Homework: Week 7

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Math 100
Due: November 17, 2017
Professor Boltje
MWF 9:20a-10:25a

Chapter 9

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Proposition:
If $a, b, c \in \mathbb{N}$ and ab, bc and ac all have the same parity, then a, b, and c all have the same parity.
Proof. Suppose $a, b, c \in \mathbb{N}$ and ab, bc and ac all have the same parity. Then we can use the following example: $a = 2$, $b = 4$, $c = 3$. We know that the product of any two natural numbers with opposite parity will be even, so ab, bc and ac are all even. But a , b and c do not all have the same parity.

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Proposition:
If A and B are sets, then $\mathscr{P}(A) \cap \mathscr{P}(B) = \mathscr{P}(A \cap B)$.
Proof. Let $Y \in \mathscr{P}(A \cap B)$. Then $Y \subseteq (A \cap B)$, so $Y \subseteq A$ and $Y \subseteq B$, and by definition of a power set, $Y \subseteq \mathscr{P}(A)$ and $Y \subseteq \mathscr{P}(B) \to Y \subseteq \mathscr{P}(A) \cap \mathscr{P}(B)$. Now let $Y \subseteq \mathscr{P}(A)$ and $Y \subseteq \mathscr{P}(B)$. So $Y \in A$ and $Y \in B \to Y \in (A \cap B) \to Y \subseteq \mathscr{P}(A \cap B)$. So $\mathscr{P}(A) \cap \mathscr{P}(B) = \mathscr{P}(A \cap B)$.

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Proposition:
If $a, b, c \in \mathbb{N}$, then at least one of $a - b$, $a + c$ and $b - c$ is even.
Proof. Suppose $a, b, c \in \mathbb{N}$. The in order for $a - b$, $a + c$, $b - c$ to all have odd outcomes, the operands must all have a parity opposite that of the other operand. So then if a is odd then b must be even, but c must be even to make $a+c$ odd, but then c must be odd to make $b - c$ odd. So we can see that c must simultaneously have two parities, which would be the case no matter what parity a and b are. It's impossible for a, b, and c to have opposite parities. Therefore, at least one of $a - b$, $a + c$ and $b - c$ is even.

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Proposition:
Suppose $a, b \in \mathbb{Z}$. If $a|b$ and $b|a$ then $a = b$.
Proof. Let $a = 1$ and $b = -1$, then $b = an, n = -1 \in \mathbb{Z}$ so $a|b$, alternatively $a = bm, m = 1 \in \mathbb{Z}$ so $b|a$, but $a \neq b$.

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Proposition:
If $X \subseteq A \cup B$, then $X \subseteq A$ or $X \subseteq B$.
Proof. Let $A = \{1, 2, 3\}, B = \{3, 4, 5\}, X = \{1, 4\}$. Then $X \subseteq (A \cup B)$ but $X \nsubseteq A$ and $X \nsubseteq B$.