General Relativity: Celestial Mechanics

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Introduction

- So what is General Relativity?
- Provides a very accurate picture of gravity.
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The Geodesic Equation and Particle Trajectories

- We're physicists and want to know how things move near these weird objects.
- We want a version of $\mathbf{F} = m\mathbf{a} = m\frac{d^2\mathbf{r}}{dt^2}$ for the space near a black hole.
- What we want is the Geodesic Equation:

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu{}_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = 0$$

Fundamentally a geodesic is a geometrical concept, the notion of a straight line in a non-standard geometry.

- A really nice example of light following a geodesic can be seen in the ray model of optics,
- Where light will actually change it's path at the interface between two mediums:

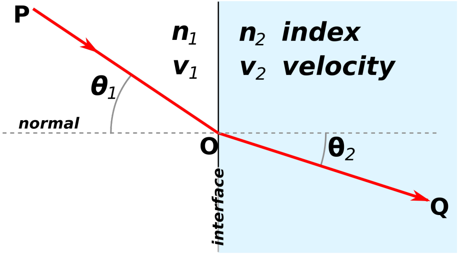

Figure: 3, a ray of light bending at an interface

Geodesic Equation Explanation

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0$$

This is a fairly complicated equation, with a lot going on.

- First off, we're using the Einstein summation convention. Which is that we sum over t, x, y, z when we have diagonals and everything else we can choose it to be either t, x, y, z.
- The important quantities in this are the proper time acceleration; that is, the acceleration measured in the particle's own frame.
- The Christoffel symbol which is some tensor containing information of the curvature of the space time.
- This is equivalent to the equations of motions for a free particle in Newtonian mechanics.

The Work Involved

- The bulk of the work involved is solving ODEs numerically
- This is done by essentially approximating the solution of the ODE at a finite set of points
- This is done by using a Taylor, or Euler, Approximation:

$$\frac{dr}{dt} = \frac{r(t_n) - r(t_{n-1})}{\Delta t} + O((\Delta t)^2)$$

Workspace

- So the bulk of my work will be utilizing these approximations on relevant ODEs or PDEs
- Coming up with suitable initial conditions and values of related constants
- Then finally implementing these into Matlab

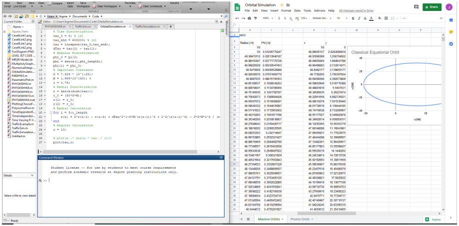

Figure: 4, My workspace

Orbits!

Comparing the geodesic equation

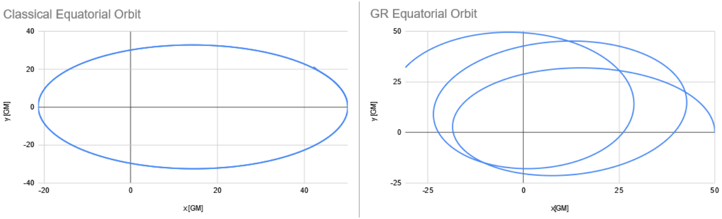

Figure: 5, The Newtonian and Einsteinian Orbits around a star.

Future Work

- Work out orbits for a spinning or charged blackhole
- Model