Depth-First Search
(DFS)

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Outline

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Elementary Graph Operations

Reachability

- **Given:** an undirected graph $G = (V, E)$, and a vertex $v \in V(G)$
- **Goal**: visit all vertices in G that are reachable from v.

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    - Similar to the preorder tree traversal.
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- In the following discussion, we shall assume that the linked adjacency list representation for graphs is used.

Outline

Depth First Search (DFS) (1/2)

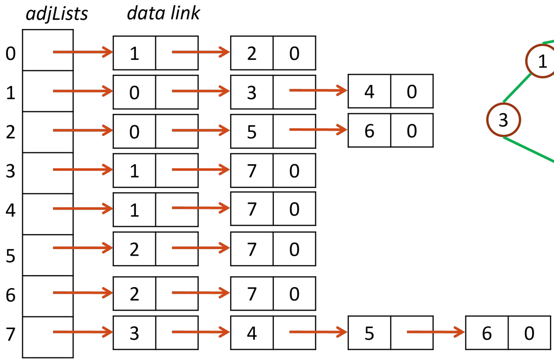
- We begin the search by visiting the start vertex, v.
- Next, we select an unvisited vertex, w, from v's adjacency lists and carry out a DFS on w.
- We preserve our current position in v's adjacency list by placing it on a **stack**.
- Eventually our search reaches a vertex, say u, that has no unvisited vertices on its adjacency list.

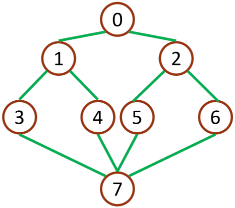
Depth First Search (DFS) (2/2)

- At this point, we remove a vertex from stack and continue processing its adjacency list.
- Previously visited vertices are discarded; unvisited vertices are visited and placed on the stack.
- The search terminates when the stack is empty.

DFS Example

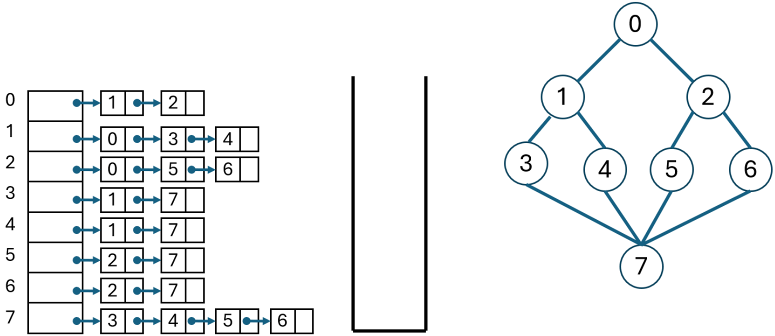
- Using a stack and recursion.
  - It resembles the preoder tree traversal.





- The DFS order: $v_0 \to v_1 \to v_3 \to v_7 \to v_4 \to v_5 \to v_2 \to v_6$.

Example (Illustration)



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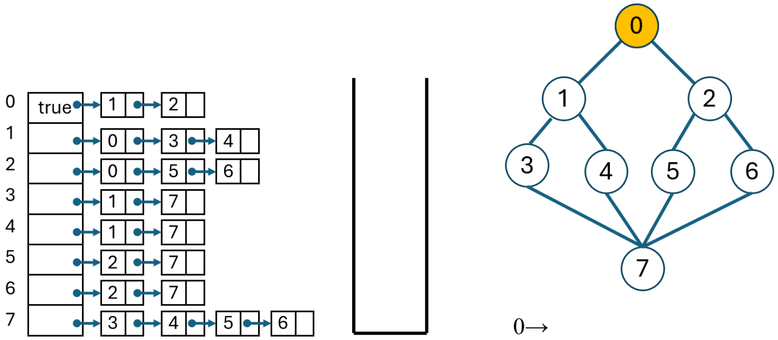
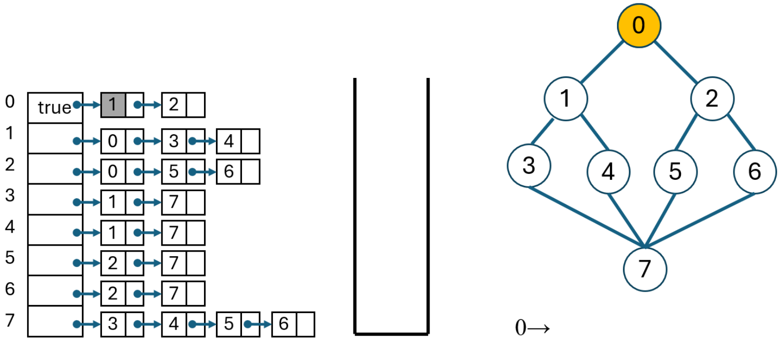


0$\rightarrow$

Example (Illustration)



$0 \rightarrow$

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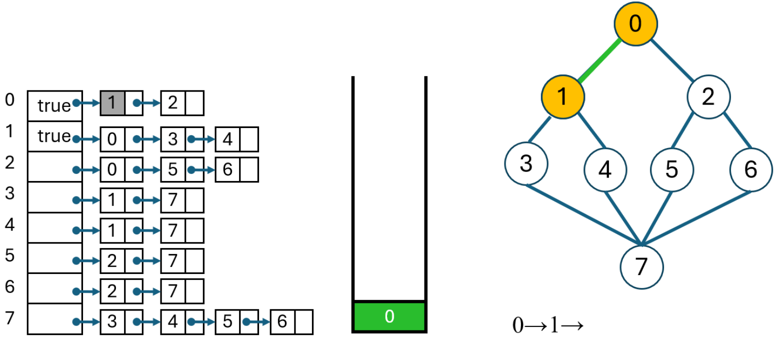

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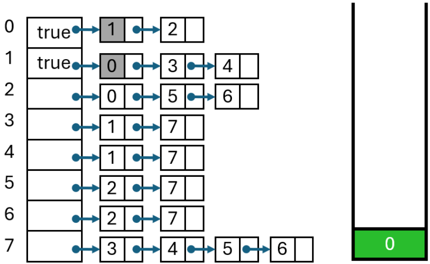

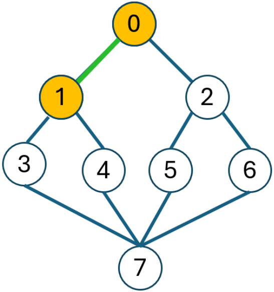

$0 \rightarrow 1 \rightarrow$

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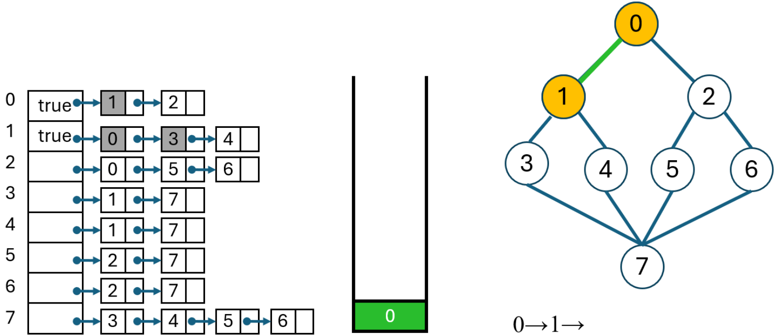

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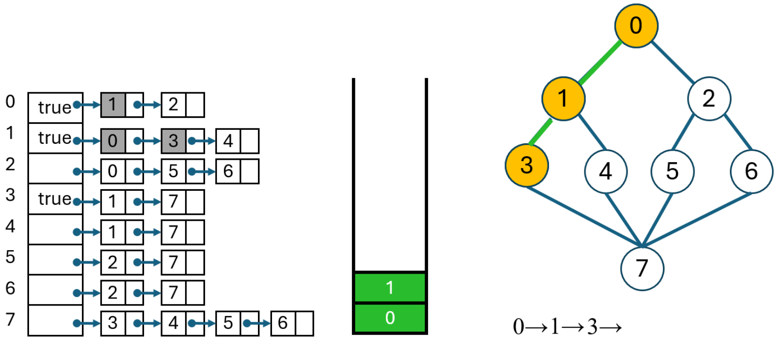

$0 \to 1 \to 3 \to$

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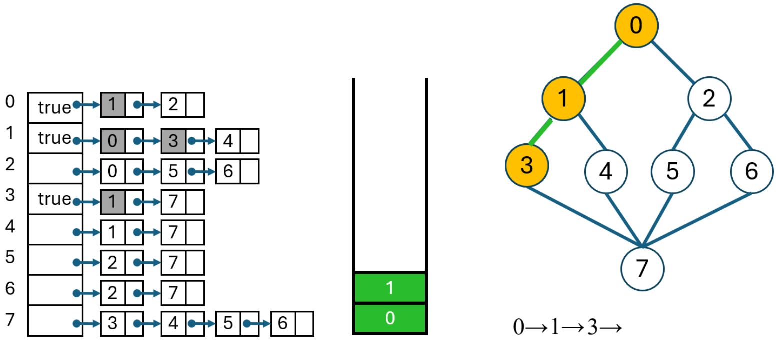

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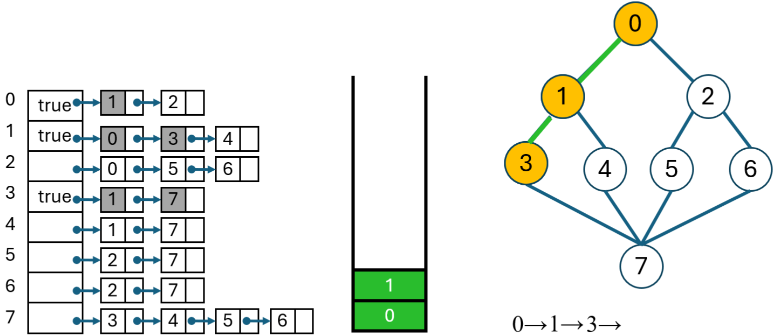

$0 \to 1 \to 3 \to$

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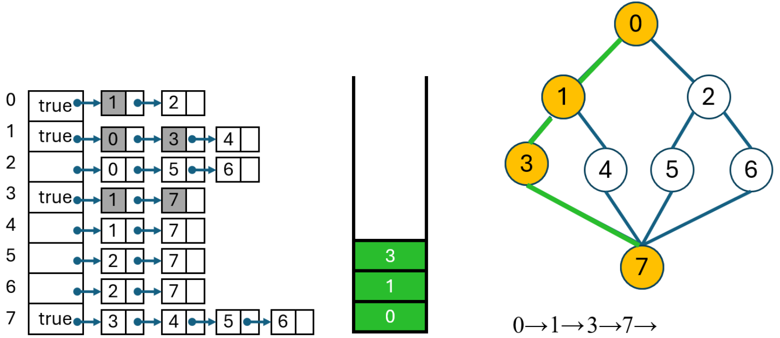

$0 \to 1 \to 3 \to 7 \to$

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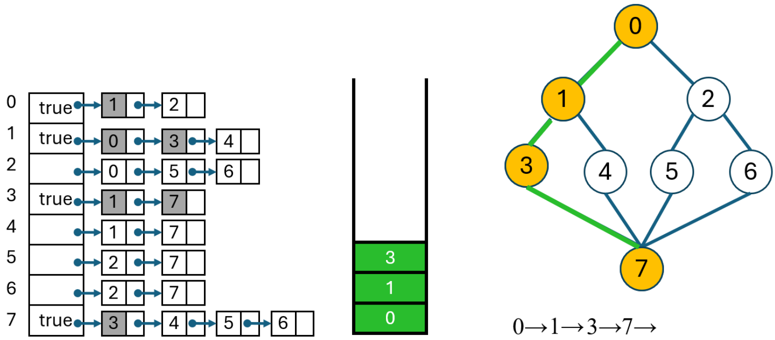

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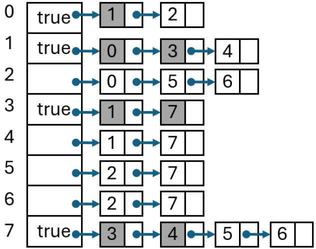

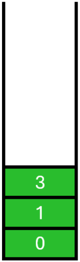

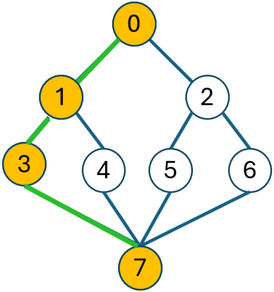

$0 \to 1 \to 3 \to 7 \to$

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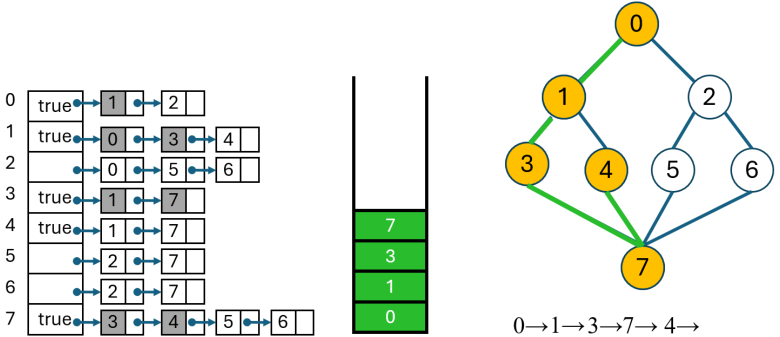

$0 \to 1 \to 3 \to 7 \to 4 \to$

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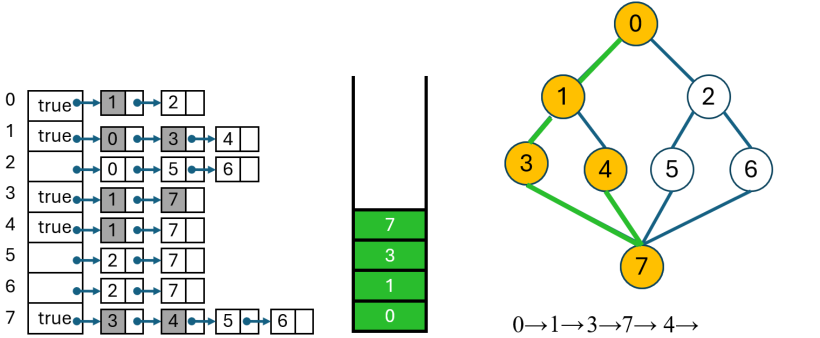

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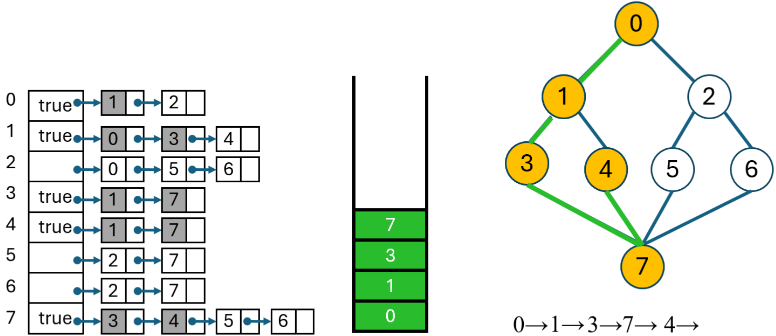

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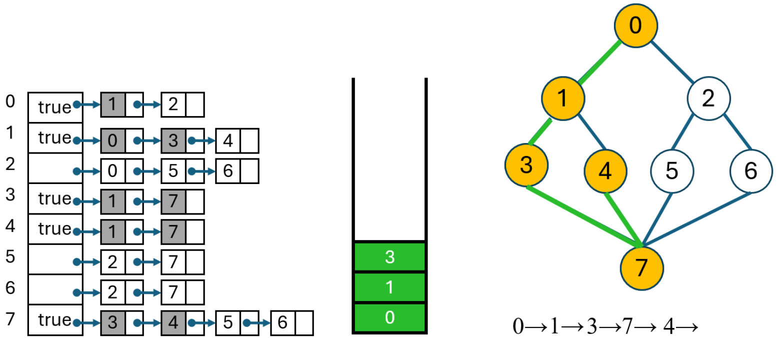

$0 \rightarrow 1 \rightarrow 3 \rightarrow 7 \rightarrow 4 \rightarrow$

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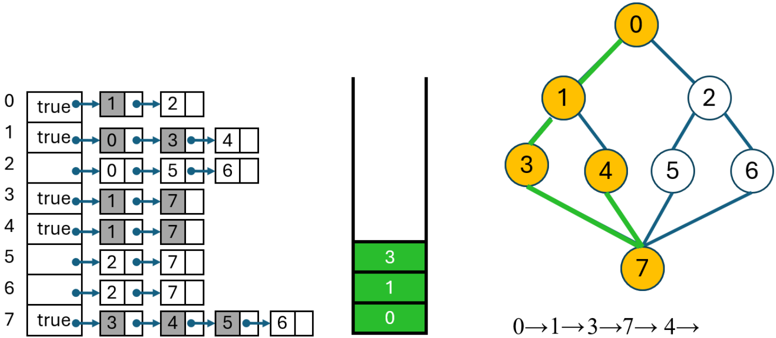

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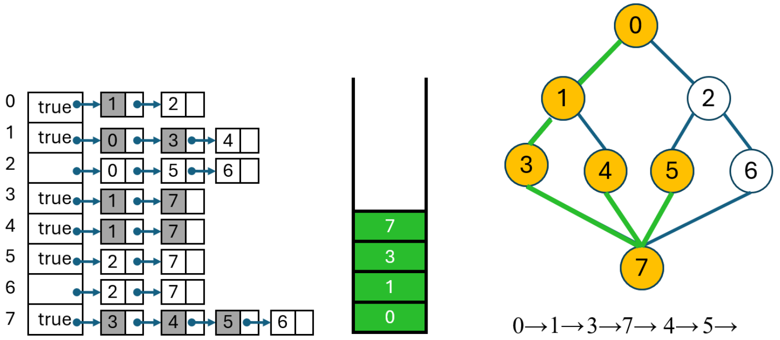

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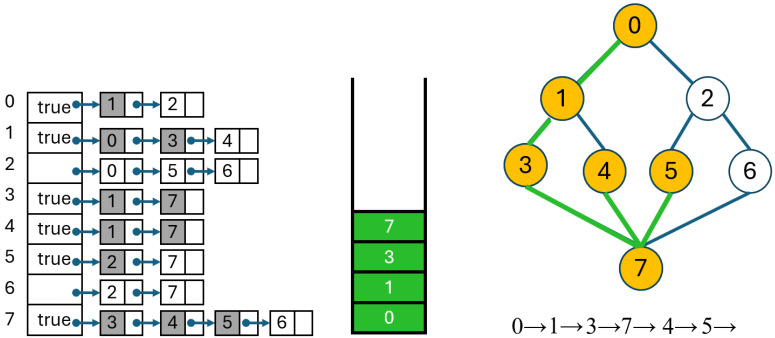

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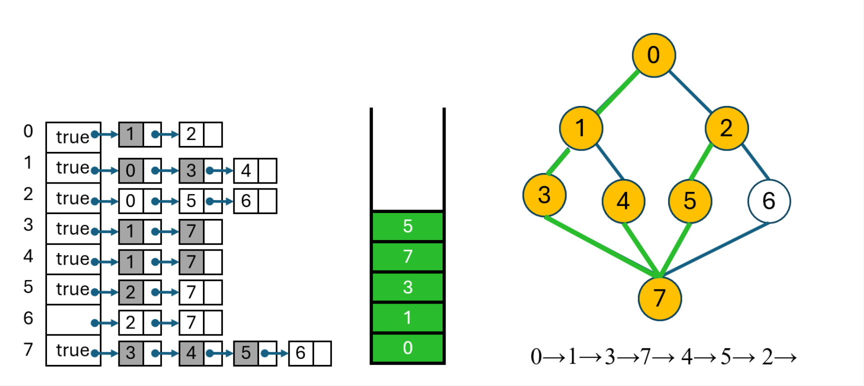

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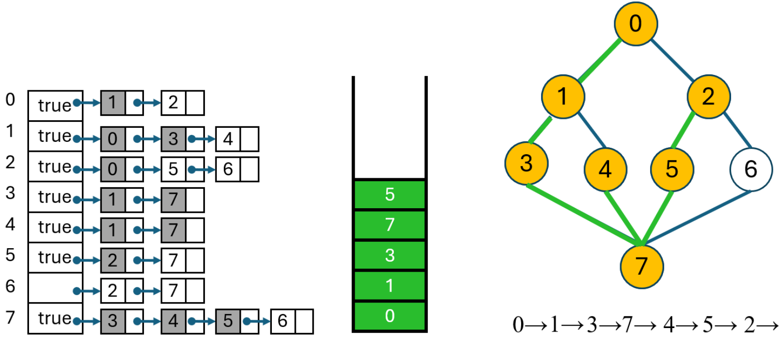

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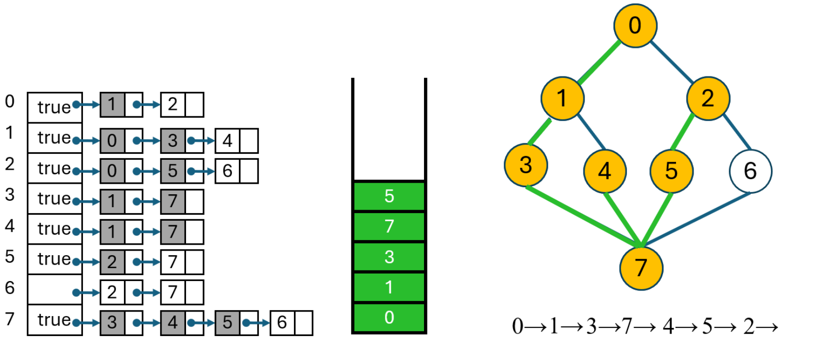

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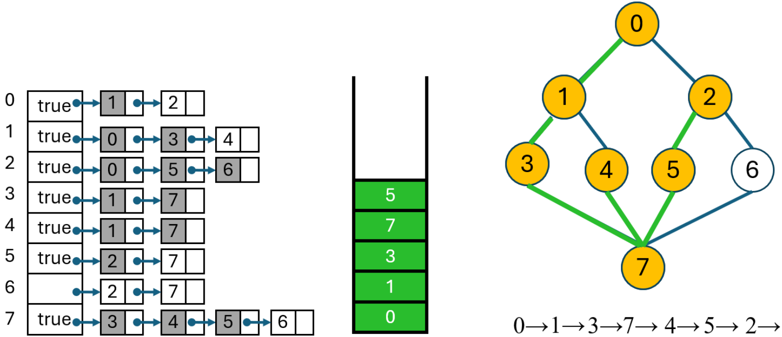

$0 \to 1 \to 3 \to 7 \to 4 \to 5 \to 2 \to$

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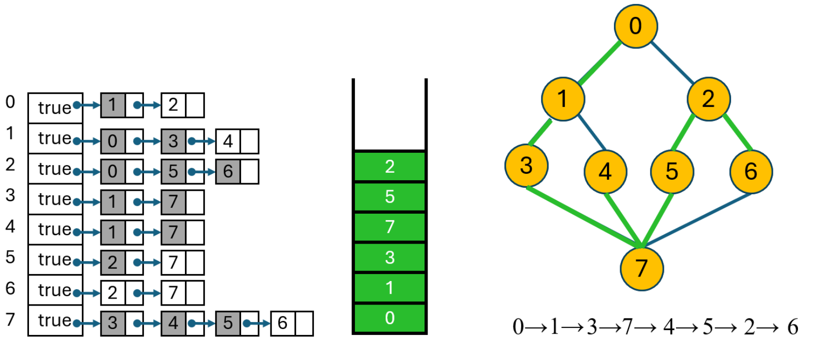

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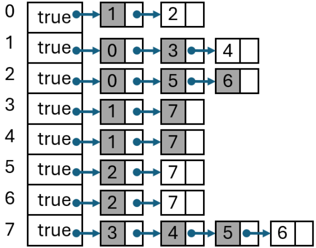

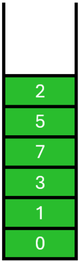

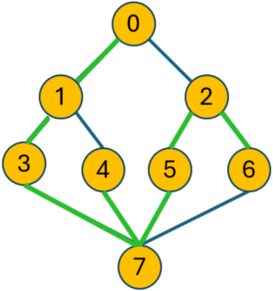

$0 \rightarrow 1 \rightarrow 3 \rightarrow 7 \rightarrow 4 \rightarrow 5 \rightarrow 2 \rightarrow 6$

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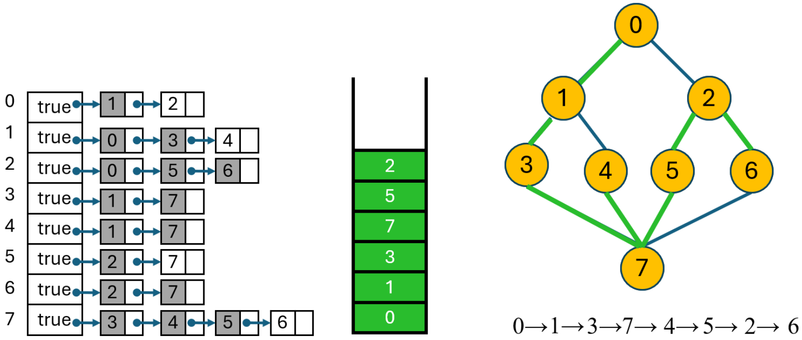

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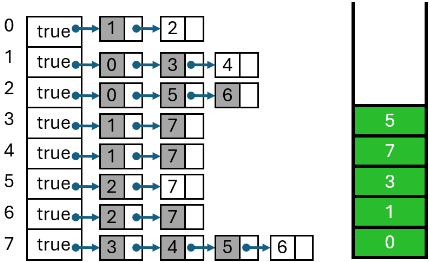

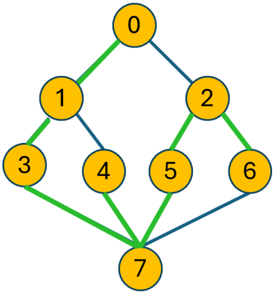

$0 \to 1 \to 3 \to 7 \to 4 \to 5 \to 2 \to 6$

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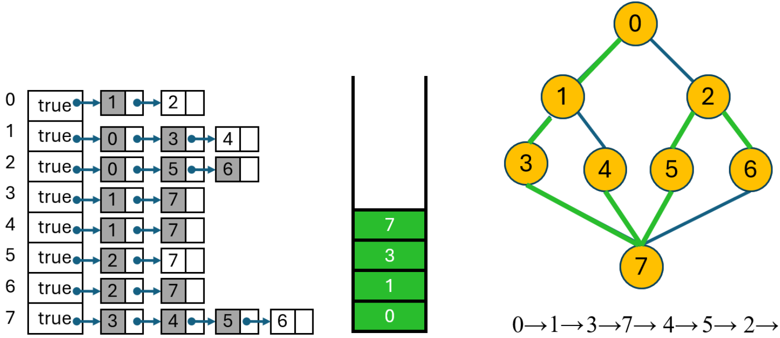

0→1→3→7→ 4→5→ 2→ 6

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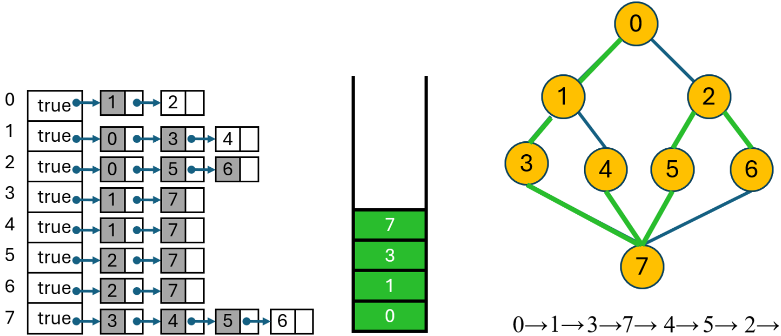

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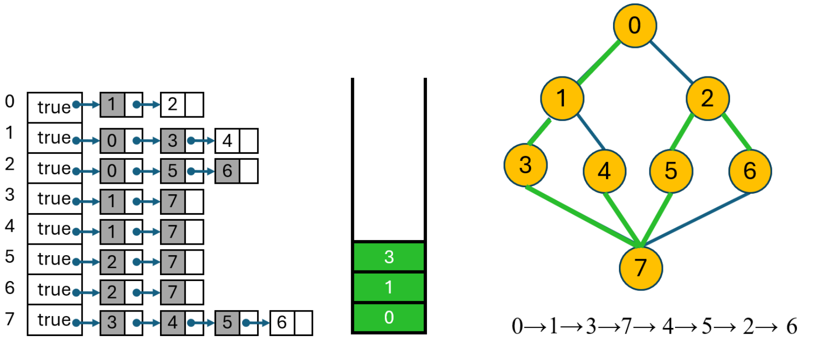

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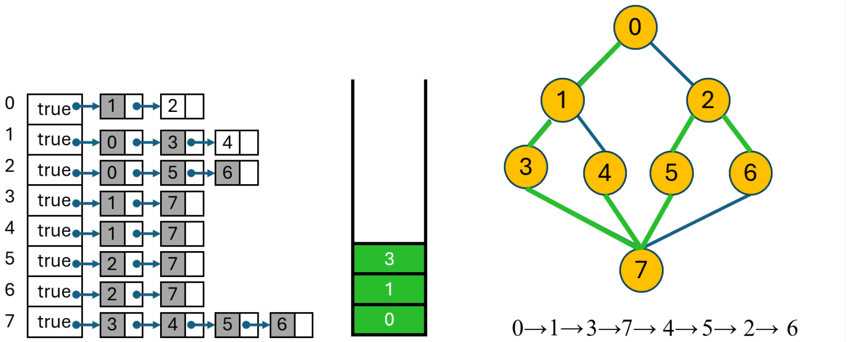

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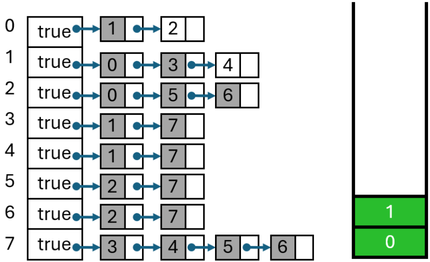

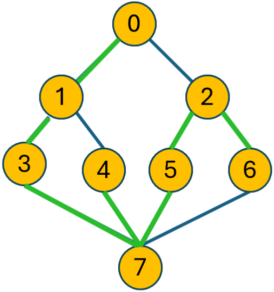

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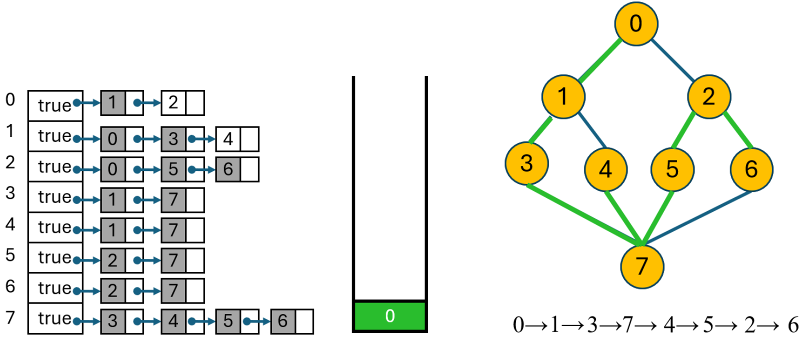

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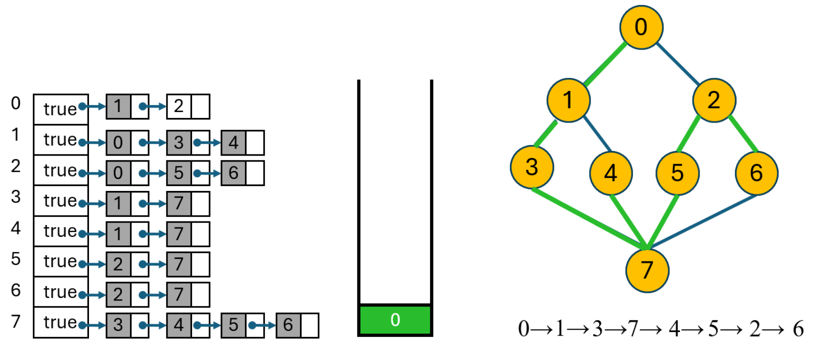

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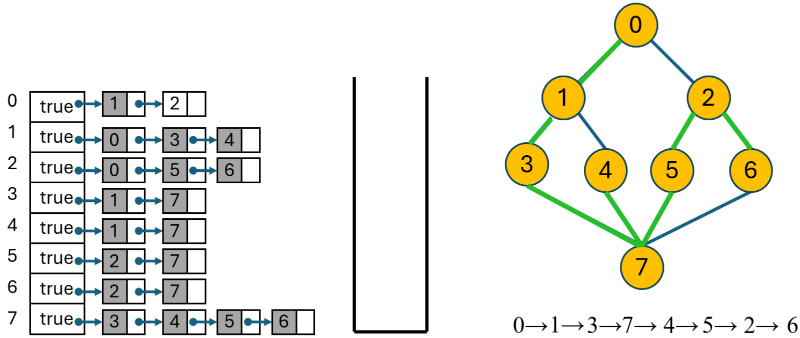

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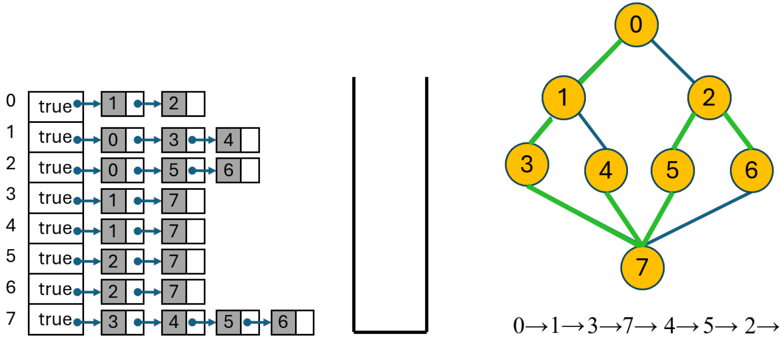

0→1→3→7→ 4→5→ 2→ 6

The Pseudocode of DFS

```
DFS(G, u) {
    u.visited = True
    for each v in G.Adj[u]
        if v.visited == False
            DFS(G, v)
}

driving main () {
    for each u in G
        u.visited = false
    for each u in G
       DFS(G, u)
}
```

DFS in C

```c
#define FALSE 0
#define TRUE 1
short int visited[MAX_VERTICES];
/* intializing to be FALSE for all */

void DFS(int v)  {
/* DFS beginning at vertex v */
    nodePointer w;
    visited[v] = true;
    printf("%5d",v);
    for(w = graph[v]; w; w = w->link)
        if (!visited[w->vertex])
            DFS(w->vertex);
}
```

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  - One needs to scan the corresponding row of the adjacency matrix.
- Total time: $O(n^2)$.

Discussions