

Linked List

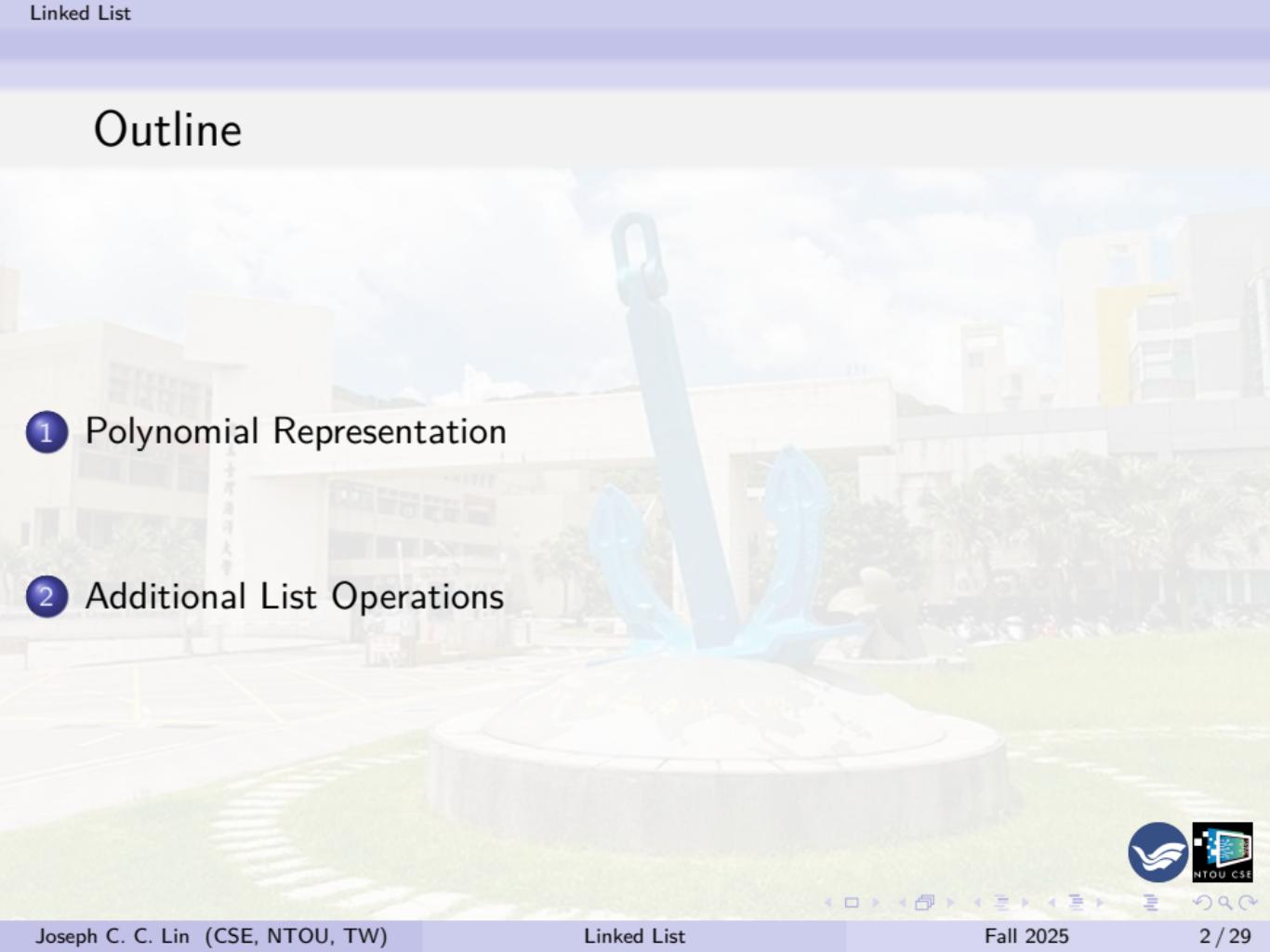
Polynomials

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Outline

- 
- 1 Polynomial Representation
 - 2 Additional List Operations

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1 Polynomial Representation

2 Additional List Operations

Goal

Represent the polynomial:

$$a_{m-1}x^{e_{m-1}} + a_{m-2}x^{e_{m-2}} + \cdots + a_0x^{e_0}.$$



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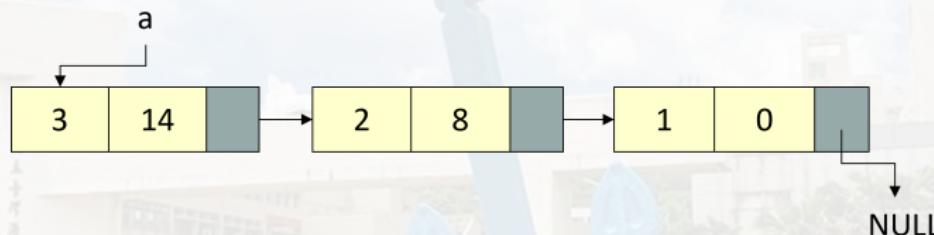
- **Idea:** Represent each term as a **node** containing
 - coefficient field
 - exponent field
 - pointer to the next term

Declaration

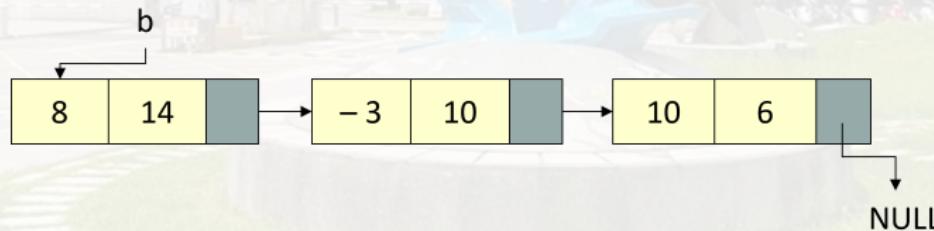
```
typedef struct polyNode *polyPointer;
struct polyNode {
    int coef;
    int expon;
    polyPointer link;
};
polyPointer a, b;
```

