Data Science for Business
Lecture #8
Cell2Cell Assignment Introduction

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Outline

Introduce the Cell2Cell Case and Industry Context

Preview the Cell2Cell Data

Assignment

Appendix with Details about Lifetime Value Calculation

Cell2Cell: Managing Customer Churn

Source: Sunil Gupta, Wagner Kamakura, Junxiang Lu, Charlotte Mason, Scott Neslin (Marketing Science Conference 2002)

Overview

Predictive modeling is a statistical process of “scoring” customers for a targeted marketing campaign.

The idea is to predict what actions customers will take, and allow firms to target their marketing efforts more effectively.

One application is to reduce the problem of customer “churn”. Annual churn rates have been estimated in the 20%-40% range for the telecommunication industry.

Customer Churn

What is Churn?

Probability the customer will leave the company at a given point in time (in our problem it will be the probability of ending their contract next month)

What is its relationship with Loyalty or Retention?

Prob(Churn) = 1 – Prob(Retention)

Wireless Industry

Tremendous growth in the wireless industry in the late 1990's. The number of subscribers doubled every two years during the 90s. Business Week (2002) and Wireless News (2002) reported:

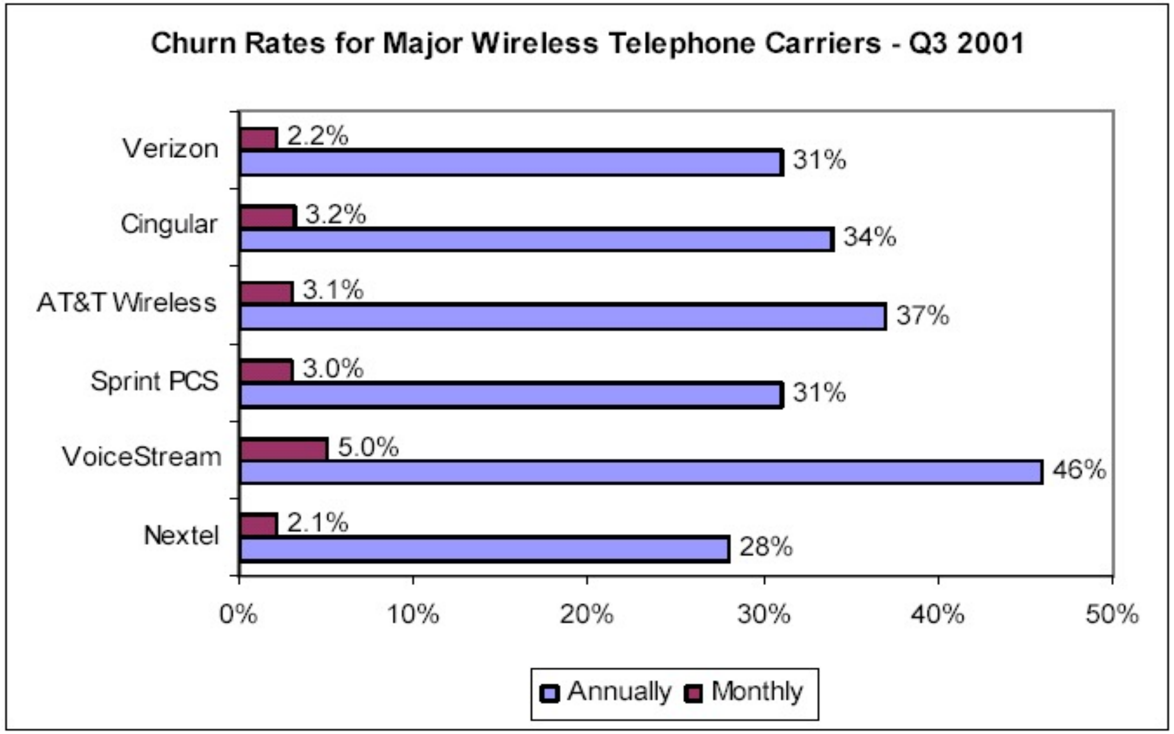
- By 2003, 25% of all telephone minutes will be accounted for by wireless services
- By 2006 the US penetration in the wireless-voice market is expected to hit 189 million subscribers, while that of the wireless-data market is expected to jump to 38 million
- Of all wireless customers, 70% are using digital networks that allow carriers to efficiently offer more appealing services
- Investment in network infrastructure has increased by 17% and the number of cells sites increased by 22.3% (upward trend in both quality and coverage)

