# MATH 117: Precalculus Algebra 2

Instructor: Joseph McGuire

09/12/2020

# Class Logistics

- Make sure you have access to the Canvas, page email me at: jmcgui05@calpoly.edu if you don't.
- My office hours are 12 - 1pm Monday/Wednesday, if you can't make it to these email me for an appointment.
- The meeting ID will always be: 760-722-9034, I'll have a waiting room open so it might take me a minute to jump on the call but I'll be there promptly.
- The 1st homework is posted and will be based on the coming slides.
- This class is at slower pace than Math 118, but prepared to work on this stuff.

# Polynomial Review

What exactly is a polynomial?

- It's a function, of the form: $p(x) = a_0 + a_1x + ... + a_nx^n$.
- The $a$-terms are constant, $x$ can be whatever number you want.
- The $a_0$ is the constant term.
- The $a_n$ is the leading term, where $n$ is the degree of the polynomial.
- If $p(x) = a_0$ ( a constant), then the polynomial has degree 0.

# Polynomial Review

Okay, but really what is a polynomial?

- It's basically just anything that is written as a sum of $(1, x, ..., x^n)$, with whatever coefficients, $a_n \neq 0$
- Quadratics are important: $ax^2 + bx + c$
- Through some playing around with these numbers: $a(x - h)^2 + k$, where $(h, k)$ is a vertex, $a > 0$ upwards, $a < 0$ downwards

# Polynomial Review

Find the maximum/minimum of the quadratic $f(x) = ax^2 + bx + c$.

- First, pull out a $a$:

$$f(x) = ax^2 + bx + c = a\left(x^2 + \frac{b}{a}x\right) + c$$

- We "complete the square":

$$a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + c - a\left(\frac{b^2}{4a^2}\right)$$

- Notice that $(n+m)^2 = n^2 + 2nm + m^2$:

$$a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + c - a\left(\frac{b^2}{4a^2}\right) = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$$

# Polynomial Review

- It's in standard form!

$$a\left(x^2+\frac{b}{a}x+\frac{b^2}{4a^2}\right)+c-a\left(\frac{b^2}{4a^2}\right)=a\left(x+\frac{b}{2a}\right)^2+c-\frac{b^2}{4a}$$

- So it has to have a vertex at $(h,k)$ where $h=-\frac{b}{2a}, k=c-\frac{b^2}{4a}$.
- But this depends on $a>0$(Minimum), or $a<0$(Maximum).

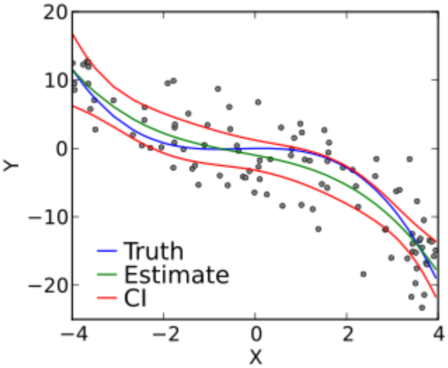
# Polynomial Review

Why is this important?

- Lot's of things depend on polynomials.
- Like motion of a falling objects:

$$x(t) = a_0 t^2 + v_0 t + x_0$$

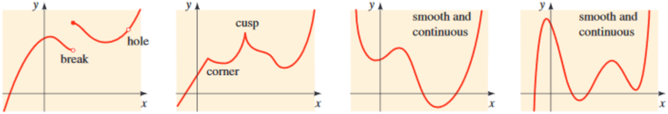
- Polynomials are very important in data fitting:



# Section 3.2: Graphing Polynomials

How can we relate what a polynomial look's like when graphed and the formula for it: $a_0 + a_1x + ... + a_nx^n$?

- Monomials: $x^1, x^2, x^3, ..., x^{100}$, just have 1-term
- Polynomials are always smooth and continuous:




Not the graph of a polynomial function | Not the graph of a polynomial function | Graph of a polynomial function | Graph of a polynomial function

FIGURE 3

How about where a polynomial equals 0?

