TEL-AVIV UNIVERSITY

The Iby and Aldar Fleishman Faculty of Engineering

The Zandman-Slamer School of Graduate Studies

Multi-Modal Multi-Objective Evolutionary Optimization with Solutions of Variable-Length

A thesis submitted toward the degree of

Master of Science in Mechanical Engineering

By

Yosef Breslav

July, 2021

The Iby and Aladar Fleischman
Faculty of Engineering
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This research was carried out in the School of Mechanical Engineering

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Abstract

Multi-modal optimization aims to provide decision-makers with alternative solutions, possibly near optimal, and not just one optimal solution. In recent years, there has been a surge of interest on extending this approach to Pareto-based multi-objective optimization.

This research focuses on solving a special kind of multi-modal multi-objective optimization problems (MMOPs) in which solutions belong to decision-spaces of various dimensions (i.e., solutions of variable length). First, this thesis introduces the research motivation, contribution, and innovation. Next, a review is provided on the state-of-the-art algorithms and methods in the area of MMOPs. According to the review, this thesis contains the first study on MMOPs that involve such problems with solutions of variable-length. Problem definition and solution framework is suggested to allow using standard multi-modal multi-objective evolutionary algorithms (MMEAs) to solve the considered type of problems. Next, a novel MMEA is introduced. The algorithm, which is termed Archived Multi-Modal NSGA-II (amm-NSGA-II), constitutes a modification to the well-known NSGA-II. Following the algorithm description, a real-life example of the considered type of problems is suggested concerning optimal antennas' layout-allocation design for a wireless communication network.

Finally, the proposed amm-NSGA-II is employed to solve different optimal antennas' layout-allocation design problems. When compared with other MMEAs, it is shown that the proposed algorithm provides not only better solution diversity in the decision-space, but also solutions with superior performance vectors. This appears to be attributed to the type of archive that is used here, which leads to recommendations concerning future research.

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Table of Contents

1. INTRODUCTION 1

1.1. Research Motivation 1

1.2. How to Solve vl-MMOPs 2

1.3. Research Contribution and Innovation 2

1.4. **Thesis Outline 3**

2. BACKGROUND 4

2.1. Multi-Objective Optimization 4

2.2. Multi-modal Multi-objective Optimization 5

2.3. Studies on MMEAs 6

2.4. State-of-the-art Conclusions 9

3. PROBLEM DEFINITION & SOLUTION APPROACH 10

4. ANTENNAS’ LAYOUT-ALLOCATION PROBLEM 11

4.1. Network Modelling 11

4.2. Main and Auxiliary Problems 13

5. SOLUTION SELECTION MEASURES AND ARCHIVING 16

6. THE PROPOSED ALGORITHM 19

6.1. Selection 21

6.2. Elitism 21

6.3. Archive 21

7. COMPARISON APPROACH AND MEASURES 22

7.1. Full-Search Approach to obtain a Reference Solution 22

7.2. Performance Indicators for the Decision-Space 22

8. EXPERIMENTS & RESULTS 24

8.1. The fl – MMOPs and Their Reference Solutions 24

8.2. Comparing amm-NSGA-II using the fl-MMOPs 29

8.3. Comparing amm-NSGA-II using vl-MMOPs 44

8.4. Discussion on the Applied Archive 48

9. SUMMARY & CONCLUSIONS 49

10. REFERENCES 50

List of Figures

Figure 1: An example of multi-modal multi-objective scenario 1
Figure 2: Pareto-front example in a Min-Min problem 5
Figure 3: Difficulty to obtain all the Pareto sets 5
Figure 4: An example of a theoretical communication operation-area 11
Figure 5: 2x2km2 operation-area with 100 edge devices and 50 buildings 14
Figure 6: Pareto-fronts of 3 different concepts 15
Figure 7: Archive size as a function of the generation number 18
Figure 8: Basic GA 19
Figure 9: Mutual elements of the fl-MMOPs 25
Figure 10: Decision Space - 1st fl-MMOP 26
Figure 11: Decision Space – 2nd fl-MMOP 26
Figure 12: Decision Space – 3th fl-MMOP 27
Figure 13: Pareto Front - 1st fl-MMOP 28
Figure 14: Pareto Front– 2nd fl-MMOP 28
Figure 15: Pareto Front– 3th fl-MMOP 29
Figure 16: The evolution of non-dominated solutions in a sub-region 31
Figure 17: Archive size as a function of number generations 31
Figure 18: Front comparison for the first problem 33
Figure 19: Front comparison for the second problem 34
Figure 20: Front comparison for the third problem 34
Figure 21: Decision space comparison for 1 antenna – 1st problem (part A) 35
Figure 22: Decision space comparison for 1 antenna – 1st problem (part B) 36
Figure 23: Decision space comparison for 1 antenna – 2nd problem (part A) 37
Figure 24: Decision space comparison for 1 antenna – 2nd problem (part B) 38
Figure 25: Decision space comparison for 1 antenna – 3rd problem (part A) 39
Figure 26: Decision space comparison for 1 antenna – 3rd problem (part B) 40
Figure 27: amm-NSGA-II: 30 runs (right) vs. single run (left) for problem No. 1 41
Figure 28: Close-up on some single run results of amm-NSGA-II for the 1st problem 42
Figure 29: Mo-Ring-PSO-SCD single run (left) vs. 30 runs (right) for the 1st problem 42
Figure 30: amm-NSGA-II (left) & Mo-Ring-PSO-SCD (right), single run, 2nd problem 43
Figure 31: amm-NSGA-II (left) & Mo-Ring-PSO-SCD (right), 30 runs, 2nd problem 43
Figure 32: Front comparison for the 1st vl-MMOP 45
Figure 33: Front comparison for the 2nd vl-MMOP 46
Figure 34: Front comparison for the 3rd vl-MMOP 47

List of Tables

Table 1: An example of the archive 17
Table 2: Test problems parameters 25
Table 3: Measures comparison – Test problem 1 30
Table 4: Measures comparison – Test problem 2 30
Table 5: Measures comparison – Test problem 3 30

List of Acronyms

amm-NSGA-II	– Archived Multi Modal NSGA-II
AS	– Archive Similarity
CD	– Crowding Distance
CON	– Connectivity
CPDEA	– Convergence Penalized Density Evolutionary Algorithm
DTC	– Discrete Total Cost
EA	– Evolutionary Algorithm
FDR	– False Discovery Rate
fl	– Fixed Length
FP	– False Positive
MMEA	– Multi-modal Multi-objective evolutionary algorithm
MMOEA/DC	– MMOEA with Dual Clustering
MMOP	– Multi-modal Multi-objective Optimization Problem
MOEA	– multi-objective evolutionary algorithm
MOEA/D-DE	– MOEA based on Decomposition
MOPSO	– Multi-Objective Particle Swarm Optimization
MORingPSOSCD	– MOPSO using Ring Topology & Special Crowding Distance
PF	– Pareto Front
PS	– Pareto Set
Sim	– Similarity
SM	– Similarity Measure
T	– Threshold
TP	– True Positive
TriMOEA-TA&R	– MMEA using Two-Archived and Recombination Strategies
vl	– Variable Length

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List of Nomenclature

n_o - Dimension of the objective-space
X_m - Set of all the feasible solutions of dimension m
m - Dimension of the decision space
X - Set of all the feasible solutions from all the considered decision-spaces
f_m - The mapping between the solutions of X_m into the objective-space
F_m - The set of all performance vectors of all the solutions of $X_{m,}$
S - Set of the considered decision-spaces
A - Operation area
J - Number of buildings
K - Number of edge devices
p_i^a - Position vectors
R_{ij} - Distance between the i-th antenna and the j-th building
T - Threshold
SS_k - Connectivity index
C_{ss} - Given coefficient
β_j - Insurance cost
C_p - Cost of sub-area p
n_p - Number of antennas in the p-th sub-area
γ - Aggregated other costs
I - Number of antennas
p_k^{ed} - Location of an edge device
p_j^b - Building location
Par - Parent
etac - The distribution index for crossover
N - Population size
Gen - Generation
#F - Rank number
P - Population
R_{safe} - Required Safety Distance

1. INTRODUCTION

1.1. Research Motivation

Multi-Objective Evolutionary Algorithms (MOEAs) have been widely used to search for Pareto-optimal solutions to multi-objective problems [1]. Commonly, in such algorithms there is no special effort to find more than one solution per each of the optimal performance vectors. However, it is well known that decision-makers might wish to consider multiple solutions of either the same optimal performance or nearly the same (i.e., equivalent solutions). In fact, in recent years, such understanding has motivated the rapid development of algorithms to solve Multi-modal Multi-objective Optimization Problems (MMOPs) [2] & [3], i.e., problems that involve equivalent solutions, as described below.

Figure 1 illustrates an example of a MMOP with two-objectives. The left graph represents a two-dimensional decision-space, and the right one represents the objective-space. It can be seen from Figure 1 that the green squares have different (x,y) values in the decision-space but share the same performance vector in the objective-space. The same situation happens with the blue stars, orange circles and yellow triangles. Such solutions are referred to as equivalent solutions, i.e., solutions that have different decision-vectors in the decision-space, while sharing the same performance-vector in the objective space.

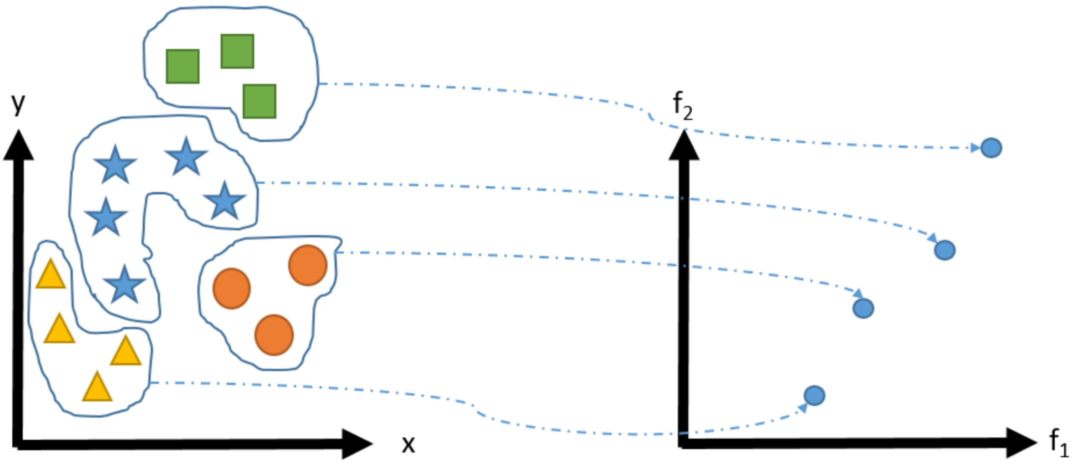


Figure 1: An example of multi-modal multi-objective scenario

It should be noted that in this example all the equivalent solutions belong to the same decision–space, i.e., solutions of fixed length. In contrast to the illustration in Figure 1, equivalent solutions may come from different decision-spaces, i.e., solutions may be of variable length (see example in Chapter 4).

According to [4], many design problems involve a decision on the optimal number of some components to be used. Such optimization problems are often described as problems of variable number of dimensions [4], also known as variable length problems. An extensive review on evolutionary approaches to solve such problems can be found in [5], which also describes many real-life variable-length problems. As demonstrated here, MMOPs may involve solutions of variable length. Such MMOPs are hereby referred to as vl-MMOPs, whereas those that are based on a decision space of a fixed dimension are hereby termed as fl-MMOPs.

To the best of our knowledge, and according to the recent review in [3], developments of Multi-modal Multi-objective Evolutionary Algorithms (MMEAs) have been restricted to solving MMOPs with solutions of fixed-length, i.e., to fl-MMOP). The above observations served as a motivation to this thesis.

1.2. How to Solve vl-MMOPs

When solving vl-MMOPs, special attention should be given to their definition and solution approach. This is primarily due to the fact that while comparable in a shared objective space, solutions from different search spaces are commonly not similar in an intuitive way. This may lead to the wrong conclusion that existing MMEAs cannot be used to solve vl-MMOPs. Here, it is suggested and demonstrated that a vl-MMOP can be solved by a simple search framework that is based on existing MMEAs. The main assumption is that the considered vl-MMOP involves a finite number of decision spaces.

According to the proposed solution approach, in Chapter 3, the considered vl-MMOP is divided into a set of n_{ds} auxiliary fl-MMOPs, where the i-th auxiliary problem involves solutions strictly from the i-th decision space. While separated by the decision spaces, all solutions of the auxiliary fl-MMOPs share the objective space of the original vl-MMOP. Such an approach assumes that equivalent solutions may belong to separated spaces but their equivalency is defined by way of their performances in the mutual objective space of the considered vl-MMOP. Solving the auxiliary problems can be done in a coupled (dependent/ simultaneous) or a decoupled (independent/sequential) approach. In this thesis the simple decoupled approach is studied.

1.3. Research Contribution and Innovation

In this thesis, a decoupled approach is suggested, which allows using existing MMEAs to solve vl-MMOPs without any modifications. The proposed procedure involves posteriori sorting of the union of solutions from all the auxiliary problems to reveal the equivalent optimal solutions of the considered vl-MMOP. The proposed solution framework has been inspired by the concept-based approach (e.g., [6]).

Another contribution of this thesis is the real-life problem example that is devised to demonstrate the considered type of vl-MMOPs. The suggested example is motivated by the need to design new wireless communication networks due to the rapid development of communication technology and the increasing use of sensors and actuators as edge devices. Optimizing the positions of the antennas and their number in such networks is one of the main challenges during the design process [7]. The optimal antennas' layout-allocation problem, which is devised here, constitutes a vl-MMOP. It involves three conflicting objectives. The first is the minimization of the number of antennas, the second is the maximization of the communication connectivity, and the third is the minimization of the involved cost.

In addition to the above contributions, this study suggests a new modification to NSGA-II, [8] for solving fl-MMOPs. While, not the first attempt to adapt NSGA-II for such problems (e.g., [9-10]), it appears that it is the first to include an unbounded archive rather than just elitism. Given that it is hard to predict the number of equivalent solutions in a given MMOP, the lack of an archive in previous attempts is considered a disadvantage. To highlight this difference, the proposed algorithm is termed Archived Multi-Modal NSGA-II, or in short amm-NSGA-II.

Finally, using the antennas' layout-allocation problem, the applicability of the proposed algorithm and solution framework is demonstrated. In addition, the results as obtained by the proposed amm-NSGA-II are compared with those obtained by several existing MMEAs. It is noted, that the use of multi-objective optimization to the design of wireless communication networks is not new, e.g., [11]. However, it appears to be the first time for studying such a design problem as a MMOP, and in particular as a vl-MMOP.

1.4. Thesis Outline

The rest of this thesis is organized as follows. Chapter 2 provides the necessary background for this thesis. Next, Chapter 3 includes the problem definition and solution approach, whereas Chapter 4 describes the suggested type of real-life antennas' layout-allocation problems. Chapter 5 explains the solution selection measures and the archiving. Chapter 6 introduces the proposed algorithm, while Chapter 7 suggests measures to compare the algorithm with existing ones. Next, Chapter 8 describes the numerical study and its results and analyses. Finally, Chapter 9 concludes this thesis.

2. BACKGROUND

This chapter is organized as follows. First, in Section 2.1, multi-objective optimization and in-particular Pareto-optimality is presented. Next, in Section 2.2, the concept of multi-modal multi-objective optimization is described. In Section 2.3, recent studies on the development of population based algorithms for such optimization problems are reviewed. Finally, the main conclusions from this review are highlighted in Section 2.4.

2.1. Multi-Objective Optimization

Without loss of generality, a multi-objective problem with *n*-dimensional decision variable vectors and *m* objectives can be defined as follows [12]:

$$\begin{cases} \min \boldsymbol{y}(\boldsymbol{x}) \\ \textbf{\textit{such that}}\ \boldsymbol{g_i}(\boldsymbol{x}) \leq \boldsymbol{0}, \quad \boldsymbol{i} = \boldsymbol{1}, \boldsymbol{2}, \dots, \boldsymbol{k} \\ \boldsymbol{h_j}(\boldsymbol{x}) = \boldsymbol{0}, \quad \boldsymbol{j} = \boldsymbol{1}, \boldsymbol{2}, \dots, \boldsymbol{l} \end{cases} \tag{1}$$

where $\boldsymbol{x} = (\boldsymbol{x_1}, \boldsymbol{x_2}, \dots, \boldsymbol{x_n}) \in \boldsymbol{X} \subset \boldsymbol{R^n}$ is an *n*-dimensional decision-vector, which is possibly subject to some inequality and equality constraints, and $\boldsymbol{y} = \boldsymbol{F}(\boldsymbol{x}) = (\boldsymbol{f_1}(\boldsymbol{x}), \boldsymbol{f_2}(\boldsymbol{x}), \dots, \boldsymbol{f_m}(\boldsymbol{x})$ is the performance vector of $\boldsymbol{x}$. In multi-objective optimization, which involves conflicting objectives, there is no one particular feasible solution that minimizes all the objectives simultaneously. Pareto-optimality considers such problems using the notion of dominance relation among solutions.

A solution $\boldsymbol{x} \in \boldsymbol{X}$ is called Pareto-optimal if there does not exist another solution that dominates it. Solution A dominates solution B, denoted as $\boldsymbol{A} \preceq$B, if both of the following conditions are satisfied:

i. $\boldsymbol{f_i}(\boldsymbol{A}) \leq \boldsymbol{f_i}(\boldsymbol{B})$ for all objectives
ii. $\boldsymbol{f_j}(\boldsymbol{A}) < \boldsymbol{f_j}(\boldsymbol{B})$for at least one objective

The Pareto-optimal set consists of the individual solutions that their performance vectors cannot be improved in any of the objectives without degrading at least one of the other objectives, i.e., they are non-dominated by any other feasible solution. The set of performance vectors of the associated solutions of the Pareto optimal set is termed the Pareto-front.

Figure 2 provides an example of a Pareto-front, for a Min-Min problem. The vectors connected by the red curve constitute the Pareto-front. Namely, they are the performance-vectors of the Pareto optimal solutions. The vector marked as C is not associated with a Pareto-optimal solution. It can easily be observed that it is dominated by the solutions with vectors A & B. On the other hand, the solutions associated with the performance-vectors A & B are not dominated by any other solution. Hence, both are included in the Pareto-optimal set.

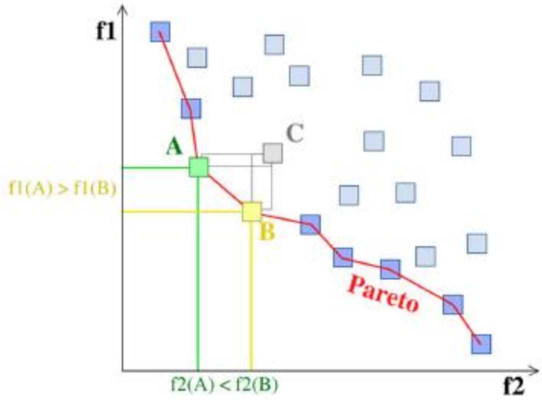


Figure 2: Pareto-front example in a Min-Min problem

2.2. Multi-modal Multi-objective Optimization

In practical applications, some multi-objective problems have different Pareto sets with the same performance vectors in the Pareto-front. Such problems are commonly termed as multimodal multi-objective optimization problems (MMOPs) [3], [13]. The solutions of MMOPs are termed equivalent solutions (see Figure 1 in the Introduction Chapter).

Most existing Evolutionary Multi-objective Optimization Algorithms (MOEAs), such as NSGA-II [8], focus on improving the diversity, spread and convergence of the solutions in the objective space. However, only a few works study the distribution of the equivalent solutions in the decision-space. In fact, it is difficult to simultaneously find and maintain all the equivalent Pareto optimal solutions. This is explained using Figure 3. Both A_1 and A_2 in the decision space correspond to A' in the objective space. Assume that A_1 and B are obtained in the decision space. B' is the point in the objective space corresponding to B. it can be seen in Figure 3 that the distance d_2 between A' and B' is very small. That is because A_1 and B are too crowded in the objective space. In the traditional multi-objective optimization algorithms, B will be deleted.

