Discovery with Data
Statistical Literacy and Critical Thinking
STAT 1600 – Course Pack

Statistics Computational Lab (SCL)

2020 - 2021 Edition

Acknowledgements:

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A general introduction to statistics with an emphasis on data analysis and graphical presentation. Extensive use will be made of the computer to prepare results. Topics may include: data collection, sampling and experimentation, measurement issues, descriptive statistics, statistical graphics, normal distribution, cross-classified data, correlation and association, formal statistical inferences, and resampling methods.

Contents

Chapter 1

Knowledge and Data

1.1 Objectives

After completing this chapter, students should be able to:

- Distinguish the difference between knowledge and data.
- Understand the process of acquiring knowledge from data.
- Be aware of common fallacies when interpreting data.
- Comprehend the meaning of the Type-I and Type-II errors.

1.2 Introduction

1.2.1 Why Data?

Information vs. Data

First, data is not information. Researchers extract information from data. The study of statistics will give us the tools (methods) to judge data validity and to interpret the information extracted from data.

Designing studies

Most scientific journals, after 1966, started to report statistical results. For example, before 1966, FDA (Food and Drug Agency) approved many drugs with written statements from a physician saying that the drug was safe, after 1966 the FDA required statistical studies to

show that the proposed drug did not harm. Later, FDA expected that the drug would be therapeutic.

Conducting research

Unless a sample is biased (we will get to that later), it will reveal the truth about the population. For example, let's suppose that a drug development team after working on a compound for several months decided to try the experimental drug themselves to test its effects before the animal safety studies were complete. (By the way, this behavior is unethical.) The team concluded that it appeared to be safe; however, a mouse study later showed that mice in the high dose group started to cannibalize themselves.

Developing critical thinking skills

Critical thinking starts with asking these type of questions: How can I confirm the validity of these data? Is the source of these data credible? What can I infer from these data? And so forth.

1.3 Knowledge and Data

There is a difference between what we know, and what we think that we know. Are vaccines safe? Will the new tax rules bring more or less revenue? Do mandatory seat belt laws save lives? Will legalizing drugs lead to more addiction? Do alternative therapies work as effectively as traditional treatments? Does exercise help prevent illness and are people who generally exercise healthier? Do taller people make more money than shorter people? [Naranjo]

We can settle most of these issues by looking at data. What do the data tell us? Even then, some problems can only be partially answered by at best incomplete data. In this course, we will lay the groundwork for writing a protocol and will discuss some fundamentals of sound scientific studies and data analysis. In the process, we hope to become better consumers of information and better judges of what is real or not, and what remains to be proven.

1.3.1 Building Knowledge Step-by-step

Scientific studies often go through the following steps. It might help to read the example in the next section, then read this again.

1. Conceptualize the problem

Suppose that you wish to study the problem of interest. The wording is usually broad. The idea at first may be too complex to understand so we look for ways to explain and describe it.

For example, perhaps we are interested in the effectiveness the iClicker within education. We will want to clarify and define education. Are we interested in investigating particular schools or grade levels? Perhaps we want to look at a specific component of education, like science or look at different methods of instruction. Clarifying specific concepts which we want to investigate and putting this into words helps bring focus to our study.

2. Operationalize the problem

As a researcher for your study you also need to come up with ways to narrow in on your problem of interest. What specifically do you want to measure? Here we are formulating the specific question we want to answer. We are also beginning to define our variables. We will need to know what we can measure to help answer our problem of interest.

Perhaps we are interested in looking at how well students are doing with several different methods of teaching science. We may look at measures of assessment like how well they did on an exam or overall in a class given the different teaching methodologies.

3. Design the study

We need to address all the components of how our study will be performed. We will likely need to identify the population of interest and to obtain a good representative sample of that population. We will need to figure out how we will select the sample and how many groups we may be comparing.

In our study of science education will we sample from several schools or several districts? If we take a sample of students from several schools, but from only one state, for example, our population would be students within that state. If we wanted to define our population as education within the continental U.S., we might need to pull randomly from schools across the U.S.

4. Collect the data

In the data collection stage, we must decide what instrument we are using (questionnaire, interview, observation). Once we have designed our study, we should be able to identify the primary variables of interest. As we collect the data, we are gathering information to help us answer the question we have proposed.

In our example of science education, we collect scores on a particular assessment (exam) to see how well students are comprehending the material.

5. Analyze the data

Using the proper statistical methods and procedures for the data is an important step in the data analysis stage. Also, checking the assumptions behind these statistical methods is critical. Are we comparing means? Proportions? Are the differences in means/proportions

statistically significant? How do we know if there is a significant difference (if the differences we find are not likely due to chance)?

In our exploration of teaching science perhaps we are comparing two different methodologies of teaching. We may then compare the overall means of the two groups.

6. Conclusions

Do our results generalize to a larger population? Check back to see how we defined the population so that we can generalize the results. Are we concluding cause-and-effect relationship, did we do an experiment, or did we observe associations? In our conclusions, we will want to identify limits to our study and suggest further research in our area.

As we summarize our study of science teaching, we could only afford to do a smaller study within a particular district. We could then relate the results of our study, based on our sample of schools, to the population of the entire school district. We may not be able to make conclusions to the whole state or the U.S. since our sample was not representative of the whole state or the U.S. Depending on how the study was defined we may have only made observations of the teachers. If we were investigating two distinct types of instruction, then this falls more within the realm of an experiment. We, of course, would still need to identify and try to control for confounding variables, which might be difficult.

7. Disseminate results

How are we sharing our results? We need to find the best approach to sharing the results of our study.

Is this information going to be made available to the schools for future educational improvement? Are we going to try to publish in a scientific or educational journal?

The Science Wheel Summarizes the above Steps

The Wheel of Science [Wallace, 1971] gives us a process to refine our thinking about the nature of social interactions. See Figure 1.1. We will define the four parts of the wheel below: Theory, Hypotheses, Observations, and Empirical Generalizations.

1.3.2 Example

Let's look at another example. Suppose we are interested in comparing the following four popular weight loss diets: Zone (balance between carbohydrates, protein and fat), Atkins (low carbohydrate, high fat, unrestricted calories), LEARN (low fat, based on national guidelines), and Ornish (low fat, high carbohydrate, unrestricted calories). How would we design a comparison study?

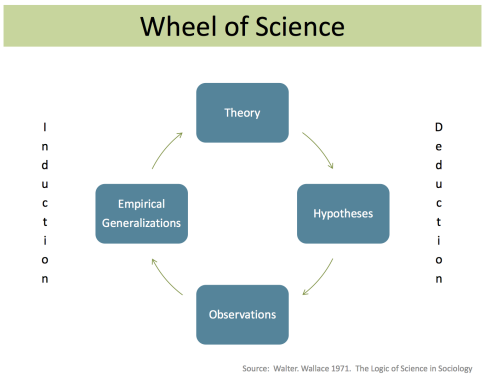


Figure 1.1: The Wheel of Science

1. Conceptualize the problem

Which weight loss program is most effective? Which one is the *most healthy*? Can unrestricted-calorie diets make you lose weight?

2. Operationalize the problem

How would we measure 'effective' and 'healthy?' Do we look at weight *loss* after two weeks? Two months? Two years? Do we compare the average weight *loss* or percentage of people who lost 15 pounds or more? How do we measure health? By cholesterol reduction? LDL cholesterol reduction? Blood pressure reduction? Glucose levels? Percentage who feel better?

3. Design the study

Where do we recruit our subjects for the study? How long will the study last? Will we include only overweight subjects? Do they choose which diet, or do we randomize the selection procedure? How do we ensure that they stay on the diet? Do we drop the cheaters from the study?

4. Collect the data

How many times will we measure their weights? How many times will we take blood samples? Urine samples? Do we send all samples to the same lab? Do we measure how strictly each subject adheres to their diet?

5. Analyze the data

Are there significant differences in average weight loss between diet groups? Are there significant differences in the percentage who lost weight? Are there differences in cholesterol changes, blood pressure changes, glucose level changes? Are there differences in long-term adherence rates to each diet?

6. Conclusions

Which diet would we recommend? Under what conditions? Are our subjects sufficiently representative to allow generalization? Are we sure the observed weight loss can be attributed to the diet?

7. Disseminate results

How would we present the results of the study? What tables and graphs would make the study easy to read and understand?

1.3.3 A final product

The following is a summary of a study reported in the Journal of the American Medical Association (JAMA), Vol. 297, No. 9 in March 2007. The paper is titled "Comparison of the Atkins, Zone, Ornish,and LEARN Diets for Change in Weight and Related Risk Factors Among Overweight Premenopausal Women The A TO Z Weight Loss Study: A Randomized Trial," by Christopher D. Gardner, PhD, Alexandre Kiazand, MD, Sofiya Alhassan, PhD, Soowon Kim, PhD, Randall S. Stafford, MD, PhD, Raymond R. Balise, PhD, Helena C. Kraemer, PhD, and Abby C. King, PhD. All authors were affiliated with the Stanford Prevention Research Center and the Department of Medicine, Stanford University Medical School.

Context

Popular diets, particularly those low in carbohydrates, have challenged current recommendations advising a low-fat, high-carbohydrate diet for weight loss. Potential benefits and risks have not been tested adequately.

Objective

To compare four weight-loss diets representing a spectrum of low to high carbohydrate intake for effects on weight loss and related metabolic variables. Design, Setting, and Participants Twelve-month randomized trial conducted in the United States from February 2003 to October 2005 among 311 free-living, overweight/obese (body mass index, 27-40) nondiabetic, premenopausal women. Intervention Participants were randomly assigned to follow the Atkins (n=77), Zone (n

= 79), LEARN (n=79), or Ornish (n=76) diets and received weekly instruction for two months, then an additional 10-month follow-up.

Main Outcome Measures

Weight loss at 12 months was the primary outcome. Secondary outcomes included lipid profile (low-density lipoprotein, high-density lipoprotein, and non-high-density lipoprotein cholesterol, and triglyceride levels), the percentage of body fat, waist-hip ratio, fasting insulin and glucose levels, and blood pressure. Outcomes were assessed at months 0, 2, 6, and 12. The Tukey studentized range test was used to adjust for multiple testing.

Results

Weight loss was greater for women in the Atkins diet group compared with the other diet groups at 12 months and mean 12-month weight loss was significantly different between the Atkins and Zone diets ($P < .05$). *Mean 12-month weight loss was as follows: Atkins, -4.7 kg (95% confidence interval [CI], -6.3 to -3.1 kg), Zone, -1.6 kg (95% CI, -2.8 to -0.4 kg), LEARN, -2.6 kg (-3.8 to -1.3 kg), and Ornish, -2.2 kg (-3.6 to -0.8 kg). Weight loss was not statistically different among the Zone, LEARN, and Ornish groups. At 12 months, secondary outcomes for the Atkins group were comparable with or more favorable than the other diet groups.*

Conclusions

In this study, premenopausal overweight and obese women assigned to follow the Atkins diet, which had the lowest carbohydrate intake, lost more weight and experienced more favorable overall metabolic effects at 12 months than women assigned to follow the Zone, Ornish, or LEARN diets. While questions remain about long-term effects and mechanisms, a low-carbohydrate, high-protein, high-fat diet may be considered a feasible alternative recommendation for weight loss.

The study tells us that all three groups lost weight. Over 12 months, the Atkins group lost the most weight without increasing cholesterol, blood pressure, the percentage of body fat, or fasting glucose levels.

The study does not tell us that if one stays on the Atkins diet long term (say, two years, or five years, or 10 years) their cholesterol, blood pressure, the percentage of body fat, or fasting glucose levels will remain favorable. If one 'cheats' on the Atkins diet periodically, what will be the effect on weight and metabolic profile? Will the same results be observed for men? For non-obese women?

Some studies are well designed; others are not. It is often up to us to make our judgments about what is true, instead of leaving this responsibility to journalists or bloggers. Whenever you hear a claim on television or in a magazine, ask yourself "What is the evidence?"

1.4 Common Fallacies

A fallacy is sometimes defined as a mistake in basic reasoning. The following are examples of fallacies:

1. **Lack of evidence fallacy**

   "There is no proof that the drug is unsafe." It allows claims to be made without providing any evidence, simply by shifting the burden of proof. The fallacy lies in the reasoning that lack of evidence means the contrary is true.

2. **Anecdotal evidence fallacy**

   "We give you testimonies of real people who ..." improved their golf game, or improved their sex life, or lost weight, or got rid of acne or achieved financial success. They make claims without comparison studies. Take a golf infomercial with testimonies from five golfers that a new driver improved their distance and accuracy. Did it? These are five golfers out of how many that they approached? These are televised shots out of how many that each took? The fallacy lies in the reasoning that existence means prevalence.

3. **Correlation equals causation fallacy**

   For example, "Married people are happier than singles," may be wrongly interpreted as "Want to be happy? Get married." It may be that happy people are the ones who tend to get married, or that high earners tend to be both happy and married. The fallacy lies in the reasoning that "two things happening together" must mean one causes the other.

1.4.1 Some fallacies in interpreting evidence

In medical studies, we use clinical trials to see if a difference exists between treatments. We use statistical analysis to determine if there is a significant difference between the treatment types. Perhaps there is very little difference, and that difference can be explained as being due to chance. It could be there is no actual difference between treatment groups. It leads us to a Type I error which can commonly occur.

A **Type I error** occurs when the researcher falsely finds a difference between treatments where no actual difference exists. Often, we use 95% confidence intervals when analyzing for statistical significance. It means that 5% of the time we may conclude a significant difference, when in fact one doesn't exist! The researcher is thus willing to accept a 5% chance that the study conclusions could be wrong. Notice, however, 5% is very small compared to the 95% assurance we have that there is a significant difference.

Another type of error that may be made is a **Type II error**. Here the researcher fails to find a difference between the treatment groups when a true difference does exist! Perhaps we

did a study with only a small sample, and we didn't see a difference between our treatment and control group. We would conclude there is no difference, which the treatment was not effective. Another lab might perform the same type of study but have a much large sample size, and once they analyze their data find that there is a true difference. If we fail to find a statistical difference when there is a difference, we have committed a Type II error. It also relates to the power of a study. The power of a study is related to the strength of the study. Can we detect an effect when there is an effect? How well was the study conducted? It also relates to our sample size. When we have a larger sample size, we will reduce the likelihood of committing a Type II error.

1.5 Key Words

- correlation
- data
- fallacies
- knowledge
- type I error
- type II error

1.6

Exercises

Exercise 1.6-1: What might be wrong about these headlines?

1. A study proclaims: "Slightly overweight people live longer than thin people."
2. A study states, "People who consider themselves depressed eat more chocolate than people who consider themselves otherwise."
3. A U.S. Census reported "More American women are living without a husband than with one." Additionally, women rated themselves happier when compared to the previous year's census. Can we then infer that living single is leading to greater happiness in women? What flaw is present here?

Exercise 1.6-2: A handful of people were recently polled on their preference of soda at a local shopping mall in the U.S. and the study indicated more people preferred Mountain Dew over Cola. It indicates Mountain Dew is exceeding Cola as the beverage of choice in mainstream America. What is wrong with this assertion?

Exercise 1.6-3: A new anti-wrinkle cream was given to several people who frequently purchase make-up products from a specific company. In a follow-up with these individuals, all of them assert the new product was highly effective. What fallacy is present here? Should the company mass produce this new product? If not, what else needs to be studied here?

Exercise 1.6-4: All the members (sample size, n=30) of a high school tennis team were given a new racket and were asked to report back on how the racket affected their performance during practices for the week. All the members of the tennis team then completed a survey, and everyone minus one individual reported that the new racket impacted their performance in a positive manner. Since we have a large enough sample size can we now conclude the new racket causes tennis players to perform better? Why or why not? Is there a fallacy here?

Exercise 1.6-5: Is this advertising claim misleading? "More than 8 in 10 Dentists recommend Colgate."

Exercise 1.6-6: You heard a company rumor that the boss was going to crack down on employees who send e-mail from work. You, recently, read a report that showed most people spend an average of two hours per day checking and sending personal e-mails from work. You say to yourself 'No way do I spend that much time.' Should this employee be concerned?

Chapter 2

Data Presentation

2.1 Objectives

After completing this part, students should be able to:

- Recognize the types of variables (categorical or numerical) and give examples of each type.
- Remember and explain the four levels of measurement and give examples of each level.
- Tell the difference between dependent and independent variables.
- Compute the relative frequency.
- Use graphical methods to display data.

2.2 Statistics and Data

Statistics refers to a collection of techniques and procedures for analyzing data. In this context, statistics may be considered a synonym of data analysis. In a narrower context, 'statistics' is sometimes synonymous with the numbers themselves, such as in 'demographic statistics' or 'baseball statistics.' 'Data' refers to any collection of measurements that a researcher makes on some number of subjects. Researchers often store these measurements in a row-and-column display called a spreadsheet. The following example shows a spreadsheet containing data taken on ten students in a class.

Each row represents a subject (or student, in this case), and each column represents a measurement. The measurement columns are also called variables because the values vary from one subject to another.

CLASS DATA						
Student	Gender	Level	GPA	Credit Hrs Taken	Transport	Hrs Slept
1	M	Sophomore	3.10	32	Car	7
2	M	Junior	3.20	66	Car	8
3	F	Senior	3.49	94	Bus	8
4	M	Senior	2.68	89	Walk	10
5	F	Junior	3.73	69	Bicycle	8
6	F	Junior	3.39	59	Car	8
7	F	Senior	3.80	86	Walk	8
8	M	Junior	3.11	75	Car	8
9	F	Sophomore	3.10	27	Car	7
10	M	Senior	3.10	96	Walk	3

2.3 Classification of variables

2.3.1 Levels of Measurement

It is useful to distinguish between **four levels of measurements** for variables, from weakest to strongest: Nominal (no ordering), Ordinal (ordering exists, but not distance), Interval (distance exists, but not ratios), and Ratio (ratios exist).

1. **Nominal Variables**

   Nominal variables are **categorical variables** that have two or more categories without having any logical sequence or order. For example, Gender is a nominal variable, since 'Male' and 'Female' are just names of categories. Additional nominal variable examples include religious affiliation, language, and nationality. There is no intrinsic ordering between these. We often use bar charts and pie charts to represent these data.

   - You cannot perform arithmetic operations like addition, subtraction, multiplication, etc.
   - No order or logical sequence is present
   - Only names

2. **Ordinal Variables**

   Ordinal variables are **categorical variables** with a clear ordering or rank. A student's level of standing (freshman, sophomore, junior, or senior) is ordinal; they are also names of categories but, unlike gender, they are rank-ordered. However, we cannot subtract them, and distances do not make sense. Other examples of ordinal variables include Likert scale items in which we rank items. Completing surveys or even end

of semester evaluations we can rank on a scale of 1-5 (often from strongly disagree to agree strongly).

- Be careful about the size of the difference between categories may not be equally spaced.
- Only order matters not the difference between categories

3. **Interval Variables**

   Interval variables are **numerical variables** where the difference between the two values is meaningful. GPA is an interval measurement; we can subtract these measurements, and distances make sense. For example, the distance from 2.3-2.4 is the same distance as 3.7-3.8. However, ratios do not make sense; is 4.0 'twice as high' as 2.0? The answer is no. The grading system would work just as well on the scale (A, B, C)=(5.0, 4.0, 3.0) instead of (4.0, 3.0, 2.0).

4. **Ratio Variables**

   Ratio variables are numerical variables with true zero. Number of credit hours is a ratio measurement. A student who has completed 90 credit hours has twice as many as 45 credit hours and three times as many as 30 credit hours. Another example is the speed of an automobile which has true zero mph when it is not moving. A speed of 40 mph is twice as fast as 20 mph.

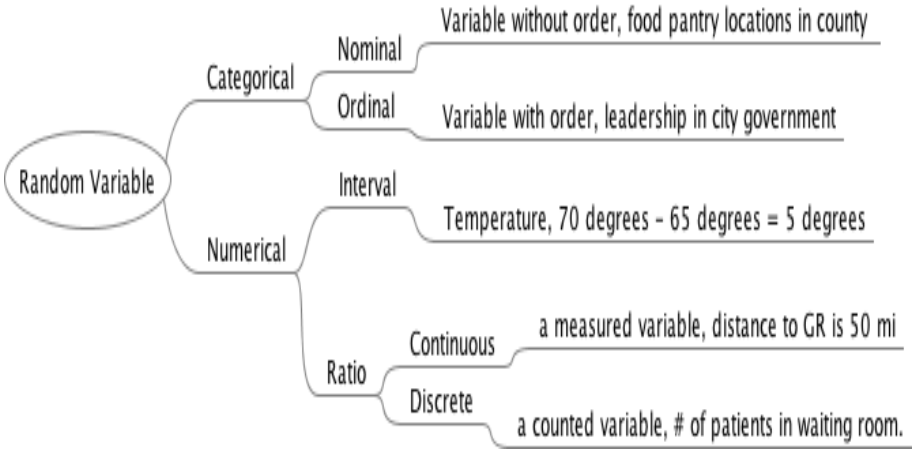
It is useful to recognize a hierarchy of information in the sense that *a measurement level contains an amount of information greater than or equal to the level below it*. At lower levels of measurement, data analyses tend to be less sensitive and sophisticated. A statistical study should aim for the highest levels of measurement possible or affordable.

2.3.2 Numerical versus categorical variables

Interval and ratio variables together are often called **numerical** variables or **quantitative** because they provide a number which measures 'quantity' (how much, how many) of something.

Nominal and ordinal variables together are often called **categorical** variables because they classify rather than count or measure. It is tempting to think of categorical variables as 'non-numerical,' but sometimes they do consist of numbers. For example, 'Social Security number' consists of numbers, but are used more as labels rather than quantities. Hence, SS number is categorical.

Figure 2.1: Mind map of a Random Variable



2.3.3 Dependent versus independent variables

Another classification for variables is dependent and independent. This distinction is relevant in studies that investigate cause and effect. The independent variable is the probable cause. We also refer to this as the predictor or explanatory variable. The dependent variable is the outcome variable that is affected by the independent variable.

In cause-and-effect studies:

- The **independent** variable is the probable cause.
- The **dependent** variable is the outcome being caused/affected.

The following is a description of a clinical study on the possible link between vaccines and autism. In particular, the study tries to find an association between the amount of a mercury-containing preservative in vaccines and measures of brain function in children. It has one independent variable (amount of mercury in vaccines) and 42 dependent variables (measures of brain function). Similar to two previously published studies, it found no evidence of neurologic problems in children exposed to mercury-containing vaccines. [Thompson, 2007]

1. **Background**

   Others have hypothesized that early exposure to thimerosal, a mercury-containing a preservative used in vaccines and immune globulin preparations are associated with neuropsychological deficits in children.

2. **Methods**

William W. Thompson enrolled 1047 children between the ages of seven and ten years and administered standardized tests assessing 42 neuropsychological outcomes. We did not assess autism-spectrum disorders. We determined exposure to mercury from thimerosal from computerized immunization records, medical records, personal immunization records, and parent interviews. We obtained the information on potential confounding factors from the interviews and medical charts. We assessed the association between current neuropsychological performance and exposure to mercury during the prenatal period, the neonatal period (birth to 28 days), and the first seven months of life.

3. **Conclusion**

   Our study does not support a causal association between early exposure to mercury from thimerosal-containing vaccines and immune globulins and deficits in neuropsychological functioning at the age of 7 to 10 years.

Among the 42 variables used to measure brain function tests on speech and language (comprehension of instructions, recalling sentences, stuttering), tests on verbal memory (no delay, short delay, long delay), tests on motor coordination (grooved pegboard, finger tapping), tests on behavior regulation (hyperactivity, inattentiveness), presence of tics (motor and phonics) and tests on intelligence (verbal IQ, full-scale IQ). Most of these are numerical test scores. Others are categorical, like the presence of stuttering (Yes-No) and tics (Yes-No).

Note that the researcher has a choice of whether to use numerical or categorical measures of brain function. Even for categorical outcomes, there is a choice between nominal (i.e., Yes-No), or ordinal (Frequently-Sometimes-Never). Many surveys like to use the ordinal five-point scale:

1. Strongly agree
2. Agree
3. Neither agree nor disagree
4. Disagree
5. Strongly disagree

It is called a *Likert item*. The scores from adding up several Likert items is said to be on a *Likert scale*. Western Michigan University's course evaluation uses a Likert scale on some topics.

2.4 Summarizing Categorical Data

The Census Bureau conducts a separate nationwide survey on population and information. Called the American Community Survey (ACS), it is "designed to provide communities a

fresh look at how they are changing." The ACS collects population and housing information every year instead of every ten years. The following data is a sample of the actual responses to the ACS from the state of Michigan and Indiana in 2008.

The variables 'State' and 'Type of Payment' are categorical. The variables 'No. of bedrooms,' 'Monthly Payment' and '12-month Household Income' are numerical.

Michigan and Indiana ACS Data
A Partial List

Household	State	No. of Bedrooms	Monthly Payment	Type of Payment	12-month Household Income
1	Michigan	2	880	Rent	11200
2	Michigan	3	990	Mortgage	80800
3	Michigan	4	750	Mortgage	87600
4	Michigan	3	1400	Mortgage	94000
5	Michigan	4	1400	Mortgage	97000
6	Michigan	1	560	Rent	6000
7	Michigan	4	900	Mortgage	95000
8	Michigan	2	0	None	39000
9	Michigan	1	380	Rent	24370
10	Michigan	2	910	Rent	54500
⋮			⋮		⋮
39	Indiana	2	200	Mortgage	46000
59	Indiana	4	200	Mortgage	38300
66	Indiana	2	190	Mortgage	18320
⋮			⋮		⋮

Table 2.1: Partial List of Michigan and Indiana ACS Data.

2.4.1 Relative Frequency Table

A *relative frequency table* gives the count for each category and the relative frequency or percentage of time in which each category occurs. Here is a relative frequency table for Monthly Payment type.

To compute Relative Frequency, divide Frequency by the total and multiply by 100%. For example,

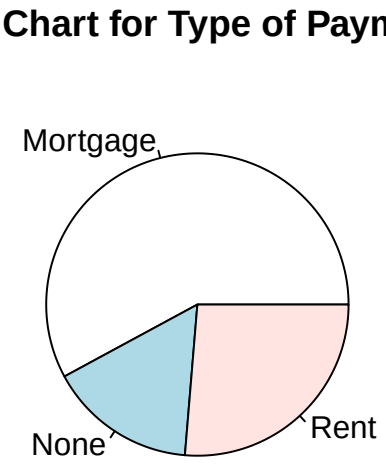
$$RF(Mortgage) = \frac{\text{Mortgage Frequency}}{\text{Total Frequency}} = \frac{44}{76} \times 100 = 57.9$$

	Frequency	Relative Frequency(%)
Mortgage	44	58
None	12	16
Rent	20	26
Total	76	100

Table 2.2: Relative Frequency Table for Type of Monthly Payment

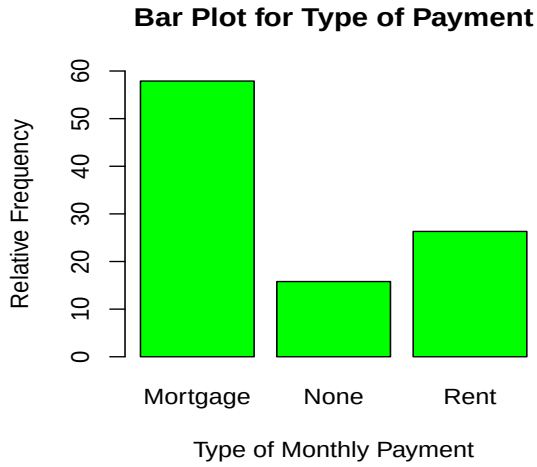
2.4.2 Pie Chart

A pie chart gives the same information as a bar chart, except in a circular shape. We also use pie charts for categorical data. Some people prefer the look of pie charts over bar charts. However, studies have shown that people have difficulty comparing the relative size of wedges. When using this type of chart, we should only consider it when the number of variables is less than six, and we will use bar charts when we have more than six. Look at the pie chart below. Is Rent less than half or more than half of Mortgage?



2.4.3 Bar Chart

A *bar chart* is a plot of the relative frequency table. We use bar charts for categorical data. The order by which the categories are presented in the bar chart is arbitrary. Sometimes they are presented in order of decreasing frequency for aesthetic purposes and is called a Pareto chart. Some computing packages present the categories alphabetically unless otherwise specified.



2.5 Summarizing numerical data

Remember that the *mean* and *median* cannot be used for nominal variables.

Consider the variable Monthly Payment in the ACS housing data on page 26. How do the numbers appear? How large are they? How variable is the data? Are there any extreme values? These questions are attempts at getting to know the data. In this section, we discuss tools for getting to know numerical data better. We have n = 64 observations available since 12 households (out of 76) do not have monthly payments. Here is a list of the monthly payments:

880, 990, 750, 1400, 1400, 560, 900, 0, 380, 910, 1200, 1200, 450, 1200, 1300, 550, 340, 770, 700, 700, 0, 0, 0, 140, 220, 0, 650, 5200, 0, 970, 740, 500, 700, 1800, 800, 1200, 1000, 1200, 200, 710, 1100, 400, 370, 670, 350, 510, 0, 900, 0, 340, 440, 1500, 0, 420, 490, 2400, 0, 500, 200, 670, 760, 720, 0, 500, 280, 190, 1000, 250, 700, 380, 740, 0, 850, 530, 290, 230

Now, we sort the observations from smallest to largest:

140, 190, 200, 200, 220, 230, 250, 280, 290, 340, 340, 350, 370, 380, 380, 400, 420, 440, 450, 490, 500, 500, 500, 510, 530, 550, 560, 650, 670, 670, 700, 700, 700, 700, 710, 720, 740, 740, 750, 760, 770, 800, 850, 880, 900, 900, 910, 970, 990, 1000, 1000, 1100, 1200, 1200, 1200, 1200, 1200, 1300, 1400, 1400, 1500, 1800, 2400, 5200

The smallest observation is $140. The largest is $5200, which is an **outlier**.

> In data analysis, an **outlier** is an observation that falls far from the rest of the data. It may need to be checked for correctness if there is any possibility of clerical error.

Can this be a typographical error that we need to correct? When we Look closely at the data, we see it belongs to observation 28, which is a mortgage for a household with a $358,000 annual income. We can infer that it is probably correct, so we leave it alone. The simple act of sorting the data already gives us more information about monthly payment. For example, we can now see that the typical monthly payment falls around $700. We also see the extent to which the $5200 value is outlying. We now turn to better ways to visualize the **spread** of this data.

2.5.1 Stem-and-Leaf Plot

Graphs are helpful in summarizing information in the data, especially for large datasets. Below, we summarize two-year average percentage of persons living in poverty (2008-2009) in the U.S.A. in a graph called a stem-and-leaf plot. The numbers on the left are stems, while the numbers on the right are called leaves.

7.4, 8.2, 8.6, 9.1, 9.2, 9.3, 9.7, 9.9, 10.1, 10.2, 10.3, 10.5, 10.5, 10.9, 11, 11, 11.1, 11.2, 11.4, 11.7, 11.7, 11.9, 12, 12.8, 12.9, 13, 13.2, 13.2, 13.3, 13.5, 13.5, 13.6, 13.9, 13.9, 14.4, 15, 15, 15.2, 15.2, 15.4, 15.4, 15.8, 16.2, 16.6, 16.9, 17, 17.1, 17.2, 19.3, 19.6, 20.6

```
The decimal point is at the |

 7 | 4
 8 | 26
 9 | 12379
10 | 123559
11 | 00124779
12 | 089
13 | 022355699
14 | 4
15 | 0022448
16 | 269
17 | 012
18 |
19 | 36
20 | 6
```

There is one leaf for each observation, so there are 51 leaves in all, one for each state. Our smallest observation is 7.4. We write it on the stem '7' as a leaf of '4.' We can infer that the stems represent tens and units digits while the leaves represent tenths digits. We write the value 20.6 on the stem '20' as a '20' and the leaf as a '6', and so forth. [Sullivan, 2013]

2.5.2 Relative Frequency Table and Histogram

Earlier in this chapter, we presented a relative frequency table for categorical data. Relative frequency tables may also be used for numerical data. A relative frequency table for monthly

payment is presented below. The class width is chosen to achieve a moderate number of class intervals.

