Homework #1

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1.)

To solve for velocity v, we set $\gamma_v = 1.01$:

$$\gamma_v = \frac{1}{\sqrt{1-(\frac{v}{c})^2}}$$

$$1.01 = \frac{1}{\sqrt{1-(\frac{v}{c})^2}}$$

$$(1.01)^2 = \frac{1}{1-(\frac{v}{c})^2}$$

$$\left(1-\left(\frac{v}{c}\right)^2\right) = \frac{1}{(1.01)^2}$$

$$\frac{v^2}{c^2} = 1 - \frac{1}{(1.01)^2}$$

$$v = \sqrt{c^2\left(1-\frac{1}{1.01^2}\right)}$$

$$v \approx 4.21 \times 10^7 \frac{m}{s} = 0.14c\frac{m}{s}$$

Thus we would need to travel at 14% of the speed of the light to feel 1% of relativity's effects.

2.)

From the stationary frame we have that Carl's spaceship is traveling at $v = 0.5c\frac{m}{s}$. Hence finding our γ_v, we get $\gamma_v = \frac{1}{\sqrt{1-(0.5)^2}} = 1.15$. Using the time dilation formula $\Delta t = \gamma_v \Delta t_0$, and treating Carl's frame as the proper time and the stationary time as Δt. We get: $\Delta t = \gamma_v \Delta t_0 \iff \Delta t_0 = \frac{\Delta t}{1.15} \iff \Delta t_0 = \frac{60}{1.15} = 52.17\ s$. Thus it takes Carl 52.17 seconds to complete his calculations.

3.)

Using the length contraction equation and taking the plane's length as measured by its passengers as the proper length, and L as the length from the observer on the Earth, we have $L = 35\ m$. Thus, we want to solve for ΔL_0:
$L = \frac{L_0}{\gamma_v} \iff L_0 = L\gamma_v \iff L_0 = 35\left(\frac{1}{\sqrt{1-(\frac{0.6c}{c})^2}}\right) = 43.75\ m$. Thus the length of the plane according to the passengers is 43.75 m.

4.)

a.

From the description, we know that Bob's frame is the stationary frame (denote as S), and Anna's is the moving (denote as S'). Then we know that the time difference for Bob is 40 [ns]. So if we treat Anna turning on the back light as our first event and the front light as out second event, then we have:
$t_2 - t_1 = \gamma_v\left(\frac{v}{c^2}x_2' + t_2'\right) - \gamma_v(\frac{v}{c^2}x_1' + t_1') = \gamma_v(\frac{v}{c^2}(x_2' - x_1') + (t_2' - t_1'))$.
In S' the length of the spaceship is 60[m], and for Anna the lights were turned on simultaneously hence: $t_2 - t_1 = \gamma_v(\frac{v}{c^2}(60) + 0) = \frac{60v\gamma_v}{c^2} > 0$. Since the difference between t_2 and t_1 is positive, that means t_2 happened after t_1. Thus event 1 occurred first, so the back light is the first to turn on according to Bob.

b.

Taking our equation from part(a.) and taking into account that the time difference was 40 ns, we get: $\Delta t = \gamma_v\frac{60v}{c^2} \iff 40 \times 10^{-9} = \frac{60v}{c^2\sqrt{1-(\frac{v}{c})^2}} \iff \frac{c^2(40\times10^{-9}}{60} = \frac{v}{\sqrt{1-(\frac{v}{c})^2}} \iff$ $(\frac{c^2(40\times10^{-9}}{60})^2 = \frac{v^2}{1-(\frac{v}{c})^2}$, using Mathematica to solve for v: $v = 5.88 \times 10^7\frac{m}{s} = 0.196c$.

5.)

a.

