Homework: Week 9

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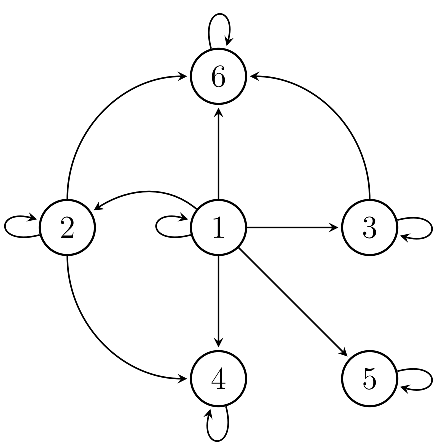
Math 100
Due: December 1, 2017
Professor Boltje
MWF 9:20a-10:25a

Section 11.0

2.

$$R = \{(1,1),(1,2),(1,3),(1,4),(1,5)\\ ,(1,6),(2,2),(2,4),(2,6),(3,3)\\ ,(3,6),(4,4),(5,5),(6,6)\}$$



12.

$y \leq x$. The relation is $\leq$.

Section 11.1

14.

Suppose $x \in A$. Since R is symmetric, then xRa and aRx, then since R is transitive then xRx.

Section 11.2

2.

$$R = \{(a,a),(b,b),(c,c),(d,d),(e,e)\\ ,(a,d),(d,a),(b,c),(c,b),(e,d)\\ ,(d,e),(a,e),(e,a)\}$$

10.

Proposition: Suppose R and A are two equivalence relations on a set A. Prove that $R \cap S$ is also an equivalence relation.

Proof. Suppose $(x, y) \in R \cap S$ is not an equivalence relation. Then $(x, y) \in R$ and $(x, y) \in S$. R and S are equivalence relations therefore $(x, y) \in R \cap S$ is an equivalence relation, but we said $(x, y) \notin R \cap S$, a contradiction.

12.

Proposition: Suppose R and S are equivalence relations on a set A, then $R \cup S$ is also an equivalence relation on A.

Proof. Let $R = \{(a, a), (b, b), (c, c), (a, b), (b, a)\}$ and $S = \{(a, a), (b, b), (c, c), (b, c), (c.b)\}$. Then $R \cup S = \{(a, a), (b, b), (c, c), (a, b), (b, a), (b, c), (c.b)\}$. Now observe that $(a, b) \in R \wedge (a, b) \in S \not\Longrightarrow (a, b) \in R \cup S$. So $R \cup S$ is not transitive, therefore it is not an equivalence relation.

14.

To show S is an equivalence relation, we must show that it's reflexive, symmetric, and transitive. To show it's reflexive, we choose an $x \in A$ and $n = 1$, Then $x_1 = x$ so then xRx since R is reflexive, thus xSx. Now suppose xSy. Then, by definition, there are elements $x_1, x_2, ..., x_n \in A$ such that $xRx_1, x_1Rx_2, ..., x_nRy$. Since R is symmetric, $yRx_n, ...x_2Rx_1, x_1Rx$, so ySx. Now suppose xSy and ySz. Since xSy then $xRa, aRa_1, ...a_nRy$. Since $ySzthenyRb, bRb_1, ..., b_mRz$ so $xRa, ..., a_nRy, yRb1, ..., b_mRz$ so xSz. Now to show that $R \subseteq S$, suppose $(x, y) \in \mathbb{R}$. Since xRy, xSy by definition $xSy \implies (x.y) \in S$. To show S is the smallest equivalence relation on A, assume T is an equivalence relation on A containing R. Suppose $(x, y) \in S$. So xSy is $xRx_1, ..., x_nRy$ so $xTx_1, ..., x_nTy$ so xTx which shows that S is a subset of T.

Section 11.3

2.

The partitions of set A are:
$\{\{a\}, \{b\}, \{c\}\}$,
$\{\{a, b\}, \{c\}\}$,
$\{\{a\}, \{b, c\}\}$,
$\{\{a, c\}, \{b\}\}$,
$\{\{a, b, c\}\}$.

4.

Proof. Suppose $x \in A$. Let $X \in P$, such that $x \in X$. Then by definition, xRx. Then suppose xRy for some $x, y \in A$, then by definition there exists a $X \in P$ such that $x, y \in X$. In particular, it also means $y, x \in X$ so yRx. Now suppose xRy and xRz for some $x, y, z \in A$. Then by definition, there exists $X_1, X_2 \in P$ such that $x, y \in X_1$ and $y, z \in X_2$. Since P is a partition of A, y can only be in one part, therefore $X_1 = X_2$. Thus $x, z \in X \to xRz$. Let $X \in P$ be a part of the partition. Then by definition xRy for any two $x, y \in X$. Also, given $a \in X$ and $b \in A - X$ then by definition a and b are not related. Thus X is an equivalence class of R. Since every element of A is in exactly one of the parts of P, there are no equivalence classes besides the ones from the partition, thus P is the set of equivalence classes of R.

Section 11.4

4.

$+$	[0]	[1]	[2]	[3]	[4]	[5]
[0]	[0]	[1]	[2]	[3]	[4]	[5]
[1]	[1]	[2]	[3]	[4]	[5]	[0]
[2]	[2]	[3]	[4]	[5]	[0]	[1]
[3]	[3]	[4]	[5]	[0]	[1]	[2]
[4]	[4]	[5]	[0]	[1]	[2]	[3]
[5]	[5]	[0]	[1]	[2]	[3]	[4]

$\cdot$	[0]	[1]	[2]	[3]	[4]	[5]
[0]	[0]	[0]	[0]	[0]	[0]	[0]
[1]	[0]	[1]	[2]	[3]	[4]	[5]
[2]	[0]	[2]	[4]	[0]	[2]	[4]
[3]	[0]	[3]	[0]	[3]	[0]	[3]
[4]	[0]	[4]	[2]	[0]	[4]	[2]
[5]	[0]	[5]	[4]	[3]	[2]	[1]

6.

No, because in $\mathbb{Z}_6$, for example, $[a]$ could be $[3]$ and $[b]$ could be $[4]$ which would be $[4] \cdot [3] = [12] = [0]$. As long as $[a] \cdot [b]$ results in a multiple of 6 then the result will be $[0]$.

8.

By definition, $[a] = [a']$ is $a \equiv a' \pmod{n}$ so $n|a-a' \to a-a' = nk, k \in \mathbb{Z}$. Similarly, $[b] = [b']$ is $b \equiv b' \pmod{n}$ so $n|b-b' \to b-b' = nm, m \in \mathbb{Z}$. Then we can say $a = a' + nk$ and $b = b' + nm$. If we add them together, we get $a + b = (a' + b') + nk + nm \to (a+b) - (a'+b') = n(k+m) \to (a+b) - (a'+b') = nh, h = (k+m) \in \mathbb{Z}$. Therefore $n|(a+b) - (a'+b') \to (a+b) \equiv (a'+b') \pmod{n}$. Therefore in $\mathbb{Z}_n$, $[a+b] = [a'+b']$.