- $p(x) = a_0 + a_1x + ... + a_nx^n$
- To find a zero we get the equation: $0 = a_0 + a_1x + ... + a_nx^n$
- If $p(c) = 0$, then we can actually rewrite this as another polynomial: $p(x) = a_0 + a_1x + ... + a_nx^n = (x - c)(b_0 + b_1x + ... + b_nx^{n-1})$
- The $b$'s are not the same as the $a$'s!

Can we use the fact that polynomials are smooth?

- If we have a polynomial $p(x) = a_0 + a_1 x + ... + a_n x^n$, and we get that $p(1) < 0$ and $p(2) > 0$.
- For $p$ to be continuous there's a point where $p(x) = 0$ in between 0 and 1!

**GUIDELINES FOR GRAPHING POLYNOMIAL FUNCTIONS**

1. **Zeros.** Factor the polynomial to find all its real zeros; these are the $x$-intercepts of the graph.
2. **Test Points.** Make a table of values for the polynomial. Include test points to determine whether the graph of the polynomial lies above or below the $x$-axis on the intervals determined by the zeros. Include the $y$-intercept in the table.
3. **End Behavior.** Determine the end behavior of the polynomial.
4. **Graph.** Plot the intercepts and other points you found in the table. Sketch a smooth curve that passes through these points and exhibits the required end behavior.

Let's zoom in around the zeros of the polynomial.

- For example, $p(x) = (x-2)^2$
- This kinds of looks like a monomial?
- It depends on the power of $(x-c)$!
- Will call this multiplicity

Polynomials near their roots will look like monomials, depending on "how many" times it occurs.

- That is, if the root $(x - c)$ has a multiplicity that's even, then it'll be cup-shaped
- If $(x - c)$ happens an odd number of times it'll be saddle-shaped

Remember long division?

- If I give you two numbers 34 and 7, how do we divide these?

We can write this as $34 = (7)4 + 6$. In general this becomes the Division Algorithm:

- for any number $x$ and $d$ there are numbers $q$ and $r$ such that $x = qd + r$
- where $r < q$, why?

Now let's apply this to polynomials $p(x) = x^2 + 2x + 1$ and $q(x) = x^2 + x + 1$

In general let's apply this to polynomials $p(x)$ and $s(x)$

- For there will always exist polynomials $q(x)$ and $r(x)$ where $p(x) = s(x)q(x) + r(x)$.
- For number we had $r < q$, now we have polynomials $r(x)$ and $q(x)$ what now?

Notice that if we do this with $s(x) = x - d$

- Remember that $\deg(r(x)) < \deg(s(x))$ or $\deg(r(x)) = 0$
- $\deg(s(x)) = 1$, so $\deg(r(x)) = 0$
- $p(x) = (x - d)p(x) + r$
- If we plug in $x = d$, then $p(d) = (d - d)p(d) + r = r$
- Anytime you divide by $x - d$ the remainder will be $p(d)$!

What if $p(d) = 0$?

- Then let's look at $p(x)$ divided by $x - d$, well we have $p(d) = 0$ is the remainder!
- So $p(x)$ has a factor of $x - d$, where $p(d) = 0$!

From the previous sections, we found that $p(x)$ can be factored by $(x - d)$ if $p(d) = 0$; that is, $d$ is a root of $p$.

- If we fully factor $p(x) = x^3 - x^2 - 14x + 24$ we'll get

$$p(x) = (x - 2)(x - 3)(x + 4)$$

- Notice that $2, 3, 4$ all divide 24 and in fact: $(-3)(-2)4 = 24$
- So the roots of $p$ are factors of the constant term

**The Rational Zeros Theorem**: If the polynomial $P(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0$ has integer coefficients (where $a_n \neq 0$ and $a_0 \neq 0$), then every rational zero of $P$ is of the form

$$\frac{p}{q}$$

where $p$ and $q$ are integers $(..., -2, -1, 1, 2, ...)$ and $p$ is a factor of the constant coefficient $a_0$ and $q$ is a factor of the leading coefficient $a_n$.