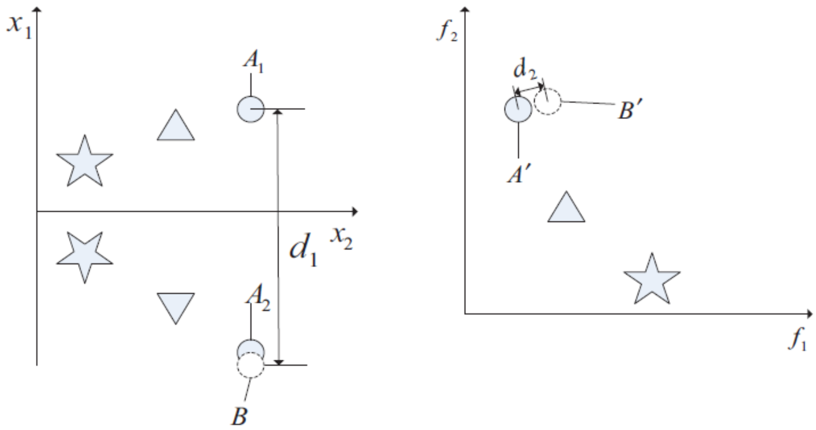


Figure 3: Difficulty to obtain all the Pareto sets

However, in the decision space, the distance d_1 between A_1 and B could be relatively large. In other words, A_1 and B are not crowded in the decision space. If we want to obtain both A_1 and A_2 at the same time, B should not be deleted. Solutions which are close to each other in the objective space may be far away from each other in the decision space.

Assume that we want to obtain all the Pareto optimal solutions in decision space and at the same time to find a high quality PF in objective space. It is not easy to improve the distribution in both the decision and the objective space simultaneously. Considering only the distribution in the decision space may deteriorate the quality of the obtained PF. As mentioned above, if all the Pareto optimal solutions in the decision space are to be maintained, then the crowding distance in the objective space should be ignored. However, without this indicator, the diversity in the objective space might not be maintained, which will deteriorate the quality of the obtained front.

2.3. Studies on MMEAs

According to the review in [3], in recent years there has been an increasing interest in the development of algorithms to solve MMOPS. The following provides some examples as of 2017.

In [15], which was published in 2017, the authors suggested a Particle Swarm Optimization (PSO) algorithm for MMOPs. The proposed method features an index-based ring topology to induce stable niches that allow the identification of a larger number of Pareto-optimal solutions with special crowding distance concept as a density metric in the continuous decision and objective spaces. During 2017, the authors in [16] proposed an evolutionary multi-objective optimization based multimodal optimization algorithm (EMO-MMO). In this paper the authors suggested to approximate multimodal fitness landscapes via multi-objectivization, thus providing and estimation of potential optimal areas. A multimodal optimization problem is transformed into a multi-objective optimization problem by adding an adaptive diversity indicator as the second optimization objective. An approximate fitness landscape is obtained via optimization of the transformed multi-objective problem using a MOEA. Then on the basis of the approximate fitness landscape, an adaptive peak detection method is proposed to find peaks where optimal solutions may be.

In 2018, TriMOEA-TA&R algorithm was suggested in [17] using two-archive and recombination strategies. The properties and relationships of decision variables are analyzed to guide the evolutionary search. The archives include convergence and diversity archives. The diversity archive employs a clustering strategy to guarantee diversity in the objective space and a niche-based clearing strategy to promote diversity in the decision space.

In 2019, another algorithm with an archive is suggested in [18] to solve MMOPs. It is termed MM-NAEMO (Multimodal Neighborhood-sensitive Archived Evolutionary Man-Objective Optimization). It uses mutation that occurs between the points which are local neighbors and each reference line always keeps at least two or more candidate solutions associated with it. Clustering is used to maintain the diversity in the population associated with each reference line, thus providing multiple Pareto sets as output. The MM-NAEMO algorithm removes the dominated points from each cluster while maintaining a minimum population for each cluster in the sub-archive. In [19], which was also published in 2019, the Multimodal Multi-objective Differential Evolution (MMODE) algorithm was suggested.

A comprehensive algorithm comparison study is given in [20]. It surveyed MMOP case studies and compared 11 different algorithms (NSGA-II/III, SPEA2, MOEA/D/D-TCH/D-PBI/D-AD, IBEA, DNEA, MO-Ring-PSO-SCD & Omni-Optimizer). The authors pointing out that standard evolutionary multi-objective algorithms show difficulties when solving multi-modal multi-objective optimization problems. Using genetic drift, they explain the decrease of diversity in the decision space during the evolutionary process. The study in [21] suggests using a density-based one-by-one update strategy. The update strategy considers diversity in both the objective and decision spaces. The harmonic average distance approach is used to estimate the global density of solutions in the decision space. Another bio-inspired method is suggested in [22]. The authors developed a self-organizing multi-modal multi-objective pigeon-inspired optimization (MMOPIO) algorithm which employs an improved pigeon-inspired optimization (PIO) based on consolidation parameters for simplifying the structure of the standard PIO. The self-organizing map is combined with the improved PIO for better control of the decision spaces.

Another MMEA is proposed in [23]. It is termed MMOEA/DC, which stands for Multimodal Multi-objective Evolutionary Algorithm with Dual Clustering. One clustering is run in decision space to gather solutions into multiple local clusters. Non-dominated solutions within each local cluster are selected to maintain local Pareto sets, and the remaining ones with good convergence in the objective space are also selected. The second clustering concerns diversity in objective space. Finally a pruning process is repeatedly run on the above clusters until each cluster has only one solution.

In [24], the authors proposed NIMMO, which is an algorithm to deal with more than three objectives. In NIMMO, the fitness calculation is performed among a child and its closest individuals in the solution space to maintain the diversity. Another way to tackle the imbalances in the decision space is described in [25]. The authors present a set of imbalanced distance minimization benchmark problems. Then propose an evolutionary algorithm using Convergence Penalized Density Evolutionary Algorithm (CPDEA), the distances among solutions in the decision space are transformed based on their local convergence quality. Their density values are estimated based on the transformed distances and used as the selection criterion.

Ring topology methods are also a common method to deal with MMOPs. The authors of [26] proposed an index-based ring topology to induce stable niches (MO_Ring_PSO_SCD) that allow the identification of a larger number of Pareto-optimal solutions and adopts a special crowding distance concept as a density metric in the decision and objective spaces. Another ring-based algorithm, [27], is MO_Ring_CSO_SCD, which uses a recently proposed meta-heuristic method competitive swarm optimizer algorithm. It also combines the ring topology, non-dominated sort strategy and special crowding distance approach.

In [28] the Self-organized Speciation based Multi Objective Particle Swarm Optimizer (SS-MOPSO) was introduced. A self-organized mechanism is proposed to reduce the complexity of the algorithm and improve the efficiency of the speciation. The self-organized mechanism increases the diversity of species and enhances the performance of the algorithm.

Some algorithms to solve MMOPs are based on NSGA-II. In [29], the authors added weighted-sum and different crowding distance mechanism to the NSGA-II. The authors study the effects of modified crowding distance and polynomial mutation operator on MMO algorithms. They provide an in-depth analysis on these modifications and apply them to NSGA-II.

In [30], another NSGA-II based algorithm is presented. The paper proposes a combination of crowding distance in the objective space, crowding distance in the decision space and a weighted-sum of both distances. The evaluation of the performances is integrated into the diversity part of NSGA-II. A grid-based mechanism is being used to measure distances between solutions in the decision space. The grid distance between two solutions is measured by Manhattan distances. A similar NSGA-II based algorithm is presented in [31].

In 2020, a Modified Particle Swarm Optimization (AMPSO) algorithm was presented in [32]. First, a dynamic neighborhood-based learning strategy is introduced to replace the global learning strategy and then, to enhance the performance of PSO, the offering competition mechanism is utilized. Another PSO algorithm is performed by adding Gaussian Sampling for multi-objective (GS-MOPSO).

The paper in [33] proposes the Gaussian sampling mechanism to form multiple neighborhoods by learning from optimal information of particles. The particles search their own neighborhoods to obtain more optimal solutions in the decision space. Moreover, an external archive maintenance strategy is proposed which allows the algorithm to maintain an archive containing better distribution and diversity of solutions.

2.4. State-of-the-art Conclusions

When considering studies as described in Section 2.3 and in the review on MMOPs in [3], one must reach the following conclusions. First, while a surge of new algorithms to solve MMOPs is apparent, these algorithms have been commonly tested on synthetic test problems and not on real-life problems. Second, neither the common definition of MMOPs, as presented in [3], nor any of its reviewed studies, includes MMOPs with solutions of variable length. These two conclusions led us to the current thesis focus, i.e., on MMOPs with variable length, as defined in Chapter 3, and on a real-life problem as suggested in Chapter 4. Finally, it should be noted that when deciding to focus on the design application area of wireless communication networks, as described in Chapter 4, we came across a few studies, which posed such a problem as a multi-objective one (e.g., [34], [35]). Yet, we could not find any such study that posed the problem as a MMOP.

3. PROBLEM DEFINITION & SOLUTION APPROACH

A recent review in [3], defines MMOPs in a form which is restricted to solutions that share one decision-space (i.e., solutions of a fixed length). In contrast, here the problem is defined such that solutions may belong to various decision-spaces of different dimensions. Let n_o be the dimension of the objective-space $\mathbb{R}^{\boldsymbol{n_0}}$ of the considered vl-MMOPs. Also, let S be the set of the considered decision-spaces, and $\boldsymbol{n_{ds}}$ be the number of such spaces (i.e., $\boldsymbol{n_{ds}} = |\boldsymbol{S}|$). Let $\boldsymbol{X_m}$ be the set of all the feasible solutions that are associated with the m-th decision-space, where n_m is the dimension of this space. In addition, let X be the set of all the feasible solutions from all the considered decision-spaces, i.e., $\boldsymbol{X} = \bigcup_{\boldsymbol{m=1}}^{\boldsymbol{n_{ds}}} \boldsymbol{X_m}$. Next, define $\boldsymbol{f_m}$:$\boldsymbol{X_m} \longrightarrow \mathbb{R}^{\boldsymbol{n_0}}$ as the mapping between the solutions of $\boldsymbol{X_m}$ into the objective-space. Also, let s_{mj} indicates the *j-th* solution of $\boldsymbol{X_m}$, and let $\boldsymbol{x_{s_{mj}}}$ represents the decision-vector of s_{mj}. Now, let $\boldsymbol{y_{s_{mj}}} = \boldsymbol{f_m}\left(\boldsymbol{x_{s_{mj}}}\right)$ be the performance vector of s_{mj}, where $\boldsymbol{y_{s_{mj}}} \in \mathbb{R}^{\boldsymbol{n_0}}$. In addition, let F_m be the set of all performance vectors of all the solutions of $\boldsymbol{X_m}$, and let F be the set of all the performance vectors of all feasible solutions from all the considered decision-spaces, i.e., $\boldsymbol{F} = \bigcup_{\boldsymbol{m=1}}^{\boldsymbol{n_{ds}}} \boldsymbol{F_m}$. As argued in [3], the formal definition of MMOPs is still controversial. In [3], based on [36], Tanabe and Ishibuchi adopted the idea of equivalent Pareto-optimal solutions and discussed both a relaxed and non-relaxed versions of this term. Following [3], the considered vl-MMOP is described as follows:

Given the aforementioned sets and mappings, find $\boldsymbol{X_e} \subseteq \boldsymbol{X}$, which contains all the equivalent Pareto-optimal solutions of X, and $\boldsymbol{F_e} \subseteq \boldsymbol{F}$ that contains the associated performance-vectors of the solutions in $\boldsymbol{X_e}$. As in [3], here equivalent solutions may include in the relaxed case not just the Pareto-optimal solutions but also any solution that has a performance vector that is within a prescribed distance from a Pareto-optimal solution. The above vl-MMOP description is general and can accommodate the two major types of equivalent solutions as described in [3]. In the current implementation we devised the problem such that there are multiple equivalent solutions without adhering to a relaxed definition of the equivalency. It is noted that, the above vl-MMOP description has been inspired by the multi-concept optimization problem, as in [6]. There are various ways to solve vl-MMOPs. As mentioned in Section 1.2, in the current study we use a simple solution approach that can be implemented using existing MMEAs. It is based on the understanding that the order of non-domination sorting does not influence the final results.

Hence, we propose to separately search the solutions within each $\boldsymbol{X_m}$ based on F_m, to obtain its set of equivalent Pareto-solutions $\boldsymbol{X_{me}} \subseteq \boldsymbol{X_m}$ and then sort the solutions within the union $\boldsymbol{X_u} = \bigcup_{\boldsymbol{m=1}}^{\boldsymbol{n_{ds}}} \boldsymbol{X_{me}}$ to obtain its set of equivalent Pareto-solutions $\boldsymbol{X_{ue}} \subseteq \boldsymbol{X_u}$. This procedure can be realized via separated searches within each $\boldsymbol{X_m}$ using existing MMEAs. However, in the current study we proposed our own algorithm for fl-MMOPs and implemented it according to the aforementioned approach.

4. ANTENNAS' LAYOUT-ALLOCATION PROBLEM

Our research includes a real-life vl-MMOP. The problem concerns a wireless communication network in which the location and amount of antennas have to be decided. First, the network modelling is described. Next, the main and auxiliary optimization problems are described.

4.1. Network Modelling

The considered operation area, $A \subseteq \mathbb{R}^2$, includes J existing buildings and K edge devices (potential customers) at fixed locations. Let a two-dimensional coordinate system X-Y be used to define a position vector of an object within the operation area A. For each of the separated auxiliary problems, the decision variables are the coordinates of the position vectors of the involved fixed number of antennas (I). The position vectors are:

$$\boldsymbol{p}_i^a \in \mathrm{A}\,,\ \ \forall \mathrm{i} \in \{1,2,\dots,\mathrm{I}\} \qquad (2)$$

The position vectors of the j-th building and of the k-th edge device are assumed to be given. These are defined as follows:

$$\boldsymbol{p}_j^b \in \mathrm{A}, \qquad \forall \mathrm{j} \in \{1,2,\dots,\mathrm{J}\} \qquad (3)$$

$$\boldsymbol{p}_k^{ed} \in \mathrm{A}, \qquad \forall \mathrm{k} \in \{1,2,\dots,\mathrm{K}\} \qquad (4)$$

The above vectors can be used to calculate the distances between each pair of objects in the operation area. Let the Euclidean distance between the i-th antenna and the k-th edge device be denoted as R_{ik}. Each such distance plays a role in the connectivity calculation of the network (see eq. 5). Similarly, let R_{ij} denotes the distance between the i-th antenna and the j-th building. These distances play a role in calculating the safety-insurance cost of the design (see eq. 6). Figure 4 illustrates an area with such a network design. Each antenna has a danger zone, which is marked by a red curve. Buildings within such a zone are drawn in red, whereas all others are in green. The figure also shows that some edge devices have communication connectivity to the antennas, whereas some, which are marked in a red color, do not.

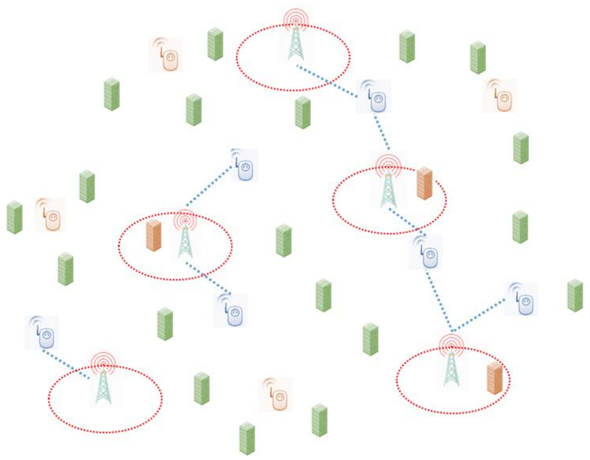


Figure 4: An example of a theoretical communication operation-area

In the considered problem there is a tradeoff between maximizing the number of potential customers (i.e., connectivity) and minimizing the total cost of the network change. Technically, the first objective translates into a desire to have a maximal number of edge-devices that can communicate with at least one antenna of the considered system.

For a system with I antennas, this number is expressed as a total connectivity measure CON. This measure is calculated assuming that the communication signal decays with increasing ${R_{ik}}^2$. To be effective, it is assumed that the signal between the i-th antenna and the k-th edge-device should be larger than a given threshold, T. The CON measure is defined as follows:

$$CON = \sum_{1<k\leq K} SS_k \qquad (5)$$

where, $SS_k = 1 \; if \; \exists \; i \in \{1,2,\dots,\mathrm{I}\} \; s.t. \frac{C_{ss}}{{R_{ik}}^2} > \mathrm{T}$, and C_{SS} is a given known coefficient.

The other objective deals with minimizing the total cost of changing the antennas. The total cost (denoted below as DTC) is assumed to be composed of several components including: safety-insurance cost, leasing cost, system maintenance cost, degradation cost, construction cost and the antennas' production cost.

The time dependent costs, e.g., the leasing cost, are calculated based on the expected life-time of the system. The cost components can be categorized into two types. The first involves costs that are location dependent including the safety-insurance cost.

$\mathrm{C_{safe}}$, and the leasing cost, C_{lease}. The other type involves all the rest of the cost components which are assumed to be location independent. These cost elements are aggregated into C_{other}, which is a function of the number of antennas, I. Let the total cost of a particular solution be:

$$DTC = \mathrm{C_{safe}} + C_{lease} + C_{other} \qquad (6)$$

This study assumes that the safety-insurance cost depends on the number of buildings that are at a radiation risk due to their proximity to the intended locations of the antennas. Namely, this cost is a function of the decisions on the antennas' locations, i.e., $C_{safe}(\boldsymbol{p_1^a}, \boldsymbol{p_2^a}, \dots \boldsymbol{p_I^a})$. A safety radius R_{safe} defines the unsafe areas around the antennas' locations. Let, $C_{ij} = 1 \; if \; R_{ij} < R_{safe}$ else $C_{ij} = 0$. Also, let β_j be the insurance cost associated with the j-th unsafe building, then:

$$\mathrm{C_{safe}} = \beta_j \sum_{\substack{0\leq\, i\, \leq I \\ 0<j<J}} C_{ij} \qquad (7)$$

The leasing cost is assumed to vary between different sub-areas of the considered operation area, i.e., $C_{lease}(\boldsymbol{p_1^a}, \boldsymbol{p_2^a}, \dots \boldsymbol{p_I^a})$. It is to be calculated according to the expected life-time of the system. Let the number of the sub-areas to be P and let the number of antennas that are located at the p-th sub-area be n_p. Also, let the leasing cost for one antenna at the p-th sub-area be C_p. Then, the leasing cost is:

$$C_{\text{lease}} = \sum_{p=1}^{P} C_p \cdot n_p \qquad (8)$$

Finally, the aggregated other costs results in:

$$C_{\text{other}} = \gamma \cdot I \qquad (9)$$

Where γ is the aggregated other costs per one antenna.

4.2. Main and Auxiliary Problems

The main network optimization problem is a vl-MMOP. This means that it aims to find all the equivalent optimal locations $\boldsymbol{p_i^a} \in A$, $\forall i \in \{1,2,\dots,I^*\}$ for the minimum number of antennas I^*, with maximum connectivity and minimum cost. Namely, the main problem is:

$$Min(I), Max\ (CON), Min(DTC) \quad (10)$$

where $1 \leq I \leq M$ and also $CON=CON(I, \boldsymbol{p_1^a}, \boldsymbol{p_2^a}, \dots \boldsymbol{p_I^a})$ and $DTC=DTC(I, \boldsymbol{p_1^a}, \boldsymbol{p_2^a}, \dots \boldsymbol{p_I^a})$.

