PSTAT174 Project

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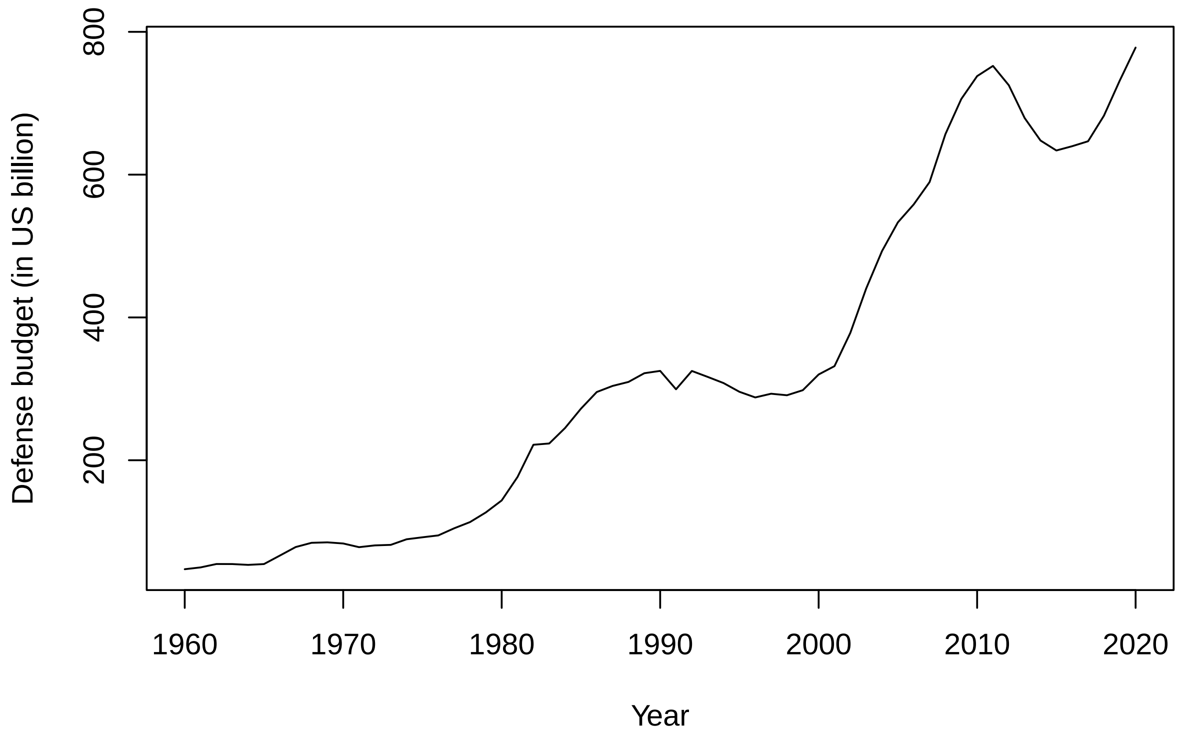
Data Importation

```r
# load data
spending_data <- read.csv("/Users/josephchang/Desktop/MilitarySpending.csv.xls")
spending_data
```

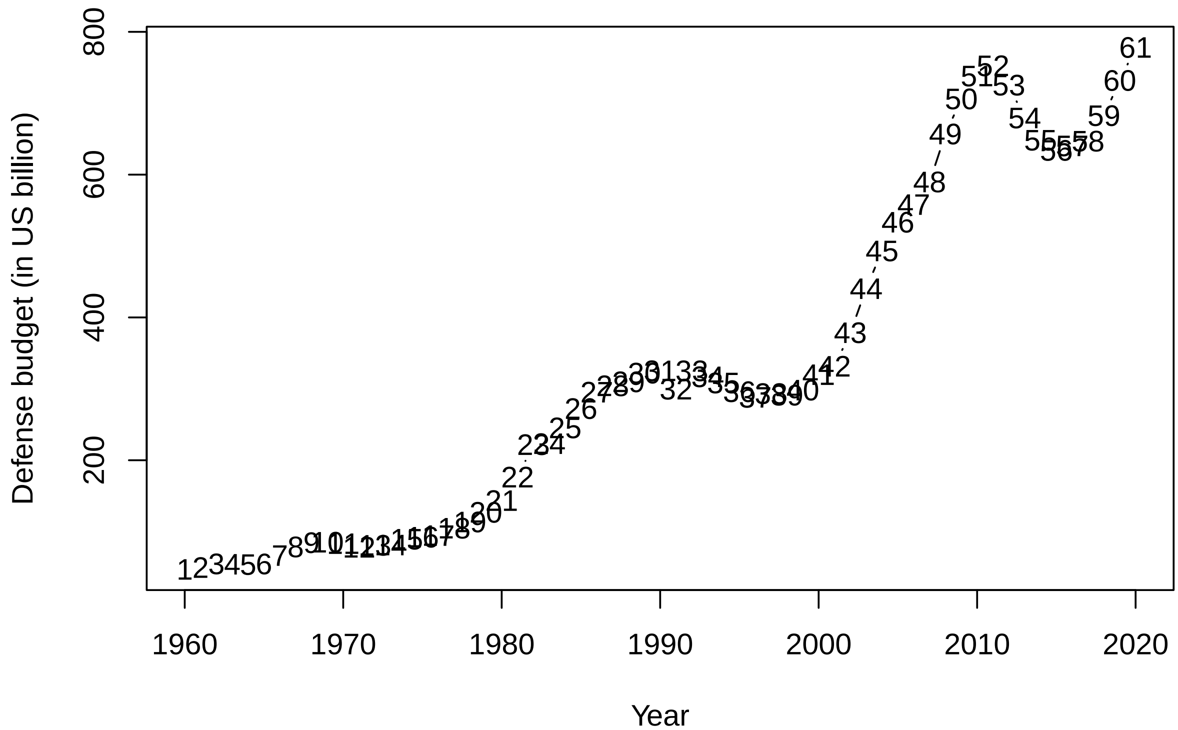
```
##    Year DefenseBudget    GDP Population
## 1  1960         47.35  543.3      180.7
## 2  1961         49.88  563.3      183.7
## 3  1962         54.65  605.1      186.5
## 4  1963         54.56  638.6      189.2
## 5  1964         53.43  685.8      191.9
## 6  1965         54.56  743.7      194.3
## 7  1966         66.44  815.0      196.6
## 8  1967         78.40  861.7      198.7
## 9  1968         84.33  942.5      200.7
## 10 1969         84.99 1019.9      202.7
## 11 1970         83.41 1073.3      205.1
## 12 1971         78.24 1164.8      207.7
## 13 1972         80.71 1279.1      209.9
## 14 1973         81.47 1425.4      211.9
## 15 1974         89.28 1545.2      213.8
## 16 1975         92.08 1684.9      216.0
## 17 1976         94.72 1873.4      218.0
## 18 1977        104.67 2081.8      220.2
## 19 1978        113.38 2351.6      222.6
## 20 1979        126.88 2627.3      225.1
## 21 1980        143.69 2857.3      227.2
## 22 1981        176.56 3207.0      229.5
## 23 1982        221.67 3343.8      231.7
## 24 1983        223.43 3634.0      233.8
## 25 1984        245.15 4037.6      235.8
## 26 1985        272.16 4339.0      237.9
## 27 1986        295.55 4579.6      240.1
## 28 1987        304.09 4855.2      242.3
## 29 1988        309.66 5236.4      244.5
## 30 1989        321.87 5641.6      246.8
## 31 1990        325.13 5963.1      249.6
## 32 1991        299.37 6158.1      253.0
## 33 1992        325.03 6520.3      256.5
```

```
## 34 1993         316.72  6858.6      259.9
## 35 1994         308.08  7287.2      263.1
## 36 1995         295.85  7639.8      266.3
## 37 1996         287.96  8073.1      269.4
## 38 1997         293.17  8577.5      272.6
## 39 1998         291.00  9062.8      275.9
## 40 1999         298.09  9630.7      279.0
## 41 2000         320.09 10252.4      282.2
## 42 2001         331.81 10581.8      285.0
## 43 2002         378.46 10936.4      287.6
## 44 2003         440.53 11458.2      290.1
## 45 2004         493.00 12213.7      292.8
## 46 2005         533.20 13036.6      295.5
## 47 2006         558.34 13814.6      298.4
## 48 2007         589.59 14451.9      301.2
## 49 2008         656.76 14712.8      304.1
## 50 2009         705.92 14448.9      306.8
## 51 2010         738.01 14992.0      309.3
## 52 2011         752.29 15542.6      311.6
## 53 2012         725.21 16197.0      313.8
## 54 2013         679.23 16784.8      316.0
## 55 2014         647.79 17527.2      318.3
## 56 2015         633.83 18238.3      320.6
## 57 2016         639.86 18745.1      322.9
## 58 2017         646.75 19543.0      325.0
## 59 2018         682.49 20611.9      326.7
## 60 2019         731.75 21433.2      328.2
## 61 2020         778.00 20940.0      330.7
```

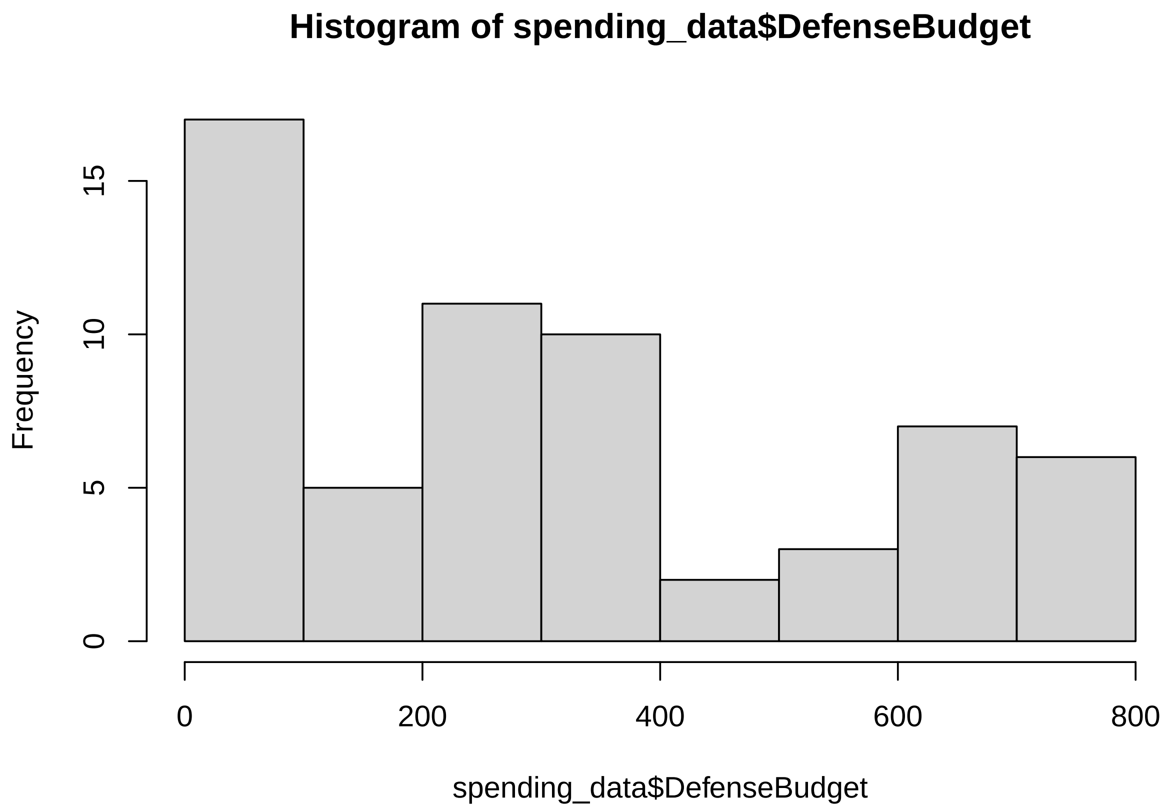
```
# plot data
plot(spending_data$Year, spending_data$DefenseBudget, ylab = "Defense budget (in US billion)", xlab = "Y
```




```
# plot time series
plot.ts(spending_data$Year, spending_data$DefenseBudget, ylab = "Defense budget (in US billion)", xlab =
```

