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Traffic Model Report

October 19, 2024

Case 1

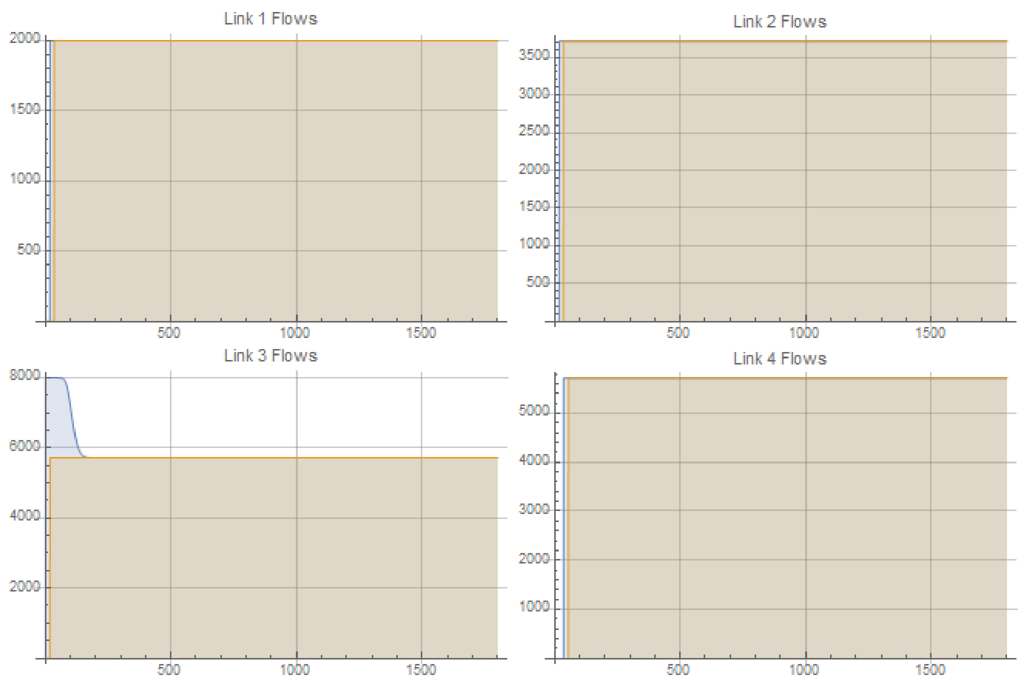
In the parameter space of (r, k) this is:

$$\{k \leq r \leq 1 : 0 \leq k \leq 1\}$$

The exact solution is given in §3.1 in Nie-Zhang.

Results

Using vales of $r = 0.35, k = 0.33$. Yielding the results (blue - inflow, orange - outflow):



Results - Updated (July 2020) Mathematica Code

Using the values of $r = 0.35, k = 0.33$, which corresponds to *Lanes* input of $\{3.3, 10, 13.3, 10\}$,

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In[12]:= Solution = TrafficModel[1, 50, -10, 3600, {3.3, 10, 13.3, 10}, {2000, 2000, 2000, 2000}, {1, 1, 1, 1}, {240, 240, 240, 240}, 0.35];
{7.62439, Null}

In[13]:= GraphingResults[Solution, 1000]
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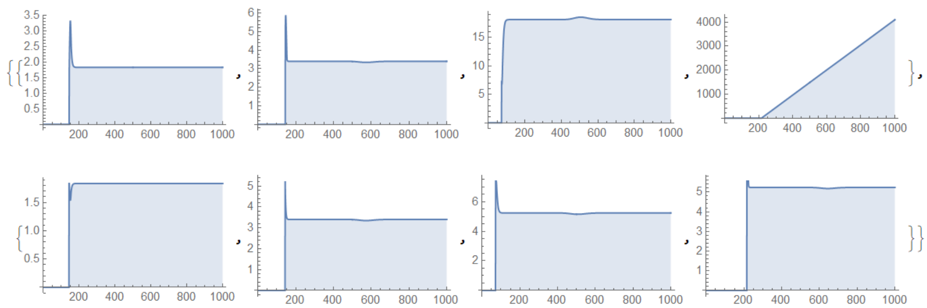
Out[13]=



Nie-Zhang Analysis

This case is trivial because both links 1 and 2 never become congested. Drivers queue upright before the diverge point, and the total discharging rate is proportional $\frac{k}{r}$.