	Frequency	Relative Frequency(%)
0-200	2	3
200-400	13	20
400-600	12	19
600-800	14	22
800-1000	8	12
1000-1200	3	5
1200-1400	6	9
1400-1600	3	5
1600-1800	1	2
2400-2600	1	2
5200-5400	1	2

Table 2.3: RF Table of Monthly Payment for ACS Housing Data

There are three things to keep in mind when constructing a frequency table. First, decide how many classes (i.e., intervals) we want. This also determines the *class width*. Try to have 5 to 15 intervals, depending on how many observations we have. Second, decide where the first interval starts. Third, decide how to avoid boundary disputes.

The last item requires us to choose a **boundary convention**. For instance, the intervals in the frequency table above could have been written as follows:

0-199	$[0-200)$
200-399	$[200-400)$
400-599	$[400-600)$
600-799	$[600-800)$
etc.	etc.

This way, it is easier to tell that \$200 belongs to the second interval, not the first. However, the table looks more complicated, harder to read. To keep the intervals simple but avoid boundary disputes, include a footnote to the table that describes the boundary convention, i.e. "Intervals contain the left endpoint but not the right." Then, we know that \$200 belongs to the second interval, not the first. Alternatively, we may use square braces and parenthesis.

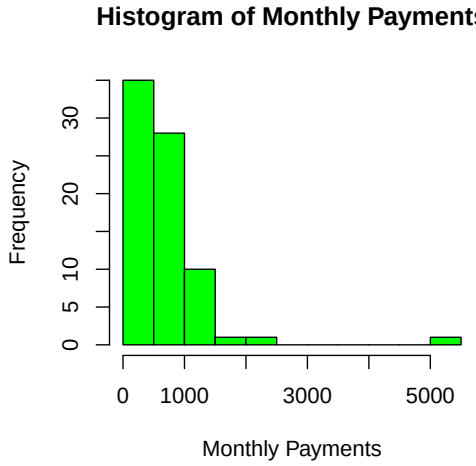
It also means that the class intervals contain the left endpoint but not the right. Which method do we prefer?

The relative frequency table is a compact numerical way to present how the data is distributed. If we plot the frequencies as columns, the resulting plot is called a *histogram*.

The histogram and stem-and-leaf plots look alike, except that the stem-and-leaf plot has

columns that go sideways instead of upwards. Stem-and-leafs are better if we want the data values themselves available from the plot. However, the histogram can handle large sample sizes easily and is more flexible in choosing class widths. For example, we may choose class widths of \$500, as follows: $[0,500),[500,1000),[1000,1500),\ldots,[5000,5500)$, which stem plots cannot do.

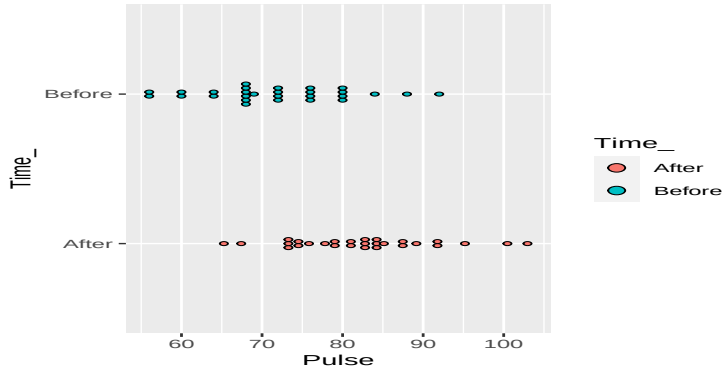
Figure 2.2: Monthly Payments



2.5.3 Dot Plot

For moderately large samples, a *dotplot* diagram is a quick way to see repeating data values. We present the dot-plots of the resting pulse rates among college students before and after exercise. As we observed using side-by-side dots plots, we find that the after exercise spread (maximum minus minimum) is wider than before exercise which makes sense. We also see that six students of the 28 had a pulse rate of 68 beats per minute.

Figure 2.3: Dot Plot



2.5.4 Box-and-Whisker Plot

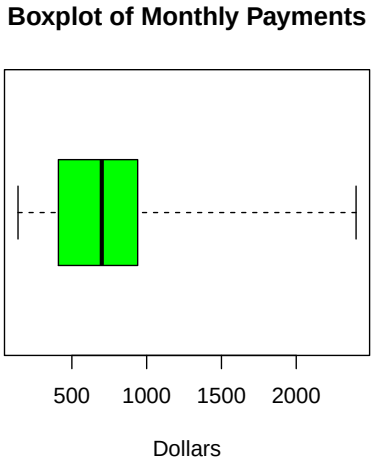
A box-and-whisker or boxplot shows useful information about the data including the measures the central tendency and the variability of the distribution. First, order the data from smallest to largest. What is the range of the first (or smaller) half of the data? The smallest quarter of the data? The next quarter? The box-and-whisker plot or boxplot is a graphical picture of the distribution of quarters of the data. Consider once again the monthly payments from the ACS housing data. We list them here sorted from smallest to largest (minus the outlier ($5200)).

140, 190, 200, 200, 220, 230, 250, 280, 290, 340, 340, 350, 370, 380, 380, 400, 420, 440, 450, 490, 500, 500, 500, 510, 530, 550, 560, 650, 670, 670, 700, 700, 700, 700, 710, 720, 740, 740, 750, 760, 770, 800, 850, 880, 900, 900, 910, 970, 990, 1000, 1000, 1100, 1200, 1200, 1200, 1200, 1200, 1300, 1400, 1400, 1500, 1800, 2400

There are $n = 63$ observations. One-quarter of the data is 63/4=15.75, or approximately 16 observations. The boxplot below gives the range of each quarter of the data: the range of the first quarter (i.e., the lowest 16 monthly payments) is the left whisker. The range of the 2nd quarter is the left part of the box, the 3rd quarter the right part of the box, and the range of the last quarter is the right whisker.

More precisely, the software draws vertical lines at (140, 400, 700, 970, 2400). Hence, the lowest 16 monthly payments lie within ($140, $400). The next set of 16 observations lies within ($400, $700). The third set lies within ($700, $970). Finally, the last 16 observations lie within ($970, $2400). The five values (140, 400, 700, 970, 2400) that divide the data into quarters and form the fences and whiskers of the boxplot are collectively called the five-number-summary of the data. They are often denoted as MIN, Q_1, MED, Q_3, and MAX respectively.

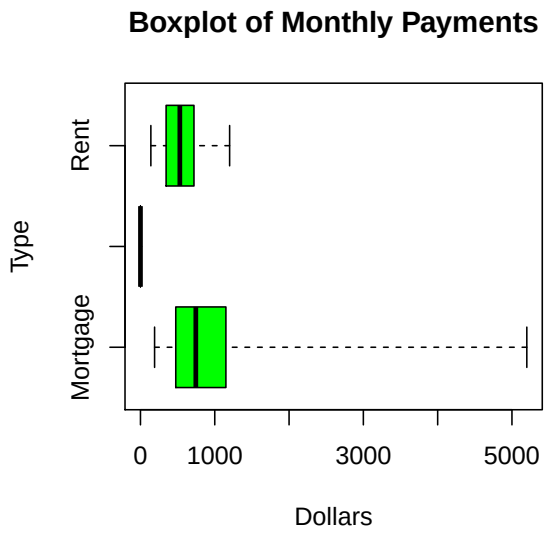
Figure 2.4: Box Plot



1. **MIN** is called the **minimum** and is the smallest of the ordered observations.
2. Q_1 is the median of the lower half of the data. Also, the is the upper boundary of the first quarter and is called the **first quartile**.
3. **MED** is the upper boundary of the second quarter and is called the **second quartile**. However, it also divides the data into lower and upper halves and is more often called the median.
4. Q_3 is the median of the upper half of the data. Also, it is the upper boundary of the third quarter and is called the **third quartile**.
5. **MAX** is the largest of the ordered observations and is called the **maximum**.

Boxplots are particularly useful for comparing two distributions side-by-side. Below are boxplots of mortgage data and rent drew parallel to each other on the same scale. When comparing data of mortgages and rental, we find that mortgages tend to be larger than rent since the median and quartiles are larger. There is also a considerable difference in the spread, with the mortgage data having a long right tail, evidence that mortgages have a higher ceiling than rent.

Figure 2.5: Box Plot



Different statistical computing packages have different ways of computing the quartiles, but the differences are minimal. In this class, we compute the quartiles as follows. First, arrange the observations from smallest (1st ordered observation) to largest (n^{th} ordered observation). Then

Q_1 is the .25(n+1)th ordered observation. MED is the .50(n+1)th ordered observation. In other words, the median is the middle value or the average of the two middle values Q_3 is the .75(n+1)th ordered observation.

If .25(n+1) is not an integer, take the average of the two adjacent ordered observations. Similarly, for MED and Q_3. Here again are the $n = 63$ ordered observations of monthly payments.

140, 190, 200, 200, 220, 230, 250, 280, 290, 340, 340, 350, 370, 380, 380, 400, 420, 440, 450, 490, 500, 500, 500, 510, 530, 550, 560, 650, 670, 670, 700, 700, 700, 700, 710, 720, 740, 740, 750, 760, 770, 800, 850, 880, 900, 900, 910, 970, 990, 1000, 1000, 1100, 1200, 1200, 1200, 1200, 1200, 1300, 1400, 1400, 1500, 1800, 2400

Since .25(63+1)=16, then Q_1 is the 16^{th} ordered observation. Hence $Q_1 = 400$. Similarly, 0.50(63+1)=32 so MED=700. Finally, .75(63+1)=48, so $Q_3 = 970$.

If $n = 64$, then .25(64+1) = 16.25, so that Q_1 would be the average of the 16^{th} and 17^{th} ordered observations. Therefore, if we return the $5200 outlier we removed, then the five-number-summary would be (140, 410, 700, 980, 5200).

2.5.5 Symmetry and Skewness

The shape of the data is often described by its symmetry or non-symmetry (also called skewness). Here are stem-and-leaf plots for *symmetric*, *right skewed*, and *left-skewed* data.

Symmetric Data

4	7
5	35
6	115
7	11112
8	467899
9	1199
10	14
11	1

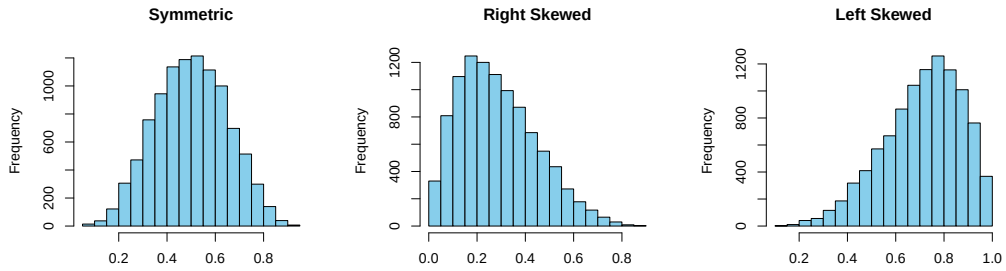
Right-Skewed Data

4	58
5	112345569
6	2445
7	17
8	5
9	2
10	5
11	2

Left-Skewed Data

4	0
5	5
6	2
7	5
8	17
9	2445
10	111245569
11	58

The term 'skew' refers to the direction of the most extended tail when you flip the stem-and-leaf sideways or draw a histogram. See histogram examples below. A long tail denotes presence of extreme or outlying observations. The left-skewed histogram contains observations that are small and outlying, while a right-skewed histogram would have observations that are large and outlying. A left skew also indicates that the smallest quarter of observations will be more spread out.



2.6 Key Words

- data
- dependent
- graphical methods
  - bar chart
  - box and whisker
  - dot plot
  - histogram
  - pie chart
  - stem-and-leaf
- independent
- statistics
- tabular methods
  - relative frequency table
- variables
  - categorical
  - numerical

2.7 Summary

- We cannot use the *mean* or *median* for nominal variables.
- We cannot use the mean for ordinal variables.

2.8

Exercises

Exercise 2.8-1: Identify the following types of data as numerical or categorical. If numerical, further classify into interval or ratio. If categorical, classify if it is nominal or ordinal.

1. The scores on exam one for Stat 1600.
2. Marital status
3. Annual income
4. Social Security Number
5. Cumulative GPA
6. Academic level (freshman, sophomore, junior, senior, other)
7. Quality (poor, fair, good, excellent)
8. Height (short, average, tall)
9. Age (years)
10. Grade $(A, B, C, \dots)$
11. Color
12. Rating of eight local plays (poor, fair, good, excellent).
13. Times required for mechanics to do a tune-up.

2	
3	4
4	56889
5	01112345567889
6	0124457
7	17
8	
9	2

Exercise 2.8-2: The 6-year graduation rate for a random sample of 30 colleges and universities in the U.S. is displayed in the stem-and-leaf plot below. Note that the stem unit is 10%, and the leaf unit is 1%. For example, the maximum value is 92%.

1. Obtain the five-number summary.
2. Obtain a boxplot and histogram for 6-year graduation rate.
3. In our opinion, which of the three plots (stem-and-leaf, boxplot, histogram) best illustrates the data? Why?

Exercise 2.8-3: A manager of a car rental company took a random sample of 100 business days over the last fiscal year and recorded the number of cars rented per day. The frequency distribution of the data is given below.

Interval	Frequency	Relative Frequency
$(20, 25]$	4	
$(25, 30]$	11	
$(30, 35]$	23	
$(35, 40]$	31	
$(40, 45]$	15	
$(45, 50]$	10	
$(50, 55]$	6	

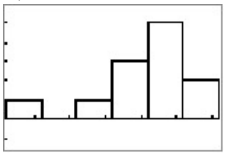
1. Fill in the relative frequencies above.

2. Draw a histogram by hand, with either the frequencies or relative frequencies as the vertical axis.
3. What interval does the median number of car rentals per day fall?
4. What percentage of business days had 30 or fewer car rentals?
5. How many business days had more than 45 car rentals?

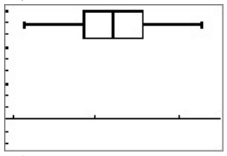
Exercise 2.8-4: The following plots represent five different samples of data. For each, describe the shape.

Figure 2.6: Describe Shapes

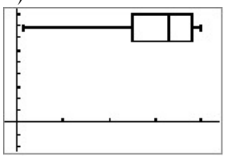
a)



b)



c)



d)

4	58
5	011245569
6	2445
7	17
8	
9	2
10	5
11	0

e)

0	4
1	889
2	11467
3	457
4	7

Exercise 2.8-5: A survey contained a question regarding marital status. The respondent checked either single, married, divorced, separated or widowed. What is the level of measurement of marital status?

1. Ordinal
2. Interval-Ratio
3. Nominal
4. I don't know

Answer:

Exercise 2.8-6: Blood alcohol content (BAC) is the concentration of alcohol present in a person's blood. What is the level of measurement of BAC?

1. Ordinal
2. Interval-Ratio
3. Nominal
4. I don't know

Answer:

Exercise 2.8-7: What type of variable is the number of robberies reported in June 2014 in Kalamazoo County?

1. Attribute
2. Discrete
3. Continuous
4. Qualitative

Answer:

Exercise 2.8-8: The team of researchers took the following items from a 2007 survey of drug use among young UK children Fuller [2008]. In how many occasions have you used or taken Cannabis? Determine whether the data are categorical or numerical and the level measurement.

____ Never
____ Once
____ 2 - 8 occasions
____ 9 - 15 occasions
____ More than 15 occasions

Answer:

Exercise 2.8-9: The researchers asked students the following questions from a 2007 survey of drug use among young UK children Fuller [2008]. Write the number of glasses of liquors (e.g., Baileys, gin, tequila, vodka) drunk in the last seven days ____. Determine whether they are categorical or numerical and the level of measurement.

Answer:

Exercise 2.8-10: The General Social Survey (GSS), conducted by the National Opinion Research Center at the University of Chicago, is a primary source of data on social attitudes in the U.S. Once each year, researchers from UC interview 1500 adults in their homes all across the country. They ask subjects their opinions about sex and marriage, attitudes toward women, welfare, foreign policy, and many other issues Trustees [2016].

The population for the GSS is

1. the 1500 persons interviewed.
2. the University of Chicago.
3. the list of questions asked.
4. all adult residents of the U.S.

Answer:

Exercise 2.8-11: A recent issue of the *New England Journal of Medicine* reported a study of all 122,754 infants born over an 8.5 year period at Parkland Hospital in Dallas, Texas, leaving out multiple births and babies with birth defects. The researchers wanted to know if there is a specific birth weight below which infant death and illness increases sharply.

The independent variable in the study is

1. death and illness.
2. infants (leaving out multiple births, etc.)
3. birth weight.
4. Parkland Hospital.

Answer:

Exercise 2.8-12: For the following research project, classify all variables in terms of levels of measurement and reveal whether they're continuous and discrete. As researchers, we should select the correct statistical method: *single variable descriptive statistic* or *a multi-variable descriptive statistic*

or *inferential statistics*. Recall that it is common for some projects to require more than one type of method.

Ten years ago, a state re-instituted the execution for first-degree murder. Researchers asked, did this change of policy reduce capital crime? A team of researchers gathered data on the number of homicides in the state for two-year periods before and after the policy change.

Answer:

Chapter 3

Descriptive Statistics

3.1 Objectives

After completing this part, students should be able to:

- Explain the concept of measures of central tendency and interpret the information they provide.
- Calculate, describe and compare the commonly used measures of central tendency: mean, $\bar{X}$ and median, MED.
- Understand other measures of central tendency.
- Select appropriate measure of central tendency based on the level of measurement and characteristics of the data (distribution).
- Explain the concept of measures of spread and the information they convey.
- Compute and explain sample standard deviation, (SD).

3.2 Estimates of Center

Suppose that a random sample of two-bedroom apartments in the Kalamazoo area yields the following data on monthly rent:

$635, $525, $500, $800, $650, $750, $555, $500, $670, $675

How much would we say is the average rent for two-bedroom apartments in Kalamazoo? In this chapter, we will discuss the sample average and several alternatives to the average when estimating 'average value' in a population.

3.2.1 The Sample Mean

The sample mean is the statistical term for 'average of the sample.' For the example above, the sample mean (denoted $\bar{X}$) is:

$$\bar{X} = \frac{635 + 525 + 500 + 800 + 650 + 750 + 555 + 500 + 670 + 675}{10} = \$626$$

so that the average rent in Kalamazoo may be estimated as \$626. Note that this is an *estimate* based on a *sample*. Therefore, it is subject to *sampling error*. Sampling error means that due to the luck of the draw, the sample average likely missed the true population average. More precisely, the two-word term 'sampling error' refers to $|\bar{X} - \mu|$, the distance by which the *sample* average $\bar{X}$ misses the population average μ.

The advantages of the sample mean:

- It is easy to understand and simple calculate.
- It is based on all the values.
- It is not based on the position in the series.

The disadvantage of the mean:

- It is always affected by outliers, which are relatively small and/or relatively large data values.

3.2.2 The Sample Median

There are alternative ways to estimate average rent if we want to avoid the effect of outliers. We can determine the sample median, instead of the sample mean. We have discussed the sample median (denoted MED) earlier in Chapter 1. It is computed as follows.

1. Let n represent the total number of observations.
2. Order the n observations from smallest to largest.
3. Then calculate $0.50(n+1)$ to locate the middle value of the dataset.
4. If $0.50(n+1)$ is an integer, then MED is the $0.50(n+1)th$ ordered observation.
5. If $0.50(n+1)$ is not an integer, then MED is the average of the two adjacent ordered observations.
6. There are $n = 10$ observations in our rental data. We first order them from smallest to largest.

$$500, 500, 525, 555, 635, 650, 670, 675, 750, 800$$

Now, $0.50(n+1) = 0.50(10+1) = 5.5$. Since this is not an integer, we average the 5^{th} and 6^{th} largest observations:

$$MED = \frac{(635+650)}{2} = \$642.50$$

The advantage of the median is that it is more robust than the mean, i.e., the median is not as affected by extreme values (outliers).

3.2.3 The Trimmed Mean

Since the mean uses all observations in the calculation, it can be strongly affected by outlying small and large values. What happens to the mean when the smallest value \$500 is replaced by \$400? It will become smaller. What happens to the median? It remains unchanged.

The trimmed mean is a compromise estimator that looks a lot like a mean but is less sensitive to extreme values. The 10%-trimmed mean removes the lowest 10% and highest 10% of the data, then take the sample mean of the remaining data. In the rental example, 10% of the data is one observation. We remove the lowest observation (\$500) and the largest observation (\$800), and take the mean of the remaining eight observations:

$$\bar{X} = \frac{635+525+650+750+555+500+670+675}{10} = \$620$$

What if 10% of the data is not an integer? For example, if $n = 23$, then 10% of n is 2.3. Since we cannot remove 2.3 observations, we will remove three observations from each end (to ensure at least 10% protection) and take the average of the middle 17 observations.

3.3 Estimate of Spread (or Uncertainty or Variation)

Recall the data on the monthly rent of two bedroom apartments in the Kalamazoo area:

$$500, 500, 525, 555, 635, 650, 670, 675, 750, 800$$

If a future student asked us "What should a two-bedroom apartment cost us in rent?" how should we reply? Knowing that the data averages \$626, we might say something like "Around \$626, give or take . . . (?)." This second number, the give or take, is important because it says how much uncertainty there is in our guess. In other words, the student's rent will probably miss \$626, but by how much? We will get to the answer in a minute. For a second example, if we were going to San Francisco for a couple of days in August, and we want to know what clothes to bring, it is not enough to know that the average temperature is 68

degrees. If it was 68 degrees, give or take 15 degrees, then we will need a sweater. If it is 68 degrees, give or take 30 degrees, we might need a winter coat.

Variation presents itself everywhere. Consider weight loss. The 77 subjects in the Atkins group lost an average of 10.5 pounds 'give or take' 15 pounds. Compare this to an average of 3.5 pounds for the Zone diet 'give or take' 14 pounds. How about in bowling? In his first seven games in a tournament in Indiana, Walter Ray Williams Jr. averaged 228, 'give or take' 34. Notice how the 'give or take' number seems to complete the description.

In this chapter, we will discuss the sample standard deviation, which is typically used as the 'give or take' number to describe spread in a group of numbers.

3.3.1 The Sample Standard Deviation

The standard deviation (or SD) is computed using a series of steps:

1. Compute the mean.
2. Subtract mean from each observation. These are the deviations from the mean or "Deviations".
3. Square the deviations from the mean. These are the "Squared Deviations".
4. Take the sum of squared deviations, this is called the "Sum of Squares" or *SS*.
5. Compute SD using the equation:

$$SD = \sqrt{\frac{SS}{n-1}}$$

where n is the number of observations.

The table below shows the computations for the monthly rent data. Notice in the last row that the sum of the deviations from the mean is 0. This will always be the case for the sum of deviations from the mean.

$$SD = \sqrt{\frac{SS}{n-1}} = \sqrt{\frac{9.764 \times 10^4}{9}} \approx 104.2$$

It is helpful to get a feel for the numbers in the second column. The first value 9 says that '635 is nine above average'. The next number -101 says that '525 is 101 below average.' Ignoring signs, the numbers in the second column represent the 'give or take' from the average. To get the actual SD, we take the Sum of Squares and divide by $(n-1)$, and last, of all take the square root of that answer. It gives us a final SD of 104.158. We say that monthly rent averages \$626, 'give or take' \$104.

Look at the second column again. Ignoring signs, these are the distances from the average. Which rental is closest to the average? The answer is \$635, which missed by 9. Which

	Rent	Deviations	Squared Deviations
1	635	9	81
2	525	-101	10201
3	500	-126	15876
4	800	174	30276
5	650	24	576
6	750	124	15376
7	555	-71	5041
8	500	-126	15876
9	670	44	1936
10	675	49	2401
Sum	6260	0	97640

rental is farthest from the average? \$800, which missed by 174. What is the average 'size of the miss?' It should be somewhere between the smallest and largest, right? It is how we interpret the SD: "On the average, the monthly rentals miss the mean by \$104."

Taken together, the mean and SD allow comparison of both relative size and spread of two groups of numbers. In the professional bowling tournament in Indiana that he won in November of 2008, here are Walter Ray Williams Jr.'s first seven games and last seven games (including the final):

Games		Mean	SD
First seven	163,231,224,238,279,239,226	228.6	34.3
Last seven	246,244,247,248,237,258,246	246.6	6.2

What do the mean and SD tell us? He averaged higher in the end but was also more consistent with a give-or-take of only 6 points! His earlier games had more massive swings: from a low of 163 to a high of 279, resulting in the SD of 34.

3.3.2 Effect of Multiplication and Addition by a Constant

Recall that monthly rent for apartments average \$626 with an SD of \$104. If the student plans to get a roommate and pay only half the rent, how much does he expect to pay? If we are thinking of dividing both numbers by two, i.e., \$313 give or take \$52, this is correct.

$$\text{Original rent: } 626 \pm 104 \qquad \text{Half the rent: } 313 \pm 52$$

Now suppose that the student does not plan to get a roommate, but his parents have agreed to contribute \$100 to rent each month. How much does the student expect to pay after a subsidy of \$100? If we are thinking of subtracting \$100 from both numbers, i.e., \$426 'give or take' \$4, there is an error in our thinking. Here the SD remains the same, i.e.,

Original rent: 626 ± 104 Subsidized rent: 526 ± 104

If we are not convinced, consider the data itself on the following table.

In general, when the data is **multiplied or divided by a positive constant**, the **same thing happens** to both the average and the SD

	Rent	Deviations	Squared Deviations
1	317.5	4.5	20.2
2	262.5	-50.5	2550.2
3	250.0	-63.0	3969.0
4	400.0	87.0	7569.0
5	325.0	12.0	144.0
6	375.0	62.0	3844.0
7	277.5	-35.5	1260.2
8	250.0	-63.0	3969.0
9	335.0	22.0	484.0
10	337.5	24.5	600.2
Sum	3130.0	0.0	24410.0

Table 3.1: Calculating the SD when rent is DIVIDED BY 2

$$SD = \sqrt{\frac{SS}{n-1}} = \sqrt{\frac{2.441 \times 10^4}{9}} \approx 52.08$$

In general, when we add or subtract a **constant** to the data, the same thing happens to the average, but the **SD remains unchanged.**

	Rent	Deviations	Squared Deviations
1	535.0	9.0	81.0
2	425.0	-101.0	10201.0
3	400.0	-126.0	15876.0
4	700.0	174.0	30276.0
5	550.0	24.0	576.0
6	650.0	124.0	15376.0
7	455.0	-71.0	5041.0
8	400.0	-126.0	15876.0
9	570.0	44.0	1936.0
10	575.0	49.0	2401.0
Sum	5260.0	0.0	97640.0

Table 3.2: Calculating the SD when rent is reduced by 100 dollars

$$SD = \sqrt{\frac{SS}{n-1}} = \sqrt{\frac{9.764 \times 10^4}{9}} \approx 104.16$$

3.3.3 Risk ratio and odds ratio

In clinical studies, statisticians frequently take ratios of proportions or probabilities (instead of differences). There are several reasons for this idea. Sometimes, the disease or medical event of interest is quite rare, i.e., $p_1 = 0.0008$. If a new treatment reduces the probability of getting the disease to 0.0006, the difference in probabilities is quite small and hard to assess ($(p_1 - p_2) = 0.0002$). In the meantime, the ratio $\frac{p_2}{p_1} = \frac{0.0006}{0.0008} = 0.75$ means that the risk of getting the disease under the new treatment has been reduced by 25%.

A technical reason for using ratios is that a ratio can control for other variables such as age and race.

3.3.4 Risk ratio

Following is the abstract of the study "Safety and Efficacy of a Recombinant Hepatitis E Vaccine."

Background

Hepatitis E virus (HEV) is a significant cause of viral hepatitis. We evaluated the safety and efficacy of an HEV recombinant protein (rHEV) vaccine in a phase 2, randomized, double-blind, placebo-controlled trial.

Methods

In Nepal, we studied 2000 healthy adults susceptible to HEV infection who were randomly assigned to receive three doses of either the rHEV vaccine or placebo at months 0, 1, and 6. Active (including hospital) surveillance was used to identify acute hepatitis and adverse events. The primary endpoint was the development of hepatitis E after three vaccine doses.

Results

A total of 1794 subjects (898 in the vaccine group and 896 in the placebo group) received three vaccine doses; researchers followed the total vaccinated cohort for a median of 804 days. After three vaccine doses, hepatitis E developed in 69 subjects, of whom 66 were in the placebo group. The vaccine efficacy was 95.5% (95% confidence interval [CI], 85.6 to 98.6). In an intention-to-treat analysis that included all 87 subjects in whom hepatitis E developed after the first vaccine dose, nine subjects were in the vaccine group, with a vaccine efficacy of 88.5% (95% CI, 77.1 to 94.2). Among subjects in a sub-group randomly selected for analysis of injection-site findings and general symptoms (reactogenicity sub-group) during the 8-day period after the administration of any dose, the proportion of subjects with adverse events was similar in the two study groups, except that injection-site pain was increased in the vaccine group ($p = 0.03$).

Conclusion

In a high-risk population, the rHEV vaccine was effective in the prevention of hepatitis E.

The data given in the 'Results' part of the abstract may be summarized as follows.

	hepatitis E		
	Yes	No	Total
Vaccine	3	895	898
Placebo	66	830	896

We may present data for comparing proportions in a 2 by 2 table.

	Disease		
	Yes	No	Total
Exposure	a	b	$a+b$
No Exposure	c	d	$c+d$

The risk ratio also called the relative risk is the ratio of probabilities

$$RR = \frac{\frac{P(Disease)}{Exposure}}{\frac{P(Disease)}{NoExposure}} = \frac{\frac{a}{a+b}}{\frac{c}{c+d}} = \frac{p_1}{p_2}$$

For our example, we can define and calculate the risk ratio as

$$RR = \frac{\frac{P(Disease)}{Exposure}}{\frac{P(Disease)}{NoExposure}} = \frac{\frac{3}{898}}{\frac{66}{896}} = \frac{0.00334}{0.07366} = 0.045$$

It means that getting the vaccine reduces our risk to only 4.5% of the original, or has 95.5% efficacy.

3.3.5 Odds ratio

The odds of an event occurring is

$$Odds = \frac{\texttt{Probability that event occurs}}{\texttt{Probability that event doesn't occur}} = \frac{p}{q}$$

For example, if you win a game a 20% of the time ($p = 0.20$), then our odds of winning is $(\frac{0.20}{0.80}) = \frac{1}{4}$. We say that we have a *1-in-4* odds of winning, or we win once for every four times we lose. If we win 80% of the time, the odds are $\frac{0.80}{0.20} = 4$. It means we have *4-in-1* odds of winning, or we win four times for every one time we lose. Here is a table of odds corresponding to various probabilities.

Probability	Odds
0.10	1/9 = 0.11
0.20	1/4 = 0.25
0.50	1/1 = 1.00
0.80	4/1 = 4.00
0.90	9/1 = 9.00

Unlike probabilities, odds can be greater than 1. The odds ratio is just the ratio of two odds (usually for comparing two groups).

$$OddsRatio = \frac{\texttt{Odds of Group 1}}{\texttt{Odds of Group 2}} = \frac{\frac{p_1}{q_1}}{\frac{p_2}{q_2}}$$

Returning to the hepatitis E study, recall the disease occurrence data:

	hepatitis E		
	Yes	No	Total
Vaccine	3	895	898
Placbo	66	830	896

The disease rate for each group is

$$\texttt{Odds(Hepatitis|Placebo)} = \frac{0.07366}{(1-0.07366)} = 0.07952$$

$$\texttt{Odds(Hepatitis|Vaccine)} = \frac{0.00334}{(1-0.00334)} = 0.00335$$

$$\texttt{Odds Ratio} = \frac{\texttt{Odds(Hepatitis|Placebo)}}{\texttt{Odds(Hepatitis|Vaccine)}} = \frac{0.07952}{0.00335} = 23.7$$

We say that "the odds of getting hepatitis is 24 times greater if we remain unvaccinated."

Odds ratios are generally easier to interpret if they are more significant than one. We can always ensure this by choosing which group to put in the numerator, i.e., the one with more substantial odds.

It is essential to understand that the odds ratio is not a ratio of likelihood or probabilities. If the disease rates for men and women are 0.80 and 0.40, respectively, then the odds ratio is

$$\frac{\frac{0.80}{0.20}}{\frac{0.40}{0.60}} = 6.00$$

In this example, men are twice as likely to get the disease but have six times the odds.

3.4 Key Words

- odds
- odds ratio
- sample mean
- sample median
- trimmed mean
- sample standard deviation

3.5

Exercises

Exercise 3.5-1: The carbon monoxide measures (in mgs) are given on 25 brands of cigarettes.