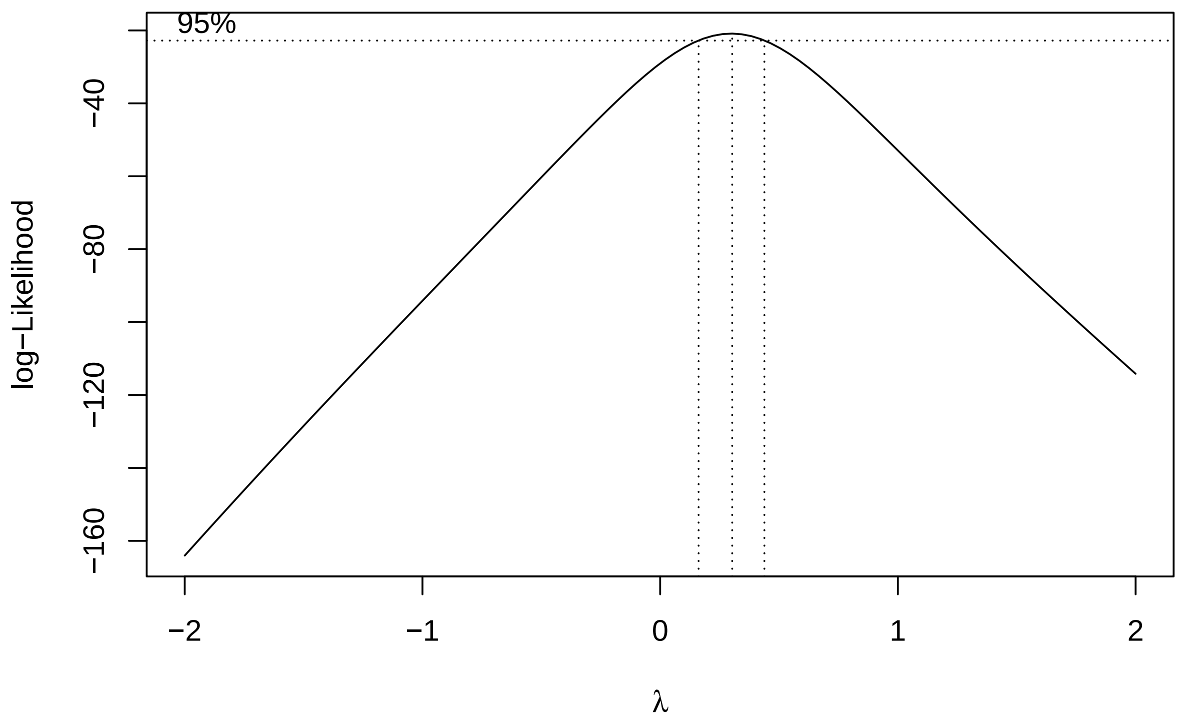



```
# Immediate observations: There seems to be a linear trend that is positive but there is no seasonality

hist(spending_data$DefenseBudget)
```

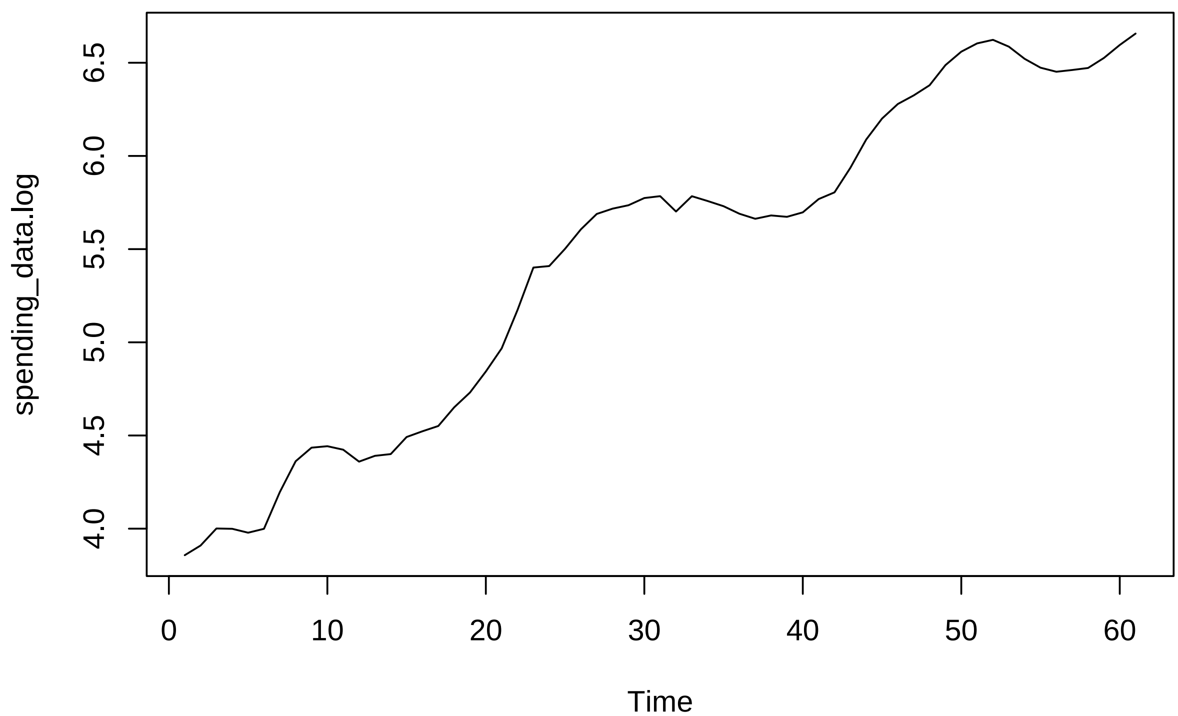


Transformation

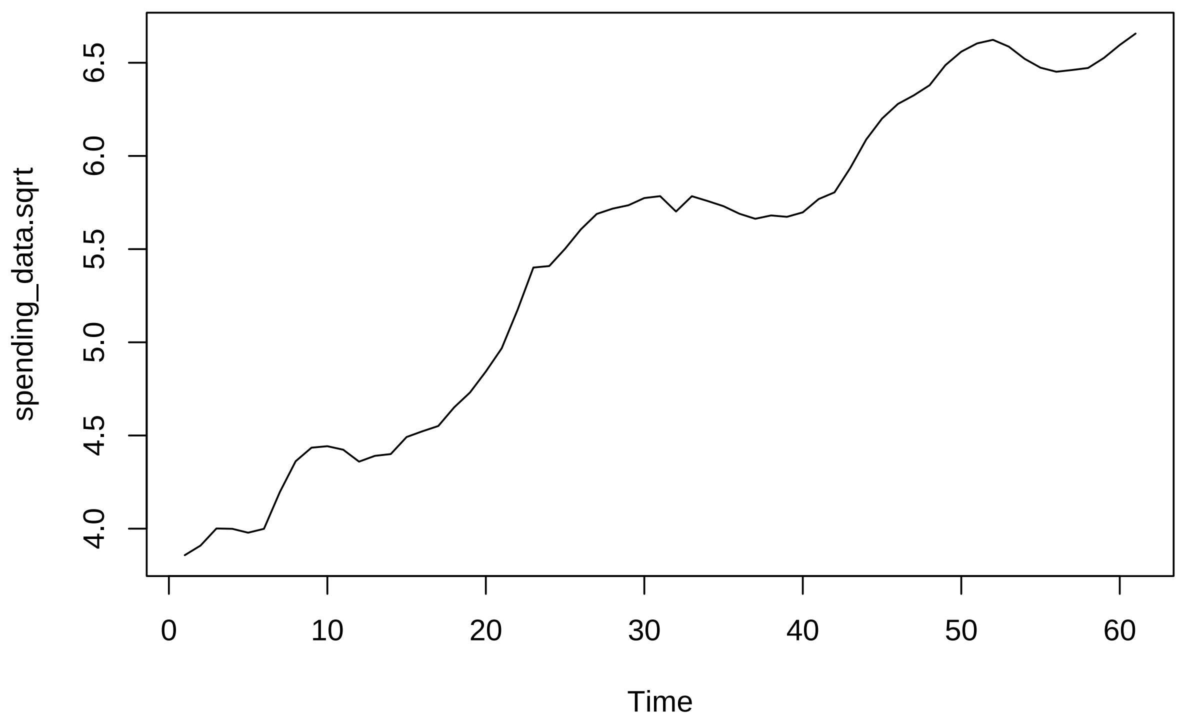
```
# Since original data is skewed I will try Box-Cox transformation
t <- 1:length(spending_data$DefenseBudget)
fit <- lm(spending_data$DefenseBudget~t)
bcTransform <- boxcox(spending_data$DefenseBudget ~ t, plotit=TRUE)
```



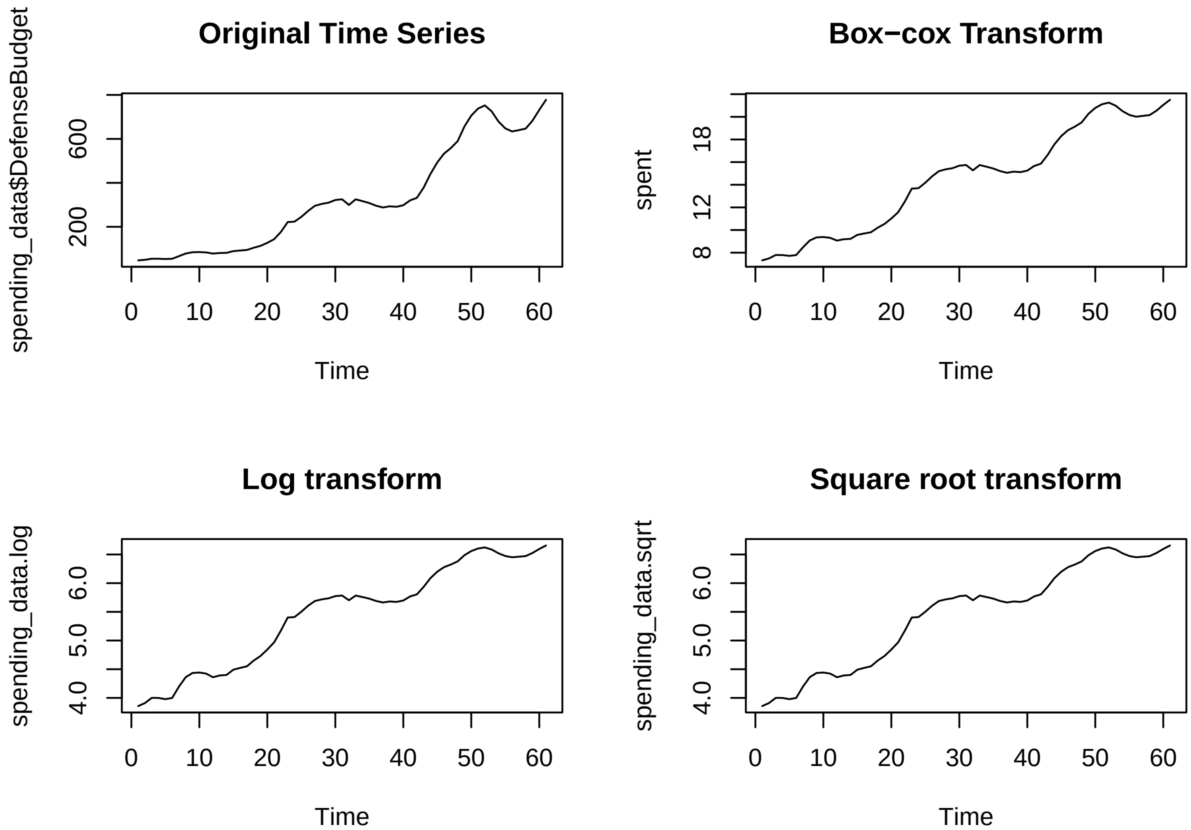

```
# best lambda
lambda = bcTransform$x[which(bcTransform$y == max(bcTransform$y))]
spent = (1/lambda)*(spending_data$DefenseBudget^lambda-1)

# comparison of best lambda vs log/sqrt transformation
spending_data.log = log(spending_data$DefenseBudget)
plot.ts(spending_data.log)
```



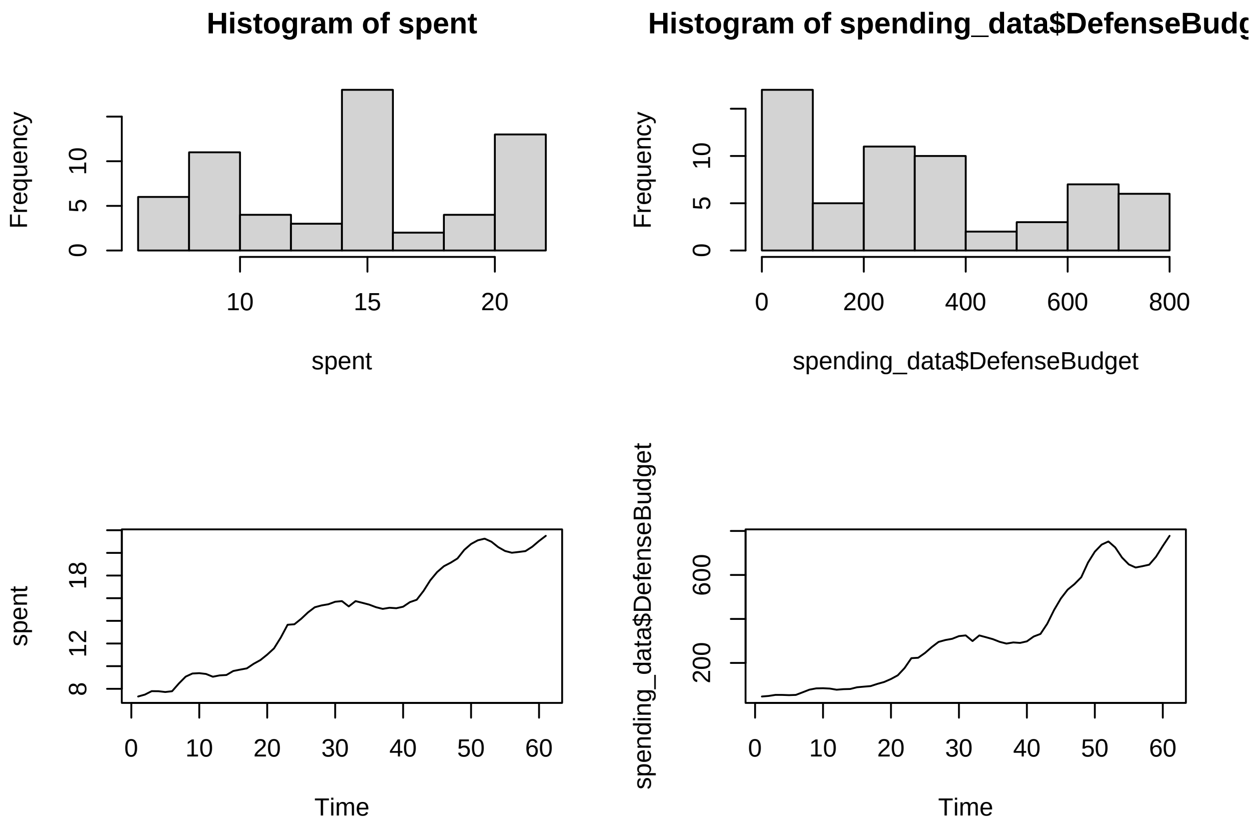

```
spending_data.sqrt = log(spending_data$DefenseBudget)
plot.ts(spending_data.sqrt)
```




```
# compare transforms on time series plot
op=par(mfrow=c(2,2))
ts.plot(spending_data$DefenseBudget, main = "Original Time Series")
ts.plot(spent, main = "Box-cox Transform")
ts.plot(spending_data.log, main = "Log transform")
ts.plot(spending_data.sqrt, main = "Square root transform")
```




```
# histogram of spent vs original
hist(spent)
hist(spending_data$DefenseBudget)
# spent seems more normal while original is skewed

# time series plot of spent vs original
plot.ts(spent)
plot.ts(spending_data$DefenseBudget)
```



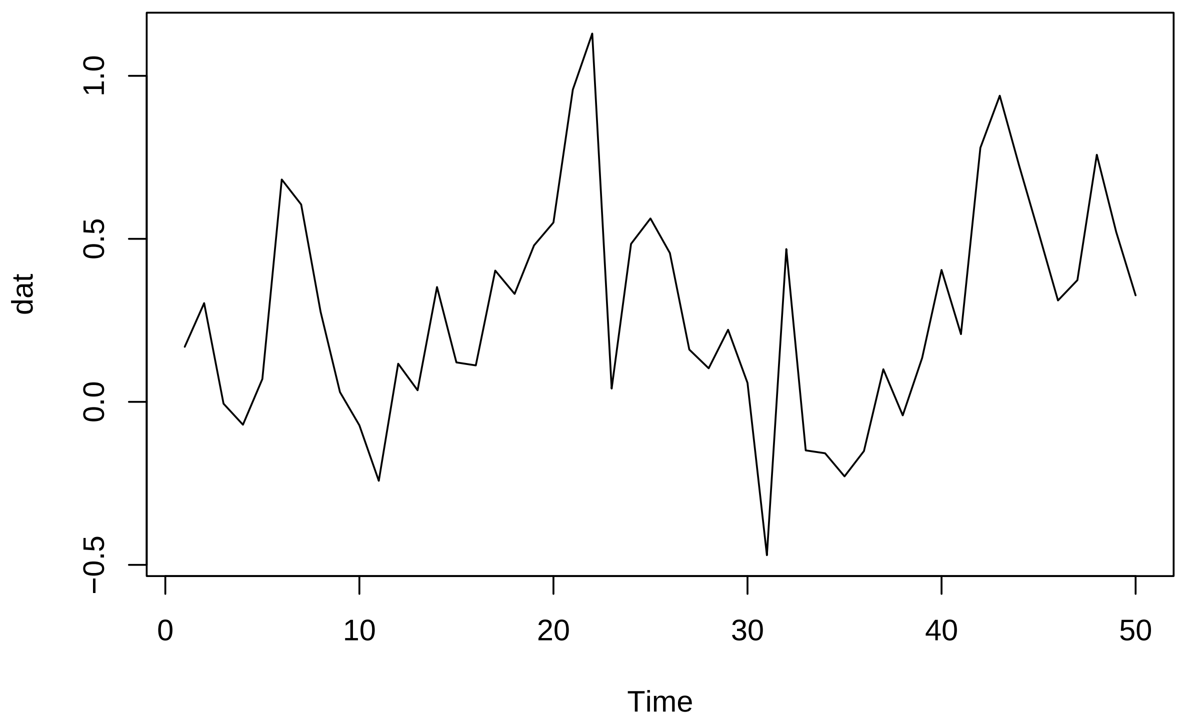

```r
# both plots look around the same, but spent seems to have more spent in the median


# Based from the histogram, I will use the spent instead of the original data


# training and testing dataset
length(spent)
```

```
## [1] 61
```

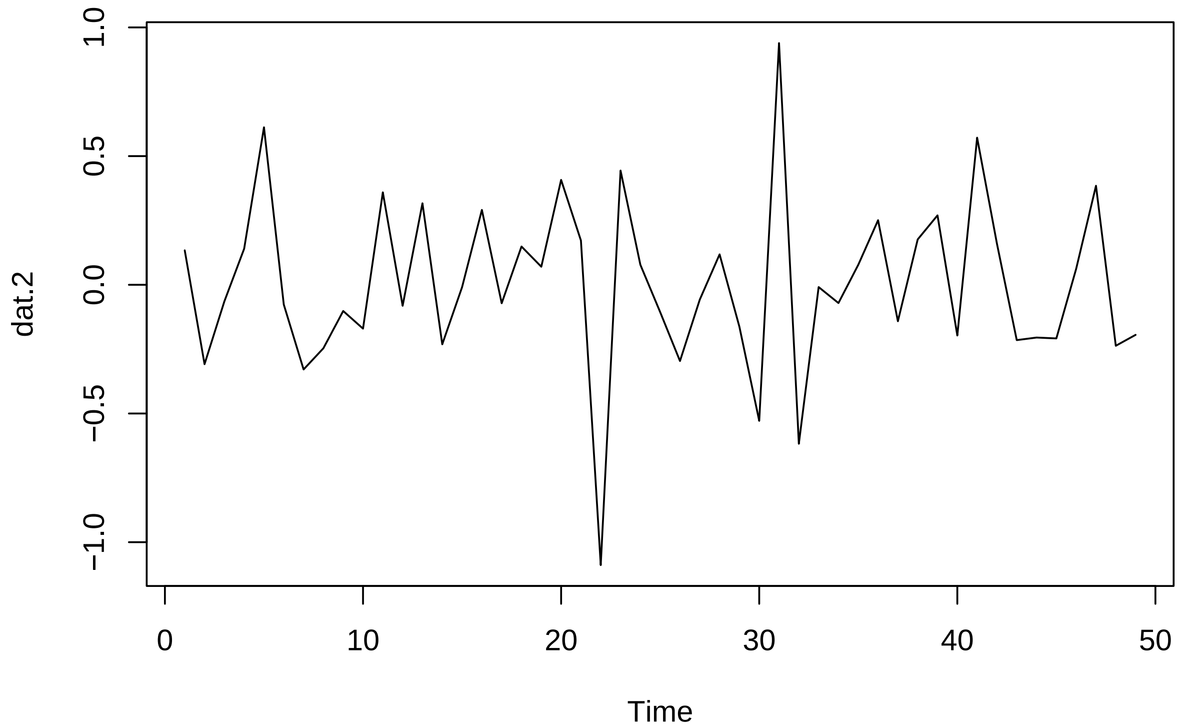
```r
spent.train = spent[c(1:51)]
spent.test  = spent[c(52:61)]
```

Differencing

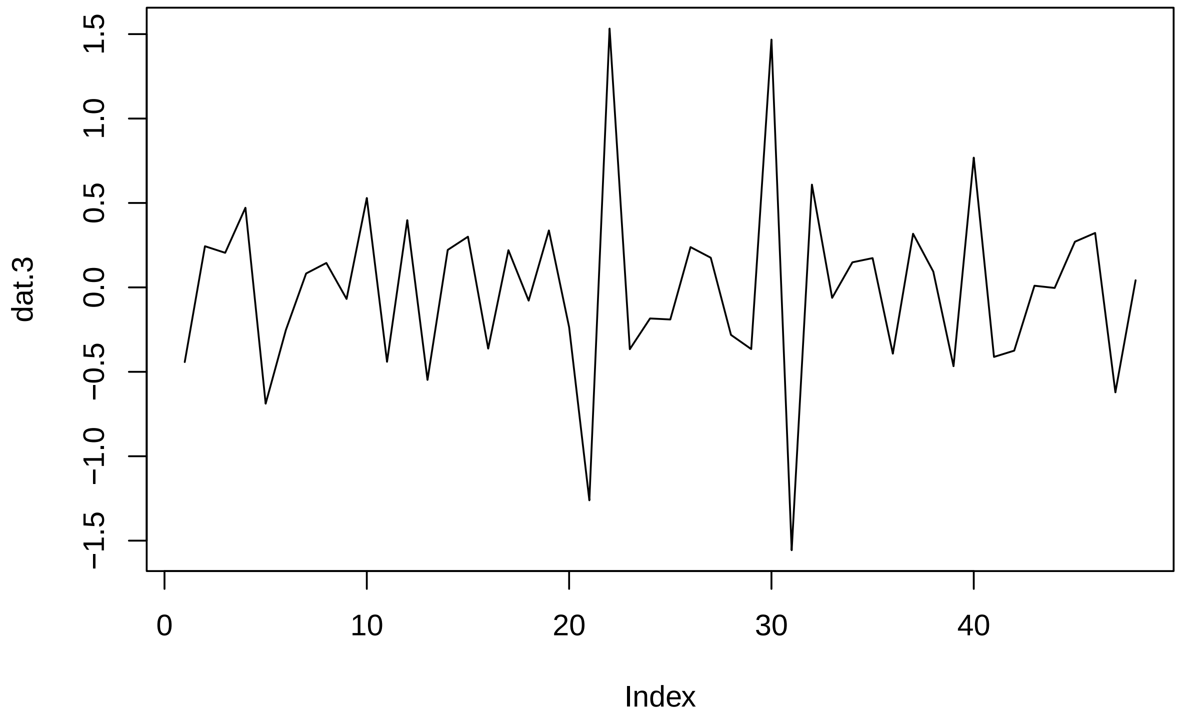
```r
# difference once and plot time series
dat <- diff(spent.train, 1)
plot.ts(dat, type= "l")
```




```
# twice differenced and plot time series
dat.2 <- diff(dat, 1)
plot.ts(dat.2, type= "l")
```




```
# three times differenced and plot
dat.3 <- diff(dat.2,1)
plot(dat.3, type= "l")
```




```r
# check variance
var(spent.train)
```

```
## [1] 15.92
```

```r
var(dat)
```

```
## [1] 0.114
```

```r
var(dat.2)
```

```
## [1] 0.1118
```

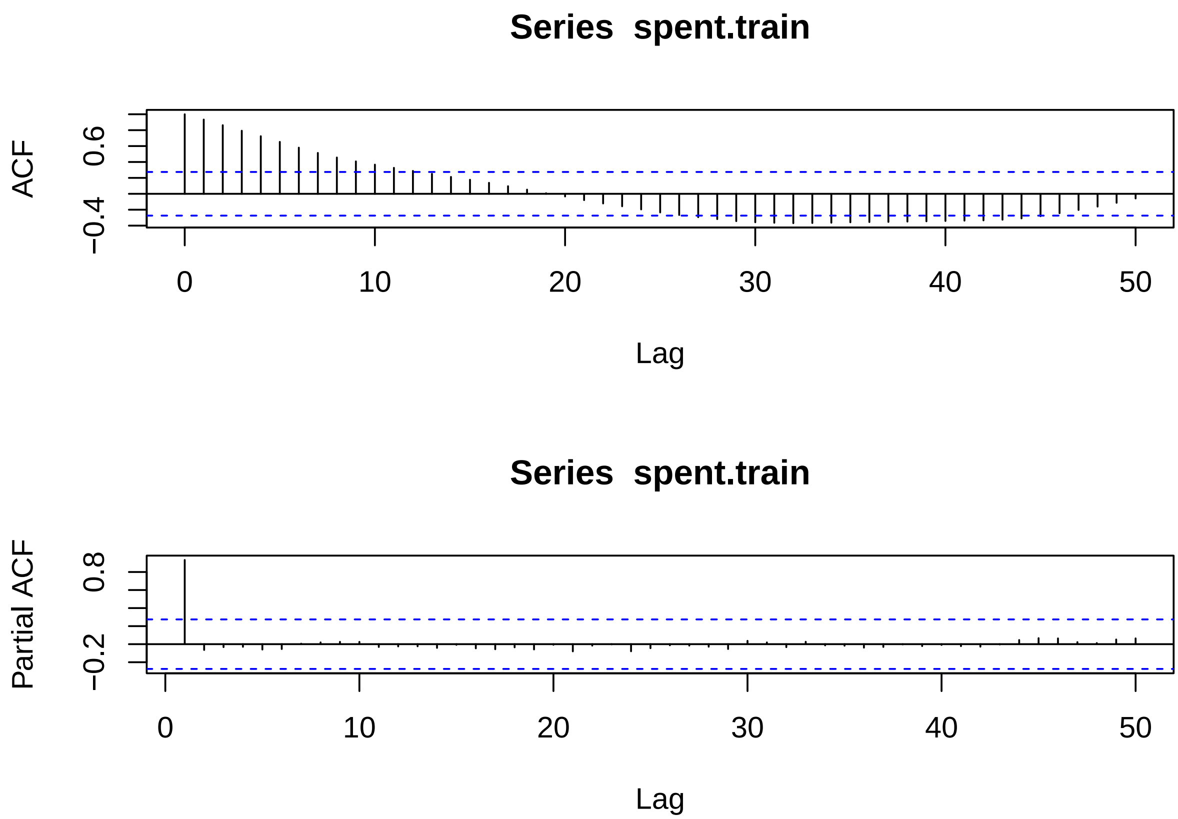
```r
var(dat.3)
```

```
## [1] 0.296
```

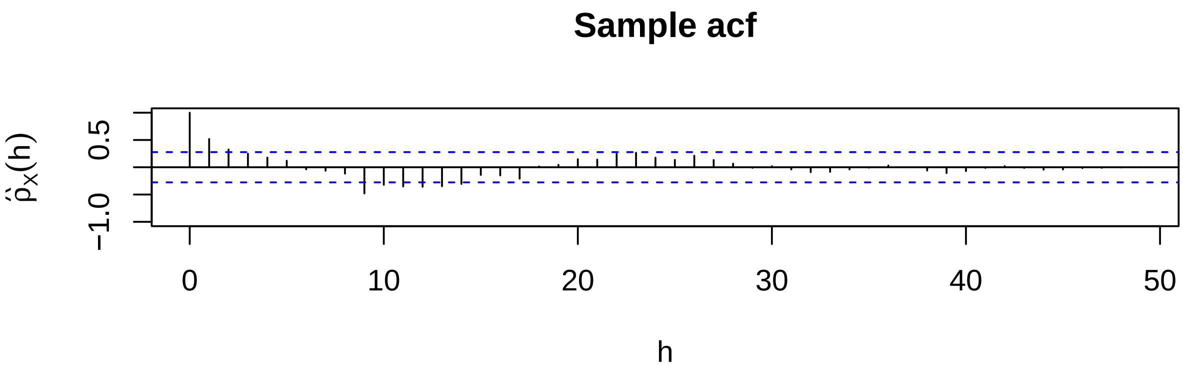
```r
# I differenced the data and found d to be 2. Based from the variance, the lowest variance is when d=2.
```

