MATH 117: Exam # 1

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Due: 10/16/2020 11:59pm

Please show your work for the following problems. These problems have solutions that are easily found online, so most of your grade will be based on explaining how we get the solution that we get.

Pick 8 of the following problems. All problems are worth 12.5 **points.**

1

Give the values of x where $x^4 - 2x^2 + 1 \geq x^2 - 10$ is true. (Hint: Quadratic Formula)

2

Prove that there are no real x-values where $x^4 - 4x^2 + 8 \leq x^2$.

3

Using polynomial long division go through dividing $6x^6+5x^5+4x^4+3x^3+2x^2+x$ by $x^2 + x + 1$. That is, what are the polynomials $q(x), r(x)$ such that

$$\frac{6x^6 + 5x^5 + 4x^4 + 3x^3 + 2x^2 + x}{x^2 + x + 1} = q(x) + \frac{r(x)}{x^2 + x + 1}.$$

4

Prove that there is no horizontal asymptote of

$$\frac{b_{n+1}x^{n+1} + b_n x^n \ldots + b_1 x + b_0}{b_n x^n + \ldots + b_1 x + b_0}.$$

5

Through polynomial division prove that the roots of $x^4 - \frac{x^3}{4} - \frac{47x^2}{6} - 4x + 6$ are $\frac{2}{3}, \frac{-3}{2}, -2, 3$.

6

Using the Descartes Rules of Signs, how many positive and negative real roots can $7x^7 - 5x^5 + 4x^4 + 3x^3 - 2x^2 + x + 1$ have? Show your work.

7

Using Desecrates Rules of Signs show that $x^4 + x^2 + 2$ has no real roots.

8

Find a polynomial $p(x)$ with real coefficients (so coefficients are not complex, that is $a + bi$ isn't a coefficient) where $p(1 - 2i) = 0$.

9

Prove that if $a - bi$ is a complex root of the polynomial with all real coefficients $p(x) = cx^3 + dx^2 + fx + g$, $(c, d, f, g$ are real numbers) that there has to be one real root of this polynomial. (Hint: Consider what would happen if there was another complex root $c + di$.)

10

Using the rational roots theorem, find where

$$\frac{3x^2 - 9x + 6}{2x^2 + 12x + 18}$$

have roots and vertical asymptotes. Then tell me what are the horizontal asymptotes of this function.

11 (Extra Credit) Replaces Your Lowest Scoring Problem.

Prove that the quadratic formula works. That is, the solutions to $ax^2+bx+c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$