{\lambda}$. We have from this section of reading that at this scale X-rays behave like waves, hence it will follows Bragg's Law and thus we would end up with the same wavelength as the electrons from part(a). Since d, θ, and the $m's$ are all the same. So we would have the energy of the photon's as:

$$E = \frac{hc}{\lambda} = \frac{hc}{0.03881 \times 10^{-9}} = 5.125 \times 10^{-15}\ J = 3.199 \times 10^{4}\ eV$$

4 #17 p.134

Determine the Compton wavelength of the electron, defined to be the wavelength it would have if its momentum were $m_e c$.

Solution:

Using the formula for the momentum of a photon and setting $p = m_e c$:

$$p = \frac{h}{\lambda} \iff \lambda = \frac{h}{m_e c} = 2.426 \times 10^{-12}\ m = 2.426 \times 10^{-3}\ nm$$

5 #18 p.134

A particle is "thermal" if it's in equilibrium with its surroundings - its average kinetic energy would be $\frac{3}{2}k_BT$. Show that the wavelength of a thermal particle is given by:

$$\lambda = \frac{h}{\sqrt{3mk_BT}}$$

Solution:

Using the relation $KE = \frac{p^2}{2m}$ and the wavelength of a matter wave, we can derive this wavelength.

Let $KE = \frac{3}{2}k_BT$, then $\frac{p^2}{2m} = \frac{3}{2}k_BT \iff \frac{\left(\frac{h}{\lambda}\right)^2}{2m} = \frac{3}{2}k_BT \iff \lambda^2 = \frac{2h^2}{6mk_BT} \iff \lambda = \frac{h}{\sqrt{3mk_BT}}$.
Thus our derivation for the wavelength of a thermal particle is complete.

6 #23 p.135

Atoms in a crystal form atomic planes at many different angles with respect to the surface. The accompanying figure shows the behaviors of representative incident and scattered waves in the Davisson - Germer experiment. A beam of electrons accelerated through 54 V is directed normally at a nickel surface, and strong reflection is detected only at an angle ϕ of 50^o. Using the Bragg law, show that this implies a spacing D of nickel atoms on the surface in agreement with the known value of 0.22 nm

Solution: To verify this using Bragg's Law, note that we must first find the wavelength. Using the same relation derived in 3, we have $\lambda^2 = \frac{h^2}{2Vm_e e} = \frac{h^2}{2(54)m_e e} \iff \lambda = 1.670 \times 10^{-10}$. Note that the angle made with the atomic plane is 65^o, hence, by Bragg's Law:

$$2D\sin\theta = m\lambda \iff 2D\sin 65^o = (1.670 \times 10^{-10}) \iff d = 9.213 \times 10^{-11}$$

Similar to problem 2, we have $\sin 25^o = \frac{9.213\times10^{-11}}{D} \implies D = 2.180 \times 10^{-10}\ m = 0.218\ nm$. Thus verifying our results.