Case 2

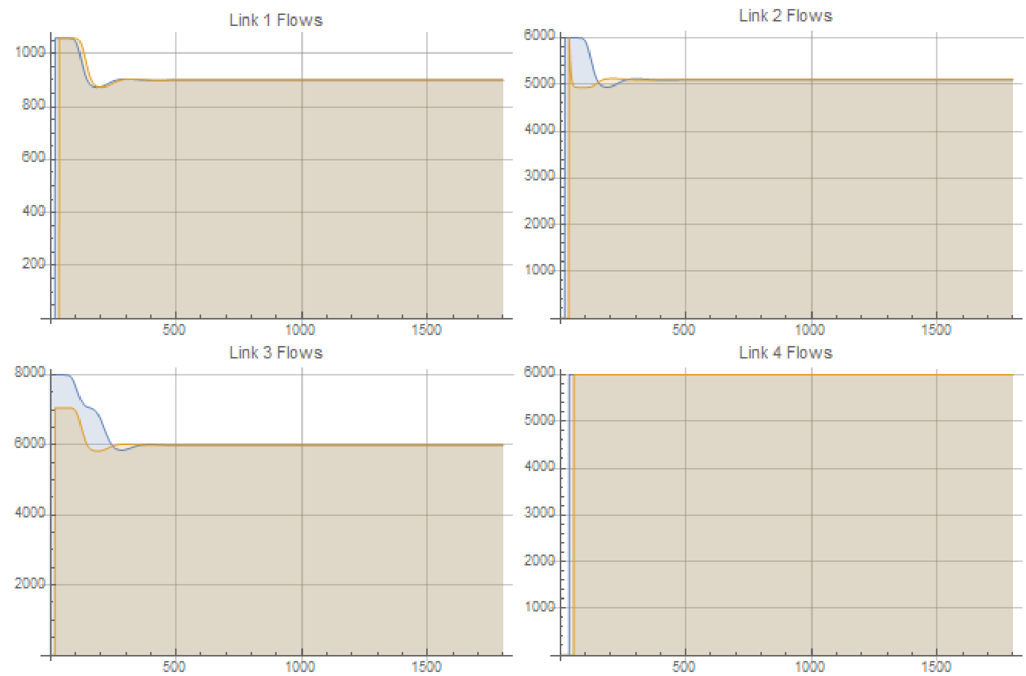
In the parameter space of (r, k) this is:

$$\left\{0 \leq r \leq \frac{k}{2k+1} : 0 \leq k \leq 1\right\}$$

The exact solution is given in §3.2.1.

Results

Using the values of $r = 0.15, k = 0.33$, gives a value of $\frac{k}{2k+1} = 0.2$. Yielding the results (blue - inflow, orange - outflow):



Results - Updated (July 2020) Mathematica Code

Using the values of $r = 0.15, k = 0.33$, which corresponds to *Lanes* input of $\{3.3, 10, 13.3, 10\}$,

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In[16]:= Solution = TrafficModel[1, 50, -10, 3600, {3.3, 10, 13.3, 10}, {2000, 2000, 2000, 2000}, {1, 1, 1, 1}, {240, 240, 240, 240}, 0.15];
         {7.50556, Null}

In[18]:= GraphingResults[Solution, 3600]
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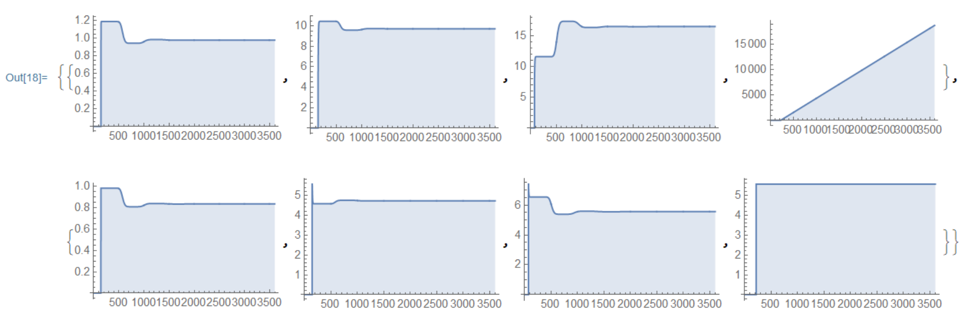


Nie-Zhang Analysis

Inflows/outflows of links 1 and 2 oscillate with decaying amplitudes and identical periods. As $t \to \infty$, the system converges to a stable solution. Link 1 never becomes congested.

Case 3

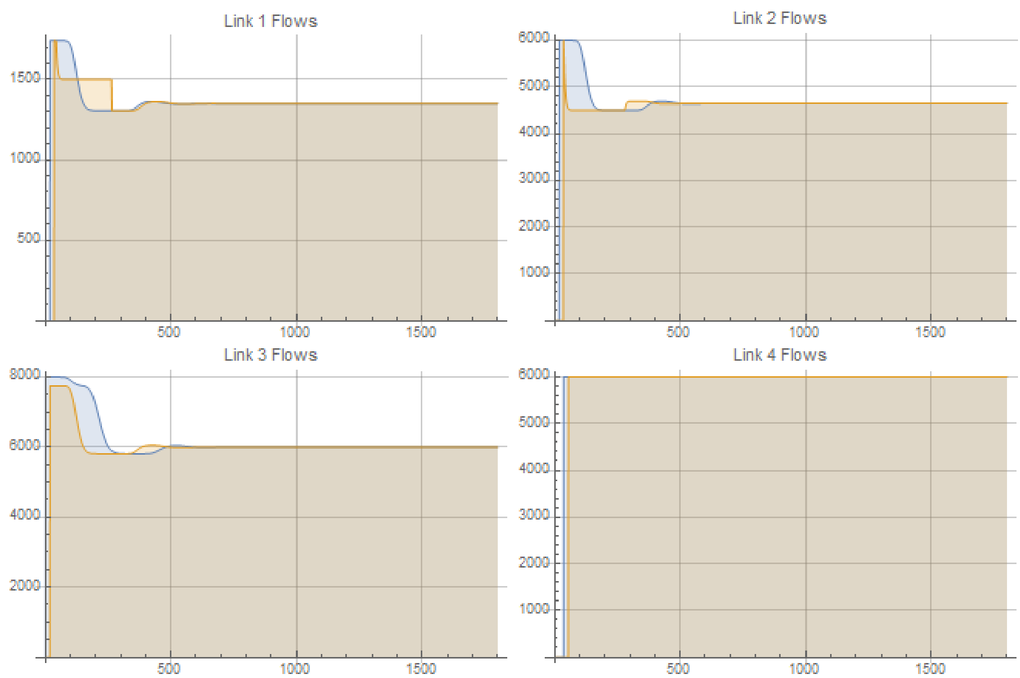
In the parameter space of (r, k) this is:

$$\left\{ \frac{k}{2k+1} \leq r \leq \frac{k}{k+1} : 0 \leq k \leq 1 \right\}$$

The exact solution is given in §3.2.2.

