Threaded Binary Tree & Heaps

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Outline

1. Threaded Binary Trees (引線二元樹)
2. Heaps

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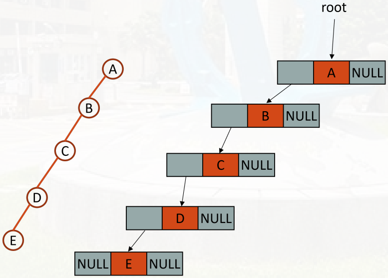
- Number of null non-null links: $n - 1$.
- Number of null links: $n + 1$.



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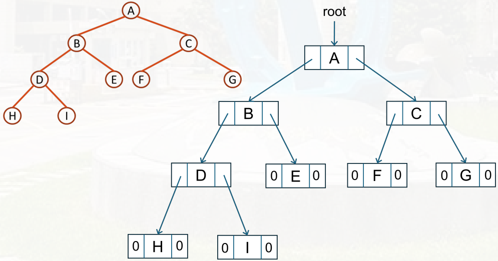
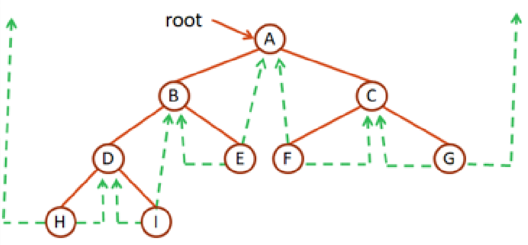


Solution

Replace the NULL links by pointers, threads, pointing to other nodes.



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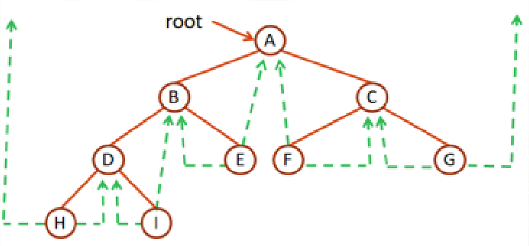


Threading Rules

- if `ptr->leftChild` is NULL, then `ptr->leftChild` = inorder predecessor (中序前行者) of `ptr`.
- if `ptr->rightChild` is NULL, then `ptr->rightChild` = inorder successor (中序後續者) of `ptr`.

To distinguish between normal pointers and threads

- Two additional fields of the node structure: left-thread, right-thread.

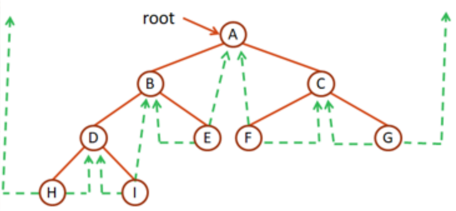
```
typedef struct threadedTree *threadedPointer;

typedef struct threadedTree {
    bool leftThread;
    threadedPointer leftChild;
    char data;
    threadedPointer rightChild;
    bool rightThread;
};
```

leftThread	leftChild	data	rightChild	rightThread

Rules of the Threading Fields

- If `ptr->leftThread == true`, `ptr->leftChild` contains a thread; Otherwise, the node contains a pointer to the left child.
- If `ptr->rightThread == true`, `ptr->righChild` contains a thread; Otherwise, the node contains a pointer to the right child.