13.6 16.6 23.5 10.2 5.4 15.0 9.0 12.3 16.3 15.4 13.0 14.4 10.0 10.2 9.5 1.5 18.5 12.6 17.5 4.9 15.9 8.5 10.6 13.9 14.9

Calculate the

1. mean
2. median
3. 10% trimmed-mean
4. standard deviation
5. mean and standard deviation (in grams)

Exercise 3.5-2: Daily high temperature for a given day are provided for the past 10 years:

2007	2008	2009	2010	2011
59	50	49	13	41
2012	2013	2014	2015	2016
46	51	53	58	47

Find the following statistics for temperature:

1. range
2. mean
3. median
4. Remove the temperature of the year 2010 from the data set and re-calculate the mean and compare it with part (2)? Is the mean closer to the median and why?

Exercise 3.5-3: Joshua has been working on programming and updating a Website for his company for the past 12 months. The following numbers represent the number of hours Joshua has worked on this Website for each of the past 12 months:

24, 25, 31, 40, 48, 40, 36, 50, 38, 35 ,42, 112

1. Calculate the mean
2. Calculate the median
3. Decide if its symmetric, skewed to the right or to the left
4. Decide which measure of center provides the most relevant information about the distribution? Why?

Exercise 3.5-4: In a study of warp breakage during the process of weaving fabric (*Technometrics*, 1982: 63). Nine specimens of yarn were tested. The number of cycles of strain to breakage was determined for each yarn specimen, resulting in the following data:

146, 194, 393, 277, 246, 282, 55, 61, 131

1. Calculate the mean
2. Calculate the median
3. Calculate the standard deviation
4. Decide if its distribution (symmetric, skewed to the right or to the left)
5. Decide which measure of center provides the most relevant information about the distribution? Why?

Exercise 3.5-5: The weekly budgets for groceries of six students are as follows:
$30, $35, $40, $28, $35, $25.
Compute for the mean and median and choose the correct answer from the choices below:

1. mean = 188, median = 30
2. mean = 32.167, median = 35
3. mean= 32.167, median = 32.5
4. mean= 188, median = 30

Answer:

Exercise 3.5-6: John got 16 and 22 on his two Stat 1600 quizzes. What score must he have on his next quiz to have the mean of exactly 20 for his three quizzes?

1. 20
2. 22
3. 18
4. 24

Answer:

Exercise 3.5-7: The following table lists the average number of cars per 1000 population for eight nations. Compute the mean and median for these data.

Nation	cars per 1000 population
United States	820
Canada	607
China	83
Russia	293
Japan	591
Mexico	275
Spain	593
United Kingdom	519

1. Which is greater in value?
2. Is there a positive skew in the data?
3. How do you know?

Exercise 3.5-8: You are a researcher for a mid-sized city. You collected six types of variables from a random sample of students from a large university. These variables include:

1. their region of birth of country
2. the extent they support marijuana legalization (7=strong, 4=neutral, 1=weak)
3. the weekly amount of money spent on cafeteria food
4. number of movies they watched per week
5. quality of cafeteria food at their university (10=excellent, 0=bad)
6. religious affiliation.

Select the appropriate measures of central
tendency for each variable.

Table 3.3: Find appropriate measures of central tendency

Student	Birth	Expense	Movies	Food	Religion
a	West	43	4	6	Cath.
b	MW	51	3	5	Other
c	South	65	14	0	Other
d	South	52	0	10	Prot.
e	North	48	1	6	Jew
f	North	62	5	8	Prot.
g	MW	47	7	1	None
h	South	45	10	2	Cath.
i	North	39	14	7	Prot.
j	North	33	0	10	Prot.

Exercise 3.5-9: Using the dataset from Table 3.3, determine the value of the central tendency measures for each variable.

Exercise 3.5-10: As a leader of a Kalamazoo social services agency that employs 20 staff members, you are concerned that your staff has an increased case load of clients compared to 10 years ago. The case load of each worker is reported in the following table (Table 3.4) for years 2005 and 2015.

Table 3.4: Case Loads Comparison

2005	50	64	55	64	60	53	56	50	51	45
	46	46	50	56	63	65	53	54	58	69
2015	57	47	46	59	59	50	57	52	41	66
	65	52	57	75	65	76	43	56	52	67

Has the average case load increased, decreased, or stayed the same?

Answer:

Exercise 3.5-11: The admissions department at WMU gave 25 randomly selected freshmen a national prejudice survey. The racial prejudice index scores will be used in three years to see if higher education affects prejudice.

Table 3.5: Freshmen Racial Prejudice Index

45	30	35	30	42
50	43	40	32	48
9	13	10	11	11
40	26	39	38	44
32	37	41	27	22

Calculate the mean and median scores of these data from Table 3.5.

Answer:

Exercise 3.5-12: The same 25 students completed the same survey during their senior year. Compute the mean and median for this second set of scores, and compare them to the scores from four years earlier. What happened?

Table 3.6: Senior Racial Prejudice Index

50	27	31	35	41
11	45	50	37	43
11	9	10	20	10
35	10	30	41	40
15	30	40	26	21

Calculate the mean and median scores of these data.

Answer:

Exercise 3.5-13: A local social service agency has started a sex education course for teen girls. The girls took a 20-question exam for general information about sex, anatomy and physiology upon entry and again after completing the course. Table 3.7 has the listing of scores of a random sample of 15 girls.

Calculate the mean and median of the pre and post test scores and comment on the results.

Answer:

Exercise 3.5-14: The acidity or alkalinity of a solution is measured using pH. A pH less than 7 is acidic; a pH greater than 7 is alkaline. The following data represent the pH in samples of bottled water and tap water.

1. Calculate the mean of the tap water.
2. Determine the median of the tap water.

Table 3.7: Pre and Post Test Results

case	posttest	pretest	difference
1	12	8	4
2	13	7	6
3	12	10	2
4	19	15	4
5	8	10	-2
6	17	10	7
7	12	3	9
8	11	10	1
9	7	5	2
10	12	15	-3
11	21	13	8
12	5	4	1
13	15	10	5
14	11	8	3
15	20	12	8

Table 3.8: Source: Emily. McCarney, Joliet College

Tap		—	Bottled	
7.64	7.45	—	5.15	5.28
7.45	7.10	—	5.09	5.26
7.47	7.56	—	5.26	5.13
7.50	7.47	—	5.20	5.26
7.68	7.52	—	5.02	5.21
7.69	7.47	—	5.23	5.24

3. Comment on the results.

Chapter 4

Threats to Valid Comparisons

4.1 Objectives

After completing this part, students should be able to:

1. Know the difference between association and causation.
2. Understand how confounders can affect the results of a study.

4.2 Hidden Confounder

In 1992, a research study "Lower extremity fractures in motor vehicle collisions: Influence of direction of impact and seatbelt use" by Dischinger, P., Cushing, B. and Kerns, T. was published in the 36th Proceedings of the Association for the Advancement of Automotive Medicine. It involved data analysis of the trauma-center population in Maryland. Some of the conclusions were:

1. there was a higher incidence of lower extremity injury in frontal collisions,
2. seatbelt use was not effective in preventing lower extremity fractures, and
3. there was a higher incidence of lower extremity fracture among women.

The conclusion (3) is interesting. It begs the follow-up question: "Why do women have higher rates of leg fractures?" Is it because they drive faster, or apply brakes more slowly, or have weaker bones? It turns out that these are false questions – they presume that gender is the variable that causes higher leg fractures.

Researchers proposed an explanation in a follow-up study with the same lead author: 'Lower extremity fractures in motor vehicle collisions: The role of driver gender and height'

by Dischinger, P.C., Kerns, T.J., Kufera, J.A. Accident Analysis & Prevention Volume 27, Issue 4, August 1995.

> Abstract: In a previous study it was noted that there was a higher incidence of lower-extremity fractures among women drivers. Analyses were based on a linkage between trauma registry and police crash report data. The present study addresses the issue of whether the differences noted are attributed to driver gender or are merely a reflection of differences in driver's height. An inverse association was noted between driver height and the incidence of lower-extremity fractures. Those shorter than average (5'7") for this population had a 64% increase in a lower-extremity fracture, which can be mainly attributed to ankle/tarsal injuries. Thus, the incidence of these injuries appears to be a function of driver height, with an increase among shorter drivers, most of whom are women.

[Dischinger, 1995]

It turns out that height was the culprit, but it initially looked like gender because height and gender have a strong link. The following *pathway graph* describes the true relationship:

$$\begin{array}{ccc} \texttt{Height} & \Rightarrow & \texttt{Leg Fracture} \\ \updownarrow & & \\ \texttt{Gender} & & \end{array}$$

But the association between height and gender led us to believe the wrong relationship:

$$\texttt{Gender} \Rightarrow \texttt{Leg Fracture}$$

The symbol $\Rightarrow$ represents "cause-and-effect" while $\updownarrow$ represents "association."

In this situation, leg fracture rate is the outcome, variable gender is the probable cause. The hidden variable height is called a **confounder**, or a confounding variable.

Confounding Variable:
A confounder or confounding variable is a third variable that is associated with both the probable cause and the outcome. It can lead us to a wrong conclusion about the cause-and-effect relationship.

4.2.1 Apples and Oranges

Now, why do unknown confounders belong in a chapter on threats to valid comparisons? They are one of the most significant sources of (often unknowingly) invalid comparisons. It seemed fair to compare leg fracture rates of men and women, didn't it? What's wrong with that? Unfortunately, concerning leg fracture rates, it was a case of comparing apples to

oranges – women as a group is shorter than men! Of course, in the 1992 study, the investigators did not know that height would make a difference. Women also tend to weigh less, smoke less, drink less, have longer hair, and wear higher heels. Which of these would make a difference, i.e., be potential confounders?

Confounders are a big problem in comparison studies. Some confounders may remain hidden, but it is critical that the researchers identify and control for potential confounders as much as possible. Does smoking cause lung cancer? When comparing smokers to nonsmokers, it is easy to show that smokers have higher lung cancer rates. But as a group, they exercise less than nonsmokers. They drink more coffee than nonsmokers. Can the exercise or the coffee or the combination be the culprit? Furthermore, smokers tend to be male, older, and drive in the winter with their car windows open. There are plenty of confounding variables even in this comparison.

4.2.2 In the news

The STATS website (http://www.stats.org/) is dedicated to correcting "scientific misinformation in the media and public policy resulting from bad science, politics, or a simple lack of information or knowledge." The following is an excerpt from an article written by Rebecca Goldin and Jing Peng in August 2010. What is the confounding variable? The probable cause? The outcome variable?

> **If you take Viagra, will you get an STD?**
> Rebecca Goldin Ph.D. and Jing Peng, August 2, 2010
>
> Judging from recent headlines, it seems clear: "Sex Diseases Tripled in Men 40 or Older Taking Viagra, Cialis, Study Says" reports Bloomberg; "Older Viagra Users More Likely to Get STDs" says the Chicago Sun-Times, presumably comparing older Viagra users with older non-Viagra users. And Health Day was even more explicit, saying "Drugs Like Viagra Linked to Higher Rates of STDs." The next logic behind the claim seems persuasively apparent: if men who take Viagra are having more sex, then they have inevitably increased their risk of catching a sexually transmitted disease (STD). But is this the case?
>
> The study compared men who took erectile dysfunction (ED) drugs with those who did not. Researchers found that ED drugs such as Viagra are linked to higher rates of STDs among older men, but not older women, especially after the introduction of Viagra in 1998. A study published in the July 6 issue of the Annals of Internal Medicine was the first to examine the relationship between ED drugs and STDs. But its findings turn out to be far different than media accounts would have you believe.
>
> The main problem in the coverage is the direct suggestion that taking Viagra is associated with STDs, as opposed to being the sort of person who takes Viagra.
>
> There is a critical distinction. To see what it means, let's go backward in time and compare STD rates among people who plan to take Viagra but haven't and

> people who are not planning on taking Viagra. This quick comparison is what the authors of the research article did; they combed medical records for people who filled prescriptions for ED drugs and compared STD rates for these people before filling their prescription to the STD rates of people who did not subsequently fill a prescription for Viagra. It turns out that the rate of STDs is higher among people who intend to take it. In other words, the drug is absent, but those people who will take Viagra within a year are already at higher risk of STDs.
>
> In fact, compared to those who don't take ED drugs, those who plan to take Viagra had a slightly higher rate for STDs, an odds ratio (OR) of 2.80; 95% confidence interval CI, 2.10 to 3.75, than those who actually take it (OR) 2.65, CI, 1.84 to 3.81, though the difference was not significant. It means that the drug had no discernable effect on STD rates for this group of men. [Goldin, 2010]

The paper goes on with detailed analysis, but the main point has been stated. To summarize the conclusions of the study:

1. Taking ED drugs do not increase the rate of STD's
2. The type of people who take ED drugs are different from people who do not.

$$\begin{array}{ccc} \texttt{Person type} & \Rightarrow & \texttt{STD} \\ \updownarrow & & \\ \texttt{ED drugs} & & \end{array}$$

instead of the (admittedly more sensational) relationship implied by the headlines.

4.3 Key Words

• association	• confounders
• cause	• effect

4.4

Exercises

Exercise 4.4-1: A study says, "Slightly overweight people live longer than thin people."

1. The headline implies cause-and-effect, not just association, i.e. "if you are thin, you should try to gain weight." When comparing lifespan of slightly overweight and thin people, can you think of possible confounders?
2. Using one confounder from your answer(s) in (1), draw a pathway graph depicting the possible relationship between confounder, possible cause, and outcome.

Exercise 4.4-2: A study that appeared in the American Journal of Cardiology (March 15, 2003) found out that generally, "heart attack survivors who owned a dog had better heart function post-heart attack than those who did not own a dog."

1. Does a dog help a heart heal faster? Can you think of possible confounders?
2. Using one confounder from your answer(s) in (1), draw a pathway graph depicting the possible relationship between confounder, possible cause, and outcome.

Exercise 4.4-3: A study says, "People who consider themselves depressed eat more chocolate than people who consider themselves otherwise."

1. The headline implies cause-and-effect, not just association, i.e., "if you are depressed, you will tend to eat more chocolate." When comparing chocolate consumption of depressed versus not-depressed, can you think of possible confounders?
2. Using one confounder from your answer(s) in (1), draw a pathway graph depicting the possible relationship between confounder, possible cause, and outcome.

Exercise 4.4-4: The next four questions refer to the following scenario. In a recent statewide election, 55 percent of the voters rejected a proposal to institute a state lottery. In a random sample of 150 urban precincts, 60 percent of the electorate rejected the proposal. Can you think of possible confounders?

Chapter 5

Study Designs

5.1 Objective

After completing this part, students should be able to:

1. Grasp the basics of good experimental design
2. Understand the importance of randomized trials (RT).
3. Comprehend the meaning of double blind, randomized controlled trials (RCT).
4. Recognize the difference between observational studies and randomized controlled trials.
5. Perceive the need for case-controlled studies.
6. Grasp the need for case-crossover studies.

5.2 Randomized trials

In the previous chapter, we covered unknown confounders, one of the most critical threats to valid comparisons in statistics. In this chapter, we will explore elements of study design that help statisticians deal with the danger of bias posed by hidden confounders. One of the most important aspects of many study designs is appropriate randomization in the selection of subjects for the study, and to different groups within the study. Without such randomization, results of data analyses can be biased. Let's see by way of example how randomization can help us address the threat of results biased by hidden confounders.

Recall the diet study discussed previously: "Comparison of the Atkins, Zone, Ornish,

and LEARN Diets for Change in Weight and Related Risk Factors Among Overweight Premenopausal Women The A TO Z Weight Loss Study: A Randomized Trial" by C.D. Gardner, et al. (JAMA, Vol. 297, pp. 969-77, March 2007)

> **Objective:**
> To compare four weight-loss diets representing a spectrum of low to high carbohydrate intake for effects on weight loss and related metabolic variables. Design, Setting, and Participants Twelve-month randomized trial conducted in the United States from February 2003 to October 2005 among 311 free-living, overweight/obese (body mass index, 27-40) nondiabetic, premenopausal women. Intervention participants were randomly assigned to follow the Atkins (n=77), Zone (n = 79), LEARN (n = 79), or Ornish (n = 76) diets and received weekly instruction for 2 months, then an additional 10-month follow-up.

Here is an example of a randomized trial or randomized experiment. Subjects entering the test are randomized, using a virtual roll of a die, into one of several treatment groups.

The randomization into groups eliminates the main threat to valid comparisons: confounders. How? By making the comparison groups similar to each other in all aspects, except for the treatment. Think about it. If you ask for volunteers to each treatment group, then most of the meat-eaters would go to the Atkins group, and most of the vegetarians would go to the Ornish group. Would it then surprise you if one group had, say, more Asians than another group? Or more smokers? Or less physical activity? Randomized assignments, instead of volunteering, is the best way to get balanced groups in the factors that we think might matter and achieves balance in all other factors we have not even considered.

Table 5.1 on page 66 presents the characteristics (averages or frequencies) of the subjects after randomization, but before treatment has started. Observe how randomization has achieved a right balance between groups in demographic and anthropometric variables (using either percentages or averages and standard deviations, respectively), and known health risk factors. All other variables, essential or not, like IQ, shoe size, and preference for country music would also tend to be balanced, thus providing a level of protection against unknown confounders.

> To safeguard against potential **confounders**, comparison groups should be **similar** in all factors except for treatment itself. **Randomization** is the best way to achieve the goal.

5.2.1 Double blind randomized controlled trials (RCT)

Scientific studies should be randomized as described above whenever possible. But it might surprise you to learn that randomization is just one way to address hidden confounders and bias in clinical trials. The present section will cover *controls and blinding*, two critical elements in study design that address confounders and bias.

Thus far, we've discussed how vital randomly assigning subjects in a study to groups within that study is to overcoming bias, but study designers should be aware that there is some luck involved in randomization. It is not always likely, but it is possible, that even the best randomization techniques leave unknown confounders present across all *subjects* within the study. We may be selecting subjects for the study from a population where they all have a confounding characteristic in common. In such a case, it would not help us much to randomly assign subjects to different groups of the study since all the groups would end up exhibiting the same bias regardless as to how we assigned the subjects. Imagine attempting to conclude whether a medication is effective in treating the common cold by studying only subjects who have some particular resistance to the cold. No patients would get severe colds, or if they did, they would not manifest many cold symptoms like coughing, sneezing, or a runny nose. Sounds hard to study such subjects, right? Yes, the immune response confounder makes these subjects hard to explore, but this problem is solvable. Scientists need more *control* in assessing such matters and their hidden confounders, so control *groups* are added to scientific studies to correct for this problem.

Control groups are groups of subjects within the study that are not expected to respond in the same way as the treated groups. For instance, control groups in medical studies are often groups of patients given inert placebos or "sugar pills" instead of active medications. Since the study directors do not provide these groups with the active medication, such groups are not expected to exhibit effects as strong as the treatment groups who receive medications. We would not expect patients who receive sugar pills to miraculously feel relief from their cold symptoms in the same way patients who receive a new alternative to pseudoephedrine might. Even when such placebo groups do exhibit effects like the expected treatment effects, which can be due to the placebo effect or other hidden confounders (different resistance to the cold), the presence of these control groups within the study gives researchers a valid point of comparison. It is true that both the placebo and medicated groups selected from our cold-resistant population of patients would be asymptomatic. However, even though there is less sneezing when comparing the medicated and unmedicated groups, the response allows scientists to determine if it is just the patients' robust immune responses or the psychological impact of taking a pill is responsible, or if the medication is exceptionally effective at treating the cold.

When one of the comparison groups in a study is a control group or placebo, randomized trials are sometimes called randomized controlled trials, or randomized controlled experiments. As much as possible, researchers try to conduct *double-blind* randomized controlled trials, when neither the doctor nor the patient knows what treatment the patient is receiving. In the common cold example, some patients get a sugar pill or placebo while others get a new alternative medication thought to treat cold symptoms. (The critical point to remember here is that in a double-blind-controlled experiment both the *patients* and *doctors* have no idea if a patient is receiving the sugar pill/placebo or the cold medicine.) This method gives each patient a pill that looks identical regardless as to whether it is the inert placebo or the active drug.

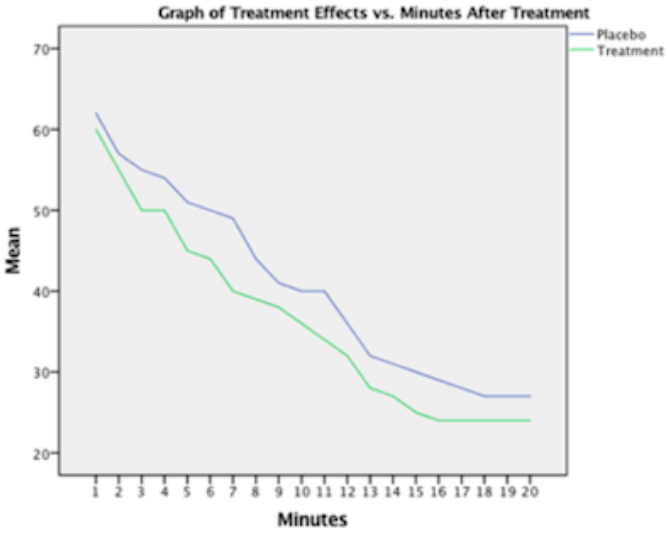


Figure 5.1: **Placebo** or **Medicine**

Can you spot the difference? – Single Blinding. Can your doctor? – Double Blinding.

This double-blinding is essential because it offers additional protection against bias. In such studies, all groups have the same frame of mind (as opposed to knowing you are not getting the new drug). Similarly, the experimenter has the same frame of mind evaluating patients from each group. You might wonder, isn't it enough to single-blind the study, to make sure that just the patients are unaware whether they received the placebo or medicinal pill? No, it is not. Imagine an experimenter who administers both the placebos and medications but does so know which is which. That researcher might unintentionally treat these two groups differently, perhaps just by spending an extra minute with patients from the medicated treatment group or by approaching them with a little more enthusiasm. It's this sort of unwitting behavior that can invite bias into the study.

5.3 Observational studies

In many cases, the treatments being compared cannot be assigned, and hence cannot be randomized. The study involving women and leg fractures is an example. We compared leg fracture rates of two groups, men, and women. Whenever a new subject enters the study,

e.g., by having a car accident, we *observe* what gender they belong to, instead of randomly assigning it to control or treated group. Studies like these are called *observational studies*, as opposed to randomized experiments. In the hierarchy of scientific evidence, observational studies are not as reliable as randomized trials are regarded as the gold standard. Since the subjects of observational studies assign themselves to a different group, there may be a selection bias that leads to confounders (like the women in the leg-fracture-study being shorter). We can control the effects of known confounders in the analysis. For example, we can compare leg fracture rates of men and women with the same heights. Investigators will need to anticipate potential confounders and control for them. We enumerate common reasons for nonrandomized studies:

1. Assigning treatment is impossible (e.g., to compare fracture rates between men and women, we cannot randomize subjects into the comparison groups).

2. Assigning treatment is unethical (e.g., to compare cancer rates of smokers and nonsmokers, we do not want to randomize subjects into smoker-nonsmoker comparison groups.) In other words, we do not want to force nonsmokers to smoke.

3. Assigning treatment is impractical (e.g., the outcome is a rare event like cancer or stroke, and a randomized trial would need too many subjects and too much time). In cases like these, a case-control study is generally the way to go.

Observational studies are conducted when **randomization** to treatment groups is **impossible**, **unethical**, or **impractical**.

Returning to the leg-fracture example, we stick to observational studies in such a case for two reasons. As described above, it's impossible to perform the research any other way with the data collected. But we also *should not* elect to collect the data in a randomized manner. In other words, we should not even try to get the bias-reducing benefits of randomization because of ethical constraints. To perform randomized clinical trials in this study of leg fractures, we would need to randomly select subjects whose legs *would* be broken for the sake of the study. Causing suffering like this is prohibited by several codes of ethics like the Nuremberg Code, a system of research ethics drafted after the Nuremberg trials of Nazi war criminals. These criminals included scientists who designed and performed inhumane experiments. Beyond the official ban, though, such research is wrong in its causing undue suffering and its infringement upon fundamental human rights.

Another reason we may conduct an observational study is that they can be more practical compared to randomized controlled trials. It can be due to the constraints of a rare disease, or the cost of a randomized study would entail. A researcher might want to study whether regular exercise can prevent a rare form of cancer. Randomization and control here would require that we randomly select patients first, group them along exercise programs (say regular, irregular, and no exercise), and wait to see how the different groups respond (do the exercise groups have significantly fewer cases of cancer?). This study might not work out at

all because, since the cancer is rare, it's possible that no or very few of our subjects will get cancer. It also may take many years before cancer would appear.

This study would be quite time-consuming and labor-intensive. Imagine all the financial resources that would be required to do all that exercise-monitoring over all the patients and years! This experiment brings us to our second point that observational studies are practical because they tend to be less costly than randomized controlled trials. Companies often use statistics to generate revenue, but they tend to do this not by randomized experiments, but by analyses of marketing data they collect through sales or surveys. Nonrandom observational studies like analyses of sales data are a relaxed approach in that all they require are a data collection apparatus and a statistician. For instance, clothing designers want to keep up with trends in consumer spending but might sell hundreds of different garments to thousands of outlets across the world. When sales of one item begin to slip, it might be difficult to notice amidst all the chaos of such a business enterprise. A statistician can help here by analyzing the proportions of revenue generated by each garment from month to month and making recommendations about which products to push when. The statistician might even find a style or color-based patterns that repeat themselves seasonally, allowing the company to adjust before their sales ever begin slipping.

5.3.1 Case-control studies

Instead of randomizing subjects into diet groups and then comparing the weight loss outcome, a case-control study would look for people in the population who lost weight, and then ask them what diet they used. Thus, you start with the *outcome* and then work back to the type of treatment. These are also classified as *retrospective* studies because they look back and compare weight loss or disease rates of various procedures. Randomized experiments are necessarily *prospective*, in the sense that you randomize treatment and then later see which groups lost more weight or had more disease.

We frequently use case-control studies because they are cheaper and easier to conduct since it generally requires a survey of a database, instead of expensive and time-consuming recruitment and handling of subjects. There are plenty of successful case-control success stories in the scientific literature.

The first study formally linking lung cancer to smoking was a 1950 case-control study "Smoking and Carcinoma of the Lung" by Richard Doll and A. Bradford Hill(British Medical Journal, 1950 September 30; 2(4682): page 739–748). Using patients in 20 hospitals in London, they found that lung-cancer patients had higher rates of smokers than a comparable control group of patients. For example, among district hospitals, 48 out of 98 lung cancer patients smoked 15 or more cigarettes daily. In contrast, only 30 out of 98 non-cancer patients smoked 15 or more cigarettes daily.

In general, case-control studies can conclude a link or 'association' but are not able to prove 'causation.' However, case-control studies provide initial evidence that can generate resources for more rigorous studies like double-blind, randomized controlled trials. In the

case of smoking and lung cancer, randomized trials are unethical, but given the eventual results of multiple studies, the scientific community now accepts that smoking causes lung cancer. [Doll, 1950]

5.3.2 Case-crossover studies

Sometimes, a subject can be its control. This method is called a case-crossover study because of subjects in the treatment group "crossover" to the control group. An example is a 1997 study linking cell phone use to car accidents: "Association between cellular-telephone calls and motor vehicle collisions" by D.A. Redelmeier and R.J. Tibshirani (The New England Journal of Medicine, 1997 Feb 13; Vol 336, pp. 453-8). The subjects were people who reported a collision to the North York Collision Reporting Centre between July 1, 1994, and August 31, 1995, Among these, 742 had cell phones and consented to participate in the study. Instead of asking each person whether they were using their cell phone during the time of the collision (an unreliable method), the investigators examined their detailed phone billing records. The results of the experiment follow below:

> Overall, 170 subjects (24 percent) had used a cellular telephone during the 10-minute period immediately before the collision, 37 (5 percent) had used the telephone during the same period on the day before the collision, and 13 (2 percent) had used the telephone during both periods. The crude analysis indicated that cellular-telephone activity was associated with a relative risk of a motor vehicle collision of 6.5 (95 percent confidence interval, 4.5 to 9.9). The primary analysis, adjusted for intermittent driving, indicated that cellular-telephone activity was associated with a quadrupling of the risk of a motor vehicle collision (relative risk, 4.3; 95 percent confidence interval, 3.0 to 6.5).

[Redelmeier, 1997]

5.4 Key Words

- randomized trial
- double blind randomized trial
- observational studies
- case-control studies
- case-crossover studies

Table 5.1: Baseline Participant Characteristics

	Atkins	Zone	LEARN	Ornish
Number of Subjects	77	79	79	76
Demographics, No.(%)				
Race/ethnicity				
Asian/Pacific Islander	7(9)	9(11)	6(8)	8(10)
Black	2(3)	7(9)	6(7)	4(5)
Hispanic	7(9)	8(10)	7(9)	11(14)
White	59(76)	52(66)	59(75)	52(69)
Other	2(3)	3(4)	1(1)	1(1)
Smokers	2(3)	4(5)	4(5)	3(4)
Age, yrs; $\bar{x}(SD)$	42(5)	40(6)	40(7)	42(6)
Education, yrs; $\bar{x}(SD)$	16(2)	16(2)	16(2)	16(2)
Physical activity (kcal/kg per day); $\bar{x}(SD)$	34(6)	34(6)	34(5)	35(7)
Anthropometrics $\bar{x}(SD)$				
Weight, kg	86(13)	84(12)	85(14)	86(10)
Body fat, %	41(6)	40(6)	38(6)	40(6)
Body mass index	32(4)	31(3)	31(4)	32(3)
Waist-hip ratio	.843(.067)	.841(.068)	.839(.066)	.840(.060)
Cardio. disease risk factors				
LDL-C, mg/dL	109(29)	114(32)	104(29)	111(27)
HDL-C, mg/dL	53(14)	52(11)	51(11)	50(11)
Triglycerides, mg/dL	125(78)	123(98)	119(73)	118(62)
Non-HDL-C, mg/dL	134(33)	139(39)	127(34)	135(33)
Ratio of total cholestrol to HDL-C	3.7(1.0)	3.8(1.1)	3.6(1.0)	3.8(1.0)
Fasting insulin, U/mL	10(7)	10(7)	10(8)	10(5)
Fasting glucose, mg/dL	92(9)	94(20)	96(17)	93(13)
Blood pressure, mmHg; $\bar{x}(SD)$				
Systolic	118(11)	115(13)	116(12)	116(10)
Diastolic	75(8)	74(9)	75(9)	75(8)
Metabolic syndrome, No. (%)	22(29)	20(25)	29(37)	27(36)

5.5

Exercises

Exercise 5.5-1: Give some examples of studies where:

1. a randomized trial is impossible
2. a randomized trial is impractical because of length of time the study would require
3. a randomized trial is impractical because of the number of subjects the study would require

Exercise 5.5-2: Suppose a doctor is interested in investigating the causes of a sporadic disorder that occurs in only 0.00001% (that is 1 in 10 million) of men. What type of study would be necessary to establish the origins of the disease? Is this type of research an appropriate choice for such a disease firmly? Why or why not?

Exercise 5.5-3: What is a double-blind study? Give an example where double-blinding is not possible.

Exercise 5.5-4: Some recent research has shown that patients who know they have taken nothing more than an inert sugar pill still experience the placebo effect. Briefly describe a study design that might replicate these results. What type of study would you choose? Should we consider different groups?

Exercise 5.5-5: What is the advantage of case-control studies over randomized controlled trials?

Exercise 5.5-6: A football organization is concerned about the number of injuries to its athletes during games. The organization designs an observational study to help decide on rule changes that will reduce the risk of injury to players. They collect data and observe that a very high proportion of injuries incurred during the first moments of the game during kickoff. The organization thus entertains the idea of changing the rules governing kickoff. Are there any hidden confounders that the organization should address before making changes to kickoff rules? What are they, and how might they be treated?

Exercise 5.5-7: What is the advantage of randomized controlled trials over case-control studies?

Exercise 5.5-8: Suppose the Kalamazoo School District is proposing a 1 mil property tax to provide a new high school to be built in the district. What is your opinion on the tax proposed tax? (Strongly in favor, in favor, neutral, against, strongly against). What are possible confounders?

Chapter 6

The Normal Distribution

6.1 Objective

After completing this part, students should be able to:

- Define and explain the concept of the **Normal Curve**.
- Convert empirical scores to z-scores and use z-scores and the Normal Distribution curve table to find areas above, below and between points of the curve and express them regarding their probabilities.
- Utilize the standard normal distribution to solve probability (chance) problems.

