

Coupling plant physiology and pest demography to understand plant-nematode interactions

Joseph Penlap, Suzanne Touzeau, Frédéric Grogard, Valentina Baldazzi

Inria Centre at Université Côte d'Azur

13th ECMTB, July 22-26, 2024, Toledo, Spain

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Root-Knot Nematodes (RKN), *Meloidogyne spp.*

- small soil worms,
- obligate root endoparasites,
- ubiquitous polyphagous pest
- 14% of global crop losses worldwide [1]
[1] Djian-Caporalino, *EPPO Bulletin*, 2012

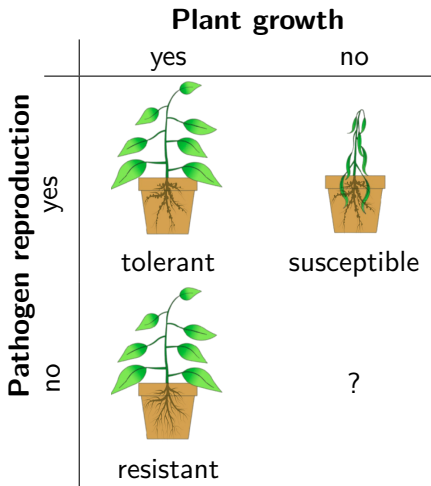


Main impacts

- root deformation (galls)
- stunted growth and wilting




Plant variability

Strong variability in plant response to RKN parasitism among crop species



Plant variability

Strong variability in plant response to RKN parasitism among crop species

Plant growth		yes	no
Pathogen reproduction	yes	 tolerant	 susceptible
	no	 resistant	?

Which
mechanisms
underlie
plant
tolerance?

Problematic

Host-Pathogen interactions

pest dynamics

Epidemiological modeling

(Tankam et al., Mathematical Biosciences, 2020)

(Nilusmas et al., Evolutionary applications, 2020)

plant physiology

Ecological modeling

(Thornley, Annals of botany, 1972)

(Dewar, Functional Ecology, 1998)

Problematic

Host-Pathogen interactions

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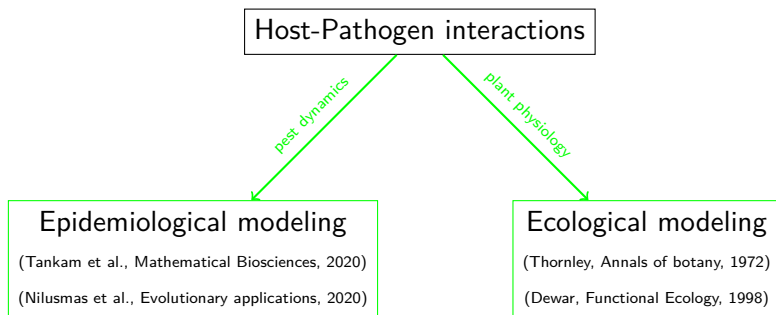
plant physiology

Ecological modeling

(Thornley, Annals of botany, 1972)

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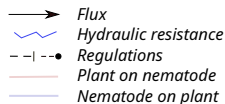
Problematic



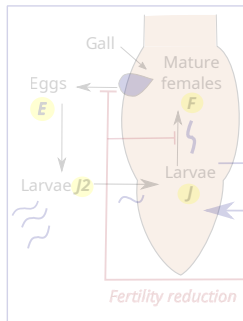
Approach

- Model coupling plant ecophysiology & pest population dynamics
- Experimental data on 2 plant species (tomato & pepper)

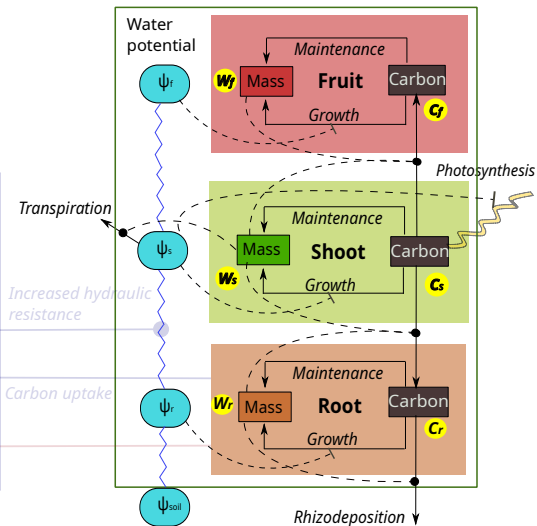
Plant model



Pest model



Plant model



Mathematical equations

$$\text{Shoot} \left\{ \begin{array}{l} \frac{dW_s}{dt} = \\ \frac{dC_s}{dt} = \end{array} \right.$$

$$\text{Root} \left\{ \begin{array}{l} \frac{dW_r}{dt} = \\ \frac{dC_r}{dt} = \end{array} \right.$$

$$\text{Fruit} \left\{ \begin{array}{l} \frac{dW_f}{dt} = \\ \frac{dC_f}{dt} = \end{array} \right.$$

Mathematical equations

$$\text{Shoot} \left\{ \begin{array}{l} \frac{dW_s}{dt} = \\ \frac{dC_s}{dt} = \underbrace{\sigma_c f(\psi_s)}_{\text{Uptake}} \end{array} \right.$$

$$- \underbrace{\frac{1}{W_s} \frac{dW_s}{dt} C_s}_{\text{Dilution}}$$

$$\text{Root} \left\{ \begin{array}{l} \frac{dW_r}{dt} = \\ \frac{dC_r}{dt} = \end{array} \right.$$

$$- \underbrace{\frac{1}{W_r} \frac{dW_r}{dt} C_r}_{\text{Dilution}}$$

$$\text{Fruit} \left\{ \begin{array}{l} \frac{dW_f}{dt} = \\ \frac{dC_f}{dt} = \end{array} \right.$$

$$- \underbrace{\frac{1}{W_f} \frac{dW_f}{dt} C_f}_{\text{Dilution}}$$

Mathematical equations

$$\text{Shoot} \left\{ \begin{aligned} \frac{dW_s}{dt} &= \\ \frac{dC_s}{dt} &= \underbrace{\sigma_c f(\psi_s)}_{\text{Uptake}} - \frac{1}{W_s} \underbrace{(T_r + T_f + T_a)}_{\text{Transport}} - \underbrace{\frac{1}{W_s} \frac{dW_s}{dt} C_s}_{\text{Dilution}} \end{aligned} \right.$$

$$\text{Root} \left\{ \begin{aligned} \frac{dW_r}{dt} &= \\ \frac{dC_r}{dt} &= \frac{1}{W_r} \underbrace{T_r}_{\text{Transport}} - \underbrace{\frac{1}{W_r} \frac{dW_r}{dt} C_r}_{\text{Dilution}} \end{aligned} \right.$$

$$\text{Fruit} \left\{ \begin{aligned} \frac{dW_f}{dt} &= \\ \frac{dC_f}{dt} &= \frac{1}{W_f} \underbrace{(T_f + T_a)}_{\text{Transport}} - \underbrace{\frac{1}{W_f} \frac{dW_f}{dt} C_f}_{\text{Dilution}} \end{aligned} \right.$$

