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Homework #2

1.) #59 p.66

Bob is on Earth. Anna is on a spacecraft moving away from Earth at $0.6c$. At some point in Anna's outward journey, Bob fires a projectile loaded with supplies out to Anna's ship. Relative to Bob, the projectile moves at $0.8c$

a.

How fast does the projectile move relative to Anna?

Solution: Taking Bob's frame to the stationary frame S, and Anna's frame to by the moving frame S', we have that $v = 0.6c$ and $u = 0.8c$. Then we want to find u'. Using the velocity transformation:

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}} = \frac{0.8c - 0.6c}{1 - \frac{0.8c \cdot 0.6c}{c^2}} = \frac{0.2c}{1 - 0.8(0.6)} = \frac{0.2}{0.52}c = 0.38c$$

Thus the projectile is moving $0.38\frac{m}{s}$ according to Anna.

b.

Bob also sends a light signal, "Greeting from Earth", out to Anna's ship. How fast does the light signal move relative to Anna?

Solution: In the this circumstance S' motion is still the same, $v = 0.6c$ but the objects motion is not $u = c$. Using the velocity transformations again:

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}} = \frac{c - 0.6c}{1 - \frac{0.6c^2}{c^2}} = \frac{0.4}{0.4}c = c.$$

Thus according to Anna, the beam of light is moving at the speed of light c.

2.) #66 p.66

In a particle collider experiment, particle 1 is moving to the right at $0.99c$, both relative to the laboratory. What is the relative velocity of the two particles according to (an observer moving with) particle 2?

Solution:

Let S be the frame relative to the laboratory and S' be the frame relative to particle 2. Then suppose that u is the speed of particle 1 according to frame 1. Then we have that $v = -0.99c$ and $u = 0.99c$. Here we choose right to be the direction of positive motion, and left to be the negative. Using the velocity transformations we get that the speed of particle 1 relative to 2 is:

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}} = \frac{0.99c - (-0.99c)}{1 - \frac{(0.99c)(-0.99c)}{c^2}} = \frac{2(0.99)c}{1 + (0.99)^2} = 0.999949c.$$

Thus according to an observer in particle 2's frame, we have that particle 1 is moving to the right at $0.99995c\frac{m}{s}$.

3.) #65 p.66

You fire a light signal at 60^o north of west.

a.

Find the velocity components of this signal according to an observer moving eastward relative to you at half the speed of light. From there, determined the magnitude and direction of the light signal's velocity according to the other observer.

Solution:

Let S be the stationary frame and S' be the frame of the ship moving eastward. Then we take the measurements given as those measurements from frame S. So then we have $v = (0.5c, 0)$ and $u = (u_x, u_y) = (-cos(60^o)c, sin(60^o)c) = (-0.5c, \frac{\sqrt{3}}{2}c)$.

Then using the velocity transformations on the x-component:

$$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} = \frac{-0.5c - (0.5c)}{1 - \frac{-0.5c(0.5c)}{c^2}} = \frac{-c}{1 + (0.5)^2} = -0.8c$$

Then using the velocity transformation on the y-component:

$$u'_y = \frac{u_y}{\gamma_v(1 - \frac{u_x v}{c^2})} = \frac{sin(60^o)c}{\frac{1}{\sqrt{(1-(\frac{0.5c}{c^2})^2}}\left(1 - \frac{(-0.5c)(0.5c)}{c^2}\right)} = \frac{\frac{\sqrt{3}}{2}c}{\frac{1}{\sqrt{1-\frac{\sqrt{3}}{4}}}(1 + 0.5^2)} = 0.6c$$

Using tangent inverse to find the angle, we get

$$\tan(\theta) = \frac{u'_y}{u'_x} = \frac{0.6c}{0.8c} \implies \arctan\left(\frac{0.6}{0.8}\right) = 36.87^o$$

Find the magnitude of the light signal: $u' = \sqrt{u'^2_x + u'^2_y} = \sqrt{(-0.8c)^2 + (0.6c^2)} = c$.

Thus, according to the other observer, we have that the light signal is traveling at the speed of light 37^o North of West.

b.

