

# Depth-First Search and Breadth-First Search (DFS & BFS)

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# Outline

1 Introduction

2 Depth First Search (DFS)

3 Breadth-First Search (BFS)

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    - Similar to the **level-order tree traversal**.

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  - Breadth-Frist Search (BFS)
    - Similar to the **level-order tree traversal**.
- In the following discussion, we shall assume that the **linked adjacency list** representation for graphs is used.

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# Depth First Search (DFS) (1/2)

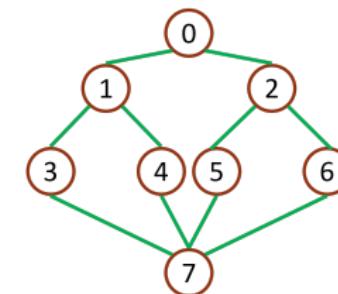
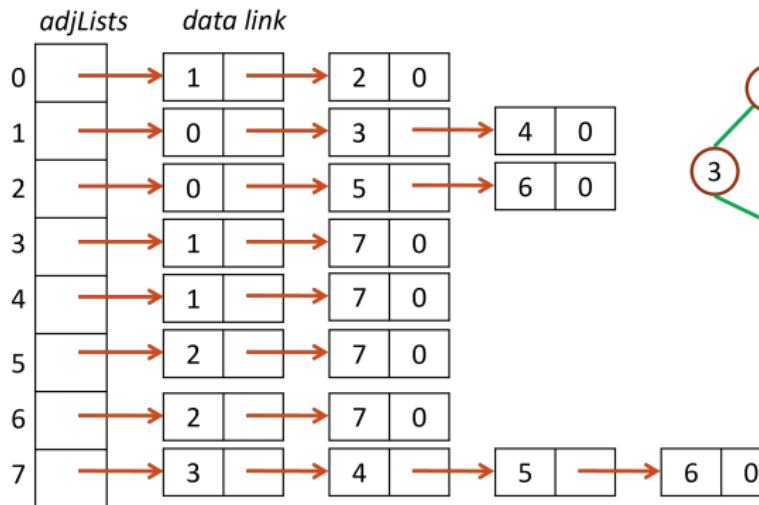
- We begin the search by visiting the start vertex,  $v$ .
- Next, we select an **unvisited** vertex,  $w$ , from  $v$ 's adjacency lists and carry out a DFS on  $w$ .
- We preserve our current position in  $v$ 's adjacency list by **placing it on a stack**.
- Eventually our search reaches a vertex, say  $u$ , that has no unvisited vertices on its adjacency list.

# Depth First Search (DFS) (2/2)

- At this point, we remove a vertex from **stack** and continue processing its adjacency list.
- Previously visited vertices are **discarded**; unvisited vertices are visited and placed on the stack.
- The search terminates when the stack is empty.

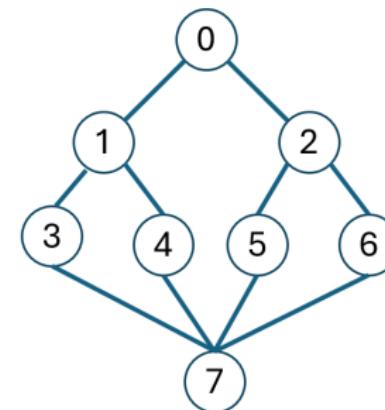
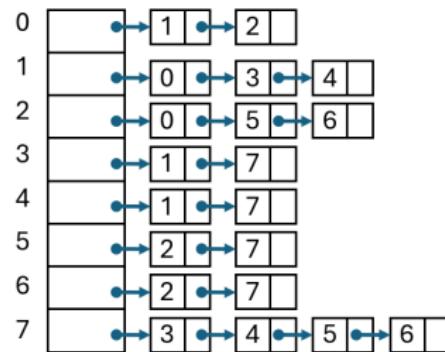
# DFS Example

- Using a stack and recursion.
- It resembles the preorder tree traversal.

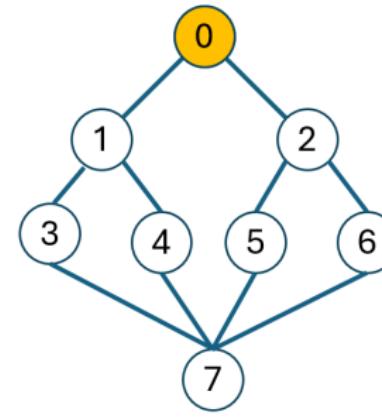
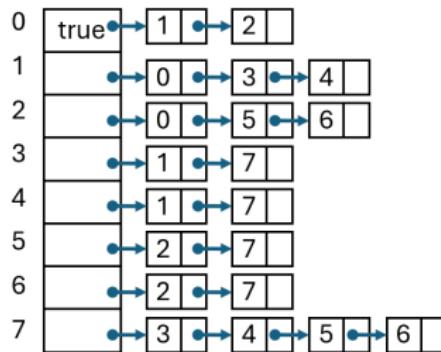


- The DFS order:  $v_0 \rightarrow v_1 \rightarrow v_3 \rightarrow v_7 \rightarrow v_4 \rightarrow v_5 \rightarrow v_2 \rightarrow v_6$ .

# Example (Illustration)

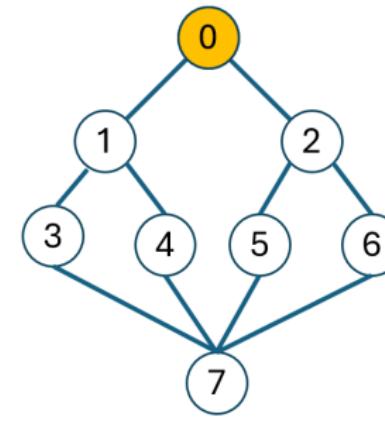
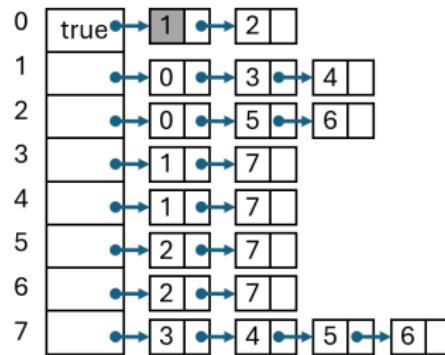


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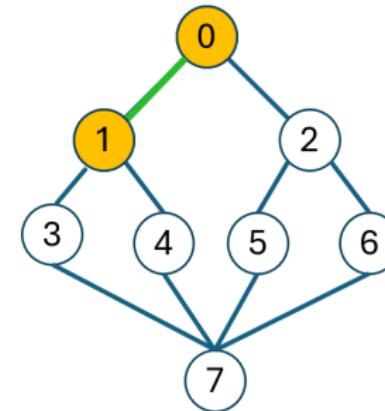
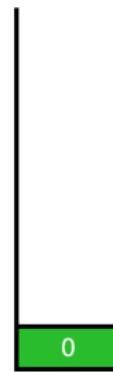
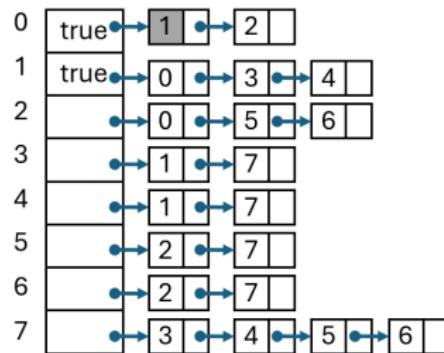
0→

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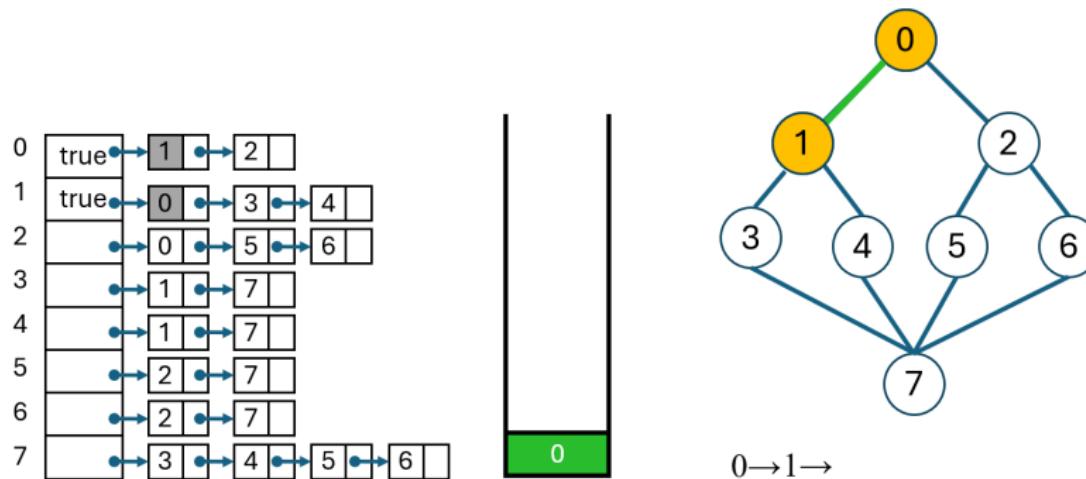


0→

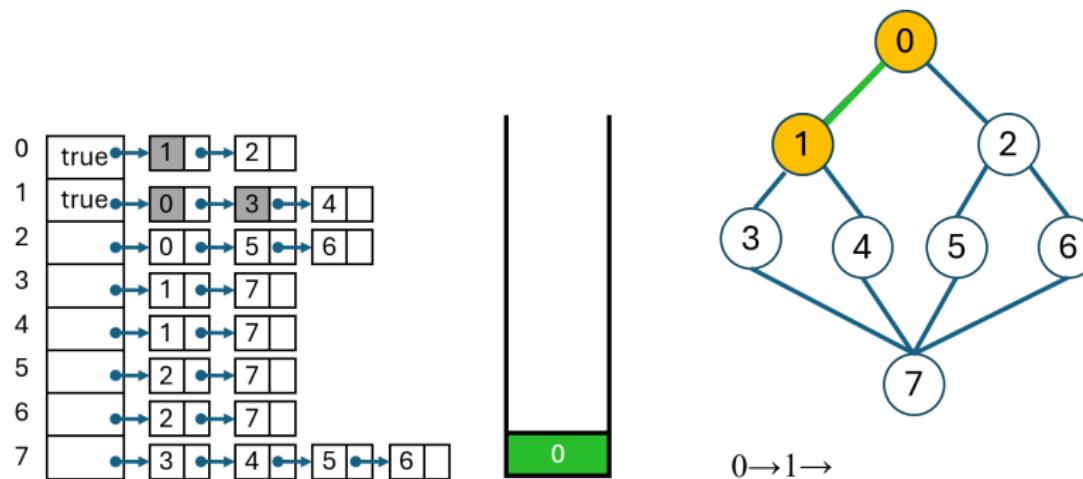
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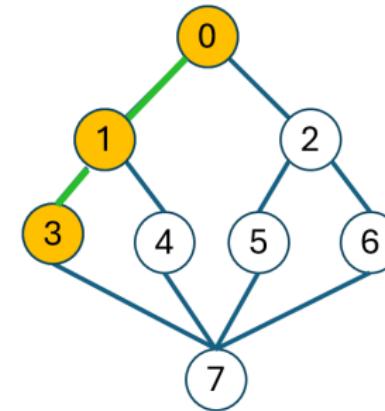
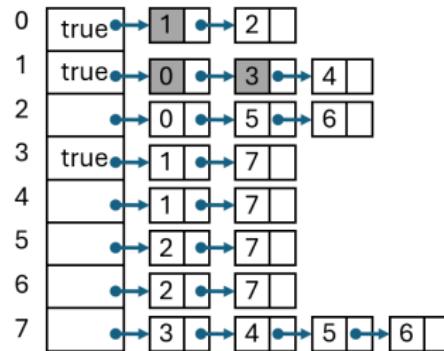
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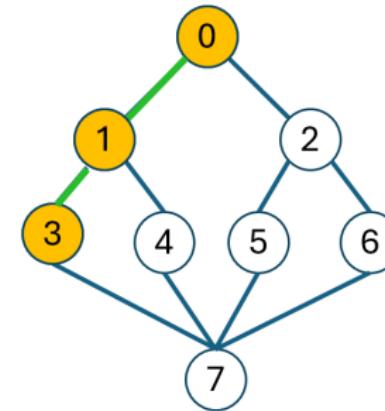
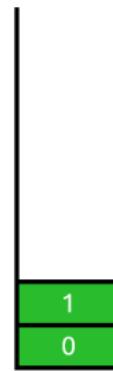
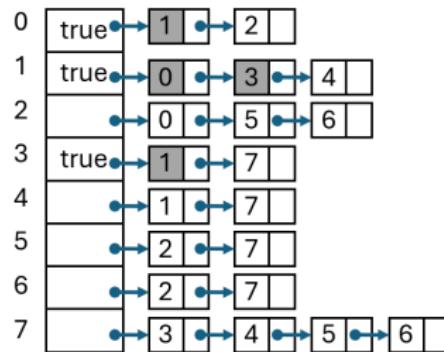
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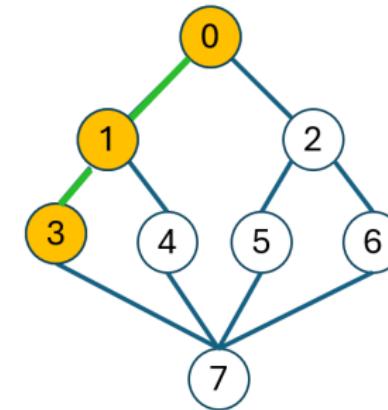
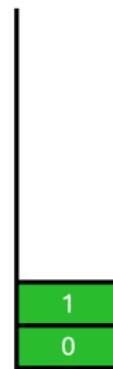
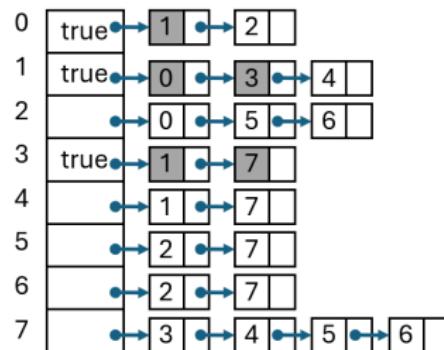


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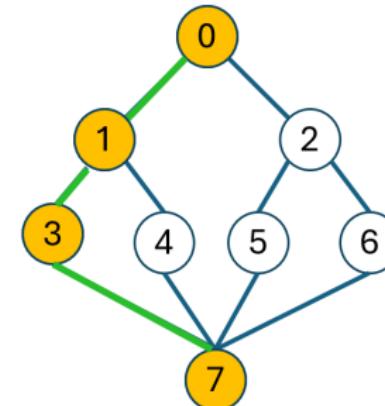
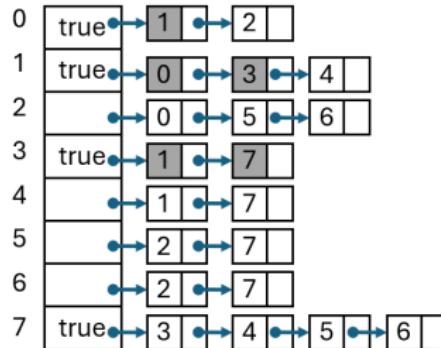


0→1→3→

# Example (Illustration)

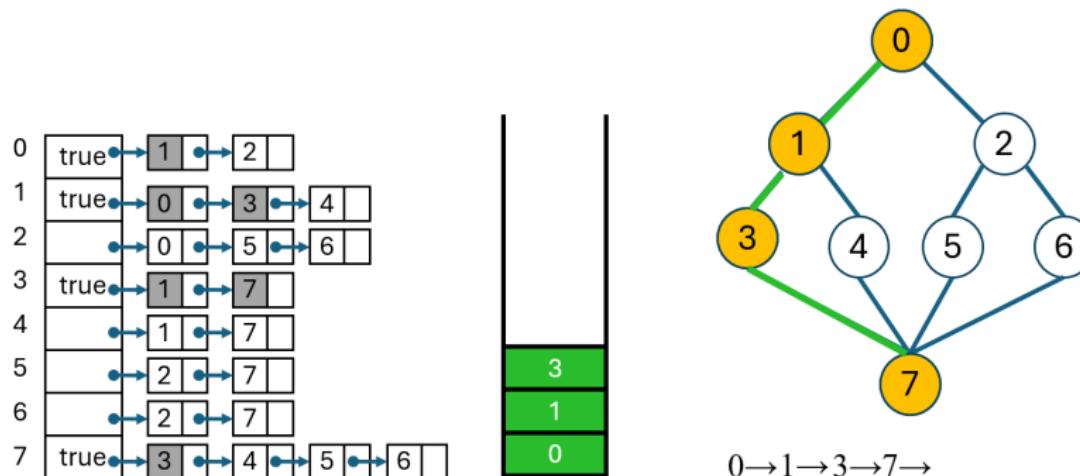


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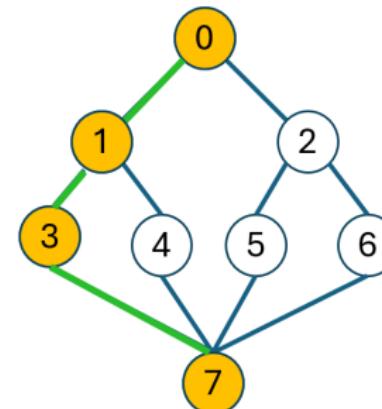
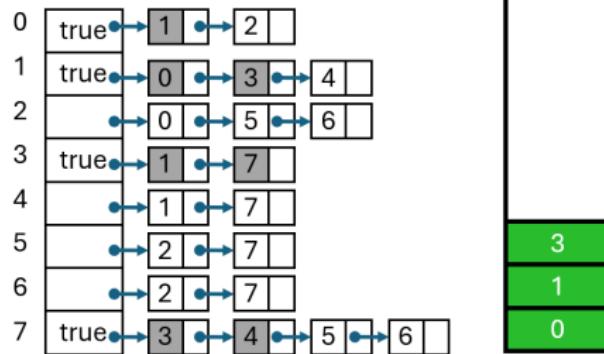


0 → 1 → 3 → 7 →

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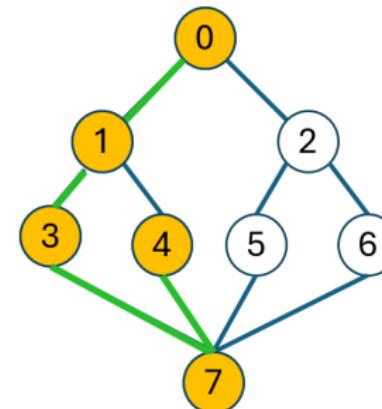
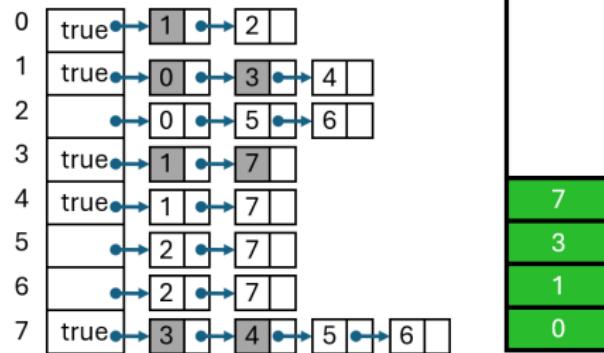


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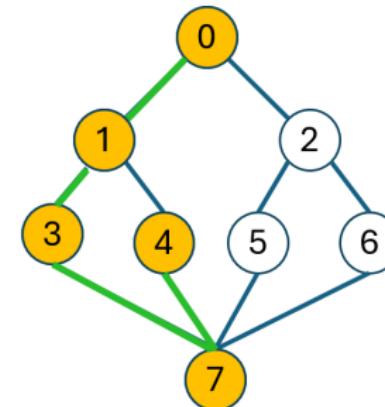
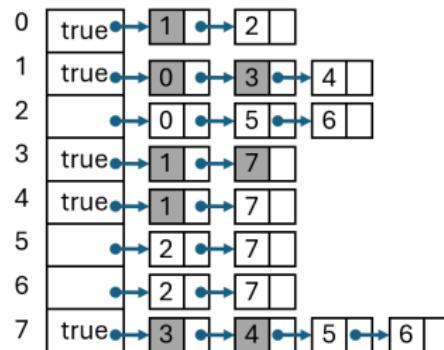
0 → 1 → 3 → 7 →

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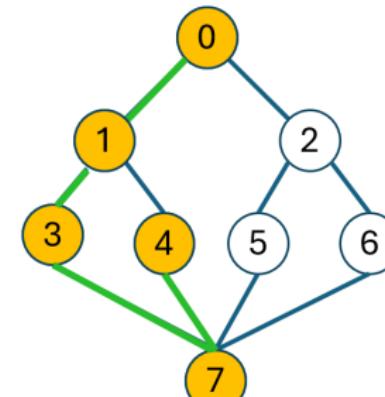
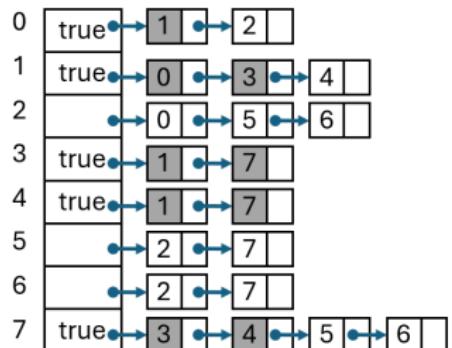
0 → 1 → 3 → 7 → 4 →

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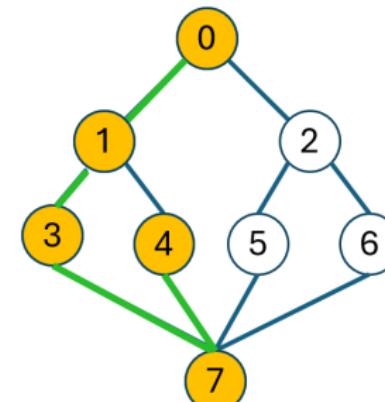
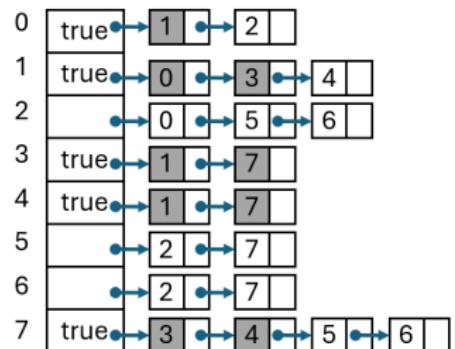
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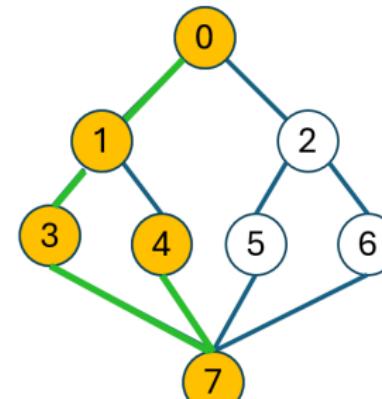
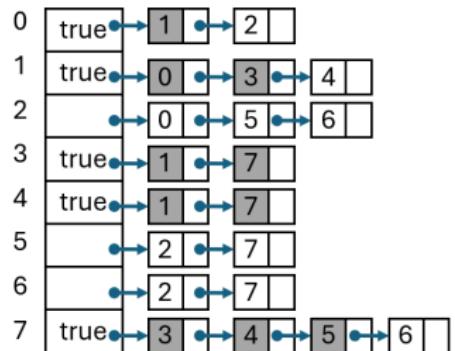


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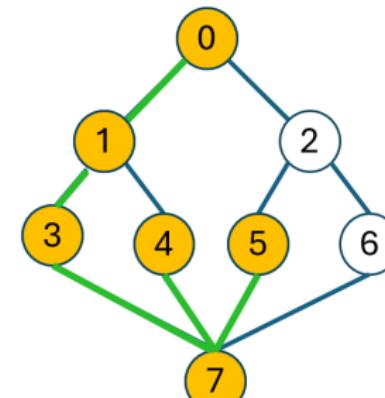
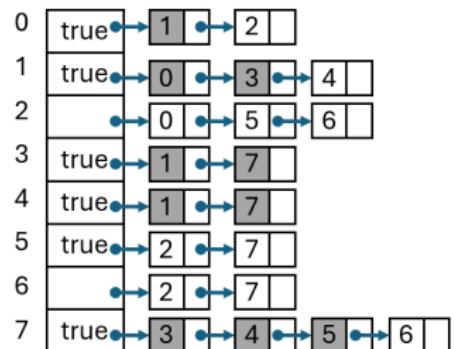


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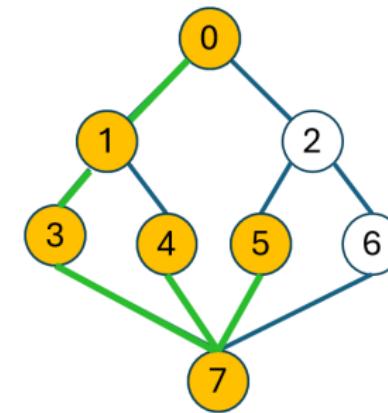
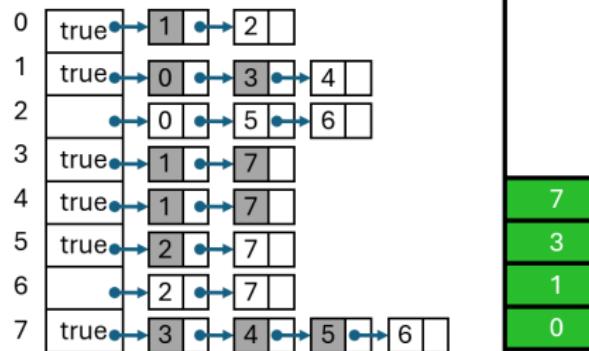
0 → 1 → 3 → 7 → 4 →

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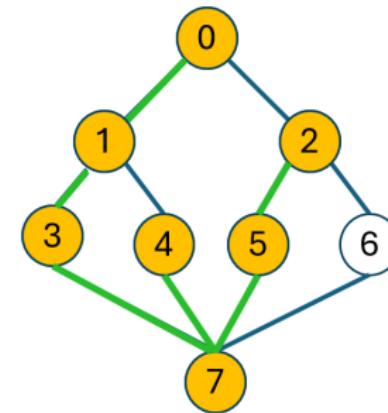
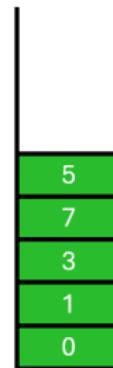
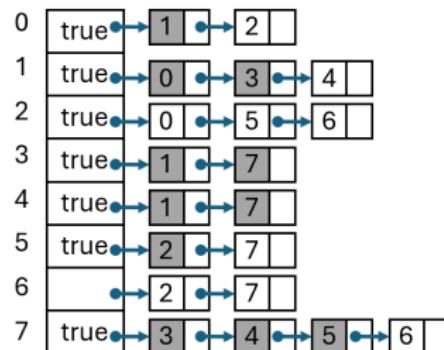
0 → 1 → 3 → 7 → 4 → 5 →

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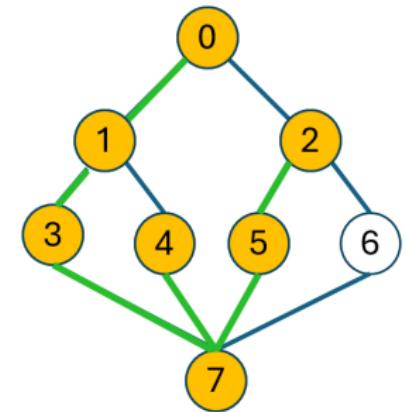
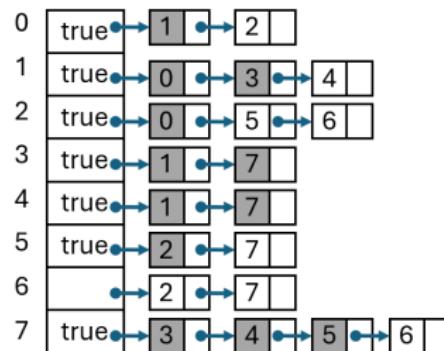
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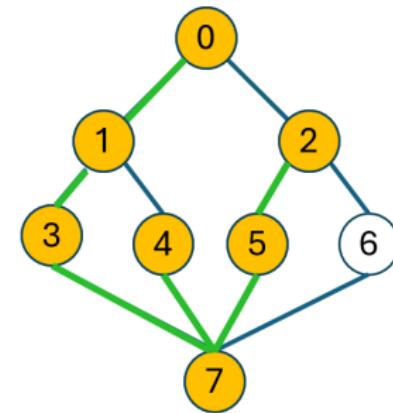
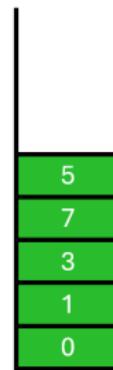
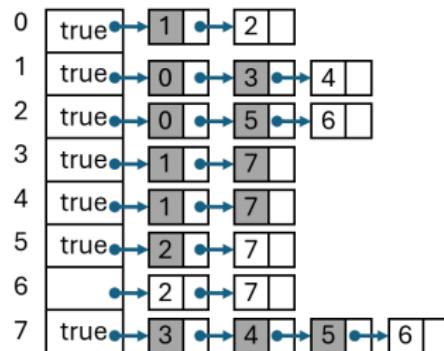
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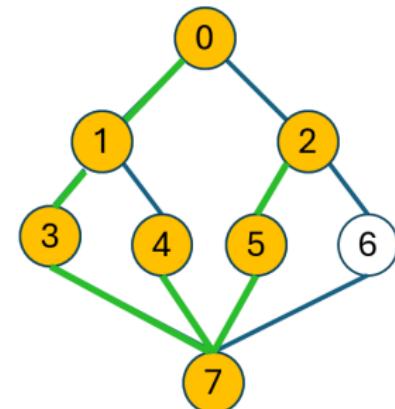
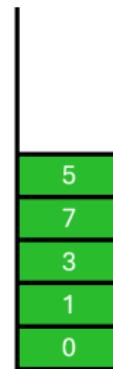
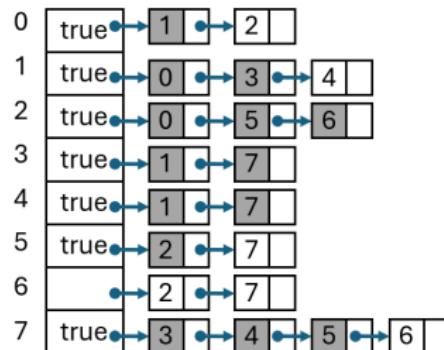


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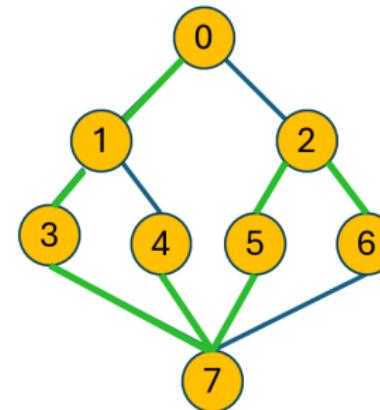
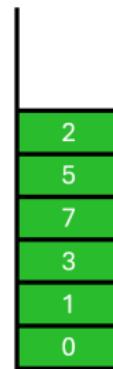
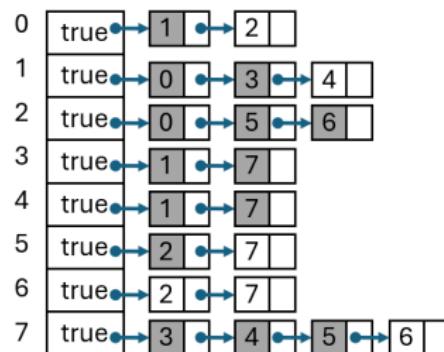


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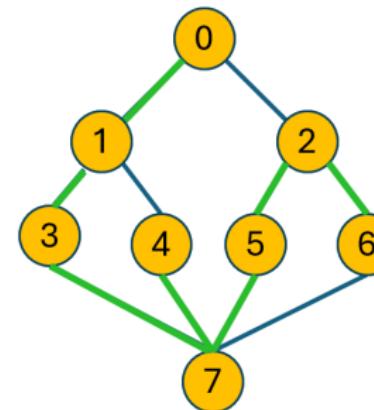
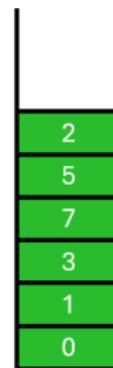
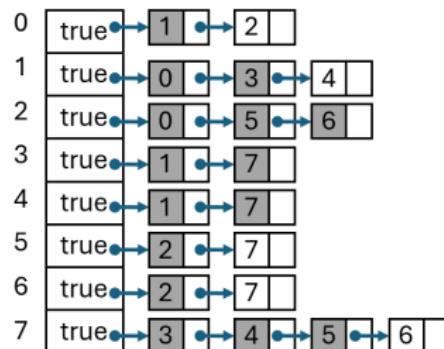
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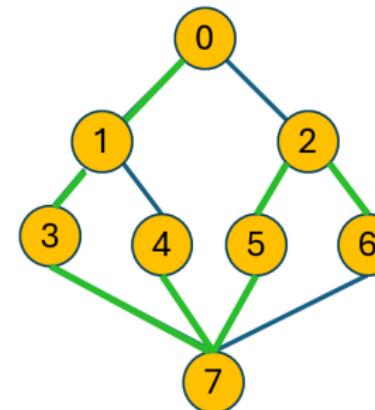
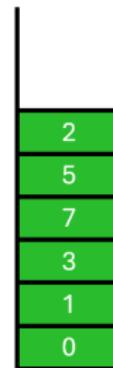
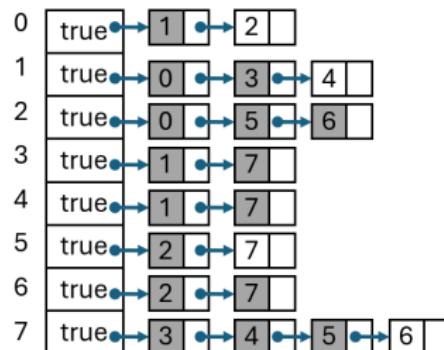
0 → 1 → 3 → 7 → 4 → 5 → 2 → 6

# Example (Illustration)



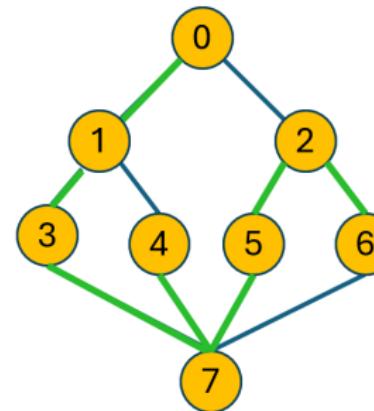
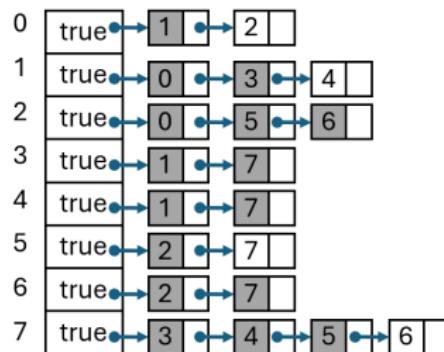
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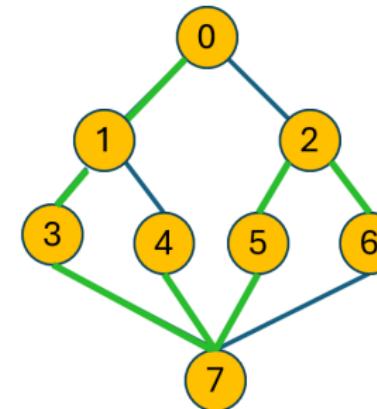
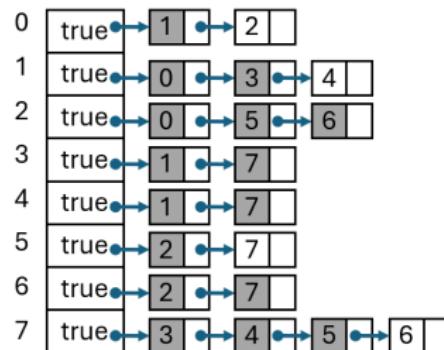
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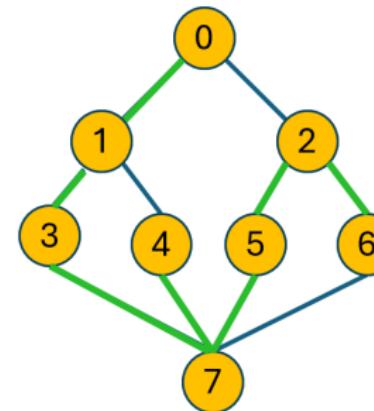
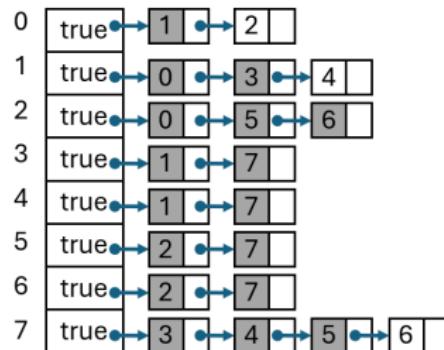
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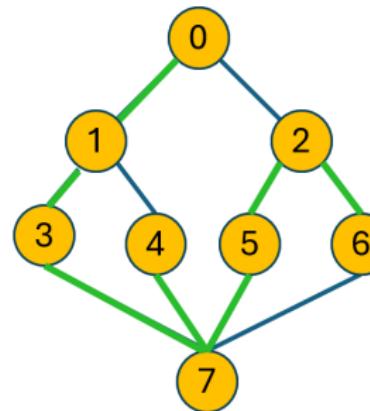
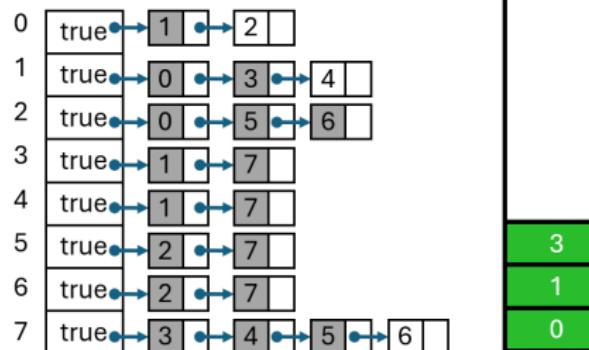
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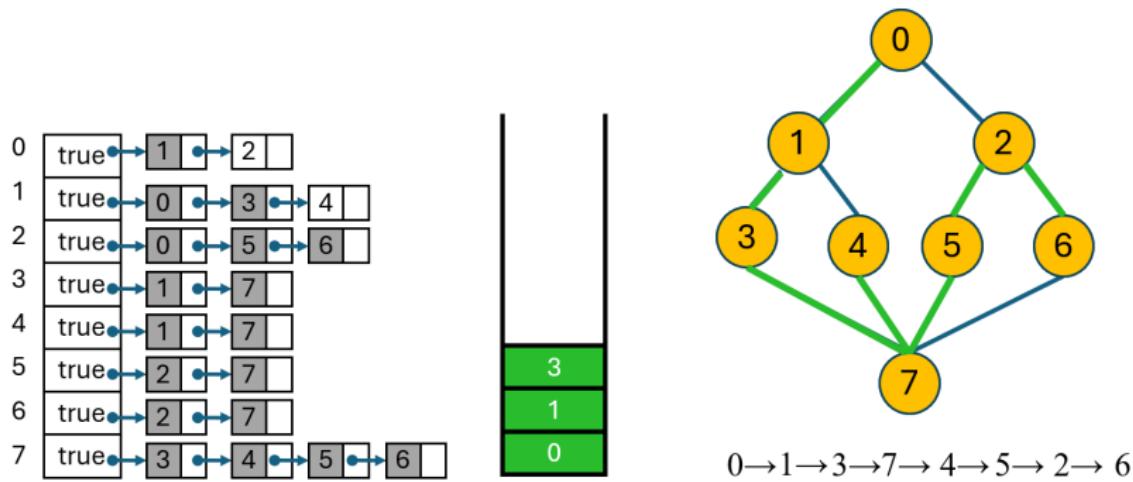
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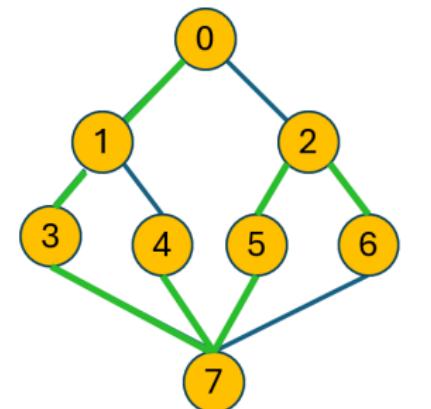
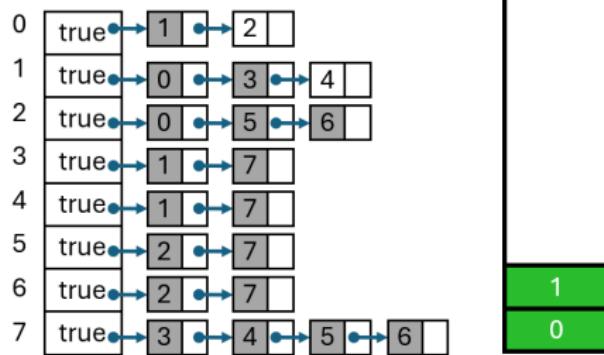


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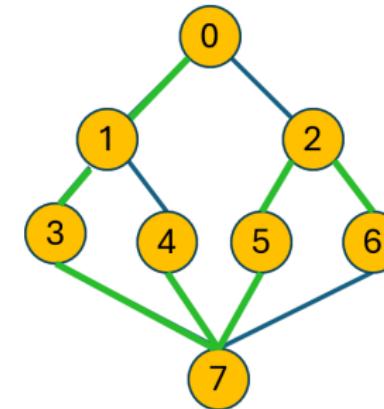
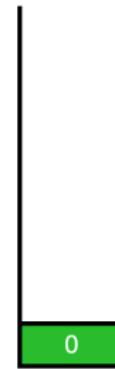
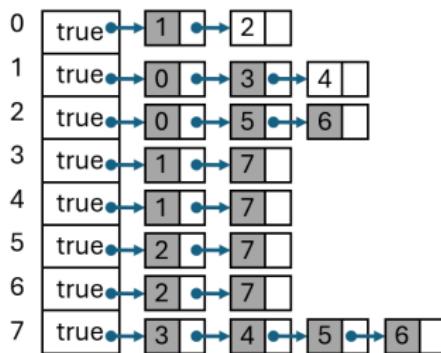


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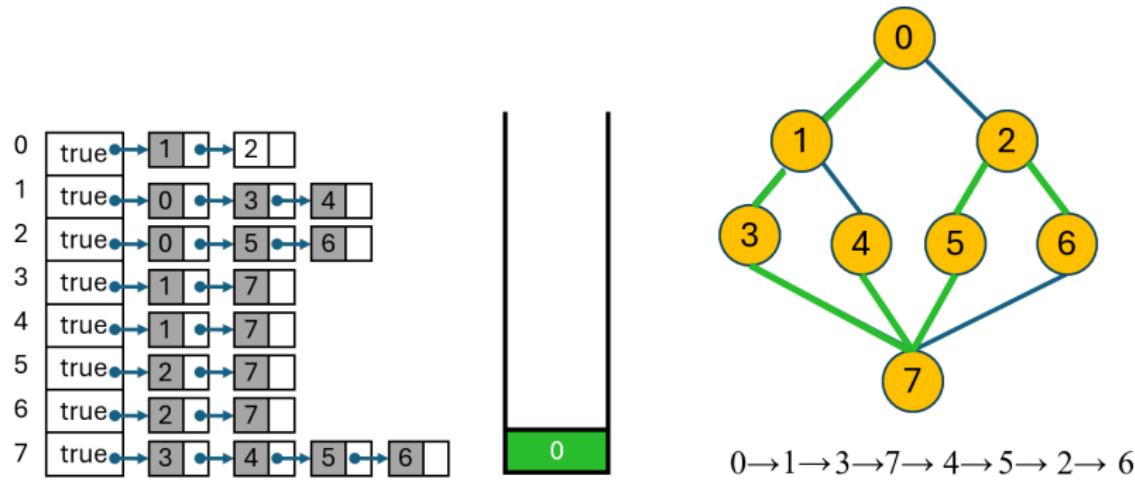
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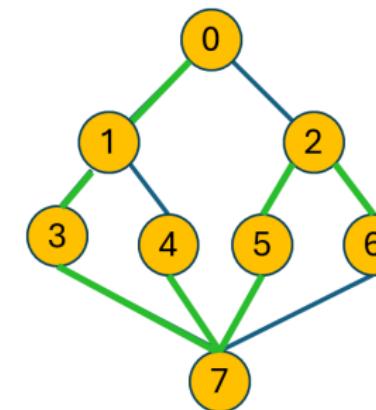
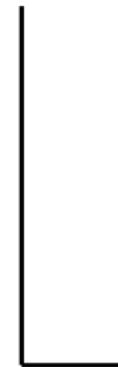
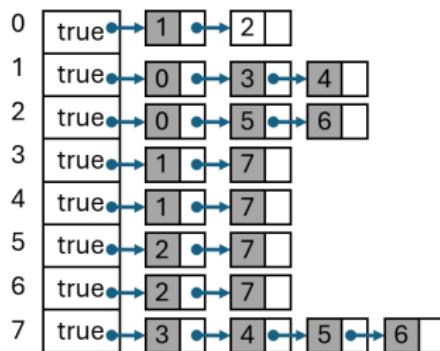


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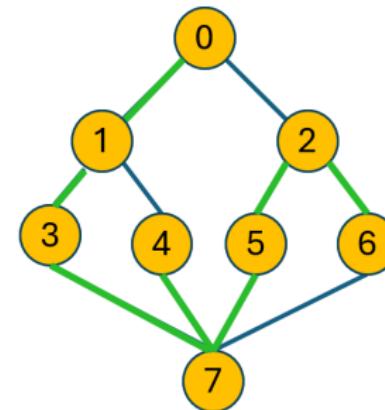
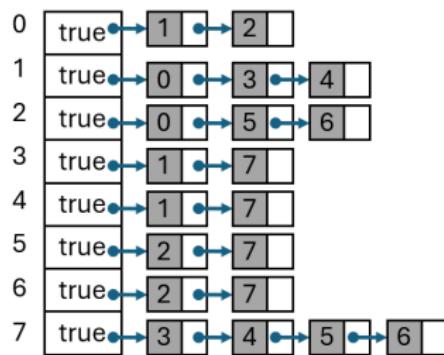
# Example (Illustration)



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# Example (Illustration)



0→1→3→7→4→5→2→6

# The Pseudocode of DFS

```
DFS(G, u) {
    u.visited = True
    for each v in G.Adj[u]
        if v.visited == False
            DFS(G, v)
}

driving main () {
    for each u in G
        u.visited = false
    for each u in G
        DFS(G, u)
}
```

# DFS in C

```
#define FALSE 0
#define TRUE 1
short int visited[MAX_VERTICES];
/* initializing to be FALSE for all */

void DFS(int v)  {
/* DFS beginning at vertex v */
    nodePointer w;
    visited[v] = true;
    printf("%5d",v);
    for(w = graph[v]; w; w = w->link)
        if (!visited[w->vertex])
            DFS(w->vertex);
}
```

# Analysis of DFS

- For  $G = (V, E)$  represented by an **adjacency list**, vertices adjacent to  $v$  can be determined in  $|N(v)|$ , where  $N(v)$  denotes the set of vertices adjacent to  $v$  in  $G$ .

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    - One needs to scan the corresponding row of the adjacency matrix.
  - Total time:  $O(n^2)$ .



# Outline

1 Introduction

2 Depth First Search (DFS)

3 Breadth-First Search (BFS)

# Breadth First Search (BFS) (1/2)

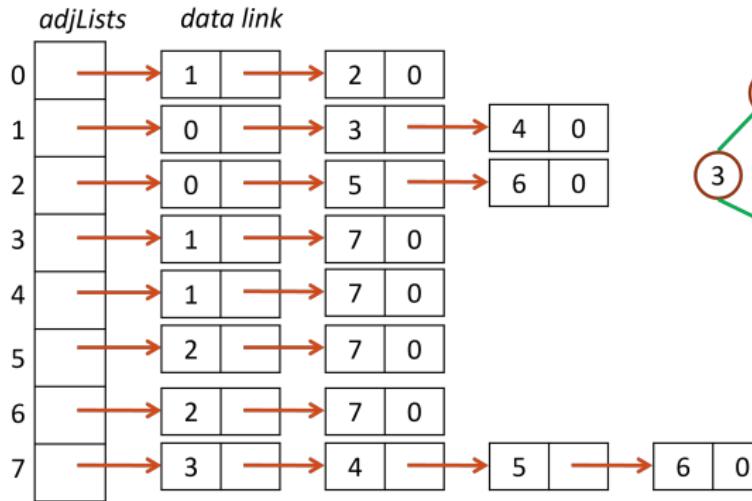
- The algorithm starts at vertex  $v$  and marks it as visited.
- Then visiting each of the vertices on  $v$ 's adjacency list.
- When we have visited all the vertices on  $v$ 's adjacency list, we visit all the unvisited vertices that are adjacent to the first vertex on  $v$ 's adjacency list.
- To implement this scheme, as we visit each vertex we place the vertex in a **queue**.

## Breadth-First Search (BFS) (2/2)

- When we have exhausted an adjacency list, we remove a vertex from the queue and proceed by examining each of the vertices on its adjacency list.
- Unvisited vertices are visited and placed on the queue; visited are ignored.
- Finish the search when the queue is empty.

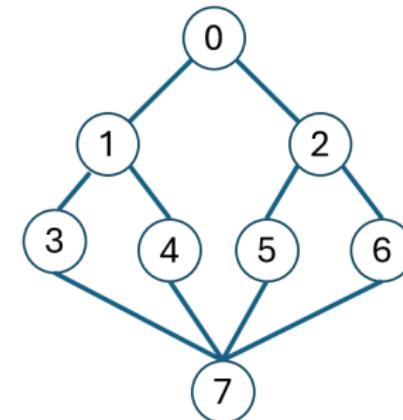
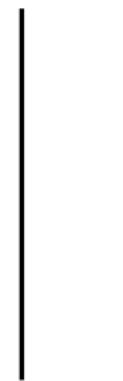
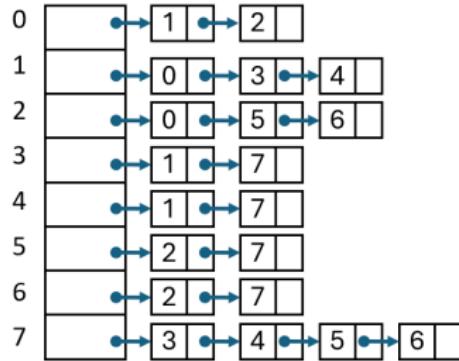
# BFS Example

- Using a queue.
- It resembles the level-order tree traversal.

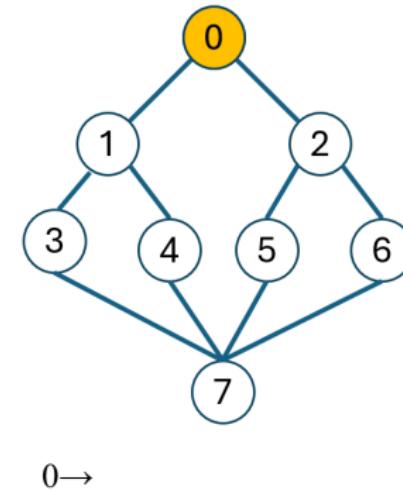
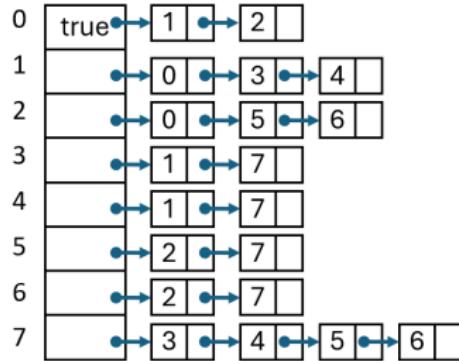


- The DFS order:  $v_0 \rightarrow v_1 \rightarrow v_2 \rightarrow v_3 \rightarrow v_4 \rightarrow v_5 \rightarrow v_6 \rightarrow v_7$ .

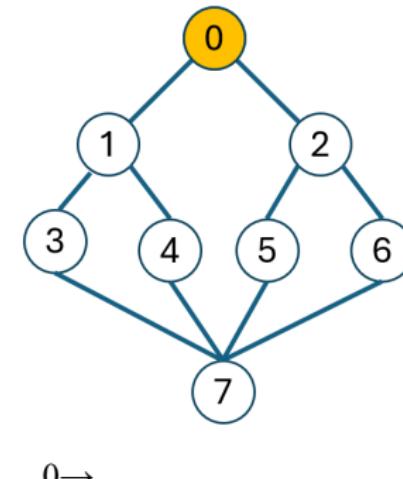
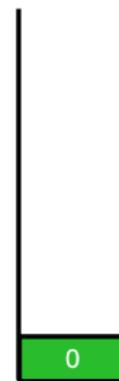
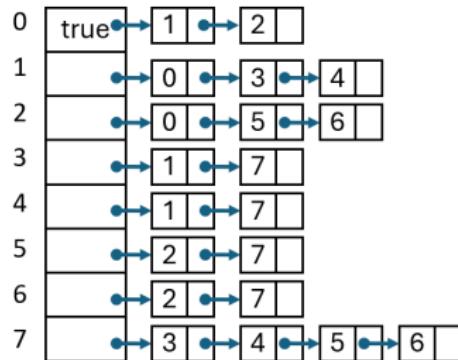
# Example (Illustration)



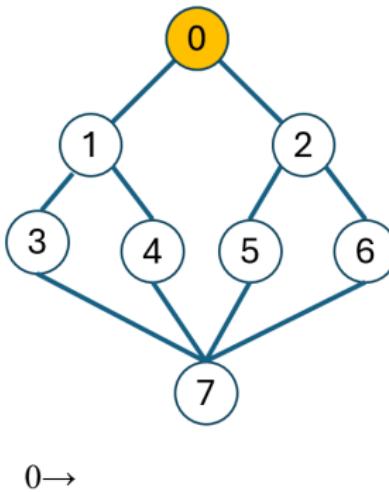
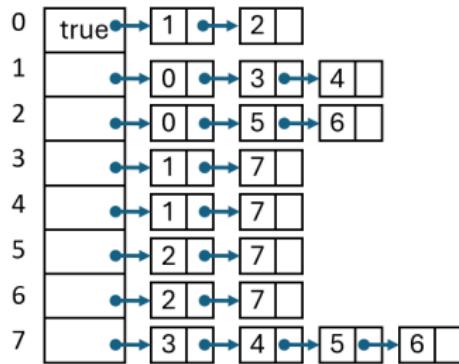
# Example (Illustration)



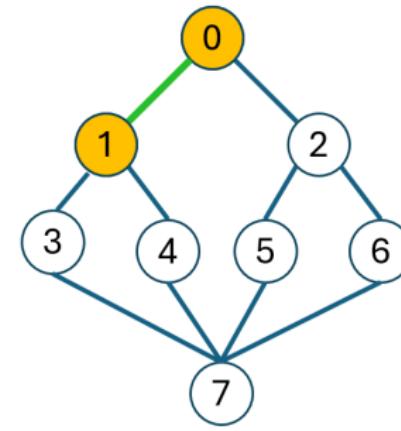
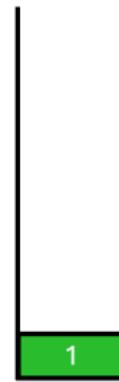
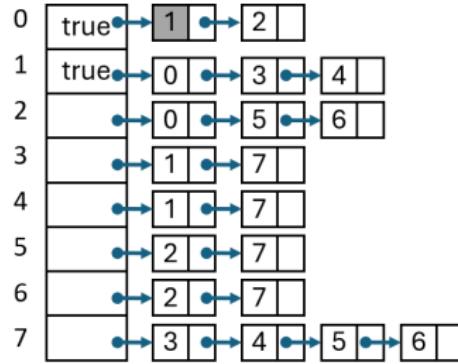
# Example (Illustration)



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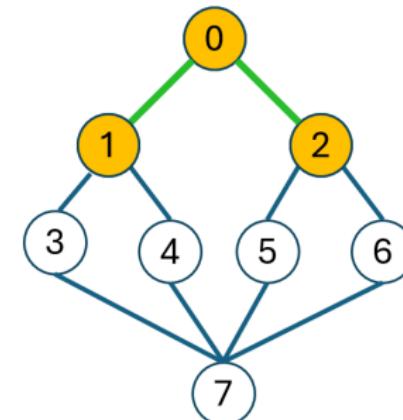
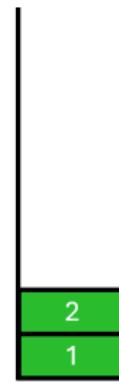
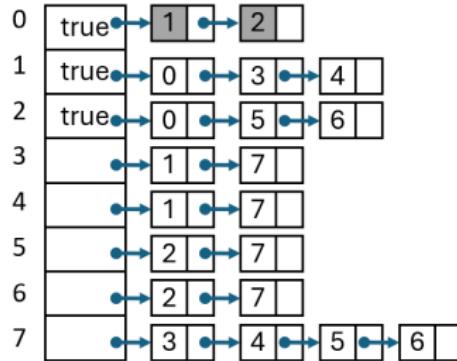


# Example (Illustration)

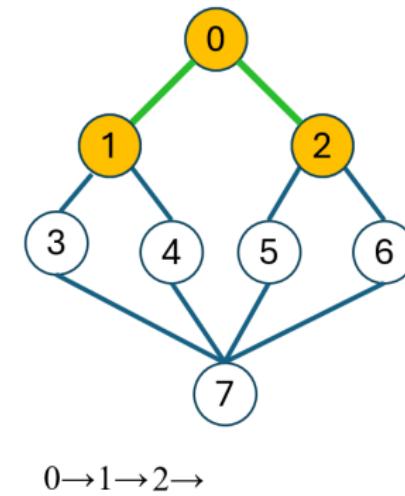
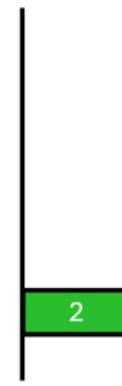
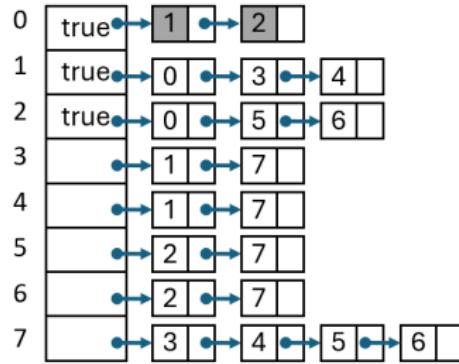


0→1→

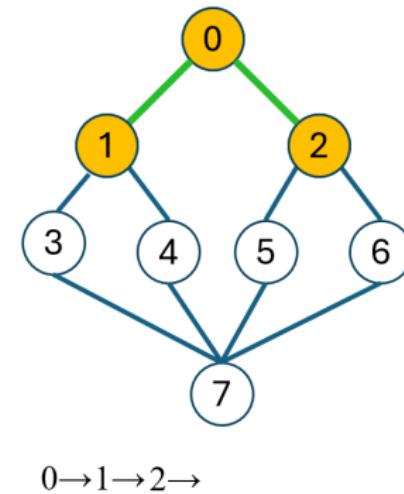
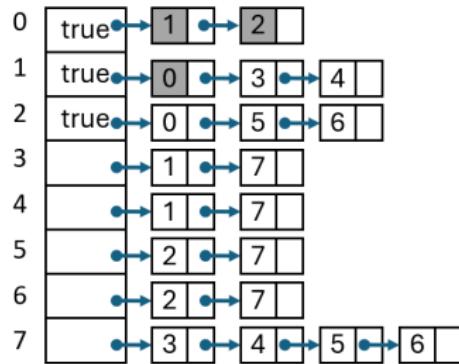
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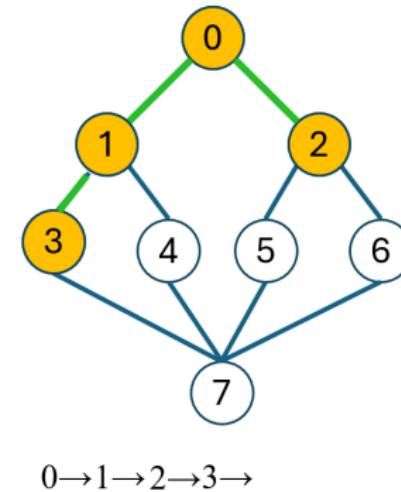
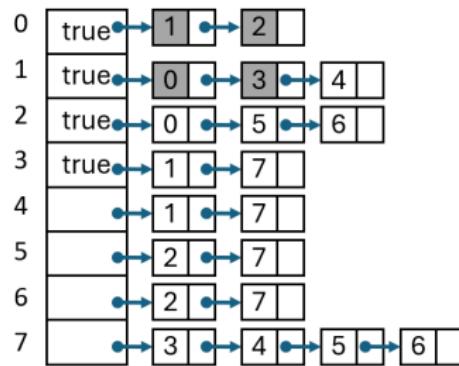
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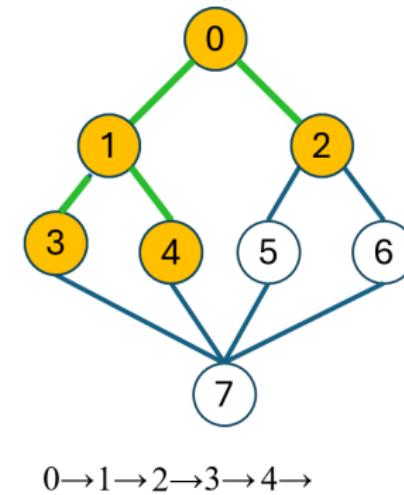
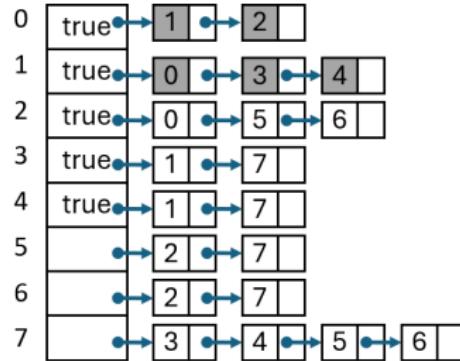
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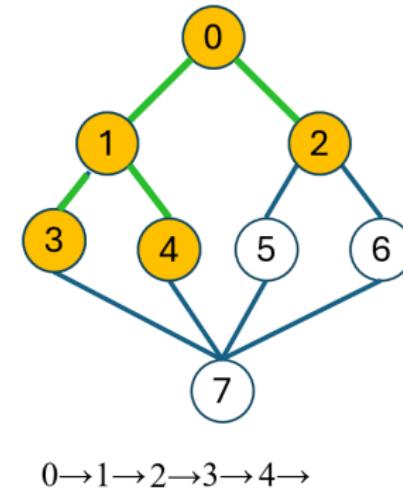
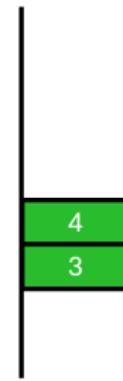
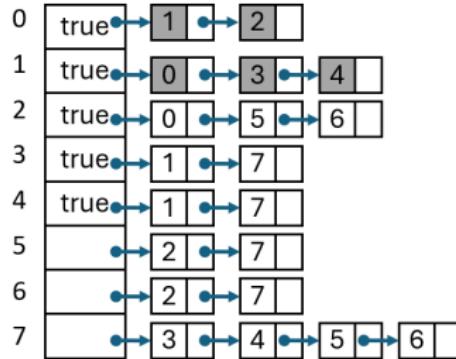
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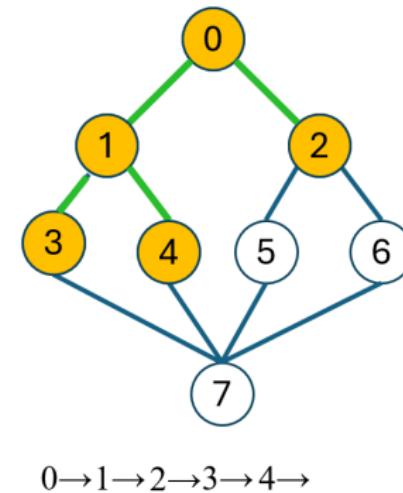
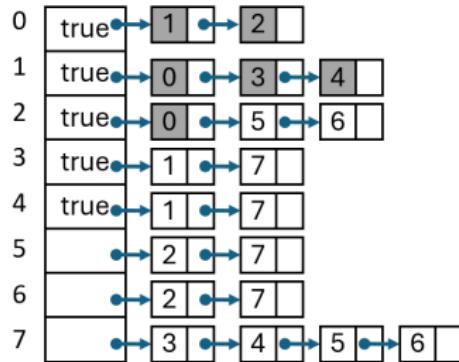
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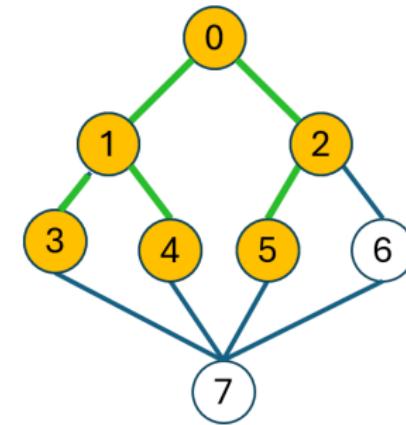
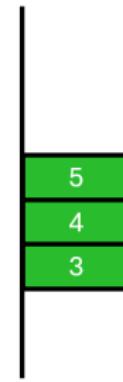
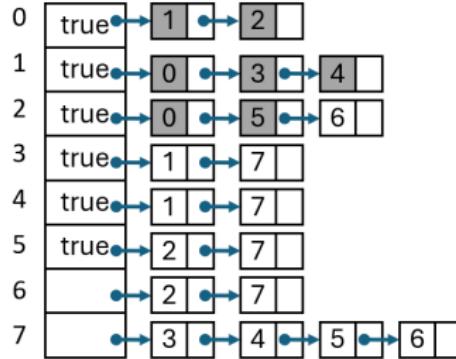
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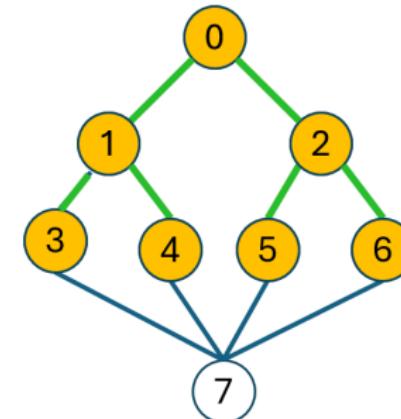
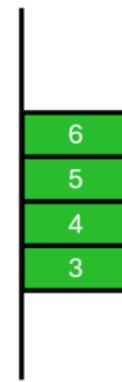
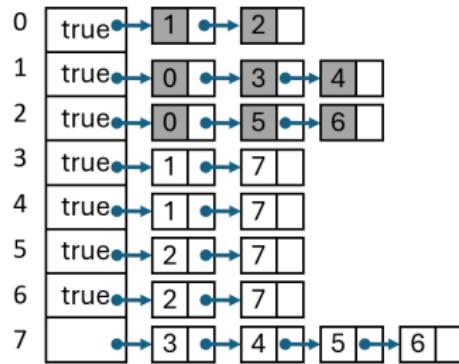
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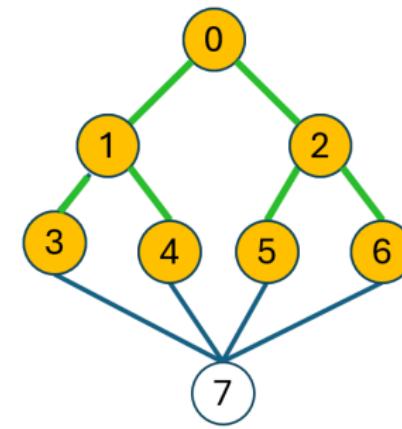
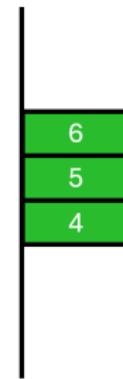
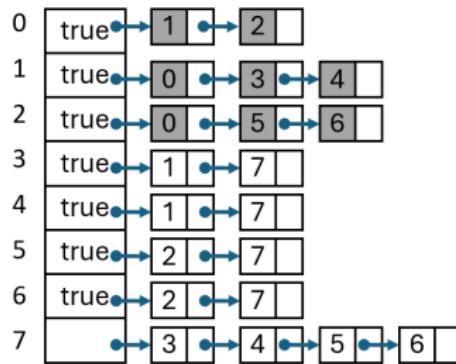


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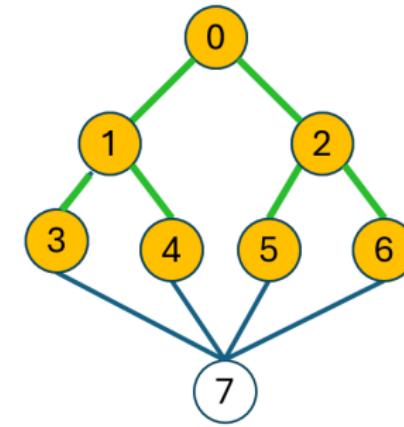
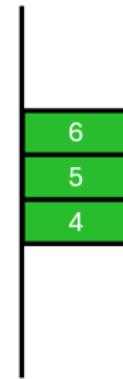
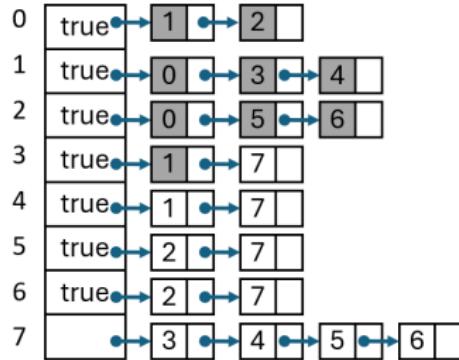
0 → 1 → 2 → 3 → 4 → 5 → 6 →

# Example (Illustration)



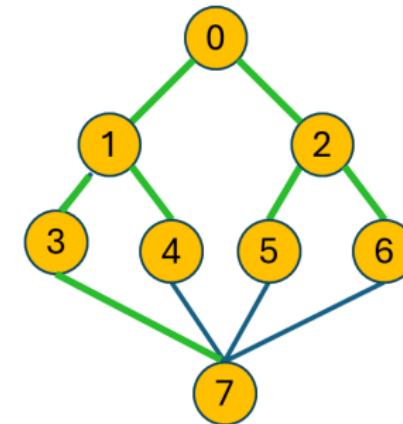
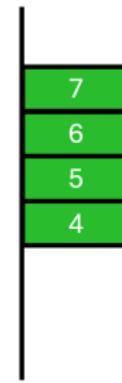
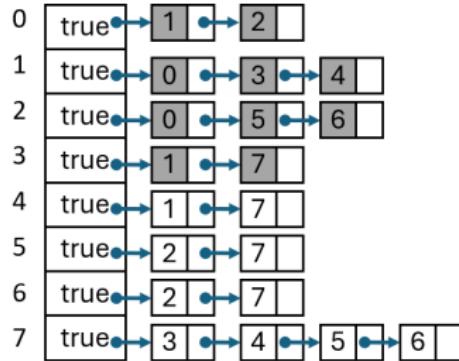
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# Example (Illustration)



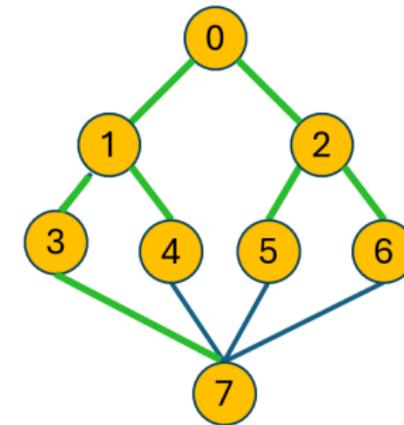
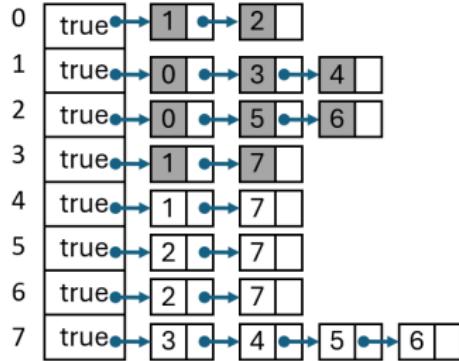
0→1→2→3→4→5→6→

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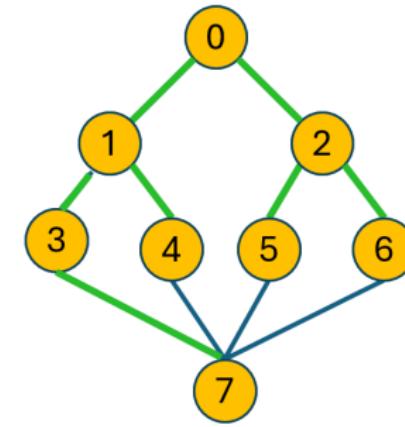
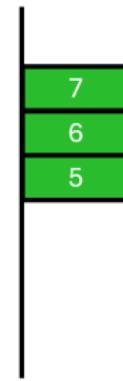
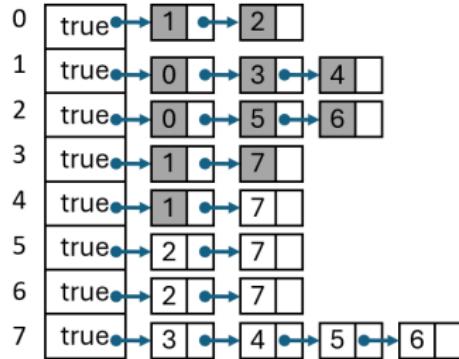
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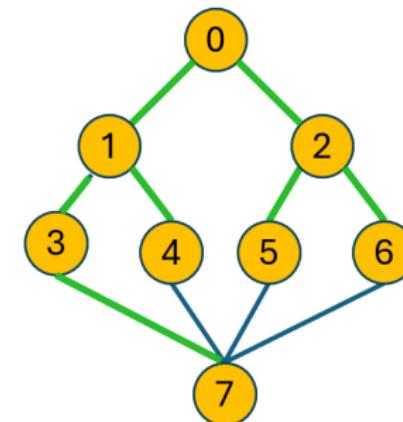
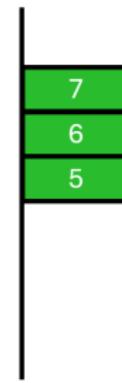
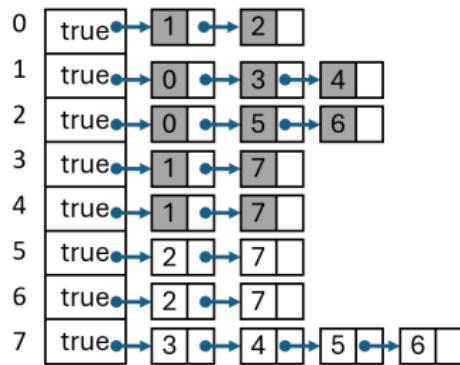
0 → 1 → 2 → 3 → 4 → 5 → 6 → 7

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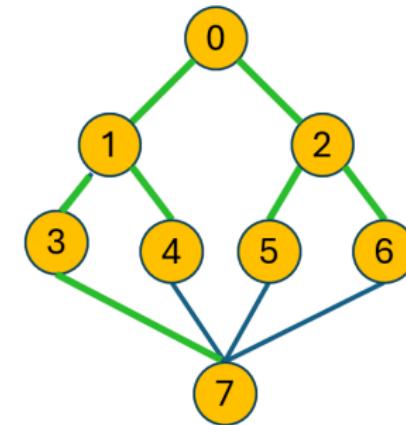
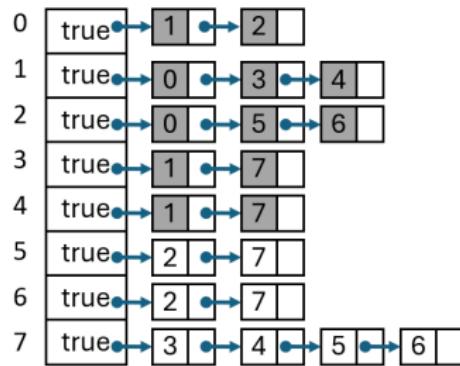
0 → 1 → 2 → 3 → 4 → 5 → 6 → 7

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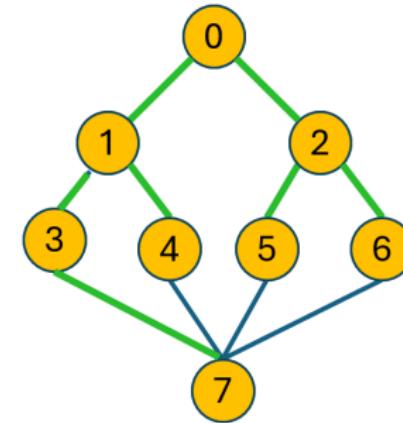
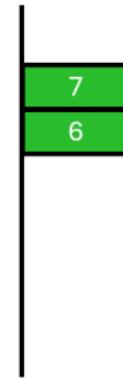
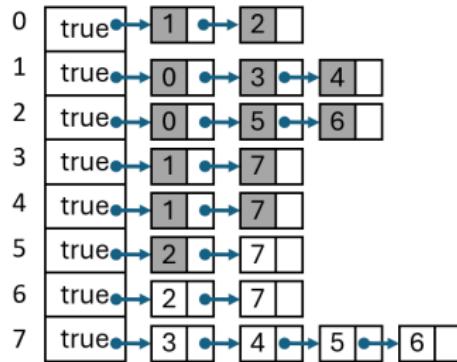
0 → 1 → 2 → 3 → 4 → 5 → 6 → 7

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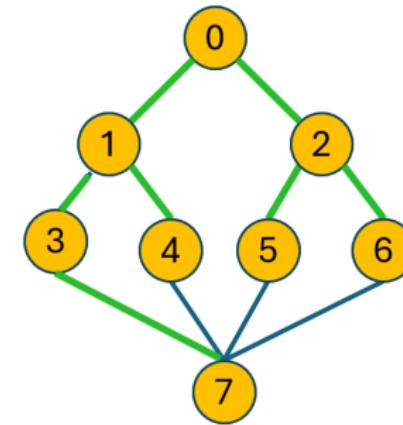
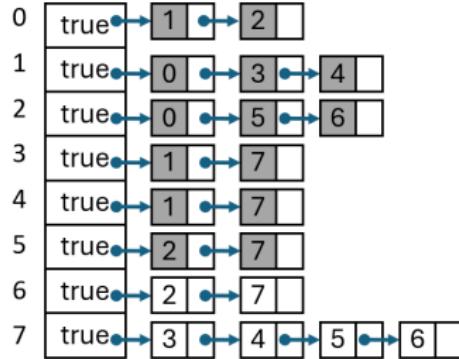


0 → 1 → 2 → 3 → 4 → 5 → 6 → 7

# Example (Illustration)

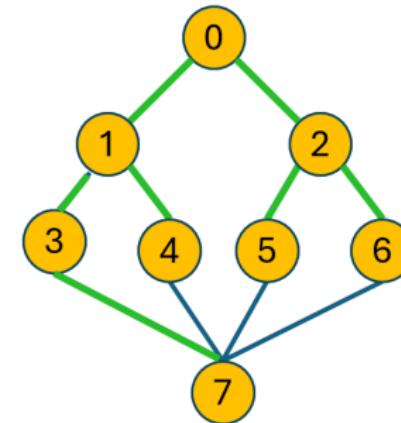
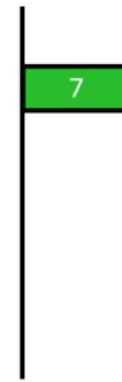
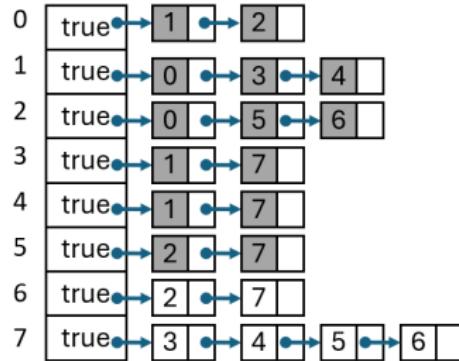


# Example (Illustration)



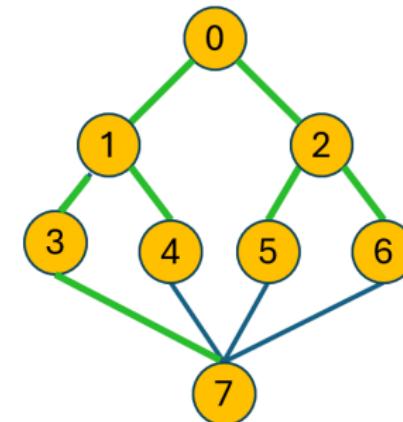
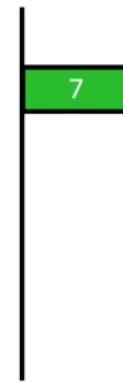
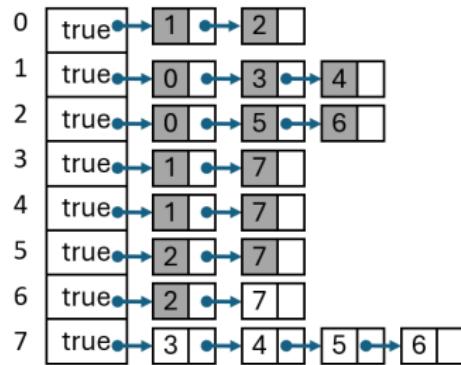
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# Example (Illustration)

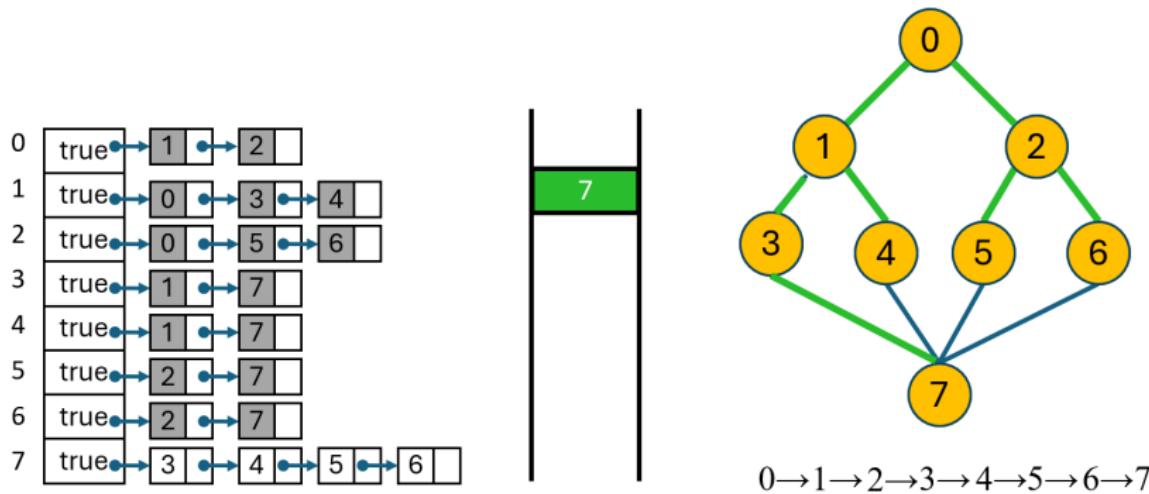


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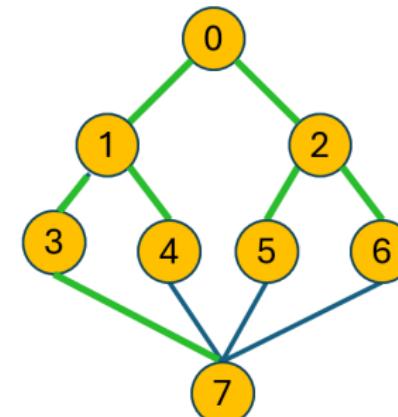
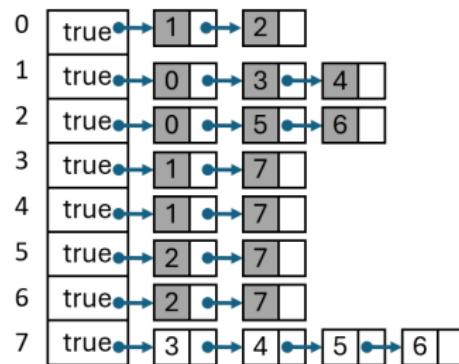
# Example (Illustration)



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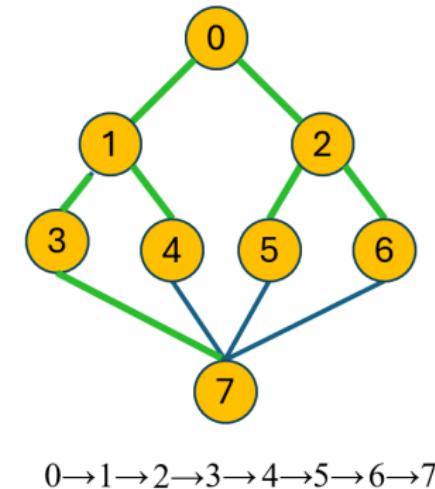
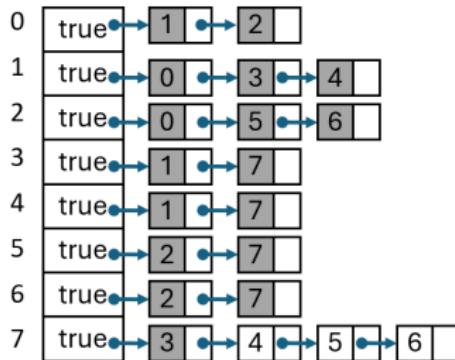


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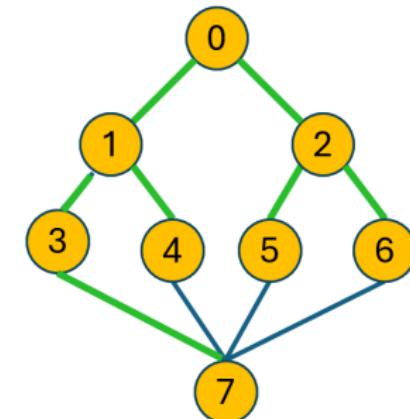
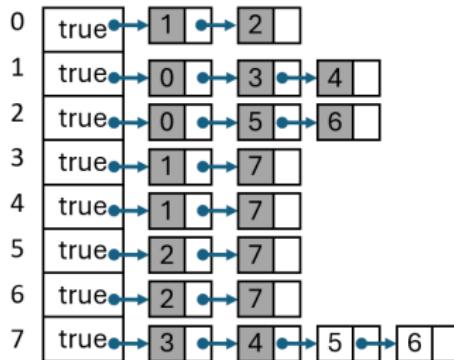


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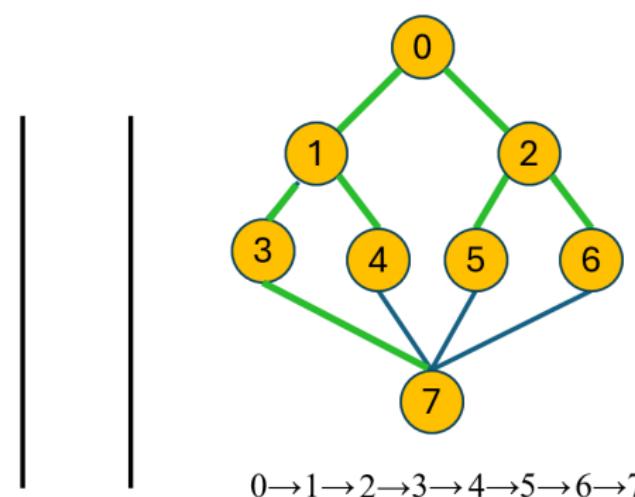
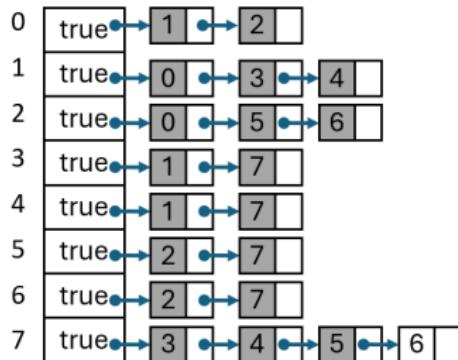


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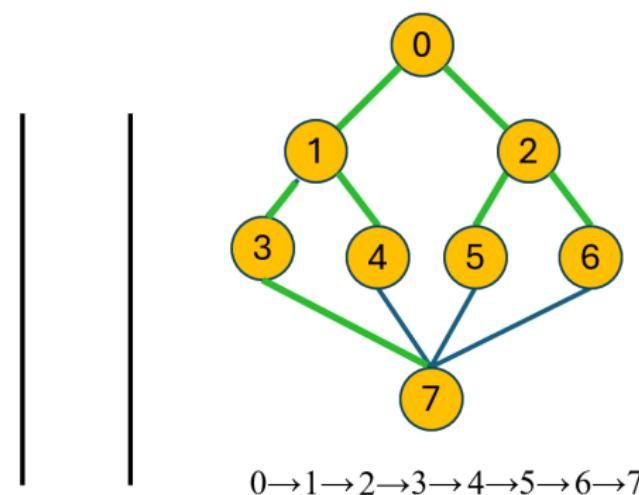
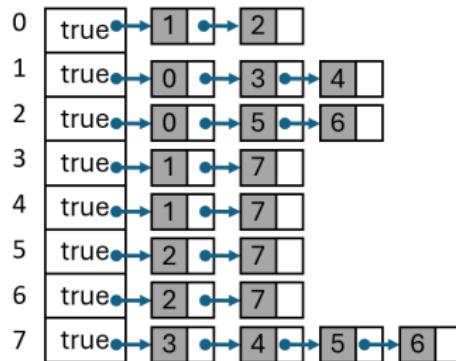


$0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6 \rightarrow 7$

# Example (Illustration)



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# The Pseudocode of BFS

```
BFS(G, u) { // let Q be the queue
    Q.enqueue(u)
    u.visited = True
    while (Q.empty() == False) { // when Q is not empty
        v = dequeue(Q)
        for all w in N(v) {
            if (w.visited == False) {
                Q.enqueue(w)
                w.visited = True
            }
        }
    }
}
driving main () {
    for each u in G
        u.visited = false
    BFS(G, u)
}
```



# BFS in C

```
void bfs(int v) {
    nodePointer w;
    front = rear = NULL; /* initialize queue */
    printf("%5d",v);
    visited[v] = TRUE;
    addq(v);

    while (front) {
        v = dequeue();
        for (w = graph[v]; w ; w->link)
            if (!visited[w->vertex]) {
                printf("%5d", w->vertex);
                enqueue(w->vertex);
                visited[w->vertex] = TRUE;
            }
    }
}
```



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- For the **adjacency matrix** representation, the while loop takes  $O(n)$  time for each vertex visited.
  - Therefore, the total time is  $O(n^2)$ .

As was true of DFS, all vertices visited, together with all edges incident to them, form a **connected component** of  $G$ .

# Discussions