Governance Decentralization Game Theory: A Deep Dive into Negotiations and the Median Voting Rule

Manda Labs

August 15, 2023

Abstract

This paper explores the governance and negotiation processes within Decentralized Autonomous Organizations (DAOs). As DAOs grow, communication ways reveal inefficiencies, necessitating innovative solutions. The study highlights the median mechanism as a voting rule that minimizes discontent within a group, satisfying desirable properties like robustness to outliers and truthfulness. The practical applications of these theoretical concepts can be used in use cases in price discovery mechanisms for grant allocations, treasury management, counter-proposal mechanisms and more.

Introduction

Decentralized Autonomous Organizations (DAOs) represent a novel form of internet-native communities based on cooperation. Vitalik Buterin defined DAOs as "a virtual entity that has a certain set of members or shareholders which, perhaps with a 67% majority, have the right to spend the entity's funds and modify its code." DAO operates as a funding mechanism that is limited by the complexity of decision making when the number of participants increases, especially for numerical parameters such as when dealing with price discovery problems.

This organizational standard offers numerous competitive [1] advantages over traditional structures, including scalability, rapid growth, enhanced retention, global impact, fair resource distribution, and the elimination of a single point of failure across various sectors such as financial institutions, investment, non-profit, research...

One of the key principles underlying DAOs is ensuring no single point of failure, achieved through decentralization that enhances resilience across social (governance) and technical (code) layers. While this increases security, it also leads to slow decision-making and high technical costs, limiting efficiency [2].

Manda aims to introduce a platform for decentralized governance and a level playing field for executing operations, designed for business efficiency. Built around two products, a voting mechanism leveraging the wisdom of the crowd and a secure trading infrastructure. Manda seeks to open new avenues of opportunities, support communities, and empower the decentralized market by facilitating operations between all actors.

Ways of negotiating

We have identified 3 ways of negotiating a price with a group. We partition them by top down, bottom up or a mixture of both.

- Top-down approach: One participant proposes its preference, the rest vote yes or no.
- Bottom up approach: Every participant of the group proposes it's preference in a simultaneous or sequential way, and then a voting rule aggregates all preferences to come up with a value.
- Flexible Top-Bottom approach: Every participant of the group delegate his preference in a simultaneous way. Each participant then proposes it's preference in a simultaneous or sequential way. A voting rule aggregates all preferences to come up with a value.

Often, DAO contributors have raised their voices about the feeling of not being represented by one proposal maker. Hence, sometimes the top-down approach is not ideal. See the communities contention around the price discovery in the FEI and RARI merger [3]. Relying on current top-down voting mechanisms (yes/no) reveal to be undemocratic, going against the ethos of decentralization. A lot of issues are being discussed today behind closed doors in a centralized way. We will highlight in a first part the median mechanism as a collaborative way to reach agreement on numerical and categorical parameters and then discuss the complexity of decision making in a second part.

The median mechanism as a voting rule for DAOs

In bargaining theory and voting, The geometric median is the most known solution of the Weber problem. It can be seen as an optimal outcome and represents a compromise that minimizes overall discontent.

Voters choose points in a space, symbolizing their preferred outcomes. The Geometric Median of these points minimizes the sum of the distances to all chosen points, yielding a compromise that reflects the least collective discontent.

It assures a minimum satisfaction in the group of 50%. The property of the geometric median that it minimizes the sum of the distances to all chosen points is its defining characteristic.

The proof involves concepts from optimization theory (convex optimization).

Let's first define some concepts:

- Let S be a set of points $\{p1, p2, \ldots, pn\}$ in a Euclidean space. We can denote the coordinates of these points as vectors of votes.
- Let X be a point in the same space, also denoted as a vector of coordinates.
- The geometric median $m(S)$ is defined as the point that minimizes the sum of the Euclidean distances from X to all points in S. The Euclidean distance between X and p_i is denoted $d(X, p_i)$, and it's equal to $\|X - p_i\|$ (the norm of the difference between X and p_i).
- So we want to find the X that minimizes $F(X) = \Sigma d(X, p_i)$ for $i = 1$ to n. This is a minimization problem.

The function $F(X)$ is a convex function, since it is a sum of convex functions (each $d(X, p_i)$ is a convex function). In a convex function, any local minimum is also a global minimum. So, to find the minimum, we need to find the point(s) where the derivative of $F(X)$ is zero.