Industry Turmoil in the early 2000s

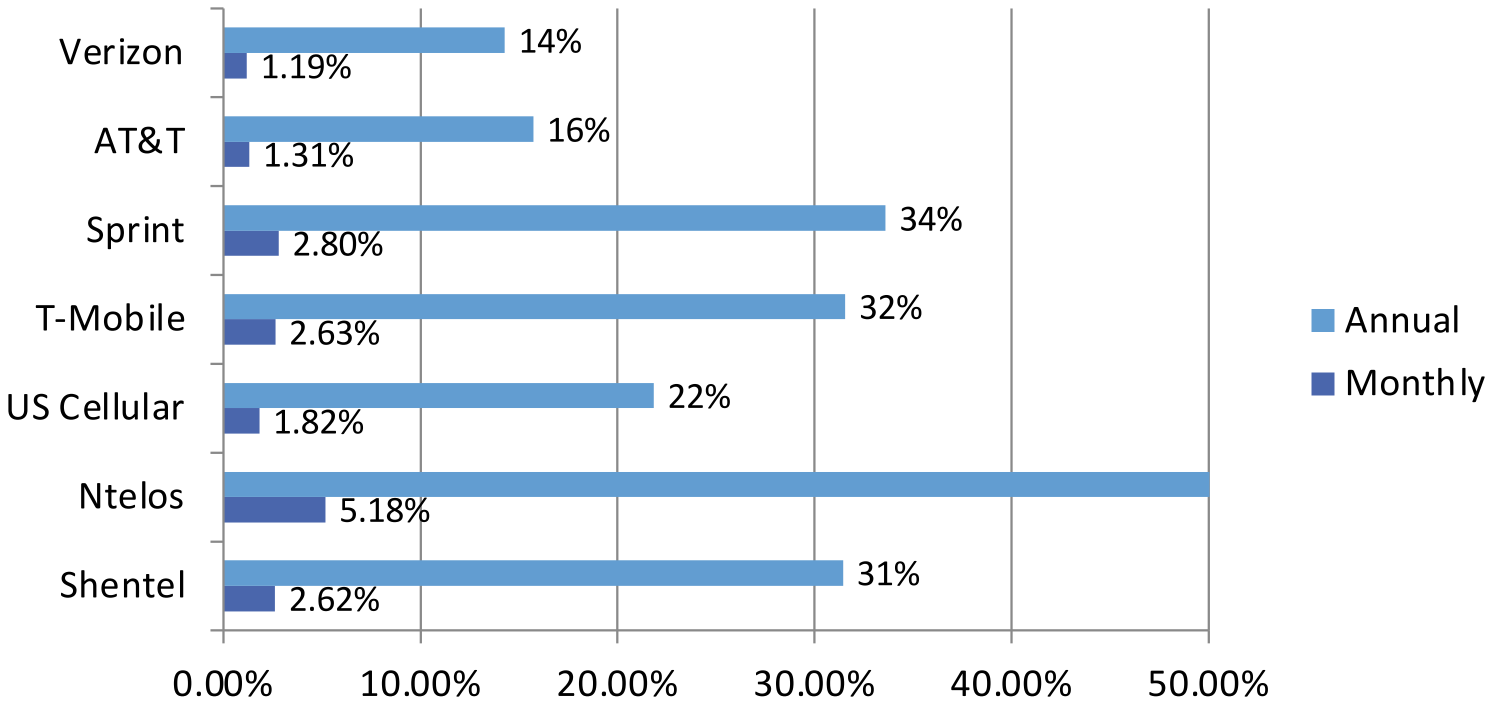
Some serious problems to industry profitability have arisen:

- Consolidation: From nearly 60 cellular companies in the early 90s to only six big players (80% of the market)
- Growth: Subscriber growths have gone from 50% to 15-20% annuals, and analysts predict a meager 10% growth rate in 2003
- Competition: Firms are engaged in devastating price wars
- Customer Strategy: industry attitudes have changed from “make big networks, get customers” to “make new services, please customers”. A retention orientation instead of an acquisition emphasis.

Customer Churn



Customer Churn (2015)



http://www.statista.com/statistics/283511/average-monthly-churn-rate-top-wireless-carriers-us/

The Elusive Customer

Firms have been able to acquire customers without much effort, but now the situation has changed. Churn – the customer's decision to end the relationship and switch to another company – has become a major concern. In 2001 the churn rates were 20-25% annually (approximately 2% per month)

Some reasons for churn:

- Variety of companies
- Similarity of offerings
- Cheap prices of handsets
- And now…portability of phone numbers

An estimate by Reuters (2002) is that the "top six US wireless carriers would have saved $207m if they had retained an additional 5% of customers open to incentives but who switched plans in the past year."