The above problem is converted into a series of M two-objective auxiliary fl-MMOPs. Each of the auxiliary problems aims to find all the equivalent optimal locations $\boldsymbol{p_i^a} \in A$, $\forall i \in \{1,2,\dots,I\}$ where I is a fixed number of antennas. The n-th auxiliary problem is defined as:

$$Max\ (CON), Min(DTC) \quad (11)$$

where I=n antennas, $CON=CON(\boldsymbol{p_1^a}, \boldsymbol{p_2^a}, \dots \boldsymbol{p_n^a})$ and $DTC=DTC(\boldsymbol{p_1^a}, \boldsymbol{p_2^a}, \dots \boldsymbol{p_n^a})$.

It should be noted that we devised the implementation of the above design problem such that it has a discrete set of performance vectors. Moreover, it was devised such that it has an infinite number of equivalent solutions. This was achieved without adhering to the relaxed equivalency approach. It is further noted that the set of all possible performances in the first objective, which concerns the total connectivity, belongs to the positive natural numbers with the upper limit of K. In other words, CON belongs to a discrete finite set. For example in Figure 4, the upper limit of the CON value is 10 (10 edge devices).

Figure 5 represents an operation-area example of 2x2km^2 with 100 edge devices and 50 buildings. In this example we examined 3 different types of solutions. The first is with 5 antennas, the second is with 10 antennas and the third with 15. Figure 6 shows the CON and DTC optimal performances for the three types of solutions. It can be seen there that even with 15 antennas the maximal CON value of 100 was not reached.

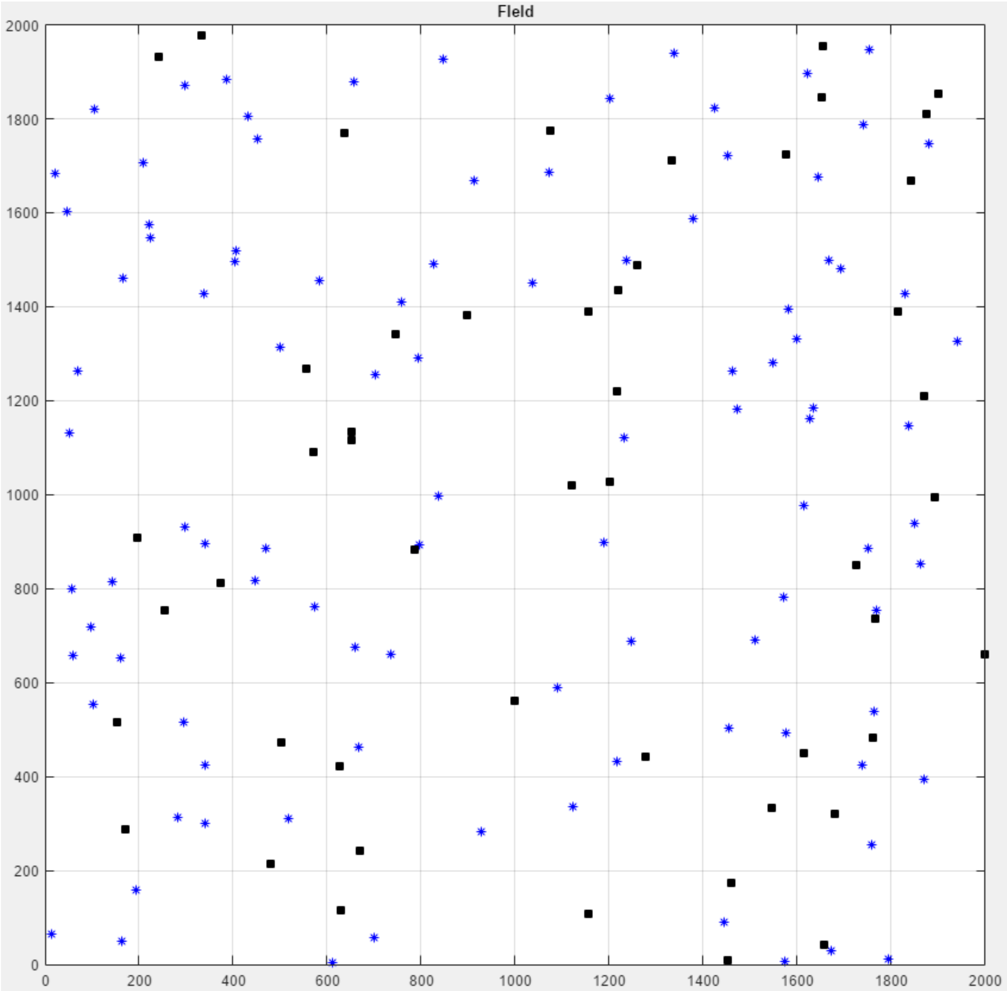


Figure 5: 2x2km2 operation-area with 100 edge devices and 50 buildings

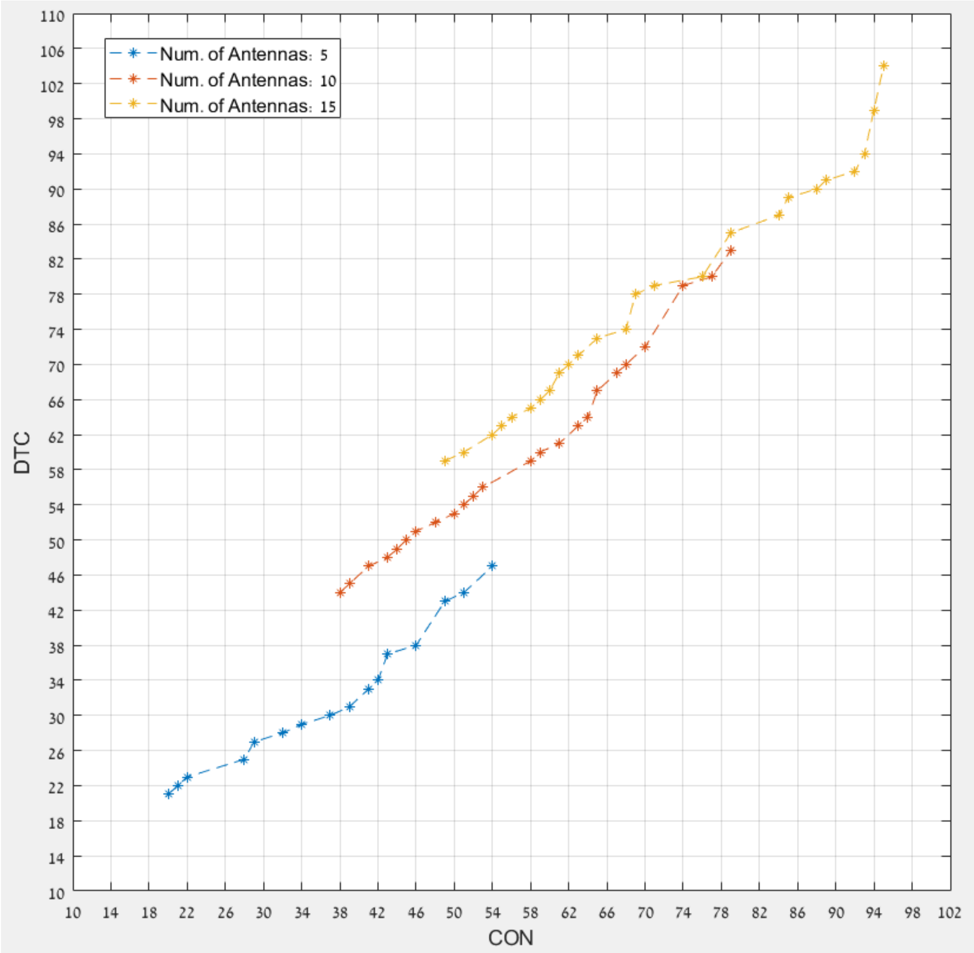


Figure 6: Pareto-fronts of 3 different concepts

5. SOLUTION SELECTION MEASURES AND ARCHIVING

Solving the main problem is done via the auxiliary problems. Each auxiliary problem is solved by the proposed evolutionary algorithm, which is described in Chapter 6. The following provides some descriptions to the measures that are used in the algorithm, when selecting solutions during the process of the evolution.

Each of the evolved solutions is to be evaluated and compared using three measures. The first is ranking, which is calculated as in NSGA-II (Deb. et al. [8]), using non-domination sorting based on the performance vectors of the compared solutions.

When the compared solutions have the same rank, they are compared by a second measure, i.e., crowding-distance, which is calculated based on the modified NSGA-II (Fortin et al. [37]). In NSGA-II, the crowding-distance of a solution is a distance measure in the objective space between the solution performance-vector and its neighboring performance vectors. The higher the value is, the better the solution is as compared with other solutions of the same rank. This aims to enhance uniform distribution of solutions along the Pareto-front. The authors in [37] noticed that the original measure does not support preservation of equivalent solutions. Hence, they suggested modifying the measure such that all solutions with the same performance vectors are assigned with the same crowding-distance value.

The third measure is a decision-space similarity value, which is proposed here. It allows a comparison between solutions, within each of the auxiliary problems, based on their similarity in the decision space. In the case of the considered problem, this means that the third measure is based on the antennas' locations in the operation area. Let the i-th individual solution be defined as an ordered set S(i) of position vectors of the involved antennas i.e.:

$$S(i) = \{\, \boldsymbol{p}^{a}_{\boldsymbol{1}(i)}, \boldsymbol{p}^{\boldsymbol{a}}_{\boldsymbol{2}(\boldsymbol{i})}, \ldots \boldsymbol{p}^{\boldsymbol{a}}_{\boldsymbol{I}(\boldsymbol{i})}\} \qquad (12)$$

Given N solutions, the similarity measure for the i-th solution is defined as:

$$SM(\mathrm{i}) = \sum_{\mathrm{j}=1}^{\mathrm{N}} \mathrm{SimMat}(\mathrm{j},\mathrm{i}) \qquad (13)$$

Where SimMat is a symmetric NXN similarity matrix, such that:

$$SimMat(\mathrm{i},\mathrm{j}) = \sum_{\mathrm{n}=1}^{\mathrm{I}} \sum_{\mathrm{m}=1}^{\mathrm{I}} \mathrm{AS}_{\mathrm{nm}}(\mathrm{i},\mathrm{j}) = \mathrm{SimMat}(\mathrm{j},\mathrm{i}) \qquad (14)$$

and,

$$\mathrm{AS}_{\mathrm{nm}}(\mathrm{i},\mathrm{j}) = 1 \text{ if } \mathrm{i} \neq \mathrm{j} \ \& \left\| \boldsymbol{p}^{a}_{m(i)} - \boldsymbol{p}^{a}_{n(j)} \right\| < \mathrm{SimDist} \qquad (15)$$
$$else \quad \mathrm{AS}_{nm}(i,j) = 0$$

During the evolution, the search algorithm uses an archive of solutions, which saves the accumulated solutions of the first rank including all equivalent solutions that have been found. Since that in the considered problem there is an infinite number of equivalent solutions, the employed archive is with an unlimited capacity, i.e. a dynamic archive. Table 1 shows an example of the archive after 50 generations in case of two antennas. The first 4 columns represent the decision space values of the two antennas i.e. – $\{X_1,Y_1\}$, $\{X_2Y_2\}$. The fifth and sixth columns are the CON and the DTC, the seventh, eighth and the ninth are the rank, crowding distance and the decision-space similarity value.

Figure 7 shows the archive size as a function of the generation number. We can see from the graph in Figure 7 that the size of the archive changes in each generation. Moreover, due to the fact that the archive saves only the first rank, it is possible that its size will get smaller, when better solutions are found.

Table 1: An example of the archive

X_1	Y_1	X_2	Y_2	CON	DTC	Rank	CD	Sim
1487.64	1347.97	295.15	1696.23	25	30	1	Inf	5
52.42	936.07	11.75	897.95	8	9	1	Inf	46
99.49	838.92	6.05	972.81	10	11	1	0.76	43
104.78	838.15	6.03	972.63	10	11	1	0.76	43
106.77	842.17	8.67	971.3	10	11	1	0.76	43
107.13	834.74	8.55	982.47	10	11	1	0.76	43
1486.44	1862.19	328.72	1690.2	22	25	1	0.51	7
32.98	686.84	63.39	318.77	12	13	1	0.48	43
33.91	668.66	61.92	324.04	12	13	1	0.48	43
78.82	809.35	9.45	307.83	12	13	1	0.48	44
79.68	816.88	9.97	295.72	12	13	1	0.48	44
81.7	808.74	9.63	291.89	12	13	1	0.48	44
107.63	807.88	9.7	298.16	12	13	1	0.48	48
107.66	807.99	9.68	286.69	12	13	1	0.48	48
107.66	807.98	9.68	286.3	12	13	1	0.48	48
107.8	807.96	9.93	297.28	12	13	1	0.48	48
108.27	808.03	9.67	282.25	12	13	1	0.48	48
111.47	806.64	9.56	240.3	12	13	1	0.48	48
111.49	806.34	9.56	240.85	12	13	1	0.48	48
112.17	806.95	9.56	195.94	12	13	1	0.48	48
113.27	807.08	9.57	245.8	12	13	1	0.48	48
114.48	806.79	9.57	244.83	12	13	1	0.48	48
206.87	1802.6	254.3	199.76	17	19	1	0.39	26
404.9	569.65	20.45	874.26	16	16	1	0.38	32
428.21	549.98	25.86	978.26	16	16	1	0.38	32
160.78	758.63	7.35	177.43	13	14	1	0.37	48
1489.15	1913.61	361.49	1684.32	20	23	1	0.37	7
1489.53	1914.22	362.94	1677.12	20	23	1	0.37	7
199.29	1650.52	193.94	200.89	19	20	1	0.36	24
165.57	722.94	56.83	277.08	15	15	1	0.33	48
154.6	285.33	2.4	884.47	15	15	1	0.33	50

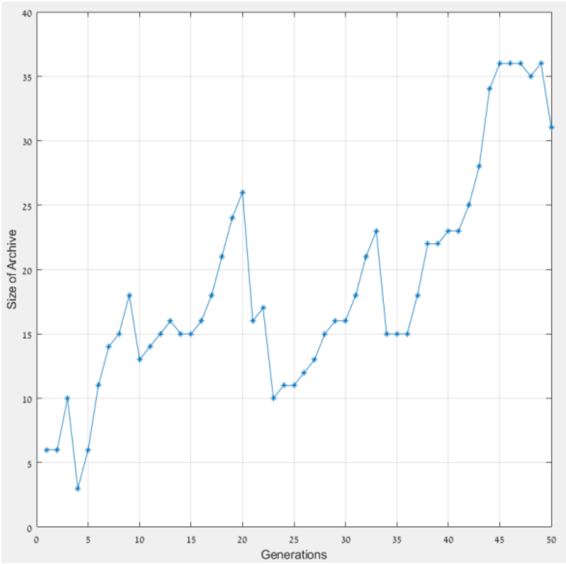


Figure 7: Archive size as a function of the generation number

6. THE PROPOSED ALGORITHM

The proposed algorithm, which is termed amm-NSGA-II, aims to find good representative sets of equivalent solutions which are associated with each of the performance vectors that represent the Pareto-front. Such a good representation aims not just at a uniform distribution of the vectors along the Pareto-front but also at a spread of evenly spaced solutions in the decision space**.** The proposed algorithm follows NSGA-II, which involves several modifications to a Genetic Algorithm (GA).

Figure 8 illustrates a basic GA. In addition to the shown elements, advanced GAs have an elite procedure and a diversity preservation procedure. The former procedure aims to ensure that new parents are not inferior to previous parents, whereas the latter procedure aims to reduce the chance of premature convergence.

In a GA each solution is evaluated based on a scalar measure (fitness). In contrast, in NSGA-II the evaluation of solutions is based on a transformation of their performance vectors into two measures, i.e., ranking and crowding distance. Using the former measure aims to provide selection pressure towards the Pareto-front. In contrast, using the latter measure aims to promote even distribution of the performance vectors of the optimal solutions along the Pareto-front. As in advanced GAs, NSGA-II also includes elitism. Finally, in NSGA-II, diversity preservation is achieved implicitly by the lexicographic selection that is based on the ranking and the crowding distance.

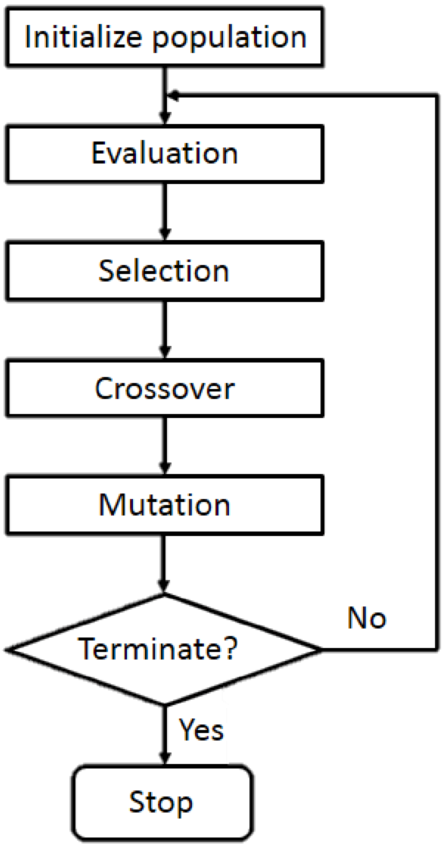


Figure 8: Basic GA

There are four main differences between amm-NSGA-II and NSGA-II. The first difference is the selection technique, which is used to create the mating pool. In contrast to NSGA-II, amm-NSGA-II uses a three level lexicographic selection that takes into considerations not only the ranking and the crowding distance, but also the decision-space similarity values of the solutions (see Chapter 5 and Section 6.1). The second difference involves a modification to the crowding distance (see Chapter 5). The third difference is the creation of the new population using elitism that is based on the aforementioned three measures (see Chapter 5 and Section 6.2). Finally, in contrast to NSGA-II, the current algorithm employs an archive that is updated at the end of each generation (see Section 6.3).

Based on [8], the Main Procedure of NSGA-II (following an initialization stage) involves four steps including:

Step 1 Combine parent and offspring populations and create $R_t = P_t \cup Q_t$. Perform a non-dominated sorting to R_t and identify different fronts: $\mathcal{F}_i$, $i = 1, 2, \ldots$, etc.

Step 2 Set new population $P_{t+1} = \emptyset$. Set a counter $i = 1$. Until $|P_{t+1}| + |\mathcal{F}_i| < N$, perform $P_{t+1} = P_{t+1} \cup \mathcal{F}_i$ and $i = i + 1$.

Step 3 Perform the `Crowding-sort`$(\mathcal{F}_i, <_c)$ procedure (described below on page 236) and include the most widely spread $(N - |P_{t+1}|)$ solutions by using the crowding distance values in the sorted $\mathcal{F}_i$ to P_{t+1}.

Step 4 Create offspring population Q_{t+1} from P_{t+1} by using the crowded tournament selection, crossover and mutation operators.

The 1st Step aims to calculate the rank of solutions that belong to a union of the parent and offspring populations. The 2nd and 3rd Steps aim to find the elite population out of the aforementioned union. The last step is a reproduction step that includes selection, crossover and mutation.

In addition to the use of a non-dominated sorting procedure in the ranking stage (Step 1), the main procedure of NSGA-II calls two sub-procedures, including crowding-sort in Step 3 and crowded tournament selection in Step 4. The details for these sub-procedures can be found in [8].

Based on the above, the Main Procedure of amm-NSGA-II is described as follows.

Step 1: As above.

Steps 2 and 3: Finding the elite population is done according to Section 6.2.

Step 4: The reproduction is done with a selection procedure as described in Section 6.1.

Step 5: Assign solutions to an archive as described in Section 6.3.

6.1. Selection

As in NSGA-II, binary tournament selection is used between individuals of the elite population. Here, a three-level lexicographic selection is employed, as compared with the two-level that are used in NSGA-II. The measures that are used for the selection are described in Chapter 5. The lexicographic selection involves ranking at the first level and then the modified Crowding-Distance (CD) in the objective-space and finally the similarity in the decision-space.

6.2.Elitism

This procedure follows the one in NSGA-II with the modification of considering not only the rank and CD, but also by the similarity measure SM. The solutions in the union set of the parent and offspring, i.e., R, are first ordered by their rank. Within the rank they are ordered by their CD, solutions of the same CD, i.e., the same performance vector, are sorted by their SM.