Mathematical equations

$$\begin{aligned}
 \text{Shoot} \quad & \left\{ \begin{aligned} \frac{dW_s}{dt} &= \underbrace{k_s f(\psi_s) \frac{C_s}{K_s + C_s} W_s}_{\text{Growth}} \\ \frac{dC_s}{dt} &= \underbrace{\sigma_c f(\psi_s)}_{\text{Uptake}} - \underbrace{\frac{1}{W_s} (T_r + T_f + T_a)}_{\text{Transport}} - \underbrace{(f_c) k_s f(\psi_s) \frac{C_s}{K_s + C_s}}_{\text{Growth}} - \underbrace{\frac{1}{W_s} \frac{dW_s}{dt} C_s}_{\text{Dilution}} \end{aligned} \right. \\
 \text{Root} \quad & \left\{ \begin{aligned} \frac{dW_r}{dt} &= \underbrace{k_r f(\psi_r) \frac{C_r}{K_r + C_r} W_r}_{\text{Growth}} \\ \frac{dC_r}{dt} &= \frac{1}{W_r} \underbrace{T_r}_{\text{Transport}} - \underbrace{(f_c) k_r f(\psi_r) \frac{C_r}{K_r + C_r}}_{\text{Growth}} - \underbrace{\frac{1}{W_r} \frac{dW_r}{dt} C_r}_{\text{Dilution}} \end{aligned} \right. \\
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 \end{aligned}$$

Mathematical equations

Shoot

$$\left\{ \begin{aligned} \frac{dW_s}{dt} &= \underbrace{k_s f(\psi_s) \frac{C_s}{K_s + C_s} W_s}_{\text{Growth}} \\ \frac{dC_s}{dt} &= \underbrace{\sigma_c f(\psi_s)}_{\text{Uptake}} - \underbrace{\frac{1}{W_s} (T_r + T_f + T_a)}_{\text{Transport}} - \underbrace{(f_c + r_{g,s}) k_s f(\psi_s) \frac{C_s}{K_s + C_s}}_{\text{Respiration growth}} - \underbrace{\frac{1}{W_s} \frac{dW_s}{dt} C_s}_{\text{Dilution}} \end{aligned} \right.$$

Root

$$\left\{ \begin{aligned} \frac{dW_r}{dt} &= \underbrace{k_r f(\psi_r) \frac{C_r}{K_r + C_r} W_r}_{\text{Growth}} \\ \frac{dC_r}{dt} &= \underbrace{\frac{1}{W_r} T_r}_{\text{Transport}} - \underbrace{(f_c + r_{g,r}) k_r f(\psi_r) \frac{C_r}{K_r + C_r}}_{\text{Respiration growth}} - \underbrace{\frac{1}{W_r} \frac{dW_r}{dt} C_r}_{\text{Dilution}} \end{aligned} \right.$$

Fruit

$$\left\{ \begin{aligned} \frac{dW_f}{dt} &= \underbrace{k_f f(\psi_f) \frac{C_f}{K_f + C_f} W_f}_{\text{Growth}} \\ \frac{dC_f}{dt} &= \underbrace{\frac{1}{W_f} (T_f + T_a)}_{\text{Transport}} - \underbrace{(f_c + r_{g,f}) k_f f(\psi_f) \frac{C_f}{K_f + C_f}}_{\text{Respiration growth}} - \underbrace{\frac{1}{W_f} \frac{dW_f}{dt} C_f}_{\text{Dilution}} \end{aligned} \right.$$

Mathematical equations

Shoot

$$\left\{ \begin{aligned} \frac{dW_s}{dt} &= \underbrace{k_s f(\psi_s) \frac{C_s}{K_s + C_s} W_s}_{\text{Growth}} \\ \frac{dC_s}{dt} &= \underbrace{\sigma_c f(\psi_s)}_{\text{Uptake}} - \underbrace{\frac{1}{W_s} (T_r + T_f + T_a)}_{\text{Transport}} - \underbrace{(f_c + r_{g,s}) k_s f(\psi_s) \frac{C_s}{K_s + C_s}}_{\text{Growth}} - \underbrace{r_{m,s} \left(\frac{C_s^n}{K_m^n + C_s^n} \right)}_{\text{Maintenance respiration}} - \underbrace{\frac{1}{W_s} \frac{dW_s}{dt} C_s}_{\text{Dilution}} \end{aligned} \right.$$

Root

$$\left\{ \begin{aligned} \frac{dW_r}{dt} &= \underbrace{k_r f(\psi_r) \frac{C_r}{K_r + C_r} W_r}_{\text{Growth}} \\ \frac{dC_r}{dt} &= \underbrace{\frac{1}{W_r} T_r}_{\text{Transport}} - \underbrace{(f_c + r_{g,r}) k_r f(\psi_r) \frac{C_r}{K_r + C_r}}_{\text{Growth}} - \underbrace{r_{m,r} \left(\frac{C_r^n}{K_m^n + C_r^n} \right)}_{\text{Maintenance respiration}} - \underbrace{\frac{1}{W_r} \frac{dW_r}{dt} C_r}_{\text{Dilution}} \end{aligned} \right.$$

Fruit

$$\left\{ \begin{aligned} \frac{dW_f}{dt} &= \underbrace{k_f f(\psi_f) \frac{C_f}{K_f + C_f} W_f}_{\text{Growth}} \\ \frac{dC_f}{dt} &= \underbrace{\frac{1}{W_f} (T_f + T_a)}_{\text{Transport}} - \underbrace{(f_c + r_{g,f}) k_f f(\psi_f) \frac{C_f}{K_f + C_f}}_{\text{Growth}} - \underbrace{r_{m,f} \left(\frac{C_f^n}{K_m^n + C_f^n} \right)}_{\text{Maintenance respiration}} - \underbrace{\frac{1}{W_f} \frac{dW_f}{dt} C_f}_{\text{Dilution}} \end{aligned} \right.$$