Managing Customer Churn

Untargeted Approach

- Product quality, mass advertising

Targeted Approach

- Reactive: Wait for customer to call
- Proactive: Contact customer in advance

Targeted Proactive Approach has advantages:

- Less expensive than untargeted
- Offer does not have to be as large as reactive
- Rewards being a customer, not being a churner

But requires accurate prediction of churn probability.

Developing a Proactive Churn Management Program

Overall approach

- Use predictive model to determine which customers are likely to churn, and what incentives should be offered to retain these customers

Process:

- Problem: What outcomes do we want?
- Data: What information do we have? Do we have the right information?
- Model: Develop a model to predict customer churn
  - Use model to understand main drivers of churn
  - Use insights to develop appropriate offers/incentives
- Recommendation: Decide which offers to target to which customers
  - Use the model to identify customers who are most likely to churn
  - Evaluate results

Cell2Cell Business Context

Customer churn in a telco

Industry / function: A telco is facing increasing levels of churn among its customers despite providing a menu of reactive promotions. (This is the US market in the early 2000s)

Business situation: The telco want a business analytics approach with the potential of improving the status quo: data on customer behavior, demographics and marketing are available but unused currently, reactive promotions are offered and measured for effectiveness but not incorporated back in the decision cycle, and no clear explanation of the reasons for churn are apparent.

Problem statement: Your task is to suggest a system design that uses all the data available synergistically and provides a clear explanation of the causes of churn with a business plan to address it

Current Situation

Acquisition costs are $200-$500 per customer and anticipated to rise as the pool of new users shrinks

Managerial focus is shifting towards customer retention from acquisition

Current monthly churn rates are 2% → Annual retention rate of 78.5%

- Monthly churn of 1.5% → Annual retention of 83.4%
- Monthly churn of 1% → Annual retention of 88.6%

Huge gains, especially among heavy users, if Cell2Cell can decrease churn

Business Overview

# No	Variable	Value	Units	Source
A.1	Total number of customers	10,000	000 customers	Text in case
A.2	Revenue per customer per month	40	USD	Table in case
A.3	Revenue per customer per year	480	USD	A.2 * 12
A.4	Current churn rate per month	2.5%		Text in case
A.5	Current churn rate per year	30.0%		A.4 * 12
A.6	Number of customers churning per year	3,000	000 customers	A.5 * A.1
A.7	Revenue churning per year	1,440	USD million	A.6 * A.3
A.8	Annual revenues	4,800	USD million	A.3 * A.1

We have a $1b USD problem here

Untargeted Retention:
Make a promotional offer to everyone to reduce churn

# No	Variable	Value	Units	Source
B.1	Value of offer	200	per customer	Assumed
B.2	% of customer base receiving offer	100%		Assumed
B.3	% of customers taking up the offer	50%		Assumed
B.4	# customers taking up the offer	5,000	000 customers	B.3 * A.1
B.5	Total cost of the offer	1,000	USD million	B.1 * B.4
B.6	Total % churners who take up the offer	30.0%		A.5
B.7	Number of churners retained	1,500	000 customers	A.6 * B.3
B.8	Revenue retained	720	USD million	B.7 * A.3
B.9	Value generated by the offer	(280)	USD million	B.8 - B.5

We cannot afford to make an attractive offer to all customers

Simple Method for Calculating LTV

We need to consider profits over the life of the customer and not just for one year

The basic idea is to calculate profitability over the life of a customer. Here is a simplified version that doesn't consider interest rates:

$$LTV = \left[\left(\text{monthly profit} \right) \times \left(\text{customer lifetime in months} \right) \right] - \left(\text{acquisition cost} \right)$$

How can you determine customer lifetime?

$$\text{Customer lifetime in months} = {}^{1}\!/_{\text{monthly churn rate}}$$

Note: this formula comes from the mean of a geometric distribution. See http://en.wikipedia.org/wiki/Geometric_distribution

The Customer's Expected Life depends inversely upon the Churn Rate

Suppose the probability of churn follows a Bernoulli model (e.g., each period either you stay with probability β or go with probability 1-β)

