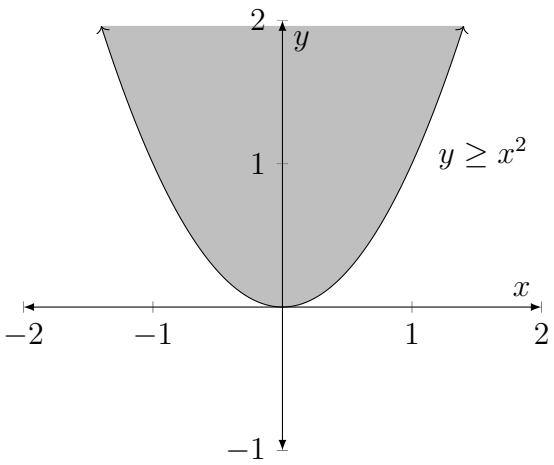
Homework: Week 2

Joseph Ismailyan

Math 100
Due: October 13, 2017
Professor Boltje
MWF 9:20a-10:25a

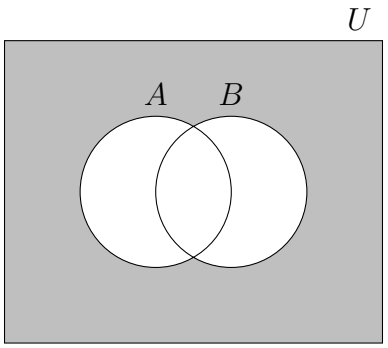
Section 1.6

6.

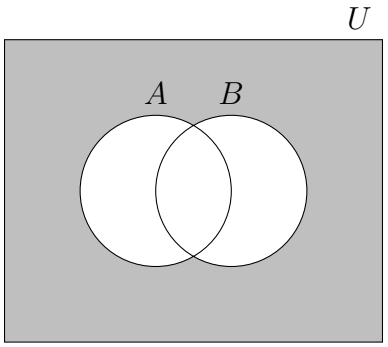


Section 1.7

$\overline{A \cup B}$



$\overline{A} \cap \overline{B}$



Answer: From the two diagrams above, we can see that the sets are equal.

Section 1.8

6(a).

Answer: $\bigcup_{i \in \mathbb{N}} [0, i+1] = [0, \infty)$

6(b).

Answer: $\bigcap_{i \in \mathbb{N}} [0, i+1] = [0, 2]$

Section 2.1

6.

Question: Some sets are finite.

Answer: It is statement because it can be proven definitely true or false.

14.

Question: Call me Ishmael.

Answer: It is a sentence because it isn't a mathematical expression.

Section 2.2

8.

Question: At least one of the numbers x and y equals 0.
$P : x$ is equal to 0.
$Q : x$ is equal to 0.

Answer: $P \vee Q$

Section 2.3

2.

Problem: Convert the following sentences to be in the form $P \implies Q$ P : A function is differentiable.
Q : A function is continuous.

Answer: $P \implies Q$

Section 2.4

4.

Problem: Convert to the form "P if and only if Q".
$P : a \in \mathbb{Q}$
$Q : 5a \in \mathbb{Q}$

Answer: $P \Leftrightarrow Q$
$a \in \mathbb{Q}$ if and only if $5a \in \mathbb{Q}$

Section 2.5

4.

Problem: Write a truth table for the logical problem: $\sim (P \vee Q) \vee \sim (P)$

P	Q	$\sim (P \vee Q)$	$\sim (P)$	$\sim (P \vee Q) \vee \sim (P)$
T	T	F	F	F
T	F	F	F	F
F	T	F	T	T
F	F	T	T	T

Section 2.5 cont.

8.

Problem: Write a truth table for the logical problem: $P \vee (Q \vee \sim R)$

P	Q	R	$(Q \vee \sim R)$	$P \vee (Q \vee \sim R)$
T	T	T	T	T
T	T	F	T	T
T	F	T	F	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	F
F	F	F	T	T

10.

Problem: Suppose the statement $((P \wedge Q) \vee R) \implies (R \vee S)$ is false. Find the truth values of P, Q, R and S.

Asnwer: Suppose A and B are statements. The only way that $(A \implies B) = False$ is if $A = True$ and $B = False$. Therefore $((P \wedge Q) \vee R)$ must be $True$ and $(R \vee S)$ must be $False$. For $(R \vee S)$ to be $False$, both R and S must be $False$. Since R is false, the statement $(P \wedge Q) \vee R)$ relies on $(P \wedge Q)$ to be true so both P and Q must be true. In conclusion:

P	Q	R	S
T	T	F	F

Section 2.6

2.

Problem: Show that the following statements are logically equivalent.
a: $P \vee (Q \wedge R)$
b: $(P \vee Q) \wedge (P \vee R)$

P	Q	R	$(Q \wedge R)$	$P \vee (Q \wedge R)$
T	T	T	T	T
T	T	F	F	T
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

P	Q	R	$(P \vee Q)$	$(P \vee R)$	$(P \vee Q) \wedge (P \vee R)$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	F	T	F
F	F	F	F	F	F

10.

Decide whether the following statements are logically equivalent:
a. $(P \implies Q) \vee R$
b. $\sim ((P \wedge \sim Q) \wedge \sim R)$
1) $(P \implies Q) \vee R = (\sim P \vee Q) \vee R$
by definition of implication
2) $(\sim P \vee Q) \vee R = \sim ((\sim P \vee Q) \vee R)$
by DeMorgan's Laws
3) $\sim ((\sim P \vee Q) \vee R) = \sim (\sim P \vee Q) \wedge \sim R$
by DeMorgan's Laws
4) $\sim (\sim P \vee Q) \wedge \sim R = (P \wedge \sim Q) \wedge \sim R$
by DeMorgan's Laws
5) $(P \wedge \sim Q) \wedge \sim R =$
$\sim ((P \wedge \sim Q) \wedge \sim R) = b$

$\therefore a \equiv b$, they're logically equivalent.