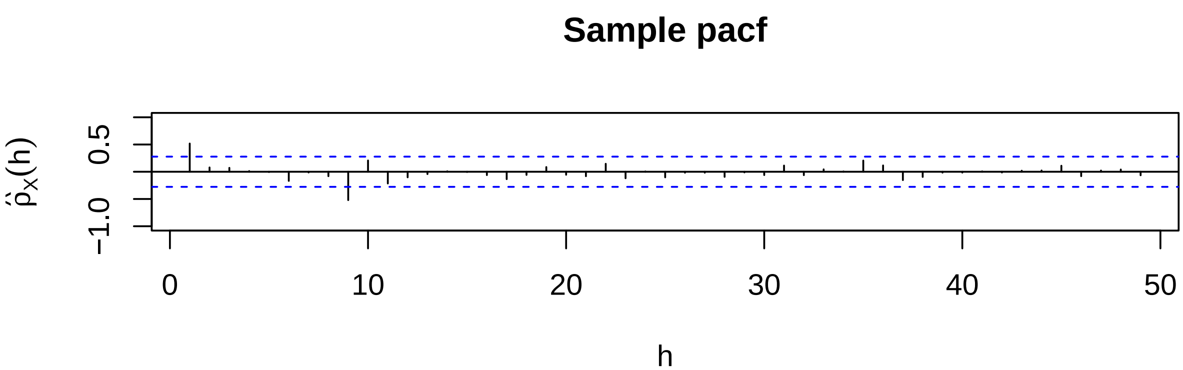
Model Identification

```r
# ACF, PACF for spent
opar <- par(no.readonly = T)
par(mfrow=c(2,1))
acf(spent.train, lag.max=100)
pacf(spent.train, lag.max=100)
```



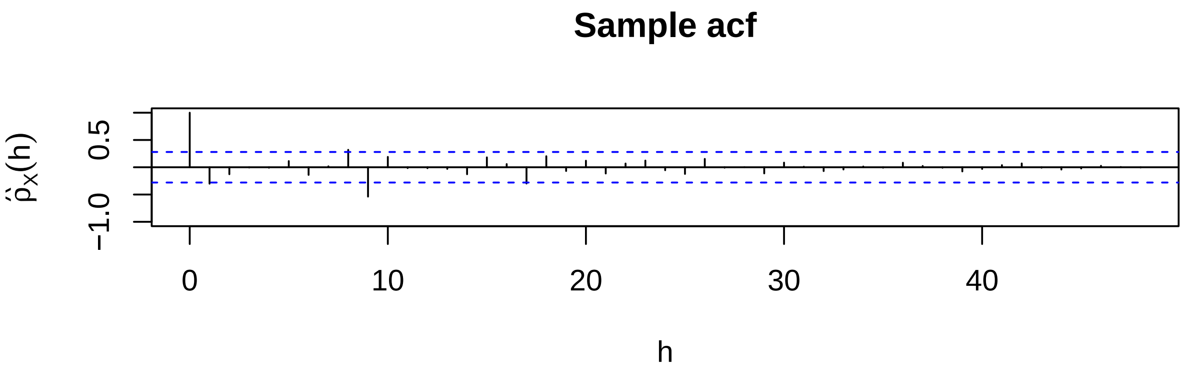

```r
par(opar)

# ACF, PACF for dat (difference once)
opar <- par(no.readonly = T)
par(mfrow=c(2,1))
acf(dat, lag.max = 100, main="Sample acf", ylim=c(-1,1),xlab="h", ylab= expression(hat(rho)[X](h)))
pacf(dat, lag.max=100, main="Sample pacf", ylim=c(-1,1),xlab="h", ylab= expression(hat(rho)[X](h)))
```

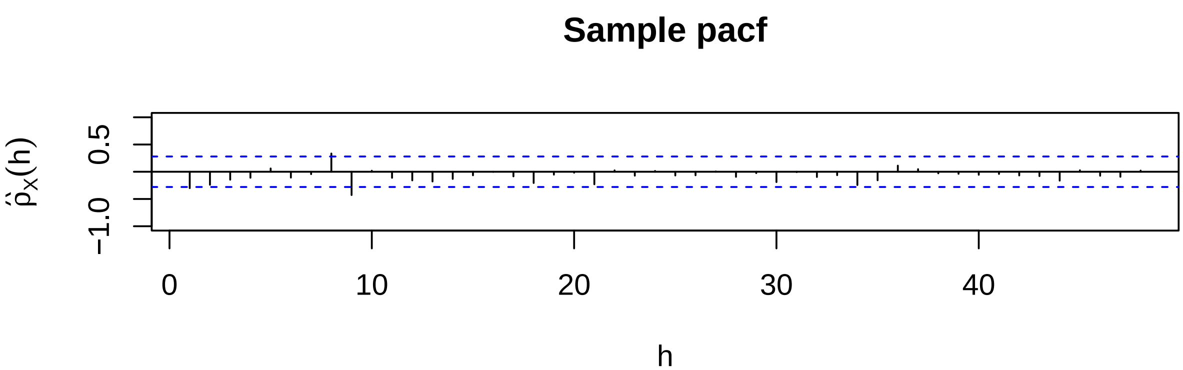





```r
# ACF, PACF for dat.2 (difference twice)
opar <- par(no.readonly = T)
par(mfrow=c(2,1))
acf(dat.2, lag.max=100, main="Sample acf", ylim=c(-1,1),xlab="h", ylab= expression(hat(rho)[X](h)))
pacf(dat.2, lag.max=100, main="Sample pacf", ylim=c(-1,1),xlab="h", ylab= expression(hat(rho)[X](h)))
```

Sample acf





Model estimation

```
# Candidate models:
df <- expand.grid(p=0:10, q=0:10)
df <- cbind(df, AICc=NA)

# Compute AICc:
for (i in 1:nrow(df)) {
  sarima.obj <- NULL
  try(arima.obj <- arima(spent.train, order=c(df$p[i],2, df$q[i]), method="ML"))
  if (!is.null(arima.obj)) { df$AICc[i] <- AICc(arima.obj) }
  # print(df[i, ])
}
```

```
## Warning in log(s2): NaNs produced

## Warning in log(s2): NaNs produced

## Warning in log(s2): NaNs produced

## Warning in arima(spent.train, order = c(df$p[i], 2, df$q[i]), method = "ML"):
## possible convergence problem: optim gave code = 1
```

```
## Warning in arima(spent.train, order = c(df$p[i], 2, df$q[i]), method = "ML"):
## possible convergence problem: optim gave code = 1

## Warning in log(s2): NaNs produced

## Warning in arima(spent.train, order = c(df$p[i], 2, df$q[i]), method = "ML"):
## possible convergence problem: optim gave code = 1

## Warning in log(s2): NaNs produced

## Warning in log(s2): NaNs produced

## Warning in log(s2): NaNs produced

## Warning in arima(spent.train, order = c(df$p[i], 2, df$q[i]), method = "ML"):
## possible convergence problem: optim gave code = 1

## Warning in log(s2): NaNs produced

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## Warning in arima(spent.train, order = c(df$p[i], 2, df$q[i]), method = "ML"):
## possible convergence problem: optim gave code = 1

## Warning in arima(spent.train, order = c(df$p[i], 2, df$q[i]), method = "ML"):
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## Warning in arima(spent.train, order = c(df$p[i], 2, df$q[i]), method = "ML"):
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## Warning in arima(spent.train, order = c(df$p[i], 2, df$q[i]), method = "ML"):
## possible convergence problem: optim gave code = 1

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## possible convergence problem: optim gave code = 1

## Warning in arima(spent.train, order = c(df$p[i], 2, df$q[i]), method = "ML"):
```

```
## possible convergence problem: optim gave code = 1

## Warning in arima(spent.train, order = c(df$p[i], 2, df$q[i]), method = "ML"):
## possible convergence problem: optim gave code = 1

## Warning in arima(spent.train, order = c(df$p[i], 2, df$q[i]), method = "ML"):
## possible convergence problem: optim gave code = 1

## Warning in arima(spent.train, order = c(df$p[i], 2, df$q[i]), method = "ML"):
## possible convergence problem: optim gave code = 1

## Warning in log(s2): NaNs produced

## Warning in arima(spent.train, order = c(df$p[i], 2, df$q[i]), method = "ML"):
## possible convergence problem: optim gave code = 1

## Warning in arima(spent.train, order = c(df$p[i], 2, df$q[i]), method = "ML"):
## possible convergence problem: optim gave code = 1

## Error in optim(init[mask], armafn, method = optim.method, hessian = TRUE,  :
##   non-finite finite-difference value [10]

## Warning in log(s2): NaNs produced

## Warning in log(s2): possible convergence problem: optim gave code = 1
```

```
df[which.min(df$AICc), ]
```

```
##    p q  AICc
## 13 1 1 26.31
```

```
df[order(df$AICc),]
```

```
##      p  q  AICc
## 13   1  1 26.31
## 12   0  1 26.86
## 27   4  2 26.97
## 80   2  7 27.14
## 21   9  1 27.60
## 23   0  2 28.04
## 100  0  9 28.47
## 10   9  0 28.47
## 32   9  2 28.60
## 81   3  7 29.19
## 22  10  1 29.23
## 3    2  0 29.40
## 28   5  2 29.62
## 38   4  3 29.63
## 39   5  3 29.79
## 15   3  1 29.96
## 67   0  6 30.01
```

```
## 91    2  8 30.11
## 25    2  2 30.12
## 2     1  0 30.17
## 24    1  2 30.30
## 34    0  3 30.30
## 14    2  1 30.33
## 111   0 10 30.36
## 51    6  4 30.43
## 4     3  0 30.68
## 101   1  9 30.79
## 112   1 10 30.96
## 79    1  7 31.20
## 11   10  0 31.47
## 45    0  4 31.70
## 54    9  4 31.87
## 40    6  3 31.93
## 43    9  3 32.03
## 33   10  2 32.04
## 52    7  4 32.12
## 57    1  5 32.17
## 48    3  4 32.25
## 46    1  4 32.31
## 26    3  2 32.38
## 50    5  4 32.38
## 92    3  8 32.41
## 63    7  5 32.41
## 36    2  3 32.42
## 5     4  0 32.52
## 62    6  5 32.57
## 35    1  3 32.65
## 68    1  6 32.66
## 78    0  7 32.68
## 1     0  0 32.70
## 61    5  5 32.86
## 60    4  5 32.88
## 83    5  7 32.95
## 31    8  2 33.00
## 20    8  1 33.26
## 47    2  4 33.33
## 56    0  5 33.50
## 102   2  9 33.78
## 82    4  7 33.86
## 93    4  8 33.91
## 90    1  8 34.16
## 58    2  5 34.17
## 69    2  6 34.26
## 49    4  4 34.43
## 71    4  6 34.58
## 42    8  3 34.67
## 6     5  0 34.74
## 16    4  1 34.77
## 65    9  5 34.80
## 84    6  7 34.90
## 37    3  3 34.98
```

```
## 74   7  6 35.01
## 70   3  6 35.23
## 19   7  1 35.26
## 89   0  8 35.45
## 44  10  3 35.53
## 55  10  4 35.63
## 29   6  2 35.86
## 72   5  6 36.03
## 9    8  0 36.16
## 104  4  9 36.25
## 94   5  8 36.53
## 7    6  0 36.76
## 59   3  5 36.81
## 66  10  5 36.93
## 73   6  6 36.95
## 30   7  2 36.98
## 41   7  3 37.18
## 17   5  1 37.21
## 113  2 10 37.22
## 103  3  9 37.22
## 76   9  6 37.44
## 114  3 10 37.84
## 53   8  4 38.10
## 64   8  5 39.12
## 95   6  8 39.13
## 8    7  0 39.24
## 105  5  9 39.25
## 18   6  1 39.42
## 115  4 10 39.55
## 75   8  6 39.86
## 85   7  7 40.30
## 87   9  7 40.91
## 77  10  6 41.07
## 106  6  9 41.53
## 96   7  8 42.88
## 116  5 10 43.06
## 107  7  9 43.63
## 117  6 10 44.63
## 98   9  8 45.26
## 86   8  7 45.28
## 88  10  7 45.41
## 108  8  9 47.35
## 118  7 10 47.49
## 97   8  8 47.82
## 99  10  8 48.81
## 119  8 10 52.18
## 109  9  9 52.60
## 110 10  9 52.60
## 120  9 10 54.31
## 121 10 10 63.93
```

```
# using principle of parsimony and lowest AICC, I will consider the two best models which are (1,1) and

# Final model 1
```

```r
fit1 <- arima(spent.train, order = c(1,2,1),
              method = "ML")
fit1
```

```
##
## Call:
## arima(x = spent.train, order = c(1, 2, 1), method = "ML")
##
## Coefficients:
##         ar1     ma1
##       0.539  -1.000
## s.e.  0.124   0.077
##
## sigma^2 estimated as 0.0833:  log likelihood = -10.03,  aic = 26.06
```

```r
# Final model 2
fit2 <- arima(spent.train, order = c(0,2,1),
              method = "ML")
fit2
```

```
##
## Call:
## arima(x = spent.train, order = c(0, 2, 1), method = "ML")
##
## Coefficients:
##          ma1
##       -0.490
## s.e.   0.144
##
## sigma^2 estimated as 0.0927:  log likelihood = -11.39,  aic = 26.78
```

```r
#source("plot.roots.R")
#plot.roots(NULL,polyroot(c(1, -0.3353, 0, -0.1612)), main="(A) roots of ma part, nonseasonal ")
```