Then the expected time until the customer churns follows a geometric model, so probability that customer churns (or dies) in period *k* is 1-β

$$\Pr(lifetime = k) = \beta^{k-1}(1-\beta)$$

The expected life of the customer is:

$$\mathrm{E}[lifetime] = \frac{1}{1-\beta}$$

Better Method for Computing LTV considers discounting future cash flows

More formally we can discount the future profit stream, but if we assume the monthly profits are constant then this simplifies:

$$LTV = \sum_{t=1}^{T} \frac{E[\text{Profit}_t]}{(1+d)^t} - A = \text{Profit}\left(\frac{1+d}{1+d\text{-}\beta}\right) - A$$

Where

β	Retention or Loyalty Rate (or Churn Probability=1-β)
d	Discount Rate (monthly)
A	Acquisition Cost
T	Time horizon
$E[\text{Profit}_t]$	Expected monthly profits (=$\beta^t \text{Profit}_t$)

Notice we make assumptions about constant churn, discount, and profits to make the problem more tractable. See Appendix to change time horizon to finite time periods of 12 or 24 months.

What is needed to compute LTV?

We need to know…

- Discount rate (say 5% per month, or 5%/12 per year)
- Profit (we don't have this, but since marginally the cost per user is close to 0, let's set Profit=Revenue)
- Churn rate (comes from our model)
- Acquisition cost (or how much we are going to spend to retain customer, if we do nothing it is $0)

Lifetime Value Calculation shows that customer value may depend more upon Retention Rate than Monthly Profits

Lifetime value of customer

$$LTV = \text{Profit}\left(\frac{1+d}{1+d-\beta}\right)$$

Assumptions

$$\text{Monthly Churn Rate} = 1-\beta = .018$$

$$\text{Monthly Retention Rate} = \beta = .982$$

$$\text{Annual Interest Rate} = 12 \times d = .05$$

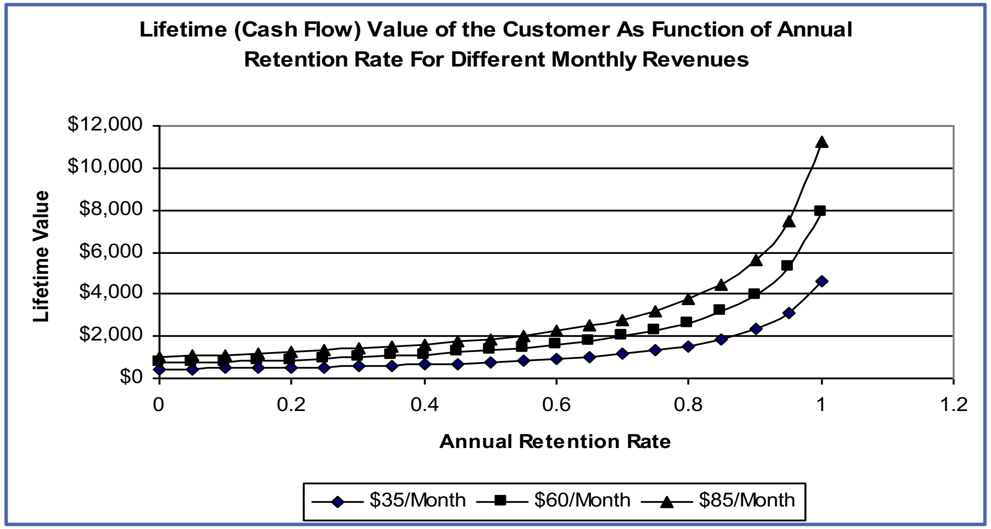
$$\text{Profit per Month} = \text{Profit} = \$50$$

Example

$$LTV = \text{Profit}\left(\frac{1+d}{1+d-\beta}\right) = \$50\left(\frac{1+(0.05/12)}{1+(0.05/12)-.982}\right) = \$2265$$

Small reductions in retention could have big impact on LTV

- Acquisition costs are $200-$500 per customer and anticipated to rise as the pool of new users shrinks
- Managerial focus is shifting towards customer retention from acquisition
- Current monthly churn rates are 2% → Annual retention rate of 78.5%
  - Monthly churn of 1.5% → Annual churn of 83.4%
  - Monthly churn of 1% → Annual churn of 88.6%
- Huge gains, especially among heavy users, if Cell2Cell can decrease churn



Cell2Cell Data

Solution Approach Using Data Science

Overall approach

- Use predictive model to determine which customers are likely to churn, and what incentives should be offered to retain these customers

Process:

1. Develop a model to predict customer churn
2. Use the model to identify customers who are most likely to churn (at least 75% more likely than average)
3. Use model to understand main drivers of churn
4. Use insights to develop appropriate offers/incentives
5. Decide which offers to target to which customers
6. Evaluate results

Data Available

Dependent variable:

- CHURN: Did the customer churn in month x?