Examples

- $a = 3x^{14} + 2x^8 + 1$



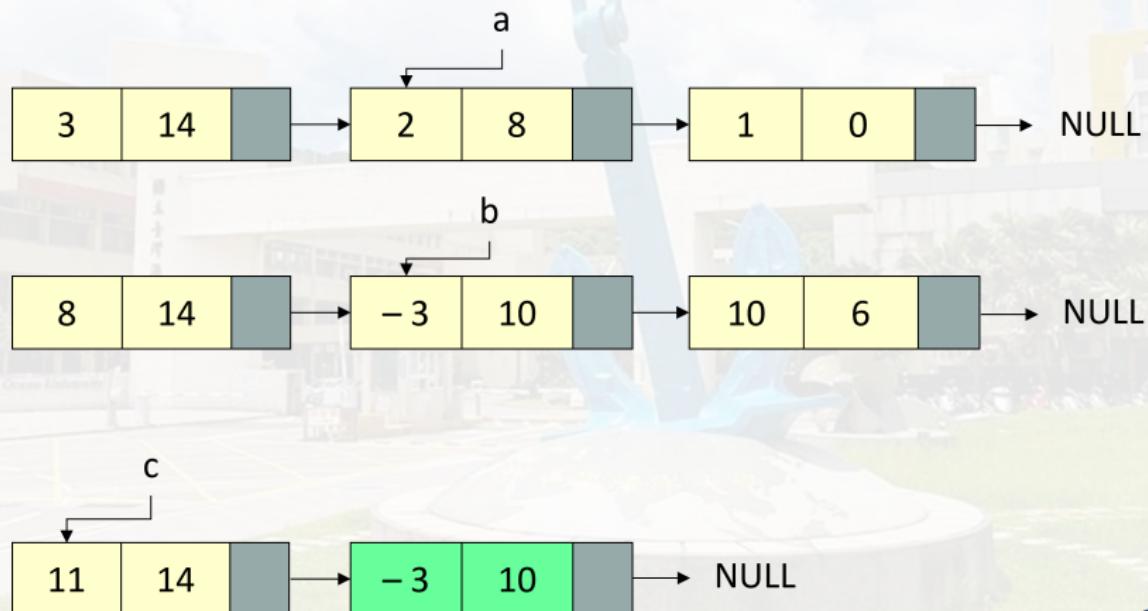
- $b = 8x^{14} - 3x^{10} + 10x^6$



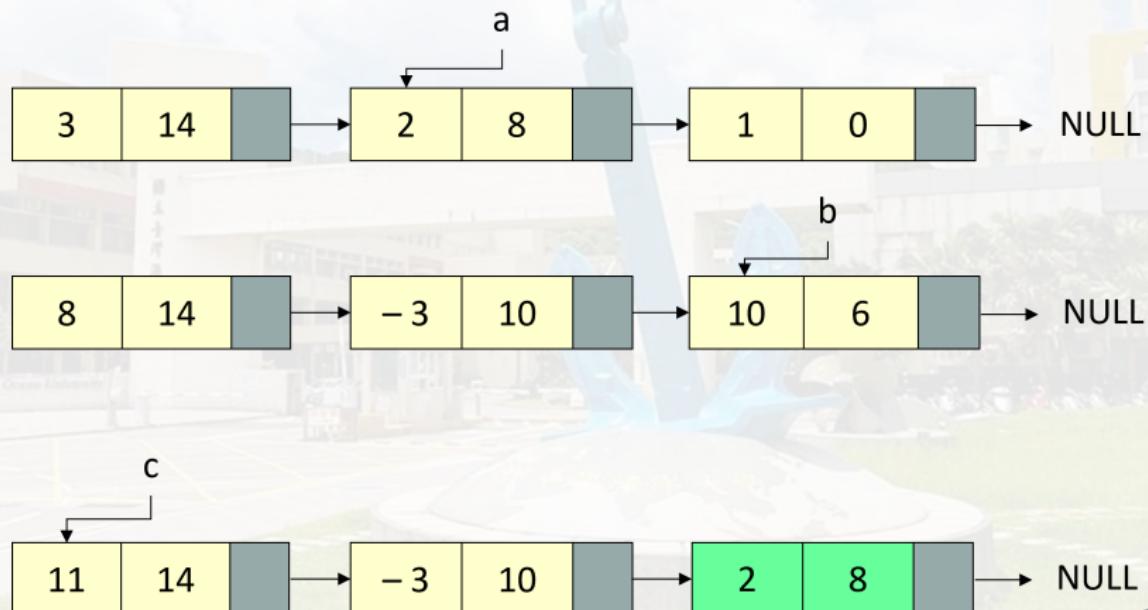
Generating the first three terms of $c = a + b$



Generating the first three terms of $c = a + b$



Generating the first three terms of $c = a + b$



Addition of Two Polynomials

```

polyPointer polyAdd(polyPointer a, polyPointer b) { /* return a polynomial which is the sum of a and b */
    polyPointer front, rear, temp;
    int sum; rear = (polyPointer)malloc(sizeof(*rear));
    if (IS_FULL(rear)) { printf("The memory is full\n"); exit(1) }
    front = rear;
    while (a && b) {
        switch (COMPARE(a->expon, b->expon)) {
            case -1: /* a->expon < b->expon */
                attach(b->coef, b->expon, &rear);
                b = b->link;
                break;
            case 0: /* a->expon = b->expon */
                sum = a->coef + b->coef;
                if (sum)
                    attach(sum, a->expon, &rear);
                a = a->link; b = b->link; break;
            case 1: /* a->expon > b->expon */
                attach(a->coef, a->expon, &rear);
                a = a->link;
        }
        /* copy rest of list a and then list b*/
        for (; a; a = a->link) attach(a->coef, a->expon, &rear);
        for (; b; b = b->link) attach(b->coef, b->expon, &rear);
        rear->link = NULL;
        /* delete extra initial node */
        temp = front; front = front->link; free(temp);
        return front;
    }
}