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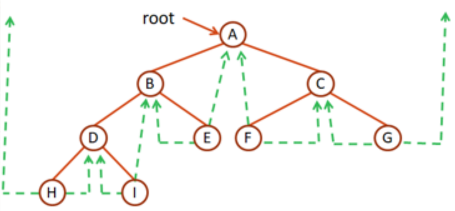
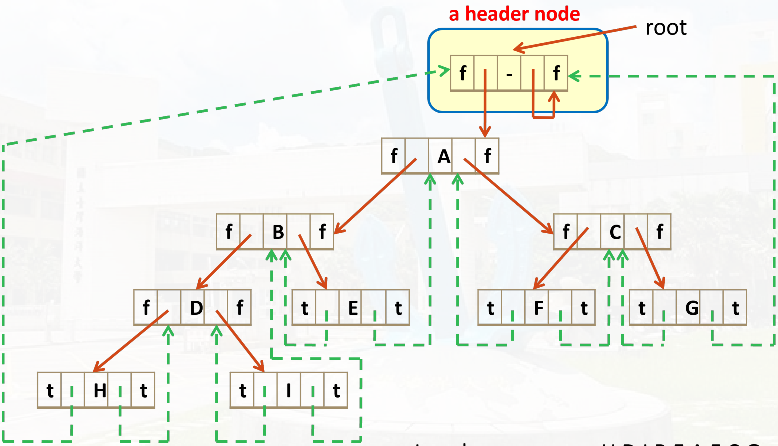


- Two dangling threads at node *H* and *G*.
  $\Rightarrow$ Use a header node to collect them!

- The original tree becomes the left subtree of the head node.



Inorder sequence: H D I B E A F C G

Representing an Empty Binary Tree

leftThread	leftChild	data	rightChild	rightThread
true	•	—	•	**false**

Finding the Inorder Successor of Node

```
threadedPointer insucc(threadedPointer tree) {
/* find the inorder sucessor of tree in a threaded
   binary tree */
    threadedPointer temp;
    temp = tree->rightChild;
    if (!tree->rightThread) // rightChild exists!
        while (!temp->leftThread)
            temp = temp->leftChild;
    return temp;
}
```

To perform an inorder traversal, we can simply make repeated calls to insucc!

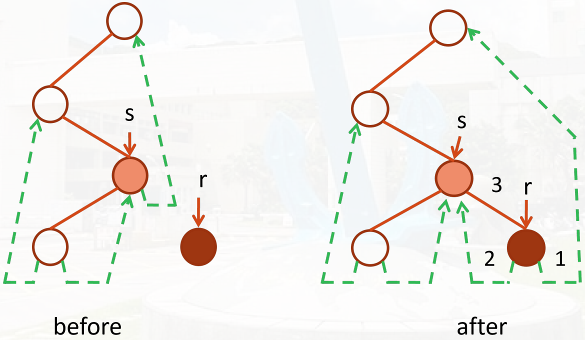
Inorder Traversal of a Threaded Binary Tree

```
void traverseInorder(threadedPointer tree) {
/* traverse the threaded binary tree inorder */
    threadedPointer temp = tree;
    while (1) {
        temp = insucc(temp);
        if (temp == tree)
            break;
        printf("%3c", temp->data);
    }
}
```

- **Note:** `temp == tree` happens when the last node is visited (then the successor becomes the header node).

Inserting *r* as the rightChild of a node *s*

- Case I: `s->rightThread == false`

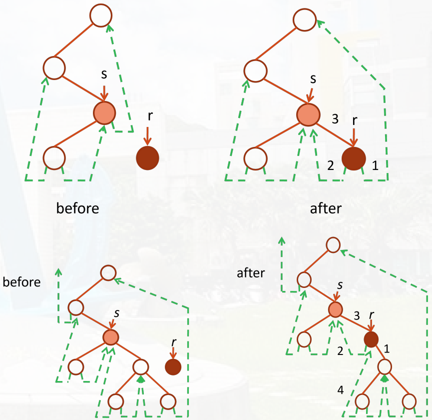


Inserting r as the rightChild of a node s

- Case II: `s->rightThread != false`

The Code for the Insertion

```
void insertRight (threadedPointer s,
                  threadedPointer r) {
/* insert r as the right child of s */
    threadedPointer temp;
    r->rightChild = s->rightChild;
    r->rightThread = s->rightThread;
    r->leftChild = s;
    r->leftThread = true;
    s->rightChild = r;
    s->rightThread = false;
    if (!r->rightThread){ // step 4
        temp = insucc(r);
        temp->leftChild = r;
    }
}
```



Outline

Heaps

Max Tree

A max tree is a tree in which

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A min tree is a tree in which

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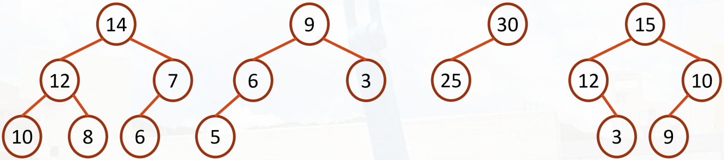
Max Heap

A complete binary tree that is also a max tree.