Mathematical equations

$$\text{Shoot} \left\{ \begin{aligned} \frac{dW_s}{dt} &= \underbrace{k_s f(\psi_s) \frac{C_s}{K_s + C_s} W_s}_{\text{Growth}} \\ \frac{dC_s}{dt} &= \underbrace{\sigma_c f(\psi_s)}_{\text{Uptake}} - \underbrace{\frac{1}{W_s} (T_r + T_f + T_a)}_{\text{Transport}} - \underbrace{(f_c + r_{g,s}) k_s f(\psi_s) \frac{C_s}{K_s + C_s}}_{\text{Growth}} - \underbrace{r_{m,s} \left(\frac{C_s^n}{K_m^n + C_s^n} \right)}_{\text{Maintenance respiration}} - \underbrace{\frac{1}{W_s} \frac{dW_s}{dt} C_s}_{\text{Dilution}} \end{aligned} \right.$$

$$\text{Root} \left\{ \begin{aligned} \frac{dW_r}{dt} &= \underbrace{k_r f(\psi_r) \frac{C_r}{K_r + C_r} W_r}_{\text{Growth}} \\ \frac{dC_r}{dt} &= \underbrace{\frac{1}{W_r} T_r}_{\text{Transport}} - \underbrace{(f_c + r_{g,r}) k_r f(\psi_r) \frac{C_r}{K_r + C_r}}_{\text{Growth}} - \underbrace{r_{m,r} \left(\frac{C_r^n}{K_m^n + C_r^n} \right)}_{\text{Maintenance respiration}} - \underbrace{r_{rh} C_r}_{\text{Rhizodeposition}} - \underbrace{\frac{1}{W_r} \frac{dW_r}{dt} C_r}_{\text{Dilution}} \end{aligned} \right.$$

$$\text{Fruit} \left\{ \begin{aligned} \frac{dW_f}{dt} &= \underbrace{k_f f(\psi_f) \frac{C_f}{K_f + C_f} W_f}_{\text{Growth}} \\ \frac{dC_f}{dt} &= \underbrace{\frac{1}{W_f} (T_f + T_a)}_{\text{Transport}} - \underbrace{(f_c + r_{g,f}) k_f f(\psi_f) \frac{C_f}{K_f + C_f}}_{\text{Growth}} - \underbrace{r_{m,f} \left(\frac{C_f^n}{K_m^n + C_f^n} \right)}_{\text{Maintenance respiration}} - \underbrace{\frac{1}{W_f} \frac{dW_f}{dt} C_f}_{\text{Dilution}} \end{aligned} \right.$$

Mathematical equations

Shoot

$$\left\{ \begin{aligned} \frac{dW_s}{dt} &= \underbrace{k_s f(\psi_s) \frac{C_s}{K_s + C_s} W_s}_{\text{Growth}} - \underbrace{\Gamma_s W_s}_{\text{Natural mortality}} - \underbrace{\Gamma_s \left(\frac{K_m^n}{K_m^n + C_s^n} \right) W_s}_{\text{Additional mortality due to carbon shortage}} \\ \frac{dC_s}{dt} &= \underbrace{\sigma_c f(\psi_s)}_{\text{Uptake}} - \underbrace{\frac{1}{W_s} (T_r + T_f + T_a)}_{\text{Transport}} - \underbrace{(f_c + r_{g,s}) k_s f(\psi_s) \frac{C_s}{K_s + C_s}}_{\text{Growth}} - \underbrace{r_{m,s} \left(\frac{C_s^n}{K_m^n + C_s^n} \right)}_{\text{Maintenance respiration}} - \underbrace{\frac{1}{W_s} \frac{dW_s}{dt} C_s}_{\text{Dilution}} \end{aligned} \right.$$

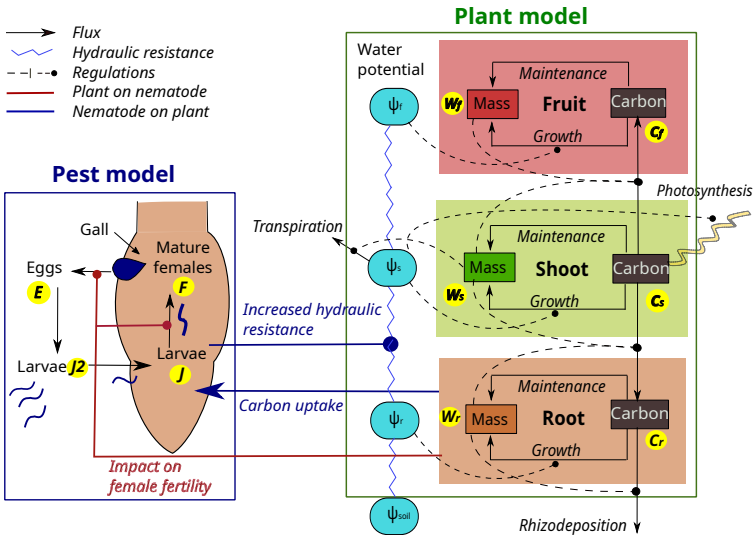
Root

$$\left\{ \begin{aligned} \frac{dW_r}{dt} &= \underbrace{k_r f(\psi_r) \frac{C_r}{K_r + C_r} W_r}_{\text{Growth}} - \underbrace{\Gamma_r W_r}_{\text{Natural mortality}} - \underbrace{\Gamma_r \left(\frac{K_m^n}{K_m^n + C_r^n} \right) W_r}_{\text{Additional mortality due to carbon shortage}} \\ \frac{dC_r}{dt} &= \underbrace{\frac{1}{W_r} T_r}_{\text{Transport}} - \underbrace{(f_c + r_{g,r}) k_r f(\psi_r) \frac{C_r}{K_r + C_r}}_{\text{Growth}} - \underbrace{r_{m,r} \left(\frac{C_r^n}{K_m^n + C_r^n} \right)}_{\text{Maintenance respiration}} - \underbrace{\zeta_{rh} C_r}_{\text{Rhizodeposition}} - \underbrace{\frac{1}{W_r} \frac{dW_r}{dt} C_r}_{\text{Dilution}} \end{aligned} \right.$$

Fruit

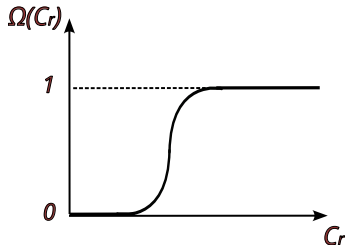
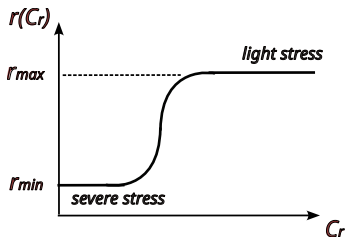
$$\left\{ \begin{aligned} \frac{dW_f}{dt} &= \underbrace{k_f f(\psi_f) \frac{C_f}{K_f + C_f} W_f}_{\text{Growth}} - \underbrace{\Gamma_f W_f}_{\text{Natural mortality}} - \underbrace{\Gamma_f \left(\frac{K_m^n}{K_m^n + C_f^n} \right) W_f}_{\text{Additional mortality due to carbon shortage}} \\ \frac{dC_f}{dt} &= \underbrace{\frac{1}{W_f} (T_f + T_a)}_{\text{Transport}} - \underbrace{(f_c + r_{g,f}) k_f f(\psi_f) \frac{C_f}{K_f + C_f}}_{\text{Growth}} - \underbrace{r_{m,f} \left(\frac{C_f^n}{K_m^n + C_f^n} \right)}_{\text{Maintenance respiration}} - \underbrace{\frac{1}{W_f} \frac{dW_f}{dt} C_f}_{\text{Dilution}} \end{aligned} \right.$$

