Homework: Week 3

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Math 100
Due: October 13, 2017
Professor Boltje
MWF 9:20a-10:25a

Section 2.7

6.

Translate to English:
$\exists n \in \mathbb{N}, \forall X \in \mathscr{P}(\mathbb{N}), |X| < n$

Translated: There exists a natural number n for every subset X of $\mathbb{N}$ where $|X| < n$.

This can be proven false with a simple counter example as follows:
$X = \{1, 2\}$ and $n = 1$ then $|X| > n$

Section 2.9

6.

Translate to SL:
For every positive number ε there is a positive number M for which
$|f(x) - b| < \varepsilon$, whenever $x > M$.

Logic:
$\forall \varepsilon \in \mathbb{R}, \varepsilon \geq 0, \exists M \in \mathbb{R}, M \geq 0,$
$(x > M) \implies (|f(x) - b| < \varepsilon)$

Section 2.10

8.

Translate: If x is a rational number and $x \neq 0$, then $\tan(x)$ is not a rational number.

SL: $(x \in \mathbb{Q} \wedge x \neq 0) \implies (tan(x) \notin \mathbb{Q})$
Negation:$x \in \mathbb{Q} \wedge x \neq 0 \wedge tan(x) \in \mathbb{Q}$

Translation (Answer): x is a rational number and $x \neq 0$ and $\tan(x)$ is a rational number.

Section 3.1

4.

Question: Five cards are dealt off of a standard 52-card deck and lined up in a row. How many such line-ups are there in which all 5 cards are of the same suit?
Answer: $13 * 12 * 11 * 10 * 9 = 154,440$

12.

Answer: $5^5 - (4^5 + (5 * 4^4)) = 821$

Section 3.2

8.

Answer: There are 4 places the odd sequence could start.
$O = \text{odd}, E = \text{even}$

$$
\begin{array}{l}
OOOOEEE \\
4\;3\;2\;1\;3\;2\;1 = 144 \\
\qquad\qquad\quad + \\
EOOOOEE \\
3\;6\;3\;2\;1\;2\;1 = 144 \\
\qquad\qquad\quad + \\
EEOOOOE \\
3\;2\;4\;3\;2\;1\;1 = 144 \\
\qquad\qquad\quad + \\
EEEOOOO \\
3\;2\;1\;4\;3\;2\;1 = 144
\end{array}
$$

Total : 576

Section 3.3

10.

Answer: $\binom{5}{3}\binom{7}{2} = 210$

12.

Answer:
$\binom{21}{10}\binom{11}{11} = 352,716$

Section 3.4

12.

Show that:

$$\binom{n}{k}\binom{k}{m} = \binom{n}{m}\binom{n-m}{k-m}$$

LHS:

$$= \frac{n!}{k!(n-k)!} * \frac{k!}{m!(k-m)!}$$
$$= \frac{n!}{m!(n-k)!(k-m)!}$$

RHS:

$$= \frac{n!}{m!(n-m)!} * \frac{n-m!}{(k-m)![(n-m)-(k-m)]!}$$
$$= \frac{n!}{m!(k-m)![n-m-k+m]!}$$
$$= \frac{n!}{m!(k-m)!(n-k)!}$$

$$\frac{n!}{m!(n-k)!(k-m)!} = \frac{n!}{m!(k-m)!(n-k)!}$$

$$\therefore \binom{n}{k}\binom{k}{m} = \binom{n}{m}\binom{n-m}{k-m}$$