Model Diagnostics for model 1

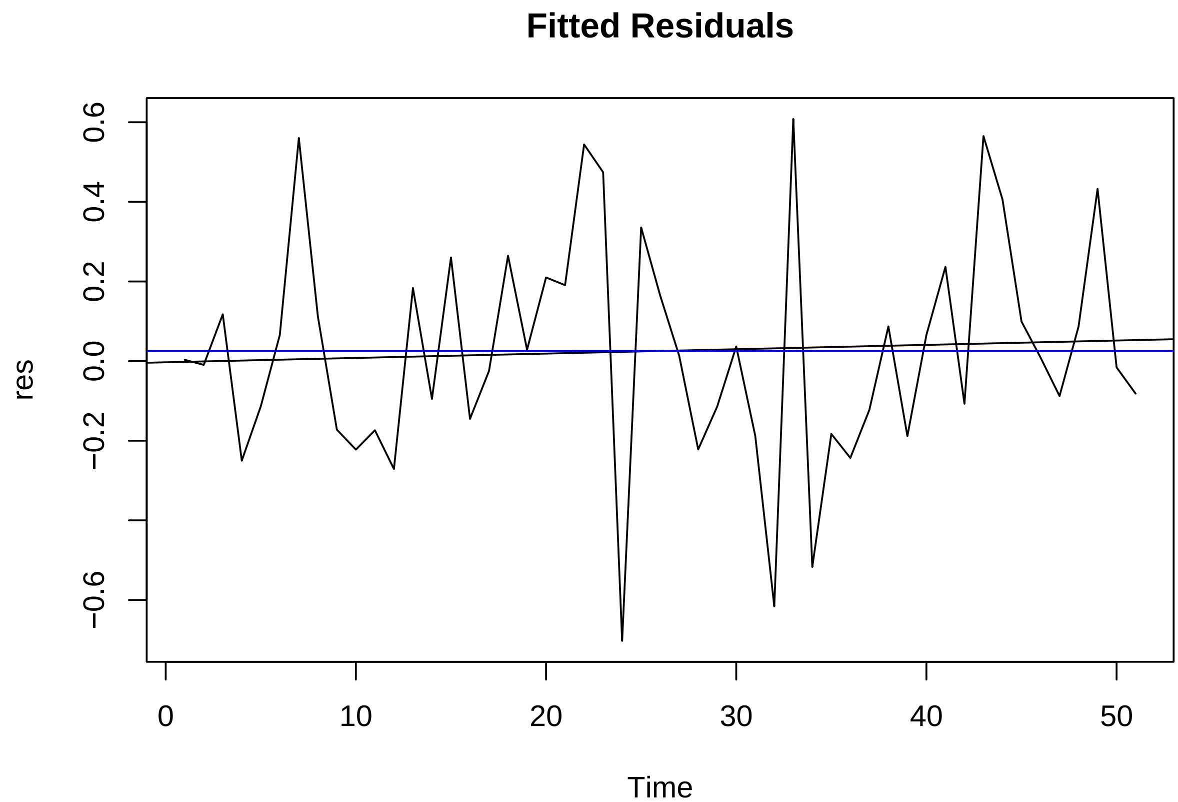
```r
# residual plots
res <- residuals(fit1)
mean(res)
```

```
## [1] 0.02542
```

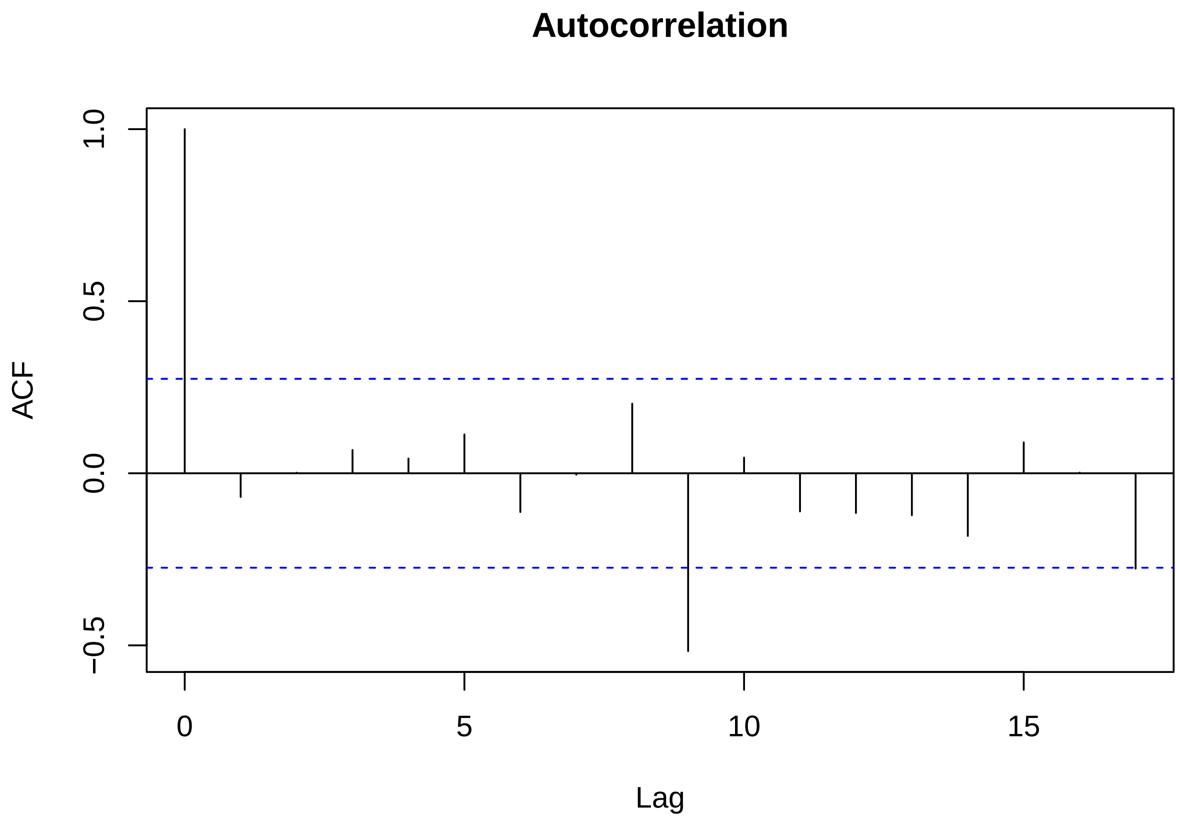
```r
var(res)
```

```
## [1] 0.08103
```

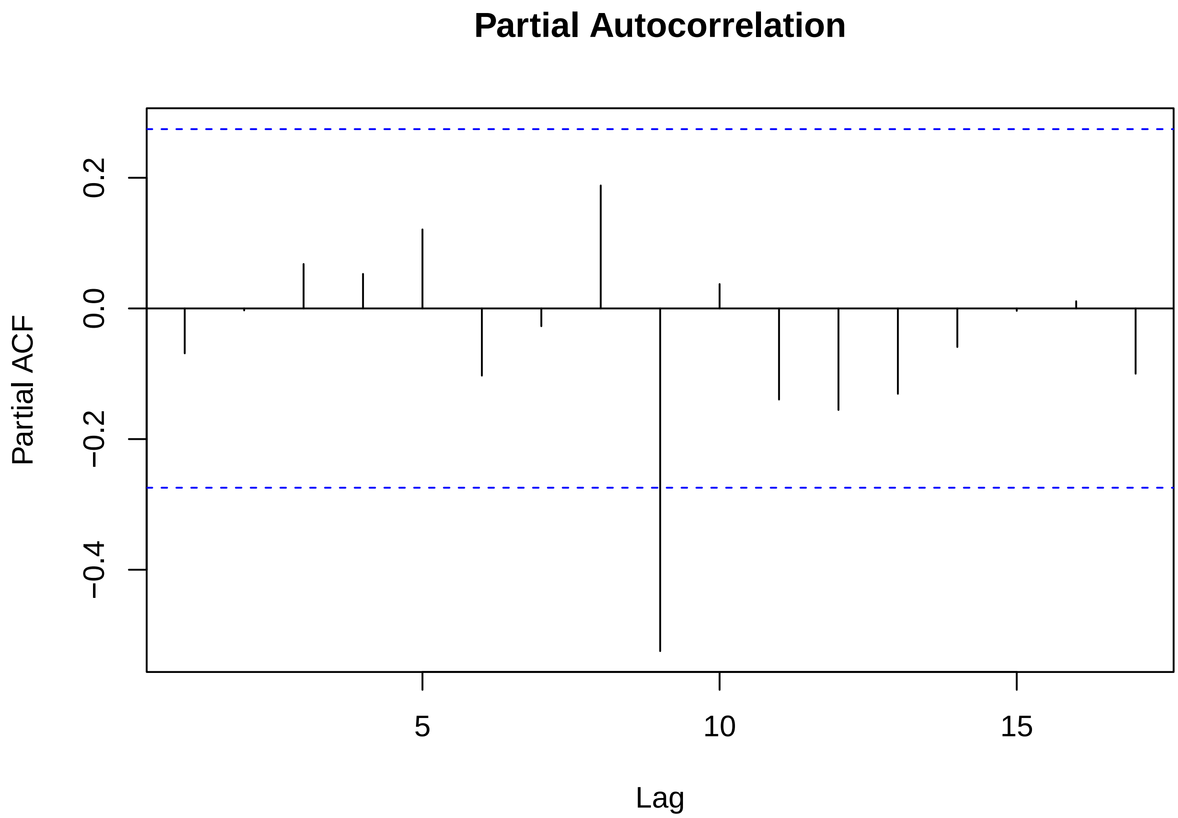
```r
# layout
par(mfrow=c(1,1))
ts.plot(res, main  = "Fitted Residuals")
t <- 1:length(res)
fit1.res = lm(res~t)
abline(fit1.res)
abline(h=mean(res), col = "blue")
```




```r
# ACF and PACF
par(mfrow=c(1,1))
acf(res, main = "Autocorrelation")
```




```
pacf(res, main = "Partial Autocorrelation")
```




```r
# Testing for independence of residuals
# lag 8 is chosen since sqrt 61 is around 8
Box.test(res, lag = 8, type = c("Box-Pierce"), fitdf = 2)
```

```
##
##  Box-Pierce test
##
## data:  res
## X-squared = 4, df = 6, p-value = 0.7
```

```r
Box.test(res, lag = 8, type = c("Ljung-Box"), fitdf = 2)
```

```
##
##  Box-Ljung test
##
## data:  res
## X-squared = 4.7, df = 6, p-value = 0.6
```

```r
Box.test(res^2, lag = 8, type = c("Ljung-Box"), fitdf = 0)
```

```
##
##  Box-Ljung test
##
```

```
## data:  res^2
## X-squared = 20, df = 8, p-value = 0.01
```

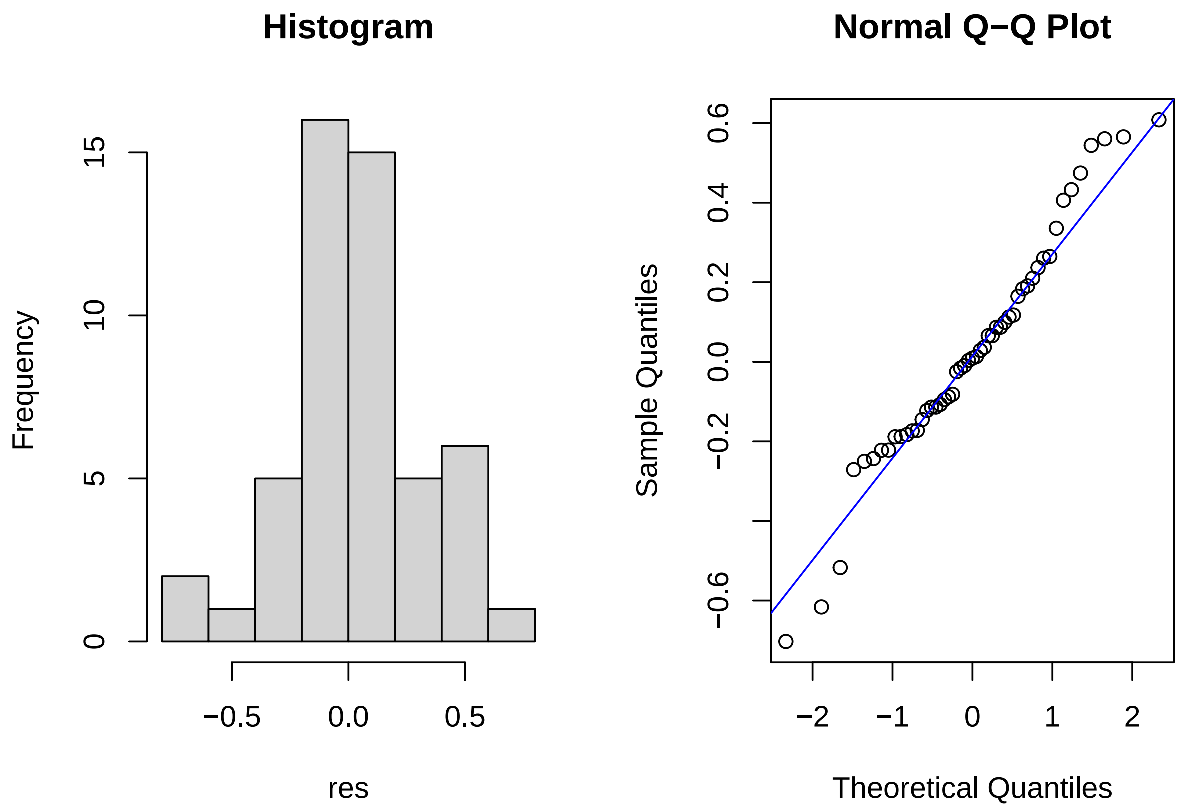
```
# test for normality of residuals
shapiro.test(res)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  res
## W = 0.97, p-value = 0.2
```

```
# yule-walker test
ar(res, aic=TRUE, order.max = NULL, method=c("yule-walker"))
```

```
##
## Call:
## ar(x = res, aic = TRUE, order.max = NULL, method = c("yule-walker"))
##
## Coefficients:
##      1       2       3       4       5       6       7       8       9
##  0.035   0.015   0.004   0.102   0.125  -0.074  -0.003   0.155  -0.525
##
## Order selected 9  sigma^2 estimated as  0.0665
```

```
# Histogram and qq plot
par(mfrow=c(1,2))
hist(res, main= "Histogram")
qqnorm(res)
qqline(res, col = "blue")
```




```
# Model 1 passes all tests but Mc-Leod Li test and residuals are normal
```

Model Diagnostics for model 2

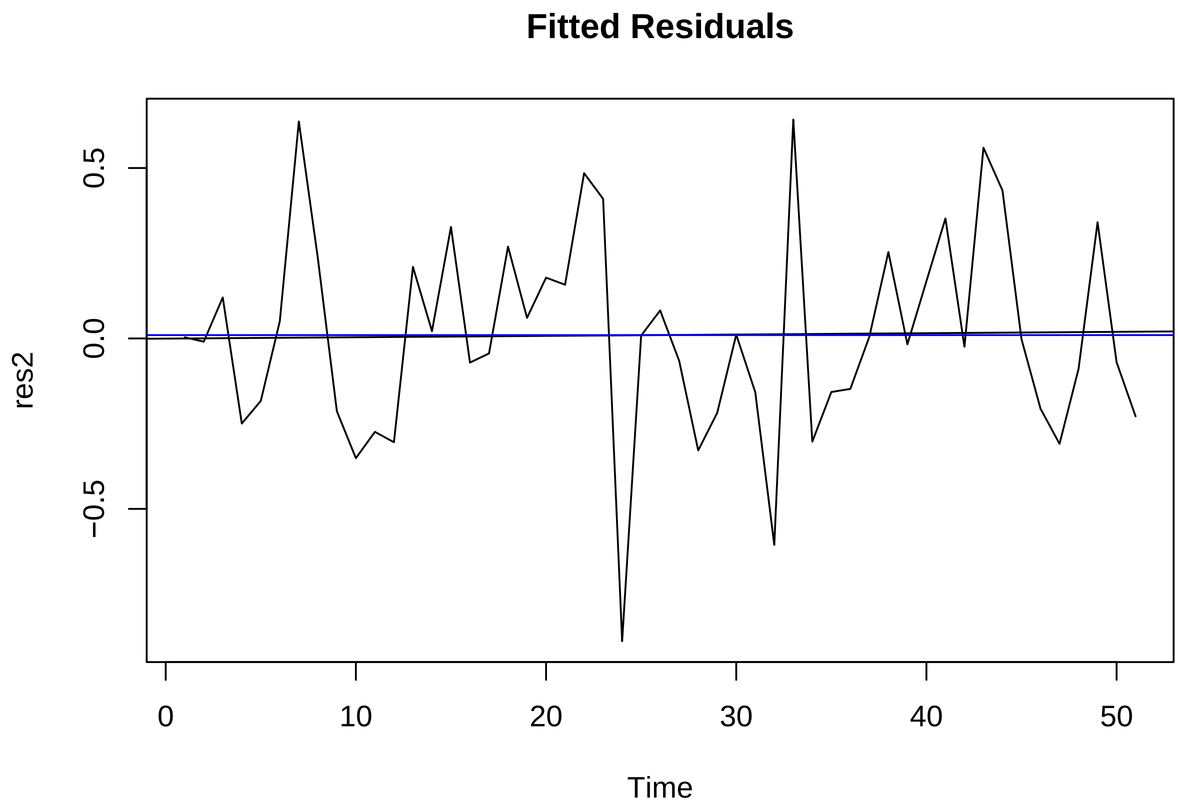
```
# residual plots
res2 <- residuals(fit2)
mean(res2)
```

```
## [1] 0.009866
```