Results

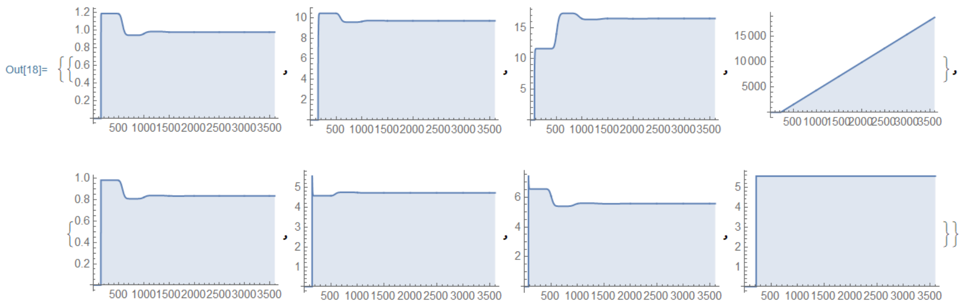
Using the values of $r = 0.225, k = 0.33$, gives $\frac{k}{2k+1} = 0.2$ and $\frac{k}{k+1} = 0.25$. Yielding the results (blue - inflow, orange - outflow):



Results - Updated (July 2020) Mathematica Code

Using the values of $r = 0.225, k = 0.33$, which corresponds to *Lanes* input of $\{3.3, 10, 13.3, 10\}$,

```
In[16]:= Solution = TrafficModel[1, 50, -10, 3600, {3.3, 10, 13.3, 10}, {2000, 2000, 2000, 2000}, {1, 1, 1, 1}, {240, 240, 240, 240}, 0.15];
         {7.50556, Null}
In[18]:= GraphingResults[Solution, 3600]
```



Nie-Zhang Analysis

his case is similar to Case II, but link 1is congested in the first period. The length of the first period is thus dependent on the queue clearance time on link 1. As a result, the first period is longer than the regular period. The oscillatory solution converges to the same stable state as $t \to \infty$.

Our Analysis

Case 4

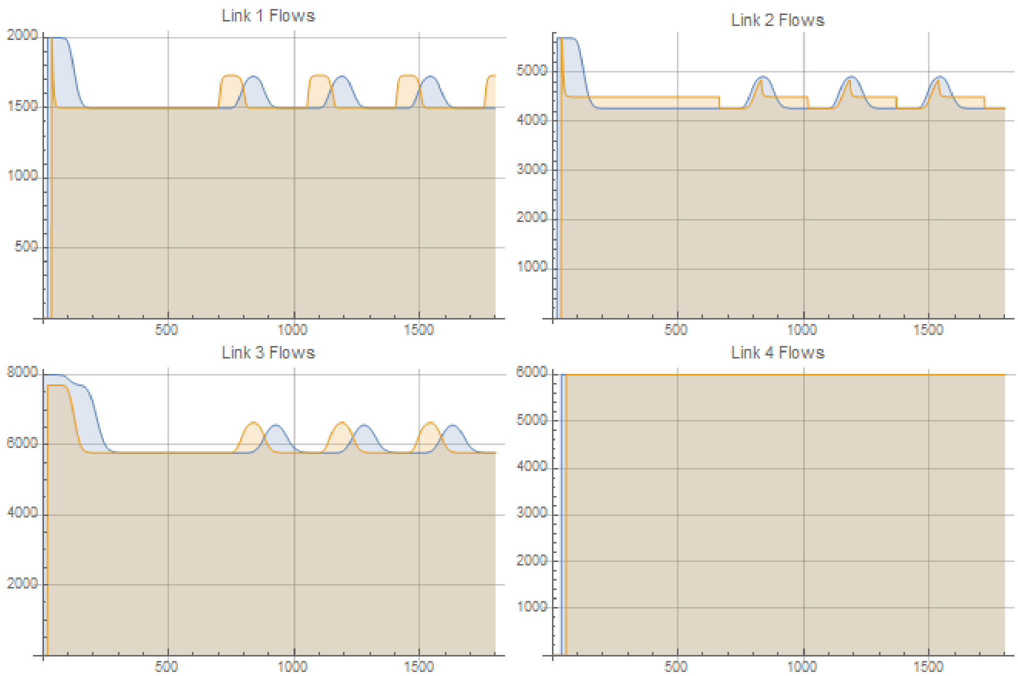
In the parameter space of (r, k) this is:

$$\left\{\frac{k}{k+1} \leq r \leq \frac{k}{1+k-k^2} = \beta : 0 \leq k \leq 0.618\right\} \cup \left\{\frac{k}{k+1} \leq r \leq 0.5 : 0.618 \leq k \leq 1\right\}$$

The exact solution is given in §3.3.2(b) starting at $\theta \geq 1$.