6.2 Using the normal curve

Best Buy wants to know how many smart fitness and GPS watches (like Fitbit, Garmin, and Apple) to order for their Kalamazoo location. Past data show that this time of year, they sell an average of 36 fitness watches per month, with a standard deviation of 8.

Problem 6.2.1

If they order 40 watches, what is the probability that they will run out of stock?

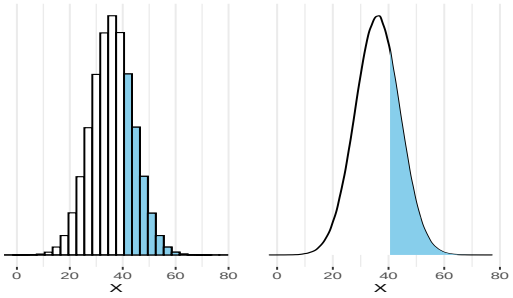
Problem 6.2.2

Given the cost of running out of stock and the storage cost of keeping too many, the manager decides to order enough smartwatches to cover customer demands 90% of the time, i.e.,

there should be 10% or less likelihood that they run out of stock. How many should they order?

Knowing the average and SD of a process (36 ± 8) gives us some understanding of what to expect. However, the sales situation above requires the computation of probabilities, or chances, as in the "chance of demand exceeding 40." This desired probability is the shaded area to the right of 40 under the histogram of demand (see the first graph in Figure 6.1). Does this look like 0.20 of the total area? 0.30? 0.40?

Figure 6.1: Probability of Demand Exceeding 40



The normal curve (or bell curve) is useful in helping us calculate probabilities like these. If we smooth out the histogram in Figure 6.1, it will look like a normal curve. More precisely, it will look like a normal curve with a mean of 36 and an SD of 8, denoted as $N(36, 8)$. So the shaded area on the left will be approximated by the shaded area under the $N(36, 8)$ curve on the right. Do the two shaded regions look the same?

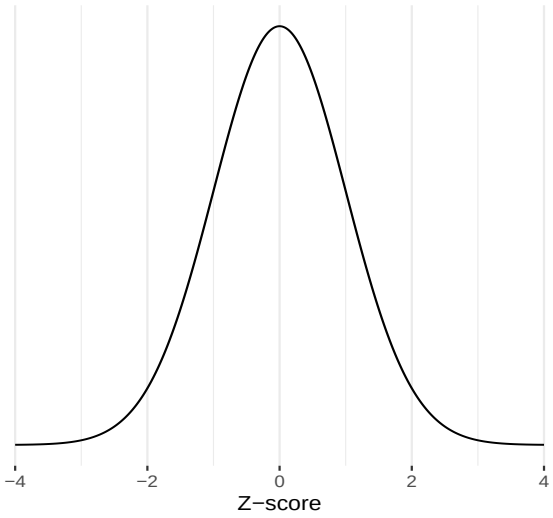
Of course, the areas look similar because, in our example, the histogram on the left looks like a normal curve. It does not have to. If it does not, then the approximation of areas using the normal curve will be wrong. We should use the normal curve approximation with caution.

The Standard Normal or Z Curve

The standard normal curve (or Z-curve) looks like Figure 6.2, and has the following properties:

1. center is at zero
2. The area under the curve satisfies the following:
   The area between -1 and +1 is 0.68
   The area between -2 and +2 is 0.95
   The area between -3 and +3 is 0.997
   The area between $-\infty$ and $+\infty$ is 1.00

Figure 6.2: Normal Z-curve



In general, area under the curve for *positive Z values* can be found using the Z-table on 6.6 on page 82 at the end of this chapter.

For practice

Using the Z-table, find …

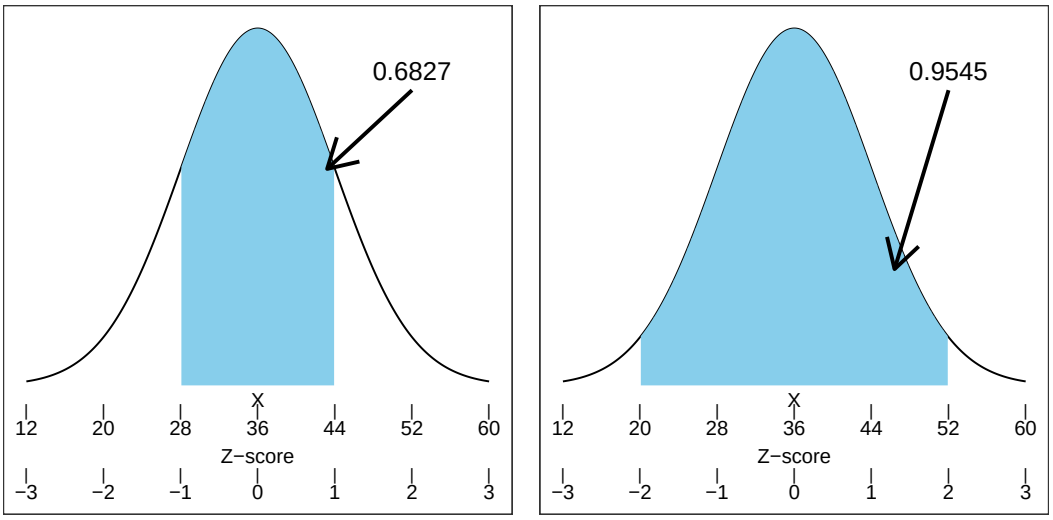
1. the area to the left of 1.2
2. the area to the left of 1.25
3. the area to the right of 1.25
4. the area to the left of -1.25
5. the area between 1.25 and 2.50

Are we ready to find the area under the normal curve in Figure 6.1? Not yet. The horizontal axis in Figure 6.1 is wrong for the Z-table, it does not have a mean of 0 (horizontal axis) and an SD of 1. Instead, it has a mean of 36 and an SD of 8. The trick is to replace the $N(36,8)$ horizontal axis with a $N(0,1)$ axis, labeled Z. See Figure 6.3. Note that $Z = 1$ whenever X is one SD above the mean. Similarly, $Z = -1$ whenever X is one SD below the mean. In general, Z is related to X as follows:

$$Z = \text{number of SD away from the mean} = \frac{X - mean}{SD} \tag{6.1}$$

What is the probability of demand exceeding 40? This is the area under the curve to the right of $X = 40$ or $Z = 0.5$ (see Figure 6.4). Using the Z-table, this area is $1 - 0.6915 = 0.3085$.

Figure 6.3: Areas within one and two SD's of the Mean *N(36, 8)*



6.3 Calculating percentiles

We restate Problem 6.2.2 on page 69 given at the start of the chapter.

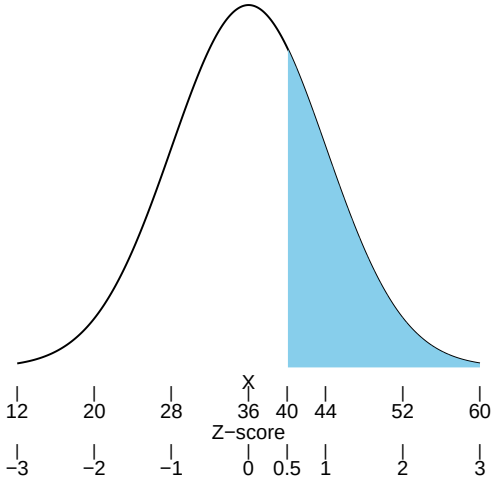
Problem 6.2.2 Given the cost of running out of stock and the value of having too many, the manager decides to order enough smartwatches to cover customer demands 90% of the time, i.e., there should be 10% or less chance that they run out of stock. How many should they order?

We want to estimate the number of smartwatches so that there is only a 10% chance of running out. Note that this is the 90th percentile of demand.

Looking at Figure 6.5, we see that we need to shade the upper 10% of the area under the curve . What X value corresponds to the boundary? When we use the normal table, the Z-value 90th percentile is 1.28. What X value corresponds to Z = 1.28? We solve equation 6.1 backward.

$$\begin{aligned} 1.28 &= \frac{X-36}{8} \\ 36+1.28(8) &= X-36+36 \\ 36+1.28(8) &= X \\ X &= 46.24 \end{aligned}$$

Figure 6.4: $P[X \geq 40]$



The manager should have at least 47 smartwatches in the store.

6.4 Calculating symmetric tail areas

In later sections, we will need to calculate tail areas (or P-values). For example, how frequently does a normal variable fall outside of 2.25 SD from the center? We know a random variable following a normal distribution falls within 1 SD from the center 68% of the time, and hence outside of 1 SD from the center only 32% of the time. What about 2.25 SD?

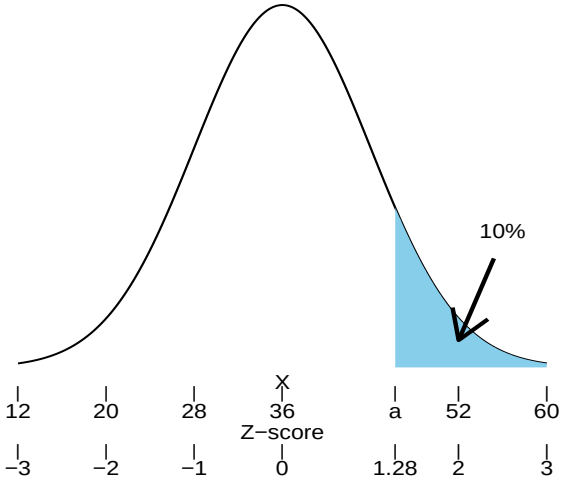
The area outside of 2.25 SD from the center is the combined area left of -2.25 and right of 2.25. From the Z-table, the area to the right of 2.25 is 1 - 0.9878 = 0.0122. Therefore, the combined tail areas is 0.0122 + 0.0122 = 0.0244. A random variable following a normal distribution falls outside of 2.25 SD from the center only 2% of the time.

For practice

What percentage of time does a normal variable fall outside:

1. 0.4 of the mean
2. 1.4 SD of the mean
3. 2.4 SD of the mean
4. 3.4 SD of the mean

Figure 6.5: The 90th Percentile: $P[X \geq a] = 0.10$



6.5 The Empirical Rule

Figure 6.3 provides a useful interpretation of the SD. We state it as follows:

Empirical Rule:
If the data histogram is approximately bell-shaped, expect around
68% of the observations will fall within **one** SD of the mean
95% of the observations will fall within **two** SD of the mean
99.7% of the observations will fall within **three** SD of the mean

6.6 Key Words

- area under the curve
- empirical rule
- normal curve
- normal Z-curve

	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.500	0.504	0.508	0.512	0.516	0.520	0.524	0.528	0.532	0.536
0.1	0.540	0.544	0.548	0.552	0.556	0.560	0.564	0.567	0.571	0.575
0.2	0.579	0.583	0.587	0.591	0.595	0.599	0.603	0.606	0.610	0.614
0.3	0.618	0.622	0.626	0.629	0.633	0.637	0.641	0.644	0.648	0.652
0.4	0.655	0.659	0.663	0.666	0.670	0.674	0.677	0.681	0.684	0.688
0.5	0.691	0.695	0.698	0.702	0.705	0.709	0.712	0.716	0.719	0.722
0.6	0.726	0.729	0.732	0.736	0.739	0.742	0.745	0.749	0.752	0.755
0.7	0.758	0.761	0.764	0.767	0.770	0.773	0.776	0.779	0.782	0.785
0.8	0.788	0.791	0.794	0.797	0.800	0.802	0.805	0.808	0.811	0.813
0.9	0.816	0.819	0.821	0.824	0.826	0.829	0.831	0.834	0.836	0.839
1	0.841	0.844	0.846	0.848	0.851	0.853	0.855	0.858	0.860	0.862
1.1	0.864	0.867	0.869	0.871	0.873	0.875	0.877	0.879	0.881	0.883
1.2	0.885	0.887	0.889	0.891	0.893	0.894	0.896	0.898	0.900	0.901
1.3	0.903	0.905	0.907	0.908	0.910	0.911	0.913	0.915	0.916	0.918
1.4	0.919	0.921	0.922	0.924	0.925	0.926	0.928	0.929	0.931	0.932
1.5	0.933	0.934	0.936	0.937	0.938	0.939	0.941	0.942	0.943	0.944
1.6	0.945	0.946	0.947	0.948	0.949	0.951	0.952	0.953	0.954	0.954
1.7	0.955	0.956	0.957	0.958	0.959	0.960	0.961	0.962	0.962	0.963
1.8	0.964	0.965	0.966	0.966	0.967	0.968	0.969	0.969	0.970	0.971
1.9	0.971	0.972	0.973	0.973	0.974	0.974	0.975	0.976	0.976	0.977
2	0.977	0.978	0.978	0.979	0.979	0.980	0.980	0.981	0.981	0.982
2.1	0.982	0.983	0.983	0.983	0.984	0.984	0.985	0.985	0.985	0.986
2.2	0.986	0.986	0.987	0.987	0.987	0.988	0.988	0.988	0.989	0.989
2.3	0.989	0.990	0.990	0.990	0.990	0.991	0.991	0.991	0.991	0.992
2.4	0.992	0.992	0.992	0.992	0.993	0.993	0.993	0.993	0.993	0.994

Table 6.1: Z Table (Positive Values)

6.7

Exercises

Exercise 6.7-1: Researchers conducted a study to determine if any link existed between cellular phone usage and the development of brain cancer (don't worry, no connection was found). Data from this study indicate that the daily cell phone usage for all users is approximately normally distributed with mean 2.4 hours and standard deviation of 1.1 hours.

1. What proportion of cell phone users are on their phones between 1 hour and 3 hours per day?
2. To be safe, suppose you decide to be in the 5th percentile of cell phone users concerning monthly usage. How much time can you spend on your phone per day?

Exercise 6.7-2: On the average, a watch battery is known to last for two years (24-months), with a standard deviation of 9 months. Assume a normally distributed population.

1. What percentage of watch batteries last more than six months?
2. What is the lifespan of a watch battery which lasts longer than 60% of all batteries?
3. What proportion of watch batteries last shorter than two years or longer than 3 1/2 years (42 months)?

Exercise 6.7-3: Suppose 95% of data coming from a normally distributed population falls between 4 and 35. Based on the empirical rule, what is the standard deviation of this sample of data? Show your work.

Exercise 6.7-4: Let's assume a normally distributed random variable with a mean of 10 and standard deviation of 2.

1. What is the probability the value is greater than 6?
2. What is the probability the value is less than 12?
3. What is the probability the value is between 6 and 12?
4. 33% is above what value?
5. 33% is below what value?

Exercise 6.7-5: The stock price for Coca-Cola (KO) is normally distributed with a mean of 42.14 and standard deviation of 1.43.

1. What is the probability that the stock price is between 39.88 and 46.01?
2. What is the probability that the stock price is above 40?
3. What is the probability that the stock price is below 40?

Exercise 6.7-6: The average adult female height is 63.8 inches with a standard deviation of 2.40. Assume the distribution is approximately normal.

1. What proportion of adult female heights is below 72?

2. 25% of adult females are greater than what height?

Exercise 6.7-7: Values in the z-table give the area under the curve to the left of the z-values on the margins. The upper margin gives the hundredth digit of z. For example:

1. The area to the left of 0.0 is?
2. The area to the left of 0.2 is?
3. The area to the left of 0.25 is?
4. The area to the left of 2.25 is?

Exercise 6.7-8: Test scores for 120 students were found to have a mean of 32. Suppose the upper bound of the middle 99.7% is 41. What is the standard deviation of this data?

1. 2
2. 2.5
3. 3
4. 4

Answer:

Exercise 6.7-9: Given a distribution of male weights has a mean of 153 lbs and standard deviation of 11 pounds, how many standard deviations away from the average weight is 173 lbs?

1. −1.82
2. 1.82
3. 1.82 lbs.
4. You cannot calculate the z-score with the information provided.

Answer:

Exercise 6.7-10: Given the distribution of
male weights has a mean of 155 and standard deviation of 12, what percent of men weight less than 143 pounds?

1. 0.1587
2. 0.3413
3. 15.87
4. 34.13

Answer:

Exercise 6.7-11: Given the distribution of male poundage with a mean of 155 and standard deviation of 12, what value represents the 90^{th} percentile for weights of men (in
pounds)?

1. 155.00
2. 170.36
3. 179.00
4. 189.13

Answer:

Exercise 6.7-12: Given the distribution of male poundage with a mean of 155 and standard deviation of 12, what value represents the 25^{th} percentile for weights of men (in
pounds)?

1. 155.00
2. 163.10
3. 146.91
4. 131.00

Answer:

Exercise 6.7-13: The mean and standard deviation of 2013 Expenditures on Health as a share of GDP are 8.30 and 3.224, respectively.
What percentage of countries had values below 7.3? Hint: find the probability of the mean being less than 7.3.

Exercise 6.7-14: The mean and standard deviation of 2013 Expenditures on Health as a share of GDP are 8.30 and 3.224, respectively.
What percentage of countries had values above 9.3? Hint: find the probability of the mean being greater than 9.3.

Exercise 6.7-15: The mean and standard deviation of 2013 Expenditures on Health as a share of GDP are 8.30 and 3.224, respectively.
What percentage of countries had values between 7.3 and 9.3? Hint: find the probability of the mean being between 7.3 and 9.3.

Exercise 6.7-16: Fast-food restaurants spend a lot of time researching the time spent servicing their customers who use the drive-through window. If they can process their orders faster, they have an opportunity to increase their profit. *QSR Magazine* studied the drive-through time for fast-food restaurants and found the Wendy's had the best time, with a mean time time of 138.5 seconds and standard deviation of 29 seconds. Let's assume that the drive-through time are normally distributed.

1. What is the probability that a random sample of cars go through the drive-through in less than 110 seconds?
2. What is the probability that a random sample of cars go through the drive-through in more than 150 seconds?
3. What is the probability that a random sample of cars go through the drive-through between 2 and 3 minutes?
4. Would it be unusual for a car to spend more than 3 minutes in the Wendy's drive-through? Why?

Chapter 7

The Binomial Distribution

7.1 Objectives

After completing this part, students should be able to:

- Use the binomial distribution to compute probabilities.
- Make use of the expected value and SD of a Binomial Random Variable.
- Apply Binomial Probabilities Using the Normal Curve.
- Understand that some Approximations Are Better Than Others.

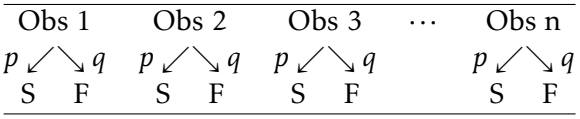
7.2 Binomial Probabilities

A sequence of n observations is called a binomial process if

1. each observation results in one of two possible outcomes (which we call success and failure)

2. the probability of success is p, and the probability of failure is q = 1 - p for all observations

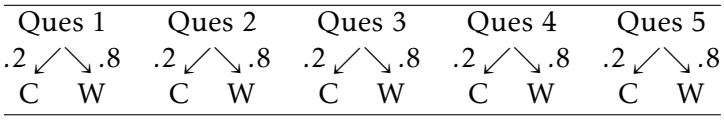
3. the observations are independent of each other.



Let X denote the total number of successes among the n observations. Then X is called a **binomial random variable** with parameters n and p. The following are all binomial random variables.

Example 1

A Stat 1600 multiple choice quiz has five questions, with each question having five choices. Let X be the number of correct answers (C) by someone who is guessing on all questions. Then X is a binomial random variable with parameter values n = 5 and p = 0.2.



Example 2

Available data shows that 40% of telephone respondents agree to be interviewed for market research surveys. Suppose the polling organization Reliable Research randomly selects and dials telephone numbers until it reaches 50 respondents. Let X be the number of respondents (out of the 50) who agree to be interviewed. Then X is a binomial random variable with parameter values n = 50 and p = 0.40.

Example 3

Historically, 20% of buyers at Best Buy who purchase smart fitness and GPS watches (like Fitbit, Garmin, and Apple) also purchase the Geek Squad's protection plan. Suppose Best Buy sold 300 smart fitness watches during the previous quarter. Let X be the number of extended protection plans that the retailer sold along with the 300 smartwatches. Then X is a Binomial random variable with parameter values n = 300 and p = 0.20.

7.3 Computing Binomial Probabilities

In Example 1, the number of correct guesses may be 0, 1, 2, 3, 4, or 5. How likely can a guesser get all five questions right? The answer is 0.0003, or about three times in 10000 attempts. How about the likelihood of getting 2 out of 5 items right? The answer is .2048, about a fifth of the time. The following **probability distribution table** gives the likelihood or probability of each possible value of X.

$P[X=0]$	$P[X=1]$	$P[X=2]$	$P[X=3]$	$P[X=4]$	$P[X=5]$
0.32768	0.4096	0.2048	0.0512	0.0064	3.2×10^{-4}

You can compute these probabilities yourself by successively substituting $j = 0, 1, 2, 3, 4,$ and 5 in the formula

$$P[X = j] = \frac{5!}{j!(5-j)!} 0.2^j 0.8^{5-j}, \text{ where } j = 0,1,2,3,4,5$$

Remember that $0! = 1$ and $(0.2)^0 = 1$. This formula is called the **binomial probability distribution function (pdf)** for $n = 5$ and p= 0.2. To compute the probabilities for Examples 2 and 3, you will need the binomial pdf for general n and p:

$$P[X = j] = \frac{n!}{j!(n-j)!} p^j q^{n-j}, \text{ where } j = 0,1,2, \cdots, n$$

Example 3

(Cont.): The retailer sold ten smart fitness and GPS watches in one day. What is the probability that customers purchased three extended Geek Squad protection plans? Using the equation above with $n = 10, p = 0.20$, and $j = 3$, we get

$$P[X = 3] = \frac{10!}{3!(10-3)!} .2^3 .8^{10-3} = 0.201$$

Five percent of customers who rent video games from Gamers Retro Rental who return the rental late. What is the probability that customers will return their rental late if 30 customers borrow a video game during the last hour?

For practice

1. 2 will be returned late
2. none will be returned late
3. 2 or fewer will be returned late
4. 5 or more will be returned late

7.4 Expected Value and SD of a Binomial Random Variable

Suppose last quarter; Best Buy sold 300 Smart fitness watches. If there is 0.20 likelihood of selling an extended protection plan with each smartwatch, the number of extended protection plans sold last quarter should be around 60 give or take 7 or so (we will compute this later).

We call the first number the expected value of the number of protection plans sold; the second number is the standard deviation. Recall that X, the number of protection plans sold, is a **Binomial random variable**. The **expected value**, denoted **E(X)**, of a binomial random variable X with parameters n and p is computed as:

$$E[X] = np$$

The expected value $E(X)$ is also called the average or mean of X, and denoted μ. The standard deviation, denoted $SD(X)$ or σ_x, of a binomial random variable is computed as:

$$SD(X) = \sigma_x = \sqrt{npq}$$

Returning to the example, since $n = 300$ and $p = 0.20$, we have $E(X) = 300(0.20) = 60$, and $SD(X) = \sqrt{300(0.20)(0.80)} = 6.93$, or approximately 60 ± 7.

The SD for random variables is interpreted similarly to the SD for a sample. If the store sells 300 smartwatches sets every quarter, they won't sell precisely 60 Geek Squad protection plans every time; sometimes they will sell more, sometimes they will sell less. By how much more, and how much less? The answer is, "By 7, on average." Similarly, a baseball player with 0.200 batting average won't necessarily get 60 hits in 300 at-bats. We expect him to get 60 hits, give or take seven hits.

For practice Suppose that 5% of video games rented at Gamers Retro Rental incur a late rental fee. If 700 videos were rented last week, the number that will incur a late rental fee should be around _____ give or take _____.

7.5 Computing Binomial Probabilities Using the Normal Curve

Beyond the empirical rule, we may apply the normal curve to approximating binomial probabilities. The key image is a plot of binomial probabilities as a histogram. For example, Figure 7.1 is a histogram of binomial probabilities for $n = 30$ and $p = 0.4$.

The height of the rectangle over, say 10, is its probability $P[X = 10]$. However, since the width of the rectangle is 1, then

$$P[X = 10] = \texttt{height of rectangle over 10} = \texttt{area of rectangle over 10}$$

What about $P[X \leq 10]$? This probability corresponds to the total area of the rectangles over and the left of 10 (shaded rectangles in Figure 7.2)

The total area equals 0.2915 using the binomial probability function to compute the area (probability) of each shaded rectangle. However, this requires repeated applications of the binomial formula (11 times, in fact). We may calculate a quick approximation of the desired probability by replacing the rectangles with a curve! See shaded area under the curve in Figure 7.2.

There are infinitely many normal curves, which one do we use to replace the rectangles? Answer: the one with the same mean and SD as the rectangles! The (binomial) rectangles have a **mean**:

$$\begin{aligned} \mu &= np \\ &= (30)(.4) \\ &= 12 \end{aligned}$$

The (binomial) rectangles have a SD:

$$\begin{aligned} \sigma &= \sqrt{npq} \\ &= \sqrt{(30)(.4)(.6)} \\ &= 2.68 \end{aligned}$$

Using the normal curve with the same mean and SD, the area to the left of 10.5 is 0.2877, which is a close estimate of the actual area under the curve is 0.2915.

Similarly, $P[X = 14] = 0.1101$ is approximated by 0.1115, the area under the normal curve between 13.5 and 14.5.

7.6 Some Approximations Are Better Than Others

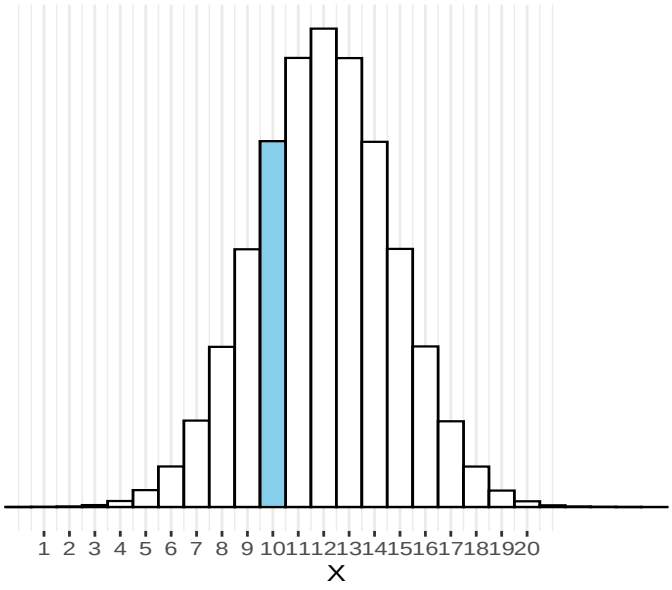
Examine the shape of the binomial histogram for n = 20 and p = 0.10 in Figure 7.3.

Since the curve is right-skewed, we will get a poor approximation of areas if we replace the rectangles by a normal curve. If the value of p were 0.90 instead of 0.10, the binomial histogram would be left-skewed. It is typical behavior of binomial histograms whenever p is either too close to 0 or too close to 1. When is it 'safe' to use the normal curve to approximate binomial probabilities? A convenient rule of thumb is as follows:

The **Normal Curve gives reasonable approximations** of binomial probabilites whenever both $np > 5$ and $nq > 5$.

We remind the reader that the normal curve approximations are still approximations. The binomial formula should be used to calculate exact probabilities whenever possible.

Figure 7.1: Histogram of Probabilities for Binomial n = 30 and p = 0.4



7.7 Key Words

- binomial distribution
- binomial experiment
- binomial random variable
- normalize binomial distribution

Figure 7.2: Approximating $P[X \leq 10]$ using Curve instead of Rectangles

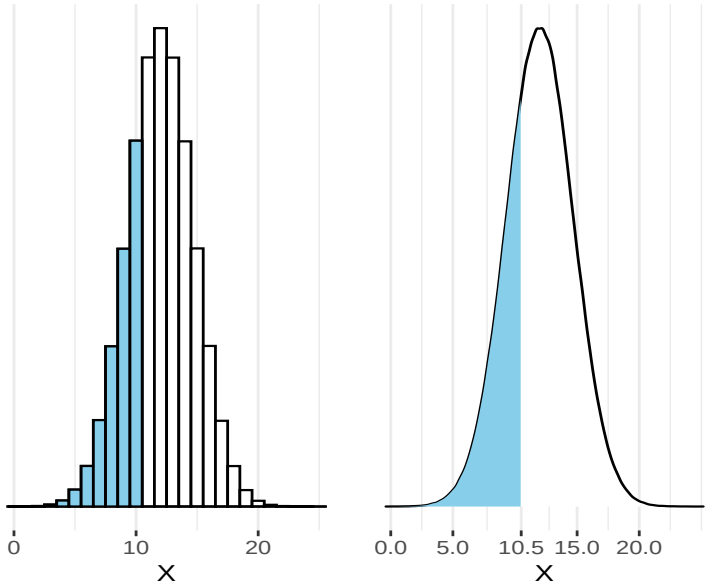
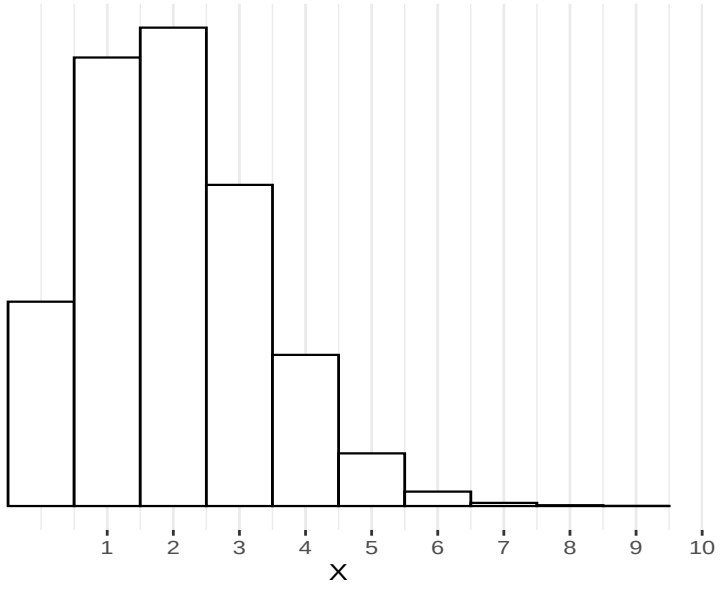


Figure 7.3: Histogram of Probabilities for Binomial n = 20 and p = 0.10



7.8

Exercises

Exercise 7.8-1: A binomial random variable with a probability of success of 0.5 and ten observations. Assume each observation is independent.

1. What is the mean and standard deviation?
2. What is the probability that there are more than five successes?
3. What is the probability that there are fewer than five successes?
4. What is the probability that there are between 1 and 3, inclusive?

Exercise 7.8-2: A statistics exam contains ten questions with five multiple choice options per question. By guessing on all questions,

1. What is the probability that at least two questions correct?
2. What is the probability that at most two questions correct?
3. What is the probability that between 1 and 3?

Exercise 7.8-3: Apple's smartphone market share is 0.146. If Apple conducts a nationwide survey of 651 smartphone users,

1. What is the probability that at least 100 of the people are Apple users?
2. What is the probability that at most 100 of the people are Apple users?
3. What is the probability that between 80 and 120?

Exercise 7.8-4: When rolling a die ten times,

1. What is the probability of rolling a six no more than three times?
2. What is the probability of rolling a six no less than three times?

Exercise 7.8-5: The career batting average of Ty Cobb is 0.3664. If Ty Cobb had eight at-bats during a doubleheader,

1. What is the probability that he gets at least seven hits?
2. What is the probability that at most one hits?
3. What is the probability that between 4 and 6?

Exercise 7.8-6: Fill in the blanks. The probability of picking the Powerball number is 0.0254. If 50 tickets are purchased, around _____ of tickets will be winners, give or take _____. Assume each pick is independent.

Exercise 7.8-7: Fill in the blanks. The probability of a defective light bulb is 0.04. If purchase order of 200 bulbs is submitted, the number of defective light bulbs in the shipment is around _____, give or take _____. Assume each light bulb is independent.

Exercise 7.8-8: An industry official claims that 60 percent of all satellite dish owners subscribe to at least two premium movie channels. In an attempt to clarify this claim, the official will poll a random sample of dish owners. Suppose the official's claim is true, and that she selected a random sample of 50 dish owners. Assuming independence,

1. what is the probability that 33 or more dish owners in the sample subscribe to at least two premium movie channels?

2. what is the probability that 25 or fewer dish owners in the sample subscribe to at least two premium movie channels?