**Proof.**

If $\frac{p}{q}$ has a rational zero, then we have to make sure that $p/q$ are in lowest terms (i.e: $2/4 = 1/2$), remember that ($q^n = qq^{n-1}$) when we pull out a $q$ we subtract from the exponent, and so we can pull that many copies of $q$ out of $q^n$ i.e $q^n = q^2 q^{n-2}$, the exponents "add" when you multiply by the same base, the base being $q$ here. $\square$

## Proof. (Cont.)

$$a_n\left(\frac{p}{q}\right)^n + ... + a_1\left(\frac{p}{q}\right) + a_0 = 0$$

$$q^n(a_n\frac{p^n}{q^n}) + ... + q_n(a_1\frac{p}{q}) + q_n a_0 = (q_n)0$$

$$a_np^n + a_{n-1}p^{n-1}q + ... + a_1q^{n-1}p + a_0q^n = 0$$

$$a_np^n + a_{n-1}p^{n-1}q + ... + a_1pq^{n-1} = -a_0q^n$$

$$p(a_np^{n-1} + a_{n-1}p^{n-2}q + ... + a_1q^{n-1}) = -a_0q^n$$

So since $p$ us a factor of the left, we have that either $p$ is a factor of $-a_0q^n$, since $p$ and $q$ aren't factors (remember we said they were reduced), then $p$ has to be a factor of $a_0$. If we use similar reasoning we get that $q$ is a factor of $a_n$. $\square$

How do we use this?

- Let's take: $q(x) = 2x^3 + x^2 - 13x + 6$.
- So we know any rational root will look like $\frac{p}{q}$
- And by the Rational Zeros Theorem we know $p$ is a factor of 6 and $q$ is a factor of 2.
- So since $6 = (1)(2)(3)$ and $2 = (1)(2)$
- So the possible values of $p = 1, 2, 3, 6$ and $q = 1, 2$
- So that the only possible rational roots after we do some reducing: $\frac{p}{q} = 1, 2, 3, 6, \frac{1}{2}, \frac{3}{2}$
- Now to plug these in and test them ...

That's nice, but what about irrational roots/zeros?

- A variation in sign of $p(x) = 2x^3 - 3x^2 - 3x - 1$, is the difference of signs of consecutive terms so 2 and $-3$ is a sign difference but $-3 \to -3$ isn't
- Descartes' Rule of Signs
- For a polynomial $P(x) = a_n x^n + ... + a_1 x + a_0$ where $a_n, ..., a_1, a_0$ are real numbers $(e.g. \pi, \frac{\pi}{2}, \frac{1}{2}, 1, 7, -13, \frac{-100000}{\pi})$
- The number of positive real roots/zeros of $P(x)$ is either equal to the number of variations in sign in $P(x)$ or is less than that by an even whole number.
- The number of negative real zeros/roots of $P(x)$ either is equal to the number of variations in sign in $P(-x)$ or is less than that by an even whole number

In plain language, the number of negative/positive roots is is less than or equal to how many times coefficients in $P(-x), P(x)$ switch signs compared to their neighbors.

- Let's try: $p(x) = x^2 - x + 1$
- Setting up a chain we see $(1 \to -1 \to 1)$ so that the number of positive roots is less than or equal to 2.
- $p(-x) = (-x)^2 - (-x) + 1 = x^2 + x + 1$
- There is no sign differences! So there are no negative roots!

# Section 3.5: Complex Zeros and The Fundamental Theorem of Algebra

Some problems don't have "real" (any number you can think of) solution:

$$x^2 + 1 = 0$$

But we want solutions for these equations. So if we act like we can solve for $x$:

$$x = \pm\sqrt{-1}$$

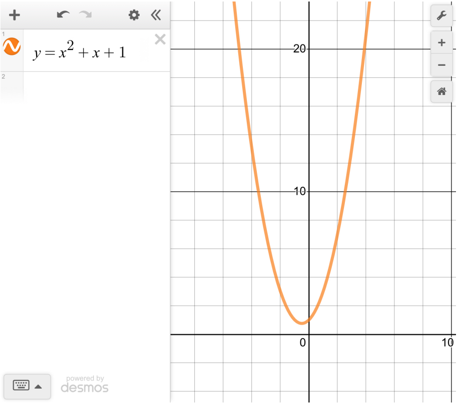
## Definition (Complex Number)

We have to extend the "real" numbers by this new number!! We'll call it $i = \sqrt{-1}$, equivalently $i^2 = -1$. So that complex numbers are $a + bi$ where $a$ and $b$ can be any "real" number.

# Section 3.5: Complex Zeros and The Fundamental Theorem of Algebra

Remember how we got to $i$: $x^2 + 1 = 0$, but that's a polynomial!! As it turns out lot's of polynomials have complex roots.

Look at $x^2 + x + 1$ and remember that we can just look at the graph of a polynomial to find its real roots.



That polynomial didn't have any roots on the graph!
But that doesn't mean it doesn't have a zero, remember $x^2 + 1$.

Well what can we say??

**Theorem (Fundamental Theorem of Algebra)**

*Every polynomial*

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0 \qquad (n \geq 1, a_n \neq 0)$$

*with complex coefficients has at least one complex zero.*

**Theorem (Complete Factorization Theorem)**

*If $P(x)$ is a polynomial of degree $n \geq 1$, then there exist complex numbers $a, c_1, c_2, \ldots, c_n$ (with $a \neq 0$) such that $P(x) = a(x - c_1)(x - c_2)\ldots(x - c_n)$.*

How do we use this?

- Let's look at $x^3 - 3x^2 + x - 3$
- $x^2(x-3) + (x-3) = (x^2+1)(x-3)$
- So the roots of this are given by:

$$x - 3 = 0$$
$$x^2 + 1 = 0$$

Remember multiplicity?

- The multiplicity of a root is how many times it occurs, $(x^2 + 2x + 1) = (x + 1)^2$ so $x = -1$ has multiplicity 2.

**Theorem (Zeros Theorem)**

*Every polynomial of degree $n \geq 1$ has exactly $n$ zeros, provided that a zero of multiplicity $k$ is counted $k$ times.*

Let's apply this to $P(x) = 3x^5 + 24x^3 + 48x$

- Factoring it: $3x(x^4 + 8x^2 + 16)$
- Factoring it again: $3x(x^2 + 4)^2$
- We know $x^2 + 4 = 0 \implies x = \pm 2i$
- $3x((x-2i)(x+2i))^2 = (3x)(x-2i)^2(x+2i)^2$
- So we have $x = 0$ has multiplicity 1, $x = 2i$ has multiplicity 2, and the same with $x = -2i$.

# Section 3.5: Complex Zeros and The Fundamental Theorem of Algebra

Notice how we ended up with $x = -2i$ and $x = 2i$ being roots in the previous problem, this isn't just a coincidence.

**Definition (Complex Conjugate)**

If we have a complex number $a + bi$, it's complex conjugate is $a - bi$

**Theorem (Conjugate Zeros Theorem)**

*If the polynomial $P$ has real coefficients and if the complex number $z = a + bi$ is a zero of $P$, then it's complex conjugate $\bar{z} = a - bi$ is also a zero of $P$.*

# Section 3.5: Complex Zeros and The Fundamental Theorem of Algebra

Let's prove this:

**Proof.**

Let $P(z) = 0$ be a root of $P$. Then note that $\overline{z^n} = (\bar{z})^n$.

$$
\begin{aligned}
P(\bar{z}) &= a_n(\bar{z})^n + \ldots + a_1\bar{z} + a_0 \\
&= \overline{a_n z^n + \ldots + a_1 z + a_0} \\
&= \bar{0} = 0.
\end{aligned}
$$

□

# Section 3.5: Complex Zeros and The Fundamental Theorem of Algebra

Finally, sometimes we don't want the complete factorization of a polynomial. Sometimes we want to stay in the real numbers. Say for $3x^5 + 24x^3 + 48x = (3x)(x^2 + 4)^2$, but we'll stop there because we want to stay in "real" land, but they still have complex zeros.

**Theorem (Linear and Quadratic Factors Theorem)**

*Any polynomial with real coefficients can be factored into a product of linear and irreducible quadratic factors with real coefficients.*

So far we have only dealt with polynomials:

$$p(x) = a_0 + a_1x^1 + a_2x^2 + \ldots + a_nx^n$$

These are things that are sums of positive integer $(0, 1, 2, \ldots, n)$ powers of $x$.

What about $\frac{1}{x} = x^{-1}$??

This is just a division of two polynomials: $p(x) = 1$ and $q(x) = x$.

Why might you use this?

In Physics, particles with mass can get increasingly faster and faster but there is a speed limit. This is model by:

$$\frac{c}{c^2 - v^2}$$

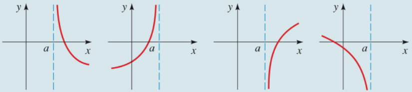
when $v \to c$ this thing will explode because $v^2 - c^2 \to 0$. This is a polynomial $p(v) = c$ divided by another polynomial, $q(v) = c^2 - v^2$.

One of the defining features of Rational Functions is that they are not as nice as polynomials. This means:

- They don't have to be "well-defined" everywhere, if $f(x) = \frac{1}{x}$, then what is $f(0)$??
- Let's first estimate this with $f(0.0000001) = \frac{1}{0.0000001} = 10000000$.
- What about $\frac{1}{1+x^2}$ at $x = 0$, is there anywhere this is badly defined?
- So we get that some rational functions are nice, some not so much.

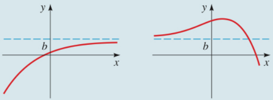
**DEFINITION OF VERTICAL AND HORIZONTAL ASYMPTOTES**

1. The line $x = a$ is a **vertical asymptote** of the function $y = f(x)$ if $y$ approaches $\pm\infty$ as $x$ approaches $a$ from the right or left.




$y \to \infty$ as $x \to a^+$ $\quad$ $y \to \infty$ as $x \to a^-$ $\quad$ $y \to -\infty$ as $x \to a^+$ $\quad$ $y \to -\infty$ as $x \to a^-$

2. The line $y = b$ is a **horizontal asymptote** of the function $y = f(x)$ if $y$ approaches $b$ as $x$ approaches $\pm\infty$.




$y \to b$ as $x \to \infty$ $\quad$ $y \to b$ as $x \to -\infty$

$f(x) = \frac{1}{x}$ has a vertical asymptote at $x = 0$, since $y \to \infty$ from the right and $y \to -\infty$ from the left. And a horizontal asymptote at $y = 0$, that is as $x \to \infty$ we get $y \to 0$.

To Desmos!

With polynomials, we put the problem of graphing them near roots as finding out the multiplicity of that root.

So we put the problem of graphing rational functions in terms of how they compare to $f(x) = \frac{1}{x}$.

Let's do this with $\frac{3x+5}{x+2}$.

- If we carry out that long division we get: $3 - \frac{1}{x+2}$
- But notice that $x \to -2, y \to -\infty$
- So we can actually write this out as $f(x) = \frac{1}{x}$ with $x$ replaced by $x + 2$ so that $f(x+2) = \frac{1}{x+2}$.
- This "shifts" the graph to the left.
- And it's inverted since we have $\frac{-1}{x+2}$
- And shifted up by 3 since we're adding by 3.
- Take a look at Desmos

This actually works for anything that looks like $f(x) = \frac{ax+c}{bx+d}$.

How to we find these asymptotes for more complicated functions?? Like

$$\frac{2x^2-4x+5}{x^2-2x+1}$$

- First we've got to simplify this if we can
- Note let's never talk about complex numbers ever again, we want real roots!!
- $\frac{2x^2-4x+5}{x^2-2x+1} = \frac{2x^2-4x+5}{(x-1)^2}$
- So we get $x \to 1^-$ and $x \to 1^+$ gives us $y \to \infty$
- This is because $2x^2 - 4x + 5$ at $x = 1$ is $2 - 4 + 5 = 3 > 0$ and the denominator goes to zero, but is never zero!!
- This gives us our vertical asymptotes, places where the function doesn't make sense

$$\frac{2x^2-4x+5}{x^2-2x+1} = \frac{2x^2-4x+5}{(x-1)^2}$$

- In general the only places where horizontal asymptotes appear are at $x \to \pm\infty$, so let's just "evaluate" the functions there
- This is kind of difficult just looking at the function though, it's always easier to look at $\frac{1}{x}$
- $\frac{2x^2-4x+5}{x^2-2x+1} = \frac{2-\frac{4}{x}+\frac{5}{x^2}}{1-\frac{2}{x}+\frac{1}{x^2}}$
- As we let $x \to \infty$, we get everything disappears except $\frac{2}{1} = 2$!!
- As we do this for $x \to -\infty$ the same thing happens!!
- So we have horizontal asymptotes at $y = 2$, since $f(x) \to 2$ as $x \to \infty$

This process will be very important!!

## FINDING ASYMPTOTES OF RATIONAL FUNCTIONS

Let $r$ be the rational function

$$r(x) = \frac{a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + \cdots + b_1 x + b_0}$$

1. The vertical asymptotes of $r$ are the lines $x = a$, where $a$ is a zero of the denominator.
2. **(a)** If $n < m$, then $r$ has horizontal asymptote $y = 0$.

   **(b)** If $n = m$, then $r$ has horizontal asymptote $y = \dfrac{a_n}{b_m}$.

   **(c)** If $n > m$, then $r$ has no horizontal asymptote.

## SKETCHING GRAPHS OF RATIONAL FUNCTIONS

1. **Factor.** Factor the numerator and denominator.
2. **Intercepts.** Find the $x$-intercepts by determining the zeros of the numerator and the $y$-intercept from the value of the function at $x = 0$.
3. **Vertical Asymptotes.** Find the vertical asymptotes by determining the zeros of the denominator, and then see whether $y \to \infty$ or $y \to -\infty$ on each side of each vertical asymptote by using test values.
4. **Horizontal Asymptote.** Find the horizontal asymptote (if any), using the procedure described in the box on page 300.
5. **Sketch the Graph.** Graph the information provided by the first four steps. Then plot as many additional points as needed to fill in the rest of the graph of the function.

As a quick aside, what if $\frac{x(x-3)}{x^2(x-3)}$, then technically we can never plug in $x = 3$, but this will look identical to $\frac{1}{x}$. So the graph will have a "hole" at this point

Not all asymptotes are vertical or horizontal, we can actually force a "slanted" asymptote when $r(x) = ax + b + \frac{R(x)}{Q(x)}$.

- Take something of the form: $r(x) = ax + b + \frac{R(x)}{Q(x)}$ where $\deg(R(x)) = \deg(Q(x))$
- We know that $x \to \infty$ then $\frac{R(x)}{Q(x)} \to 0$ so that all that's left over is $ax + b$
- That's a line
- Desmos!

If we have $x^4 - x^2 = 0$ we can figure this out with our tools we've developed so far. Descartes's Rule of Signs, Rational Roots Theorem, testing values to find out which are negative versus positive.

But what about $x^4 - x^2 > 0$, well in turns out this is the same problem as $x^4 - x^2 = 0$ with $>$ taking the place of $=$.

What do I mean by this?

- $x^4 - x^2 > 0$
- $x^2(x^2 - 1) > 0$
- $x^2(x-1)(x+1) > 0$

So $x^2$ is always positive, for real numbers. So when is $x - 1 > 0$ and $x + 1 > 0$.

We'll get the inequalities:

$$x > 1 \text{ and } x > -1$$

So combining these two we just found that $x^4 - x^2 > 0$ is only true for $-1 < x < 1$.

But hold on we still have $x^2 = 0$ at $x = 0$. So we messed up this is only true for $-1 < x < 0$ and $0 < x < 1$.

This process will hold in general:

**SOLVING POLYNOMIAL INEQUALITIES**

1. **Move All Terms to One Side.** Rewrite the inequality so that all nonzero terms appear on one side of the inequality symbol.
2. **Factor the Polynomial.** Factor the polynomial into irreducible factors, and find the **real zeros** of the polynomial.
3. **Find the Intervals.** List the intervals determined by the real zeros.
4. **Make a Table or Diagram.** Use test values to make a table or diagram of the signs of each factor in each interval. In the last row of the table determine the sign of the polynomial on that interval.
5. **Solve.** Determine the solutions of the inequality from the last row of the table. Check whether the **endpoints** of these intervals satisfy the inequality. (This may happen if the inequality involves $\leq$ or $\geq$.)

But this chapter isn't titled Polynomial Inequalities.
What if we want to compare two polynomials? Say $x^2 - 2x - 3$ and $1 - 2x$. Then if we want to find the point's where $1 - 2x$ is greater that $x^2 - 2x - 3$, that's the inequality:

$$1 - 2x \geq x^2 - 2x - 3$$

$$\frac{1 - 2x}{x^2 - 2x - 3} \geq 1$$

We can then put this in terms of something that looks like a "root" finding problem:

$$\frac{1-2x}{x^2-2x-3}-1\geq 0$$

Doing the algebra

$$\frac{4-x^2}{x^2-2x-3}\geq 0$$

Difference of squares: $a^2-x^2=(a-x)(a+x)$ and plug and check $x^2-2x-3$ with $x=-1$ and $x=3$ ( could also use the rational roots theorem...)

$$\text{So we have } \frac{(x-2)(x+2)}{(x-3)(x+1)} \geq 0.$$

- This is a much easier problem to solve
- Because we know this graph will have zeros at $x = 2, -2$ and vertical asymptotes at $x = -1, 3$
- So just check any point on the intervals $(-\infty, -2), (-2, -1), (-1, 2), (2, 3), (3, \infty)$.
- An alternative way of doing this is stopping before factoring and looking $\frac{4-x^2}{x^2-2x-3} \geq 0$.
- Just ask the questions $4 - x^2 \geq 0$ and $x^2 - 2x - 3 \geq 0$. When are these simulatenously true??

# Applications of Polynomials: Data Fitting

You've probably heard of linear regression.

- The idea behind this is take a data set $\{(a_1, b_1), \ldots, (a_n, b_n)\}$.
- The $a$'s could be something like position/age/number of Taco Bell visits in the last month and $b$'s could be the air temperature/wealth/Your average heart rate for the Month.
- Real data is typically random in some way, we don't like true randomness though, we like predictability.
- So we fit this data to a curve, a function, a polynomial!!

# Applications of Polynomials: Data Fitting

- A linear regression fits this data $(a_1, b_1), \ldots, (a_n, b_n)$ to a line $mx + a$.
- The process by which we do this is called interpolation and regression.
- The quantification of how "well" the data is fit to the line is called the $R$-value

Why does this matter?

- This allows us to determine relationship between things, called correlations.
- Google Sheets Example

# Applications of Polynomials: Approximation Using Polynomials

Polynomials are really nicely behaved functions, unlike $\sin(x)$ or as we'll see $e^x$ or $\log(x)$. So we like using polynomials instead of these functions, but how?

Desmos!