MATH 117: Sample Final Exam

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October 19, 2024

Attempt all of the following problems, these will help prepare you for the content on the final exam. This material is meant to build your capability in doing similar problems, but the content on the final may be very different than what you see here. One thing I can guarantee is the final exam will not be this long, this is meant to be a good, but not total, cumulative review for the course. I will be posting solutions to these in a video and be posting the notes from these videos, I recommend before watching these that you attempt these problems on your own.

1

Use long division to find the quotient and remainder for the following polynomials.

1.1

$2x^5 + 4x^4 - x^3 - x^2 + 7$ is divided by $2x^2 - 1$.

1.2

$x^2 - 3x + 7$ is divided by $x - 2$.

1.3

$4x^3 + 2x^2 - 2x - 3$ is divided by $2x + 1$.

2

Find the zeros of the following polynomials, and write them in factored form

2.1

$$x^3 + 2x^2 - 13x + 10$$

2.2

$$x^3 + 11x^2 + 8x - 20$$

2.3

$$4x^4 - 37x^2 + 9$$

3

Solve the following rational and polynomial inequalities.

3.1

$$\frac{(1-x)^2}{\sqrt{x}} \geq 4\sqrt{x}(x-1)$$

3.2

With $0 < a < b < c$ being constants:

$$\frac{x^2 + (a-b)x - ab}{x+c} \leq 0.$$

3.3

With $a < b < c < d$ being constants:

$$(x-a)(x-b)(x-c)(x-d) \geq 0.$$

4

Solve the following logarithmic and exponential equations.

4.1

$$5\ln(3-x) = 4$$

4.2

$$\log_2(x+2) + \log_2(x-1) = 2$$

4.3

$$2^{2/\log_5(x)} = \frac{1}{16}$$

4.4

$\log_2(\log_3(x)) = 4$

5

A hot bowl of soup is served at a dinner party. It starts to cool according to Newton's Law of Cooling, so its temperature at time t is given by

$$T(t) = 65 + 145e^{-0.05t}$$

where t is measured in minutes and T is measured in ^oF.

1. What is the initial temperature of the soup?
2. What's the temperature after 10 min?
3. After how long will the temperature be 100^oF?

6

Radium-221 has a half-life of 30 s. How long will it take for 95% of a sample to decay?

7

The following system of equations have unique solutions. Using Gaussian elimination or Jordan-Gauss elimination solve the following systems of equations.

7.1

$$\begin{cases} 2x_1 + x_2 & = 7 \\ 2x_1 - x_2 + x_3 & = 6 \\ 3x_1 - 2x_2 + 4x_3 & = 11 \end{cases}$$

7.2

$$\begin{cases} 2y + z & = 4 \\ x + y & = 4 \\ 3x + 3y - z & = 10 \end{cases}$$

8

The following system of equations are either inconsistent or dependent. Determine which, and if they're dependent solve for the complete solution in ordered pair notation with parameter t (s & t if there are more than one parameter)

8.1

$$\begin{cases} y - 5z & = 7 \\ 3x + 2y & = 12 \\ 3x + 10z & = 80 \end{cases}$$

8.2

$$\begin{cases} -2x + 6y - 2z & = -12 \\ x - 3y + 2z & = 10 \\ -x + 3y + 2z & = 6 \end{cases}$$

9

For the following matrices, find a pattern in the powers of the matrices, that is, for a square matrix A, compute $A, A^2, A^3, \ldots$ until you detect a pattern.

9.1

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

9.2

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

10

For the following matrices, determine for what values of x does the matrix have an inverse. For the values of x where the inverse does exists, calculate its inverse.

10.1

$$\begin{bmatrix} 2 & x \\ x & x^2 \end{bmatrix}$$

10.2

$$\begin{bmatrix} x & 1 \\ -x & \dfrac{1}{x-1} \end{bmatrix}$$

11

Solve the following matrix equations by finding the inverse of the coefficient matrix.

11.1

$$\begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} x & y & z \\ u & v & w \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \end{bmatrix}$$

11.2

$$\begin{bmatrix} 0 & -2 & 2 \\ 3 & 1 & 3 \\ 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} x & u \\ y & v \\ z & w \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 6 & 12 \\ 0 & 0 \end{bmatrix}$$

12

Assume $a, b, c > 0$ are constants. Find the determinants of the following matrices.

12.1

$$\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

12.2

$$\begin{bmatrix} a & a & a \\ 0 & b & b \\ 0 & 0 & c \end{bmatrix}$$

12.3

$$\begin{bmatrix} a & 0 & a \\ 0 & b & 0 \\ c & 0 & c \end{bmatrix}$$