Exercise 7.8-9: According to the American Lung Assocation (ALA) 90% of adult smokers started their habit before they were 21 years old. Suppose researchers randomly selected 10 smokers 21 years or older and recorded their answer about when they started to smoke tobacco.

1. What is the probability that fewer than 8 of them started smoking before 21 years of age?

2. What is the probability that at least 8 of them started smoking before 21 years of age?

Chapter 8

Sampling Distribution of the Proportion

8.1 Objective

After completing this part, students should be able to:

- Describe and use the sampling distribution of the proportion.
- Make and interpret confidence interval of proportions.

8.2 The Sample Proportion

Suppose a student guesses at the answer to every question in a 300-question exam. If he gets 60 questions correct, then his proportion of correct guesses is 60/300 = 0.20. If he gets 75 questions correct, then his proportion of correct guesses in 75/300 = 0.25. The proportion of correct guesses is simply the number of correct guesses divided by the total number of questions.

Similarly, if Best Buy's Geek Squad replacement plan sells 60 extended warranties with 300 smartwatches sold, then its protection plan sales rate is 60/300 = 0.20. If it sold 75 protection plans, this is a sales rate of 75/300 = 0.25. The warranty or protection plan sales rate is merely the number of warranties sold divided by the total number of smartwatches sold.

Now, let X denote the number of successes out of a sample of n observations. If each response is a success with probability p independently of the other observations, then X is a binomial random variable with parameters n and p. Furthermore, the proportion of successes in the sample is also a random variable and is computed as

$$\hat{p} = \frac{X}{n} = \frac{\textit{Number of successes}}{\textit{Number observations in the sample}}$$

Since X is expected to be around np give or take $\sqrt{npq}$, then X/n is expected to be around np/n give or take $\sqrt{npq}/n$, or p give or take $\sqrt{pq/n}$. Make sure that you agree with the last statement before moving on. It may help to think of this analogy: Suppose annual rainfall in Kalamazoo is expected to be around 24 inches give or take 6 inches. How do we change the measurement from inches to feet? We divide both numbers by 12! In feet, annual rainfall in Kalamazoo is expected to be around 24/12 give or take 6/12, or 2 feet give or take 0.5 feet. Now read the first sentence of this paragraph one more time.

Going back to the Best Buy example, the number of protection plans sold is expected to be around 60 ± 7. Thus, the proportion of plans sold is supposed to be about $60/300 \pm 7/300$, or 0.20 ± 0.02.

We summarize the formulas for the mean and SD of X and $\hat{p}$ in the following table.

Random Variable	Mean	SD
X	np	$\sqrt{npq}$
$\hat{p}$	p	$\sqrt{\frac{pq}{n}}$

Excercise 1

If the local Best Buy store sold 1200 smartwatches last year,

1. the proportion of sets sold with extended protection plans should be around 0.20, give or take _____.
2. the percentage of watches sold with extended protection plans should be around 20%, give or take _____.

Data analysis sometimes involves percentages instead of proportions. Proportions and percentages are two ways of saying the same thing (e.g., we refer to 1/5 as either 0.20 or 20%). How do we convert a proportion to a percentage? We multiply by 100%. To avoid repetition, we present all statistical formulas in proportions. As Exercise 1 shows, we will convert all the answers to percentages in the end.

Historically, 5% of customers return their video game rentals from Gamers Retro Rental late.

Excercise 2

1. Gamers Retro Rental rented out 100 video games yesterday. The percentage that will be returned late should be around 5%, give or take _____.

2. Gamers Retro Rental rented out 700 video games last week. The percentage that will be returned late should be around 5%, give or take _____.

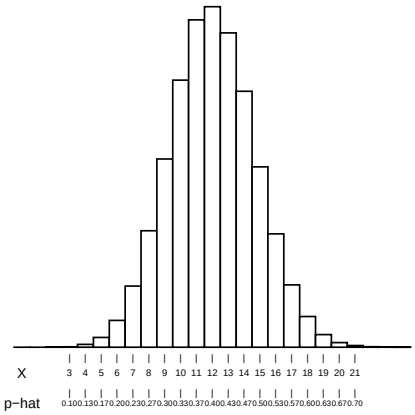
Exercise 2 is an example of the law of large numbers. A more straightforward illustration involves a coin toss. If you toss a coin repeatedly, which tends to get closer to 50% heads: 100 tosses or 700 tosses? The correct answer is 700. As the number of tosses increases, the closer we expect to get to 50%. The reason for this, as the Exercise shows, is the smaller give or take value. The sample size, n, lies in the denominator of the SD of $\hat{p}$. Therefore, the larger the sample size, the smaller the SD, which happens to be the give-or-take value.

The Law of Large Numbers for Sample Percentages:
The sample percentage tends to get closer to the true percentage as the size increases.

8.2.1 Sampling Distribution of p-hat is Approximately Normal

Since $\hat{p} = \frac{X}{n}$, the sampling distribution of $\hat{p}$ looks the same as that of X except for different numbers on the horizontal axis. For $n = 30$ and $p = 0.4$, the probability histogram of X and $\hat{p}$ is shown in Figure 8.1.

Figure 8.1: Probability of Histogram and p-hat



Therefore, like the binomial, the sampling distribution of $\hat{p}$ may be approximated by a normal curve with the correct mean and SD.

Example: Toss a fair coin 50-times. What is the chance of getting 60% or more heads?

Solution: The question is equivalent to 'What is the probability that$\hat{p}$ exceed 0.60?' Using the mean and SD given in the formula above with $n = 50$ and $p = 0.50$, the sample proportion is expected to be around 0.50 give or take $\sqrt{\frac{0.5(0.5)}{50}}$ or 0.50 give or take 0.07. With a mean of 0.50 and a SD 0.07, the area to the right of 0.60 (under the normal curve with mean 0.50 and SD 0.07) is 0.0766. Thus, the proportion of heads will exceed 0.60 fewer than 8 percent of the time.

Example 3 If Best Buy sold 1200 smartwatches last year, the percentage of smartwatches sold with extended protection plans is expected to be around 20%, give or take _____. Estimate the likelihood that it sold protection plans with more than 25% of those watches.

8.3 Estimating the Population Proportion p

The Best Buy computations in the previous section assume that we know the protection plan sales rate is $p = 0.20$. In data analysis, population parameters like p are typically unknown and estimated from the data. Consider estimating the proportion p of the current WMU graduating class who plan to go to graduate school. Suppose we take a sample of 40 graduating students and suppose that 6 out of the 40 are planning to go to graduate school. Then our estimate is $\hat{p} = \frac{6}{40} = 0.15$ of the graduating class plan to go to graduate school. Now $\hat{p}$ is based on a sample, and unless we got fortunate, chances are the 0.15 estimate missed. By

how much? On the average, a random variable misses the mean by one SD. From the previous section, the SD of $\hat{p}$ equals $\sqrt{\frac{pq}{n}}$. It follows that the expected size of the miss is $\sqrt{\frac{pq}{n}}$. This last term is the *standard error of estimation of the sample proportion*, or simply **standard error (SE)** of the proportion.

However, since we don't know p, we cannot calculate this SE. In a situation like this, statisticians replace p with $\hat{p}$ when calculating the SE. The resulting quantity is called the *estimated standard error of the sample proportion*. In practice, however, the word "estimated" is dropped and estimated SE is called merely the SE.

The population p is **estimated using the sample proportion** $\hat{p}$. This estimate tends to miss by an amount called the **standard error (SE)** of $\hat{p}$.

$$SE_{\hat{p}} = \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

Exercise 4

Fill in the following blanks.

1. If 6 out of 40 students plan to go to graduate school, the proportion of all students who plan to go to graduate school is estimated as _____. The standard error of this estimate is _____.
2. If 54 out of 360 students plan to go to graduate school, the proportion of all students who plan to go to graduate school is estimated as _____. The standard error of this estimate is _____.

Exercise 4 shows the effect of increasing the sample size on the SE of the sample proportion. Multiplying the sample size by a factor of 9 (from 40 to 360) makes the SE decrease by a factor of 3. In the formula for the SE of $\hat{p}$, the sample size appears (i) in the denominator, and (ii) inside a square root. Therefore, multiplying the sample size by a specific factor divides the SE of $\hat{p}$ by the square root of that factor.

As the sample size n **increases**, the $SE_{\hat{p}}$ **decreases** like the square root of the sample size.

8.4 Estimating Population Proportion Using Intervals

Variables tend to miss their expected value but should be within *one* SD 68% of the time, and within 1.96 SD 95% of the time. Since the SE of $\hat{p}$ is simply an estimate of the SD, then we can write $|\hat{p} - p| \leq 1.96(SE)$ or that p is inside the interval $\hat{p} \pm 1.96(SE)$ 95% of the time. In other words, the interval $\hat{p} \pm 1.96(SE)$ contains the true value of p with 95% certainty. This method gives us an interval estimate of p.

95% Confidence Interval for p:
A 95% confidence interval estimate for the population proportion p is given by

$$\hat{p} \pm 1.96\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

The term $1.96\sqrt{\frac{\hat{p}\hat{q}}{n}}$ is the 95% margin of error, ME.

Try the following problems.

Exercise 5

1. If 6 out of 40 students plan to go to graduate school, the proportion of all students who plan to go to graduate school is estimated as _____. The margin of error is _____.
2. Calculate a 95% confidence interval estimate for the true proportion p of WMU students who plan to go to graduate school.
3. If 54 out of 360 students plan to go to graduate school, calculate a 95% confidence interval estimate for the true proportion p of WMU students who plan to go to graduate school.

8.5 Sample Size for Estimating the Population Proportion

If 9 out of 25 randomly selected WMU students live in Southwest Michigan, the 95% confidence interval for the true proportion is $\hat{p} \pm 1.96\sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.36 \pm 0.19$. This result says that the true proportion can be as low as 0.17 or as high as 0.55. If we wanted to reduce the margin of error from 0.19 to some value ME, then we set the formula for margin of error equal to ME, i.e. $ME = 1.96\sqrt{\frac{\hat{p}\hat{q}}{n}}$. Solving for n gives the result we need.

To be 95% confident that the sample proportion is within a distance of ME of the true proportion p, choose a sample size equal to

$$n = (1.96)^2 \frac{\hat{p}\hat{q}}{ME^2}$$

where $\hat{p}$ is an estimate based on historical data or a pilot study. The quantity ME is called the 95% margin of error for p.

Example:

Suppose we want to reduce the margin of error for estimating the population proportion from 0.19 to 0.10. Using the estimate $\hat{p} = 0.36$ based on the initial sample, the sample size we need is: $n = (1.96)^2 \frac{(0.36)(0.64)}{0.10^2} = 89$. To verify that this is the correct sample size, the 95% confidence interval would be computed (if the sample proportion remained at 0.36) as $0.36 \pm (1.96)\sqrt{\frac{0.36(0.64)}{89}} = 0.36 \pm 0.10$.

8.6 Key Words

- sample proportion
- sampling distribution
- standard error
- 95% confidence interval
- margin of error

8.7

Exercises

Exercise 8.7-1: Suppose that 20% of students in a large university are graduate students. If a random sample of 125 students are randomly selected, what is the probability that 25% or more of the sample are graduate students?

Exercise 8.7-2: A sample of 100 individuals showed that 20% experienced gastrointestinal problems after consuming 10 grams of sorbitol, a common artificial sweetener. Attach a standard error to this estimate.

Exercise 8.7-3: Let's say that we take a sample of 100 observation has 20 successes.

1. What is the estimate of the population proportion?
2. What is the standard error of this estimate?
3. What is the 95% margin of error?
4. What is the 95% confidence interval?

Exercise 8.7-4: When a flight experiences fewer no-shows than expected, some passengers are 'bumped' from their flights (are denied boarding). These incidents can reflect poorly on customer satisfaction. Suppose United Airlines (for example) would like to estimate the actual proportion of involuntarily bumped passengers across all domestic flights in the industry. In a pilot study of 500 domestic passengers, 33 were involuntarily bumped.

1. What is the estimate of the population proportion?
2. What is the standard error of this estimate?
3. What is the 95% margin of error?
4. What is the 95% confidence interval?

Exercise 8.7-5: An appliance manufacturer offers maintenance contracts on its major appliances. A manager wants to know what fraction of buyers of the company's convection ovens are also buying the maintenance contract with the oven. From a random sample of 120 sales slips, 31 of the oven buyers opted for the contract.

1. The proportion of customers who buy the contract along with their oven is estimated as _____.
2. Calculate a standard error for the estimate in (1).
3. Calculate a 95% confidence interval estimate for the true proportion of customers who buy the contract along with their oven.

Exercise 8.7-6: Wiley Publications has determined that out of a sample of 5,511 of its publications for 2012, 1,754 of them are pirated in some form.

1. What is the estimate of the population proportion?

2. What is the standard error of this estimate?

3. What is the 95% margin of error?

4. What is the 95% confidence interval?

Exercise 8.7-7: Researchers who were concerned if doctors were consistently adjusting dosages for the weight of elderly patients studied 2000 prescriptions. They found that for 600 of the prescriptions, the doctors failed to change the dosages.

1. Doctors fail to adjust dosage for an estimated _____ percent of prescriptions.

2. Calculate a standard error for the percentage in (1).

3. Calculate a 95% margin of error for the percentage in (1).

4. Calculate a 95% interval estimate for the true proportion

5. Calculate a 95% confidence interval for the true percentage of prescriptions where doctors fail to adjust dosages.

Exercise 8.7-8: Suppose researchers are interested in the potential sample size of an experiment to investigate gastrointestial problems after consuming 10 grams of sorbitol. What is the sample size that researchers need if they suspect that 20% of people who experience gastrointestial problems after consuming 10 grams of sorbitol? They want 95% confidence and and margin of error of 8%.

Exercise 8.7-9: The CDC is recommending that everybody wash their hands multiple times per day during the COVID-19 pandemic. The students from STAT 1600 decided to conduct a survey of students on the WMU campus. They decided on a sample size of 100 students. They found that 82 washed their hands multiple times per day.

1. What is the point estimate for the proportion?

2. What is the standard error?

3. What is the 95% confidence interval for the proportion of students who say they wash their hands multiple times per day?

Chapter 9

Comparing Two Proportions

9.1 Objectives

After completing this part, students should be able to:

- Estimate the difference between independent proportions.
- Calculate and interpret confidence interval of the difference between proportions.
- Understand the concept of risk ratio and odds ratio.
- Calculate and interpret risk ratio and odds ratio confidence intervals.

9.2 Estimating the difference between independent proportions

Has retention rate at WMU changed over time? Suppose that a random sample of 200 entering students in 1989 showed 74% were still enrolled three years later. Another random sample of 200 entering students in 1999 showed that 66% were still enrolled three years later. This difference constitutes an 8% change in 3-year retention rate. However, the 8% difference is based on random sampling and is only an estimate of the actual difference. What is the likely size of the error of estimation?

Changing notation from the percentage to proportions and taking the difference of 0.74 - 0.66, we get 0.08 to compare retention rates. The proportions of 0.74 and 0.66 are *independent* proportions, in the sense that we base them on separate and independent groups of students. The SE of the difference is

$$SE_{(\hat{p}_1-\hat{p}_2)} = \sqrt{SE_{\hat{p}_1}^2 + SE_{\hat{p}_2}^2}$$

Whenever the two proportions are independent. Applying equation SE of $\hat{p} = \sqrt{\frac{\hat{p}\hat{q}}{n}}$ twice, we have $SE_{\hat{p}_1} = \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1}}$ and $SE_{\hat{p}_2} = \sqrt{\frac{\hat{p}_2\hat{q}_2}{n_2}}$. Substituting into the formula above, $SE_{(\hat{p}_1-\hat{p}_2)} = \sqrt{SE_{\hat{p}_1}^2 + SE_{\hat{p}_2}^2}$, we get:

Standard Error of the Difference between two independent Proportions

$$SE_{(\hat{p}_1-\hat{p}_2)} = \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

Continuing with the retention rate example, we let $\hat{p}_1 = 0.74$, $\hat{p}_2 = 0.66$, $n_1 = 200$, $n_2 = 200$ so that

$$SE_{(\hat{p}_1-\hat{p}_2)} = \sqrt{\frac{0.74(0.26)}{200} + \frac{0.66(0.34)}{200}} = 0.045$$

Thus, the difference in retention rate is estimated by $0.74 - 0.66 = 0.08$ with a standard error of 0.045. Changing notation back to a percentage and with less technical language, the drop-in retention rate is estimated to be 8%, give or take 4.5% or so. We also could have computed $SE_{(\hat{p}_1-\hat{p}_2)}$ in three steps. First by using $SE_{\hat{p}}$ twice,

$$\begin{aligned} SE_{\hat{p}_1} &= \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1}} \\ &= \sqrt{\frac{0.74(0.26)}{200}} \\ &= 0.031 \end{aligned} \qquad \begin{aligned} SE_{\hat{p}_2} &= \sqrt{\frac{\hat{p}_2\hat{q}_2}{n_2}} \\ &= \sqrt{\frac{0.66(0.34)}{200}} \\ &= 0.033 \end{aligned}$$

Then using $SE_{(\hat{p}_1-\hat{p}_2)} = \sqrt{SE_{\hat{p}_1}^2 + SE_{\hat{p}_1}^2} = \sqrt{(0.031)^2 + (0.033)^2} = 0.045$

9.2.1 Using a confidence interval

The difference of two proportions is a random variable with an expected value and spread. The 68% and 95% rules apply, i.e. the estimated difference $\hat{p}_1 - \hat{p}_2$ should be close to the true value – within *one* SE 68% of the time, and within 1.96 SE's 95% of the time. Following the same reasoning as before,

$$(\hat{p}_1 - \hat{p}_2) \pm 1.96(SE_{(\hat{p}_1-\hat{p}_2)})$$

should contain the true difference $p_1 - p_2$ with 95% level of confidence. Substituting

$$SE_{(\hat{p}_1 - \hat{p}_2)} = \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

, we get the following formula:

95% Confidence Interval for $p_1 - p_2$

$$(\hat{p}_1 - \hat{p}_2) \pm 1.96\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

For retention rate, the difference between 1989 and 1999 the department estimated as 0.08 with SE = 0.045. Therefore, a 95% confidence interval for the change is

$$0.08 \pm 1.96(0.045)$$

or $0.08 \pm 0.088 = (-0.008, 0.168)$. Rounding off to $(-0.01, 0.17)$, we say that the drop-in retention rate from 1989 to 1999 is between -0.01 and 0.17 with 95% confidence. Note that **zero** is contained or is not been excluded from the interval, making it still a possibility that there is no real change in retention rate, just chance variability.

9.3 Statistical significance

Let $\hat{p}_1$ be the proportion of heads in 50 tosses of a coin. Let $\hat{p}_2$ be the proportion of heads in the next 50 tosses of the same coin. Will $\hat{p}_1$ and $\hat{p}_1$ be equal? Not likely. They will tend to differ, due to "the luck of the draw" or chance variability.

The table below shows partial data from an occupation survey by the Census Bureau. In this 2009 survey, regular 'cooks' were a separate classification from 'chefs or head cooks.' Note that even though 37% of cooks were women, only 16% of chefs or head cooks were women. Is the difference just luck of the draw, or due to something else besides chance?

	Women	Men	Total
Cooks	441	762	1203
Chefs or Head Cooks	45	245	290

Statistics help decision-making in cases like these by assessing how much chance variability to expect between two proportions. Following the calculations of the section above, we have

difference:

$$\begin{aligned}\hat{p}_1 - \hat{p}_2 &= \frac{441}{1203} - \frac{45}{290} \\ &= 0.37 - 0.16 \\ &= 0.21\end{aligned}$$

with a standard error:

$$\begin{aligned}SE_{(\hat{p}_1-\hat{p}_2)} &= \sqrt{\frac{0.37(0.63)}{1203} + \frac{0.16(0.84}{290}} \\ &= 0.026\end{aligned}$$

The 95% confidence interval for $p_1 - p_2$ is

$$0.21 \pm 1.96(0.026) = (0.16, 0.26)$$

Thus, even allowing for 1.96 SE's of chance variability, the actual difference between proportions is at least 0.16 (and could be as large as 0.26). This difference means that the interval does not contain or it excluded 0 from the range of possible values. When this happens, statisticians say that the differences are *statistically significant*.

If the confidence interval for $\hat{p}_1 - \hat{p}_2$ excludes **zero**, then the difference is **statistically significant.**

9.3.1 The P-value

For convenience, let us continue the example of the previous section.

Question If chance alone was at work, how likely will we get a difference of 0.21 between two proportions?

Answer Very small, less than 0.0001 (or 1 in 10,000).

The 'likelihood of getting 0.21 by chance' is called a P-value. The fact that it is minimal means we should exclude the option that 'chance alone is at work.'

The actual probability calculation is beyond the scope of this class. Let's say that random variables very rarely go past 4 SE's from their expected values (less than 1 in 10,000 times). Since the SE for the difference is 0.026, the observed difference $\hat{p}_1 - \hat{p}_2 = 0.21$ is not just 1, nor 2, but 8 SE's from 0. This difference cannot be chance alone. Something else is at work.

The Rule for p-value:
If the p-value ≤ 0.05, the difference is *statistically significant.*
If the p-value ≤ 0.01, the difference is called *highly significant.*

In the occupation example, we can say that the percentage of women head chefs is lower than that of regular cooks. Furthermore, the difference is highly significant.

9.4 Key Words

- p-value
- risk ratio
- odds
- odds ratio
- confidence interval

9.5

Exercises

Exercise 9.5-1: In a study of drug usage by students at a large university, researchers obtained the data regarding hard liquor experience of smokers and nonsmokers.

	Drug Use		
	Once or more	Never	Total
Smokers	23	18	41
Nonsmokers	15	56	71

1. Estimate the difference in the percentage of drug use between smokers and nonsmokers.
2. Calculate a standard error for our estimate in (1).
3. Calculate a 95% confidence interval for the difference in the percentage of drug use between smokers and nonsmokers.
4. Interpret the above confidence intervals above in context. Is it significant? Why or why not?

Exercise 9.5-2: Time magazine reported the result of a telephone poll of 800 adult Americans. The reporter asked Americans the following question: "Should the federal tax on cigarettes be raised to pay for health care reform?"

	Federal Tax on Cigarettes	
Status	Yes	No
Smoker	41	154
Nonsmoker	351	254

1. Estimate the difference in the percentage of Americans who supported the federal tax on cigarettes between smokers and non-smokers.
2. Calculate a standard error for our estimate in (1).
3. Calculate a 95% confidence interval for the difference in the percentage of Americans who supported the federal tax on cigarettes between smokers and non-smokers.
4. Interpret the above confidence intervals above in context. Is it significant? Why or why not?

Exercise 9.5-3: The age at which a woman gives birth to her first child may be an essential factor in the risk of later developing breast cancer. An international study conducted by WHO selected women with at least one birth and recorded if they had breast cancer or not and whether they had their first child before their 30^{th} birthday or after. In a sample of 3220 women who had their first child after their 30^{th} birthday, 683 developed breast cancer. Whereas, in a sample of 10245 women who had their first child before their 30^{th} birthday, 1483 developed breast cancer.

1. Estimate the difference in the percentage of developing breast cancer between women who had their first child after their 30^{th} birthday and before their 30^{th} birthday.
2. Calculate a standard error for our estimate in (1).

3. Calculate a 95% confidence interval for the difference in the percentage developing breast cancer between women who had their first child after their 30^{th} birthday and before their 30^{th} birthday.

4. Interpret the above confidence intervals above in context. Is it significant? Why or why not?

Exercise 9.5-4: In a sample of 200 surgeons, 15% thought the government should control health care. Whereas, in a sample of 200 general practitioners, 21% thought the same.

1. Estimate the difference in the percentage of those who think the government should control health care between surgeons and general practitioners.

2. Calculate a standard error for our estimate in (1).

3. Calculate a 95% confidence interval for the difference in the percentage of those who think the government should control health care between surgeons and general practitioners.

4. Interpret the above confidence intervals above in context. Is it significant? Why or why not?

Exercise 9.5-5: A Pew Research Center poll asked randomly selected subjects if they agreed with the statement that "It is morally wrong for married people to have an affair." Among the 386 women surveyed, 347 agreed and among the 359 men, 305 agreed. Use a 95% confidence level if the difference in significant.

CI for Two Proportion			
Sample	X	N	Sample p
1 (women)	347	386	0.898964
2 (men)	305	359	0.849582

Difference = p(1) - p(2)
Estimate for difference: -0.0493816
95% CI for difference: (0.00172287, 0.0970402)
Test for difference = 0 (vs not = 0): Z = 2.04, p-val = .042

1. Determine the difference in the percentage of those who thought that it is wrong for married people to have an affair.

2. Calculate a standard error for our estimate in (1).

3. Determine a 95% confidence interval for the difference in the percentage of those who thought that it is wrong for married people to have an affair.

4. Interpret the above confidence intervals in context. Is it significant? Why or why not?

Exercise 9.5-6: Suppose half of the public safety officers in Kalamazoo, Michigan, completed the Investigative Procedures Curriculum (IPC). The other half did not attend the IPC. Did the course increase their effectiveness in clearing crimes by arrest? The following table reports the results of the survey:

	Clearing Crimes		
	Cleared	Uncleared	Total
Trained	75	85	160
Untrained	49	66	115

1. Determine the difference in the percentage of those who completed the investigative procedures Curriculum (IPC)
2. Calculate a standard error for our estimate in (1).
3. Determine a 95% confidence interval for the difference in the percentage of those who completed the investigative procedures Curriculum (IPC)
4. Interpret the above confidence intervals above in context. Is it significant? Why or why not?

Exercise 9.5-7: In October 1947, the Gallup organization surveyed 1100 adult Americans, and asked "Are you a total abstainer from alcoholic beverages?" Of the 1100 adults surveyed, 407 indicated that they were total abstainers. In 2010, they asked the same question of 1100 adult Americans, and 333 indicated that they were total abstainers.

1. Determine the difference in the percentage of those who were total abstainers in 1947 and 2010.
2. Calculate a standard error for our estimate in (1).
3. Determine a 95% confidence interval for the difference in the percentage of those who were total abstainers in 1947 and 2010
4. Interpret the above confidence intervals above in context. Is it significant? Why or why not?

Chapter 10

More Comparing of Two Proportions

10.1 Objective

After completing this part, we should be able to:

- Set-up the hypotheses testing for proportions
- Elucidate Type I and Type II errors
- Articulate the conclusions to a broader audience.

10.2 Example 1

10.2.1 Is your colleague Cheating?

Situation: During a lunch break, a colleague wants to play a game where you each flip a coin six times. If the result of the flip heads, you win, if the result of the flip is tails your colleague wins. Suppose the outcome of six plays of the game is T, T, T, T, T, T. Did your colleague cheat?

Method: Deciding whether your colleague cheated, we need to determine the chance of getting six tails in a row. Let's assume that we are playing with a fair coin so that $P[tail] = P[Head] = 0.5$ so that each flip is independent. Next, we should ask, "is it unusual to have six tails in a row with a fair coin?"

Solution: We need to find the probability of achieving six tails in a row, assuming independence and fairness.

$$\begin{aligned} P[\texttt{six tails in a row}] &= P[\texttt{ T and T and T and T and T and T}] \\ &= P[T] \cdot P[T] \cdot P[T] \cdot P[T] \cdot P[T] \cdot P[T] \\ &= 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \\ &= (0.5)^6 \\ &= 0.0156 \end{aligned}$$

If we flipped a *fair* coin six times, 100 different times, we expect between 1 and 2 of the 100 experiments to result in all tails where each experiment consists of six flips of the coin. So what we observed it is possible but highly unlikely. Therefore, we can make one of two conclusions:

1. Your colleague is not cheating and very lucky.
2. Your colleague did not use a *fair* coin and cheated.

Is your colleague cheating, or did we see an unusual event?
Here, we use *hypothesis testing*. We assume reality (in this case, the $P[tail] = 0.5$).
We gather data (sample) to determine whether the data contradicts our assumption.

10.2.2 Define Hypotheses Testing

Definition: Hypotheses testing is a process of using sample data and probability to test the characteristics of one or more populations. For example, is there a difference in the birth weights between boys and girls.
Here is how we use hypotheses testings.

1. Make a statement about the nature of the population(s).
2. Formulate an Analysis Plan.
    - Sampling Distribution is approximately normal
    - Choose a level of significance equal to 0.05.
    - Decide the number of observations.
    - Select a test method.
3. Analyze the data.
4. Interpret the Results.
5. Communicate our results to the audience.

Definition:

The Null Hypothesis (H_0) is a statement about the status quo. For example, the proportion of (boys and girls) births are equal.

Definition:

The Alternate Hypothesis (H_a) is a statement that we are trying to contradict the H_0.

$$\begin{aligned} H_0 : P_1 &= P_2 \\ H_a : P_1 &\neq P_2 \end{aligned}$$

10.3 Testing Hypotheses

Two Proportions from Independent Samples.

When the difference, between the observed proportions of sample A and sample B, is similar; we can expect no significance, i.e., $(\hat{p}_a - \hat{p}_b) \sim 0$. On the other hand, when the proportion is dissimilar, we should expect a significant difference. We will examine this process shortly.

10.3.1 Example 1

In a survey of drug use among students at WMU, data compared usage of alcohol and smoking. Table 10.1 presents the results of the study:

Table 10.1: Smoking by using Hard Liquor

	Smoker	
Hard-Liquor Use	Yes	No
Once or more	56	15
Never	18	23
Total	74	38

In this study, the researchers are testing the proportion between non-smokers and smoker. They believe that the ratio of non-smokers between and smokers are different when drinking hard liquor. So they set-up an experiment to test the hypotheses at $\alpha = 0.05$. We should be thinking about the two-tailed test.

- If the difference between the proportions of smokers and non-smokers when consuming hard liquor, then we might assume that the proportion of smokers also consume hard liquor.

- Smoking is an independent variable.

Now we can consider the above model to solve the situation.

1. State the hypotheses
   - $H_0 : P_1 = P_2$, where P_1 is the proportion who smoke.
   - $H_a : P_1 \neq P_2$, where P_2 is the proportion who do not smoke.
2. Formulate an Analysis Plan.
   (a) Sampling Distribution is approximately normal
   (b) Choose a level of significance equal to 0.05.
   (c) Decide the number of observations. ($T_1 = 74$ smokers and $T_2 = 38$ non-smokers)
   (d) Compute the standard error of the difference between the two proportions:

$$\begin{aligned} SE_{\hat{p}_1 - \hat{p}_2} &= \sqrt{SE_{\hat{p}_1}^2 + SE_{\hat{p}_2}^2} \\ &= \sqrt{\frac{\frac{N_1}{T_1} * (1 - \frac{N_1}{T_1})}{T_1} + \frac{\frac{N_2}{T_2} * (1 - \frac{N_2}{T_2})}{T_2}} \end{aligned}$$

   (e) Calculate the test statitic:

$$z_0 = \frac{\frac{N_1}{T_1} - \frac{N_2}{T_2}}{SE_{\hat{p}_1 - \hat{p}_2}}$$

3. Make a statement about the nature of the population(s).
4. Formulate an Analysis Plan.
   - Sampling Distribution is approximately normal
   - Choose a level of significance equal to 0.05.
   - Decide the number of observations.
   - Select a test method.
5. Analyze the data.
6. Interpret the Results.

- If there is a relationship between drinkers of hard liquor and smokers, then we would assume that more students who never smoked are also non-drinkers of hard liquor. Also, those who drink hard liquor are likely to be smokers.
- In other words, The proportion of drinkers of hard liquor who smoke is going to be different from the proportion of drinkers of hard liquor who do not smoke.
- Note: non-smoking (ns) and smoking (s) are the two independent variables.

Now let's consider the five-step model introduced above.

- Step 1. Read the problem carefully.
- Step 2. Stating the Hypotheses

$$\begin{aligned} H_0 : P_s &= P_{ns} \\ H_a : P_s &\neq P_{ns} \end{aligned}$$

- Step 3. Deciding the Sampling Distribution and Set up the Critical Region.

$$\begin{aligned} \text{Sampling Distribution} &= \text{Approximately Normal} \\ \text{Alpha } (\alpha) &= 0.05 \\ \text{Critic Value (CV)} &= 1.959964 \end{aligned}$$

- Step 4. Computing the Standard Error.

$$\begin{aligned} SE_{\hat{p}_1 - \hat{p}_2} &= \sqrt{SE_{\hat{p}_1}^2 + SE_{\hat{p}_2}^2} \\ &= \sqrt{\frac{\frac{56}{74} * (1 - \frac{56}{74})}{74} + \frac{\frac{15}{38} * (1 - \frac{15}{38})}{38}} \\ &= 0.0936743 \end{aligned}$$

- Calculate the test statitic:

$$\begin{aligned} z_0 &= \frac{\frac{56}{74} - \frac{15}{38}}{0.0936743} \\ &= 3.8646667 \end{aligned}$$

- Step 5. Producing a Decision and Interpreting the Results of the Test.
  - Since the test statistic (3.8646667) is greater than CV (1.959964), therefore, reject the null hypothesis ($H_0 : P_s = P_{ns}$).
  - The interpretation is that there is evidence that the proportion of smokers is different from the proporton of non-smoker when drinking hard liquor.

10.3.2 Example 2

We have classified a random sample of 100 social work majors regarding whether the Council on Social Work Education has accredited their undergraduate programs (the *independent variable*) and whether they are working in social work positions within three months of graduation (the *dependent variable*).

Table 10.2: Employment Status

	Working as Social Worker	
Accredited	No	Yes
No	35	10
Yes	25	30
Total	60	40

From Table 1.2, Can we conclude from these data at a 5 percent level of significance that the proportion of persons working as a Social Worker who graduated from an accedited program is equal the proportion of persons working as a Social Worker who graduated from a non-accedited program?

Now let's consider the five-step model presented above.

- Step 1. Carefully, read the problem description.
- Step 2. Stating the Hypotheses

$$\begin{aligned} H_0: &\quad P_{wSW} = P_{nSW} \quad \texttt{graduated from an accedited program} \\ H_a: &\quad P_{wSW} \neq P_{nSW} \end{aligned}$$

- Step 3. Deciding the Sampling Distribution and Set up the Critical Region.

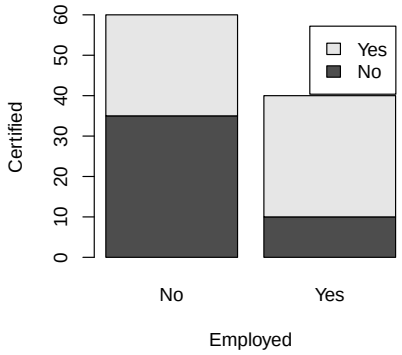
$$\begin{aligned} \text{Sampling Distribution} &= \text{normal distribution} \\ \text{Alpha } (\alpha) &= 0.05 \\ \text{Critic Value} &= \pm 1.959964 \end{aligned}$$

- Step 4. Computing the Test Statistic.

There is another way to look at this data.

Table: Student graduated from an Accredited Program by Working as Social Worker.

	Working as a S.W.	
Accredited	No	Yes
No	35	10
Yes	25	30
Total	60	40

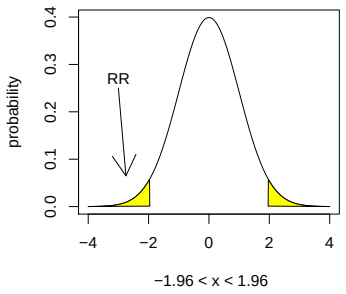


$$\begin{aligned} SE_{\hat{p}_1-\hat{p}_2} &= \sqrt{SE_{\hat{p}_1}^2 + SE_{\hat{p}_2}^2} \\ &= \sqrt{\frac{\frac{25}{60} * (1-\frac{25}{60})}{60} + \frac{\frac{30}{40} * (1-\frac{30}{40})}{40}} \\ &= 0.0934795 \end{aligned}$$

- Calculate the test statitic:

$$\begin{aligned} z_0 &= \frac{\frac{25}{60} - \frac{30}{40}}{0.0934795} \\ &= -3.5658424 \end{aligned}$$

Note: The adjacent bell-shaped curve notes the rejection regions (RR) with a significant level 5 percent. In this case, the test statistic is -3.5658424. So it falls in the RR, and we will reject our null hypotheses.



- Step 5. Producing a Decision and Interpreting the Results of the Test.
  - Since the absolute value of the test statistic ($|-3.5658424|$) is greater than CV (1.959964), therefore, **reject** the null hypothesis ($H_0 : P_w SW = P_{nSW}$).
  - The interpretation is that there is evidence that the proportion of social workers working in their field graduated from an accredited program.

10.4 Types of Errors

A **type I error** is rejecting a **true null hypothesis**. This error is a false positive. We commit a type I error when we reject a relationship when it is true. An example of a Type I error is rejecting the statement MMR vaccine is effective against mumps.

A **type II error** is failing to reject a **false null hypothesis**. This error is a false negative. We commit a type II error when we fail to reject a relationship when it is false. An example of a Type II error is a blood test designed to detect clots in patients who have a clot and does not detect it.

When comparing two proportions, concluding the proportions were different when in reality they were not different would be a type I error; concluding the proportions were not different when in reality they were different would be a type II error.

The following table summarizes the above definitions.

Table 10.3: Possible Hypotheses Test Outcomes

	The Truth	
Decision	H_0 is True	H_0 is False
Fail to reject H_0	Correct decision	$P[typeII] = \beta$
Reject H_0	$P[typeI] = \alpha$	Correct decision

10.5 Key Words

- Dependent
- Hypotheses Testing
- Hypothesis
- Independent
- Two sample proportions

10.6

Exercises

Exercise 10.6-1: A local city has outsourced some of its planning function. The contractor has built a database that describes the neighborhoods in the city and has developed a model so that we rate each area for their "quality of life" such as noise, open spaces, walkable areas, etc. She has surveyed a random sample of these districts about their level of satisfaction with their neighborhoods. Do the results of the study agree with the contractor's objective ratings of quality with the respondents' self-reports of appreciation?

	Quality of Life	
Satisfaction	Low	Higher
Low	13	14
Higher	13	60
Total	26	74

- Is the difference between the proportions of quality of life (Low and Moderate or higher) when satisfaction is low?
- Calculate the column percentages for the table to assess the pattern of the relationship. Which group is most likely to say their satisfaction is high?

Answer:

Exercise 10.6-2: The drug Drug Z is a vaccine meant to prevent a certain type of cancer. It is typically administered to middle-aged subjects starting around 45 years of age. In a randomized, double-blind clinical trial of Drug Z, the subjects were randomly divided into two groups. Subjects in group 1 received Drug Z, while subjects in group 2 received a control vaccine. After the first dose, 170 of 701 subjects in the experimental group (group 1) experienced fever as a side effect. After the first dose, 76 of 622 of the subjects in the control group (group 2) experienced fever as a side effect.

- Does the evidence suggest that the proportion of subjects in group 1 experienced fever as a side effect that is different from the subjects in group 2 at $\alpha = 0.05$ level of significance?

Answer:

Exercise 10.6-3: **Tattoos** The Harris Poll conducted a survey in which they asked "How many tatoos do you curently have on your body?" Of 1205 males surveyed, 181 responded that they had at least one tattoo. Of the 1097 females surveyed, 143 responded that they had at least one tatoo.

- Does the evidence suggest that the proportion of males that have at least one tattoo is different from females that have at least one tattoo at $\alpha = 0.05$ level of significance?

Answer:

Exercise 10.6-4: In October 1947, the polling company, Gallup, asked 1100 American adults "Are you a total abstainer from alcoholic beverages?" Of 1100 surveyed, 407 said they were total abstainers. In 2010, the company asked the same question of 1100 different American adults, 333 indicated that they were total abstainers.

- Has the proportion of American adults who totally abstain from alcohol changed? Use the significance level (α) of 5 percent.

Answer:

Exercise 10.6-5: The polling company, Sullivan, asked respondents, "Would you be willing to pay higher taxes if the tax revenue went toward deficit reduction?" Of 100 males surveyed, 32 were willing to pay more taxes if the tax revenue went toward deficit reduction. The company asked the same question of 100 females, 22 indicated that they would be willing to pay more taxes if the tax revenue went toward deficit reduction.

- Is there significant evidence to suggest the proportions of males and females who are willing to pay higher taxes to reduce the deficit differs at the $\alpha = 0.05$ level of significance?

Answer:

Exercise 10.6-6: In March 2003, the Pew Research Group surveyed 1503 adult Americans and asked, "Do you believe the U.S.A. made the right decision to use military force in Iraq?" Of the 1508 adult Americans surveyed, 1086 stated the U.S.A. made the right decision. In August 2010, the Pew Research Group asked the same question of 1508 adult Americans and found that 608 believed the U.S.A. made the right decision.

- Does the evidence suggest the proportion of American adults believed the U.S. made the right decidion to invade Iraq in 2003 is different from American adults believed the U.S. made the right decidion to invade Iraq in 2010 at a significance level of 0.05?.

Chapter 11

Sampling Distribution of the Mean

11.1 Objectives

After completing this part, students should be able to:

- Define and explain the behavioral properties of the sample average
- Estimate the population mean
- Calculate and interpret confidence interval of the population mean
- Determine the sample size for estimating the population mean

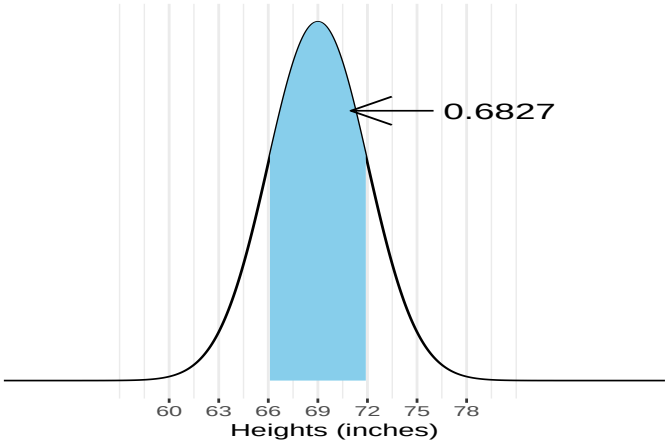
11.2 Behavioral Properties of the Sample Average

What percentage of adult men are between 5'6" and 6' tall? Population surveys have shown that men's heights are approximately normally distributed with mean 5'9" and SD 3". Thus, the percentage of men between 5'6" and 6' is estimated as 68%, the percentage within 1 SD of the mean. See Figure 10.1 below.

If the population of men is randomly assigned into groups of 9, and the average heights are computed for each group, what percentage of groups average between 5'6" and 6' in height? Is the answer approximately 68%? No. In fact, more than 99% of the groups will average between 5'6" and 6', even though only 68% of individuals do. Why? Because averages tend to include tall, short and medium heights – therefore averages tend to fall closer to the middle than individuals. (Think about this: We put in a hat the names of all the men in the class. We will win $20 if the names we draw average over 6 feet tall. Would we instead draw 1 or 2 names? What are our chances of winning if we draw nine names?)

The following is a small simulation study of the behavior of the sample mean. Fifty

Figure 11.1: The Percentage of Men's Heights between 66 and 72 inches



samples are drawn (each containing $n = 9$ individuals) from a population with mean 69 inches and SD 3 inches. For each sample, we calculate the average. Observe that none of the samples average over 71 inches, even though many individuals do.

Table 11.1: Heights of 50 samples of Nine Men

Sample	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	Ave
1:	65.5	66.8	68.9	67.8	71.9	66.8	71.0	73.1	62.6	68.27
2:	68.6	71.2	72.6	64.3	70.9	70.0	69.0	69.8	62.4	68.75
3:	67.4	67.9	67.1	68.2	70.7	68.3	67.2	68.7	67.0	68.04
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
SD:	3.17	2.56	2.66	3.25	2.94	2.56	3.10	3.33	3.62	0.92
Mean:	68.53	68.18	68.92	68.51	69.26	68.72	68.66	68.86	68.11	68.64

The first lesson of this chapter says:

> Averages are less variable than individuals

Do we see this in the simulation study? To make it easier to see, look at the SD of each column. The SD of individuals tend to be around 3.0 (the actual value), but the SD of averages is much smaller. How much smaller? We now state the central lesson of the chapter.

> SD of $\bar{X} = \frac{\texttt{SD of individuals}}{\sqrt{n}}$

Since the individuals in the simulation study have SD of 3 inches, the SD of 9-member averages is:

$$\bar{X}_{n=9} = \frac{\texttt{SD of individuals}}{\sqrt{n}} = \frac{3}{\sqrt{9}} = 1.0$$

Therefore, the real value for the SD of the last column is 1.0. The simulated SD is 0.92, which is close.

Example 1

Suppose that men's heights are normally distributed with a mean 5'9" and a SD 3".

1. What percentage of men are over 5'11" tall?
2. Select a man at random. What is the probability that he is over 5'11" tall?
3. If we calculate the average height for all possible samples of size nine that we can take, what percentage of averages will exceed 5'11"?
4. Given one randomly selected sample of size 9, what is the probability that the average height will exceed 5'11"?
5. Given a sample of size 25, what is the probability that the average height will exceed 5'11"?
6. Given a sample of size nine, the average height of the sample will exceed _____ with probability 0.90
7. 90% of samples of size nine will have an average height exceeding _____.

11.3 Estimating the Population Mean

Consider estimating the average GPA (call this μ) of the approximately 23,000 WMU undergraduates. In the absence of the complete database, we may wish to estimate μ by taking a random sample of, say, $n = 25$ students and computing the sample average (call this $\bar{X}$). Suppose $\bar{X} = 3.05$. Now, unless we got fortunate with the random sample, chances are the 3.05 estimate missed the true value μ. By how much?

On the average, a random variable misses its expected value by one standard deviation. We expect $\bar{X}$ to miss by how much? one SD of $\bar{X}$. Using the equation above, we can estimate this by $\frac{s}{\sqrt{n}}$, since the sample standard deviation s estimates the SD of individuals.

The population mean (μ) is estimated using the sample mean ($\bar{X}$). The estimate tends to miss the μ by an amount called the *standard error (SE)* of the mean which is calculated as $\frac{s}{\sqrt{n}}$:

$$SE_{\bar{x}} = \frac{s}{\sqrt{n}}$$

Returning to the GPA example, suppose that the sample of $n = 25$ students yielded an average GPA of 3.05 and a standard deviation of 0.40. Then the WMU population average GPA is estimated as 3.05 with a standard error of $\frac{s}{\sqrt{n}} = \frac{0.40}{\sqrt{25}} = 0.08$.

Example 2

What is the average length of stay for undergraduate students at WMU?

1. Suppose 25 graduating students were randomly selected and asked about their length of stay. Suppose that the sample averaged 5.3 years, with an SD of 1.5 years. Then the WMU average stay is estimated as _____ years give or take _____ years or so.

2. A second sample of 100 students were interviewed. The mean and SD for the second sample were also 5.3 years and 1.5 years, respectively. Calculate an estimate for the WMU average stay and provide a standard error for your estimate.

Similar to the SE for proportions, the formula for the SE of the mean has the sample size (i) in the denominator, and (ii) inside the square root sign. Therefore, increasing the sample size by a factor of 4 makes the standard error decrease by a factor of $\sqrt{4}$.

The standard error (SE) of the mean decreases like the square root of the sample size (n).

11.4 Estimating the Population Mean Using Intervals

Variables tend to miss their expected value, but should be within 1 SD 68% of the time, and within 1.96 SD's 95% of the time. Changing notation for SD to SE, we get

$$|\bar{X} - \mu| \leq 1.96(SE)$$

95% of the time, where SE is given by the equation in section 10.3. As a consequence,

$$\texttt{Population mean, } \mu \texttt{ is inside the interval } \bar{X} \pm 1.96(SE)$$

95% of the time. This method gives an interval estimate of μ.

95% Confidence Interval for μ
A 95% Confidence Interval estimate for the population mean μ is given by

$$\bar{X} \pm 1.96\frac{s}{\sqrt{n}}$$

The term $1.96\frac{s}{\sqrt{n}}$ is called the 95% margin of error.

11.5 Sample Size for Estimating the Population Mean

If we wanted to reduce the margin of error to some value ME, then we set the formula for margin of error equal to ME, i.e., $ME = 1.96\frac{s}{\sqrt{n}}$. Solving for n gives the result we need.

In order to be 95% confident that the sample mean is within a distance ME of the true population μ, choose a sample size equal to

$$n = \frac{1.96^2 s^2}{ME^2}$$

where s is the standard deviation based on historical data or a pilot sample. The quantity ME is called the 95% margin of error for μ.

11.6 Key Words

- sample mean
- population mean
- standard error
- confidence interval
- margin of error

11.7

Exercises

Exercise 11.7-1: The national average on a science test for tenth graders has a mean of 210 and a standard deviation of 28.

1. What percentage scored over 220? (You may assume that the histogram of scores looks approximately like the normal curve.)
2. One tenth grader is randomly selected. What is the chance that he/she scored over 220?
3. A random sample of 40 tenth graders are selected. What is the chance that this group will average over 220? (Should this be smaller, larger, or approximately equal to (2)?)
4. A larger sample of 100 tenth graders is selected. What is the probability that this group will average over 220? (Should this be smaller, larger, or approximately equal to (3)?

Exercise 11.7-2: A sample of 16 observations is taken from a distribution with mean of 10 and standard deviation 2.

1. Suppose the sample mean is 10.5. What is the standard error of this estimate?
2. What is the 95% margin of error?
3. What is the 95% confidence interval?
4. What is the probability the sample mean is greater than 9?
5. What is the probability the sample mean is less than 9?
6. What is the probability the sample mean is between 8 and 10?
7. 33% of sample means are above what value?
8. 33% of sample means are below what value?

Exercise 11.7-3: Safe Skies Airline took a random sample of 25 flights to estimate the average time that arriving passengers wait for luggage at the carousel. The sample average was found to be 16.2 minutes with a standard deviation of 4 minutes. The population
average waiting time is estimated as _____ minutes give or take _____ minutes or so.

Exercise 11.7-4: The normal human body temperature is on average 98.6^oF with a standard deviation of 1.0^oF. Researchers take a random sample of 35.

1. What is the mean of this estimate?
2. What is the standard error of this estimate?

Exercise 11.7-5: A manufacturing company's profits depend on the cost of materials. One material of interest is carbon fiber, which is used to make golf shafts and fishing rods. The cost per pound (in dollars) was recorded for ten randomly selected days from the first six months of 2002. The data follow:

7.6, 7.8, 8.8, 7.3, 6.6, 7.5, 6.7, 8.6, 7.4, 7.7

1. Calculate an estimate for the average cost per pound during the first six months of 2002.
2. Calculate a standard error for our estimate in (1).

Exercise 11.7-6: The average annual rainfall in Mawsynram, India (the wettest place on Earth) is 467.35 inches with a standard deviation of 5.12 inches. A random sample of 100 is taken.

1. What happens to the distribution of sample means as the sample size increases?
2. What is the mean of this distribution?
3. What is the standard error of this distribution?

Exercise 11.7-7: In a study published in the *American Journal of Psychiatry* (157.737-744, May 2000), researchers wanted to measure the effect of alcohol on the development of the hippocampal region in adolescents. The hippocampus is the portion of the brain responsible for long-term memory storage. The researchers randomly selected 12 adolescents with alcohol use disorders. They wanted to determine whether the hippocampal volumes in the alcoholic adolescents were less than the normal volume of 9.02 cubic-centimeters (cm^2). An analysis of the sample data revealed that the hippocampal volume is approximately normal with $\bar{x} = 8.10$ and $s = 0.7$.

1. What happens to the distribution of sample means as the sample size increases?
2. What is the mean of this distribution?
3. What is the standard error of this distribution?

Chapter 12

Comparing Two Means

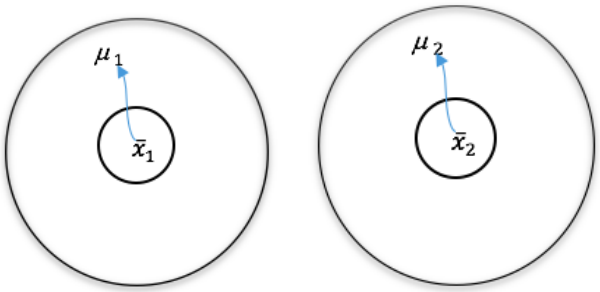
12.1 Objectives

After completing this part, students should be able to:

- Compare two population averages when the samples are independent using confidence intervals.
- Compare two population averages when the data are dependent (paired) using confidence intervals.

12.2 Estimating the Difference between Independent Means

Suppose we sample two means $\bar{x}_1$ and $\bar{x}_2$ from two populations with unknown means μ_1 and μ_2. Think of it as a picture:



As you can see, the sample circles including $\bar{x}_1$ and $\bar{x}_2$ are smaller than the entire population circles including μ_1 and μ_2. In other words, we do not have all the information we

need to be sure about the exact values of μ_1 and μ_2. However, as we have learned, statistics allows for sample means to estimate population means as depicted by the arrows. What's new in this chapter is that here we learn statistics can do more than allow us to estimate values for μ_1 and μ_2 separately. We can also use statistics to investigate how the population means differ from one another in tandem. We do this by performing *hypothesis tests* on and using confidence intervals centered around the difference between the sample means $\bar{x}_1$ and $\bar{x}_2$. Let's now turn to some formulaic examples of such tests and intervals.

Is there "grade inflation" in WMU? How does the average GPA of WMU students today compare with, say ten years ago? Suppose a random sample of 100 student records from 10 years ago yields an average sample GPA of 2.90 with a standard deviation of 0.40. A random sample of 100 current students today yields a sample average of 2.98 with a standard deviation of 0.45. The difference between the two-sample means is $2.98 - 2.90 = 0.08$. Is this proof that GPA's are higher today than ten years ago? Well … first, we need to account for the fact that 2.98 and 2.90 are not the correct averages but that we compute the average from random samples. Therefore, 0.08 is not the exact difference, but merely an estimate of the actual difference. By how much will it miss?

Note that we took differences $2.98 - 2.90$ to compare average GPA. The two averages 2.98 and 2.90 are independent, in the sense that we base them on separate and independent groups of students. The SE of the difference is

$$SE_{(\bar{x}_1 - \bar{x}_2)} = \sqrt{SE_{\bar{x}_1}^2 + SE_{\bar{x}_2}^2} \tag{12.1}$$

whenever the two means are independent. Equation (12.1) is similar to the equation from chapter 9, the $SE_{\hat{p}_1 - \hat{p}_2} = \sqrt{(SE_{\hat{p}_1^2} + SE_{\hat{p}_2^2})}$, for proportions.

Now, applying the equation from chapter 10, $SE_{\bar{X}}$ which is calculated as $\frac{s}{\sqrt{n}}$ twice, we get

The standard error of the difference between two independent means.

$$SE_{(\bar{x}_1 - \bar{x}_2)} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \tag{12.2}$$

Continuing with the example, let $\bar{X}_1 = 2.98$ and $\bar{X}_2 = 2.90$. Then the sample standard deviations are $s_1 = 0.45$ and $s_2 = 0.40$. The sample sizes are $n_1 = 100$ and $n_2 = 100$.

$$SE_{(\bar{x}_1 - \bar{x}_2)} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{0.45^2}{100} + \frac{0.40^2}{100}} = 0.06$$

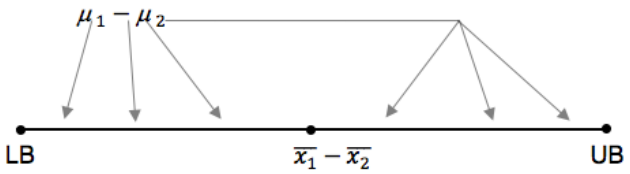
Therefore, we can state the conclusion of the study as follows: "The average GPA of WMU students today is .08 higher than 10 years ago, give or take .06 or so." We also could have used equation (12.1) instead of (12.2) in calculating the standard error:

$$
\begin{aligned}
SE_{\bar{X}_1} &= \frac{s_1}{\sqrt{n_1}} = \frac{0.45}{\sqrt{100}} = 0.045 \\
SE_{\bar{X}_2} &= \frac{s_2}{\sqrt{n_2}} = \frac{0.40}{\sqrt{100}} = 0.040 \\
SE_{\bar{X}_1 - \bar{X}_2} &= \sqrt{0.045^2 + 0.040^2} = 0.06
\end{aligned}
$$

12.2.1 Using a confidence interval

The following two sections discuss the formulas and concepts necessary for calculation and interpretation (respectively) of confidence intervals on the difference between two independent means. Let's start with the concepts and then proceed to some formulas and examples.

Confidence Interval - Concepts:



The interval between lower and upper bounds (LB, UB) includes some possible values of $\mu_1 - \mu_2$ (depicted by the many arrows). We need an interval around the central estimate $\bar{x}_1 - \bar{x}_2$ because of variation between samples and populations (which was depicted using concentric circles at the start of this chapter). This interval can be thought of as the "wiggle room" needed to estimate $\mu_1 - \mu_2$ using only $\bar{x}_1 - \bar{x}_2$.

Here we show how to use this interval, and it is necessary to talk about some basic properties of a difference first. For any two numbers A and B, there are three possibilities when evaluating their difference:

1. If $A - B$ is a positive number, then A is greater than B. Consider the numbers 4 and 3. If we take $4 - 3 = 1$, the answer is greater than zero.

2. If $A - B$ is a negative number, then B is higher than A. For instance if we take $3 - 4 = -1$, the answer is negative.

3. If $A - B$ is 0, then A = B. Consider the numbers 4 and 4: $4 - 4 = 0$.

The same kind of reasoning holds for all the possible values of $\mu_1 - \mu_2$ between LB and UB depicted above:

1. If all the values from LB to UB are positive, then μ_1 is significantly greater than μ_2.

2. If all the values from LB to UB are negative, then μ_1 is significantly less than μ_2.

3. If zero is between LB and UB (inclusive), then μ_1 is not significantly different from μ_2.

Note that this is to say that the same reasoning and terminology outlined in the section on statistical significance in chapter 9 apply in the new case of a difference. If the confidence interval for $\mu_1 - \mu_2$ does not contain 0, then 0 has been effectively excluded from the range of possible values.

When the confidence interval for $\mu_1 - \mu_2$ does not contain **zero**, we say that the difference is statistically **significant**.

Confidence Interval - Calculations and Examples:

The difference of two means is a random variable with expected value and spread. The 68% and 95% rules apply, i.e. the estimated difference of $\bar{x}_1 - \bar{x}_2$ should be within 1 SE of the true value 68% of the time, and within 1.96 SE's 95% of the time. Following the usual reasoning,

$$(\bar{X}_1 - \bar{X}_2) \pm 1.96 SE_{(\bar{X}_1 - \bar{X}_2)}$$

should contain the true difference $(\mu_1 - \mu_2)$ with 95% confidence. Substituting (12.2), we get the following formula.

95% confidence interval for $(\mu_1 - \mu_2)$:

$$(\bar{X}_1 - \bar{X}_2) \pm 1.96 \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \tag{12.3}$$

For grade inflation, we have

$$(2.98 - 2.90) \pm 1.96 \sqrt{\frac{0.45^2}{100} + \frac{0.40^2}{100}}$$

$$0.08 \pm 1.96(0.06)$$

$$(-0.04, 0.20)$$

We say that the difference in GPA averages is between –.04 and .20 with 95% confidence. Note that 0 has not been excluded, making simple chance variability a viable explanation for the observed difference.

Table 12.1: Mean Changes (SD) in Body Mass Index by Diet Group & Time

Time (months)	Atkins ($n = 77$)	Zone ($n = 79$)	Ornish ($n = 76$)
2	-1.60(0.98)	-0.76(0.99)	-0.95(0.90)
6	-2.16(2.14)	-0.73(0.90)	-0.85(1.60)
12	-1.65(2.54)	-0.53(2.00)	-0.77(2.14)

12.2.2 Statistical Significance

Let us revisit the diet study mentioned earlier. The following table contains the mean changes in body mass index (weight in kilograms divided by height in meters squared) for the Atkins, Zone and Ornish diets. Now, compare the Atkins and Zone diets at 12 months:

On the average, Atkins lost how much more body mass index points than Zone? The estimate of $(\mu_1 - \mu_2)$ is

$$(\bar{X}_1 - \bar{X}_2) = (-0.53) - (-1.65) = 1.12$$

The standard error is

$$SE_{(\bar{X}_1 - \bar{X}_2)} = \sqrt{\frac{(2.00)^2}{79} + \frac{(2.54)^2}{77}} = 0.367$$

The 95% confidence interval for $(\mu_1 - \mu_2)$ is

$$1.12 \pm 1.96(0.367)$$

$$(0.84, 1.84)$$

Thus, even allowing for 1.96 SE's of chance variability, Atkins lost at least .40 more body mass index points than Zone (and could be as large as 1.84). When the confidence interval for $(\mu_1 - \mu_2)$ does not contain zero, we say that the difference is statistically significant.

12.2.3 The P-value

Continuing the example of the previous section, we might ask "Can't the difference in averages $\bar{x}_1 - \bar{x}_2 = 1.12$ be explained by chance variability, rather than diet effect?"

The answer is "Yes, 1.12 can occur by chance, but with very tiny probability." How small? Well, if the actual difference were 0, and the SE is 0.367, then the value 1.12 is

$$\frac{1.12}{0.367} = 3.05$$

SE's from the expected value using the normal curve, random variables fall as far as 3.05 or more SE's from the expected value with approximately 0.0022 probability. Since this number (also called the P-value) is quite small, it makes it hard to believe that the

actual difference is zero. Hence, we conclude that statistically, the two means are different. Alternatively, we can say that the means are *significantly* different.

12.3 Paired data (before-and-after)

In this section, we will discuss a common problem in data analysis: comparing before and after measurements. Consider the possible weight loss data in Table 12.2.

Table 12.2: Weight in pounds before and after after 12 months on diet

Subject	Before	After
1	180	155
2	192	187
3	205	194
4	166	176
5	220	205
6	177	172
7	189	173
Ave:	189.9	180.3
SD:	18.1	16.4

Using the notation of the previous section estimating the difference between independent means, we have

$$\begin{array}{ll} \bar{X}_1 = 189.9 & \bar{X}_2 = 180.3 \\ s_1 = 18.1 & s_2 = 16.4 \\ n_1 = 7 & n_2 = 7 \end{array}$$

What is the estimate of mean weight change after 12 months? $\bar{x}_1 - \bar{x}_2 = 9.6$ pounds, right? What is the standard error of this estimate? Using equation (12.2)

$$SE_{(\bar{x}_1 - \bar{x}_2)} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{(18.1)^2}{7} + \frac{(16.4)^2}{7}} = 9.2 \text{ pounds}$$

right? **wrong!**

We should not use equations (12.1) or (12.2) because the two means are **not** independent, i.e., they are not calculated on independent samples. The use of the plural 'samples' is in itself wrong because we do not have two samples, we only have one! We need to watch out for this. How many samples are there? Before-and-after data generally consist of only one sample of subjects, each measured twice.

So how do we calculate an estimate and standard error of average weight loss? By calculating the amount of change from Before to After. The computed value amounts to taking differences, as shown in Table 12.3.

Table 12.3: Weight in pounds before and after after 12 months on diet

Subject	Before	After	Difference
1	180	155	25
2	192	187	5
3	205	194	11
4	166	176	-10
5	220	205	15
6	177	172	5
7	189	173	16
Ave:			9.6
SD:			11.1

Compare Table 12.3 to Table 12.2. We have reduced the summary statistics to a single sample, appropriately. The relevant statistics are now:

$$\bar{X} = 9.6, s = 11.1, n = 7$$

What does the sample mean $\bar{X} = 9.6$ estimate? It estimates the average change, right? To be specific, it estimates the average weight loss from Month 0 to Month 12. What is the standard error of the estimate? Since it is just another average, the appropriate procedure is given by

$$SE_{\bar{X}} = \frac{s}{\sqrt{n}} = \frac{11.1}{\sqrt{7}} = 4.2 \text{ pounds}$$

Completing the analysis, the 95% confidence interval for average change is

$$9.6 \pm 1.96(4.2)$$

$$(1.4, 17.8)$$

Since the interval does not contain zero, the average weight change is statistically significant.

12.3.1 Paired Data

Paired data are data in which natural matchings occur. For example, when researchers collect two measurements (one before treatment and one after treatment) from a subject, then we have paired data, and the analysis should follow as described above.

For example, we might want to compare the head injury of drivers versus passengers in car crashes. In a study, automobiles were crashed into a wall at a speed of 35 MPH with dummies in the driver and front passenger seat. The head injury criterion (HIC) was measured. Following is a selection of cars and their HIC values.

Company	Driver	Passenger
Acura Integra 87	599	597
Audi 80 89	600	515
Chevrolet Camaro 91	585	583
Ford Escort 87	551	418
Honda Accord LX 91	562	539
Toyota Corolla Fx 88	593	397
Volvo 740 GLE 88	519	445

Since we uniquely match each pair of observations (e.g., 599 and 597) to each other (driver and passenger HIC in the same car and crash), this is paired data and deserves paired data analysis.