Predictor variables:

- Customer Behavior
  - Revenue, Usage, Months in service, Handset type, …
- Demographics
  - Location, Age, Credit Rating, Occupation, Income, …
- Marketing Variables
  - Customer care, Buys via mail order, Responds to mail offers, Number of calls previously made to rentation team, …

Customer Behavior

1	Revenue	Mean monthly revenue
2	Mou	Mean monthly minutes of use
3	Recchrge	Mean total recurring charge
4	Directas	Mean number of director assisted calls
5	Overage	Mean overage minutes of use
6	Roam	Mean number of roaming calls
7	Changem	% Change in minutes of use
8	Changer	% Change in revenues
9	Dropvce	Mean number of dropped voice calls
10	Blckvce	Mean number of blocked voice calls
11	Unansvce	Mean number of unanswered voice calls
13	Threeway	Mean number of threeway calls
14	Mourec	Mean unrounded mou received voice calls
15	Outcalls	Mean number of outbound voice calls
16	Incalls	Mean number of inbound voice calls

17	Peakvce	Mean number of in and out peak voice calls
18	Opeakvce	Mean number of in and out off-peak voice calls
19	Dropblk	Mean number of dropped or blocked calls
20	Callfwdv	Mean number of call forwarding calls
21	Callwait	Mean number of call waiting calls
23	Months	Months in Service
24	Uniqsubs	Number of Uniq Subs
25	Actvsubs	Number of Active Subs
27	Phones	# Handsets Issued
28	Models	# Models Issued
29	Eqpdays	Number of days of the current equipment
44	Refurb	Handset is refurbished
45	Webcap	Hanset is web capable
74	Setprcm	Missing data on handset price
75	Setprc	Handset price (0=>missing)

Demographics

26	Csa	Communications Service Area
30	Customer	Customer ID
31	Age1	Age of first HH member
32	Age2	Age of second HH member
33	Children	Presence of children in HH
34	Credita	Highest credit rating - a
35	Creditaa	High credit rating - aa
36	Creditb	Good credit rating - b
37	Creditc	Medium credit rating - c
38	Creditde	Low credit rating - de
39	Creditgy	Very low credit rating - gy
40	Creditz	Lowest credit rating - z
41	Prizmrur	Prizm code is rural
42	Prizmub	Prizm code is suburban
43	Prizmtwn	Prizm code is town
46	Truck	Subscriber owns a truck
47	Rv	Subscriber owns a recreational vehicle
48	Occprof	Occupation - professional

49	Occcler	Occupation - clerical
50	Occcrft	Occupation - crafts
51	Occstud	Occupation - student
52	Occhmkr	Occupation - homemaker
53	Occret	Occupation - retired
54	Occself	Occupation - self-employed
55	Ownrent	Home ownership is missing
56	Marryun	Marital status unknown
57	Marryyes	Married
58	Marryno	Not Married
62	Travel	Has traveled to non-US country
63	Pcown	Owns a personal computer
64	Creditcd	Possesses a credit card
67	Newcelly	Known to be a new cell phone user
68	Newcelln	Known not to be a new cell phone user
70	Incmiss	Income data is missing
71	INCOME	Income (0=>missing)
72	Mcycle	Owns a motorcycle
73	Creditad	Number of adjustments made to customer credit rating

Marketing Variables

12	custcare	Mean number of customer care calls
59	mailord	Buys via mail order
60	mailres	Responds to mail offers
61	mailflag	Has chosen not to be solicited by mail
65	retcalls	Number of calls previously made to retention team
66	retaccpt	Number of previous retention offers accepted
69	refer	Number of referrals made by subscriber
76	retcall	Customer has made call to retention team