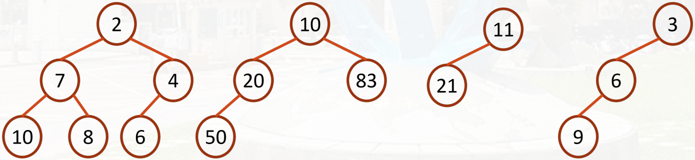
Min Heap

A complete binary tree that is also a min tree.

Examples: Max & Min Trees



Max Trees



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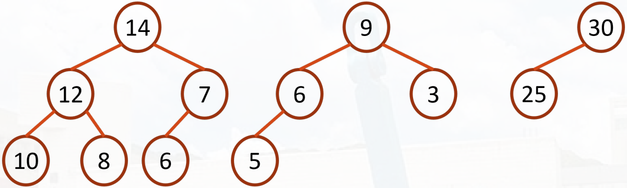
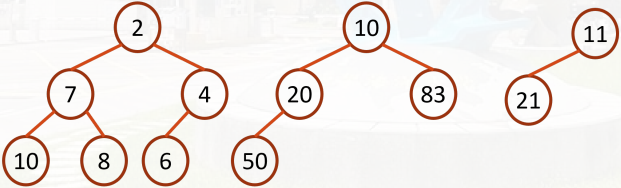


Max Heaps



Min Heaps

The Key Application: Priority Queues

- Heaps are frequently used to implement priority queues.
- In this kind of queue,
  - the element to be deleted is the one with highest (or lowest) priority.
  - at **any time**, an element with arbitrary priority can be **inserted** into the queue.

Insertion into a Max Heap

- The bubbling process.
  - It begins at the new node of the tree and moves toward the root.

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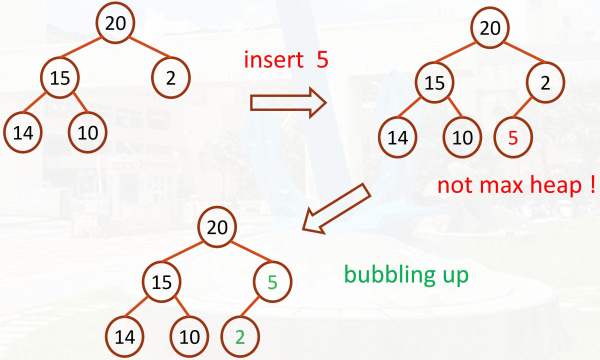

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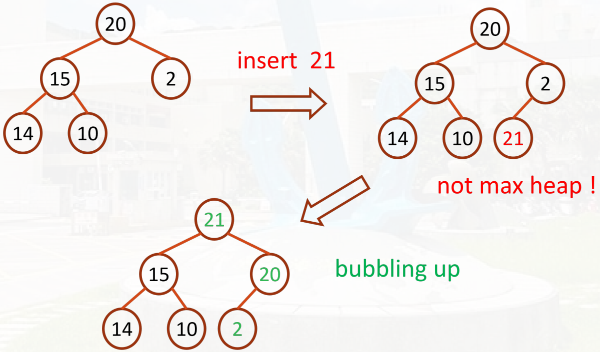

The Code for Insertion into a Max Heap

- Consider the following declarations:

```c
#define MAX_ELEMENTS 200  /* maximum heap size+1 */
#define HEAP_FULL (n) (n == MAX_ELEMENTS -1)
#define HEAP_EMPTY (n)  (!n)
typedef struct {
    int key;
    /* other fields */
} element;
element heap[MAX_ELEMENTS];
int n = 0;
```