However, the function $F(X)$ is not differentiable at every point because it involves absolute values (the norm $\|X - p_i\|$ is equivalent to the absolute value when considering distances). Yet, this issue can be resolved by the subdifferential calculus, a generalization of differential calculus for convex functions that are not everywhere differentiable.

By a general result from convex analysis, a point m is a minimizer of $F(X)$ if and only if 0 is in the subdifferential of $F(X)$ at m.

Therefore, we know that m is a geometric median if and only if 0 is in the subdifferential of $F(X)$ at m. In simpler terms, it means that m is a point where the "pull" of each point p_i in S, weighted by its distance to m, is balanced.

The satisfaction of a group can be thought of as a decreasing function of the sum of these distances, and therefore, by minimizing this sum, we are maximizing the group's satisfaction. This explains why the geometric median is the compromise that reflects the least collective discontent.

Strategic Properties of the Median Mechanism

1. **Existence:** For any finite set of points in a Euclidean space, a Geometric Median exists.
   Proof: The sum of Euclidean distances to a set of points is a continuous, strictly convex function, which means it has a unique minimum.

2. **Uniqueness:** The Geometric Median is unique whenever the points are not collinear.
   Proof: The function defined by the sum of distances to the points is strictly convex, and hence the minimum is unique.

3. **Robustness:** The geometric median has a breakdown point of 0.5 (or up to quorum). That is, up to half of the sample data may be arbitrarily corrupted, and the median of the samples will still provide a robust estimator for the location of the uncorrupted data.

4. **Truthfulness:**

   Demonstrating that the Nash equilibrium of the median mechanism is reached when participants truthfully report their individual values involves illustrating that no player can improve their outcome by deviating from truth-telling, given that all other players are also reporting truthfully. Here's a step-by-step breakdown:

Set-up

Each participant i has a private value v_i drawn from an interval I of real numbers. The mechanism M maps from I^n (the set of all possible private value profiles) to I. In the median mechanism, $M(v_1, v_2, \ldots, v_n)$ is the median of the values $v_1, v_2, \ldots, v_n$.

Consider that all participants can report values different than v_i (let's call it v_i').

- If $v_i < M(v)$ and $v_i' > v_i$, the outcome stays the same or increases.
  - **Base Case:** $n = 3$, In a 3-player game with ascending values ($v_1 \leq v_2 \leq v_3$) and median v_2, if a player with a value $v_1 < v_2$ increases their reported value to v_1', the median stays v_2 as long as $v_1' \leq v_2$. If $v_1' > v_2$, the median might increase, which does not benefit the player who wanted a lower median.
  - **Inductive Hypothesis:** In a n-player game with ascending values ($v_1 \leq v_2 \leq \ldots \leq v_n$) and median $v_{\lceil n/2 \rceil}$, if a player i with a value $v_i < v_{\lceil n/2 \rceil}$ increases their reported value to v_i', the median stays $v_{\lceil n/2 \rceil}$ as long as $v_i' \leq v_{\lceil n/2 \rceil}$. If $v_i' > v_{\lceil n/2 \rceil}$, the median might increase, which does not benefit the player who wanted a lower median.
  - **Inductive Step:** We now need to show that the statement holds for $n+1$ players. Consider a $(n+1)$-player game with ascending values ($v_1 \leq v_2 \leq \ldots \leq v_n \leq v_{n+1}$). The median is $v_{\lceil (n+1)/2 \rceil}$. Let's consider a player i such that $v_i < v_{\lceil (n+1)/2 \rceil}$. If v_i changes their reported value to $v_i' > v_i$, two scenarios can occur:
    (a) If $v_i' \leq v_{\lceil (n+1)/2 \rceil}$, the median remains $v_{\lceil (n+1)/2 \rceil}$, which is analogous to the n-player game from the inductive hypothesis.
    (b) If $v_i' > v_{\lceil (n+1)/2 \rceil}$, then the median might change. If the change doesn't affect the ordering of the middle number(s), the median remains the same. This is consistent with the inductive hypothesis.

    Thus, the statement holds for the $(n+1)$-player game.
    By the principle of mathematical induction, the statement is true for all natural numbers $n \geq 3$.