One observation of our data

```
> print(cell2cell[1,])
  Revenue    Mou Recchrge Directas Overage Roam Changem  Changer   Dropvce   Blckvce
1 23.9975 219.25     22.5   0.2475       0    0 -157.25 -18.9975 0.6666667 0.6666667
  Unansvce Custcare Threeway   Mourec Outcalls Incalls Peakvce Opeakvce  Dropblk
1 6.333333        0        0 97.17667        0       0      58       24 1.333333
  Callfwdv  Callwait Months Uniqsubs Actvsubs Csa Phones Models Eqpdays Customer Age1
1        0 0.3333333     61        2        1 SEA      2      2     361  1000001   62
     Age2 Children Credita Creditaa Creditb Creditc Creditde Creditgy Creditz Prizmrur
1 43.8689        0       1        0       0       0        0        0       0        0
  Prizmub Prizmtwn Refurb Webcap Truck Rv Occprof Occcler Occcrft Occstud Occhmkr
1       1        0      0      1     0  0       1       0       0       0       0
  Occret Occself Ownrent Marryun Marryyes Marryno Mailord Mailres Mailflag Travel
1      0       0       0       0        0       1       1       1        0      0
  Pcown Creditcd Retcalls Retaccpt Newcelly Newcelln Refer Incmiss Income Mcycle
1     1        1        1        0        0        0     0       0      4      0
  Creditad Setprcm   Setprc Retcall Churn Age1miss Age2miss
1        0       0 29.98999       1     1        0        1
> print(cell2cell[1,varlist])
  Churn Eqpdays Months Recchrge Revenue Csa Customer Age1    Age2 Mailflag Retcall
1     1     361     61     22.5 23.9975 SEA  1000001   62 43.8689        0       1
```

Summary of data

```
> summary(cell2cell[,varlist])
     Churn           Eqpdays            Months         Recchrge          Revenue       
 Min.   :0.0   Min.   :  -5.0   Min.   : 6.00   Min.   :-11.29   Min.   : -5.862  
 1st Qu.:0.0   1st Qu.: 212.0   1st Qu.:11.00   1st Qu.: 30.00   1st Qu.: 33.439  
 Median :0.5   Median : 342.0   Median :17.00   Median : 44.99   Median : 48.553  
 Mean   :0.5   Mean   : 392.8   Mean   :18.83   Mean   : 46.27   Mean   : 58.634  
 3rd Qu.:1.0   3rd Qu.: 532.0   3rd Qu.:24.00   3rd Qu.: 59.99   3rd Qu.: 70.778  
 Max.   :1.0   Max.   :1823.0   Max.   :61.00   Max.   :349.57   Max.   :861.105  

      Csa             Customer            Age1            Age2          Mailflag      
 OTH     : 9695   Min.   :1000001   Min.   :18.00   Min.   :18.00   Min.   :0.00000  
 NYC     : 4417   1st Qu.:1025206   1st Qu.:38.00   1st Qu.:43.87   1st Qu.:0.00000  
 LAX     : 2628   Median :1049847   Median :43.17   Median :43.87   Median :0.00000  
 SFR     : 2143   Mean   :1049981   Mean   :43.17   Mean   :43.87   Mean   :0.01507  
 APC     : 1914   3rd Qu.:1075003   3rd Qu.:48.00   3rd Qu.:43.87   3rd Qu.:0.00000  
 DAL     : 1807   Max.   :1099995   Max.   :99.00   Max.   :99.00   Max.   :1.00000  
 (Other):17396                                                                      
    Retcall       
 Min.   :0.00000  
 1st Qu.:0.00000  
 Median :0.00000  
 Mean   :0.04043  
 3rd Qu.:0.00000  
 Max.   :1.00000  
```

Compare Churners and Loyal Customers

```
> describeBy(cell2cell[,varlist[-6]],group=cell2cell$Churn,fast=TRUE)
```

Descriptive statistics by group

group: 0

	vars	n	mean	sd
Churn	1	20000	0.00	0.00
Eqpdays	2	20000	363.64	249.60
Months	3	20000	18.61	9.92
Recchrge	4	20000	47.82	24.33
Revenue	5	20000	59.23	44.46
Customer	6	20000	1051311.75	29659.55
Age1	7	20000	43.45	10.44
Age2	8	20000	44.08	9.28
Mailflag	9	20000	0.02	0.12
Retcall	10	20000	0.03	0.16

group: 1

	vars	n	mean	sd
Churn	1	20000	1.00	0.00
Eqpdays	2	20000	422.04	260.39
Months	3	20000	19.05	9.31
Recchrge	4	20000	44.72	22.89
Revenue	5	20000	58.04	43.66
Customer	6	20000	1048650.78	27845.72
Age1	7	20000	42.89	10.38
Age2	8	20000	43.66	9.17
Mailflag	9	20000	0.01	0.12
Retcall	10	20000	0.06	0.23

Discussion

What is the relationship between churn and ...

