

Binary Search Trees & Forests

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Outline

- 1 Binary Search Trees
- 2 Forests

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1 Binary Search Trees

2 Forests

Binary Search Tree (BST) (1/2)

For searching, insertions and deletions...

Binary search tree provides a better performance than any of the data structures studied so far.



Binary Search Tree (BST) (2/2)

BST

A binary search tree (BST) is a binary tree which may be empty.

If it is not empty, then it satisfies the following properties:

- Each node has **exactly one key** and the **keys in the tree are distinct**.
- The keys (if any) in the **left** subtree are **smaller** than the key in the root.
- The keys (if any) in the **right** subtree are **larger** than the key in the root.
- The left and right subtrees are also binary search tree.



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- The left and right subtrees are also binary search tree.
 - a flavor of recursion?

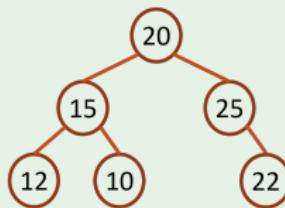


Examples of BST

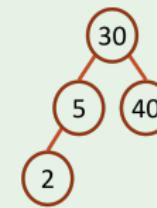
- Which one of the following trees is BST? Which one is NOT?

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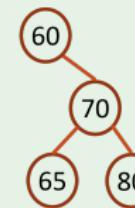
- Which one of the following trees is BST? Which one is NOT?



(a)



(b)



(c)



(d)

Recursive Search of a BST

```
element* search(treePointer root, int key) {  
    /* return a pointer to the node that contains key,  
       if there is no such node, return NULL. */  
    if (!root) return NULL;  
    if (k == root->data.key) return &(root->data);  
    if (k < root->data.key)  
        return search(root->leftChild, k);  
    return search(root->rightChild, k);  
}
```

- The time complexity of the search function is $O(h)$, where h is the height of the binary search tree.

Iterative Search of a BST

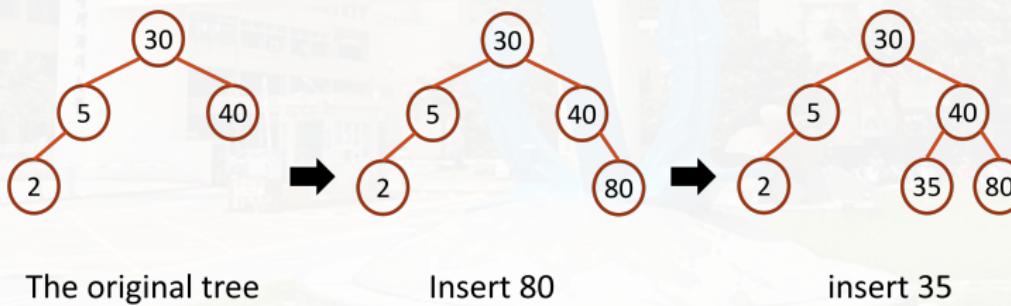
```
element* iterSearch(treePointer tree, int k) {
    /* return a pointer to the node that contains key,
       if there is no such node, return NULL. */
    while (tree) {
        if (k == tree->data.key) return &(tree->data);
        if (k < tree->data.key)
            tree = tree->leftChild;
        else
            tree = tree->rightChild;
    }
    return NULL;
}
```

- The time complexity of the search function is $O(h)$, where h is the height of the binary search tree.



Inserting into a BST

- Verify that the key is different from those existing elements.
- If the search is **unsuccessful**, insert the element at the point **the search terminated**.



A Modified Searching Function

```
modifiedSearch(treePointer *node, int k)
```

- If the BST is empty, then return NULL.
- If the key k exists in the BST, return NULL.
- Otherwise, return the pointer of the last node in the BST.

Inserting a Dictionary Pair into a BST

```
void insert(treePointer *node, int k, iType theItem) {  
    /* If k is in the tree pointed at by "node", do nothing;  
       otherwise, add a new node with data = (k, theItem) */  
    treePointer ptr, temp = modifiedSearch(*node, k);  
    if (temp || !(*node)) { /* k is not in the tree */  
        ptr = (treePointer)malloc(sizeof(node));  
        ptr->data.key = k;  
        ptr->data.item = theItem;  
        ptr->leftChild = ptr->rightChild = NULL;  
        if (*node) /* insert as child of temp */  
            if (k < temp->data.key)  
                temp->leftChild = ptr;  
            else  
                temp->rightChild = ptr;  
        else // the case of empty tree  
            *node = ptr;  
    }  
}
```

Deletion from a BST

- Case 1: **leaf**
 - delete the node and set the pointer from the parent node to NULL.
- Case 2: **having only one child**:
 - delete the node and change the pointer from the parent node to the single-child node.
- Case 3: **having two children**:
 - replaced by the **largest** element in its **left** subtree, or
 - replaced by the **smallest** element in its **right** subtree.

Illustration (Case 1 & 2)

Case 1:



Case 2:



Illustration (Case 3)

Case 3:

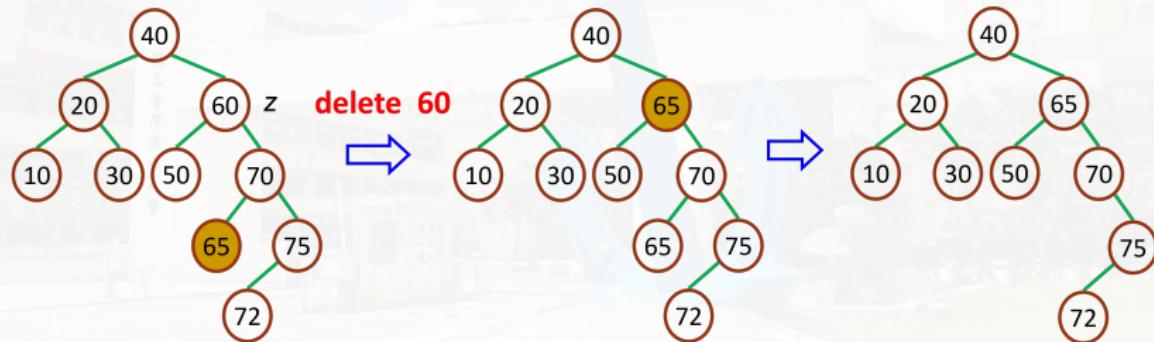
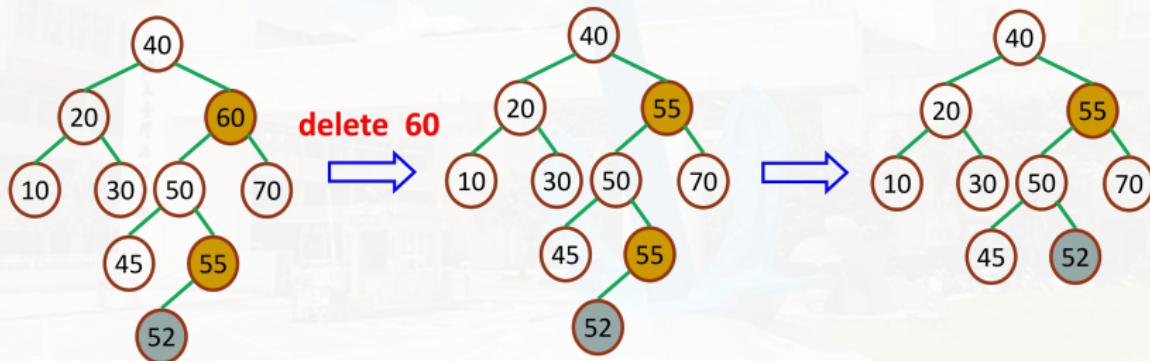


Illustration (Case 3)

Case 3:



Time Complexity Analysis of Deleting a Node in a BST

- The case: Deleting a nonleaf node that has two children.
- We can verify (Exercise) that, in both ways, it is originally in a node with a degree of at most one.
 - Check the largest and smallest elements in a subtree.
- The time complexity for case 3 is $O(h)$ (h : the height of the BST).
- A deletion can be performed in $O(h)$ time.

Outline

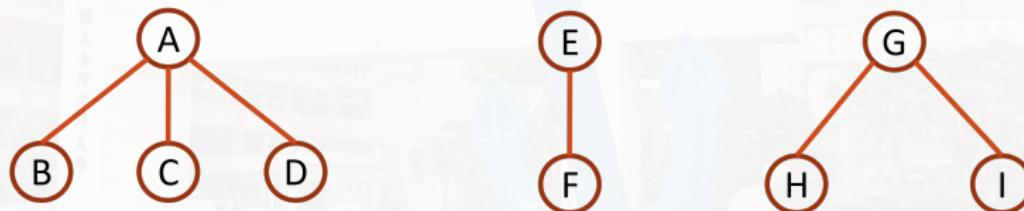
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2 Forests

Forest

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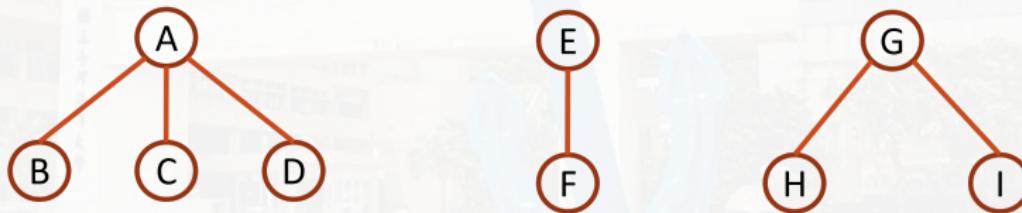
A forest is a set of $n \geq 0$ disjoint trees.



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Question

Is a binary tree a forest?

Rule of Transforming a Forest into a Binary Tree

- If T_1, T_2, \dots, T_n is a forest of disjoint trees T_1, T_2, \dots, T_n , then the binary tree corresponding to this forest, denoted by $B(T_1, T_2, \dots, T_n)$,
 - is empty if $n = 0$.
 - has root equal to $\text{root}(T_1)$;
 - has left subtree equal to $B(T_{11}, T_{12}, \dots, T_{1m})$, where $T_{11}, T_{12}, \dots, T_{1m}$ are the subtrees of $\text{root}(T_1)$; and
 - has right subtree $B(T_2, \dots, T_n)$.

Binary Tree Representation of a Forest



Discussions

