

Securin Assessment

Name: Joseph Aron Babu Pappachan

Department: Information Technology

College Name: Loyola ICAM College of Engineering and Technology

Problem Statement: The Doomed Dice Challenge

You are given two six-sided dice, Die A and Die B, each with faces numbered from 1 to 6. You can only roll both the dice together & your turn is guided by the obtained sum.

Example: Die A = 6, Die B = 3. Sum = $6 + 3 = 9$

You may represent Dice as an Array or Array-like structure.

Die A = [1, 2, 3, 4, 5, 6] where the indices represent the 6 faces of the die & the value on each face.

Part-A:

1. How many total combinations are possible? Show the math along with the code!

Ans: 36 (by multiplying the size of the both dice we get $6*6 = 36$)

2. Calculate and display the distribution of all possible combinations that can be obtained when rolling both Die A and Die B together. Show the math along with the code! Hint: A 6 x 6 Matrix

Ans:

All Possible Combinations:

[(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)]

[(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)]

[(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)]

[(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)]

[(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)]

[(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)]

Sum of Combination:

[2, 3, 4, 5, 6, 7]

[3, 4, 5, 6, 7, 8]

[4, 5, 6, 7, 8, 9]

[5, 6, 7, 8, 9, 10]

[6, 7, 8, 9, 10, 11]

[7, 8, 9, 10, 11, 12]

3. Calculate the Probability of all Possible Sums occurring among the number of combinations from (2).

Example: $P(\text{Sum} = 2) = 1/X$ as there is only one combination possible to obtain $\text{Sum} = 2$. Die A = Die B = 1.

Ans:

$P(\text{Sum} = 2) = 0.0278$, $P(\text{Sum} = 3) = 0.0556$, $P(\text{Sum} = 4) = 0.0833$,
 $P(\text{Sum} = 5) = 0.1111$, $P(\text{Sum} = 6) = 0.1389$, $P(\text{Sum} = 7) = 0.1667$,
 $P(\text{Sum} = 8) = 0.1389$, $P(\text{Sum} = 9) = 0.1111$, $P(\text{Sum} = 10) = 0.0833$,
 $P(\text{Sum} = 11) = 0.0556$, $P(\text{Sum} = 12) = 0.0278$

Output of Part-A:

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Part-A Results:
Total Combinations: 36

Distribution Matrix:
All Possible Combinations:
(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)
(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)
(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)
(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)
(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)
(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)

Sum of Combination:
[2, 3, 4, 5, 6, 7]
[3, 4, 5, 6, 7, 8]
[4, 5, 6, 7, 8, 9]
[5, 6, 7, 8, 9, 10]
[6, 7, 8, 9, 10, 11]
[7, 8, 9, 10, 11, 12]

Probability of Sums:
P(Sum = 2) = 0.0278
P(Sum = 3) = 0.0556
P(Sum = 4) = 0.0833
P(Sum = 5) = 0.1111
P(Sum = 6) = 0.1389
P(Sum = 7) = 0.1667
P(Sum = 8) = 0.1389
P(Sum = 9) = 0.1111
P(Sum = 10) = 0.0833
P(Sum = 11) = 0.0556
P(Sum = 12) = 0.0278
```

Part-B:

Now comes the real challenge. You were happily spending a lazy afternoon playing your board game with your dice when suddenly the mischievous Norse God Loki (You love Thor too much & Loki didn't like that much) appeared.

Loki dooms your dice for his fun removing all the "Spots" off the dice.

However, Loki has doomed your dice with the following conditions:

- **Die A cannot have more than 4 Spots on a face.**
- **Die A may have multiple faces with the same number of spots.**
- **Die B can have as many spots on a face as necessary i.e. even more than 6.**

But in order to play your game, the probability of obtaining the Sums must remain the same!

So if you could only roll $P(\text{Sum} = 2) = 1/X$, the new dice must have the spots reattached such that those probabilities are not changed.

Input:

Die_A = [1, 2, 3, 4, 5, 6] & Die_B = Die_A = [1, 2, 3, 4, 5, 6]

Output:

A Transform Function `undoom_dice` that takes (Die_A, Die_B) as input & outputs New_Die_A = [?, ?, ?, ?, ?, ?], New_Die_B = [?, ?, ?, ?, ?, ?] where,

No New_Die A[x] > 4

Ans:

New Dice are
(1, 2, 2, 3, 3, 4)
(1, 3, 4, 5, 6, 8)

Logic:

The problem's logic lies in maintaining the total of 42 spots after the Loki's restriction. The first approach that comes to the mind is to brute force it with all the combinations of the dice A with 6 faces and spots of [1,2,3,4] and dice B with 6 faces and spots of 1 to 11. After brute-forcing the output had multiple redundant values with different positions. But still gave the answer arrays in the time complexity of n to the power of 6 by 6 nested for loops.

After brute-forcing the recursion and memorization i.e. Dynamic programming concept is used to store the combinations and the redundant values have been removed. This reduced the space and time complexity.

After finding all the possibilities of the dice A and B now they are passed to a function iteratively to find the sum probabilities and compares with the original probability.

In order to reduce the complexity further the condition of sum of all numbers in the both dice equals to 42 is used. And after some analysis it is clear that there needs to be [1,4] and [1,8] in dice A and B respectively, Because 2 and 12 can be made only with these numbers under the Loki's restriction. so the free spots in both dice is reduced to 4 and the values needs to be given as input is reduced in die B is reduced from 12 to 6. This reduced the time complexity drastically.

Output of Part-B:

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New Dice are:  
1 2 2 3 3 4 1 3 4 5 6 8
```