Results

Using the values of $r = 0.26, k = 0.33$, gives values of $\frac{k}{k+1} = 0.25$ and $\beta = 0.272$. Also $\frac{c(1-k\theta)}{1+k} = 76$ and $\theta = 2.84$ (Remember these are scaled by number of lanes). Yielding the results (blue - inflow, orange - outflow):



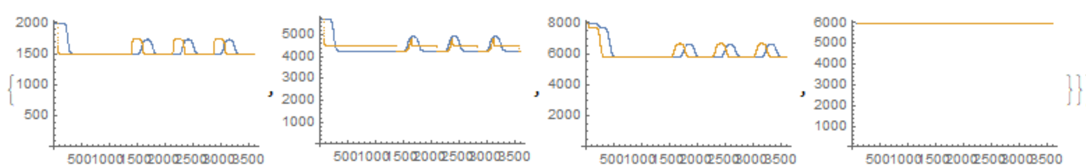
Nie-Zhang Analysis

This case produces stop-and-go oscillations with identical periods and amplitudes after the first period. Queue will develop and die out repeatedly on link 2 as the oscillation goes on. As a result,the length of the period in this case is longer than those of Cases V and VI because of the time required to clear the queue on link 2. The outflow of link 1 oscillates between $\frac{kc}{1+k}$ and $\frac{c(1-k\theta)}{1+k}$.

Our Analysis

Results - Updated (July 2020) Mathematica Code

Using the values of $r = 0.225, k = 0.33$, which corresponds to *Lanes* input of $\{1, 3, 4, 3\}$,



Case 5

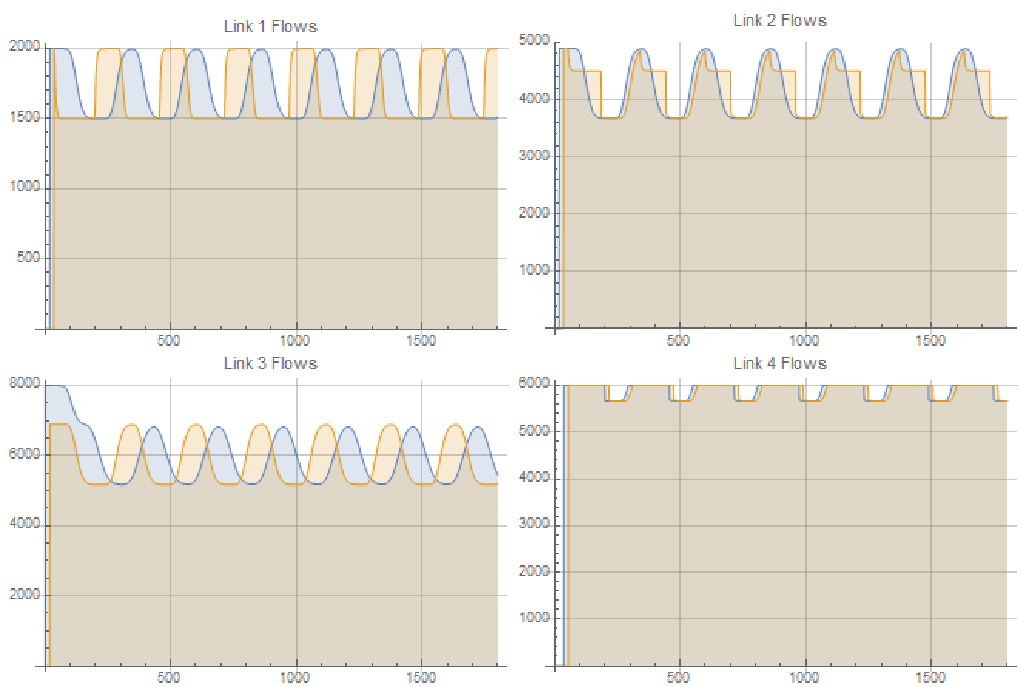
In the parameter space of (r, k) this is:

$$\left\{\beta = \frac{k}{1+k-k^2} \leq r \leq \frac{k+k^2}{1+k+k^2} = \alpha : 0 \leq k \leq 0.618\right\}$$

The exact solution is given in §3.3.2(a)

Results

Using the $r = 0.29, k = 0.33$, giving the values of $\alpha = 0.307$ and $\beta = 0.272$. With $\tau_{wv} \approx 110$ Yielding the results (blue - inflow, orange - outflow):

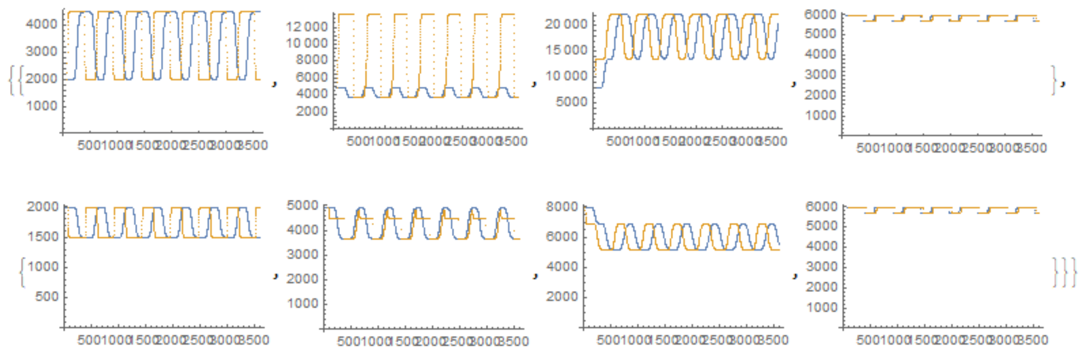


Nie-Zhang Analysis

The nondecaying stop-and-go oscillations produced in this case have the following features:(1) the length of the periods equals $2\tau_{wv}$, (2) the out-flow of link 1 oscillates between $\frac{kc}{1+k}$and kc, and(3) link 2 is congested only in the first period. Interestingly, the stable oscillatory solution (the length of periods and amplitudes) is independent of the diversion ratio r.