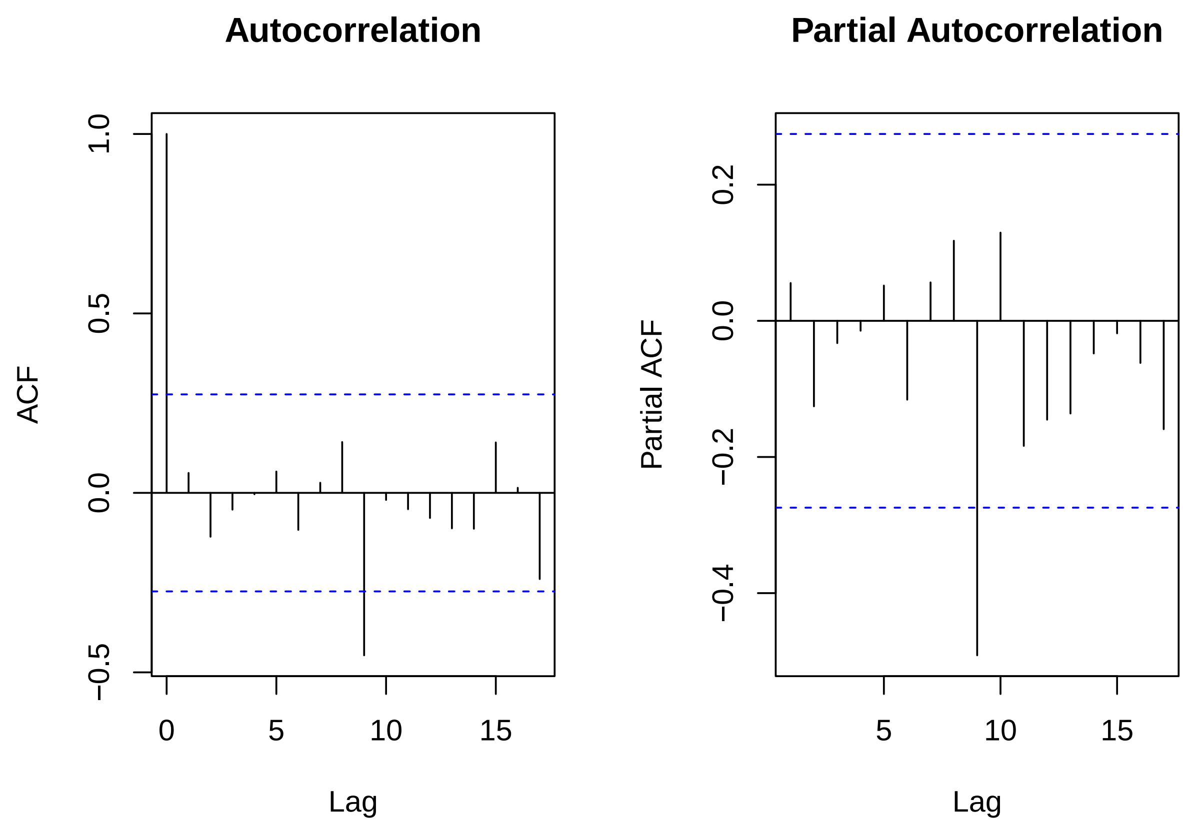
```
var(res2)
```

```
## [1] 0.09073
```

```
# layout
par(mfrow=c(1,1))
ts.plot(res2, main  = "Fitted Residuals")
t <- 1:length(res2)
fit2.res2 = lm(res2~t)
abline(fit2.res2)
abline(h=mean(res2), col = "blue")
```




```
# ACF and PACF
par(mfrow=c(1,2))
acf(res2, main = "Autocorrelation")
pacf(res2, main = "Partial Autocorrelation")
```




```r
# Testing for independence of residuals
Box.test(res2, lag = 8, type = c("Box-Pierce"), fitdf = 1)
```

```
##
##  Box-Pierce test
##
## data:  res2
## X-squared = 2.8, df = 7, p-value = 0.9
```

```r
Box.test(res2, lag = 8, type = c("Ljung-Box"), fitdf = 1)
```

```
##
##  Box-Ljung test
##
## data:  res2
## X-squared = 3.3, df = 7, p-value = 0.9
```

```r
Box.test(res2^2, lag = 8, type = c("Ljung-Box"), fitdf = 0)
```

```
##
##  Box-Ljung test
##
## data:  res2^2
## X-squared = 9.4, df = 8, p-value = 0.3
```

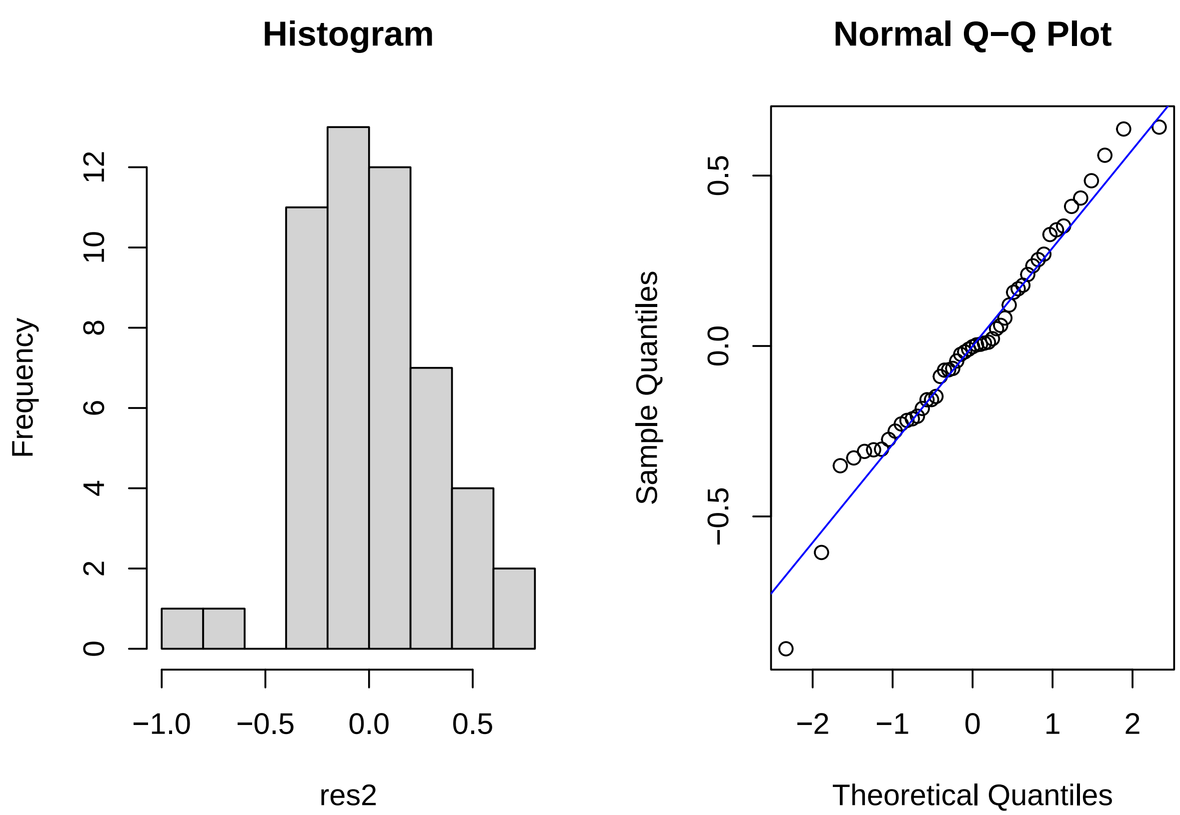
```r
# test for normality of residuals
shapiro.test(res2)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  res2
## W = 0.98, p-value = 0.4
```

```r
# yule-walker test
ar(res2, aic=TRUE, order.max = NULL, method=c("yule-walker"))
```

```
##
## Call:
## ar(x = res2, aic = TRUE, order.max = NULL, method = c("yule-walker"))
##
##
## Order selected 0  sigma^2 estimated as  0.0907
```

```r
# Histogram and qq plot
par(mfrow=c(1,2))
hist(res2, main= "Histogram")
qqnorm(res2)
qqline(res2, col = "blue")
```



```r
# residuals pass Box.test and residuals are normal
```

```r
# I will use fit2 as the final model
fit2
```

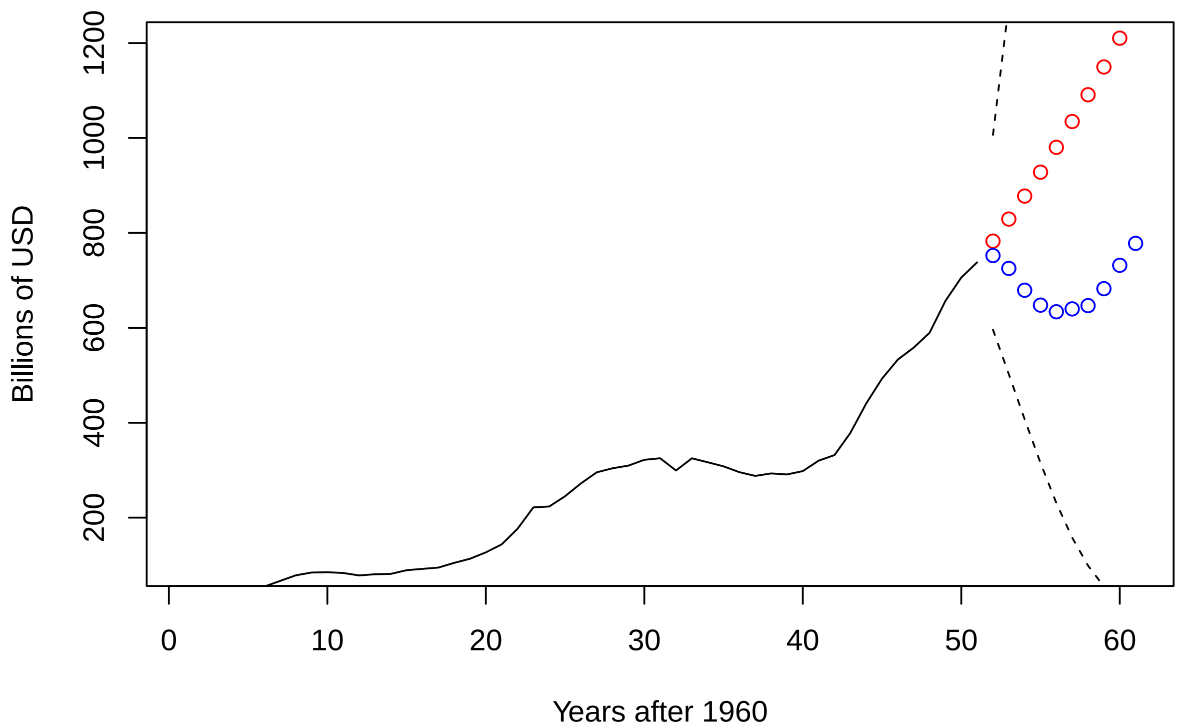
```
##
## Call:
## arima(x = spent.train, order = c(0, 2, 1), method = "ML")
##
## Coefficients:
##           ma1
##        -0.490
## s.e.   0.144
##
## sigma^2 estimated as 0.0927:  log likelihood = -11.39,  aic = 26.78
```

Final model should be (1-B^2)Xt = (1-0.49B)Zt, Zt ~ WN(0, 0.0927)

Data Forecasting

```r
# Predict 10 future observations and plot
par(mfrow=c(1,1))
m <- length(spent.train)
mypred <- predict(fit2, n.ahead=10)
ts.plot(c(lambda*spent.train +1)^(1/lambda), xlim=c(1, length(lambda*spent.train)+10), ylim=c(100,1200)

points((m+1):(m+10), col="red", (lambda*mypred$pred +1)^(1/lambda))
points((m+1):(m+10), col="blue", (lambda*spent.test +1)^(1/lambda))
lines((m+1):(m+10),
      (lambda*mypred$pred + 1.96*mypred$se+1)^(1/lambda), lty=2)
lines((m+1):(m+10),
      (lambda*mypred$pred - 1.96*mypred$se+1)^(1/lambda), lty=2)
```


Billions of USD
200
400
600
800
1000
1200
0
10
20
30
40
50
60
Years after 1960