MATH 117: HW # 8

Instructor: Joseph McGuire

Due: 11/21/2020 11:59pm

Please show your work for the following problems. These problems have solutions that are easily found online, so most of your grade will be based on explaining how we get the solution that we get. There are 3 pages to this homework.

1

Find two 2×2 matrices A and B such that: $A \neq 0$ and $B \neq 0$, but $AB = 0$, where 0 is the 2×2 matrix:

$$0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Verify that $AB = 0$ with a calculation.

2

Find the inverse of the matrices listed below.

2.1

$$\begin{bmatrix} a & -a \\ a & a \end{bmatrix}$$

with $a \neq 0$. Show your work.

2.2

$$\begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{bmatrix}$$

where $abcd \neq 0$, that is, $a \neq 0$, $b \neq 0$, $c \neq 0$, $d \neq 0$. Show your work.

3

Solve the system of equations by converting to a matrix equation and using the inverse of the coefficient matrix, as we did in Example 6.

$$\begin{cases} 5x + 7y + 4z &= 1 \\ 3x - y + 3z &= 1 \\ 6x + 7y + 5z &= 1 \end{cases}$$

Show your work.

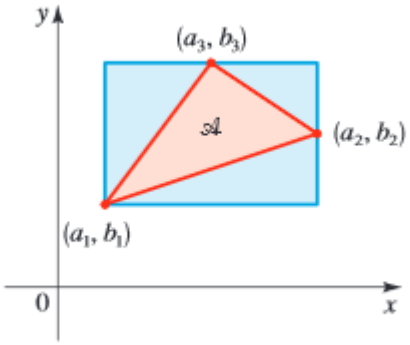
4

Solve for x in

$$\begin{vmatrix} a & b & x-a \\ x & x+b & x \\ 0 & 1 & 1 \end{vmatrix} = 0.$$

Show your work.

5



The figure above shows a triangle in the plane with vertices $(a_1, b_1), (a_2, b_2), (a_3, b_3)$.

5.1

Find the coordinates of the vertices of the surrounding rectangle, and find its area. Show your work.

5.2

Find the area of the red triangle by subtracting the areas of the three blue triangles from the area of the rectangle. Show your work.

5.3

Use your answer to part(b) to show that the area $\mathcal{A}$ of the red triangle is given by

$$\mathcal{A} = \pm\frac{1}{2}\begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$$

Show your work.