Results - Updated (July 2020) Mathematica Code

Using the values of $r = 0.29, k = 0.33$, which corresponds to *Lanes* input of $\{1, 3, 4, 3\}$,



Case 6

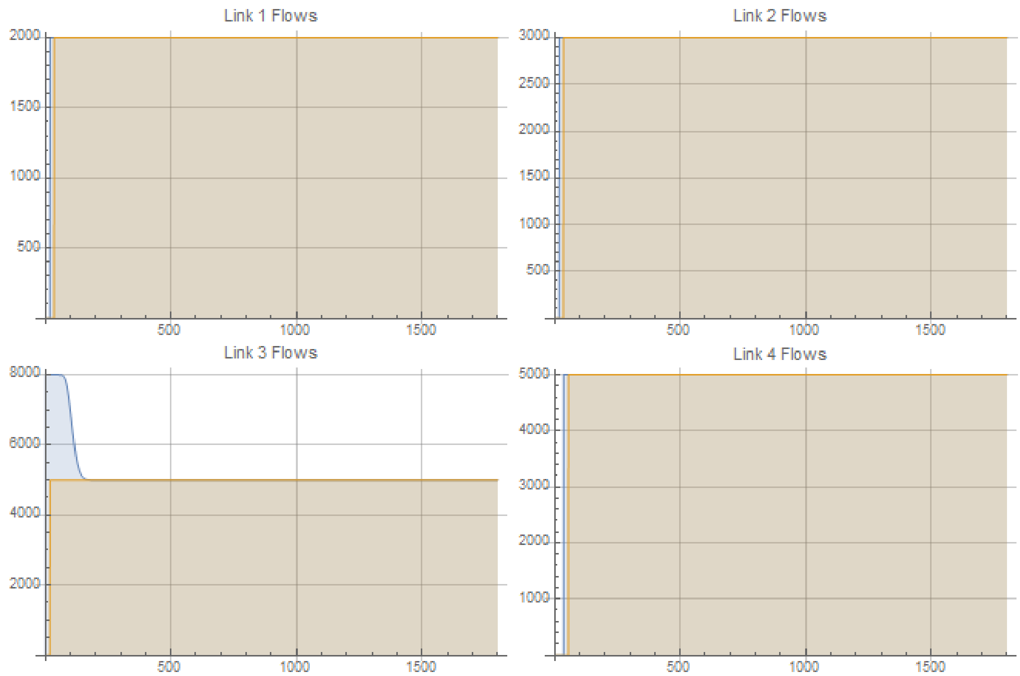
In the parameter space of (r, k) this is:

$$\left\{\alpha = \frac{k + k^2}{1 + k + k^2} \leq r \leq k : 0 \leq k \leq 0.5\right\} \bigcup \left\{\alpha = \frac{k + k^2}{1 + k + k^2} \leq r \leq 0.5 : 0.5 \leq k \leq 0.618\right\}$$

The exact solution is given in §3.3.1(a) .

Results

Using the values of $r = 0.4, k = 0.33$, gives values of $\beta = 0.272$. Yielding the results (blue - inflow, orange - outflow):

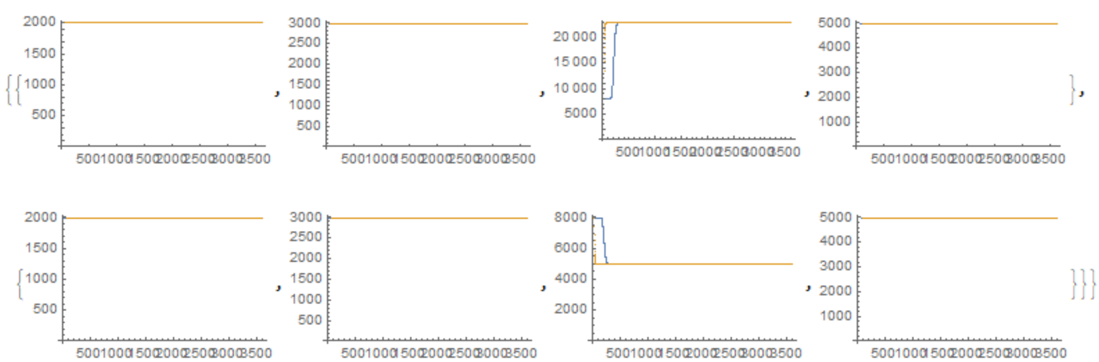


Nie-Zhang Analysis

This case is similar to Case V except (1) link 2 never becomes congested, and (2) the outflow of link 1 switched between $c(1 - k\theta)$ and kc.