Integrated plant-pest model



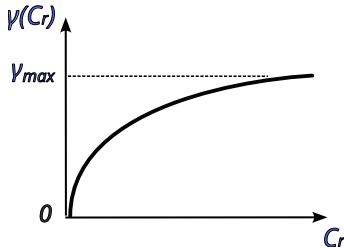
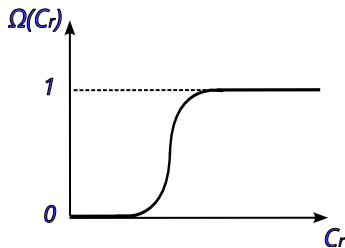
Plant status on RKN development

$$\begin{aligned}
 \text{Free} \quad & \left\{ \begin{aligned} \frac{dE}{dt} &= \underbrace{r(C_r) F}_{\text{Egg laying}} - \underbrace{h E}_{\text{Egg hatching}} - \underbrace{\mu_e E}_{\text{Mortality}} \\ \frac{dJ_2}{dt} &= \underbrace{h E}_{\text{Egg hatching}} - \underbrace{\beta J_2 W_r}_{\text{Larvae infection}} - \underbrace{\mu_{J_2} J_2}_{\text{Mortality}} \end{aligned} \right. \\
 \text{RKN} \quad & \left\{ \begin{aligned} \frac{dJ}{dt} &= \underbrace{\Omega(C_r) \beta J_2 W_r}_{\text{RKN entry}} - \underbrace{\eta J}_{\text{Maturation}} - \underbrace{\mu_j J}_{\text{Mortality}} \\ \frac{dF}{dt} &= \underbrace{\eta J}_{\text{Maturation}} - \underbrace{\mu_F F}_{\text{Mortality}} \end{aligned} \right.
 \end{aligned}$$



RKN effects on plant growth

$$\begin{aligned}
 \left. \begin{aligned}
 \frac{dW_r}{dt} &= \underbrace{k_r f(\psi_r) \frac{C_r}{K_r + C_r} W_r}_{\text{Growth}} - \underbrace{\gamma_r \left(1 + \frac{K_m^n}{K_m^n + C_r^n}\right) W_r}_{\text{Mortality}} - \underbrace{\Omega(C_r) \epsilon \beta J_2 W_r}_{\text{Infected roots}} \\
 \frac{dG}{dt} &= \underbrace{\Omega(C_r) \epsilon \beta J_2 W_r}_{\text{Gall formation}} + \underbrace{k_g f(\psi_r) \frac{C_r}{K_r + C_r} G}_{\text{Growth}} - \underbrace{\Gamma_r G}_{\text{Natural mortality}} \\
 \frac{dC_r}{dt} &= \underbrace{\frac{1}{(W_r + G)} T_r}_{\text{Transport}} - \underbrace{\left(f_c + r_{g,r}\right) \left(\frac{k_r W_r + k_g G}{W_r + G}\right) f(\psi_r) \frac{C_r}{K_r + C_r}}_{\text{Growth}} - \underbrace{r_{m,r} \left(\frac{C_r^n}{K_m^n + C_r^n}\right)}_{\text{Maintenance respiration}} - \underbrace{c_{rh} C_r}_{\text{Rhizodeposition}} \\
 &\quad - \underbrace{\frac{C_r}{(W_r + G)} \left(\frac{dW_r}{dt} + \frac{dG}{dt}\right)}_{\text{dilution}} - \underbrace{\gamma(C_r) F}_{\text{RKN feeding}}
 \end{aligned} \right\} \text{Root}
 \end{aligned}$$



Calibration of models

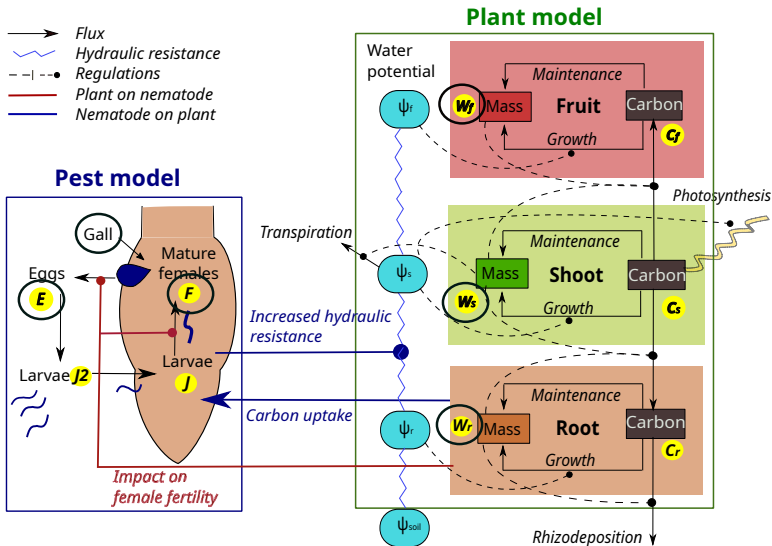
Steps

- select parameters to be estimated based on **sensitivity analysis**
- least squares **criterion** between model simulations and data
- find parameter values that minimize the criterion
 - Global search: ARS (Adaptive Random Searches)
 - Local search: Nelder-Mead from *minimize()* python module

Strategy

- calibration of plant model on control data
- calibration of plant-RKN model on inoculated plants & RKN data

Which data?



+ weekly number of leaves

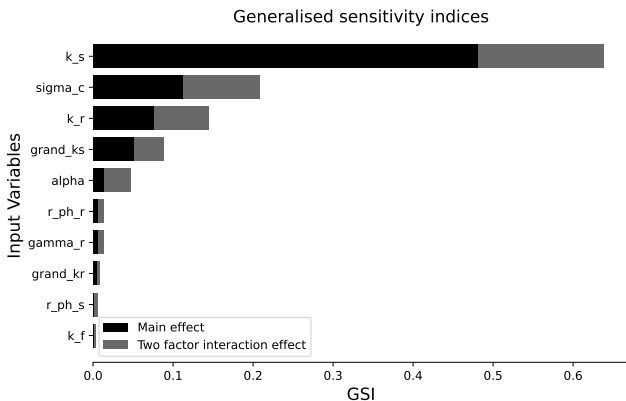
Calibration of plant model

Global sensitivity analysis on plant model

Features

- **goal:** identify the most influential parameters
- **inputs:** 26 parameters, **outputs:** W_s , W_r , W_f , C_s , C_r , C_f