Find the components according to a different observer, moving westward relative to you at half the speed of light.

Note that in this situation, we now have the following parameters: $u_x = -0.5c$, $u_y = \frac{\sqrt{3}}{2}c$, and $v = (-0.5c, 0)$.

Then the doing the same calculation in part (a):

$$u'_x = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} = \frac{-0.5c + (0.5c)}{1 - \frac{-0.5c(0.5c)}{c^2}} = 0$$

$$u'_y = \frac{u_y}{\gamma_v(1 - \frac{vu_x}{c^2})} = \frac{\frac{\sqrt{3}}{2}c}{\frac{1}{\sqrt{1-(\frac{-0.5c}{c})^2}}(1 - \frac{-0.5c(-0.5c)}{c^2})} = 1c$$

Hence we have the speed of the light signal is traveling at the speed of light in the positive $y - axis$ direction.

4.) #70 p.67

What are the momentum, energy, and kinetic energy of a proton at 0.8c?

Solution:

Momentum: Our relativistic momentum is given by $p = \gamma_u mu$. Our mass is the mass of a proton: $m_p = 1.67 \times 10^{-27}$ and our speed is $u = 0.8c$. Plugging that in:

$$p = \gamma_u mu = \frac{1}{\sqrt{1 - (\frac{u}{c})^2}} m_p(0.8c) = \frac{1}{\sqrt{1 - (\frac{0.8c}{c})^2}} (1.67 \times 10^{-27})(0.8c) = 6.68 \times 10^{-19} \frac{m \cdot kg}{s}$$

Energy: Our relativistic energy is given by $E = \gamma_u mc^2$. In this situation we have $m = m_p = a.67 \times 10^{-27}$ and $u = 0.8c$. Plugging that in:

$$E = \gamma_u mc^2 = \frac{1}{\sqrt{1 - (\frac{0.8c}{c})^2}} m_p c^2 = 2.505 \times 10^{-10} J$$

Kinetic Energy: Our relativistic kinetic energy is given by $(\gamma_u - 1)mc^2 = E - E_0$, where $E_0 = m_o c^2$ is our rest energy. Hence we have:

$$E_{Kinetic} = E - E_0 = 2.505 \times 10^{-10} - m_o c^2 = E - 2.505 \times 10^{-10} - m_p c^2 = 1.002 \times 10^{-10} J$$

5.) #85 p.67

An electron accelerated from rest through a potential difference V acquires a speed of $0.9998c$. Find the value of V.

Solution: First, note that due to conservation of energy we have $U = KE$. Next note that by definition we have $U = qV$. Thus we have $qV = KE$. Using the relativistic definition of KE:

$KE = (\gamma_u - 1)mc^2 = (\frac{1}{\sqrt{1-(\frac{0.9998c}{c})^2}})m_e c^2 = 4.02 \times 10^{-12}$

Finally using the fact that $KE = eV$.

$$V = \frac{KE}{e} = \frac{4.02 \times 10^{-12} J}{1.602 \times 10^{-19} C} = 2.50 \times 10^7 V$$

Thus we have the potential must be at $V = 2.50 \times 10^7 V$ to accelerate to this speed.

6.) #93 p.68

Consider the collisions of two identical particles, each mass m_0. In experiment A, a particle moving at $0.9c$ strikes a stationary particle.

a.

What is the total kinetic energy before the collision?

Solution: Let Particle 1: be the stationary particle and Particle 2: be the particle moving at $0.9c$. Then let E_{K_1} be the kinetic energy of particle 1 and E_{K_2} be the kinetic energy of particle 2, both of which before the collision. Then the total kinetic energy before the collision is given by: $E_K = E_{K_1} + E_{K_2}$.

Note that we have $E_{K_1} = 0$ and since for Particle 2 we have $u = 0.9c$, the kinetic energy for particle 2 is given by $E_{k_2} = (\gamma_u - 1)m_0c^2 = \frac{m_0c^2}{\sqrt{1-(\frac{0.9c}{c})^2}} = 1.294m_0c^2$.

b.

In experiment B, both particles are moving at a speed u (relative to the lab), directly toward one another. If the total kinetic energy before the collision in experiment B is the same as that in experiment A, what is u?

Solution: Assuming that the net kinetic energy of the system is equal to the result of part(a), then we have the following equation:

$$1.294m_0c^2 = KE_1 + KE_2 = (\gamma_u - 1)m_0c^2 + (\gamma_u - 1)m_0c^2$$

Where KE_1 and KE_2 is the kinetic energy of particles 1 and 2, respectively. Note that this follows from the fact that both particles share the same speed u and the same mass m_0. Solving for u:

$$1.294m_0c^2 = 2(\gamma_u - 1)m_0c^2 \iff \frac{1.294}{2} = \frac{1}{\sqrt{1-(\frac{u}{c})^2}} - 1 \iff u = 0.7946c$$

Thus, for Experiment B, we have that the particles must be moving at 0.7945c to maintain the kinetic energy from part(a).

c.

In both experiments, the particles stick together. Find the mass of the resulting single particle in each experiment. In which is more of the initial kinetic energy converted to mass?

Solution:

For experiment A:

We have that the total initial energy of the system as $E_i = m_0c^2 + \gamma_{0.9}m_0c^2 = 3.294m_0c^2$.

After the collision, we have the energy of $E_f = \gamma_u mc^2$. Where m is our new mass and u is our new speed.

By conservation of energy, we have $3.294m_0c^2 = \gamma_u mc^2 \iff 3.294m_0 = \gamma_u m$.

Switching to conservation of momentum we get:

$\gamma_{0.9}m_0(0.9c) = \gamma_u mu$

Substituting in our solution from the conservation of energy:

$2.294m_0(0.9c) = 3.294m_0u \iff u = 0.6267c$

Going back to the conservation of momentum equation and introducing u:

$$2.294m_0(0.9c) = 1.2833m(0.6267c) \iff m = 2.57m_0$$

Experiment B:

Finding the total energy before the collision: $E_i = 2\gamma_{0.7946}m_0c^2$.

Finding the total energy after the collision: $E_f = \gamma_u mc^2$, where m and u are our new mass and speed, respectively.

Thus we get by conservation of energy: $E_i = E_f \iff 2\gamma_{0.7946}m_0 = \gamma_u m$

Switching back to conservation of momentum we get:

$\gamma_{0.7946}m_0(+0.7946c) + \gamma_{0.7946}m_0(-0.7946c) = \gamma_u mu$

Hence we have *zero* momentum, and hence a final speed of zero for the new mass.

Going back to the conservation of energy piece of the problem:

$$2(\gamma_{0.7946})m_0c^2 = (1)mc^2 \iff m = 3.294m_0$$

Hence it is experiment B that gains the most mass converted from kinetic energy.

7.) #95 p.68

In the frame of reference shown, a stationary particle of mass m_0 explodes into two identical particles of mass m moving in opposite directions at $0.6c$. Momentum is obviously conserved in this frame. Verify explicitly that it is conserved in a frame of reference moving to the right at $0.6c$

Solution: First, let the particle moving to the left be called particle 1 and the right particle 2. Clearly, in this new frame we have the initial particle is moving to the left at $-0.6c$. And in our new frame particle 2 has zero speed.

Finding particle 1's new relativistic speed:

$$u' = \frac{-1.2c}{1 - \frac{uv}{c^2}} = -0.882c$$

So we have an initial momentum of $p_i = \gamma_{0.6} m_0(-0.6c) = (-0.75)m_0 c$.

Finally in our new frame we have a momentum of: $p_f = \gamma_{0.882} m(-0.882c)$, where m is our new mass.

Since we can't rely on conservation of momentum, let's solve for m is terms of m_0 by energy conservation in our original frame:

$$E_i = m_0 c^2 = 2\gamma_{0.882} mc^2 = E_f$$

$$m = 0.4m_0$$

Hence, we have a final momentum of:

$$p_f = (-0.882c)(2.122)(0.4)m_0 = -0.75m_0 c$$

They both line up, as they should! And conservation of momentum is preserved.