12.4 Key Words

- independent sample experiment
- paired difference experiment
- confidence intervals
- sampling distribution

12.5

Exercises

Exercise 12.5-1: Do credit cards with no annual fee charge higher interest rates (APR) than cards that have annual fees? Among 29 cards surveyed, 17 had no annual fees while 12 charged an annual fee. Among the cards with no annual fee, the average APR was 19% (SD=8%). Among cards with an annual fee, the average APR was 17% (SD=3%).

1. Estimate the difference in APR.
2. Calculate a standard error for your estimate in (1).
3. Calculate a 95% confidence interval for the difference in APR between the two groups.
4. Are interest rates significantly higher for cards with no annual fees?
5. What is the P-value for comparing the two averages? Is the difference significant?

Exercise 12.5-2: 100 students graduating
with Bachelor degrees in engineering make an average of $70,000 with a standard deviation of $5,000 when entering the workforce. 68 students graduating with Bachelor
degrees in statistics make an average of
$65,000 with a standard deviation of $3,000 when entering the workforce. Assuming the two samples of students are independent, answer the following questions:

1. What is the difference between the sample averages (engineers – statisticians)?
2. What is the standard error of the estimate for the true difference in average entrance salaries between all engineering and statistics BA graduates?
3. What is a 95% confidence interval for the difference in sample averages?
4. Do these statistics suggest that average entrance salaries for stats vs. engineering students are significantly different?

Exercise 12.5-3: A Junior at Southwest Michigan college is debating whether to pursue an MBA after her Bachelor degree in management. She interviews some people she knows in the workforce and was able to obtain their salaries. The annual salaries (in dollars) are summarized in the following table:

Degree	Aver	SD	Sample Size
Bachelor	$48,286	$416	12
Master	$59,496	$675	7

1. Estimate the difference in average salary between the two groups.
2. Calculate a standard error for our estimate in (1).
3. Calculate a 95% confidence interval for the average difference in salary.
4. Are MBA salaries significantly higher?
5. What is the P-value for comparing the two averages? Is the difference significant?

Exercise 12.5-4: A group of charity workers employed by a large foundation track the donations made to the foundation every month. They note that the average donations for August were somewhat higher than the average donations for September, and they want to know if this fact should worry them moving forward. Performing the correct statistical test could determine whether the average difference in donations between August and September was significant. However, they must first decide whether their August and September donations are independent or not. If we were advising the charity workers, what would we tell them?

Exercise 12.5-5: A new gasoline additive is supposed to make gas burn more cleanly and increase gas mileage in the process.
Consumer Protection Anonymous conducted a mileage test to confirm this. They took seven of their cars, filled it with regular gas, and drove it on I-94 until it was empty. They repeated the process using the same cars, but using the gas additive. The recorded gas mileage follows:

Additive	1	2	3	4	5	6	7
Without	22	15	18	28	12	25	18
With	26	19	17	34	17	25	22

1. Calculate the mean difference in mileage between the two fuel types.

2. Calculate a standard error for our estimate in (1).

3. Calculate a 95% confidence interval for the mean
mileage difference.

4. Does the data support the claim of higher gas mileage?

Exercise 12.5-6: The group of charity workers from the above question decides on a statistical test based on the sage wisdom we previously offered. They perform a test for significance of the average difference, and it yields a p-value of 0.50. As their stats advisor, how would we interpret this p-value concerning whether the average difference in donations for August versus September was significant?

Exercise 12.5-7: Suppose a shoe company wants to test material for the soles of shoes. For each pair of shoes the new material is placed on one shoe and the old material is placed on the other shoe. After a given period of time a random sample of 16 pairs of shoes is selected. The wear is measured on a 10 point scale (higher is better) with the following results. The average of the differences is $\bar{X}_n - \bar{X}_o = 0.4$ and it standard deviation is $s_{diff} = 1.6$.

1. Determine the mean difference in the sole-wear between the two material types.

2. Calculate a standard error for our estimate in (1).

3. Calculate a 95% confidence interval for the mean sole-wear difference.

4. Does the data support the claim that the new material gives superior wear?

Exercise 12.5-8: Ramp metering is an engineering experiment that studies how automobiles enter an expressway to stop for a

	RMon	RMoff
Mean	40.67	34.53
SD	10.040	9.561
n	15	15

Table 12.4: Ramp Metering

short time before they join the flow of traffic. The theory is that ramp metering directs the number of vehicles on the expressway and the number of vehicles accessing the expressway, resulting in a freer flow, which results in a faster travel times. To prove the hypothesis whether ramp metering affects travel time, traffic engineers in Kalamazoo, Michigan, set up an experiment on a section of expressway where ramp meters were installed. The response variable was the speed of vehicles. A random sample of 15 vehicles on the expressway for a Tuesday at 5 p.m. with the ramp meters on and a second random sample of 15 vehicles on a different Tuesday at 5 p.m. with the ramp meters off resulted in the following speed summaries(in miles per hour).

1. Determine the mean difference between ramp meters on and ramp meters off.

2. Calculate a standard error for our estimate in (1).

3. Calculate a 95% confidence interval for the mean ramp meter on-off difference.

4. Does the data support the claim that ramp metering improves traffic flow?

Chapter 13

More Comparing Two Means

13.1 Objectives

After completing this part, students should be able to:

- Set-up and perform hypothesis testing for means
- Explain Type I and Type II errors
- Articulate the conclusions to a broader audience.

13.2 Overview and Example 1

13.2.1 Grade Inflation Revisited

The previous chapter outlines a situation where some grade inflation might occur at WMU over the last ten years. The method used to analyze this situation was a confidence interval. Instead of a confidence interval, let us return to that example, and try a different form of analysis: the hypothesis test using test statistic, and p-value. The new method will have the same result as the confidence interval. However, we must learn both methods since many disciplines use both statistical analyses. Let us review the relevant statistics.

Suppose a random sample of 100 student records from 10 years ago yields an average sample GPA of 2.90 with a standard deviation of 0.40. A random sample of 100 current students today yields a sample average of 2.98 with a standard deviation of 0.45. The difference between the two-sample means is $2.98 - 2.90 = 0.08$. We will use the following notation: $n_1 = 100$, $\bar{x}_1 = 2.90$, $s_1 = 0.40$, $n_2 = 100$, $\bar{x}_2 = 2.98$, and $s_2 = 0.45$.

What are the two possibilities here?

1. There is no grade inflation at WMU, our sample is in error.

2. There is grade inflation at WMU, our sample is not in error.

Although we cannot say for sure which of the possibilities is true, we can calculate the probability of each possibility. To do so, we will (as always) assume the normal distribution holds, assume the two means are independent, calculate a standard error, calculate a test statistic, and find a p-value.

13.2.2 Define Hypotheses Testing

Definition: Hypotheses testing is a process of using sample data and probability to test the characteristics of one or more populations. In the present example, the populations are all the students at WMU during the first sampling, and all students at WMU during the second sampling.

Here is how we use hypotheses tests:

1. Make a statement about the nature of the population(s).
2. Formulate an Analysis Plan.
    - Sampling Distribution is approximately normal
    - Choose a level of significance equal to 0.05.
    - Decide the number of observations.
    - Select a test method.
3. Analyze the data.
4. Interpret the Results.
5. Communicate our results to the audience.

Definition:

The Null Hypothesis (H_0) is a statement about the status quo. It says there is no difference.

Definition:

The Alternative Hypothesis (H_a) is a statement that contradicts the (H_0). It says there is a difference.

If the null hypothesis is true, then in a sense nothing special is going on. In the present example, the null hypothesis is that there is no grade inflation at WMU. Can we reject this null hypothesis and state that it is probably the case that there is grade inflation at WMU? If so, then we would be affirming the alternative hypothesis We will find out soon. For now let us write that:

$$H_0 : \mu_1 = \mu_2$$
$$H_a : \mu_1 \neq \mu_2$$

The greek letter μ is a common symbol for a population mean. This is convenient notation that really says the same thing as the above text: the null hypothesis is that the two population means are equal, or that there is no difference between WMU grades at time 1 and time 2; the alternative hypothesis is that the two population means are not equal, or that there is a difference between WMU grades at time 1 and time 2. Often in the real world of statistical analyses, the statement of the alternative hypothesis is a little more difficult. An alternative hypothesis could be $<$, $>$, or $\neq$, i.e., it could be either one or two-tailed, but in our course we keep things simple for you and simply always use $\neq$.

13.3 Calculation and Analysis for Example 1

When the difference, between the observed means of sample A and sample B, is similar; we can expect no significance, i.e., $(\bar{x}_1 - \bar{x}_2) \sim 0$. On the other hand, when the means are dissimilar, we should expect a significant difference. The bigger the difference, the more likely we are to find significance and conclude with the alternative hypothesis. Recalling that our difference in the grade inflation example is $(2.98 - 2.90) = 0.08$, we might expect there really is not a significant difference here, but let us perform the full analysis to find out.

1. State the hypotheses
   - $H_0 : \mu_1 = \mu_2$, there is no grade inflation.
   - $H_a : \mu_1 \neq \mu_2$, there is grade inflation.
2. Formulate an Analysis Plan:
   (a) We assume the sampling Distribution is approximately normal
   (b) Choose a level of significance equal to 0.05 (this could be any value ≤ 1, but again, we will keep things simple for this course and always select 0.05 or 95 percent confidence).
   (c) Determine the number of observations: $n_1 = 100, n_2 = 100$.
   (d) Selecting a standard error based on the above assumptions:

$$\begin{aligned} SE_{\bar{x}_1 - \bar{x}_2} &= \sqrt{SE_{\bar{x}_1} + SE_{\bar{x}_2}} \\ &= \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \end{aligned}$$

(e) Selecting a test method (statistic):

$$\begin{aligned} Z_0 &= \frac{\bar{x}_1 - \bar{x}_2}{SE_{\bar{x}_1 - \bar{x}_2}} \\ &= \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \end{aligned}$$

3. Analyze the data. Let us calculate the above:

$$\begin{aligned} Z_0 &= \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \\ &= \frac{2.98 - 2.90}{\sqrt{\frac{0.45^2}{100} + \frac{0.40^2}{100}}} \\ &= \frac{0.08}{0.06} \\ &\approx 1.3333 \end{aligned}$$

4. Interpret the Results.

   - The test statistic is relatively small at $Z_0 = 1.3333$. Recall that we decided on a significance level (α) of 0.05. This level of significance implies we'd need at least $|Z| = 1.96$ to conclude there is a significant change between the two sampling times.
   - We cannot conclude a significant difference, so we do not reject the null hypothesis. Our sample does not show there is significant grade inflation at WMU.
   - Note: We have to be careful here. This is not quite the same as saying there is no grade inflation at WMU. Instead, we say that it's not likely there is grade inflation given our sample data. We just don't have enough evidence for the alternative hypothesis that there is grade inflation.
   - Could we have used the p-value instead to decide whether to reject the null hypothesis? Yes. The p-value of these two-tailed tests is the probability of obtaining an $|Z|$ value larger than that of the test statistic. In notation: p-value $= P[|Z| > |Z_0|]$. This probability can be obtained from the Z-table by finding the area of the curve less than a negative Z_0 or greater than a positive Z_0, then multiplying this area by 2. In our example, the probability obtained from the Z-table using $Z_0 = 1.33$ is 0.0918. p-value $= 2 \times 0.0918 = 0.1836$. Recall now that our significance level was set at 0.05 at the outset of this problem. We use the decision rule that the

p-value must be less or equal to the significance level in order to reject the null hypothesis. Since our p-value 0.1836 is much greater than 0.05, therefore, we do not reject the null hypothesis (as we expected). Once again we conclude there is not enough evidence to decide there is grade inflation at WMU, whether using the test statistic, the p-value, or the previous chapter confidence interval.

13.4 Example 2

13.4.1 The Question

In their paper *Modeling wine preferences by data mining from physicochemical properties*, [1] Cortez et al. present a dataset on wine qualities and preferences. Many interesting variables are included such as the pH, fixed acidity, alcohol content, and quality score for thousands of sampled wines. For the remainder of this chapter, we will use this dataset to compare means

So what questions can we ask about the wines using this dataset? There are many interesting topics to choose from since the dataset contains 13 variables across 6497 different sampled wines. Let's start with grouping by color and go from there; the dataset contains 1599 red wines and 4898 white wines. Do you think there may be a difference in the fixed acidity of red vs. white wines? Let's find out.

The Question:

Do the averages of fixed acidity for white vs. red wines significantly differ?

13.4.2 Working Through Example 2

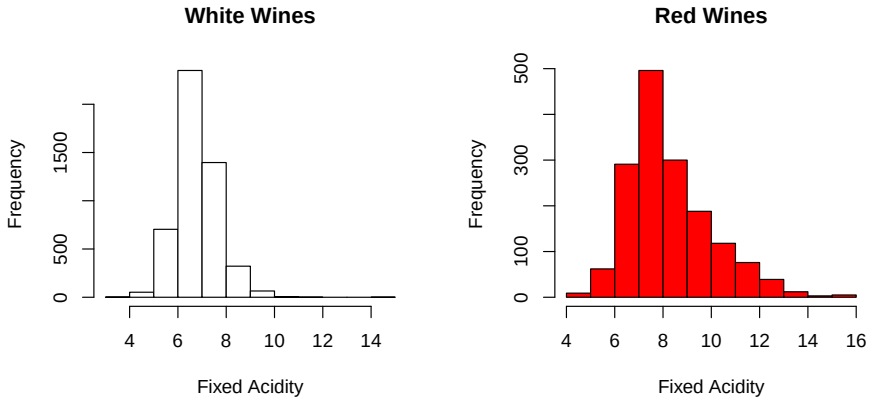
1. Make a statement about the nature of the population(s).

   We did this above. Let's note here that the two populations, red vs. white wines, are independent of one another as required by the mathematics taught here.

2. Formulate an Analysis Plan.

   - Sampling Distribution is approximately normal
     Are the samples roughly normal? Let's check by constructing histograms:

[1] P. Cortez, A. Cerdeira, F. Almeida, T. Matos and J. Reis. Modeling wine preferences by data mining from physicochemical properties. In Decision Support Systems, Elsevier, 47(4):547-553, 2009.



Although a little right skewed, we can treat these as normal for our purposes. It is always a good idea to check the data before proceeding. We also know the samples are separate.

- Choose a level of significance equal to 0.05.

  Recall this implies that we'll use a benchmark $|Z| = 1.96$ to draw our conclusions at the end.

- Determine the number of observations.

  As noted above, there are 6497 different sampled wines in this dataset. 1599 are red, and 4898 are white.

- Select a test method.

  We'll use the same Z_0 as before:

$$\begin{aligned} Z_0 &= \frac{\bar{x}_1 - \bar{x}_2}{SE_{\bar{x}_1 - \bar{x}_2}} \\ &= \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \end{aligned}$$

3. Analyze the data.

   - $H_0 : \mu_w = \mu_r$, there is no difference in fixed acidity between red and white wines.
   - $H_a : \mu_w \neq \mu_r$, there is a difference in fixed acidity between red and white wines.

   We will use R software to calculate the necessary sample statistic:

   Finally we calculate the test statistic:

Table 13.1: Summary Statistics

	Wines	
	White(w)	Red(r)
Mean	6.8547877	8.3196373
SD	0.8438682	1.7410963
N	4898	1599

$$\begin{aligned} Z_0 &= \frac{\bar{x}_w - \bar{x}_r}{\sqrt{\frac{s_w^2}{n_w} + \frac{s_r^2}{n_r}}} \\ &= \frac{6.85 - 8.32}{\sqrt{\frac{0.844^2}{4898} + \frac{1.741^2}{1599}}} \\ &\approx -32.42 \end{aligned}$$

Note that this time we obtained a negative Z_0 because of the way we took the difference in the numerator. There is nothing wrong with this. We could have switched the numerator around and obtained its positive counterpart just as easily.

4. Interpret the Results.

   This $Z_0 = -32.42$ is an extreme Z value. It falls very far off the Z table. This is much less than $Z = -1.96$, so we reject the null hypothesis with great force. We might also consider the p-value. Although the p-value cannot be obtained from the Z-table since it does not go beyond -4, we can use R software to calculate it. The p-value is 1.43e-230, diminishingly smaller than 0.05, so we reject the null hypothesis. The fixed acidities are quite different given the large sample size. We are much more than 95 percent confident that white wine and red wine differ in their fixed acidities according to this analysis.

5. Communicate our results to the audience.

   Many different options are available to you here such as the histograms, sample statistics, test statistic, and p-value. Perhaps the most difficult point to make is the significance of our Z_0 since it is quite far from zero, and the p-value quite small. To help with this, we can say we are over 30 standard errors from no difference between these two means! We could also say that the p-value has over 200 zeros past the decimal!

13.4.3 Large Samples, Free Data, and Free Software

The wine dataset used above is freely available online and fun to play with. One reason it was selected for this chapter was to demonstrate the impact a larger sample can have on

your analysis. As you saw above, a large sample tends to make even small differences appear extremely significant. Another reason it was selected was to demonstrate the computational power you can get out of the free R software used to do the calculations. The reader is highly encouraged to download both the dataset and R software to do some explorations of their own. Please do not hesitate to ask your instructor for help downloading or using R during your coursework with us.

13.5 Key Words

- Dependent
- Independent
- Sampling Distribution
- Normal Distribution
- Hypothesis Testing
- Null Hypothesis
- Alternative Hypothesis
- Standard Error
- Test Statistic
- P-value
- Significance Level
- Two Sample Means

13.6

Exercises

Exercise 13.6-1: The wine quality dataset used in this chapter includes a variable on alcohol content. The average alcohol content of red wine is 10.423, while that of white wine is 10.5143. The respective standard deviations are 1.0657 and 1.2306. The respective sample sizes are 1599 and 4898.

- Set up a hypothesis test. What are the two hypotheses?
- What is the difference between means?
- What is the standard error of the difference?
- What is the test statistic Z_0?
- What is the p-value?
- What should you conclude?

Exercise 13.6-2: The wine quality dataset used in this chapter includes a variable on quality. The average quality score of red wine is 5.636, while that of white wine is 5.8779. The respective standard deviations are
0.8076 and 0.8856. The respective sample sizes are 1599 and 4898.

- Set up a hypothesis test. What are the two hypotheses?
- What is the difference between means?
- What is the standard error of the difference?
- What is the test statistic Z_0?
- What is the p-value?
- What should you conclude?

Exercise 13.6-3: True or False? Suppose a two-tailed Z_0 test statistic is calculated and it is positive. Then the p-value will be the area below this Z_0.

Exercise 13.6-4: True or False? Suppose a two-tailed Z_0 test statistic is calculated and it is negative. Then the p-value will be the area below this Z_0.

Exercise 13.6-5: True or False? If we obtain a test statistic that is either much greater than 1.96 or much smaller than -1.96, then we fail to reject the null hypothesis.

Exercise 13.6-6: True or False? If we obtain a p-value that is much larger than 0.05, then we fail to reject the null hypothesis.

Exercise 13.6-7: 100 students graduating
with Bachelor degrees in engineering make an average of $70,000 with a standard deviation of $5,000 when entering the workforce. 68 students graduating with Bachelor
degrees in statistics make an average of
$65,000 with a standard deviation of $3,000 when entering the workforce. Assuming the two samples of students are independent, answer the following questions:

- Set up a hypothesis test. What are the two hypotheses?
- What is the difference between means?

- What is the standard error of the difference?
- What is the test statistic Z_0?
- What is the p-value?
- What should you conclude?

Chapter 14

Categorical Variables: Association or Independence

14.1 Objectives

After completing this part, students should be able to:

- Compute expected frequencies.
- Understand the difference between association and independence.
- Test for a statistical association.

14.2 Association versus independence in an r x c table

Is there an association between gender and height? Yes, males tend to be taller than females. A more formal way of saying this is 'height distribution for males tends to be different from females.' Is there an association between shoe size and height? Yes, 'height distribution for men who wear size 12 is different from those who wear size 8.' Is there an association between GPA and height? No, 'height distribution tends to be the same for 3.0 students as well as 3.5 students.'

Two variables A and B are said to be **associated** if the distribution of B tends to change with the level of the A variable. Otherwise, they are said to be **independent** variables.

Therefore, height is associated with gender and shoe size, but independent of GPA.

If we are thinking, "association and independence are the same," we are almost correct. The difference is about design. In the test of independence, we collect observational units at random from a population, and the two categorical variables are observed for each unit.

In the test of association, we collect the data by randomly sampling from each sub-group **separately**. (Say, 100 Democrat, 100 Republican, 100 Independent, and so on.) The null hypothesis is that each sub-group shares the same distribution of another categorical variable. (Say, "chain smoker", "occasional smoker", "non-smoker".) The difference between these two tests is subtle yet important.

Now consider the following 3 by 4 table. Researchers followed 189 students entering a business school program as part of attrition (i.e., drop out, transfer) study. The students were cross-classified according to 4 categories of high school GPA ($2.0 - 2.5, 2.5 - 3.0, 3.0 - 3.5, 3.5 - 4.0$) and three categories of attrition outcomes ('did not return for the 2nd year,' 'returned for second but not for a 3rd year,' 'returned for 3rd year'). Is there an association between HS GPA and college attrition?

Table 14.1: Retention versus HS GPA

	GPA			
Returned	2.0 – 2.5	2.5 – 3.0	3.0 – 3.5	3.5 – 4.0
No –2nd yr	25	3	4	6
No – 3rd yr	14	7	4	6
Yes – 3rd yr	41	7	28	44

To analyze whether attrition and GPA are independent, we will analyze whether attrition distribution remains the same regardless of GPA level. Let us start by looking at the 1st column (worst HS grades) and 4th column (best HS grades). Do the distributions look the same? The answer seems to be 'no' - a bigger proportion of the 1st column never returned for their second year. In other words, the value '25' in the very first cell is too large, implying that 'poor grades seems to be associated with first-year attrition.' If grades and attrition were independent, Table 14.1 should have looked more like Table 14.2.

Table 14.2: Expected counts (if independent)

	GPA			
Returned	2.0 – 2.5	2.5 – 3.0	3.0 – 3.5	3.5 – 4.0
No –2nd yr	16	3	7	11
No – 3rd yr	13	3	6	9
Yes – 3rd yr	51	11	23	36

The table shows expected counts under independence. Observe that the row and column totals of the two tables are the same

Furthermore, note that 20.1% of the data is in the first row, 16.4% in the second row, and 65.5% in the third row. If we apply the same percentage breakdown to each column, we get

Rounding off gives us the expected frequencies in Table 14.2.

Table 14.3: Expected counts (if independent)

	GPA					
Returned	2.0 – 2.5	2.5 – 3.0	3.0 – 3.5	3.5 – 4.0	Total	Percent
No – 2nd yr					38	(20.1%)
No – 3rd yr					31	(16.4%)
Yes – 3rd yr					120	(63.5%)
Total	80	17	36	56	189	(100%)

$80 \times .201 = 16.08$	$17 \times .201 = 3.42$	$56 \times .201 = 7.24$	$56 \times .201 = 11.26$
$80 \times .164 = 13.12$	$17 \times .164 = 2.79$	$56 \times .164 = 5.90$	$56 \times .164 = 9.18$
$80 \times .635 = 50.80$	$17 \times .635 = 10.80$	$56 \times .635 = 22.86$	$56 \times .635 = 35.56$

14.2.1 Testing for statistical association

Statisticians will conclude 'independence' if Tables 14.1 and 14.2 are close and conclude 'association' if they are far from each other. We measure closeness and fairness by subtraction and squaring, as follows:

$$\chi^2 = \frac{(25-16.08)^2}{16.08} + \frac{(3-3.42)^2}{3.42} + \cdots + \frac{(44-35.56)^2}{35.56} = 23.42$$

Note that if Tables 14.1 and 14.2 are the same, then the χ^2 statistic (pronounced 'chi-square') in (14.1) will be zero (0). If the two tables are far apart, the χ^2 statistic will be large. Statisticians use the following rule.

If the $\chi^2 > b$, then conclude statistic association

Otherwise, conclude independence. The number b is called a critical value and depends on the dimensions of the table. Let r be the number of rows, and c be the number of columns. Let

$$df = (r-1) \times (c-1)$$

be a parameter called the degrees of freedom. Then b is given by the following table:

df	1	2	3	4	5	6	7	8	9	10
b	3.84	5.99	7.81	9.49	11.07	12.59	14.07	15.51	16.92	18.31

In our example on grades and attrition, we have r=3 rows and c=4 columns, so that

$$df = (3-1) \times (4-1) = 6$$

so, the line between statistical association and independence is drawn at b=12.59. Since $\chi^2 = 23.42$ from (14.1), then $\chi^2 > 12.59$. We conclude that there is significant association between high school GPA and college attrition rate.

14.3 Key Words

- chi-square test for independence
- contingency table

14.4

Exercises

Exercise 14.4-1: In a study of drug usage by students at a large university, data was obtained regarding hard liquor experience of smokers and nonsmokers.

Hard-Liquor Use	Nonsmokers	Smokers
Once or more	15	23
Never	56	18

Is liquor use associated with smoking? Conduct a chi-square test to assess significance of association.

Exercise 14.4-2: During the filming of an original comedy special, Netflix monitored whether audience members who received free tickets laughed during the show (LDS). The results follow:

	LDS		
Free Tickets	Yes	No	Total
Yes	17	1	18
No	28	43	71
Total	45	44	89

1. Calculate the expected count for audience members who did not receive a free ticket and did not laugh during the show.
2. Calculate the expected count for audience members who did not receive a free ticket and did laugh during the show.
3. Calculate the expected count for audience members who did receive a free ticket and did laugh during the show.
4. Calculate the expected count for audience members who did receive a free ticket and did not laugh during the show.
5. Calculate the chi-square test statistic.
6. What do you conclude?

Exercise 14.4-3: A study investigating the association between size of cars and country found the following frequency counts:

	US	Japan	UK	France
Economy	21	24	33	55
Compact	27	35	37	40
Full size	36	11	12	4
Luxury	15	3	7	8

Is there evidence of a significant relationship between size of car and country, or are the two variables independent?

Exercise 14.4-4: Suppose Netflix held another special, collected data, and had a statistician calculate and interpret the chi square test statistic. However, this time, the statistician found insignificant differences between observed and expected counts for all those who did and did not laugh with and without free tickets. What is the appropriate conclusion in this case?

Exercise 14.4-5: Computer-controlled cameras are being used to ticket automobile drivers for speeding and running red lights. These devices are operated by private firms and have an incentive to pull in as many drivers as they can. Although approximately 70% of the motorists stoically accept and pay

these tickets, others resent this procedure and fight the ticket. A frequency table with marginal totals is given below.

	Volation		
Ticket	Run Red Light	Speeding	Total
Pay			140
Fight			60
Total	60	140	200

1. Compute the table of expected frequencies.
2. Suppose we know that 1/3 of those who were ticketed for running a red light fought the ticket. Is this enough information to conduct a test of association or independence between the two variables?
3. Using the information in (b), compute the chi-square statistic for testing independence or association between the two variables.
4. What is the correct degrees of freedom to use?
5. What is the conclusion of your test?

Exercise 14.4-6: A recent national poll asked female and male Americans whether they are pro life or pro choice when it comes to abortion issues. The results of the survey are as follows:

	Results	
Gender	Anti-choice	Pro-choice
Women	239	249
Men	196	199

1. What is the hypothesis?
2. Calculate the chi-square test statistic.
3. What are the degrees of freedom?
4. What is the p-value?
5. What is the conclusion of your test?

Chapter 15

Correlation

15.1 Objectives

After completing this part, students should be able to:

- Interpret a scatter plot.
- Compute and interpret the correlation coefficient r

The following data appeared in the Wall Street Journal in 1984. Advertisements were selected by an annual survey conducted by Video Board Tests, Inc., a New York ad-testing company, based on interviews with 20,000 adults who were asked to name the most outstanding TV commercial they had seen, noticed, and liked. We based the retained impressions on a survey of 4,000 adults, in which regular product users were asked to cite a commercial they had seen for that product category in the past week. 'TV Ad Budget' was the 1983 advertising budget in $ millions. 'Impressions' is the estimated number of million impressions per week.

Figure 15.1 shows a scatterplot of Impressions Score (Y) versus Ad Spending (X). Note that the points seem to fall around a line sloped upwards loosely. We say that there is a positive linear association or a linear relationship between spending and the number of impressions made.

If the points fall around a straight line sloped downwards, we say that there is a negative association.

Researchers often express the direction and the strength of association in a single number called the (Pearson) *correlation coefficient*. Typically denoted by r, the correlation coefficient r is a number between -1 and $+1$, inclusive. A value of $r = 0$ means that no linear association exists; the points either look like a random scatter or fall around a horizontal line. A value of $r = +1$ indicates a perfectly linear relationship; all the points fall on a straight line sloped upwards. If $r = -1$, all the points fall on a straight line sloped downwards. The correlation

	TV Ad				
Company	Ad Spending	Impressions	Company	Ad Spending	Impressions
Miller Lite	50.1	32.1	Bud Lite	45.6	10.4
Pepsi	74.1	99.6	ATT/Bell	154.9	88.9
Stroh's	19.3	11.7	Calvin Klein	5.0	12.0
Fed'd Express	22.9	21.9	Wendy's	49.7	29.2
Burger King	82.4	60.8	Polandoid	26.9	38.0
Cola-Cola	40.1	78.6	Shasta	5.7	10.0
McDonald's	185.9	92.4	Meow Mix	7.6	12.3
MCI	26.9	50.7	Oscar Meyer	9.2	23.4
Diet Cola	20.4	21.4	Crest	32.4	71.1
Ford	166.2	40.1	Kibbles 'n Bits	6.1	4.4
Levi's	27.0	40.8			

between TV Ad Budget and Impressions in the data above is +0.65.

Figure 15.2 shows various scatterplots with different correlations. Although 0.5 is halfway between 0 and 1, note that the plot corresponding to $r = 0.5$ barely shows a pattern of association. In practice, plots can show correlations up to 0.3 purely by accident (e.g., the correlation between GPA and, say, shoe size). When correlation reaches +1 or −1, all points fall on a straight line.

15.2 Computing the Pearson Correlation Coefficient

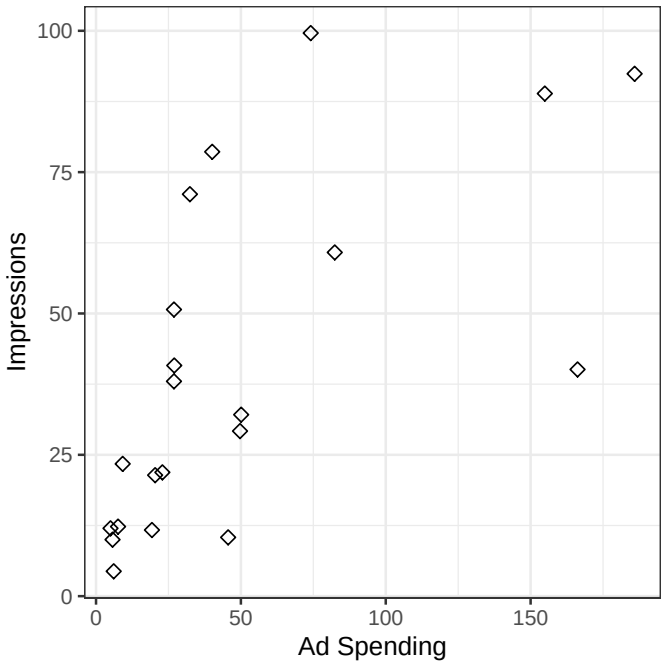
The following numerical example shows how to calculate r. We will break the calculations down into four steps.