Results - Updated (July 2020) Mathematica Code

Using the values of $r = 0.4, k = 0.33$, which corresponds to *Lanes* input of $\{1, 3, 4, 3\}$,



Case 7

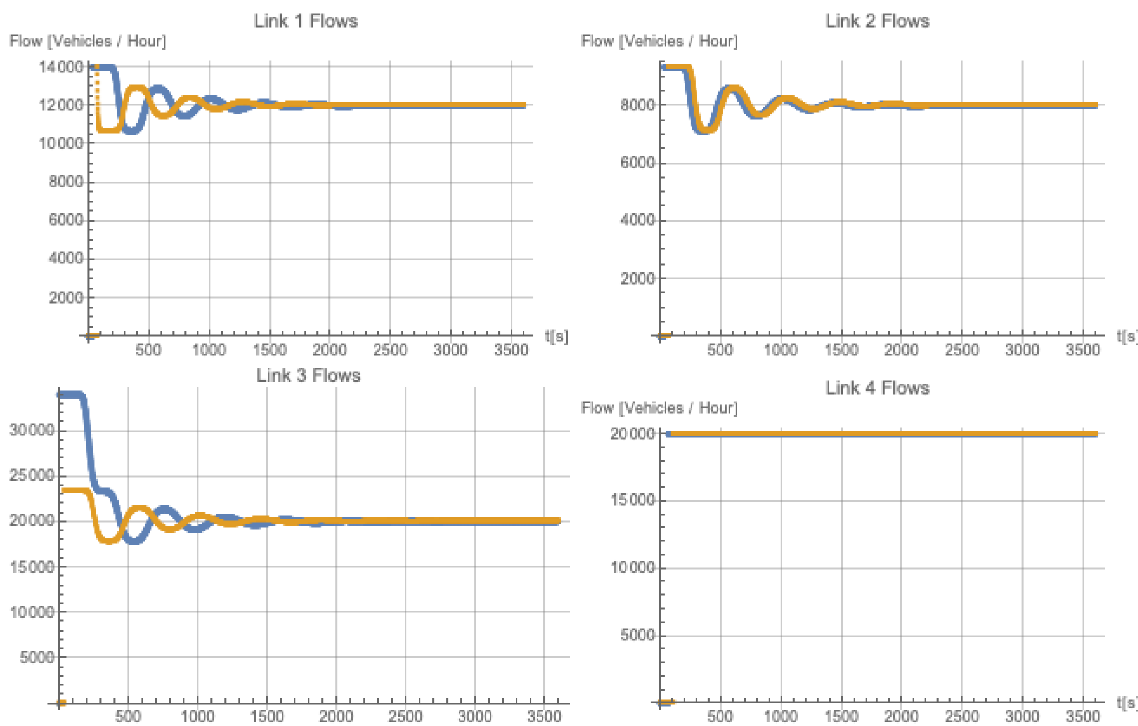
In the parameter space of (r, k) this is:

$$\{0.5 \leq r \leq k : 0.5 \leq k \leq 0.618\} \bigcup \left\{ \beta = \frac{k}{1+k-k^2} \leq r \leq k : 0.618 \leq k \leq 1 \right\}$$

The exact solution is given in §3.3.1(b).

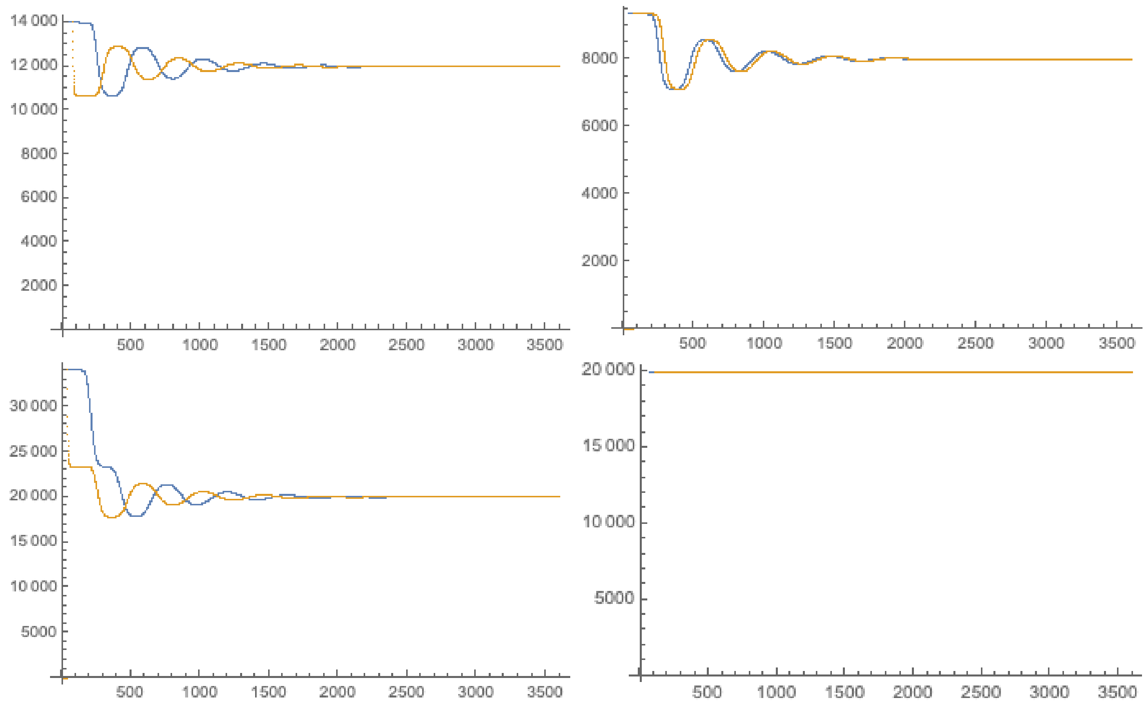
Results - Original Mathematica Code

Using the values of $r = 0.6, k = 0.7$, gives the values $\beta = 0.578$. Yielding the results (blue - inflow, orange - outflow):