The Code for Insertion into a Max Heap

```
void push (element item, int *n) {
/* insert item into a max heap of current size *n */
    int i;
    if (HEAP_FULL(*n)) {
        printf("The heap is full.\n");
        exit(EXIT_FAILURE);
    } // O(1) time
    i = ++(*n); // getting the position for the new item
    while ((i != 1) && (item.key > heap[i/2].key)) {
        heap[i] = heap[i/2];
        i /= 2;
    } //  O(lg n) time
    heap[i] = item; // O(1) time
}
```

- The time complexity of the insertion: $O(\lg n)$.

Deletion from a Max Heap

- When an element is to be deleted from a max heap, it is ALWAYS taken from the root of the heap.

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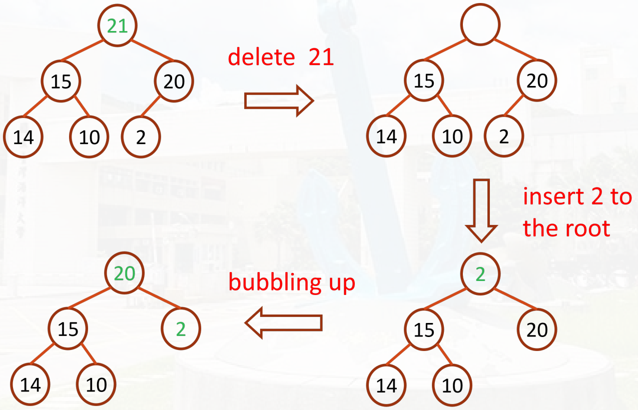
- When an element is to be deleted from a max heap, it is ALWAYS taken from the root of the heap.
- The steps of deletion from a Max heap:
  - delete the root node.
  - insert the last node into the root.
  - use the bubbling up process to ensure that the resulting heap remains a max heap.

Illustration of Deletion from a Max Heap



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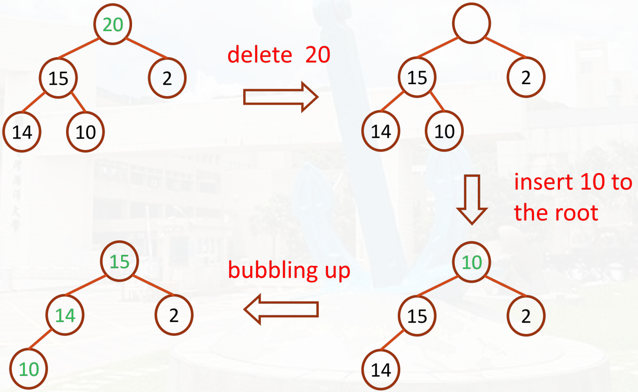

The Code for Deletion from a Max Heap

```
element pop(int *n) {
/* delete element with the highest key from the heap */
    int parent, child;
    element item, temp;
    if (HEAP_EMPTY(*n)) {
        fprintf(stderr, "The heap is empty\n");
        exit(EXIT_FAILURE);
    }
    /* save value of the element with the highest key */
    item = heap[1];
    /* use last element in heap to adjust the heap */
    temp = heap[(*n)--];
    parent = 1;
    child = 2;
    while (child <= *n) { // O(lg n) time
    /* find the larger child of the current parent */
        if ((child < *n) && (heap[child].key < heap[child+1].key))
            child++;
        if (temp.key >= heap[child].key) break;
        /* move to the next lower level */
        heap[parent] = heap[child];
        parent = child;
        child *= 2;
    }
    heap[parent] = temp;
    return item;
}
```

Time Complexity of the Deletion from a Max Heap

- Delete the root node: $O(1)$.
- Insert the last node to the root: $O(1)$.
- Since the height of the heap is $\lceil \lg(n+1)) \rceil$, the while loop is iterated for $O(\lg n)$ times.
- Thus, the overall time complexity: the time complexity of the deletion: $O(\log n)$.

Discussions