1. Calculate the mean and SD of X and Y.

Sample	X	Y
1	1	3
2	3	9
3	4	7
4	4	9
5	5	15
6	7	11
Average	4	9
SD	2	4

2. Calculate the Z-scores from equation (6.1) on page 70.

Figure 15.1: Plot of Impressions vs. Ad Budget



$$Z_x = \frac{X - \bar{X}}{s_x} = \frac{X - 4}{2} \text{ and } Z_y = \frac{Y - \bar{Y}}{s_y} = \frac{Y - 9}{4}$$

3. Multiply the Z-scores and add up.

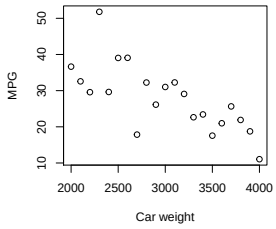
Z_x	Z_y	$Z_x Z_y$
-1.5	-1.5	2.25
-0.5	0	0
0	-0.5	0
0	0	0
0.5	1.5	0.75
1.5	0.5	0.75
Average		3.75

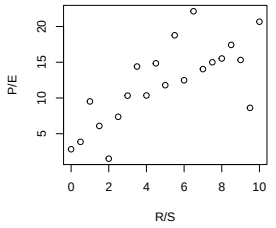
4. Finally, the correlation is the sum divided by $(n-1)$.

$$r = \frac{3.75}{6-1} = 0.75$$

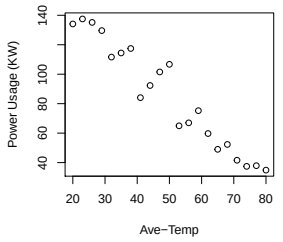
Figure 15.2: Correlation Plots

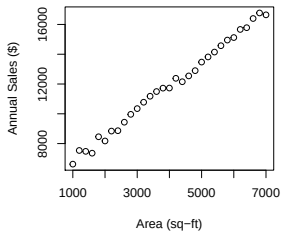
Correlation plots (from top to bottom, r = -0.7302, 0.7322)





Correlation plots (from top to bottom, r = -0.9679, 0.9971)





We summarize the whole process in the following formula.

The Correlation coefficient r:

$$r = \frac{\sum Z_x Z_y}{n-1} \tag{15.1}$$

Consider the Ad Spending example at the start of this chapter. Many of the (X, Y) points are simultaneously above average since companies that have higher than average Advertising Spending also have higher than average Impressions.

Both $X - \bar{X}$ and $Y - \bar{Y}$ are positive for these companies; therefore, Z_x and Z_y are both positive, and the product $(Z_x)(Z_y)$ is positive for these companies. Most of the remaining companies have lower than average Spending and lower than average Impressions. Both Z_x and Z_y are negative for these companies, but the product $(Z_x)(Z_y)$ is still positive! Hence the numerator in (15.1) tends to be a large positive number for the Ad Spending data.

If the points were sloped downwards, then high X-values tend to go with low Y-values, and many points have a negative product $(Z_x)(Z_y)$. This procedure shows how the correlation formula (15.1) works.

If X and Y tend to be simultaneously above average or simultaneously below average, then the correlation coefficient will be **positive**.
If the data is paired above-average X values with below-average Y values, then the correlation coefficient will be **negative**.

15.3 Key Words

- pearson correlation coefficient
- scatter plot

15.4

Exercises

Exercise 15.4-1: Consider the following data.

Table 15.1:

X	Y
-2	0
2	3
5	10
-1	1
6	15

1. Compute the correlation between X and Y.
2. Compute the correlation between Y and X.
3. Add 5 to Y, so the new values are 5, 8, 15, 6, 20. Now compute the correlation between X and Y. Is the correlation smaller, larger, or the same as before?
4. Multiply Y by 5, so the new values are 0, 15, 50, 5, 75. Now compute the correlation between X and Y. Is the correlation smaller, larger, or the same as before?
5. Multiply Y by -1, so that the new values are 0, -3, -10, -1, -15. Now compute the correlation between X and Y. Is the correlation smaller, larger, or the same as before?

Exercise 15.4-2: Suppose we have two variables, X and Y. If an increase in X results in the same increase in Y, then what is the correlation coefficient *r*?

Exercise 15.4-3: Suppose we have two variables, X and Y. If a decrease in X results in the same increase in Y, then what is the correlation coefficient *r*?

Exercise 15.4-4: Suppose that we plot two variables, X, and Y in a scatterplot, and we observe that the points appear to be clustered around a line of best fit that is tilted upward from left to right. What is the possible range of the correlation coefficient *r*?

Exercise 15.4-5: Suppose we have two variables, X and Y. The points (x, y) are plotted in a scatterplot, and it is observed that the points appear to be clustered around a line of best fit that is tilted downward from left to right. What is the possible range of the correlation coefficient r?

Exercise 15.4-6: Suppose we have two variables, X, and Y. The products of all their respective Z scores are calculated, and none of them are negative. What can we conclude about the correlation coefficient *r*?

Exercise 15.4-7: Suppose we have two variables, X and Y. The products of all their respective Z scores are calculated, and all the products are negative. What can we conclude about the correlation coefficient *r*?

Exercise 15.4-8: Research performed at NASA and led by Emily R. Morey-Holton measured the lengths of the right humerus

and right tibia in 11 rats that were sent to space on Spacelab Life Sciences 2. The following data were reported.

Table 15.2: Leg Length: NASA

Right	
Humerus(mm)	Tibia(mm)
24.80	36.05
24.59	35.57
24.59	35.57
24.29	34.58
23.81	34.20
24.87	34.73
25.90	37.38
26.11	37.96
26.63	37.46
26.31	37.75
26.84	38.50

1. Draw a scatter diagram treating the length of the right humerus as the explanatory variable and the length of the right tibia as the response variable.

2. Compute the linear correlation coefficient between the length of the right humerus and the length of the right tibia.

3. Does a linear relation exist between the length of the right humerus and the length of the right tibia?

Chapter 16

Linear Regression

16.1 Objectives

After completing this part, students should be able to:

- Understand the basic concept of simple linear regression.
- Learn to compute the least square point estimates of the slope and y-intercept.
- Compute the confidence interval of the mean.

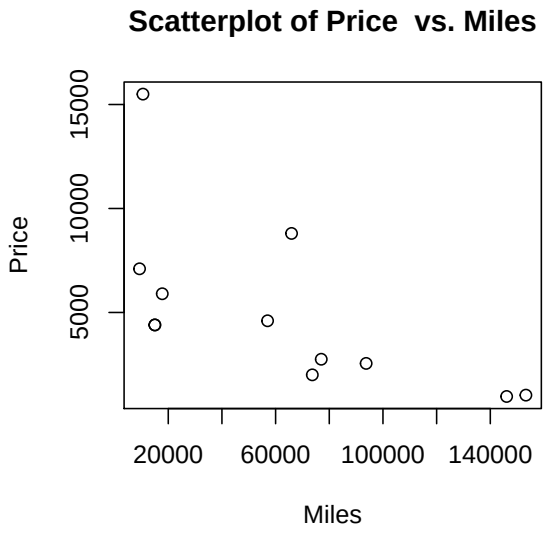
16.2 Simple Linear Regression

The following table contains data on winning bid price for 12 Saturn cars on eBay in July 2002. The car mileage is also given, and the cars have been arranged in increasing order of Miles.

Here is a scatterplot of the data. Since Price depends on Miles (not the other way around), we let Price be the Y-variable, or the *response variable*. Miles is the X-variable, or the *explanatory variable*.

NULL

Car	Miles	Price
1	9300	7100
2	10565	15500
3	15000	4400
4	15000	4400
5	17764	5900
6	57000	4600
7	65940	8800
8	73676	2000
9	77006	2750
10	93739	2550
11	146088	960
12	153260	1025



Problem:

Based on the data, how much do we expect to get for a Saturn car that has been driven 60,000 miles?

Simple linear regression is a data analysis technique that tries to find a *linear* pattern in the data. We then use this line for prediction.

Notice that the points seem to fall around a *straight line* sloping downwards. Can we draw this line? We will discuss one way to do this, called the *least squares* (LS) method. For now, suppose that the LS line has already been computed (we will do this later). The LS line is overlayed on the scatterplot looks like Figure 16.1.

The formula for this line, in the form $Y = a + bX$, is

$$\texttt{Predicted Price} = 8136 + (-0.05127)(\texttt{Miles})$$

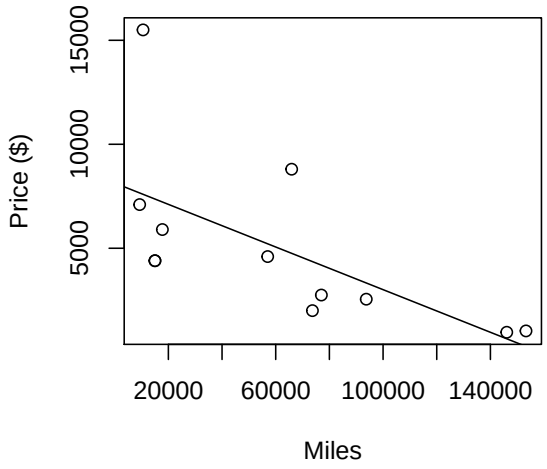
The *slope* of the line is -0.05127, which means that predicted Price tends to drop 5 cents for every additional mile driven or about $512.70 for every 10,000 miles. The *Y-intercept* of the line is $ 8136; this should not be interpreted as the predicted price of a car with 0 mileage because the range of the data does not include cars with 0 miles.

We can now use the line to predict the selling price of a car with 60000 miles. What is the height or Y value of the line at $X = 60000$? The answer is

$$\texttt{Predicted Price} = 8136 + (-0.05127)(60000) = \$5059.80$$

alternately, about $5000 or $5100 or so.

Figure 16.1: Least Squares (LS) Regression Line is overlayed on Scatterplot



16.3 Calculating the Least Squares Regression Line

One way to calculate the regression line is to use these five statistics:

$$\bar{X}, s_x, \bar{Y}, s_y, \text{ and } r$$

(i.e., the mean and SD of X, the mean and SD of Y, and the correlation between X and Y.)

The least square regression line is given by the equation

$$\texttt{Predicted} = a + bX$$

where the slope b and the intercept a are calculated as

$$b = r\frac{s_y}{s_x} \tag{16.1}$$

$$a = \bar{Y} - b\bar{X} \tag{16.2}$$

Next, we will perform the calculations for the Saturn Price data.

Car	Miles	Price ($)
1	9300	7100
2	10565	15500
3	15000	4400
4	15000	4400
5	17764	5900
6	57000	4600
7	65940	8800
8	73676	2000
9	77006	2750
10	93739	2550
11	146088	960
12	153260	1025
Average	61195	4999
SD	50989	4079
r	-0.641	

Using the formulas for slope and intercept in equation (16.1 and 16.2)

$$\begin{aligned} b &= r\frac{s_y}{s_x} = -0.641\frac{4079}{50989} = -0.05127 \\ a &= \bar{Y} - b\bar{X} = 4999 - (-0.05127)(61195) = 8136 \end{aligned}$$

so that the regression line is

$$PREDICTED = a + bX = 8136 + (-0.05127)X$$

Regarding the original variable names, the regression line is

$$\texttt{Predicted Price} = 8136 + (-0.05127)\texttt{Miles}$$

16.4 More on Simple Regression

Why is this called the least squares line? Example best shows the answer.

The first car has $Miles = 9300$. What is its predicted price? The predicted value is

$$\texttt{Predicted Price} = 8136 + (-0.05127)(9300) = 7659.35$$

This predicted value missed the actual selling price $Y = 7100$. By how much? By

Car	Miles	Price ($)	PRED	RES = Y - PRED
1	9300	7100	7659.35	-559.35
2	10565	15500	7594.49	7905.51
3	15000	4400	7367.11	-2967.11
4	15000	4400	7367.11	-2967.11
5	17764	5900	7225.41	-1325.41
6	57000	4600	5213.82	-613.82
7	65940	8800	4755.47	4044.53
8	73676	2000	4358.85	-2358.85
9	77006	2750	4188.13	-1438.13
10	93739	2550	3330.24	-780.24
11	146088	960	646.36	313.64
12	153260	1025	278.66	746.34

$$\texttt{Residual} = 7100 + 7659.35 = -559.35$$

The negative value means actual value is too low. This difference is called the residual.

Small residuals (ignoring the sign) are good because this means the prediction was close (Car 1 above was predicted well, but Car 2 was not – the selling price is almost double what was predicted). Therefore, a prediction line is okay if it gives residuals that are as small as possible.

The sum of squared residuals is

$$SSE = (-559.35)^2 + \cdots + (746.34)^2 = 107805718.50$$

and is a measure of 'overall size' of the residuals. In the Saturn Price data, $SSE = 107,805,718$.

The least square line given by (16.1) will have a smaller SSE than any other straight line.

This means that if you use any other intercept and slope combination besides $(a, b) =$ (8136,.05127), the new set of predicted values and residuals will give an SSE that is larger than, or at best equal to 107,805,718.

16.5 A 95% Confidence Interval for Slope

Is there a linear relationship between X and Y? It seems evident that selling price (Y) responds to a car's mileage (X), but in science, relationships are often not too noticeable and

need confirmation by data. For example, does an individual's systolic blood pressure (Y) tend to increase with their cholesterol level (X)? Is there a relationship between one's total number of years of education (X) and income (Y)? In this section, we will investigate the strength of linear relationships by looking at the slope estimate. Since the slope represents how much Y responds to changes in the X-value, we will calculate a 95% confidence interval for the slope, and examine whether it excludes 0. If it does, then we can rule out the likelihood that the slope is 0. Thus, we conclude that there is a significant linear relationship between X and Y.

We start by stating the formula for standard error:

The slope estimate b tends to miss the true value β by an amount called the *standard error* of the slope, denoted SE of b and calculated as:

$$SE_b = \sqrt{\frac{(1-r^2)s_y^2}{(n-2)s_x^2}} \tag{16.3}$$

The interval estimate is the familiar $b \pm 1.96(SE)$. It is formally calculated as follows.

A 95% confidence interval estimate for the slope of the regression line is given by:
The slope estimate b tends to miss the true value β by an amount called the *standard error* of the slope, denoted SE of b, and calculated as:

$$b \pm 1.96\sqrt{\frac{(1-r^2)s_y^2}{(n-2)s_x^2}} \tag{16.4}$$

If this interval excludes 0, then the likelihood of zero slopes is ruled out, and we conclude that there is a significant linear relationship between X and Y.

Returning to our Saturn car price example, recall that $b = -0.05127$. The standard error of this estimate is

$$SE_b = \sqrt{\frac{(1-(-0.641)^2)(4079)^2}{(12-2)(50989)^2}} = 0.01942$$

The 95% confidence interval is

$$-.05127 \pm 1.96(0.01942)$$

$$(-.09, -.01)$$

Since this interval excludes 0, we conclude a significant relationship between car mileage and selling price.

16.6 Key Words

- slope
- y-intercept
- regression
- residuals
- confidence interval

16.7

Exercises

Exercise 16.7-1: Consider the following data:

X	Y
-2	0
2	3
5	10
-1	1
9	5

1. Calculate the regression line for predicting Y from X.
2. Draw the scatterplot with an overlaid regression line.
3. Add 5 to Y, so the new values are 5, 8, 15, 6, 20. Calculate the new regression line.
4. Multiply Y by 5, so the new values are 0, 15, 50, 5, 75. Calculate the new regression line.

Exercise 16.7-2: Several children are observed, and their ages (in years) and vocabularies (the estimated number of words that each child knows) are recorded. A child psychologist wants to create a model that relates these two variables.

1. Which variable should be explanatory, and which response?
2. A regression equation is calculated as Y = 836X + 451. What is the slope of this regression equation?
3. A regression equation is calculated as Y = 836X + 451. What is the intercept of this regression equation?
4. If a 12-year-old child knows 8,000 words, then what is the residual for this child based on the above regression equation?
5. If a child has a negative residual, then do they fall above or below the predicted number of words known for average children their age (above or below the regression line)?
6. If a 14-year-old child knows 14,000 words, then what is the residual for this child based on the above regression equation?
7. If a child has a positive residual, then do they fall above or below the predicted number of words known for average children their age (above or below the regression line)?

Exercise 16.7-3: Moviegoers are monitored for their level of anxiety while watching a new horror movie billed to be the "scariest movie of all time." The intensity of a scene in the movie and anxiety are both measured on numerical scales of 0 to 100. A producer for the movie finds that the correlation coefficient between intensity and anxiety is 0.63, the standard deviation of intensity is 30, and the standard deviation of anxiety is 35.

1. What is the slope of the regression equation that predicts anxiety based on intensity?

2. How do you properly interpret the slope calculated in part 1?
3. If the intercept is 15, then what is the regression equation?
4. What is the predicted value of anxiety for a scene measured at 90 intensity units?
5. If a moviegoer experiences anxiety measured at 82 units during a scene measured at 90 intensity units, then what is the residual for that moviegoer?
6. How do you properly interpret the esidual in part 5?

Exercise 16.7-4: How well does the number of beers a student drinks predict his or her blood alcohol content (BAC)? Sociology researchers, at Ohio State University, wanted to know if there is a relationship between the amount of beer consumed and BAC. The researchers assigned the number of cans of
beer to each student. After each student had consumed the assigned number of beers, thirty minutes later, an officer of the law measured the students BAC.
Anonymous [2016] One student drank nine beers. You see from the scatter plot that his BAC was about %. A scatterplot of the data appears below.

Coefficients[a]

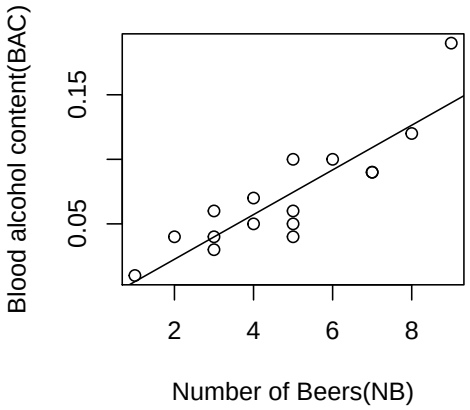
Model		Unstandardized Coefficients		Standardized Coefficients		
		B	Std. Error	Beta	t	Sig.
1	(Constant)	-.012	.013		-.892	.387
	No.beers	.017	.003	.875	6.746	.000

a. Dependent Variable: BAC

1. 0.017
2. 0.17
3. 1.7
4. 17
5. 170

Answer:

Exercise 16.7-5: How well does the number of beers a student drinks predict his or her blood alcohol content (BAC)? Sociology researchers, at Ohio State University, wanted to know if there is a relationship between the amount of beer consumed and BAC. The researchers assigned the number of cans of
beer to each student. After each student had consumed the assigned number of beers, thirty minutes later, an officer of the law measured the students BAC.
Anonymous [2016] A scatterplot of the data appears below. The scatterplot shows



1. a weak negative relationship.
2. a moderately high negative correlation.

3. almost no connection.
4. a small positive correlation.
5. a moderately high positive straight-line relationship between some beers and BAC.

Answer:

Exercise 16.7-6: How well does the number of beers a student drinks predict his or her blood alcohol content (BAC)? Sociology researchers, at Ohio State University, wanted to know if there is a relationship between the amount of beer consumed and BAC. The researchers assigned the number of cans of
beer to each student. After each student had consumed the assigned number of beers, thirty minutes later, an officer of the law measured the students BAC.
Anonymous [2016] A scatterplot of the data appears below. A plausible value of the correlation between number of beers and blood alcohol content, based on the scatterplot, is

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.875[a]	.765	.748	.02184

a. Predictors: (Constant), No.beers

1. $r = -0.875$.
2. $r = -0.765$.
3. r close to 0.
4. $r = 0.765$.
5. $r = 0.875$.

Answer:

Exercise 16.7-7: STEP 1: In the next six tasks, we will use the data from GSS2014 in this exercise. We have been asked to examine the relationship between a person's height and a person's income. Here we have *income* which is an interval-ratio type of variable while *height* is also an interval-ratio type of variable. These types of variables require that we use a Simple Linear Regression (SLR) analysis. So step 1 in the process of analysis is to choose the an independent and dependent variables. Note: In the GSS2014 dataset, *height* is known as HEIGHT and *income* is known as rincom06.

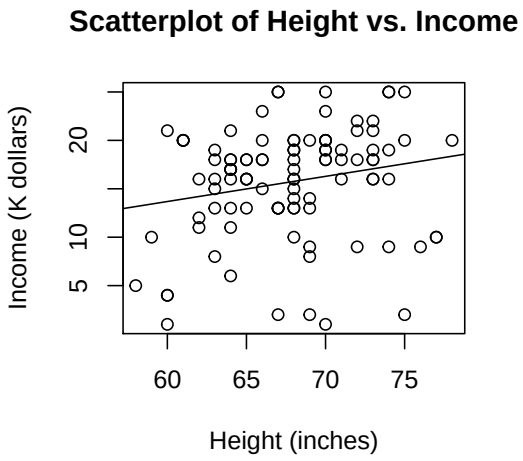


Figure 16.2: Respondent's Height vs. Income

```
Call:
lm(formula = Income ~ Height)

Residuals:
    Min      1Q  Median      3Q     Max
-15.584  -2.505   1.365   3.580   9.495

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -1.9100     8.4638  -0.226   0.8219
Height        0.2599     0.1246   2.086   0.0395 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 5.63 on 98 degrees of freedom
```

```
Multiple R-squared:  0.04253,Adjusted R-squared:  0.03276
F-statistic: 4.353 on 1 and 98 DF,  p-value: 0.03954
```

Answer:

Exercise 16.7-8: STEP 2: Using the information from exercise four, we must state the hypotheses about its relationship between *income* and *height*. Based on our experience, how should we state our hypotheses?

Answer:

Exercise 16.7-9: STEP 3: Using the information from exercise four, determine the coefficient of determination. Recall that this value is also referred to as r-squared.

Answer:

Exercise 16.7-10: STEP 4: Using the information from exercise four, record the independent, dependent variables, and the correlation coefficient.

	Independent
Dep. Var.	1. _____
1. _____	_____

Exercise 16.7-11: STEP 5: Using the information from exercise four, describe the results of the independent variable. Identify variables that we tested, their strength and direction of the relationship. We should distinguish the relationship in general terms and refer the statistical value in parentheses. Also not whether the hypotheses were supported.

Answer:

Exercise 16.7-12: STEP 1: In the next six tasks, we will use a sample of 100 subjects from the GSS2014 data in this exercise. We have been asked to examine the relationship between a person's income, age, and "not married" and a person's happiness. Here we have variables which are interval-ratio type of variable. These types of variables require that we use a Simple Linear Regression (SLR) analysis. So step 1 in the process of analysis is choosing independent and dependent variables. Note: In the GSS2014 dataset, *happy* is known as GENERAL HAPPINESS, *income06* is known as TOTAL FAMILY INCOME, *age* is known as AGE OF RESPONDENT, and *absingle* is known as NOT MARRIED.

	1	2	3	4
1	1.00	-0.08	-0.17	-0.11
2	-0.08	1.00	0.01	0.04
3	-0.17	0.01	1.00	-0.20
4	-0.11	0.04	-0.20	1.00

```
Call:
lm(formula = Happiness ~ Age + Income + NotMarried)

Residuals:
     Min       1Q   Median       3Q      Max
-1.14845 -0.67070  0.07182  0.31814  1.27401

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  2.731628   0.394257   6.929 4.86e-10 ***
Age         -0.003342   0.004490  -0.744    0.458
Income      -0.022483   0.011574  -1.943    0.055 .
NotMarried  -0.207050   0.142644  -1.452    0.150
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.6782 on 96 degrees of freedom
Multiple R-squared:  0.05558,Adjusted R-squared:  0.02607
F-statistic: 1.883 on 3 and 96 DF,  p-value: 0.1376
```

What are the dependent and independent variables?

Answer:

Exercise 16.7-13: STEP 2: Using the information from exercise nine, we must state the hypotheses about its relationship between *Happiness*, *income*, and *age*. Based on our experience, how should we state our hypotheses?

Answer:

Exercise 16.7-14: STEP 3: Using the information from exercise 12, determine the coefficient of determination.

Answer:

Exercise 16.7-15: STEP 4: Using the information from exercise 12, record the independent, dependent variables, and the correlation coefficients.

	Independent			
Dep. Var.	1. ____	2. ____	3. ____	4. ____
1. ____	____			
2. ____	____	____		
3. ____	____	____	____	
4. ____	____	____	____	____

Exercise 16.7-16: STEP 5: Using the information from exercise 12, record the independent, dependent variables, and the correlation coefficients.

Answer:

Exercise 16.7-17: STEP 1: In the next six tasks, we will use all of the data from GSS2014 in this exercise. We have been asked to examine the relationship between a person's income, age, and "not married" and a person's happiness. Here we have variables which are interval-ratio type of variable. These types of variables require that we use a Simple Linear Regression (SLR) analysis. So step 1 in the process of analysis is choosing independent and dependent variables. Note: In the GSS2014 dataset, *happy* is known as GENERAL HAPPINESS, *income06* is known as TOTAL FAMILY INCOME, *age* is known as AGE OF RESPONDENT, and *absingle* is known as NOT MARRIED.

	1	2	3	4
1	1.00	0.02	-0.00	-0.01
2	0.02	1.00	0.00	0.02
3	-0.00	0.00	1.00	0.06
4	-0.01	0.02	0.06	1.00

```
Call:
lm(formula = Happiness ~ Age + Income + NotMarried)

Residuals:
    Min      1Q  Median      3Q     Max
-1.2435 -0.6599  0.1324  0.2881  1.3956

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  2.3173217  0.0923286  25.099   <2e-16 ***
Age          0.0001357  0.0009617   0.141    0.888
Income      -0.0264172  0.0028697  -9.205   <2e-16 ***
NotMarried  -0.0290415  0.0336223  -0.864    0.388
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.6378 on 1516 degrees of freedom
Multiple R-squared:  0.05344,Adjusted R-squared:  0.05156
F-statistic: 28.53 on 3 and 1516 DF,  p-value: < 2.2e-16
```

What are the dependent and independent variables?

Answer:

Exercise 16.7-18: STEP 2: Using the information from exercise 17, we must state the

hypotheses about its relationship between *Happiness*, *income*, and *age*. Based on our experience, how should we state our hypotheses?

Answer:

Exercise 16.7-19: STEP 3: Using the information from exercise nine, determine the coefficient of determination.

Answer:

Exercise 16.7-20: STEP 4: Using the information from exercise 17, record the independent, dependent variables, and the correlation coefficients.

	Independent			
Dep. Var.	1. ____	2. ____	3. ____	4. ____
1. ____	____			
2. ____	____	____		
3. ____	____	____	____	
4. ____	____	____	____	____

Exercise 16.7-21: STEP 5:
Using the information from exercise 17, record the independent, dependent variables, and the correlation coefficients.

Chapter 17

Workshops

Exercises

Exercise 17.-1:

Workshop 1A, Submitted by:

Name: _____ Signature: _____

Name: _____ Signature: _____

Name: _____ Signature: _____

Name: _____ Signature: _____

Table 17.1: A partial list of students registered for Stat 1600 during Fall 2016 of their Major course of study.

	Class						
Major	A	B	C	D	Web	Total	RF
Anthropology	2			1		3	
Art		5	3	1		9	
Aviation			1	1		2	
Biology		3		1	1	5	
Business	1					1	
Communication	5	1	4	3	4	17	
Criminal Justice		3	2	1		6	
Data Science		1				1	
Education		2			2	4	
English	2	3			1	6	
Foreign Lang.		1	1			2	
Geography		1			1	2	
Graphic Design	1		1			2	
History	1		1			2	
Journalism		1	1		1	3	
Mathematics		2				2	
Music	2		7	3	3	15	
Nursing		1				1	
Physics				1		1	
Psychology	12	9	9	8	6	44	
Social Work	6	6	6	5	3	26	
Sociology		1		1		2	
Total	32	40	36	26	22	156	

Using the data from Table 17.1,

1. Construct a relative frequency table for total student majors (column above).
2. The highest percentage of students fall under what major?
3. What percentage of students are Art majors?
4. What percentage of students' majors fall in both Sociology and Psychology?
5. How does the percentage of students who are communications majors in Class A compare to the percentage of communication majors overall for total students?

Exercise 17.-2:

Workshop 1B, Submitted by:

Name: _____ Signature: _____

Name: _____ Signature: _____

Name: _____ Signature: _____

Name: _____ Signature: _____

Using the data from Table 17.2,

Table 17.2: A partial list of students registered for Stat 1600 during Fall 2016 of their Major course of study.

	Class						
Major	A	B	C	D	Web	Total	RF
Anthropology	2			1		3	
Art		5	3	1		9	
Aviation			1	1		2	
Biology		3		1	1	5	
Business	1					1	
Communication	5	1	4	3	4	17	
Criminal Justice		3	2	1		6	
Data Science		1				1	
Education		2			2	4	
English	2	3			1	6	
Foreign Lang.		1	1			2	
Geography		1			1	2	
Graphic Design	1		1			2	
History	1		1			2	
Journalism		1	1		1	3	
Mathematics		2				2	
Music	2		7	3	3	15	
Nursing		1				1	
Physics				1		1	
Psychology	12	9	9	8	6	44	
Social Work	6	6	6	5	3	26	
Sociology		1		1		2	
Total	32	40	36	26	22	156	

1. Using the column 'RF' above, construct a relative frequency table for student majors in the Web Class only.

2. The highest percentage of students fall under what major?

3. What percentage of students are Music majors?

4. What percentage of students are found in the majors of Psychology, Social Work and Sociology (combined)?

5. How does the percentage of students who are education majors in the Web class compare to the percentage of education majors overall for total students? Which is higher?

Exercise 17.-3:

Workshop 1C, Submitted by:

Name: _____ Signature: _____

Name: _____ Signature: _____

Name: _____ Signature: _____

Name: _____ Signature: _____

Hospital-acquired infections are a serious concern for patients in hospitals and can be caused by bacterial, viral, and fungal pathogens. Suppose hospitals in the state of Michigan reported the following number of infections and deaths from infection below for the past year.

Table 17.1: Hospital-acquired Infections

Type of infection	Number of Infections	Number of Deaths
Pneumonia	184	79
Bloodstream	258	65
Surgical site	361	12
Urinary tract	833	103
Other	287	54

Using the data from Table 17.1,

1. What type of variable is 'type of infection' numerical or categorical?
2. We can further describe this variable as _____.
3. Construct a relative frequency table for the number of infections (show percentages to two places past the decimal).
4. Complete a bar chart for the relative frequency table above complete with title and axis Labels.
5. What other type of graph can we use for this variable type?

Exercise 17.-4:

Workshop 2A, Submitted by:

Name: _____ Signature: _____

Name: _____ Signature: _____

Name: _____ Signature: _____

Name: _____ Signature: _____

The average test grades of 19 students are as follows (on a scale from 0 to 100, with 100 being the highest score):

91 (A), 95 (A), 96 (A), 82 (B), 94 (A), 75 (C), 91 (A), 79 (C), 92 (A), 100 (A), 89 (B), 93(A), 91 (A), 86 (B), 93 (A), 72 (C), 74 (C), 93 (A), 90 (A)

Table 17.2: Distribution of Grades

grades	frequency	relative frequency
A		
B		
C		

1. Fill in the frequency and relative frequency table above
2. Create a bar chart and pie chart for the above data
3. What percentage of students got a grade of 'A'?
4. Looking at the bar chart in part a, identify the shape of the data (symmetric, right-skewed, left-skewed)?

Exercise 17.-5:

Workshop 2B, Submitted by:

Name: _____	Signature: _____
Name: _____	Signature: _____
Name: _____	Signature: _____
Name: _____	Signature: _____

The following 50 test scores for Stat 1600 students are found below. Based on the intervals provided, complete the relative frequency table and answer the following questions.

84 88 76 44 80 83 51 93 69 78
49 55 78 93 64 84 54 92 96 72
97 37 97 67 83 93 95 67 72 67
86 76 80 58 62 69 64 82 48 54
80 69 62 67 66 73 77 82 84 86

Table 17.3: Distribution of Test Scores

Grades	frequency	relative frequency
30-39		
40-49		
50-59		
60-69		
70-79		
80-89		
90-99		

1. Fill in the frequency and relative frequency above.
2. Complete a stem and leaf plot with the stem representing the 10's place and using 3-9.
3. Draw a histogram for the test scores using the intervals in the relative frequency table above.
4. What percentage of students had scores of 59 or less?

Exercise 17.-6:

Workshop 2C, Submitted by:

Name: _____ Signature: _____

Name: _____ Signature: _____

Name: _____ Signature: _____

Name: _____ Signature: _____

The numbers of absences for 20 Stat 1600 students are shown below (on a scale from 0 to 30).

Sample: 10, 0, 1, 3, 15, 6, 2, 1, 0, 21, 25, 11, 9, 7, 4, 5, 12, 28, 17, 8

Sorted: 0, 0, 1, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 15, 17, 21, 25, 28

Table 17.4: Distribution of Absences

Number of absences	Frequency	Relative frequency
0-9		
10-19		
20-29		

1. Fill in the frequency and relative frequency above.
2. Complete a stem and leaf plot with the stem representing the 10's place and using 0-2.
3. Draw a histogram for the absences using the intervals in the relative frequency table above.
4. What percentage of students had number of absences less than 20?

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