Results - Updated (July 2020) Mathematica Code

Using the values of $r = 0.6, k = 0.7$, which corresponds to *Lanes* input of $\{7, 10, 17, 10\}$, produces the following output for link flows:



Nie-Zhang Analysis

Inflows/outflows of links 1 and 2 oscillate with decayed amplitudes and identical periods. When $t \to \infty$, the system settles at a stable state, where link 1 is congested. In this case, link 2 never becomes overcritical.

Case 8

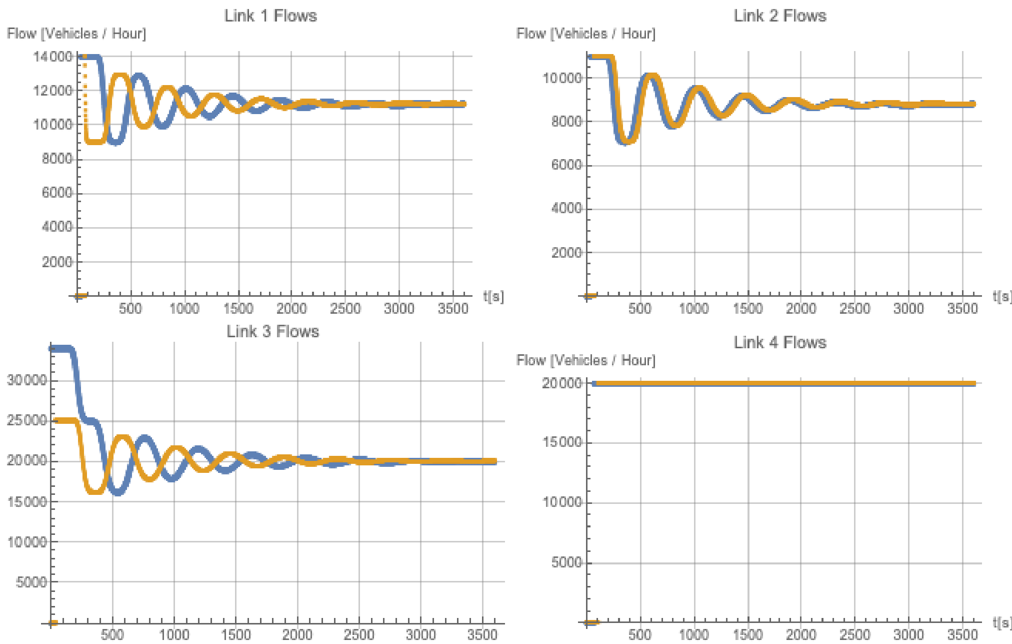
In the parameter space of (r, k) this is:

$$\left\{\alpha = \frac{k + k^2}{1 + k + k^2} \leq r \leq \frac{k}{1 + k - k^2} = \beta : 0.618 \leq k \leq 1\right\}$$

The exact solution is also??? given in §3.3.1(b).

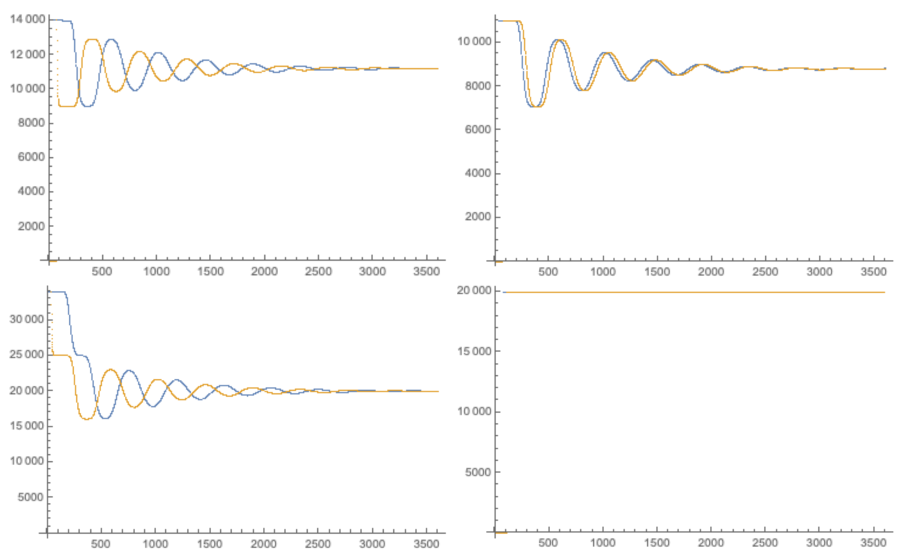
Results - Original Mathematica Code

Using the values of $r = 0.56, k = 0.7$ gives values of $\alpha = 0.543, \beta = 0.578$. Yielding the results (blue - inflow, orange - outflow):



Results - Updated (July 2020) Mathematica Code

Using the values of $r = 0.56, k = 0.7$, which corresponds to *Lanes* input of $\{7, 10, 17, 10\}$, produces the following output produces the following output for link flows:



Nie-Zhang Analysis

This case is similar to Case VII.