Plant traits that most affect **root biomass** (W_r) dynamics

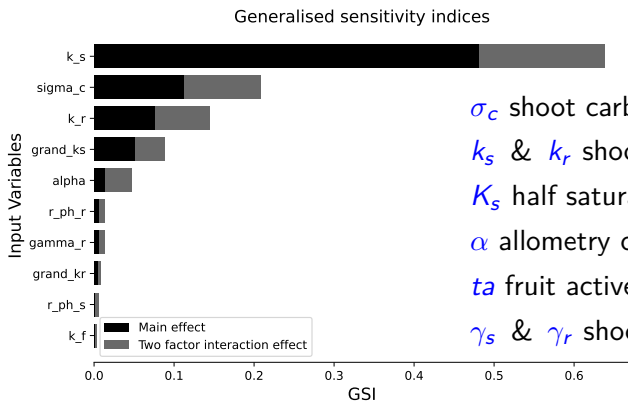


Global sensitivity analysis on plant model

Features

- **goal:** identify the most influential parameters
- **inputs:** 26 parameters, **outputs:** W_s , W_r , W_f , C_s , C_r , C_f

Plant traits that most affect **root biomass** (W_r) dynamics



σ_c shoot carbon fixation rate

k_s & k_r shoot and root growth rates

K_s half saturation coef. of shoots

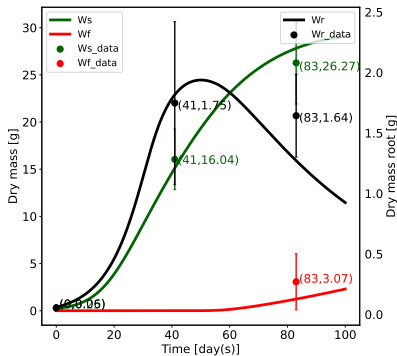
α allometry coefficient

ta fruit active transport rate

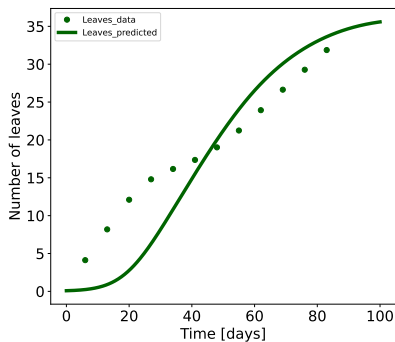
γ_s & γ_r shoot & root mortality rates

Tomato parameter estimation

Tomato dynamics

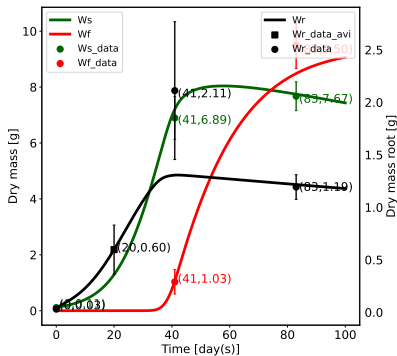


Predicted number of leaves

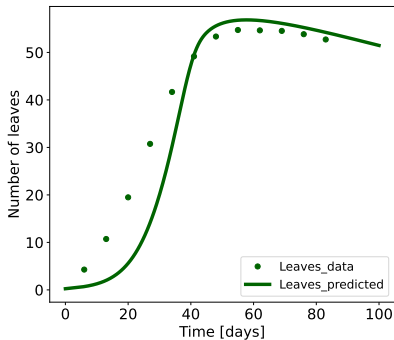


Pepper parameter estimation

Pepper dynamics



Predicted number of leaves



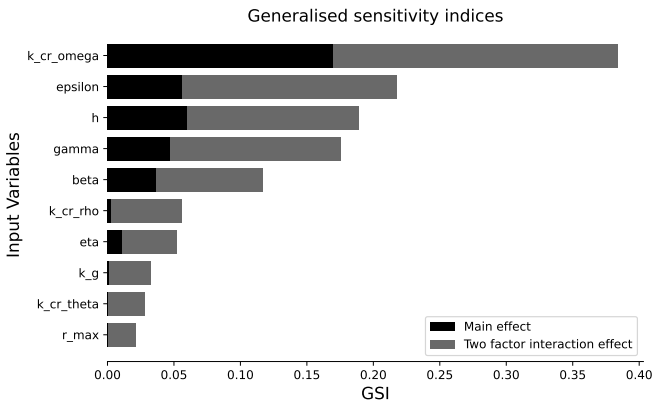
Calibration of full model

Global sensitivity analysis on full model

Features

- **inputs:** 15 parameters, **plant parameters are fixed**
- **outputs:** W_s , W_r , W_f , C_s , C_r , C_f , E_g , J_2 , J , F , G

Interaction traits that most affect **J-larvae** dynamics



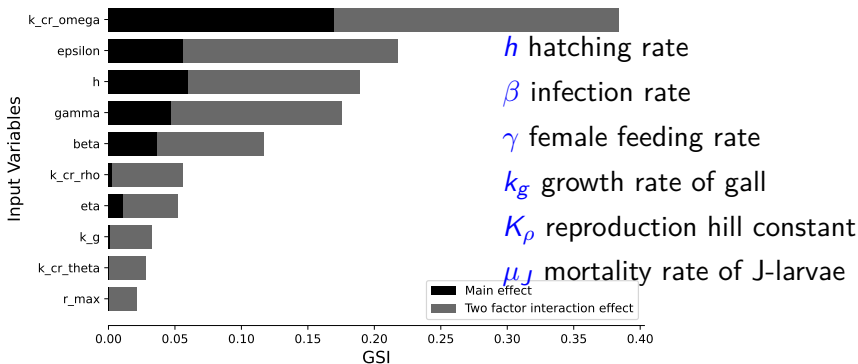
Global sensitivity analysis on full model

Features

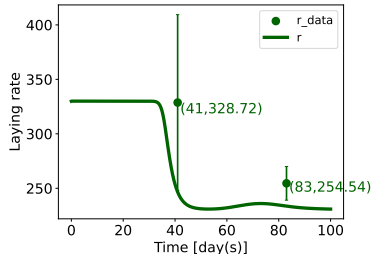
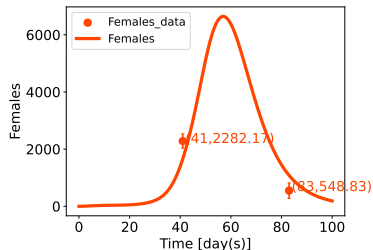
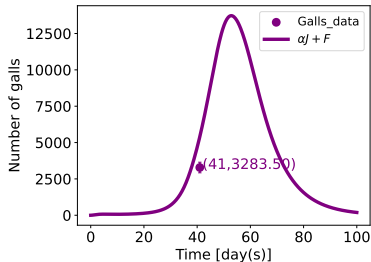
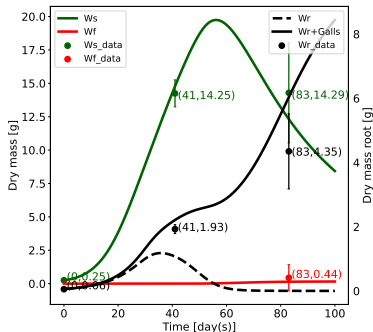
- **inputs:** 15 parameters, **plant parameters are fixed**
- **outputs:** W_s , W_r , W_f , C_s , C_r , C_f , E_g , J_2 , J , F , G

