Joseph C. McGuire
Dr. Scott Severson

Homework #2

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Problem 1.12

Calculate the average volume per molecule for an ideal gas at room temperature and atmospheric pressure. Then take the cube root to get an estimate of the average distance between molecules. How does this distance compare to the size of a small molecule like N_2 or H_2O?

Solution:

Using the ideal gas law, $T = 300\ K$ and $P = 101325\ Pa$:

$$PV = NkT$$

$$(101325)\ [Pa]\ V\ [m^3] = N(1.381 \times 10^{-23}\ [J/K])(300\ K)$$

$$\frac{V}{N} = \frac{1.38 \times 10^{-23}(300)}{101325}\ m^3 = 4.089 \times 10^{-26}\ m^3$$

Then average distance is:

$$< d >= 3.445 \times 10^{-9}\ m = 3.445\ nm$$

The radius of a molecule of N_2 is around $100\ pm = 100 \times 10^{-12}$ and for H_2O is about $300\ pm = 300 \times 10^{-12}$, so this is about 30 times farther away than the entire radius of a molecule.

Problem 1.18

Calculate the rms speed of a nitrogen molecule at room temperature.

Solution:

Nitrogen has an atomic weight of $14.01\frac{g}{\text{mole}}$, meaning that the average weight of one molecule of Nitrogen is: $\frac{14.01}{N_A} = 2.325 \times 10^{-23}\frac{\text{g}}{\text{molecule}}$. So then we have:

$$v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3k(300)}{m}} = 23.12\frac{m}{s}$$

1.23

Calculate the total thermal energy in a liter of helium at room temperature and atmospheric pressure. Then repeat the calculation for a liter of air.

Solution:

We know: $U_{\text{Thermal}} = \frac{f}{2}NkT = \frac{f}{2}(PV)$, by The Ideal Gas Law. So we have atmospheric pressure and a volume of 1 liter, additionally Helium is monoatomic, so has $f = 3$. Hence we have:

$$U_{\text{Thermal}} = \frac{3}{2}(101325\ Pa)(0.001\ m^3) = 151.98\ J$$

The only difference for air is that it is diatomic, so $f = 5$. So we have:

$$U_{\text{Thermal}} = \frac{5}{2}(101325\ Pa)(0.001\ m^3) = 253.31\ J$$

Problem 1.28

Estimate how long it should take to bring a cup of water to boiling temperature in a typical 600-watt microwave oven, assuming that all the energy ends up in the water. (Assume any reasonable initial temperature for the water.) Explain why no heat is involved in the process.

Solution: Assume the water is at a temperature of 10^oC, then we know one calorie will raise one gram of water by 1 ^oC. One cup of water is about 236 ml of water is about 1 gram. So then we have 236 grams of water and energy of 600 Joule per second. 1 calorie $= 4.18\ J$, so the microwave generating about 143 calories per second. So then to get from 10^oC to 100^oC, we need about $236 * 90 = 21240$ calories, then it would be about $21240/143\ s = 148.5\ s$ to heat up this cup of water.

Change in thermal energy here is all work, because of the energy entering the system in the form of high energy waves that then cause the material to heat up on interaction. This is similar to stirring, or other forms of work.