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Homework #3

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1 #17 p.93

Light of 300 nm wavelength strikes a metal plate, and photoelectrons are produced moving as fast as $0.002c$.

a.

What is the work function of the metal?

Solution:

Note that we have the relation $KE_{max} = hf - \phi$ for the photoelectric effect. In this case, since we have $0.002c < 0.05c$ we are fine using classical mechanics/ Thus we have $\frac{1}{2}mv^2 = hf - \phi$. Given that the wavelength of the light is $300nm$ this gives us that $f = \frac{c}{\lambda} = \frac{c}{300\times10^{-9}}$. We also have the velocity, so we can solve for our work function ϕ:

$$\phi = hf - \frac{1}{2}m_e v^2 = \frac{hc}{\lambda} - \frac{1}{2}m_e v^2 = 4.99 \times 10^{-19}[J] = 3.12[eV]$$

Thus the work function ϕ is $3.12\,eV$.

b.

What is the threshold wavelength for this metal?

Solution:

Note that this is given when the photoelectrons have $KE_{max} = 0$. This gives:

$$0 = hf - \phi \iff phi = \frac{hc}{\lambda} \iff \lambda = \frac{hc}{\phi} = 3.986 \times 10^{-7}[m] = 399[nm]$$

Thus the threshold wavelength for this metal is $399\,nm$.

2 #23 p.93

Light of wavelength 590[nm] is barely able to eject electrons from a metal plate. What would be the speed of the fastest electrons ejected by $\frac{1}{3}$ the wavelength?

Solution:

Again we have $KE_{max} = hf - \phi$. Note that by the behavior described we can say that $0 \approx hf - \phi \iff \phi \approx \frac{hc}{\lambda_o}$ where $\lambda_0 = 590[nm]$ is our threshold wavelength. Hence $\phi = 3.371 \times 10^{-19}\, J$.

So now we can plug it back into the relation $KE_{max} = \frac{hc}{\lambda} - \phi$. We get:

$$\frac{1}{2}m_e v^2 = \frac{hc}{\frac{1}{3}\lambda_0} - \phi \iff \frac{1}{2}m_e v^2 = 3\frac{hc}{\lambda_0} - \phi \iff \frac{1}{2}m_e v^2 = 3\phi - \phi \iff \frac{1}{2}m_e v^2 = 2\phi$$

Solving for v:

$$v^2 = \frac{4\phi}{m_e} = 1.480 \times 10^{12} \iff v = 1.2167 \times 10^6 \left[\frac{m}{s}\right] = 0.004c\,\frac{m}{s}$$

Thus the fastest electrons are ejected at $0.004c$ when we have third of this original wavelength.

3 #24 p.93

With light of wavelength 520 nm, photoelectrons are ejected from a metal with a maximum speed of $1.78 \times 10^5 \, \frac{m}{s}$.

a.

What wavelength would be needed to give a maximum speed of $4.81 \times 10^5 \, \frac{m}{s}$.
Solution:
Using the relation $KE_{max} = \frac{hc}{\lambda} - \phi$ to solve for our unknown work function, where we know $v = 1.78 \times 10^5 \, \frac{m}{s}$ and $\lambda = 520 \, nm$:

$$\frac{1}{2} m_e v^2 = \frac{hc}{\lambda} - \phi \iff \phi = \frac{hc}{\lambda} - \frac{1}{2} m_e v^2 = 3.6807 \times 10^{-19}$$

Now sitting $v = 4.81 \times 10^5 \, \frac{m}{s}$ we'll solve for λ:

$$\frac{1}{2} m_e v^2 = \frac{hc}{\lambda} - \phi \iff \lambda = \frac{hc}{\frac{1}{2} m_e v^2 + \phi} = 4.20 \times 10^-7 = 420 \, [nm].$$

Hence to achieve this speed the light must have a wavelength of 420 nm.

b.

Can you guess what metal it is?
Solution:
Converting our work function ϕ to eV:

$$\phi = 3.6807 \times 10^{-19} \, J = 2.30 \, eV$$

This lines up with Sodium as the metal involved here.

4 #25 p.93

You are an early 20th Century experimental physicist and do not know the value of Planck's constant. By a suitable plot of the following data, and using Einstein's explanation of the photoelectric effect ($KE = hf - \phi$ where h isn't known), determine Planck's constant.

Solution:

We can rearrange Einstein's explanation to get a linear function:

$$KE = hf - \phi$$

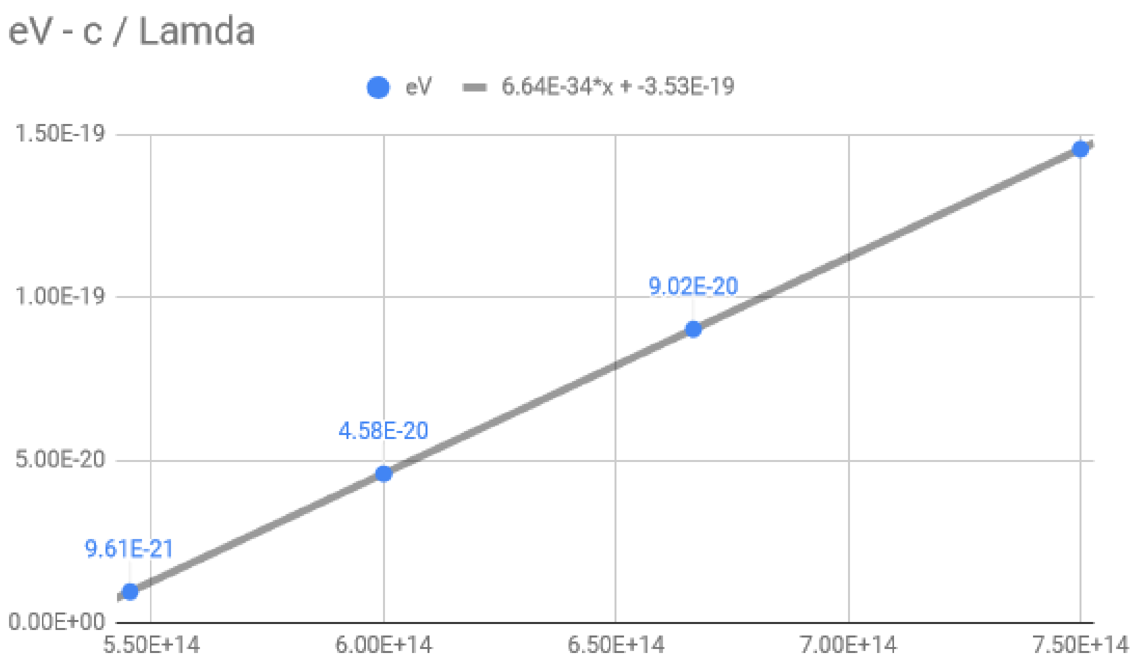
Note that by conservation of energy we have $KE = U$, where U is the potential energy of the system.

Noting that $qV = U$ we get: $qV = hf - \phi$. In this case we are only dealing with electrons so $q = e$, where e is the fundamental charge of an electron. Hence we have:

$$eV = h\frac{c}{\lambda} - \phi$$

So we can determine the Planck constant by finding the slope of this graph:

Plugging in our V values to eV and wavelengths in to $f = \frac{c}{\lambda}$:



Thus we know that $h \approx 6.64 \times 10^{-34}\ J \cdot s$.

5 #28 p.94

A television picture tube accelerates electrons through a potential difference of $30,000\,V$. Find the minimum wavelength to be expected in X-Rays produced in this tube. (Picture Tubes incorporate shielding to control X-Ray emission.)

Solution:

From section 3.3 we have the relation $qV = \frac{hc}{\lambda}$, for the production of electrons. So we know q is the fundamental charge of an electrons and $V = 30\,kV$, so we will solve for λ:

$$eV = \frac{hc}{\lambda} \iff \lambda = \frac{hc}{eV} = 4.139 \times 10^{-11}\,m = 4.139 \times 10^{-2}\,nm = 0.04139\,nm$$

Thus we have a minimum wavelength of $\lambda = 0.04139\,nm$.

6 #31 p.94

A 0.057[nm] X-ray photon "bounces off" an initially stationary electron and scatters with a wavelength of 0.061 nm. Find the directions of scatter of

a.

The photon

Solution:

Note that we have $\lambda' = 0.061\, nm$ and $\lambda = 0.057\, nm$. Using the Compton effect equation: $\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$:

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta) \iff \cos\theta = 1 - \frac{(\lambda' - \lambda)m_e c}{h}$$

$$\theta = 130.4^o$$

Hence we have that the photon scatters 130.4^o from the positive x-axis (pointing in the right direction).

b.

The electron

Solution:

Here we want to solve for the angle ϕ given the equations:

$$\begin{cases} \frac{h}{\lambda} = \frac{h}{\lambda'}\cos 130.4^o + \gamma_u m_e u \cos\phi \\ \gamma_u m_e u \sin\phi = \frac{h}{\lambda'}\sin 130.4^o \\ \frac{hc}{\lambda} + m_e c^2 = \frac{hc}{\lambda'} + \gamma_u m_e c^2 \end{cases}$$

Using the third equation we can solve for u:

$$u = \sqrt{c^2 - \left(\frac{m_e c^2}{\frac{h}{\lambda} - \frac{h}{\lambda'} + m_e c}\right)^2} = 2.237 \times 10^7\, \frac{m}{s} = 0.0746c\, \frac{m}{s}$$

Plugging this back into the second equation:

$$\gamma_u m_e u \sin\phi = \frac{h}{\lambda'}\sin 130.4^o \iff \sin\phi = \frac{\frac{h}{c}\sin 130.4^o}{\gamma_u m_e u}$$

$$\phi = 23.88^o$$

Hence the electron is scattering in the 23.88^o direction from the positive x-axis.

7 #32 p.94

A 0.065 nm X-ray source is directed at a sample of carbon. Determine the maximum speed of scattered electrons.

Solution:

Note that using the Compton effect, we have the greatest speed of a scattering occurs when the photon scatters backwards, so when $\theta = 180^o$. Plugging this into the Compton effect relation:

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta) \iff \lambda' = \frac{h}{m_e c}(2) + \lambda = 6.985 \times 10^{-11}\, m$$

Using the conservation of energy observed in the Compton effect:

$$h\frac{c}{\lambda} - h\frac{c}{\lambda'} = \gamma_u m_e c^2 - m_e c^2$$

Using this to solve for u:

$$\gamma_u = 1 - \frac{\frac{hc}{\lambda'} - \frac{hc}{\lambda}}{m_e c^2} \implies u = 2.16 \times 10^7\, \frac{m}{s} = 0.0719c\, \frac{m}{s}$$

Thus the maximum speed of a scattered electron is $0.072c\, \frac{m}{s}$.