Interaction traits that most affect **J-larvae** dynamics

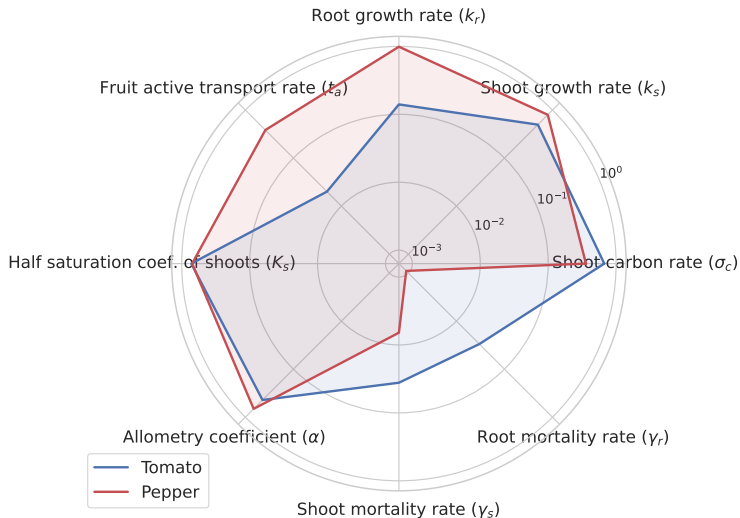
Generalised sensitivity indices



Calibration on tomato



Plant parameter patterns (Tomato vs Pepper)

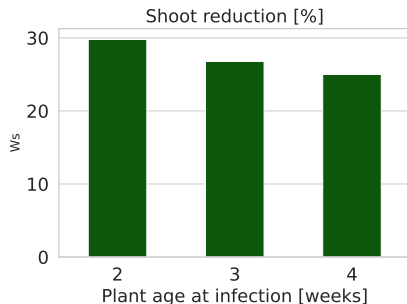


Shoot reduction vs plant age at infection

a. Cucumber dynamics
(Experimental results¹)



b. Tomato dynamics
(Model simulations)



¹Kayani et al., *Crop Protection*, 2017

Conclusion

➤ Modeling

- mechanistic model of plant functioning
- plant model coupled to RKN population dynamics

➤ Calibration

- plant parameters (8/26) & interaction parameters (6/15)
- better estimates for pepper than tomato
- estimation of pepper interaction parameters [ongoing]
- comparison of dynamical patterns of 2 species: insights on the origin of tolerance [ongoing]



{thank you}

Global sensitivity analysis (GSA)

Goal: Identifying the most influential parameters

inputs : all parameters, **outputs** : model variables along time (vector)

Steps

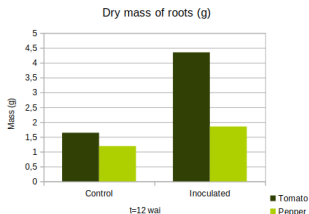
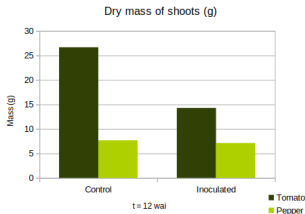
1. fractional factorial design to explore parameter space
2. PCA to reduce output and capture its variability
3. ANOVA to compute sensitivity indices (SI)

$$SI_{..} = \frac{SS_{..}}{TSS}, \quad GSI = \sum_{k=1}^{components} SI_k \times inertia_k$$

SS = sum of squares, TSS = total sum of squares

Data from ArchiNem project

- 2 plant species: tomato, pepper
- 2 plant categories:
 - control
 - inoculated by nematodes
- 6 replicates (90 plants)
- weekly number of leaves
- destructive measures, 3 points in time:
 - plant: fresh and dry masses for shoot, root and fruit
 - nematodes: number of galls, egg masses and egg per egg masses

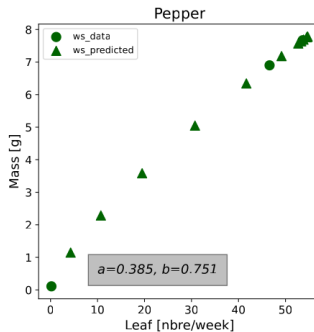
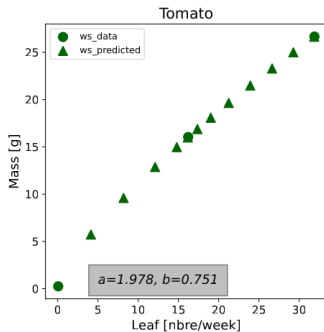


Allometric relationship

Equation

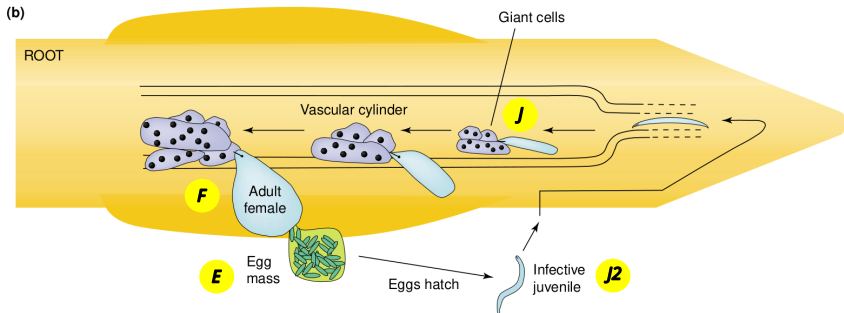
$$W_s^{\text{leaf}} = a L^b,$$

a , b allometric coefficients & weekly number of leaves (L).



Specific impacts

- clonal reproduction
- reduced water and nutrient uptake
- hijacking of plant resources (carbon)



Criterion for estimating plant parameters

Allometric relationship

$$W_s^{\text{leaf}} = a L^b,$$

a , b allometric coefficients & weekly number of leaves (L).