Taking the Earth's measurements of 4[km] to be the stationary frame, we can use the length-contraction formula to solve for first how far the muon is traveling in its frame: $L = \frac{L_0}{\gamma_v} = \frac{4\times10^3}{\sqrt{1-(\frac{0.93c}{c})^2}} = 1.47\ km$. Now that we have the distance the muon travels, we can solve for the time it takes the muon to reach the Earth:
$v = \frac{\Delta x}{\Delta t} \iff 0.93c = \frac{1.47\times10^3}{\Delta t} \iff \Delta t = 5.27\ \mu s.$
Thus plugging that into the function given: $N(5.27) = N_0(e^{\frac{-5.27}{2.2}}) = (0.091)N_0$.
Thus we only have 9.1% of the original muons that we started off with, by the time they hit the earth.

b.

Working in classical mechanics, we would start off with:
$v = \frac{\Delta x}{\Delta t} \iff 0.93c = \frac{4\times10^3}{\Delta t} \iff \Delta t = 14.3\ \mu s.$
Plugging that into our function:
$N(14.3) = N_0 e^{\frac{-14.3}{2.2}} = (0.0015)N_0$. Thus we would only have 0.15% of the original muons by the time they hit the Earth, using classical mechanics.

6.)

First note the only motion occurring here in parallel to the x-axis, thus we have for the stationary frame (S) and the moving frame according the plank (S'), y = y'. Thus the only length contraction occurring here will be in the x-direction. So first note that we can break L_0 into components, $(L_{0,x}, L_{0,y}) = (L_0\cos\theta_0, L_0\sin\theta_0)$. Then for the new length of the board L in frame S, we have the components of L as:
$(L_x, L_y) = \left(\frac{cos(\theta_0)L_0}{\gamma_v}, sin(\theta_0)L_0\right)$, since out only motion is in the x-direction, hence y is unaffected by length contraction. From here, finding the new angle theta, we take the tangent of the new angle:
$\tan\theta = \frac{sin(\theta_0)L_0}{L_x} \iff \tan\theta = \frac{\sin\theta_0 L_0}{\frac{\cos\theta_0 L_0}{\gamma_v}} \iff tan(\theta) = \tan\theta_0\gamma_v \iff \theta = \arctan(tan(\theta_0)\gamma_v).$
Now looking back at L and we take the magnitude of this:
$L = \sqrt{L_x^2 + L_y^2} = \sqrt{\frac{(cos(\theta_0)L_0)^2}{\gamma_v} + (sin(\theta_0)L_0)^2} = \left(\frac{cos^2(\theta_0)L_0^2}{\gamma_v^2} + sin^2(\theta_0)L_0^2\right)^{\frac{1}{2}} =$
$L_0\sqrt{\frac{cos^2(\theta_0)}{\gamma_v^2} + sin^2(\theta_0)} = L_0\sqrt{\frac{cos^2(\theta_0)}{\frac{1}{\sqrt{1-(\frac{v}{c})^2}}} + sin^2(\theta_0)} = L_0\sqrt{(1-(\frac{v}{c})^2)cos^2(\theta_0) + sin^2(\theta_0)} =$
$L_0\sqrt{cos^2(\theta) + sin^2(\theta) - cos^2(\theta_0)(\frac{v}{c})^2} = L_0\sqrt{1-(\frac{v\ cos(\theta_0)}{c})^2}.$

Thus we have derived the length and angle of the plank as observed from its new moving frame.

7.)

So we have the pions are both moving a total of 13 [m] and only surviving 26 [ns] in our stationary frame. Thus we can set up the following:
$v = \frac{13}{\Delta t} \iff \Delta t = \frac{13}{v}$ as well as a time dilation formula:
$\Delta t = \frac{\Delta t_0}{\sqrt{1-(\frac{v}{c})^2}} \iff \frac{26\times10^{-9}}{\sqrt{1-(\frac{v}{c})^2}} = \Delta t$.
Combining the two: $\frac{13}{v} = \frac{26\times10^{-9}}{\sqrt{1-(\frac{v}{c})^2}}$.
Using Mathematica to solve for $v = 2.57 \times 10^8 \frac{m}{s}$. Thus the pions are traveling at $2.57 \times 10^8 \frac{m}{s}$ through the laboratory.