Case 9

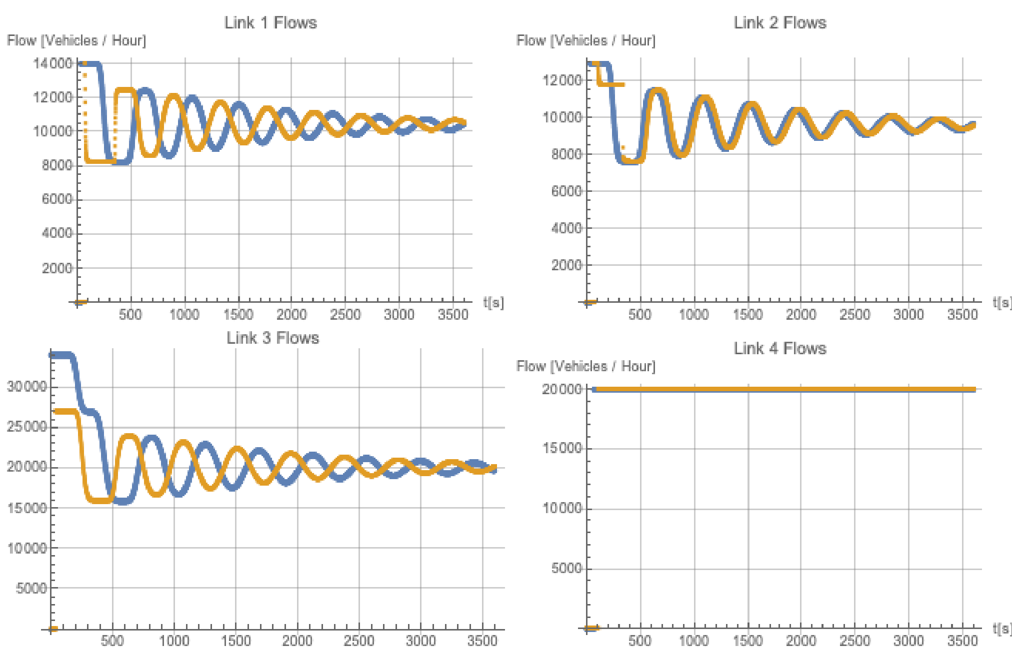
In the parameter space of (r, k) this is:

$$\left\{0.5 \leq r \leq \frac{k+k^2}{1+k+k^2} = \alpha : 0.618 \leq k \leq 1\right\}$$

The exact solution is given in §3.3.2(b) starting at $\theta < 1$.

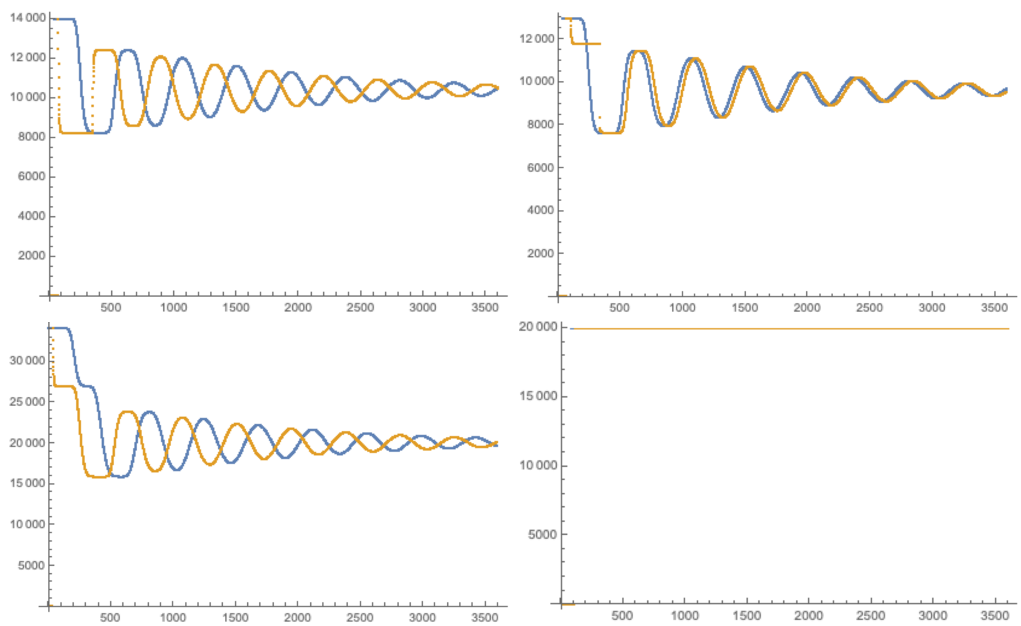
Results - Original Mathematica Code

Using the values of $r = 0.52, k = 0.7$, giving the value of $\alpha = 0.543$. Yielding the results (blue - inflow, orange - outflow):



Results - Updated (July 2020) Mathematica Code

Using the values of $r = 0.52, k = 0.7$, which corresponds to *Lanes* input of $\{7, 10, 17, 10\}$, produces the following output produces the following output for link flows:



Nie-Zhang Analysis

This case is similar to Case VII except that a queue will develop on link 2 in the first period. As a result, the first period is longer and the magnitude of inflows/outflows is slightly different