The new population is selected using these orders until reaching the population size. First, solutions are taken from the first rank. If there is no room to all solutions of that rank, solutions are selected according to their CD and by selecting the one that has a smaller SM among solutions of the same CD. When there is still a room this procedure of selecting individuals based on their CD and SM is repeated using solutions that where not selected so far, i.e., equivalent solutions of the next smaller SM. If there is room to all solutions of the first rank, and these are less than the population size, the procedure continues to the solutions of the next rank.

6.3. Archive

In contrast to NSGA-II, the proposed algorithm contains an archive. First, when the archive is empty, solutions of the first rank of the elite population enter the archive. However, if the archive already contains solutions, then those of the first rank of the current elite population are united with the archive solutions. Next, the non-dominated solutions of the union are kept in the archive including all equivalent solutions.

7. COMPARISON APPROACH AND MEASURES

7.1. Full-Search Approach to obtain a Reference Solution

In general, the considered problem has no analytical solution. Moreover, as shown in the 'Experiments & Results' chapter, it may have an infinite set of equivalent solutions. Yet, any numerical solution is bound to produce only a finite set of solutions. Hence, testing the algorithm is somewhat problematic.

To cope with this situation, we propose to create a reference solution by solving a modified problem in which the decision-space is discretized using a uniform grid, which is defined by a resolution vector $\boldsymbol{\varepsilon}$.

Next, the discretized problem is solved by a full-search approach to produce a reference set. In the 'Experiments & Results' chapter, we applied this approach to three fl-MMOPs which involve just one antenna.

7.2. Performance Indicators for the Decision-Space

The following indicators are used in this study to comapre solutions:

7.2.1. Inverted Generational Distance (IGD)

The Inverted Generational Distance indicator evaluates the quality of an obtained solution set in comparison with a pre-specified reference point set. The indicator is based on the distance between a solution and a reference point.

$$\mathbf{IGD} = \frac{\sum_{\mathbf{p}\in\mathbf{p}^*} \mathbf{dist}(\mathbf{p}, \mathbf{PF})}{|\mathbf{p}^*|} \quad (\mathbf{16})$$

7.2.2. False Discovery Rate (FDR)

False discovery rate is a statistical approach used in multiple hypotheses testing to make statistical inference. Commonly, FDR is defined as the expected proportion of false discoveries, i.e., incorrectly rejected null hypothesis, among all discoveries [38]. Inspired by this approach, we propose the following *FDR* indicator to be used when comparing the algorithms:

$$\boldsymbol{FDR} = \frac{\boldsymbol{FP}}{\boldsymbol{TP} + \boldsymbol{FP}} \qquad (\mathbf{17})$$

where FP is the number of "False-Positive" equivalent solutions and TP is the number of "True-Positive" equivalent solutions. Since the reference solutions are based on discretization using a grid whereas the obtained solutions by the algorithm are not, then the definitions of FP and TP must be relaxed.

Here each of the obtained solution is transformed into the closest corner-point of the grid. Then the transformed solution is compared with the reference solutions, which are calculated on these corners. If a reference solution exists on that corner, the solution is considered TP. If there is no reference solution at the corner of the transformed solution, then the solution is declared FP.

7.2.3. Coverage

The 'Coverage' measure calculates the ratio between the number of TP solutions and the number of solutions in the reference set.

$$Coverage = \frac{TP}{|PS|} \qquad (18)$$

8. EXPERIMENTS & RESULTS

This chapter aims to demonstrate, test and compare the suggested algorithm. The chapter is organized as follows. First, in Section 8.1, three fl-MMOPs and their reference solutions, as obtained by the full-search approach, are described. Next, in Section 8.2, these fixed length problems are used to investigate and compare the proposed algorithm with seven different algorithms including: Tri-MOEA-TA&R [17], MMOEA/DC [23], CPDEA [25], MO-Ring-PSO-SCD [26], SS-MOPSO [28], MOEA/D-DE [34] and NSGA-II [8]. It should be noted that the last two algorithms are not MMEAs, but rather regular MOEAs.

In Section 8.3, three vl-MMOPs are presented and used to demonstrate and compare the algorithm. These problems are solved by the solution approach as presented in Chapter 3. Finally in Section 8.4, in view of the obtained results, a discussion is provided on the need for an unbounded archive and its significance.

8.1. The fl – MMOPs and Their Reference Solutions

This section presents three simple fl-MMOPs and their reference solutions as obtained by the use of the full-search approach (see Section 7.1). Each of these test problems involves one antenna only. The design of these problems with one antenna was motivated by the desire to demonstrate the obtained solutions in a clear way.

Each of the fl-MMOPs involves a square operation area of 2000X2000m^2, as shown in Figure 9. This figure presents all the mutual elements for the three test problems including the buildings, which are shown by black squares, and the sub-areas, which are shown as rectangles. The C_p (leasing cost per one antenna in each sub-area) is given as a number at the upper-left corner of each sub-area. Table 2 provides a summary of all the other parameters as used in the three problems.

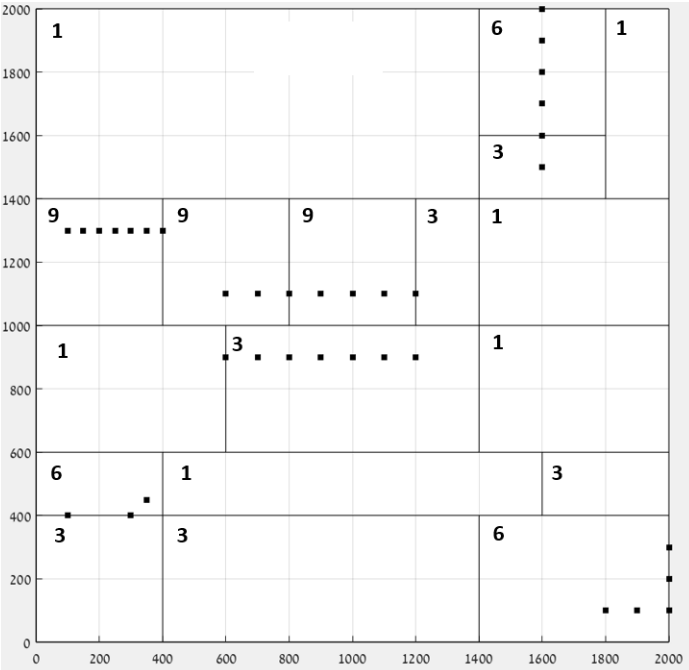


Figure 9: Mutual elements of the fl-MMOPs

Table 2: Test problems parameters

Parameter	Parameter Sign	Test Problem 1	Test Problem 2	Test Problem 3
Num. of Buildings	J	35	35	35
Num. of Edge Devices	K	26	38	63
Connectivity Coefficient	C_{ss}	9000	9000	9000
Connectivity Threshold	T	0.1	0.1	0.1
Other costs per one antenna	γ	1	1	1
Insurance of building j	β_j	1	1	1
Safety radius	$\mathbf{R_{safe}}$	600m	600m	600m

The non-mutual elements of the three fl-MMOPs are the edge devices. These devices, which differ by their amount and locations, are illustrated as blue asterisks in Figure 10, 11 & 12 for the 1st, 2nd and 3th problems, respectively. These figures also depict the resulting optimal antenna locations, i.e., the equivalent reference solutions, as obtained by the full-search procedure using the resolution vector $\boldsymbol{\varepsilon}=(5, 5)^T$. The optimal antenna locations are shown as colored regions, where each color corresponds to a unique performance vector, as detailed in the legend. For example, in Figure 10, all the equivalent Pareto optimal solution within the red region, are associated with the same performance vector in the obtained Pareto-front, i.e., *CON*=3 and *DTC*=7.

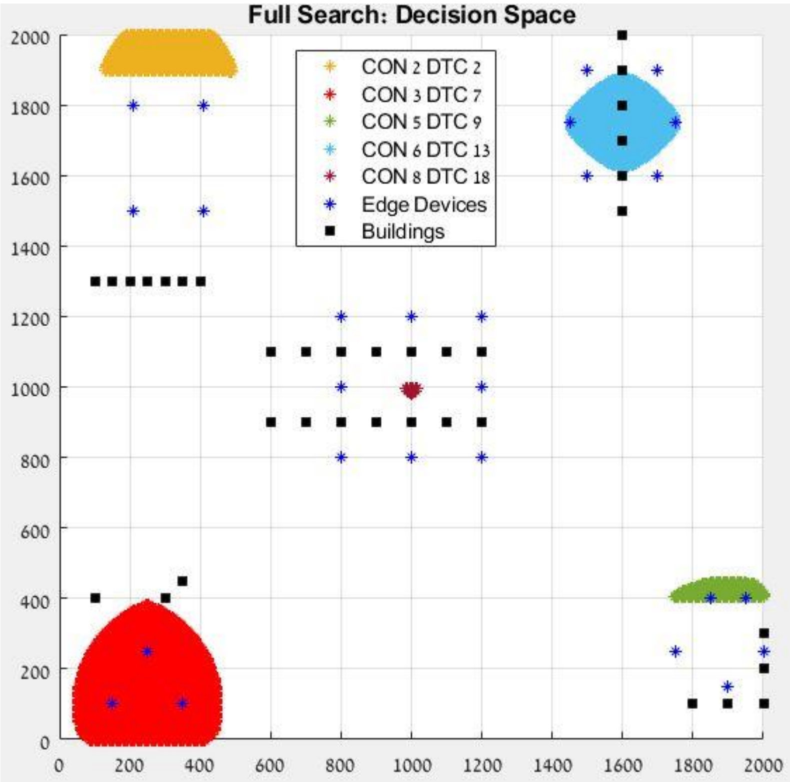


Figure 10: Decision Space - 1st fl-MMOP

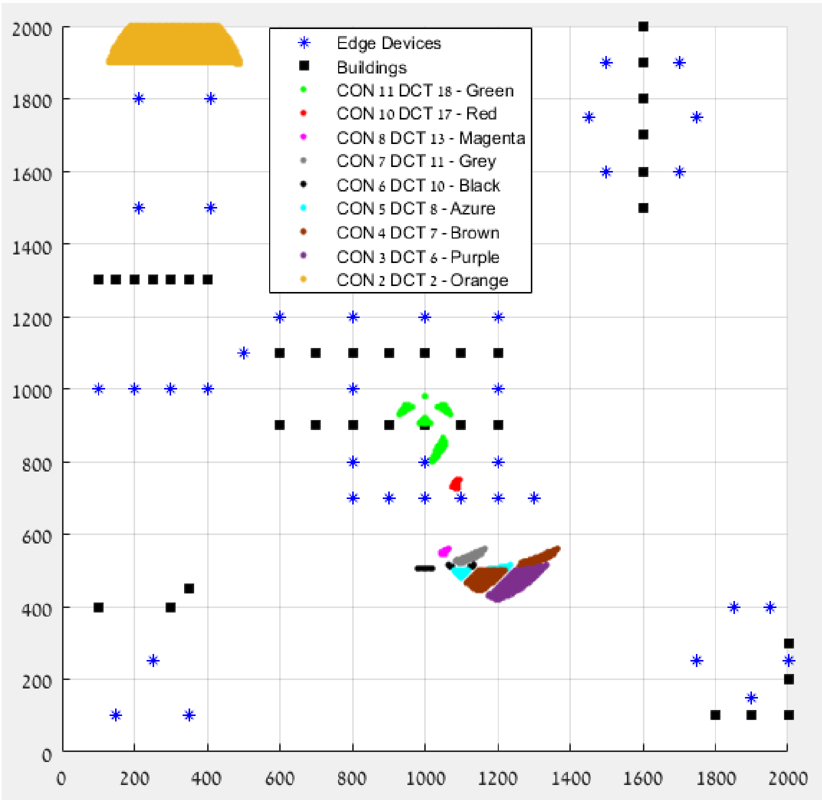


Figure 11: Decision Space – 2nd fl-MMOP

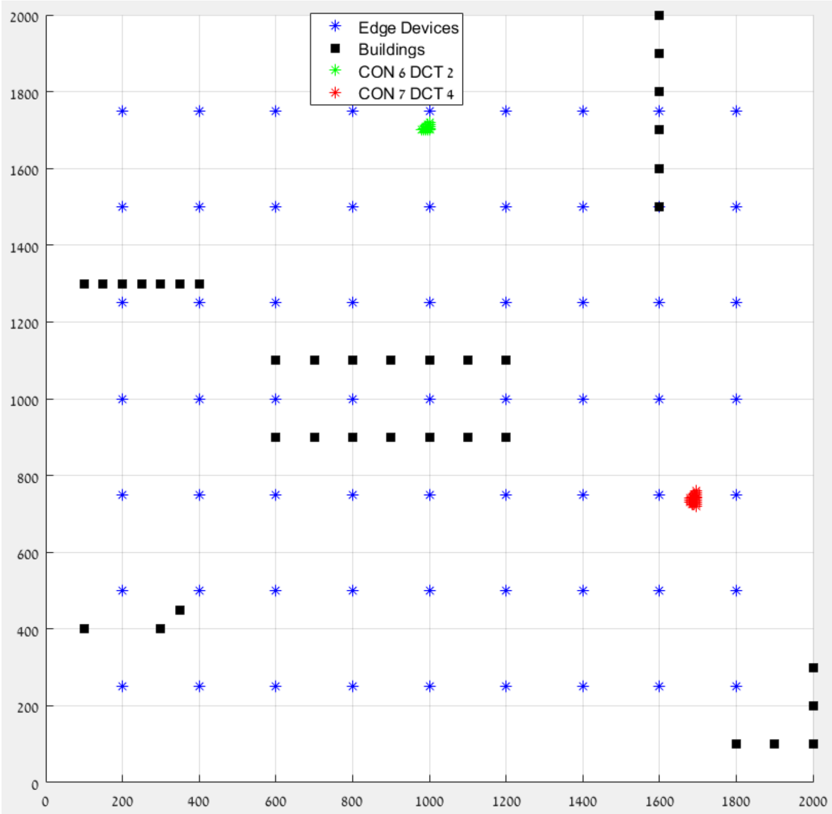


Figure 12: Decision Space – 3th fl-MMOP

Figures 13, 14 & 15 show the Pareto-fronts as obtained by the full-search, for the 1st, 2nd and 3th problem, respectively. In these figures, the performance vectors of the fronts are shown as blue squares. As can be easily observed from the figures, the 1st problem has five vectors in the front; the second has nine, whereas the third has only two. These vectors correspond to the five, nine and two colored regions, in Figure 10, 11 & 12, respectively.

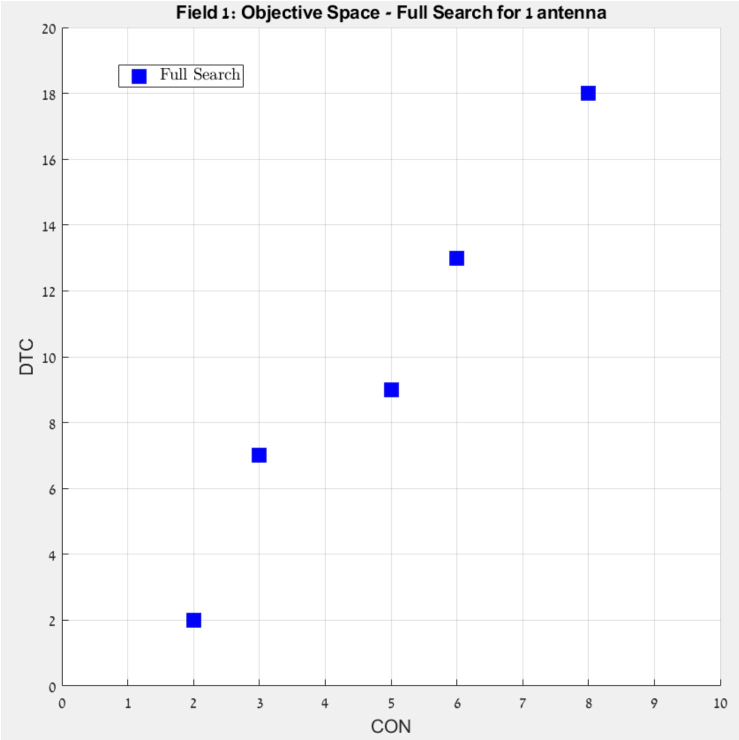


Figure 13: Pareto Front - 1st fl-MMOP

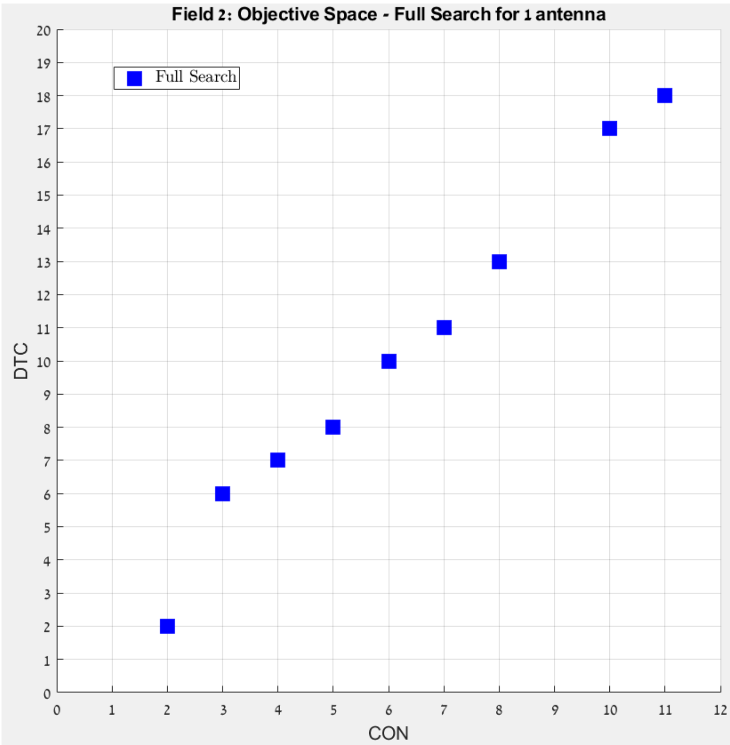


Figure 14: Pareto Front– 2nd fl-MMOP

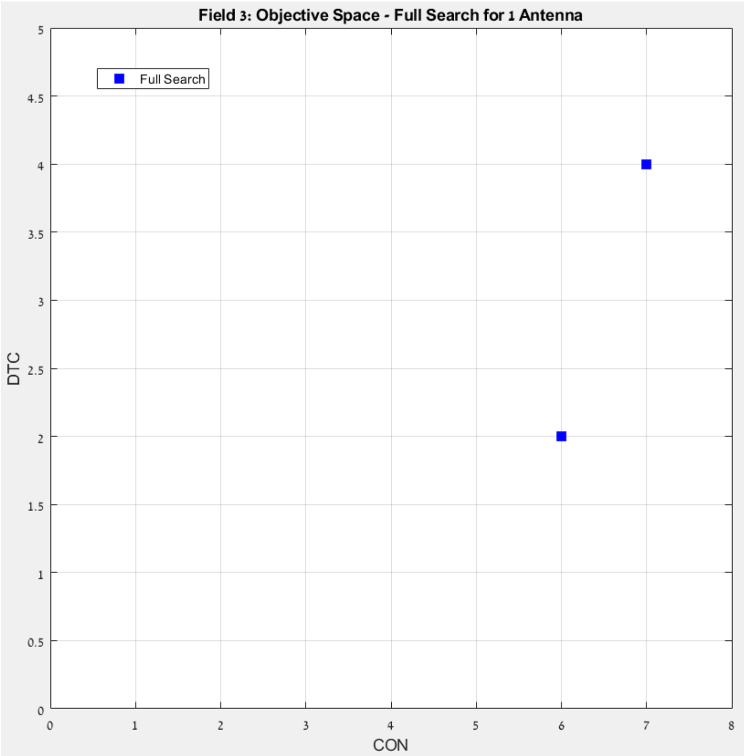


Figure 15: Pareto Front– 3th fl-MMOP

8.2. Comparing amm-NSGA-II using the fl-MMOPs

This section involves a comparison of the results of solving the fl-MMOPs, which are described in the previous section, as obtained by the proposed algorithm and by the following seven algorithms: Tri-MOEA-TA&R [17], MMOEA/DC [23], CPDEA [25], MO-Ring-PSO-SCD [26], SS-MOPSO [28], MOEA/D-DE [34] and NSGA-II [8]. As noted above, only the first five were specifically designed as MMEAs, rather than just MOEAs, with the aim to cope with multi-modality. The reference solutions are described in Section 8.1. To make statistical inference, each algorithm was applied 30 times on each of the problems, with a population size of 10 and for 200 generations.

In addition to using the decision-space indicators FDR and Coverage, which are described in Section 7.2, the IGD indicator [39] was used to compare the fronts in the objective-space. The statistical inference is based on the Wilcoxon test [40].

8.2.1. Summary of the Statistical Results

Tables 3-5, shows the results of the algorithm comparisons for the three fl-MMOPs, respectively. It can be seen that amm-NSGA-II is superior in all comparisons ('-' sign), except for two cases in which there is no statistical conclusion ('=' sign). This superiority can be explained by the applied small population size and the fact that, unlike the other algorithms, the archive of the proposed algorithm has no size limitation (see discussion in Section 8.5).

Table 3: Measures comparison – Test problem 1

		amm-NSGA-II	CPDEA	MMOEA/DC	MO-Ring-PSO-SCD	MOEA/D-DE	NSGA-II	SS-MOPSO	TriMOEA-TA&R
IGD	mean	**0.2711**	1.6352 -	1.3392 -	0.2995 =	1.4171 -	1.4828 -	3.6808 -	2.1408 -
	Var.	0.1903	0.4997	0.5809	0.1904	0.7340	0.6599	0.7338	0.7017
	Med.	0.2000	1.5185	1.0770	0.2000	1.2492	1.2770	3.5675	1.9744
FDR	mean	**0.1277**	0.42 -	0.2267 -	0.4833 -	0.5367 -	0.34 -	0.9133 -	0.6567 -
	Var.	0.029	0.061	0.0655	0.0587	0.0272	0.0494	0.0253	0.0572
	Med.	0.0991	0.4	0.1	0.5	0.5	0.4	0.8	0.6
Coverage	mean	**0.0702**	0.00074 -	8.94e-04 -	0.00092 -	5.3e-04 -	7.63e-04 -	0.00012 -	0.00081 -
	Var.	2.867e-04	2.59e-04	8.7624e-05	2.2938e-04	3.64e-06	6.6087e-08	3.3390e-06	3.4937e-04
	Med.	0.0748	0.00069	0.001	0.00083	5.78e-04	6.9412e-04	0.00039	0.00079

Table 4: Measures comparison – Test problem 2

		amm-NSGA-II	CPDEA	MMOEA/DC	MO-Ring-PSO-SCD	MOEA/D-DE	NSGA-II	SS-MOPSO	TriMOEA-TA&R
IGD	mean	**0.4886**	1.4493 -	0.5069 -	0.8527 -	0.4929 -	0.773 -	3.5129 -	2.3941 -
	Var.	0.2371	0.1231	0.5695	0.0193	0.2966	0.5873	0.8144	0.6082
	Med.	0.4024	1.4507	0.2452	0.8504	0.4484	0.5266	3.3910	2.2384
FDR	mean	**0.2943**	0.5367 -	0.302 =	0.3633 -	0.314 -	0.395 -	0.743 -	0.6564 -
	Var.	0.065	0.0327	0.0391	0.0155	0.0734	0.0694	0.0511	0.0198
	Med.	0.2604	0.5	0.3	0.4	0.3	0.3	0.7	0.7
Coverage	mean	**0.031**	0.0029 -	0.0038 -	0.0043 -	0.0035 -	0.0037 -	0.0018 -	0.0041 -
	Var.	3.256e-04	4.25e-04	9.853e-05	3.2483e-04	5.92e-06	1.7467e-06	1.3385e-06	2.3978e-05
	Med.	0.0264	0.0029	0.00401	0.00411	0.003512	0.004	0.0016	0.00372

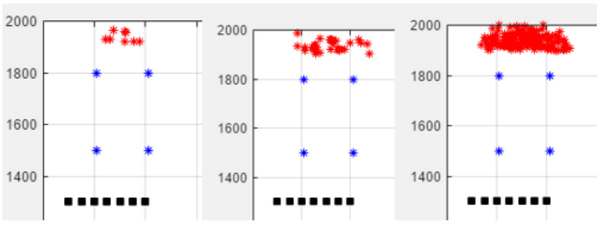
Table 5: Measures comparison – Test problem 3

		amm-NSGA-II	CPDEA	MMOEA/DC	MO-Ring-PSO-SCD	MOEA/D-DE	NSGA-II	SS-MOPSO	TriMOEA-TA&R
IGD	mean	**0.312**	0.9368 -	0.5592 -	0.6051 -	0.5713 -	0.7796 -	0.8914 -	0.7495 -
	Var.	0.132	0.2777	0.2785	0.1771	0.1193	0.1203	0.221	0.2721
	Med.	0	1	0.5	0.5	0.7071	0.7071	0.7071	0.7071
FDR	mean	**0.314**	0.9833 -	0.8733 -	0.6067 -	0.4700 -	0.63 -	0.8233 -	0.8767 -
	Var.	0.2026	0.0021	0.0055	0.1475	0.0594	0.0973	0.024	0.0074
	Med.	0	1	0.9	0.7	0.5	0.8	0.8	0.9
Coverage	mean	**0.2219**	0.0044 -	0.0333 -	0.1035 -	0.1395 -	0.0974 -	0.0082 -	0.0325 -
	Var.	0.012	0.0001	0.0004	0.0102	0.0041	0.0067	0.0044	0.0005
	Med.	0.2632	0	0.0263	0.0789	0.1316	0.0526	0.0005	0.0263
Sum Tables	+/-/=	**7/0/2**	**0/9/0**	**0/8/1**	**0/8/1**	**0/9/0**	**0/9/0**	**0/9/0**	**0/9/0**

8.2.2. Arcive Growth versus Generations

One of the main advantages of the proposed algorithm is the dynamic archive which is not bounded. Hence, once an equivalent non-dominated solution is found it is not lost. As a result, the archive grows and enables the decision-maker to choose from many possible solutions.

Figure 16 shows an example, based on the 1st problem, on how the equivalent non-dominated solutions evolve in a sub-region of the operation-area. In this sub-region the performance vector is CON = 2 and DTC = 2. The figure shows the solutions as accumulated in the archive in generations 10, 50 and 200. As can be seen in Gen. = 10 there are 9 solutions, in Gen. = 50 there are 26 and in Gen. = 200 the archive contains 191 solutions. When compared with the same sub-region in Figure 10, i.e., the reference solution, it can be observed that in Gen =200 there is almost a full coverage at the considered sub-region.



Gen. = 10 Gen. = 50 Gen. = 200

Figure 16: The evolution of non-dominated solutions in a sub-region

Figure 17, shows a typical growth of the archive size as a function of the generation number. It can be seen that the archive is growing during most of the process. If the algorithm finds a better solution, all the dominated solutions are eliminated from the archive, and as a result it might temporarily get smaller.

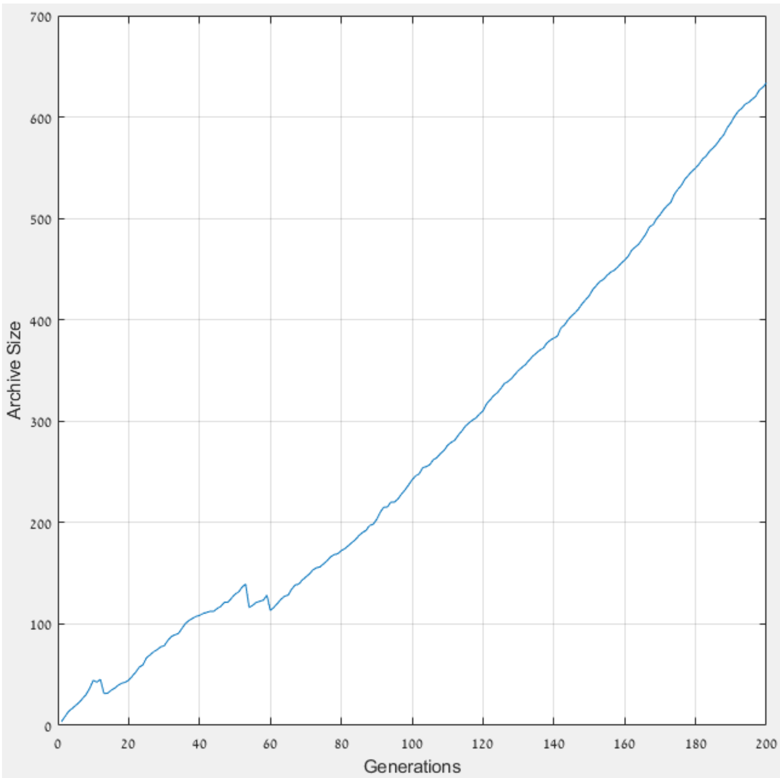


Figure 17: Archive size as a function of number generations

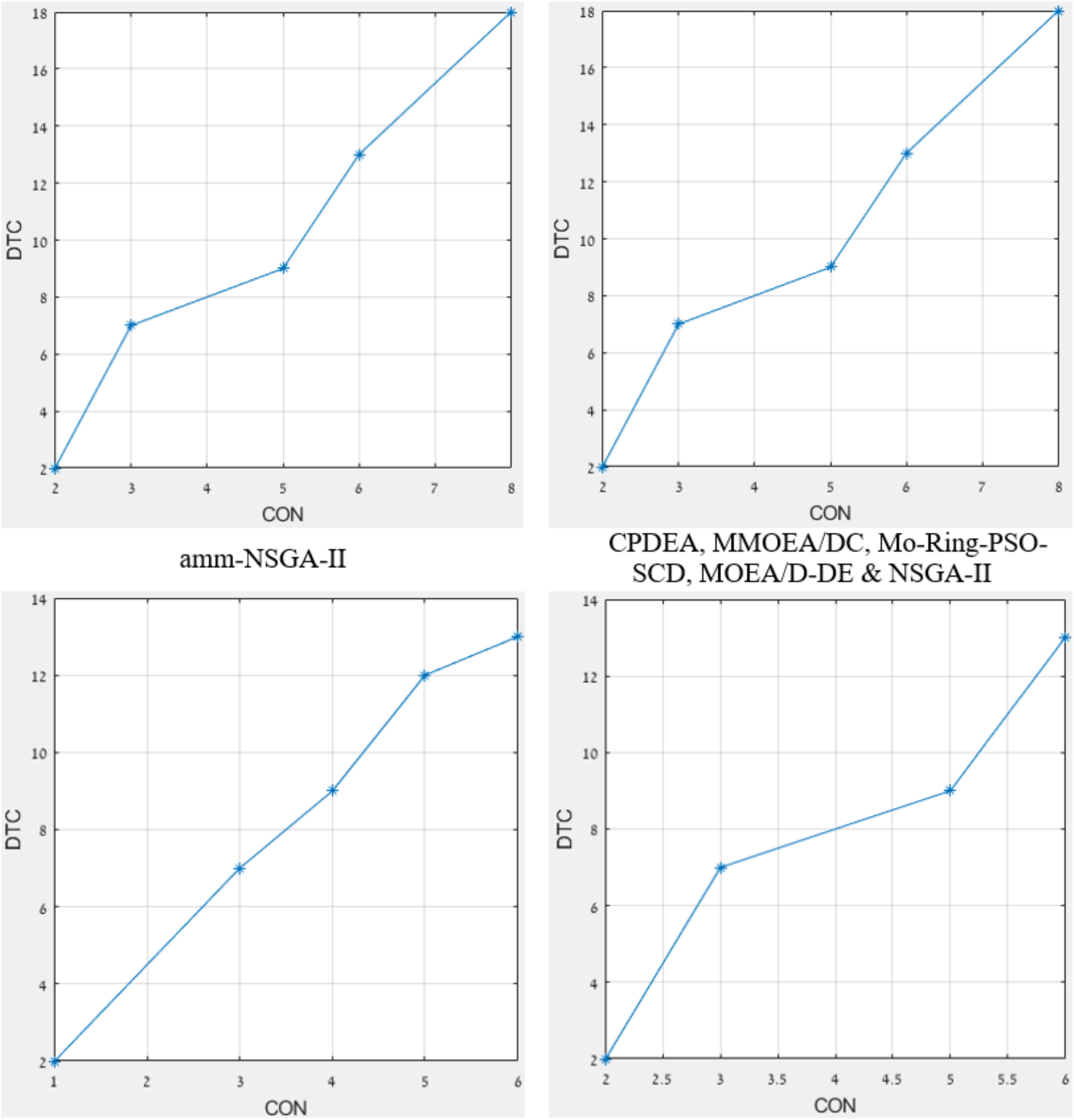
8.2.3. Front Comparisons

For each of the different algorithms, Figures 18, 19 and 20 show the obtained fronts for the 1st, 2nd and 3rd problems, respectively (vectors of the fronts are marked by asterisks). It is noted that each of the shown fronts is a result of combining and sorting all the fronts from the 30 runs.

Figure 18 includes 4 panels. The top-left panel shows the results of the proposed algorithm. The top-right panel corresponds to CPDEA, MMOEA/DC, Mo-Ring-PSO-SCD, MOEA/D-DE & NSGA-II. In fact, these algorithms, as well as amm-NSGA-II, were able to find the same front for the 1st problem, which is equal to the reference front in Figure 13. The bottom-left and the bottom-right panels correspond to SS-MOPSO and Tri-MOEA-TA&R, respectively. When compared to Figure 13, it can be concluded that these two algorithms did not find the reference front for the 1st problem.

Figure 19 provides the obtained fronts for the 2nd problem. It can be seen from the top-left and top-right panels that the proposed algorithm as well as MOEA/D-DE, NSGA-II, CPDEA, MMOEA/DC & Mo-Ring-PSO-SCD reached the same front, which is also the reference front of the 2nd problem (as can be seen from Figure 14). In contrast, the bottom-left and bottom-right panels show the fronts as obtained by Tri-MOEA-TA&R and SS-MOPSO, respectively. Clearly, these two algorithms did not reach the reference front of the 2nd problem.

Figure 20 shows the resulting fronts for the 3rd problem. The left panel shows the results of CPDEA, MMOEA/DC, Mo-Ring-PSO-SCD, MOEA/D-DE, NSGA-II, SS-MOPSO & Tri-MOEA-TA&R, whereas the right panel shows the resulting front by amm-NSGA-II. In fact, for the 3rd problem, all the algorithms reached the front, which is the same as the reference front (see Figure 15).



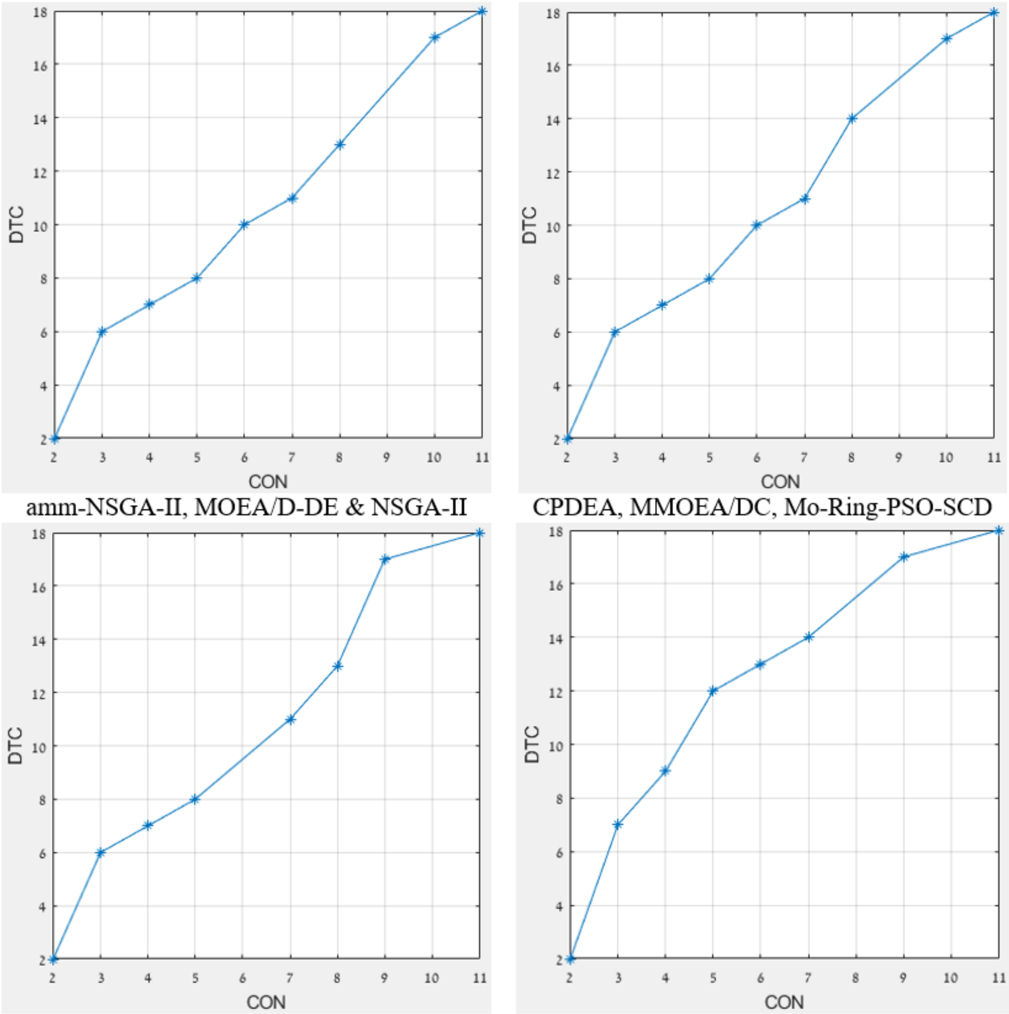
amm-NSGA-II

CPDEA, MMOEA/DC, Mo-Ring-PSO-SCD, MOEA/D-DE & NSGA-II

SS-MOPSO

Tri-MOEA-TA&R

Figure 18: Front comparison for the first problem



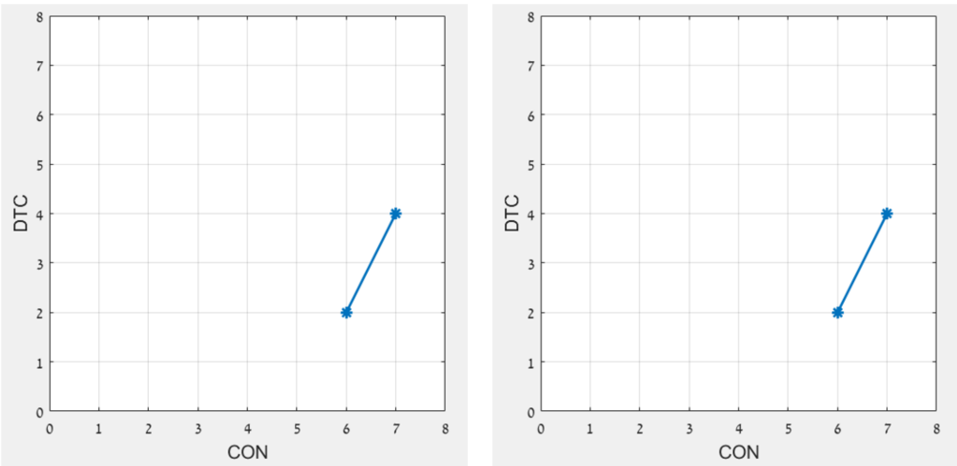
amm-NSGA-II, MOEA/D-DE & NSGA-II

CPDEA, MMOEA/DC, Mo-Ring-PSO-SCD

Tri-MOEA-TA&R

SS-MOPSO

Figure 19: Front comparison for the second problem



CPDEA, MMOEA/DC, Mo-Ring-PSO-SCD, MOEA/D-DE, NSGA-II, SS-MOPSO &Tri-MOEA-TA&R

amm-NSGA-II

Figure 20: Front comparison for the third problem

8.2.4. *Decision-space Comparisons*

Figures 21-26 show the decision-space results for the three problems, marked by green asterisks, as obtained by the various algorithms. It is noted that each of the shown solutions is a result of combining and sorting all the fronts from the 30 runs.

The results for the 1st problem are shown in Figure 21 (part A) & Figure 22 (part B). Figure 21 includes 4 panels. The top-left panel shows the results of the proposed algorithm, the top-right corresponds to CPDEA, the bottom-left corresponds to MMOEA/DC, and the bottom-right corresponds to Mo-Ring-PSO-SCD. Although, these algorithms, as well as amm-NSGA-II, were able to find the same front for the 1st problem, which is equal to the reference front in Figure 13, their decision-space distribution is limited unlike our proposed algorithm. This advantage of the proposed algorithm is probably attributed to its dynamic archive.

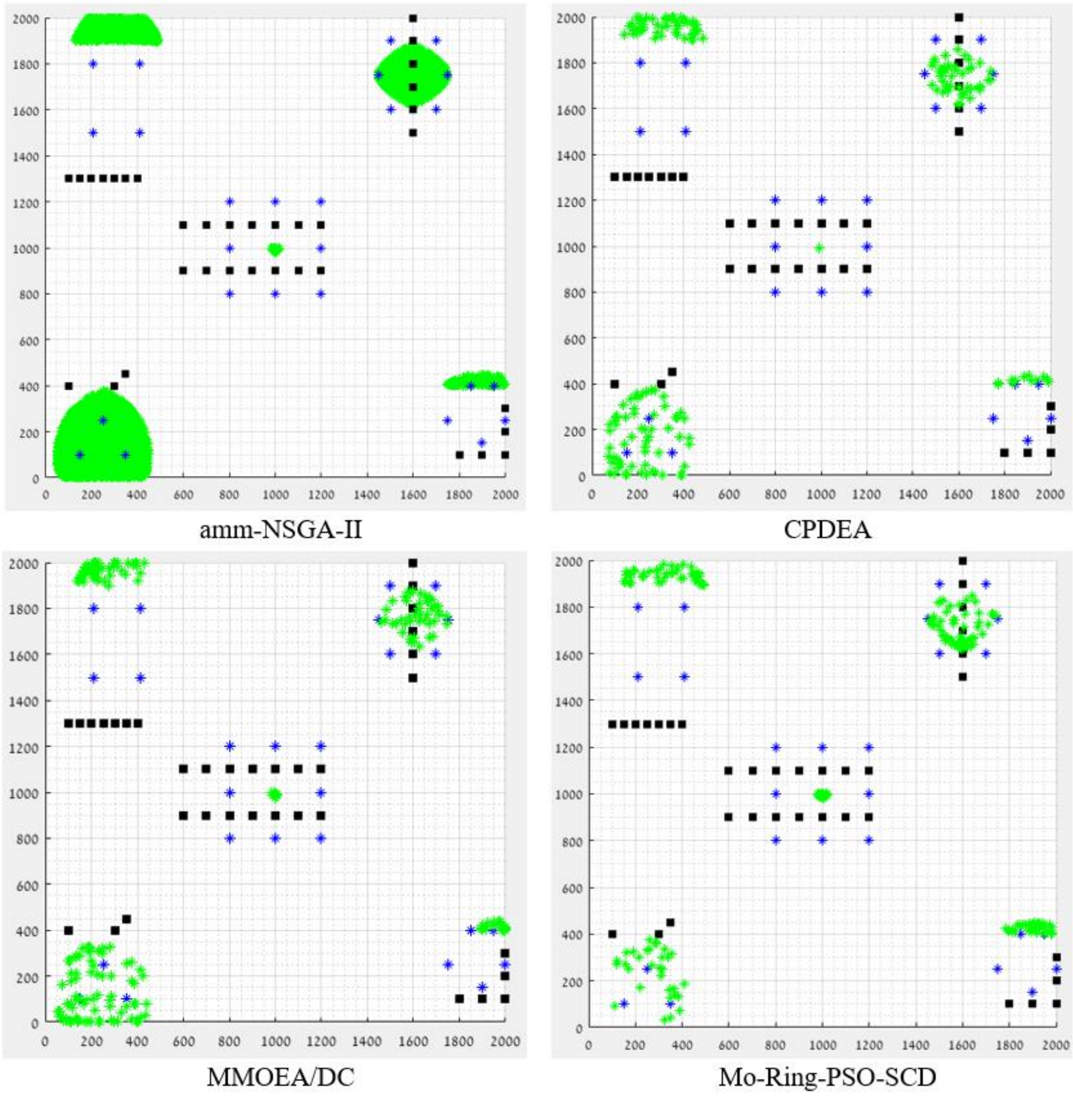


Figure 21: Decision space comparison for 1 antenna – 1st problem (part A)

The 4 panels of Figure 22 provide the decision space solutions of the 1st problem as obtained by the rest of the algorithms. The top-left corresponds to MOEA/D-DE, the top-right is NSGA-II, the bottom-left corresponds to SS-MOPSO and the bottom-right for the Tri-MOEA-TA&R. It can be seen that the distribution of SS-MOPSO solutions (bottom-left) is relatively very scattered. This fits the fact that the front, which was obtained by this algorithm, is less good than those obtained by the other algorithms (Figure 18, bottom-left panel).

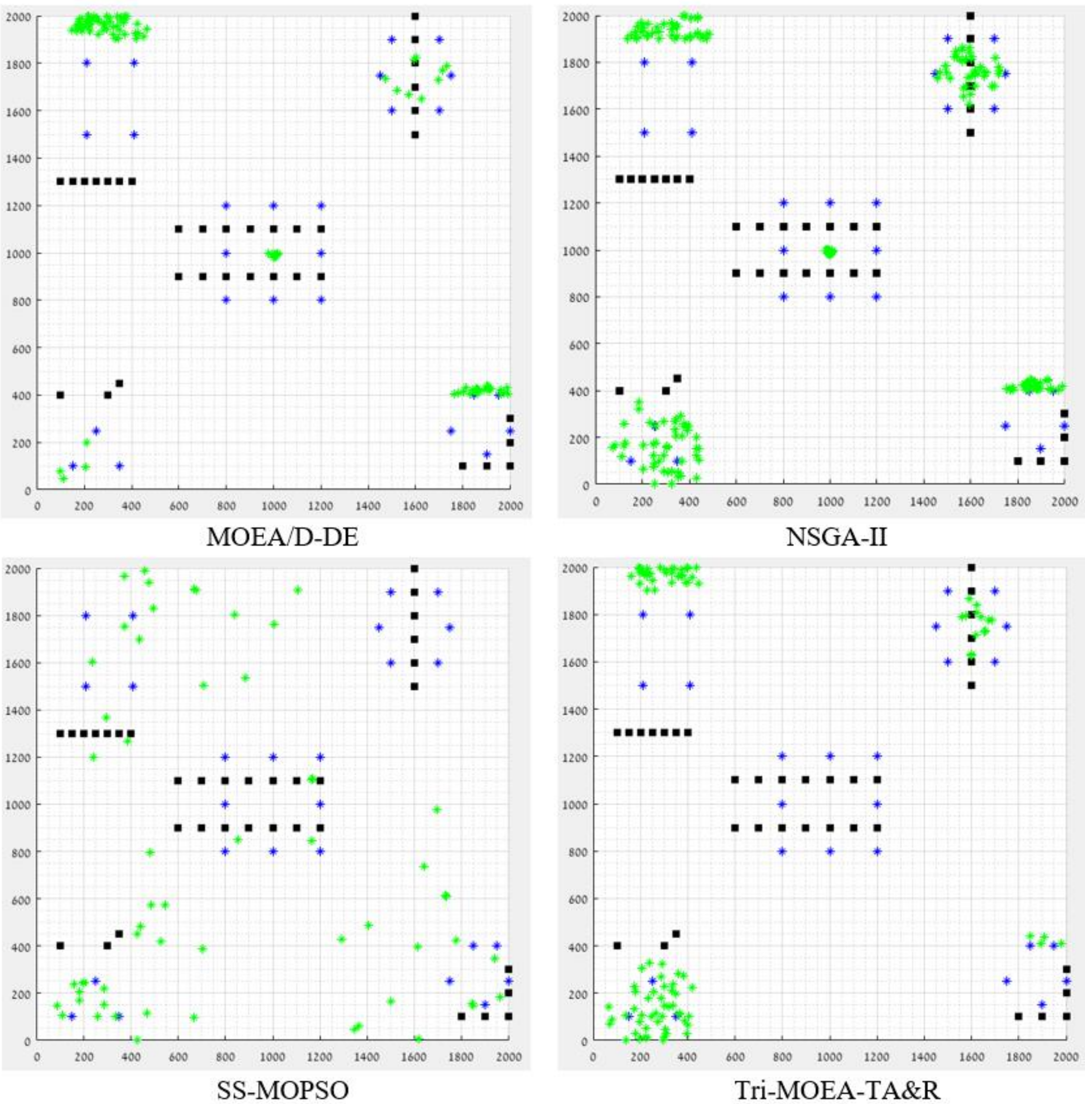


Figure 22: Decision space comparison for 1 antenna – 1st problem (part B)

Figure 23 includes 4 panels with some results for the 2nd problem. The top-left panel shows the results of the proposed algorithm, the top-right corresponds to CPDEA, the bottom-left corresponds to MMOEA/DC, and the bottom-right panel corresponds to Mo-Ring-PSO-SCD.

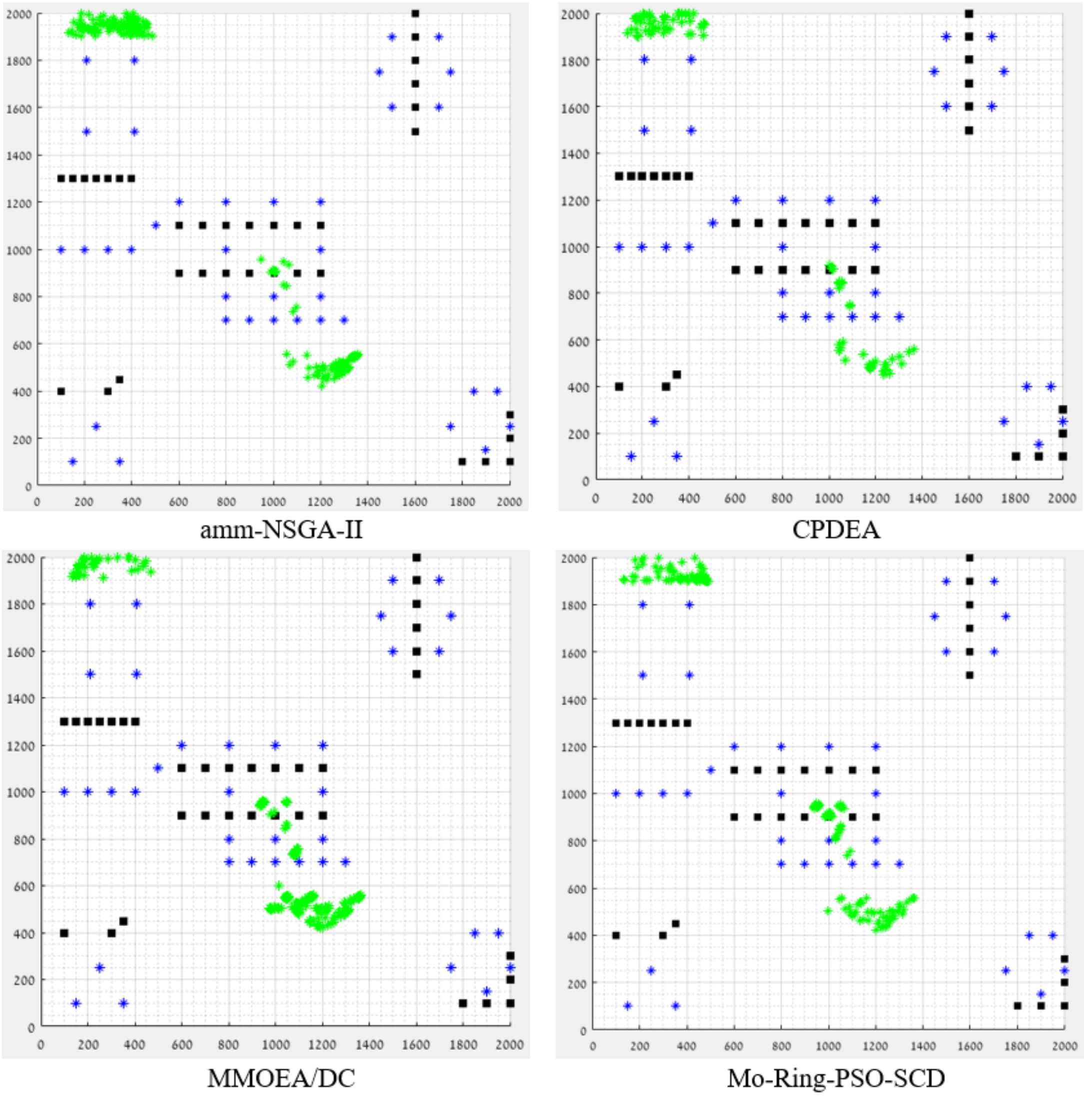


Figure 23: Decision space comparison for 1 antenna – 2nd problem (part A)

Figure 24 shows four additional panels for the 2nd problem. These include the decision space solutions, as obtained by the rest of the algorithms. The top-left panel corresponds to MOEA/D-DE, the top-right corresponds to NSGA-II, the bottom-left corresponds to SS-MOPSO, and the bottom-right panel corresponds to Tri-MOEA-TA&R. Once again, the distribution of SS-MOPSO solutions (bottom-left) is very scattered, causing poor Pareto-front as seen in Figure 19.

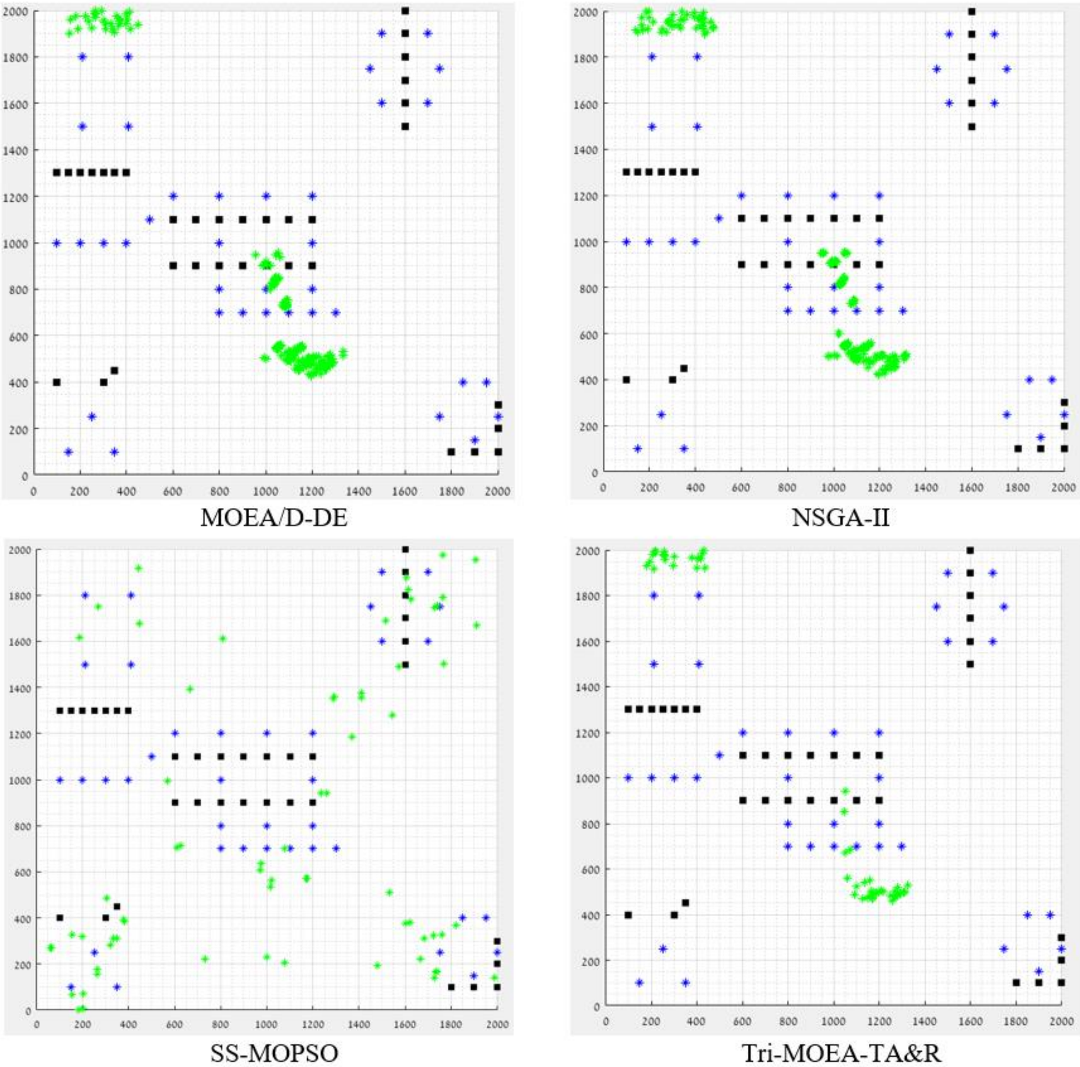
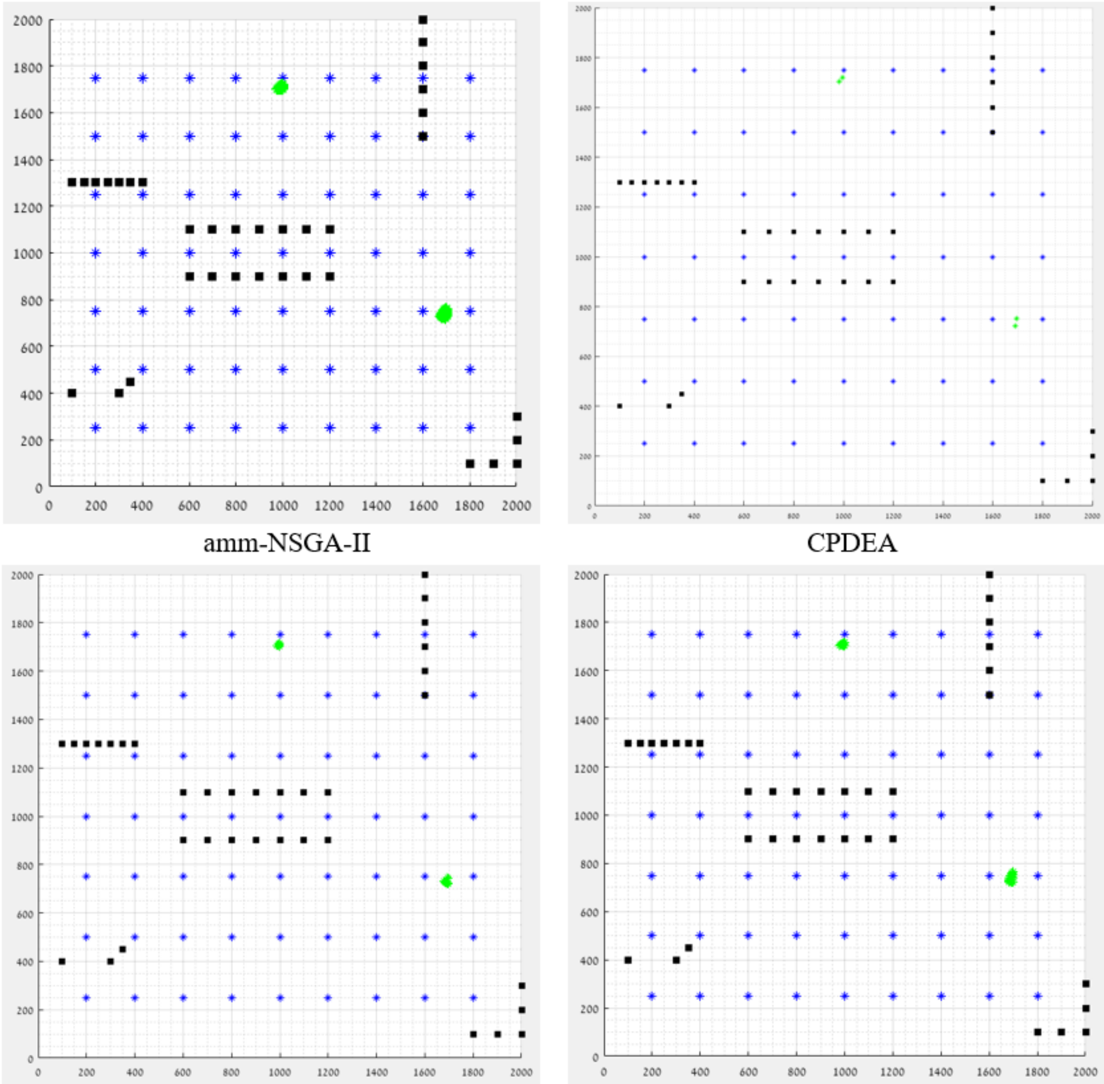


Figure 24: Decision space comparison for 1 antenna – 2nd problem (part B)

Figures 25 and 26, include the corresponding panels for the 3th problem. Once again, it can be seen that the proposed algorithm (top-left in Figure 25) has more solutions than those obtained by the other algorithms (the green areas).



amm-NSGA-II CPDEA

MMOEA/DC MO-Ring-PSO-SCD

Figure 25: Decision space comparison for 1 antenna – 3rd problem (part A)

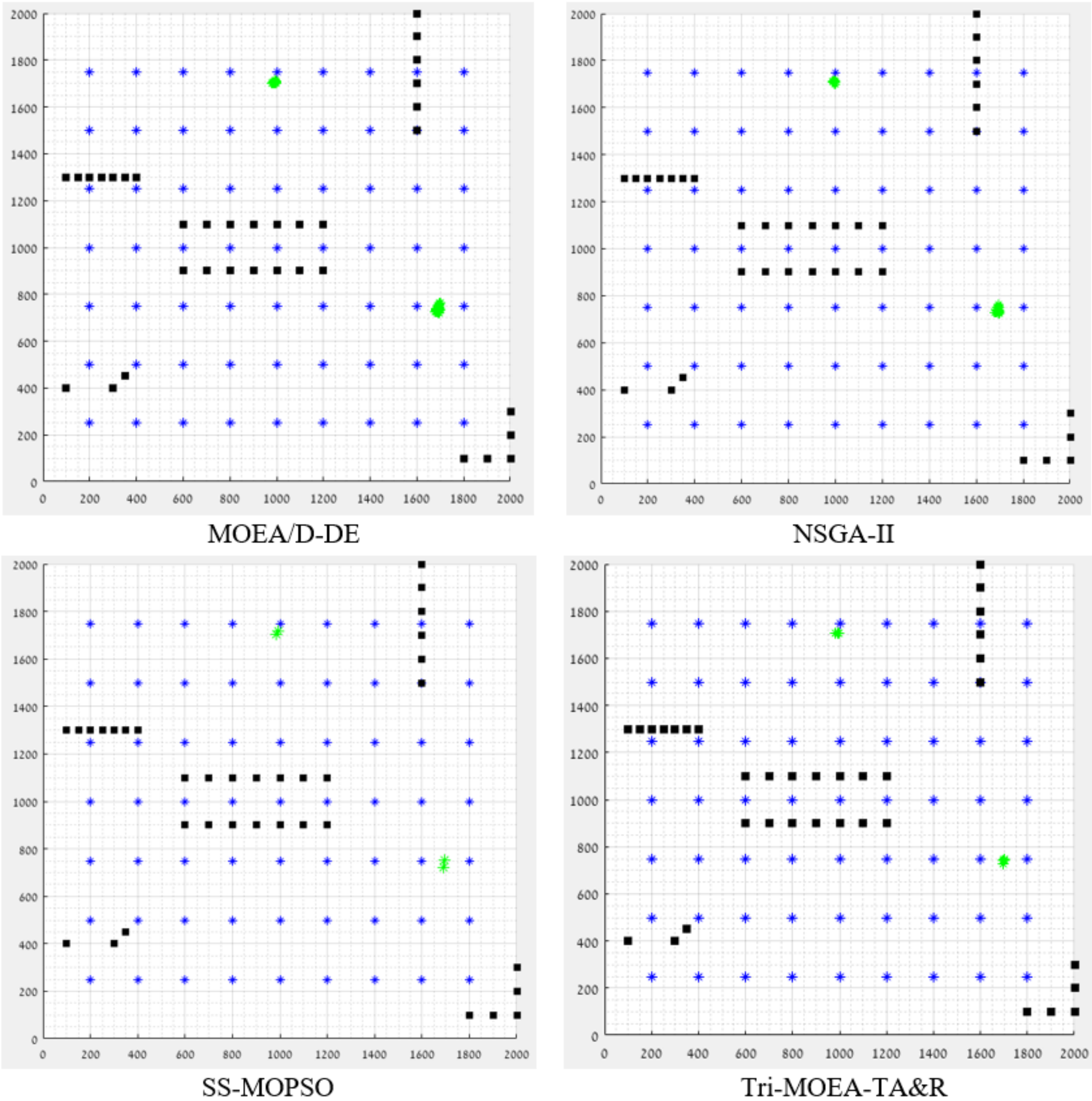


Figure 26: Decision space comparison for 1 antenna – 3^{rd} problem (part B)

8.2.5. *Equivalent Solutions – Single versus Thirty Runs*

This section shows a number of examples of the differences between the results of a typical single run and those accumulated from 30 runs. In the following figures, the green color represents the obtained results from the tested algorithm and the red color represents the full-search results, i.e., the greener the better.

Figure 27 and Figure 28 show such results as obtained by the proposed algorithm. Figure 29 shows such results, as obtained by Mo-Ring-PSO-SCD.

Clearly, the proposed algorithm shows much better results. A comparison of some accumulated results from the 30 runs of the proposed algorithm and Mo-Ring-PSO-SCD is shown in Figure 30, and in Figure 31 for a typical one run. These also confirm the superiority of amm-NSGA-II. Similar results were found when compared with the other algorithms.

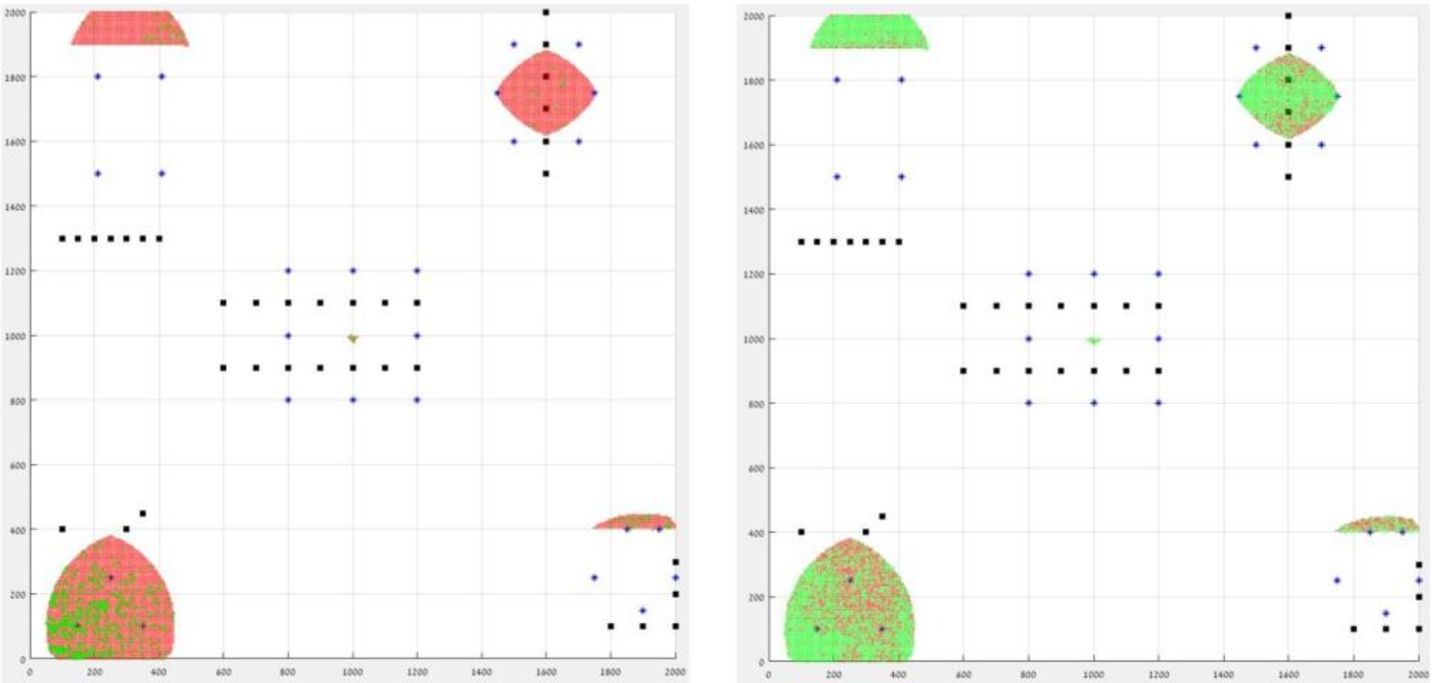


Figure 27: amm-NSGA-II: 30 runs (right) vs. single run (left) for problem No. 1

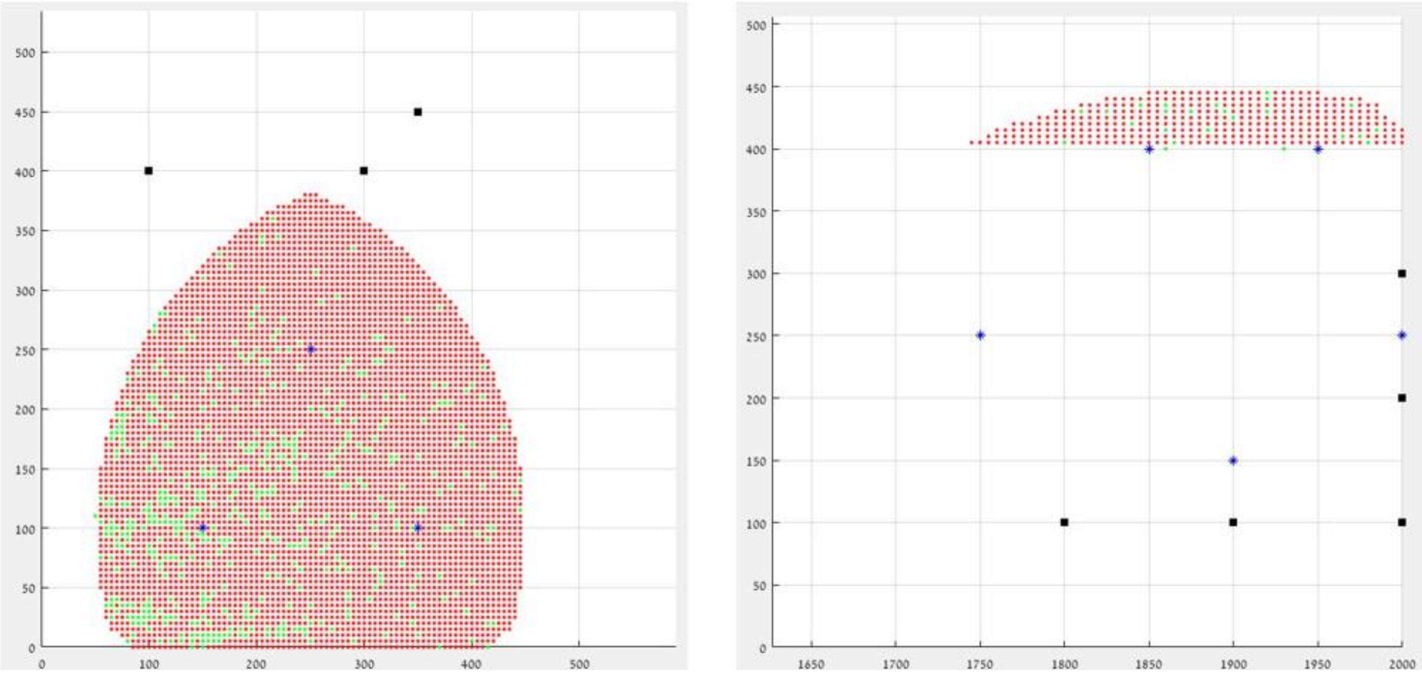


Figure 28: Close-up on some single run results of amm-NSGA-II for the 1st problem

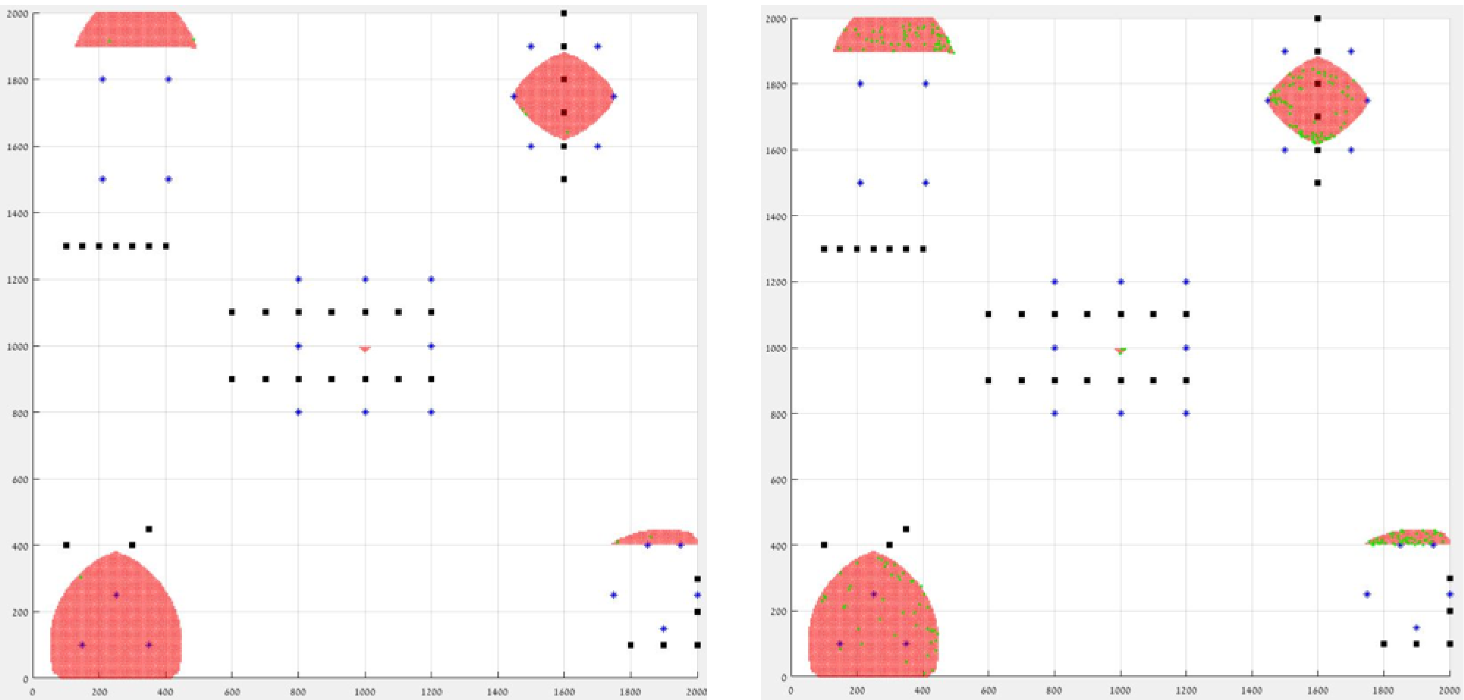


Figure 29: Mo-Ring-PSO-SCD single run (left) vs. 30 runs (right) for the 1st problem

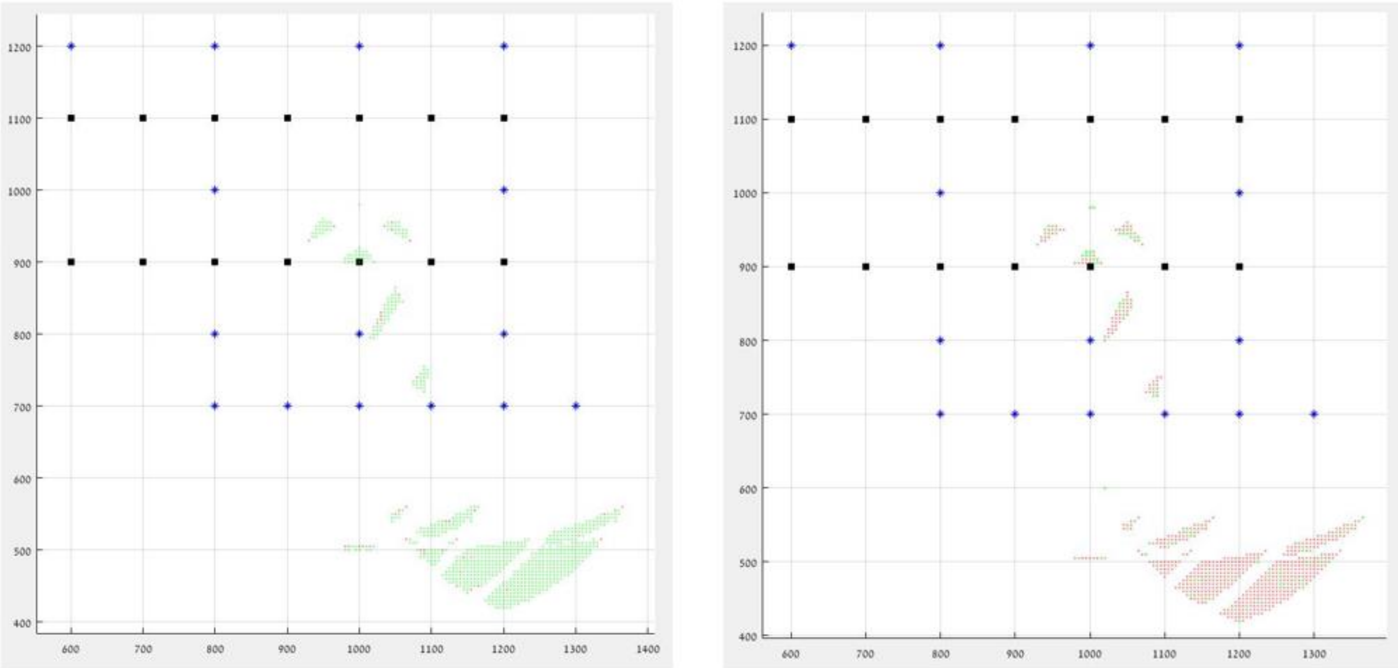


Figure 31: amm-NSGA-II (left) & Mo-Ring-PSO-SCD (right), 30 runs, 2nd problem

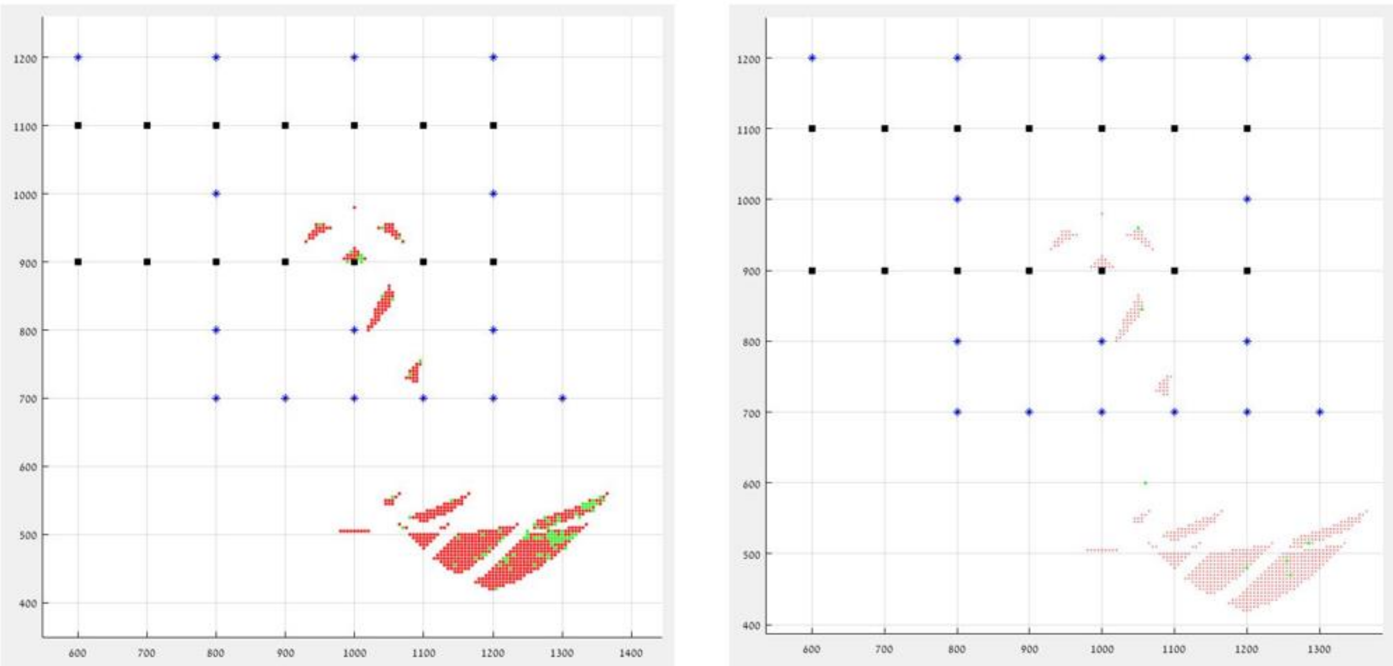


Figure 30: amm-NSGA-II (left) & Mo-Ring-PSO-SCD (right), single run, 2nd problem

8.3. Comparing amm-NSGA-II using vl-MMOPs

Here, the algorithms are compared on the three vl-MMOPs, which are versions of the three fl-MMOPs of the pervious sections. In the variable-length versions of the problems, all the parameters are equal to those in the fl-MMOPs (see Table 2 in Section 8.1). The difference from the fl-MMOPs is the requirement to also minimize the number of antennas within a given constraint. In the first vl-MMOP, the number of antennas is constrained to 8, in the 2nd problem it is constrained to 14 whereas in the 3rd problem it is constrained to 15. These numbers correspond to achieving full connectivity.

To solve the vl-MMOPs, the procedure that is suggested in Chapter 3, is used. Namely, each vl-MMOP is converted to a set of auxiliary fl-MMOPs that differs by the number of antennas. Each of the algorithms was run 30 times for each of the auxiliary problems.

Figures 32-34 show the resulting fronts for the 1st, 2nd and 3rd problem, respectively. The shown fronts result from sorting the union of the runs for each of the algorithms. In these figures, each of the performance vectors is associated with a number that indicates the obtained number of antennas. With increasing numbers of antennas, the results of amm-NSGA-II are superior as compared with those of the other algorithms.

However, to reach a statistical conclusion, a statistical study should be performed (as done in the previous section). Here, however, such a study is avoided for the reason which is described in the following discussion.

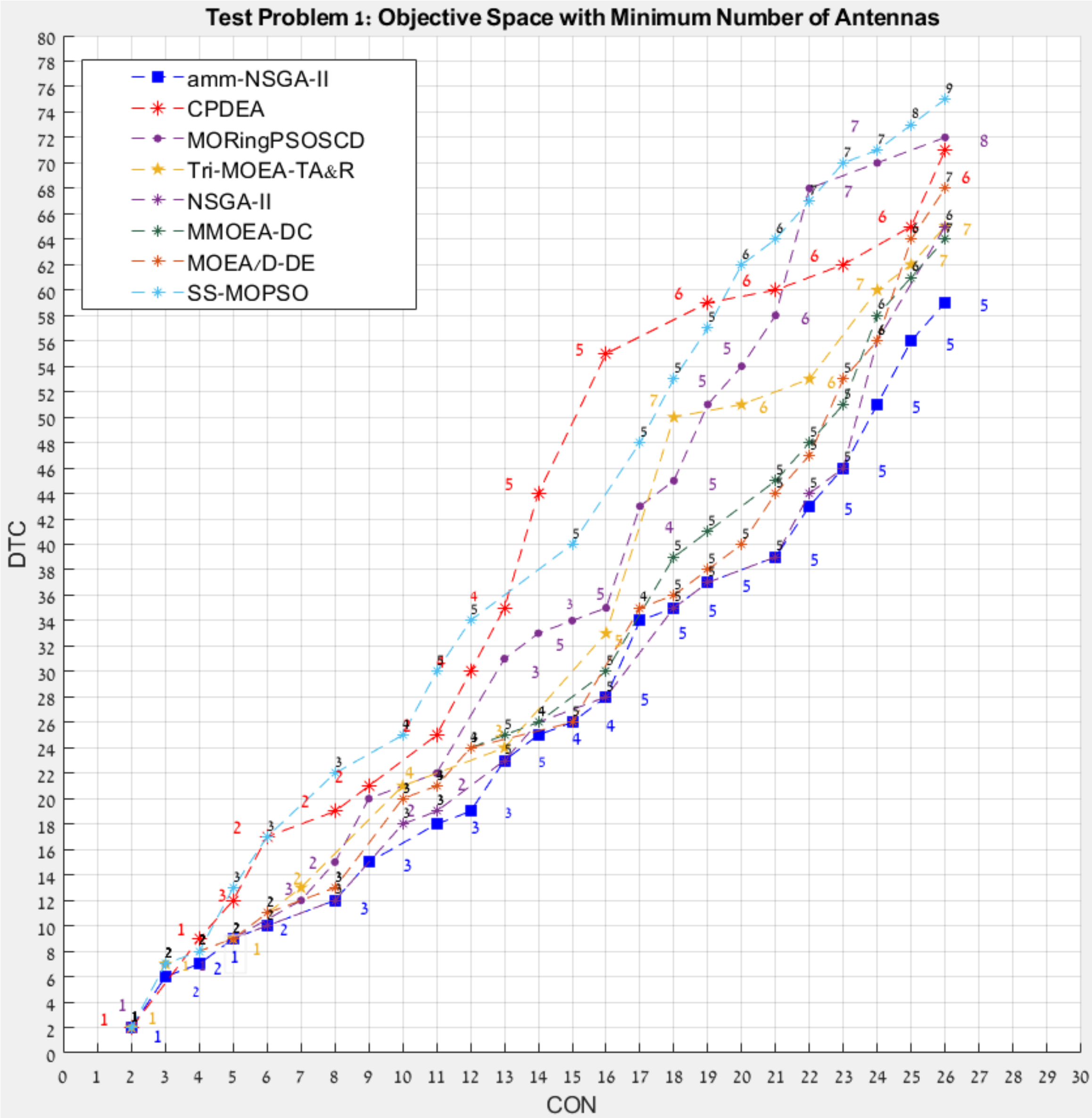


Figure 32: Front comparison for the 1st vl-MMOP

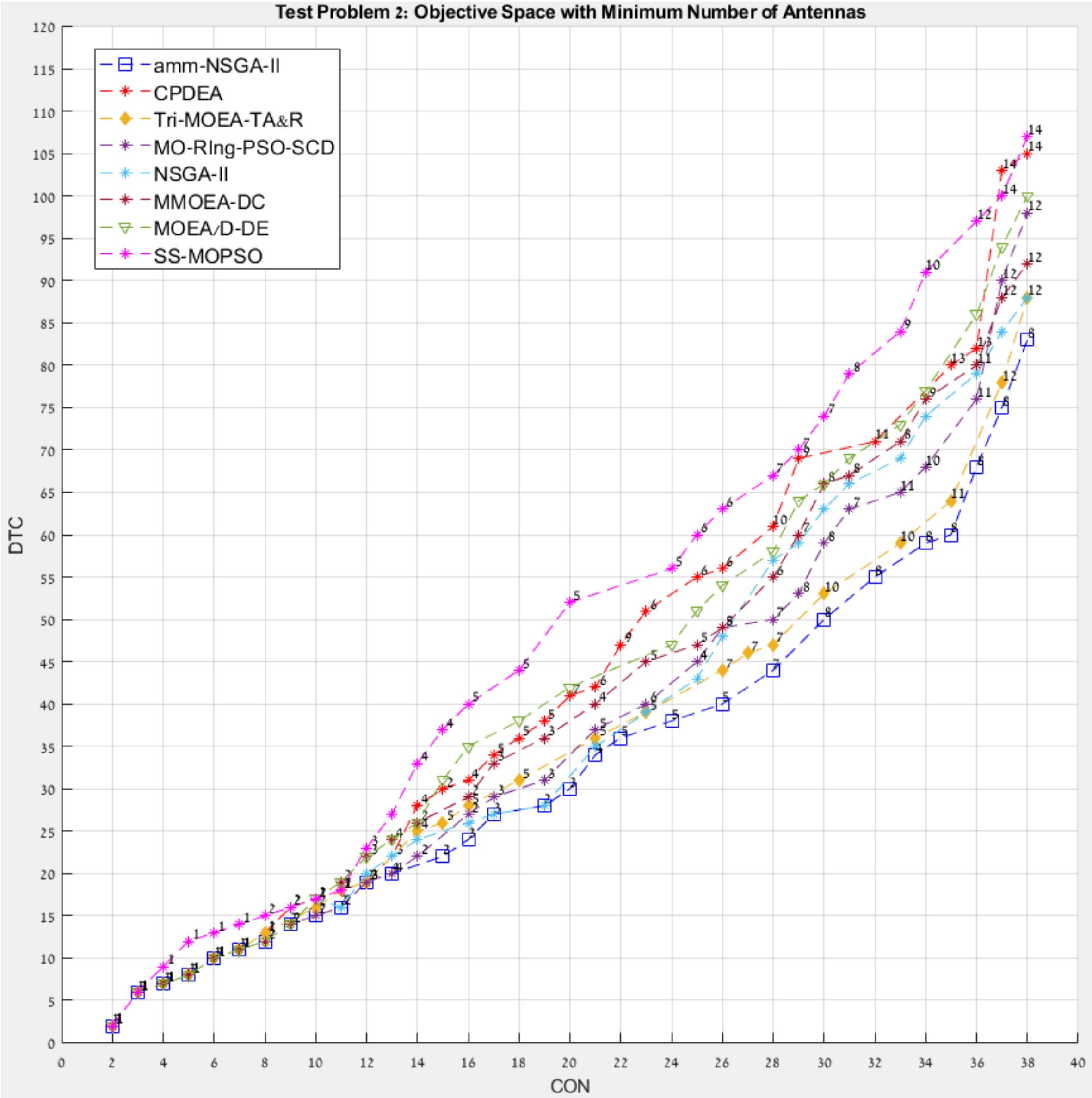


Figure 33: Front comparison for the 2nd vl-MMOP

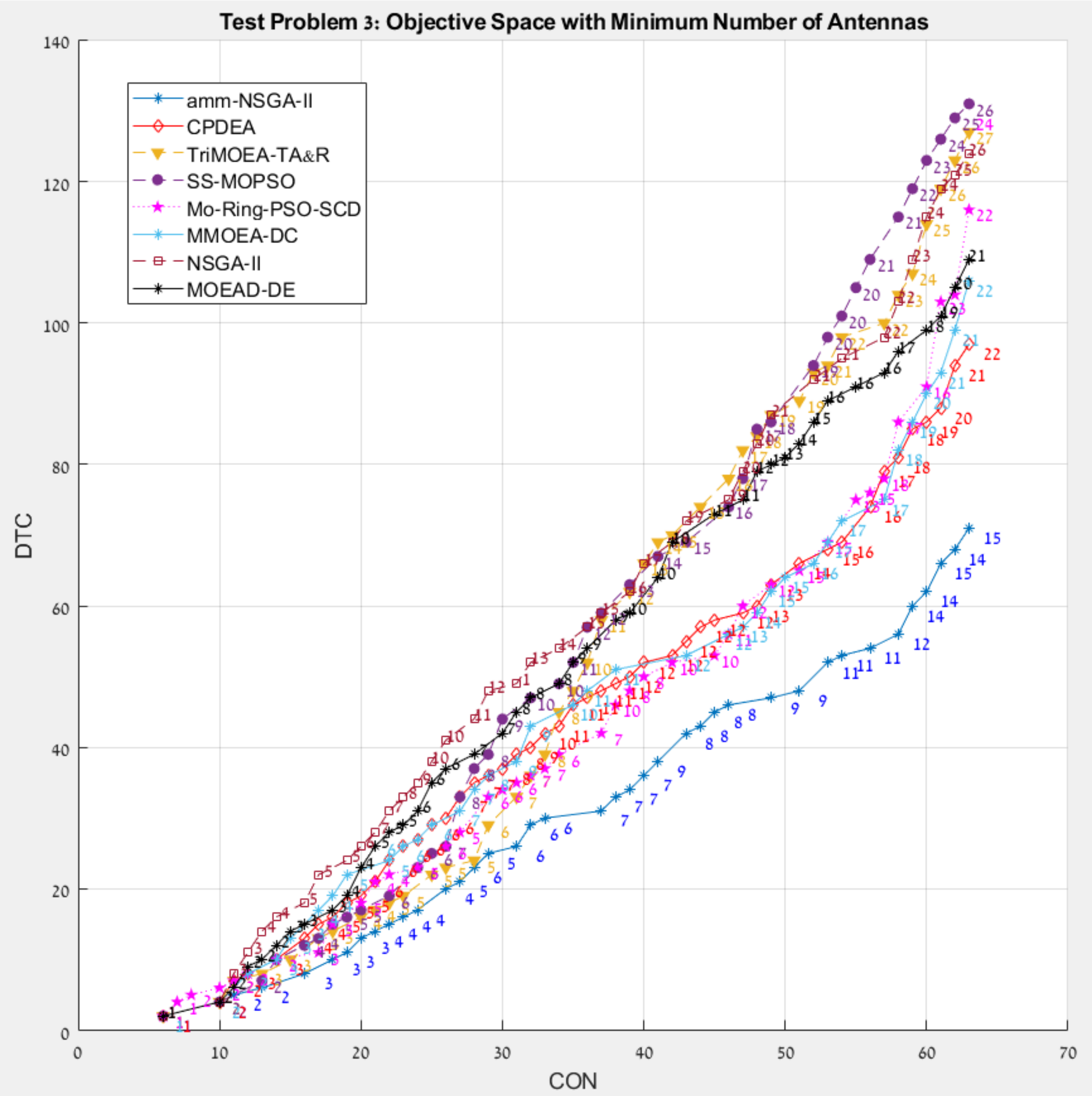


Figure 34: Front comparison for the 3rd vl-MMOP

8.4. Discussion on the Applied Archive

It is suspected that the superiority of amm-NSGA-II over the rest of the tested algorithms can be explained by the small population size and the fact that, unlike the other algorithms, the proposed algorithm does not have a limitation on the archive size. Given this understanding, it can be suggested that statistical inference about superiority should be done only under more fair conditions.

Namely, amm-NSGA-II should be compared with other MMEAs that include a similar unbounded archive. At present, however, most MMEAs do not have an unbounded archive [3]. Moreover, it appears that there is no study which is dedicated to the comparison of MMEAs with unbounded archives.

This situation is reflected in the work presented in [41]. The MMEA, which has been suggested in [41], is considered in [3] to be one of the few MMEAs to have an unbounded archive. Nevertheless, while having an unbounded population, the actual procedure that is used in [41] is that of a fixed-size archive. Namely, as noted in [41], for a fair comparison of the proposed algorithm, a selection procedure was applied to narrow down the population solutions into a fixed number of solutions that are actually archived.

It is a well-known assertion that decision-makers should not be overloaded with alternative solutions. In fact, some studies in psychology suggest that the number of alternatives that human may handle is seven plus minus two [42]. Based on such understanding, it can be argued that when solving a MMOP, special care should be taken in order to not expose the decision-makers to an excessive number of solution alternatives. This seems to be in contradiction with the use of an unbounded archive. Yet, it should be noted that while figures such as Figures 11-13, show a very large number of solution alternatives, such a large number of solutions can be grasped by humans. The reason is that the solution representation involves a limited number of clusters (areas). In other words, when observing the figures, humans can cognitively distinguish between each of the clusters of solutions and could eventually investigate the involved areas to select a preferred position for the antenna.

9. SUMMARY & CONCLUSIONS

This research suggests a new approach to deal with Multi-Modal Multi-objective Problems (MMOPs) with solutions of variable-length, i.e., vl-MMOPs. It is argued and demonstrated that such problems can be handled by existing Multi-Modal Evolutionary Algorithms (MMEAs), using a special procedure. In addition, a modification to NSGA-II is suggested to develop a new MMEA with unbounded archive.

The proposed algorithm, which is termed amm-NSGA-II, is demonstrated and compared to existing algorithms using optimal antennas layout-allocation design problems for a wireless communication network. It is concluded that for the studied problems, the proposed algorithm appears superior to the compared MMEAs.

Using the results, a discussion is provided with respect to the need for an unbounded archive for MMOPs. It is suggested that for a truly fair comparison, there is a need for MMEAs with unbounded archives. Yet, at present most existing MMEAs might not be suited for such a fair comparison with the proposed algorithm. This is left for future investigation.

Future research may also include the development of algorithms for a simultaneous coupled search within the various decision-spaces of vl-MMOPs, as well as the development of benchmark vl-MMOPs and possibly also dedicated performance indicators.

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תקציר

אופטימיזציה מולטי-מודלית היא אופטימיזציה שמטרתה לתת למקבלי ההחלטה פתרונות אלטרנטיבים, ולא רק פתרון אופטימלי בודד. בשנים האחרונות גברה ההתעניינות בהרחבת גישה זו לאופטימיזציה מרובת-מטרות.

מחקר זה מתמקד בפתרון של בעיות מיוחדות של אופטימיזציה רב-מודלית ומרובת-מטרות שבהן הפתרונות הינן ממרחבים בעלי מימדים-שונים (כלומר, פתרונות מאורכים שונים). תחילה התזה מציגה את התרומה, המוטיבציה והחדשנות של המחקר. לאחר מכן, מבוצעת סקירה של האלגוריתמים והשיטות המתקדמות ביותר לפתרון בעיות אופטימיזציה רב-מודלית ומרובת מטרות (MMOPs) . על פי הסקירה, תזה זו מהווה עבודת מחקר ראשונה לבעיות MMOPs עם פתרונות מאורכים שונים.

בתזה זו מוצעת הגדרת הבעיה ומסגרת חישובית לפתרונה שמאפשרת שימוש באלגוריתמים אבולוציונים סטנדרטיים לפתרון MMOPs. לאחר מכן, מוצע אלגוריתם חדש שנקרא Archived Multi-Modal NSGA-II (amm-NSGA-II). אלגוריתם זה מהווה מודיפיקציה לאלגוריתם המפורסם NSGA-II כדי לאפשר פתרון ל- MMOPs עם אורך פתרונות משתנה. לאחר הצגת האלגוריתם, מוצע סוג בעיות כדוגמה לבעיות "מהחיים האמיתיים" שעוסקות בפיזור אופטימלי של אנטנות בתכן רשתות תקשורת אלחוטית.

לבסוף, האלגוריתם המוצע – amm-NSGA-II מופעל על מספר בעיות מהסוג הנ"ל. כאשר האלגוריתם הושווה לאלגוריתמים אחרים, הסתבר שהאלגוריתם המוצע מספק לא רק פיזור פתרונות טוב יותר, אלא גם פתרונות בעלי ווקטורי ביצועיים טובים יותר. דבר זה נובע כנראה מסוג הארכיב שבו נעשה שימוש באלגוריתם המוצע, מה שמוביל להמלצות לגבי מחקר עתידי.

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תחת הנחייתו של ד"ר עמירם מושיוב

יולי 2021