```

Attach a new node to the end of a list

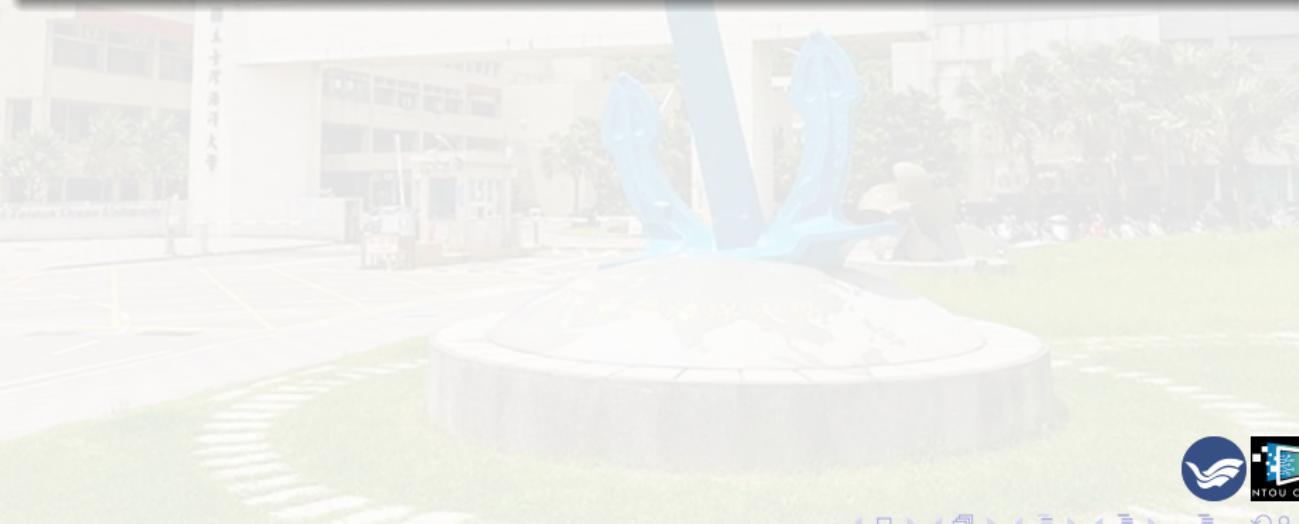
```
void attach(float coefficient, int exponent, polyPointer *ptr) {  
    /* create a new node with coef = coefficient and expon = exponent,  
       attach it to the node pointed by ptr and update ptr to point  
       to this new node */  
    polyPointer temp;  
    temp = (polyPointer)malloc(sizeof(*temp));  
    if (IS_FULL(temp)) {  
        printf("The memory is full\n");  
        exit(1);  
    }  
    temp->coef = coefficient;  
    temp->expon = exponent;  
    (*ptr)->link = temp;  
    *ptr = temp;  
}
```

Analysis of polyAdd

Consider

$$A(x) = a_{m-1}x^{e_{m-1}} + a_{m-2}x^{e_{m-2}} + \cdots + a_0x^{e_0},$$

$$B(x) = b_{n-1}x^{f_{n-1}} + b_{n-2}x^{f_{n-2}} + \cdots + b_0x^{f_0}$$



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- coefficient additions:

$$0 \leq \#\text{coefficient additions} \leq \min\{m, n\}$$

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- exponent comparisons:

- In each iteration, either pointer a or b or both move to the next term(s).
- The maximum number of exponent comparisons is $m + n$

- create new nodes for C :



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- The maximum number of terms in C is



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- create new nodes for C :

- The maximum number of terms in C is $O(m + n)$.



Erasing Polynomials

- One by one, free the nodes pointed by ptr.

```
void erase(polyPointer *ptr) {
    /* erase the polynomial pointed to by ptr */
    polyPointer temp;
    while (*ptr) {
        temp = *ptr;
        *ptr = (*ptr)->link;
        free(temp);
    }
}
```

A Question

Can we free all the nodes of a polynomial more efficiently?



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Can we free all the nodes of a polynomial more efficiently?

- Consider the **circular list representation**.

Circular List Representation

Circular List

A singly linked list in which the link field of the last node points to the first node is called a circular list.

$$3x^{14} + 2x^8 + 1:$$



- In order to obtain an efficient erase algorithm, we maintain a list of nodes that have been freed.

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A singly linked list in which the link field of the last node points to the first node is called a circular list.

$$3x^{14} + 2x^8 + 1:$$



- In order to obtain an efficient erase algorithm, we maintain a list of nodes that have been **freed**.
- Only when the list is empty do we need to use `malloc` to create a new node.

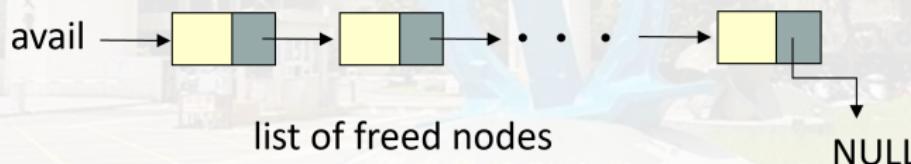
Functions to deal with freed nodes of polynomials

- Let `avail` be a variable of type `polyPointer` which points to the `first` node in the list of freed nodes.



Functions to deal with freed nodes of polynomials

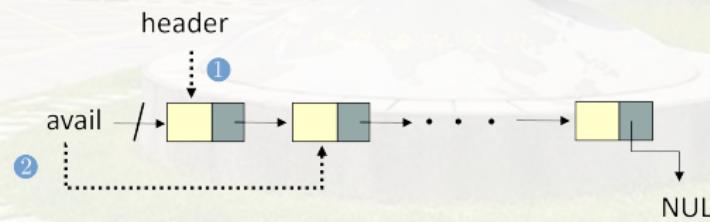
- Let `avail` be a variable of type `polyPointer` which points to the `first` node in the list of freed nodes.
- Instead of using `malloc()` and `free()`, we now use `getNode()` and `retNode()`.



getNode()

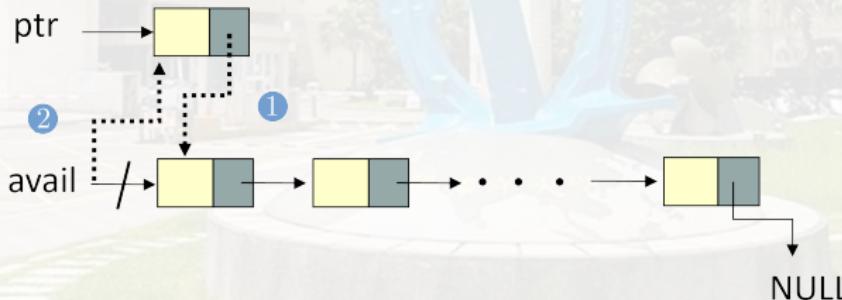
- To avoid the case of zero polynomials, we introduce a header into each polynomial.

```
polyPointer getNode(void) {
    /* provide a node header for use */
    polyPointer header;
    if (avail) {
        header = avail;
        avail = avail->link;
    } else
        header = malloc(sizeof(*header));
    return header;
}
```



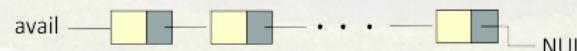
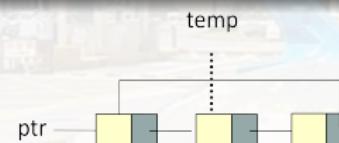
retNode()

```
void retNode(polyPointer ptr) {  
    /* return a node to the available list */  
    ptr->link = avail;  
    avail = ptr;  
}
```



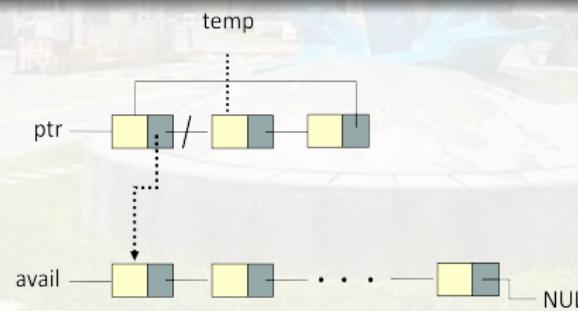
Erasing a circular list (independent of the number of nodes)

```
void cir_erase(polyPointer *ptr) {  
    // erase the circular list pointed by ptr  
    polyPointer temp;  
    if (*ptr) {  
        temp = (*ptr)->link;  
        (*ptr)->link = avail;  
        avail = temp;  
        *ptr = NULL;  
    }  
}
```



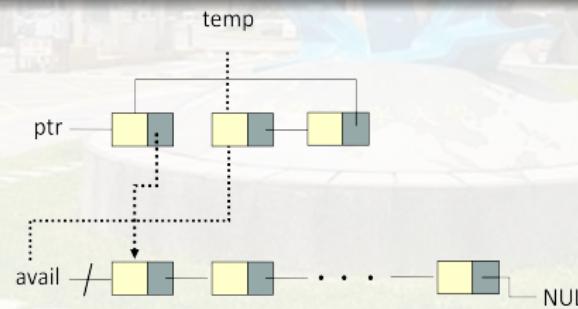
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        *ptr = NULL;  
    }  
}
```



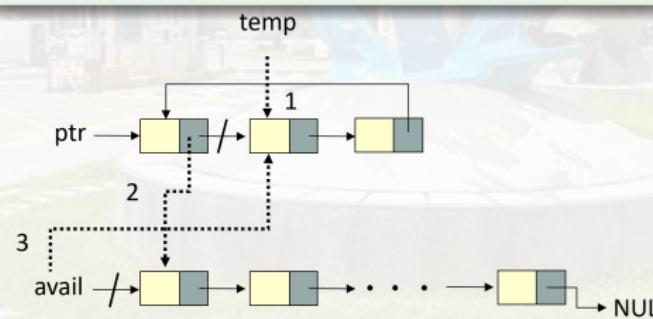
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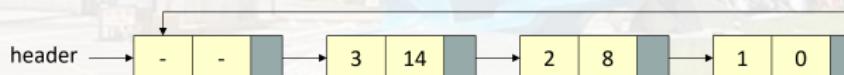
Polynomials with header nodes (Examples)

- Note: zero polynomial:

- Coefficients are zero
- No terms
- Undefined degree.



(a) Zero polynomial

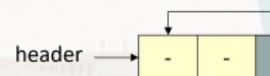


(b) $3x^{14} + 2x^8 + 1$

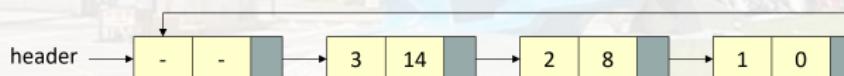
Polynomials with header nodes (Examples)

- Note: zero polynomial:

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(a) Zero polynomial



(b) $3x^{14} + 2x^8 + 1$

- We only keep nonzero terms.

Check if a polynomial is a zero polynomial

```
bool is_zero(const polyPointer header) {
    return header && header->next == header;
}
```

Outline

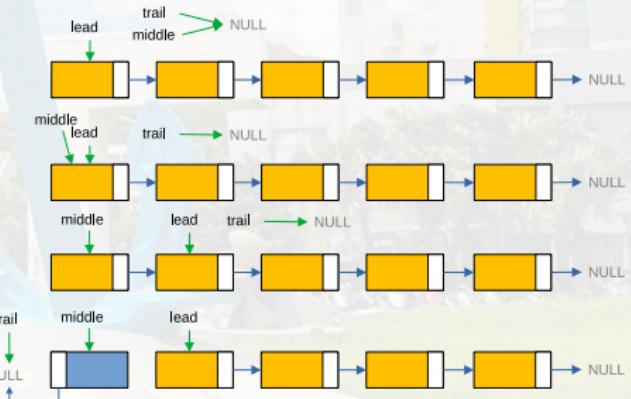
- 1 Polynomial Representation
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Additional List Operations

- a chain (singly linked list in which the **last** node has a **null link**):
 - inversion (反轉)
 - concatenation (連接) of two chains
- a circular list (singly linked list in which the **last** node **points to the first node**):
 - insert a node at the front of a circular list
 - determine the length of a circular list

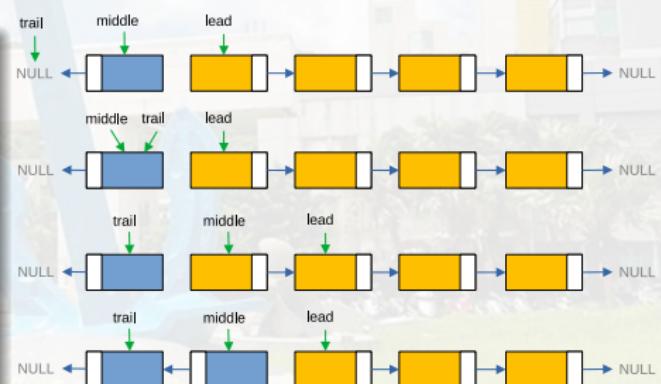
Inversion of a chain

```
listPointer invert (listPointer lead) {  
    // invert the list pointed to by lead  
    listPointer middle, trail;  
    middle = NULL;  
    while (lead) {  
        trail = middle;  
        middle = lead;  
        lead = lead->link;  
        middle->link = trail;  
    }  
    return middle;  
}
```



Inversion of a chain

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listPointer invert (listPointer lead) {  
    // invert the list pointed to by lead  
    listPointer middle, trail;  
    middle = NULL;  
    while (lead) {  
        trail = middle;  
        middle = lead;  
        lead = lead->link;  
        middle->link = trail;  
    }  
    return middle;  
}
```



- Quick notes (loop invariant):

- Initialization: “lead” at the first node; “middle” and “trail” at NULL.

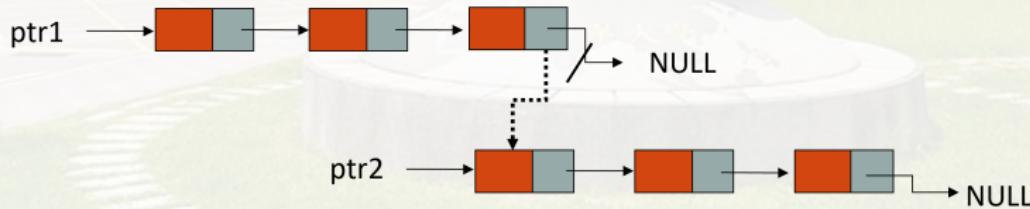
- **Maintenance:**

- Reverse “middle”'s next to be “trail”.
 - List starting from “lead” to the tail (i.e., NULL): the list that have not been inverted yet.

- Termination: “lead” at NULL.

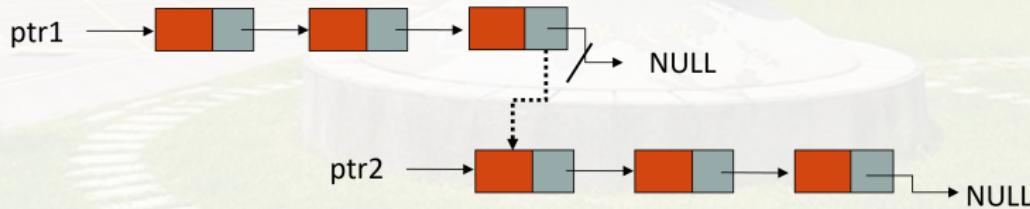
Concatenation of two singly linked lists

```
listPointer concatenate (listPointer ptr1, listPointer ptr2) {  
    /* produce a new list that contains the list  
     * ptr1 followed by the list ptr2. */  
    listPointer temp;  
    // check for empty lists  
    if (!ptr1) return ptr2;  
    if (!ptr2) return ptr1;  
    /* neither list is empty, find end of first list */  
    for (temp = ptr1; temp->link; temp = temp->link);  
    // linked end of first to start of second  
    temp->link = ptr2;  
}
```



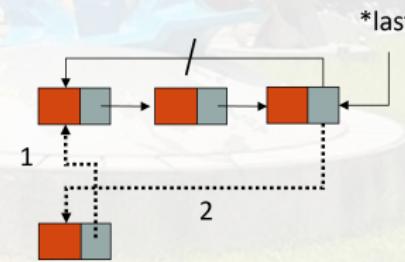
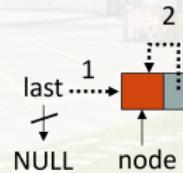
Concatenation of two singly linked lists (while-loop)

```
listPointer concatenate (listPointer ptr1, listPointer ptr2) {  
    /* produce a new list that contains the list  
     * ptr1 followed by the list ptr2. */  
    listPointer temp;  
    // check for empty lists  
    if (!ptr1) return ptr2;  
    if (!ptr2) return ptr1;  
    /* neither list is empty, find end of first list */  
    while (temp->link) temp = temp->link;  
    // linked end of first to start of second  
    temp->link = ptr2;  
}
```



Inserting at the front of a circular list

```
void insertFront (listPointer *last, listPointer node) {  
    /* insert node at the front of the circular list  
       whose last node is last */  
    if (!(*last)){ // list is empty, change last to point to new entry  
        *last = node;  
        node->link = node;  
    } else { // list is not empty, add a new entry at front  
        node->link = (*last)->link;  
        (*last)->link = node;  
    }  
}
```



Find the length of a circular list

```
int length (listPointer last) {
    /* find the length of circular list last */
    listPointer temp;
    int count = 0;
    if (last) {
        temp = last;
        do {
            count++;
            temp = temp->link;
        } while (temp != last);
    }
    return count;
}
```

Discussions

