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Homework #4

Problem 2.1

Suppose you flip four fair coins.

(a)

Make a list of all the possible outcomes, as in Table 2.1.
Solution:

Coin 1	Coin 2	Coin 3	Coin 4
T	T	T	T
T	T	T	H
T	T	H	T
T	H	T	T
H	T	T	T
T	T	H	H
T	H	H	T
H	H	T	T
T	H	T	H
H	T	T	H
H	T	H	T
H	H	H	T
H	H	T	H
H	T	H	H
T	H	H	H
H	H	H	H

(b)

Make a list of all the different "macrostates" and their probabilities.
Solution:
Their would be five macrostates they would be:

Number of Heads	Number of Tails	Probability
4	0	1/16
3	1	4/16
2	2	6/16
1	3	4/16
0	4	1/16

(c)

Compute the multiplicity of each macrostate using the combinatorial formula 2.6, and check that these results agree with what you got by brute-force counting.

Solution:

Number of Heads	Number of Tails	Probability
4	0	$\binom{4}{4} = 1$ ✓
3	1	$\binom{4}{3} = 4$ ✓
2	2	$\binom{4}{2} = 6$ ✓
1	3	$\binom{4}{1} = 4$ ✓
0	4	$\binom{4}{0} = 1$ ✓

Problem 2.3

Suppose you flip 50 fair coins.

(a)

How many possible outcomes (microstates) are there?

Solution:

$2^{50} = 1125899906842624$

(b)

How many are there of getting exactly 25 heads and 25 tails?

Solution:

$\Omega(50, 25) = \binom{50}{25} = 126410606437752$

(c)

What is the probability of getting exactly 25 heads and 25 tails?

Solution:

$$\frac{\binom{50}{25}}{2^{50}} = 0.11$$

(d)

What is the probability of getting exactly 30 heads and 20 tails?

Solution:

$$\Omega(50, 30) = \binom{50}{30} = 47129212243960$$
$$\frac{\Omega(50, 30)}{2^{50}} = 0.0419$$

(e)

What is the probability of getting exactly 40 heads and 10 tails?

Solution:

$$\Omega(50, 40) = \binom{50}{40} = 5136139085$$
$$\frac{\Omega(50, 40)}{2^{50}} = 9.126 \times 10^{-6}$$

(f)

What is the probability of getting 50 heads and no tails?

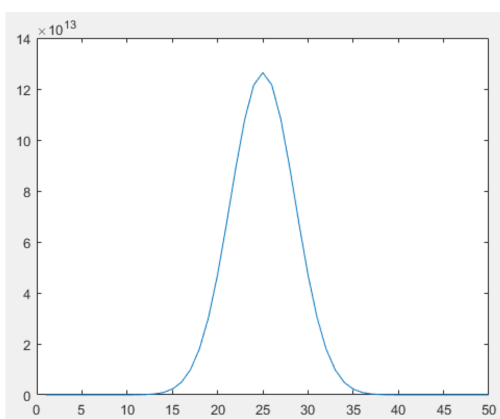
Solution:

$$\Omega(50, 50) = 1$$
$$\frac{1}{2^{50}} = 8.88 \times 10^{-16}$$

(g)

Plot a graph of the probability of getting n heads, as a function of n.

Solution:



Problem 2.5

For an Einstein solid with each of he following values of N and q, list all of the possible microstates, count them, and verify formula 2.9.

(a)

$N = 3, q = 4$

Solution:

This corresponds to a system of 3 oscillators with four allowable energy units, so that our distribution looks like:

	Total Energy = 0	
Oscillator 1	Oscillator 2	Oscillator 3
0	0	0

	Total Energy = 1	
Oscillator 1	Oscillator 2	Oscillator 3
1	0	0
0	1	0
0	0	1

	Total Energy = 2	
Oscillator 1	Oscillator 2	Oscillator 3
0	0	2
0	2	0
2	0	0
1	1	0
1	0	1
0	1	1

	Total Energy = 3	
Oscillator 1	Oscillator 2	Oscillator 3
0	0	3
0	3	0
3	0	0
0	2	1
2	1	0
2	0	1
0	1	2
1	0	2
1	2	0
1	1	1

	Total Energy = 4	
Oscillator 1	Oscillator 2	Oscillator 3
0	3	1
0	1	3
0	2	2
0	0	4
0	4	0
1	0	3
1	3	0
1	2	1
1	1	2
2	1	1
2	2	0
2	0	2
3	1	0
3	0	1
4	0	0

The formula predicts:

Total Energy of 0: $\Omega(3,0) = \binom{2}{0} = 1\checkmark$

Total Energy of 1: $\Omega(3,1) = \binom{3}{1} = 3\checkmark$

Total Energy of 2: $\Omega(3,2) = \binom{4}{2} = 6\checkmark$

Total Energy of 3: $\Omega(3,3) = \binom{5}{3} = 10\checkmark$

Total Energy of 4: $\Omega(3,4) = \binom{6}{4} = 15\checkmark$

Problem 2.6

Calculate the multiplicity of an Einstein solid with 30 oscillators and 30 units of energy. (Do not attempt to list all the microstates.)

Solution:

$$\Omega(30, 30) = \binom{30 + 30 - 1}{30} = 59132290782430712 \approx 5.91 \times 10^{16}$$