Cost function

$$\begin{aligned} \text{err} = & \alpha_s \left[\frac{1}{2} \sum_{i=1}^2 \left(\frac{(W_s^{\text{sol}})_i - (W_s^{\text{obs}})_i}{(W_s^{\text{obs}})_i} \right)^2 + \frac{1}{2} \sum_{i=1}^2 \left(\frac{(W_r^{\text{sol}})_i - (W_r^{\text{obs}})_i}{(W_r^{\text{obs}})_i} \right)^2 + \left(\frac{W_f^{\text{sol}} - W_f^{\text{obs}}}{W_f^{\text{obs}}} \right)^2 \right] \\ & + \underbrace{\beta_s \left[\frac{1}{12} \sum_{i=1}^{12} \left(\frac{(W_s^{\text{sol}})_i - (W_s^{\text{leaf}})_i}{(W_s^{\text{leaf}})_i} \right)^2 \right]}_{\text{Additional data of } W_s \text{ obtained via allometry}} \end{aligned}$$

- α_s and $\beta_s = (1 - \alpha_s)$ are the weighted coefficients
- W_k^{sol} and W_k^{obs} are model predictions and data respectively.
 $k := \{s = \text{shoot}, r = \text{root}, f = \text{fruit}\}$

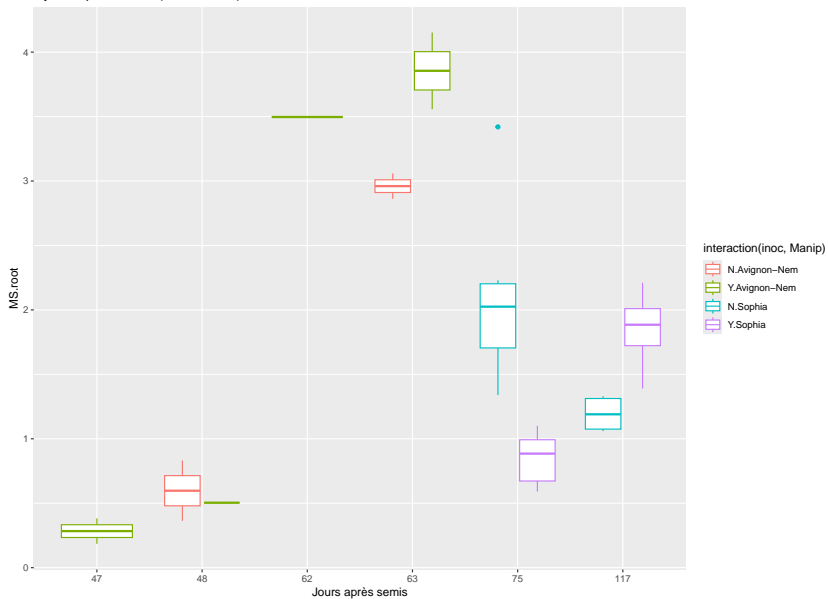
Criterion for estimating interaction parameters

Cost function

$$\begin{aligned}
 err = & \underbrace{\alpha_c \left[\frac{1}{2} \sum_{i=1}^2 \left(\frac{(W_s^{sol})_i - (W_s^{obs})_i}{(W_s^{obs})_i} \right)^2 + \frac{1}{2} \sum_{i=1}^2 \left(\frac{(W_r^{sol})_i - (W_r^{obs})_i}{(W_r^{obs})_i} \right)^2 + \left(\frac{W_f^{sol} - W_f^{obs}}{W_f^{obs}} \right)^2 \right]}_{\text{Inoculated plant}} \\
 & + \underbrace{\beta_c \left[\left(\frac{G^{sol} - G^{obs}}{G^{obs}} \right)^2 + \frac{1}{2} \sum_{i=1}^2 \left(\frac{F_i^{sol} - F_i^{obs}}{F_i^{obs}} \right)^2 + \frac{1}{2} \sum_{i=1}^2 \left(\frac{r_i^{sol} - r_i^{obs}}{r_i^{obs}} \right)^2 \right]}_{\text{Nematodes}}
 \end{aligned}$$

- α_c and $\beta_c = (1 - \alpha_c)$ are the weighted coefficients
- G^{sol} , F^{sol} , r^{sol} are nematode model predictions of galls, females and egg-laying
- G^{obs} , F^{obs} , r^{obs} are nematode data

Dynamique Poivron (sain/inoculé)

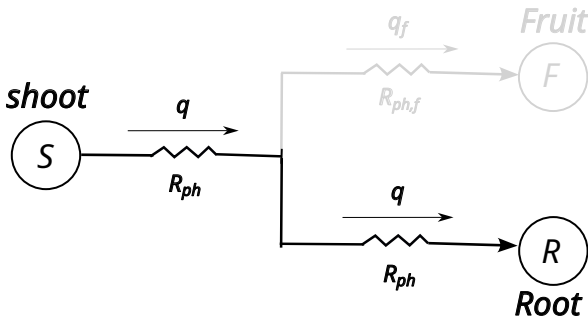


Carbon transport (T)

$$\begin{aligned}
 \text{Shoot} \left\{ \begin{aligned} \frac{dW_s}{dt} &= \underbrace{k_s f(\psi_s) \frac{C_s}{K_s + C_s} W_s}_{\text{Growth}} - \underbrace{\Gamma_s \left(\frac{K_m^n}{K_m^n + C_s^n} \right) W_s}_{\text{Additional mortality due to carbon shortage}} \\ \frac{dC_s}{dt} &= \underbrace{\sigma_c f(\psi_s)}_{\text{Uptake}} - \underbrace{\frac{1}{W_s} (T_r + T_f + T_a)}_{\text{Transport}} - \underbrace{(f_c + r_{g,s}) k_s f(\psi_s) \frac{C_s}{K_s + C_s}}_{\text{Growth}} - \underbrace{r_{m,s} \left(\frac{C_s^n}{K_m^n + C_s^n} \right)}_{\text{Maintenance respiration}} - \underbrace{\frac{1}{W_s} \frac{dW_s}{dt} C_s}_{\text{dilution}} \end{aligned} \right. \\
 \text{Root} \left\{ \begin{aligned} \frac{dW_r}{dt} &= \underbrace{k_r f(\psi_r) \frac{C_r}{K_r + C_r} W_r}_{\text{Growth}} - \underbrace{\Gamma_r W_r}_{\text{Natural mortality}} - \underbrace{\Gamma_r \left(\frac{K_m^n}{K_m^n + C_r^n} \right) W_r}_{\text{Additional mortality due to carbon shortage}} \\ \frac{dC_r}{dt} &= \underbrace{\frac{1}{W_r} T_r}_{\text{Transport}} - \underbrace{(f_c + r_{g,r}) k_r f(\psi_r) \frac{C_r}{K_r + C_r}}_{\text{Growth}} - \underbrace{r_{m,r} \left(\frac{C_r^n}{K_m^n + C_r^n} \right)}_{\text{Maintenance respiration}} - \underbrace{c_{rh} C_r}_{\text{Rhizodeposition}} - \underbrace{\frac{1}{W_r} \frac{dW_r}{dt} C_r}_{\text{dilution}} \end{aligned} \right. \\
 \text{Fruit} \left\{ \begin{aligned} \frac{dW_f}{dt} &= \underbrace{k_f f(\psi_f) \frac{C_f}{K_f + C_f} W_f}_{\text{Growth}} - \underbrace{\Gamma_f W_f}_{\text{Natural mortality}} - \underbrace{\Gamma_f \left(\frac{K_m^n}{K_m^n + C_f^n} \right) W_f}_{\text{Additional mortality due to carbon shortage}} \\ \frac{dC_f}{dt} &= \underbrace{\frac{1}{W_f} (T_f + T_a)}_{\text{Transport}} - \underbrace{(f_c + r_{g,f}) k_f f(\psi_f) \frac{C_f}{K_f + C_f}}_{\text{Growth}} - \underbrace{r_{m,f} \left(\frac{C_f^n}{K_m^n + C_f^n} \right)}_{\text{Maintenance respiration}} - \underbrace{\frac{1}{W_f} \frac{dW_f}{dt} C_f}_{\text{dilution}} \end{aligned} \right.
 \end{aligned}$$

