Math 119 — Exam 2 — Joseph McGuire

Student Name (print): ______________________________

This exam contains 4 pages (including this cover page) and 5 questions. The total number of possible points is 25. Enter your answers in the space provided. Draw a box around your final answer.

- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations may still receive partial credit.
- **Clearly identify your answer for each problem.**
- **No calculators or outside help allowed, unless it is with your instructor.**

Do not write in the table to the right.

Question	Points	Score
1	5	
2	5	
3	5	
4	5	
5	5	
Total:	25	

1. (5 points) Solve the **equation**

$$\sin(2\theta) = 2\sin(\theta)$$

for all angles θ on the interval $[0, 2\pi)$. You can use your calculator here, be careful using it though. Important: There are 2 solutions. Show work.

Solution: $2\cos(\theta)\sin(\theta) = 2\sin(\theta)$, at this point $\sin(\theta) = 0$ is a solution, we know there are two points $x = 0, \pi$, where this is true. So now assume $\sin(\theta) \neq 0$ then $\iff \cos(\theta) = 1 \iff \theta = 0$, that's already an answer!

2. (5 points) Prove

$$\sin^2(2\theta) = 4(\sin^2(\theta) - \sin^4(\theta)).$$

Show work.

Solution:

$$(2\sin(\theta)\cos(\theta))^2 = 4\sin^2(\theta)\cos^2(\theta) = 4\sin^2(\theta)(1 - \sin^2(\theta)).$$

 Points earned: ______ **out of 10 points**

3. (5 points) Convert $(-5, 5)$ from rectangular coordinates to polar coordinates. You can use a calculator here, be careful though. Show work.

Solution: Using our formuli:

$$r^2 = 50 \iff r = \pm\sqrt{50} \qquad \text{and} \qquad \tan(\theta) = -1 \iff \theta = -\frac{\pi}{4},$$

so this has polar coordinates: $(-\sqrt{50}, \frac{-\pi}{4})$ or $(\sqrt{50}, \pi - \frac{\pi}{4})$

4. (5 points) Using De Moivre's Theorem compute:

$$(4 + 3i)^{100}.$$

You don't need to evaluate the trigonometric function or evaluate any large powers like r^{100}, just set it up. Show your work.

Solution: The polar form of this complex number is($r = \sqrt{4^2 + 3^2} = 5$ and $\theta = \tan^{-1}\frac{3}{4}$):

$$4 + 3i = 5(\cos(\tan^{-1}\frac{3}{4}) + i\sin(\tan^{-1}\frac{3}{4})).$$

De Moivre's Theorem gives us:

$$(4 + 3i)^{100} = 5^{100}(\cos(100\tan^{-1}\frac{3}{4}) + i\sin(100\tan^{-1}\frac{3}{4})).$$

 Points earned: ______ **out of 10 points**

5. (5 points) Find the complex solutions to the equation:

$$z^4 = -2.$$

Just set up the solutions: $w_k = \ldots$ and don't actually plug in the values of k, but tell me which k values we would use. Show your work.

Solution: Polar form of -2 is:

$$-2 = 2(\cos(\pi) + i\sin(\pi)).$$

So that the roots are given by:

$$w_k = 2^{1/4}(\cos\frac{\pi + 2\pi k}{4} + i\sin\frac{\pi + 2\pi k}{4}) \qquad k = 0, 1, 2, 3$$

Points earned: ______ out of 5 points