Preliminaries/ x: element (8 ex bas A ex XT = 8 n A. x in Set A: X & A. Mills riotal x is not in Set A: A X & A . SUA. $\{1,2,3,4\} = \{1,3,2,4\} = \{1,4,2,3\} = \cdots$ empty set . { } or \$ difference a number of elements in A 另一種集合表示法 cardinality, [A] => A={x|x>3 and xeZ} N, Z, Q, R.C Subset

o NA, B are two sets such that for any DXEA > XEB (OIR there ex Then A is a subset of Bessel = That is, ACB or B2A2 18287 - 8 note: A & B [4.8] = 80 A.00 ASB ACB

Operations on Sets for sets A, B. ANB= {X | XEA and XEB} "intersection" A > X A + S A A S · AUB = {x/x = A or x = B} " union" = 140 (1) = 14 (8 (1)) A-B = { X | X \in A and X \in B} " difference" $\overline{A} = \{x \mid x \in A\}$ complement" Example:

Try to test these in Python Suppose (= {1,2,...,9,10}. 8 x (A x) A= {1,2,3,4} SU. B= {3,4,5,6} SU SO 10 02A Then 10 ANB = {3,4} (2) AUB = {1,2,3,4,5,6} (3) A-B = {1,29

(4) A = {5, 6, 7, 8, 9, 109

Suppose A, Az, --, An are n sets $A_{1} = A_{1} \wedge A_{2} \wedge A_{3} \wedge \cdots \wedge A_{n}$ = {x | x ∈ Ax, for all i = 1, 2, ..., n} (Cicini UAi = AIUAZUITTUAN = {x | x ∈ Ai, for some i=1,2, ..., n} 55A f(S) = [f(a) | (x = S) = Cartesian product (下代特) Sets A, B Sets A, B A x B = {(a, b) | a ∈ A, b ∈ B} Sordered pair (有序數對) (nee (1-1) = f(1) = [24 fobs what the good. it 5'= [49] then f'(5') = [-3,-2]u[2,3]

we An An are needs Function A, B: sets. O.An. AniA = A fi A Bland APR [x] = $\chi \rightarrow f(\chi)$ A: domain of f 7 7751 UsAUIA = 141 Bicodomain of f 1/2 256 if SEA, f(S) = [f(x) | x = S} if S'SB, f'(S') = {xeA | f(x)eS'} $f(A) = \{f(x) \mid x \in A\}$: range of f(1) $f: (-5, 5] \rightarrow \mathbb{R}$ $f(x) = \chi^2$ range of f: [0, 25] if S = [1,3], then f(S) = [1,9] if S'=[4,9], then f'(S') = [-3,-2]U[2,3] One-to-One Onto Det To stell metrogra Suppose f: A -> B is a function (1) if $f(x) = f(y) \Rightarrow x = y$ then f is "one-to-one" (x) (top) (2) if f(A) = B. Then f is "onto"

YYEB, $\exists x \in A$, s.t. f(x) = yOne-to-one: injection) and bijection (9 +) (x)= 2(f(x)) = 9(x))= P Example: $f: [-1, 1] \longrightarrow (0, 1], f(x) = x^2$ Since f(-1) = f(1) = 1, f is NoT one-to-Suppose f: A - B is a fueling talt dons A = 8 : E = 7: gof = IA and fof = IA Then of is invertible

Composition July 3

Suppose $f: A \mapsto B$, $g: B \mapsto C$ are two functions Then $g \circ f: A \mapsto C$ is defined as $(g \circ f)(x) = g(f(x))$

A > If > for g (for)

machine

machine

machine

machine

Example: $f(x) = \chi^{2}$, $g(x) = \chi^{2}$ $f: R \mapsto R$, $g: R \mapsto R$ $(g \circ f)(x) = g(f(x)) = g(\chi^{2}) = e^{\chi^{2}}$ $(f \circ g)(x) = f(g(x)) = f(\chi^{2}) = e^{\chi^{2}}$

Invertible function 7 14 3186

Suppose $f: A \rightarrow B$ is a fuction if $\exists g: B \rightarrow A$ such that $g \circ f = I_A$ and $f \circ g = I_B$ Then f is invertible

In: A -> A such that IA(x) = x, 4x eA

f is invertible function (=) f is one-to-one and onto

mote: (f-1)-1 = f f is also
invertible Example $f: \mathbb{R} \mapsto \mathbb{R} , f(x) = 2x + 3$ =) f is one-to-one and onto. (heck.) $f^{-1}: \mathbb{R} \to \mathbb{R}$, de de bas x + 3 = 1Let 2x + 3 = 1 $f^{-1}(x) = \frac{x-3}{x}$ $(3+d)+\alpha = 3+(d+0) + C = \alpha+(b+c)$ tost was I & 1.0 E w サAGEF、Ota P 中国を発性主意 D=D.N.b.a in the first arb=0 to ske fact is fact, ato I set such that a.b. - | ARESE B Ad'P : Cet " (P+c) = d'P+ d. C

Field Be examples: R, R, C, C2,... Suppose F is a set, and we define / hove addition and multiplication operations on F such that
(1) Ya, b E F, a+b E F 力·/乘法 打閉性 and a.be Fine by w ya,b∈F, a+b=b+a and a.b=b.a 9-9 加承法交换性 (3) \da,b,c \in T, (a+b)+c = a+(b+c) and (a.b)· C = a. (b.c) * @ 33.500 4 3 o, I & I such that +a∈F, 0+a= a か/報送單位元素 Co Yaef, 目beF such that atb=0 か流気気 的 faet, ato, 3 bet such that a.b=1 泰弦交流 (7) Ya, b, cef, a. (b+c) = a. b+ a. c 亚江至如《经分的建 Then We say F is a field. Example Z2 = fo, 1] + 0 1 0 0 0

Recall: F2: R2, 4 R" .: 1R" , 6

Span and Linear Independence / Te Recall:

Rec Rhing Rh, the in my my or more for some FI If 9- (2.5-11) 2+ (1-12) 3+ (5 4-17) 17 = 201+927 11 Suppose Man Xin Kef sp++ p2- = 1 (18) 5- or the spent for a policy - B: