Joseph C. McGuire
Dr. Scott Severson

Homework #1

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Problem 1.1

The Fahrenheit temperature scale is defined so that ice melts at 32^oF and water boils at 212^oF.

a.)

Derive the formulas for converting from Fahrenheit to Celsius and back.

Solution:

Pairing Fahrenheit with Celsius, we know two points: $(32^oF, 0^oC)$ and $(212^oF, 100^oC)$. We know then the slope between the two is: $m = \frac{100-0}{212-32} = \frac{5}{9}$. Then the line between the two points is Celsius $- y_0 = m($Fahrenheit $- x_0)$, where (x_0, y_0) is some point of the two. Let that point be $(32^oF, 0^oC)$, then we have:

$$\text{Celsius} = \frac{5}{9}\left(\text{Fahrenheit} - 32\right)$$

Solving for Fahrenheit:

$$\text{Fahrenheit} = \frac{9}{5}\text{Celsius} + 32$$

b.) What is absolute zero on the Fahrenheit scale?

Solution:

We know that absolute zero in Celsius is -273^oC, hence by the above formula we have absolute zero in Fahrenheit is -459.4^oF.

Problem 1.7

When the temperature of liquid mercury increases by one degree Celsius (or one kelvin), its volume increases by one part in 5500. The fractional increase in volume per unit change in temperature (when the pressure is held fixed) is called the thermal expansion coefficient, β:

$$\beta \equiv \frac{\Delta V/V}{\Delta T}$$

(where V is volume, T is temperature, and Δ signifies a change, which in this case should really be infinitesimal if β is to be well defined). So for mercury, $\beta = 1/5500\ \mathrm{K}^{-1} = 1.81 \times 10^{-4}\ \mathrm{K}^{-1}$. (The exact value varies with temperature but between $0^o C$ and $200^o C$ the variation is less than 1%.)

a.)

Get a mercury thermometer, estimate the size of the bulb at the bottom, and then estimate what the inside diameter of the tube has to be in order for the thermometer to work as required. Assume that the thermal expansion of the glass is negligible.

Solution: Using the definition of β and the fact that the volume of a cylinder is $\pi r^2 h = \frac{\pi d^2 h}{4}$, where h is the height of the thermometer and d is the diameter of the thermometer. Assuming the height of the thermometer is fixed, then we know $\Delta V = \frac{\pi (dd)^2 h}{4}$, where dd is the change in the internal diameter. Hence we have:

$$(\Delta T)(V)\beta = \Delta V \iff (\Delta T)\left(\frac{\pi d^2 h}{4}\right)\beta = \left(\frac{\pi (dd)^2 h}{4}\right) \iff \frac{d^2(\Delta T)}{5500} = (dd)^2$$

Assuming a change in temperature of 1 $[K]$, and the nearest thermometer has a diameter of 0.1 [cm] then we have:

$$\frac{(0.1)^2}{5500} = (dd)^2 \iff dd = 1.34 \times 10^{-2}\ [\mathrm{cm}]$$

So the inner diameter has to be 1.34×10^{-2} [cm] for such a thermometer.

b.)

The thermal expansion coefficient of water varies significantly with temperature: It is $7.5 \times 10^{-4}\ \mathrm{K}^{-1}$ at 100^oC, but decreases as the temperature is lowered until it becomes *zero* at 4^oC. Below 4^oC it is slightly *negative*, reaching a value of $-0.68 \times 10^{-4}\ \mathrm{K}^{-1}$ at 0^oC. (This behavior is related to the fact that ice is less dense than water.) With this behavior in mind, imagine the process of a lake freezing over, and discuss in some detail how this process would be different if the thermal expansion coefficient of water were always positive.

Solution:

Using the relation from part(a): $(\Delta T)(V)\beta = \Delta V$. Knowing the above about the β coefficient for water, this relation gives us that in a lake freezing (meaning $\Delta T < 0$) and $\beta < 0$ for $T < 4^oC$, then we actually see that the volume of the water will expand as the water freezes. If β were always positive for water, then the result of a lake freezing would mean that $\Delta T < 0$. Hence we would have that the volume of the water in the lake would actually shrink in size as the water freezes.

Problem 1.8

For a solid, we also define the **linear thermal expansion coefficient**, α, as the fractional increase in length per degree:

$$\alpha \equiv \frac{\Delta L/L}{\Delta T}$$

a.

For steel, α is 1.1×10^{-5} K^{-1}. Estimate the total variation in length of a 1-km steel bridge between a cold winter night and a hot summer day.

Solution:

Using Santa Rosa as reference, the hottest day in Santa Rosa is recorded as $45^oC = 318\ K$ and the record low is $-13^oC = 260\ K$. Hence we have on an extreme extreme day that the variation in the length of 1 km bridge is: $1.1 \times 10^{-5} = \frac{\Delta L}{318-260} \iff \Delta L = 0.000638$ km $=$ 0.638 m

b.)

The dial thermometer in Figure 1.2 uses a coiled metal strip made of two different metals laminated together. Explain how this works.

Solution:

Using the definition of α, we get that:

$$(\Delta T)\alpha(L) = \Delta L$$

Hence for a positive α we will get that heat will expand the metal the springs by length ΔL, hence the coils will expand and temperature will be recorded as increasing.

Problem 1.10

Estimate the number of air molecules in an average-sized room.

Solution:

1 mole is defined as being about 22.4 liters of air, or equivalently as 0.0221 m^3 of air. So if we're in room with a width and length of 5 m and a height of 3 m, then the room would be about 125 m^3. So then the number of moles in the room of air would be about $125/0.0221 \approx 5656$ moles. One mole has an Avogardo's number amount of molecules, so then we have the number of molecules in this size of a room is: $5656 * N_A = 3.41 \times 10^{27}$ air molecules in a room.