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Problem Set #4

1.

Show that the differential $x^2\ dy - (y^2 + xy)\ dx$ is not exact, but that $\frac{1}{xy^2}[x^2\ dy - (y^2 + xy)\ dx]$ is exact.

Solution:

$$\frac{\partial}{\partial y}[y^2 + xy] = 2y + x$$
$$\frac{\partial}{\partial x}[x^2] = 2x$$

Thus this differential is inexact.

First, note that $\frac{1}{xy^2}[x^2\ dy - (y^2 + xy)\ dx] = \frac{x}{y^2}\ dy - (x + \frac{1}{y})\ dx$.

$$\frac{\partial}{\partial x}\frac{x}{y^2} = y^{-2}$$
$$\frac{\partial}{\partial y}(-x - y^{-1}) = y^{-2}$$

Because these are equal we have that this solution is exact.

2.

If $s = t^u$, find $\frac{\partial s}{\partial u}$ and $\frac{\partial s}{\partial t}$.

Solution:

$$\begin{aligned}
s &= t^u \\
s &= e^{\ln(t^u)} \\
s &= e^{u\ln(t)} \\
\frac{\partial s}{\partial u} &= \ln(t)e^{u\ln(t)} \\
&= \ln(t)e^{\ln(t^u)} \\
&= \ln(t)t^u \\
\frac{\partial s}{\partial t} &= (u)t^{u-1}
\end{aligned}$$

3.

A possible equation of state for a gas takes the form

$$P \cdot V = R \cdot T \cdot e^{-\frac{\alpha}{VRT}}$$

in which α and R are constants. P, V, T are the pressure, volume, and temperature of the gas, respectively. Show that

$$\left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_V = -1$$

Solution: