MATH 320: Presentation Problem

Joseph C. McGuire
Dr. J. Morris

December 06, 2018

Let H_1 and H_2 be subgroups of a group G. Prove that $H_1 \cap H_2$ is a subgroup of G.

Proof. Let G be a group and H_1 and H_2 be subgroups of G. Note then that $e \in H_1$ and $e \in H_2$, where e is the identity element of G. Hence $e \in H_1 \cap H_2$, thus $H_1 \cap H_2 \neq \emptyset$. Next we will show that $H_1 \cap H_2$ is a subgroup using Theorem 3.2 (Two-Step Subgroup Test).
($a^{-1} \in H_1 \cap H_2$)

Let $a \in H_1 \cap H_2$, then $a \in H_1$ and $a \in H_2$. Thus, since H_1 and H_2 are subgroups of G we have $a^{-1} \in H_1$ and $a^{-1} \in H_2$, thus $a^{-1} \in H_1 \cap H_2$.
($ab \in H_1 \cap H_2$)

Let $a, b \in H_1 \cap H_2$. Then we have $a \in H_1$, $a \in H_2$, $b \in H_1$, and $b \in H_2$. Furthermore, since H_1 and H_2, we have $ab \in H_1$ and $ab \in H_2$. Hence $ab \in H_1 \cap H_2$.

Thus, by Theorem 3.2 (Two-Step Subgroup Test), we have $H_1 \cap H_2$ is a subgroup of G. □