- If $v_i > M(v)$ and $v_i' < v_i$, the outcome depends on the actual value of v_i'. If $v_i' \geq M(v)$, the outcome remains the same; if $v_i' < M(v)$, the median might decrease, which does not benefit the player who wanted a higher median.
- If $v_i = M(v)$, any deviation from truthfulness shifts the median away from the individual's private value, leaving them worse off.

In each case, the participant can't improve their outcome by deviating from truth-telling, therefore it is strategy-proof for participants to report their true value.

The complexity of group decision making on numerical parameters

Assumptions & notations:

- There are n total participants in the negotiation
- Among these, m participants have unique voting power and whose identities matter, we will call them significant players.
- M is proportional to n ($m = k \cdot n$, where $k \in \mathbb{R}[0, 1]$)
- The remaining $n - m$ participants are identical, and their individual preferences do not matter.
- Each participant can choose from p different options.

Now that we set the stage, we can tackle the complexity problem of the negotiation as follows. One way to measure the complexity of a negotiation is by looking at the number of possible agreements (or 'states') that could be made.

For example:

For the significant voters, each can choose from p options, and since their identities matter, they contribute p options each. This lead to p^m possible states.

For the identical voters $(n - m)$, since their identities don't matter, they collectively contribute a combination of choices among the p options. This yields a count of $C((n - m)p, n - m)$, which leads to the number of states that these significant players voters can generate.

Therefore, the total number of possible states for the negotiation would be $p^m \cdot C((n - m)p, n - m)$.

Translating this into Big O notation would give:

- If $k \ll 1$ (The significant voters are a very small proportion of the total voters), the complexity is denominated by the term for the identical voters, suggesting that the decision making is largely driven by the majority's opinion.

- If k is not negligible, the complexity becomes influenced by the term for the significant voters, indicating that they play a substantial role in the outcome of the negotiation.

The study dives into the intricacies of group decision-making by distinguishing between significant and identical voters. By considering the number of possible outcomes (states) stemming from these voters' choices, the study quantifies the complexity of the negotiation process. It hints that when significant voters are a minority, decisions are primarily driven by the majority. However, as the proportion of significant voters increases, their influence becomes more pronounced. This research paves the way for further exploration on the viability of bottom-up vs. top-down negotiation approaches. Current efforts are centered on simulating real-world scenarios, varying parameters like connection strengths between participants, the quality of decision-making at each layer, the time required for decisions to propagate, and more, to glean insights into centralized and decentralized governance structures.

Conclusion

Decentralized Autonomous Organizations (DAOs) present a revolutionary model for internet-native communities. As they expand, their governance models and negotiation processes face the challenge of efficiently reconciling the diverse preferences of their participants. We argue that the median mechanism is a promising solution that strikes a balance by minimizing collective discontent. Through the median voting rule, we found out this method ensures robustness and truthfulness, thus optimizing group decision-making in DAOs. Despite its merits, it's important to acknowledge that no mechanism is flawless, and the median mechanism does have its constraints, which shall be studied further. Nevertheless, its application holds significant potential in redefining decision-making processes in the rapidly evolving landscape of DAOs. Future exploration should probe further into diving deeper into the strategic properties and limitations of the median voting rule and others. We hope to better understand the strategic properties of voting rules, and find what voting rules are strategy-proof for people to report their true values. In voting rules that is not strategy-proof, we should find the equilibrium of those mechanisms. Finally, we aim to tackle the problems to address the problem of efficiency of voting rules relative to an omniscient social planner.

References

1 Llyr, B. (2022, February 8). Re-envisioning corporations: How DAOs and blockchain can improve the way we organize. World Economic Forum.

2 Chohan, U. W. (2017). The Decentralized Autonomous Organization and Governance Issues. Available at SSRN. [Online] Available: https://ssrn.com/abstract=3082055

- Myerson, R. B., Satterthwaite, M. A. (1983). Efficient Mechanisms for Bilateral Trading. Journal of Economic Theory, 29(2), 265-281.
- Procaccia, A. D., Tennenholtz, M. (2013). Approximate mechanism design without money. ACM Trans. Econ. Comp., 1(4), Article 18.
- Schummer, J., Vohra, R. V. (2007). Mechanism Design without Money. In "Algorithmic Game Theory" (pp. 243-266). Cambridge University Press. DOI: 10.1017/CBO9780511800481.012

Websites

3 https://tribe.fei.money/t/fip-51-fei-rari-token-merge/3642/12