Chapter 11: Magnetic Forces and Fields

Section 11.1: Magnetism and Its Historical Discoveries

Section 11.2: Magnetic Fields and Lines

The magnitude of the electromagnetic force is determined by the force that a particle experiences when moving through this field, it is proportional to the amount of charge q, the speed of the charged particle v, and the magnitude of the applied magnetic field. The direction of this force is perpendicular to both the direction of the moving charged particle and the direction of the applied magnetic force. Defining B as the magnetic field, then we have:

$$\vec{F} = q\vec{v} \times \vec{B} \tag{1}$$

Using the geometric definition of a cross-product we get:

$$||F|| = |q|||v||||B||sin(\theta) \tag{2}$$

where θ is the angle between the velocity and the magnetic field.
Note that the units of this are $\frac{1N}{A \cdot m}$, we define this as follows:

$$1[T] = 1\frac{[N]}{[A] \cdot [m]} \tag{3}$$

A smaller unit called a **Gauss[G]**, is defined by $1[G] = 10^{-4}[T]$. The direction of the force is given by the right-hand rule for a cross-product.
An important note is that there is no magnetic force on static charges.

Example 11.1: An Alpha-Particle Moving in a Magnetic Field

An alpha-particle ($q = 3.2 \times 10^{-19}[C]$) moves through a uniform magnetic field whose magnitude is $1.5[T]$. The field is directly parallel to the positive z-axis of the rectangular coordinate system of Figure 11.5. What is the magnetic force on the alpha-particle when it is moving (a) in the positive x-direction with a speed of $5.0 \times 10^4[m/s]$? (b.) In the negative y-direction with a speed of $5.0 \times 10^4[m/s]$? (c.) In the positive z-direction with a speed of $5.0 \times 10^4[m/s]$? (d.) With a velocity $\vec{v} = (2.0\hat{\imath} - 3.0\hat{\jmath} + 1.0\hat{k}) \times 10^4[m/s]$?
(a.) $\vec{F} = 3.2 \times 10^{-19} * 5.0 \times 10^4 * 1.5 * sin(90^o) = 2.4 \times 10^{-14}(-\hat{\jmath})$.
(b.) $2.4 \times 10^{-14}(-\hat{\imath})$
(c.) 0, since $sin(0^o) = 0$.
(d.) $\vec{F} = q\vec{v} \times \vec{B} = (3.2 \times 10^{-19})((2.0, -3.0, 1.0) \times (0, 0, 1.5) = (3.2 \times 10^{-19})(-4.5, -3, 0) =$
$(-1.44 \times 10^{-18}, -9.6 \times 10^{-19}, 0)$

Representing Magnetic Fields

The field lines emerge from the north pole, loop around to the south pole, and continue through the bar magnet back to the north pole.
(1.) The direction of the magnetic field is tangent to the field line at any point in space. A small compass will point in the direction of the field line.
(2.) The strength of the field is proportional to the closeness of the lines. It is exactly proportional to the number per unit area perpendicular to the lines (called the areal density).
(3.) Magnetic field lines can never cross, meaning that the field is unique at any point in space.
(4.) Magnetic field lines are continuous, forming closed loops without a beginning or end. They are

directed from the north pole to the south pole.
Importantly too, is that the north and south poles cannot be separated.

Section 11.3: Motion of a Charged Particle in a Magnetic Field

With a charged particle moving perpendicular to a uniform B-field. Since the magnetic field is perpendicular to the direction of travel, the charged particle will follow a curved path until the charged particle completes a circle. Hence the force exerted is that of a centripetal force i.e $F_c = \frac{mv^2}{r}$. Recalling, that the force is perpendicular we also get, $F = qvB$. Hence

$$qvB = \frac{mv^2}{r} \tag{4}$$

Solving for r, we get:

$$r = \frac{mv}{qB} \tag{5}$$

The time for the particle to complete this path is defined as the Period(T), as discussed previously. Noting that $T = \frac{2\pi r}{v}$. We get:

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{qB} = \frac{2\pi m}{qB}. \tag{6}$$

If the velocity is not perpendicular to the magnetic field, then we can compare each component of the velocity separately with the magnetic field. The component of the velocity perpendicular to the magnetic field produces a magnetic force perpendicular to both this velocity and the field:

$$v_{perp} = vsin(\theta), v_{para} = vcos(\theta). \tag{7}$$

The parallel motion determines the pitch p of the helix, which is the distance between the adjacent turns. This distance equals the parallel component of the velocity times the period:

$$p = v_{para}T. \tag{8}$$

This is called the Helical motion.