Group	Variable	Description
Customer Behavior	Eqpdays	Number of days using the current handset
	Months	Months in service
	Recchrge	Mean of total recurring charge
	Revenue	Mean of monthly revenue
Demographics	Csa	Communications service area
	Customer	Customer ID #
	Age1	Age of first HH member
	Age2	Age of second HH member
Marketing Variables	Mailflag	Customer chosen not to be solicited by mail
	Retcall	Customer has made call to retention team

Calibration and Validation Samples

Sample	Number of Observations	Churn %
Calibration	40,000	50%
Validation	31,047	1.96%

Notice that churn in the original dataset is rare (e.g., only ~2% of customers churn), so if we have 40,000 observations then we would only have 800 churners.

A simple model of always retain would be right 98% of the time, which clearly is not an interesting model. Better to over-sample churners so our predictive model will be more sensitive to detecting differences between churners and loyal customers.

Understanding oversampling

In our dataset we observe a 50% probability of being a churner. In the full dataset there is a 2.0% probability of being a churner. This is known as oversampling.

It is used to deal with the rare-event problem. Consider that a 98.0% accurate classifier for churn is to always predict retention. While this fits well overall, we care about predicting churn. Oversampling focuses the training on predictive positive (churn) cases not negative (retention) ones.

Adjusting probability predictions due to oversampling

Suppose we predict the probability of an event as *p* and the probability the event does not happen as *q*

Compute the following values:

A = *p* / (Oversampled % of events / Original % of events)

B = *q* / (Oversampled % of non-events / Original % of non-events)

The adjusted probabilities (e.g., probability in original sample) are:

p' = A/(A+B)

q' = B/(A+B)

<u>Example</u>

Before sampling: Churn has 2% probability

Post sampling: Churn has 50% probability

Suppose our oversampled prediction is *p*=70%

A = 0.70 / (50% / 2.0%) = 0.70 / 25.0 = 0.028

B = 0.30 / (50% / 98.0%) = 0.30 / .588 = 0.616

p' = 0.028 / (0.028 + 0.588) = 0.028 / 0.616 = 0.045 or 4.5%

	A	B	C
1	before	2.0%	98.0%
2	after	50.0%	50.0%
3			
4	p	70%	
5	q	30%	
6			
7	A	0.028	
8	B	0.588	
9	A+B	0.616	
10			
11	p'	4.5%	
12	q'	95.5%	
13			
14	lift after	1.4	
15	lift before	2.3	

See "oversampling_adjustments.xlsx"

Understanding the effects of oversampling for Logistic Regression

Oversampling does not effect the slopes but it does effect the intercepts (See oversampling_logistic.R for simulated example)

- The ranking of predictions is not affected (e.g., “most likely” is the same in both datasets), but the predictions are too high (e.g., intercept is too great)
- Oversampling does not affect sensitivity or specificity but false positive and negative rates are affected
- ROC curve is not affected
- Gain and Lift charts are affected since the proportion of events is changed – lift in our sample is likely understated.

For example, suppose the predicted probability in top decile is 70%. After oversampling, ratio is 50:50. The lift on the sample data is 70%/50% = 1.4. After adjusting the probability, the adjusted probability score is 4.5%. The lift on the original data is 2.3 (4.5% / 2%).

Assignment

Required Output for this Assignment

A PowerPoint presentation that communicates the following to the Chief Marketing Officer and his team:

- What predicts whether a customer is going to churn?
  - Communicate both the results from a logistic regression and decision tree
  - Tell a story in plain English about why customers are leaving
- Which model should we use?
- How can you translate your insights about churn from your model into a proactive retention campaign?
  - Recommend actions that can be taken proactively to keep customers that are likely to leave. Relate your strategy to your model
- How profitable will your proactive campaign be when applied to a sample of 31,047 users in a hold-out sample that you have never seen before?
  - What is the right metric to optimize? Retained customers, profits over one year, or LTV?

Part 1

Analyze customer churn using Classification & Regression Tree (or Decision Tree) and a Logistic Regression using the information provided to you. To help you complete this analysis briefly answer these questions in your presentation:

1. Purpose – what is the marketing purpose of your task? How do you think the company could use your results to target customers who are likely to churn? Before you being the analysis name three relationships that you expect to see between churn and the predictive variables. (Hint: focus on the direction of their influence, do you think a high/low value of this variable will result in more or less churn?)

2. Estimate at decision tree that predicts CHURN. Prepare a graphic, visualization or table that summarizes the relationships that you have found. (Hint: use the provided script, you may want to change the settings used in the script to change the complexity of the decision tree.)