Carbon transport (T)

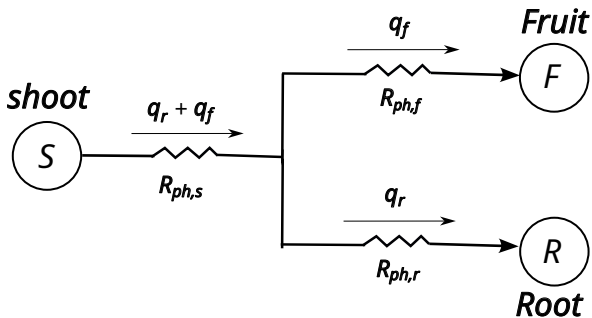
The carbon flow³ T in phloem vessels $T = q C_s$, with q the volume flow rate



Then,

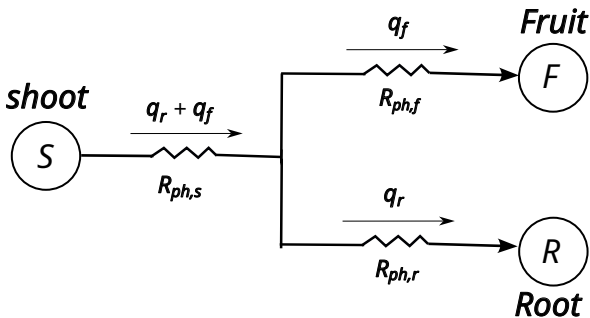
$$\begin{cases} (q_r + q_f)R_{ph,s} + q_r R_{ph,r} = (C_s - C_r) \\ (q_r + q_f)R_{ph,s} + q_f R_{ph,f} = (C_s - C_f) \end{cases} \quad \left\{ \begin{array}{l} T = \frac{(C_s - C_r)}{R_{ph}} C_s \\ R_{ph,..} = \frac{r_{ph,..}}{W_{\alpha,..}} \end{array} \right.$$

³Minchin et al., *Journal of Experimental Botany*, 1993

Carbon transport (T)

Therefore,

$$\begin{cases} (q_r + q_f)R_{ph,s} + q_r R_{ph,r} = (C_s - C_r) \\ (q_r + q_f)R_{ph,s} + q_f R_{ph,f} = (C_s - C_f) \end{cases}$$

Carbon transport (T)

Therefore,

$$\begin{cases} (q_r + q_f)R_{ph,s} + q_r R_{ph,r} = (C_s - C_r) \\ (q_r + q_f)R_{ph,s} + q_f R_{ph,f} = (C_s - C_f) \end{cases}$$

$$\begin{cases} T_r = q_r C_s \\ T_f = \left(\frac{W_s^n}{I^n + W_s^n} \right) q_f C_s \end{cases}$$

Water transport

$$\begin{aligned}
 \text{Shoot} \left\{ \begin{aligned} \frac{dW_s}{dt} &= \underbrace{k_s f(\psi_s) \frac{C_s}{K_s + C_s} W_s}_{\text{Growth}} - \underbrace{\Gamma_s W_s}_{\text{Natural mortality}} - \underbrace{\Gamma_s \left(\frac{K_m^n}{K_m^n + C_s^n} \right) W_s}_{\text{Additional mortality due to carbon shortage}}, \\ \frac{dC_s}{dt} &= \underbrace{\sigma_c f(\psi_s)}_{\text{Uptake}} - \underbrace{\frac{1}{W_s} (T_r + T_f + T_a)}_{\text{Transport}} - \underbrace{(f_c + r_{g,s}) k_s f(\psi_s) \frac{C_s}{K_s + C_s}}_{\text{Growth}} - \underbrace{r_{m,s} \left(\frac{C_s^n}{K_m^n + C_s^n} \right)}_{\text{Maintenance respiration}} - \underbrace{\frac{1}{W_s} \frac{dW_s}{dt} C_s}_{\text{dilution}} \end{aligned} \right. \\
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 \text{Fruit} \left\{ \begin{aligned} \frac{dW_f}{dt} &= \underbrace{k_f f(\psi_f) \frac{C_f}{K_f + C_f} W_f}_{\text{Growth}} - \underbrace{\Gamma_f W_f}_{\text{Natural mortality}} - \underbrace{\Gamma_f \left(\frac{K_m^n}{K_m^n + C_f^n} \right) W_f}_{\text{Additional mortality due to carbon shortage}}, \\ \frac{dC_f}{dt} &= \underbrace{\frac{1}{W_f} (T_f + T_a)}_{\text{Transport}} - \underbrace{(f_c + r_{g,f}) k_f f(\psi_f) \frac{C_f}{K_f + C_f}}_{\text{Growth}} - \underbrace{r_{m,f} \left(\frac{C_f^n}{K_m^n + C_f^n} \right)}_{\text{Maintenance respiration}} - \underbrace{\frac{1}{W_f} \frac{dW_f}{dt} C_f}_{\text{dilution}} \end{aligned} \right.
 \end{aligned}$$

Water transport

Transpiration process E guiding water flow, $E = \sigma_W W_s f(\psi_s)$,

$$\psi_r = \psi_{sol} - R_{sr}E, \quad \psi_s = \psi_r - R_{xy}E, \quad \psi_f = \psi_s.$$

where ψ_{sol} the soil water potential and $R_{..}$ resistances.

$$R_{sr} = \frac{r_{sr}}{W_r^{\alpha_r}}, \quad R_{xy,..} = \frac{r_{xy,..}}{W_{..}^{\alpha_{..}}}.$$

Water regulation function

$$f(\psi) = \frac{\psi^n}{K^n + \psi^n}$$