Example 11.2: Beam Deflector

A research group is investigating short-lived radioactive isotopes. They need to design a way to transport alpha-particles (helium nuclei) from where they are made to a place where they will collide with another material to form an isotope. The beam of alpha-particles ($m = 6.64 \times 10^{-27}[kg], q = 3.2 \times 10^{-19}[C]$) bends through a 90-degree region with a uniform magnetic field of $0.050[T]$. (a.) In what direction should the magnetic field be applied? (b.) How much time does it take the alpha-particle to traverse the uniform magnetic field region?

(a.) Down

(b.) $T = \frac{2\pi \cdot 6.64\times 10^{-27}}{3.2\times 10^{-19}\cdot 0.050} = 2.6 \times 10^{-6}[s]$, since this is only the quarter of the circle we have $0.25 \cdot T = 6.5 \times 10^{-7}[s]$.

Example 11.3: Helical Motion in a Magnetic Field

A proton enters a uniform field of $1.0 \times 10^{-4}[T]$ with a speed of $5 \times 10^5[m/s]$. At what angle must the magnetic field be from the velocity so that the pitch of the resulting helical motion is equal to the radius of the helix?

From $p = v_{para}T$ we get, $r = vcos(\theta)\frac{2\pi m}{qB}$ iff $\frac{mv_{perp}}{qB} = 5 \times 10^5 cos(\theta)\frac{2\pi 1.67\times10^{-27}}{1.602\times10^{-19}1.0\times10^{-4}}$ iff $2\pi = tan(\theta)$ iff $\theta = 81.0^o$.

Section 11.4: Magnetic Force on a Current-Carrying Conductor

Since a moving charge experience a magnetic field, we ask what this will do in an electric wire. To determine the direction of the magnetic field generated by an electrical current: point your thumb in direction of the current, and wrapping your hands is the direction of the magnetic field that is induced here.
Calculating the Magnetic Force
Considering an infinitesimal band of wire we have the volume of the band is $V = A \cdot dl$, depending on the material, n is the charge carriers per unit volume, so the number of charge carriers in a section is $nA \cdot dl$. If the charge carriers move with the drift velocity, the current is $I = neAv_d$. The magnetic force on any single charge carrier is $e\vec{v}_d \times \vec{B}$, so the total magnetic force over an infinitesimal piece of wire is:

$$d\vec{F} = (nA \cdot dl)e\vec{v}_d \times \vec{B}. \tag{9}$$

If we define dl to be the vector of length dl pointing along $\vec{v}_d$, which means we get:

$$d\vec{F} = neAv_d\vec{dl} \times \vec{B} \tag{10}$$

or,

$$d\vec{F} = I\vec{dl} \times \vec{B}. \tag{11}$$

Differentiating both side, we get a more useful expression:

$$\vec{F} = I\vec{l} \times \vec{B}. \tag{12}$$

Note that to find the direction of this we must use the standard right-hand rule, and not the curl one.
Example 11.4: Balancing the Gravitational and Magnetic Forces on a Current-Carrying Wire
A Wire of length 50 cm and mass 10 g is suspended in a horizontal plane by a pair of flexible leads (Figure 11.13). The wire is then subjected to a constant magnetic field of magnitude 0.50 T, which is directed as shown. What are the magnitude and direction of the current in the wire needed to remove the tension in the supporting leads?
We want to balance out the gravitational forces and magnetic, hence $mg = IlB$ iff $0.01 \cdot g = I \cdot 0.5 \cdot 0.50$ and solving for I we get: $I = 0.39[A]$.
Example 11.5: Calculating Magnetic Force on a Current-Carrying Wire
A long, rigid wire lying along the y-axis carries a 5.0[A] current flowing in the positive y-direction. (a.) If a constant magnetic field of magnitude 0.3[T] is directed along the positive x-axis, what is the magnetic force per unit length on the wire? (b.) If a constant magnetic field of 0.30 [T] is directed 30 degrees from the +x-axis towards the +y-axis, what is the magnetic force per unit length on the wire?
(a.) $\frac{F}{l} = IBsin(\theta) = 5.0 \cdot 0.30 \cdot sin(90^o) = 1.5[\frac{N}{m}](-\hat{k})$
(b.) $\vec{F} = I\vec{l} \times \vec{B} = (5.0)l\hat{j} \times (0.40cos(30^o)\hat{i} + 0.30sin(30^o)\hat{j}) = -1.30\hat{k}[\frac{N}{m}]$
Example 11.6: Force on a Circular Wire
A circular current loop of radius R carrying a current I is placed in the xy-plane. A constant uniform magnetic field cuts through the loop parallel to the y-axis (Figure 11.14). Find the magnetic force on the

upper half of the loop, the over half of the loop, and the total force on the loop.
Recall (11) and that $dl = R \cdot d\theta$, hence $dF = IRBd\theta$, and integrating that from $-\pi$ to 0. We have:

$$\int_{-\pi}^{0} IBRsin(\theta)d\theta = -IBR\int_{0}^{\pi} sin(\theta)d\theta = IBR(cos(\theta))\Big|_{0}^{\pi} = IBR(-1-(1)) = -2IBR \tag{13}$$

Chapter 12: Sources of Magnetic Fields

Section 12.1: The Biot-Savart Law

The Biot-Savart law states that at any point P (Figure 12.2), the magnetic field $d\vec{B}$ due to an element $d\vec{l}$ of a current-carrying wire is given by:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l} \times \hat{r}}{r^2} \tag{14}$$

The constant μ_0 is known as the permeability of free space and is exactly:

$$\mu_0 = 4\pi \times 10^{-7}[T \cdot m/A] \tag{15}$$

The infinitesimal wire segment $d\vec{l}$ is the same direction as the current I (assumed positive), r is the distance from $d\vec{l}$ to P ad $\hat{r}$ is a unit vector that points from $d\vec{l}$ to P. The direction of $d\vec{B}$ is determined by applying the right-hand rule to vector product $d\vec{l} \times \hat{r}$. The magnitude of $d\vec{B}$ is

$$dB = \frac{\mu_0}{4\pi}\frac{Idlsin(\theta)}{r^2} \tag{16}$$

where θ is the angle between $d\vec{l}$ and $\hat{r}$.
The Biot-Savart Law: The magnetic filed $\vec{B}$ due to an element $d\vec{l}$ of a current-carrying wire is given by

$$\vec{B} = \frac{\mu_0}{4\pi}\int_{wire}\frac{Id\vec{l} \times \hat{r}}{r^2} \tag{17}$$

Example 12.1: Calculating Magnetic Fields of Short Current Segments
A short wire of length 1.0 cm carries a current of 2.0 A in the vertical direction (Figure 12.3). The rest of the wire is shielded so it does not add to the magnetic field produced by the wire. Calculate the magnetic field at point P, which is 1 meter from the x-direction.
$\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta lsin(\theta)}{r^2} = 10^{-7}\left(\frac{2(0.01)sin(89.4^o)}{1^2}\right) = 2.0 \times 10^{-9}[T]$, with direction into the page.
Example 12.2: Calculating Magnetic Field of a Circular Arc of Wire
A wire carries a current I in a circular arc with radius R swept through an arbitrary angle θ (Figure 12.4). Calculate the magnetic field at the center of this arc at point P.
$\int_0^\theta \frac{\mu_0}{4\pi}\frac{IRd\theta}{R^2}$ if and only if $B = \frac{\mu_0 I\theta}{4\pi R}$

Section 12.2: Magnetic Field Due to a Thin Straight Wire

Considering the magnetic field due to the current element $Id\vec{x}$ located at the position x. Using Biot-Savart's Law we have the induced magnetic field is given by (17), to find the direction, we can just apply the first right-hand rule on the cross-product. At a point P, therefore, the magnetic fields due to all current elements have the same direction. This means that we can calculate the net field there by evaluating the scalar sum of the contributions of the elements. With $|d\vec{x} \times \hat{r}| = dx(1)sin(\theta)$, we have from the Biot-Savart law:

$$B = \frac{\mu_0}{4\pi} \int_{wire} \frac{I \sin\theta dx}{r^2}. \tag{18}$$

The wire is symmetrical about point O, so we can set the limits of the integration from zero to infinity and double the answers, rather than integrate from negative infinity to positive infinity. Based on the picture and geometry, we can write expressions for r and $sin(\theta)$ in terms of x and R, namely:

$$r = \sqrt{x^2 + R^2}$$

$$sin(\theta) = \frac{R}{\sqrt{x^2 + R^2}]}$$

Substituting these expression into Equation 12.5, the magnetic field integration becomes

$$B = \frac{\mu_0 I}{2\pi} \int_0^\infty \frac{Rdx}{(x^2 + R^2)^{3/2}}$$

.

Evaluating the integral yields

$$B = \frac{\mu_0 I}{2r\pi R} \left[\frac{x}{(x^2 + R^2)^{1/2}} \right]_0^\infty$$

Substituting the limits gives us the solution

$$B = \frac{\mu_0 I}{2\pi R}$$

.

The magnetic field lines of the infinite wire are circular and centered at the wire (Figure 12.6), and they are identical in every plane perpendicular to the wire. Since the field decreases with distance from the wire, the spacing of the field lines must increase correspondingly with distance. The direction of this magnetic field may be found with a second form of the right-hand rue (illustrated in Figure 12.6). If you hold the wire with your right hand so that your thumb points along the current, then your wrap around the wire in the same sense as $\vec{B}$. The direction of the field lines can be observed experimentally by placing several small compass needles on a circle near the wire, as illustrated in Figure 12.7. When there is no current in the wire, the needles align with Earth's magnetic field. However, when a large current is sent through the wire, the compass needles all point tangent to the circle. Iron filings sprinkled on a horizontal surface also delineate the field lines, as shown in Figure 12.7.

Example 12.3: Calculating Magnetic Field due to Three Wires

Three wires sit at the corners of a square, all carrying currents of 2 amps into the page as shown in Figure 12.8. Calculate the magnitude of the magnetic field at the other corner of the square, point P, if the length of each side of the square is 1[cm].

Chapter 1: The Nature of Light

Section 1.2: The Law of Reflection

The Law of Reflection:

$$\theta_r = \theta_i$$

In empirical terms, this is the angle of reflection is equal to the angle of incidence.

Section 1.3: Refraction

The changing of a light ray's direction (loosely called bending) when it passes through substances of different refractive indices is called **refraction** and is related to changes in the speed of light,

$$v = \frac{c}{n}$$

Note that the path of refraction is always reversible. If the ray is moving from a greater indices to a lower index, then we will see the ray moving closer to the surface normal line. This will give us the **Law of Refraction**:

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

Where the angles θ_1 and θ_2 are the angles between the respective light and the surface normal.

Example 1.2: Determining the Index of Refraction

Find the index of refraction for medium 2 in Figure 1.13(a), assuming medium 1 is air and given that the incident angle is 30.0^o and the angle of refraction is 22.0^o.

Solution: Using the law of refraction we have $n_1 sin(\theta_1) = n_2 sin(\theta_2)$, where in this example we have $1 \cdot sin(30.0^o) = n_2 \cdot \sin(22.0^o)$. Solving for n_2, we get

$$n_2 = \frac{sin(30.0^o)}{sin(22.0^o)} = 1.335$$

Example 1.3: A Larger Change in Direction

Suppose that in a situation like that in Example 1.2, light goes from air to diamond and that the incident angle is 30.0^o. Calculate the angle of refraction θ_2 in the diamond.

Solution: Using the fact that n_2 for a diamond is $n_2 = 2.419$, we solve for θ_2 and get

$$\arcsin\left(\frac{sin(30.0^o)}{2.419}\right) = 11.9^o$$

Section 1.4: Total Internal Reflection

A standard mirror only reflects 90% of the light that it receives, however, it is possible to achieve a 100% reflection using refraction. Recall that from the previous section we have if $n_1 > n_2$, then $\theta_2 > \theta_1$. Note that the largest possible refraction angle is 90^o. The incident angle that causes this to occur is called the **Critical Angle** θ_c. When this occurs we have **Total Internal Reflection** inside of the incident material. Hence it must obey the Law of Reflection, $\theta_c = \theta_r$. Recall that from Snell's Law we also have

$$n_1 sin(\theta_c) = n_1 sin(\theta_r)$$

. Since $\theta_r = 90^o$ we have $sin(\theta_r) = 1$. Hence we end up with

$$\theta_c = sin^{-1}\left(\frac{n_2}{n_1}\right), n_1 > n_2$$

Example 1.4: Determining a Critical Angle

What is the critical angle for light traveling in a polystyrene (a type of plastic) pipe surrounded by air? The index of refraction for polystyrene is 1.49.
Solution: So here we have $n_1 = 1.49$ and $n_2 = 1$. Hence

$$\theta_c = sin^{-1}\left(\frac{1}{1.49}\right) = 42.16^o$$

. An application of this principle can be seen with Fiber Optics.

Chapter 2: Geometric Optics and Image Formation

Section 2.2: Spherical Mirrors

A **Curved Mirror** can form images that may be larger or smaller than the object and may form either in front of the mirror or behind it. If the reflecting surface is the outer side of the sphere, the mirror is a **Convex Mirror**. If the inside surface is the reflecting surface, it is called a **Concave Mirror**. On a parabolic mirror, the rays of light will be reflected parallel to the optical axis, the point where they meet is called the **Focal Point**. The distance from the mirror to the focal point is called the **Focal Length** of the mirror. Here we will derive the relationship between the radius and the focal point. Note that if we denote the center of the circle C, the focal point F, and P as the perimeter of the mirror, we have $R = CF + FP$, the line CP is a normal surface to the mirror, hence we will end up with θ being the angle from any point of the mirror to the center. Note that if we form a triangle CFX, where X is any point on the mirror, we will end up with an isosceles triangle, using the small-angle approximation $(\sin(\theta) \approx \tan(\theta) \approx \theta)$, we will end up with

$$R = 2f$$

where f is the focal length of the mirror.

$$tan(\theta) = \frac{h_0}{d_0}$$

or

$$tan(\theta') = -tan(\theta) = \frac{h_i}{d_i}$$

$$\frac{1}{d_0} + \frac{1}{d_i} = \frac{1}{f}$$

$$m = \frac{h_i}{h_0} = \frac{-d_i}{d_0}$$