3. Estimate a logistic regression and decision tree that predicts CHURN. Prepare a table that summarizes the relationships that you have found. At a minimum include the following columns in your table: Variable, Parameter Estimate, Importance, and Meaning. The meaning should provide a description of the effect in plain English (e.g., if the parameter for "Eqpdays" is "0.0010527", you could make a statement like "For every extra month (30 days) that a customers odds of churning versus not churning goes up by 3%.").

4. Which model do you find is best? Be sure to justify your assessment based upon the performance of each of these models in the test sample using a confusion matrix and/or lift in the top decile.

5. Discuss how you would use your model to decide what proactive offers you would give to four specific customers.

Setup for Part 2

To help you compute the LTV calculations you are provided a spreadsheet that has encoded the logistic or decision tree models.

These spreadsheets allow you to conduct “what if” simulations by changing the input vector and observe the effect on the output.

The spreadsheets are provided to make you computations easier – but if you want to only use R then go ahead.

Part 2

1. Translate your model into a proactive retention campaign. Use your predictive model to design a strategy using your best model that you believe would increase retention (decrease churn rate) amongst mobile phone customers. Be specific in identify which of the 31,047 customers to target and what offers to make.
2. Recommend actions that can be taken proactively to keep customers that are likely to leave. Explain how your strategy relates to your model. Be specific in your recommendations about who to target, what to offer and how you will communicate the offer to the customers (phone, email, text, mail, or other).
3. What gain in profits or LTV would you expect from your proactive retention campaign? (Hint: you will need to make assumptions about the cost of your offers and how consumers will respond to potential promotional offers.)

Cell2cell Assignment Objectives

Using predictive models to understand churn (Part 1 of assignment)

- Tell a story about customer churn from your model
- Oversampling to ease model construction
- Build and test a variety of predictive models (logistic regression, decision trees, …)
- Comparing and choosing models based upon performance, actionability and communicability

Predicting Profits (Part 2 of assignment)

- Move from a predictive model to implementation of a pro-active retention campaign
- Making predictions with adjustments for oversampling
- Computing expected profits of your recommended campaign
- How will consumers respond to offers?

Appendix

Computing LTV for the first 12 months

Sum of geometric series drives lifetime value calculation

The sum of a geometric series is:

$$S = 1 + \delta + \delta^2 + \delta^3 + \cdots$$
$$\delta S = \quad \delta + \delta^2 + \delta^3 + \cdots$$
$$\Rightarrow \; S - \delta S = 1 \;\Rightarrow\; S = \frac{1}{1-\delta}. \quad \text{Assumes that } |\delta| < 1$$

If we want to sum a finite geometric series then:

$$\begin{aligned} S_T &= 1 + \delta + \delta^2 + \delta^3 + \cdots + \delta^T \\ &= S - \left(\delta^{T+1} + \delta^{T+2} + \cdots\right) \\ &= S - \delta^{T+1} S = S(1 - \delta^{T+1}) \\ &= \frac{1 - \delta^{T+1}}{1-\delta} \end{aligned}$$

Our LTV Calculation

If profits, costs and retention rates are both constant then:

$$LTV = \sum_{t=1}^{T} \frac{E[\text{Profit}_t]}{(1+d)^t} - A = \sum_{t=1}^{T} \frac{\text{Profit} \times \Pr(\text{Alive}_t)}{(1+d)^t} - A$$

$$= \sum_{t=1}^{T} \frac{\text{Profit} \times \beta^t}{(1+d)^t} - A = \text{Profit} \times \sum_{t=1}^{T} \left(\tfrac{\beta}{1+d}\right)^t - A$$

$$= \text{Profit} \times \left(1 + \tfrac{\beta}{1+d} + \left(\tfrac{\beta}{1+d}\right)^2 + \left(\tfrac{\beta}{1+d}\right)^3 + \cdots + \left(\tfrac{\beta}{1+d}\right)^T\right) - A$$

$$= \text{Profit} \times \left(\frac{1 - \left(\tfrac{\beta}{1+d}\right)^T}{1 - \tfrac{\beta}{1+d}}\right) - A = \left(1 - \left(\tfrac{\beta}{1+d}\right)^T\right) \times \text{Profit} \times \left(\frac{1+d}{1+d-\beta}\right) - A$$

If $T \to \infty$ then $\left(\tfrac{\beta}{1+d}\right)^T \to 0$ and $LTV = \text{Profit} \times \left(\frac{1+d}{1+d-\beta}\right) - A$

Notice to compute LTV for $T = 12$ months calculate $LTV_{12} = \left(1 - \left(\tfrac{\beta}{1+d}\right)^{12}\right) LTV$