

Basic Concepts:

Performance Analysis & Measurement

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Fall 2024



Outline

- 1 Performance Analysis
- 2 Performance Measurement



Outline

1 Performance Analysis

2 Performance Measurement



Criteria for judging a program:

- Meet the original specification?
- Work correctly?
- The documentation.
- Does the program effectively use functions to create logic units?
- Code readability.
- Efficient usage of storage?
- Acceptable running time?



Performance Analysis

machine **independent**

- Space complexity
 - The amount of memory that it needs to run to completion.
- Time complexity
 - Computing time



Space complexity

$$S(P) = c + S_P(I),$$

P : the program; I : the input.

- ★ $S_P(I)$ can be represented by $S_P(n)$ if n is the only instance characteristic.



Space complexity

$$S(P) = c + S_P(I),$$

P : the program; I : the input.

- ★ $S_P(I)$ can be represented by $S_P(n)$ if n is the only instance characteristic.
- Fixed space requirement: c .
 - Independent of the characteristics of the inputs and outputs.
 - Instruction space.
 - Space for simple variables, fixed-size structured variable and constants.
- Variable Space Requirement ($S_P(I)$)
 - depend on the instance characteristic I .
 - For instance, additional space when the program uses recursion.
 - values of inputs and outputs associated with I .



Example

- Assume that the integers are stored in an **array** 'list', such that the i th integer is stored in the i th position `list[i]`.

```
float abc(float a, float b, float c) {  
    return a + b + b * c + (a + b - c) / (a + b) + 4.00;  
}
```

- Fixed space requirement (c): 16.
 - Three float numbers: a, b, c and one return float number.
- $S_{abc}(l) = 0$. (for only fixed space requirements)



Example

```
float sum(float list[ ], int n) {  
    float temp = 0;  
    int i;  
    for (i=0; i<n; i++)  
        temp += list[i];  
    return temp;  
}
```

- In this program, $S_{\text{sum}}(I) = 0$.
- C Programming Language: passing the address of the first element of `list[]` (instead of copying).



Example (recursive)

```
float rsum(float list[ ], int n) {  
    if (n) return rsum(list, n-1);  
    return list[0];  
}
```

- Total variable space: $S_{\text{rsum}}(l) = 12n$.
 - parameter list[]: array pointer: 4 bytes.
 - parameter n: integer: 4 bytes
 - return address (internally used): 4 bytes.
- The recursive version has a **far greater** overhead than its iterative counterpart.



Time Complexity: $T(P) = c + T_P(I)$

- Compile time: c
 - Independent of the characteristics of the input and output.
 - Once the correctness of the program is verified, it can run without recompilation.
- Run time: $T_P(I)$ (what we are really concerned about)
 - E.g., $T_P(n) = c_a \cdot \text{ADD}(n) + c_s \cdot \text{SUB}(n) + c_l \cdot \text{LDA}(n) + c_{st} \cdot \text{STA}(n)$.
 - ADD, SUB, LDA, STA: the number of additions, subtractions, loads and stores.
 - c_a, c_s, c_l, c_{st} : the time needed to perform each operation (constants).



Time Complexity - Program Step (1/2)

- ★ machine independent

Program Step

a syntactically or semantically meaningful program segment whose execution time is independent of the instance characteristics.

- Example of **ONE program step**
 - $a = 2;$
 - $a = 2*b + 3*c/d - e + f/g/a/b/c;$



Time Complexity - Program Step (1/2)

Methods to compute the number of program steps

- Creating a global variable, say, count.
- Tabular method:
 - Compute the contribution of a statement:
 $\# \text{ program steps per execution} \times \text{frequency}.$
 - Add up the contribution of all statements.



Example

```
float sum(float list[ ], int n) {  
    float tempSum = 0; count++; /* for assignment */  
    int i;  
    for (i = 0; i < n; i++) {  
        count++; /* for the "for" loop */  
        tempSum += list[i]; count++; /* for assignment */  
    }  
    count++; /* last execution of "for" */  
    count++; /* for return */  
    return tempSum;  
}
```

- $\text{count} = 2n + 3$ (steps).



Example (Tabular Method)

Statements	s/e	Frequency	Total Steps
float sum(float list[], int n) {	0	0	0
float tempsum = 0;	1	1	1
int i;	0	0	0
for(i=0; i <n; i++)	1	n+1	n+1
tempsum += list[i];	1	n	n
return tempsum;	1	1	1
}	0	0	0
Total			$2n + 3$



Asymptotic Notation

- Issues: determining the “exact” step count of a program could be difficult or complicated.



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Asymptotic Notation

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- ▷ Using asymptotic notations: O, Ω, Θ, \dots
- A motivating example:
- $c_3 n < c_1 n^2 + c_2 n$ when n is sufficiently large.
 - For $c_1 = 1, c_2 = 2, c_3 = 100, c_1 n^2 + c_2 n \leq c_3 n$ for $n \leq 98$.
 - For $c_1 = 1, c_2 = 2, c_3 = 1000, c_1 n^2 + c_2 n \leq c_3 n$ for $n \leq 998$.



Big- O Notation

Definition ($O(\cdot)$)

$f(n) = O(g(n))$ iff there exist positive constants c and $n_0 \in \mathbb{N}$ such that $f(n) \leq g(n)$ for all $n \geq n_0$.

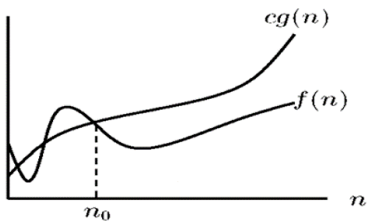


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- $g(n)$ is an **upper bound** on $f(n)$.
 - The smaller $g(n)$ is, the more informative it would be!



Big- Ω Notation

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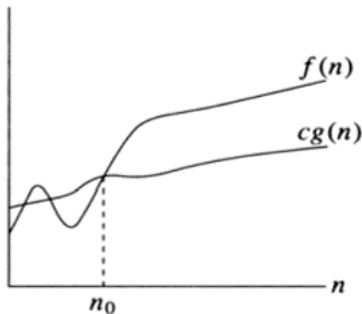


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- $g(n)$ is an **lower bound** on $f(n)$.
 - The larger $g(n)$ is, the more informative it would be!



Examples (Big- O)

- $3n + 2 = O(n)$.



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 - $100n + 6 \leq 101n$ for $n \geq 10$.



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 - $10n^2 + 4n + 2 \leq 11n^2$ for $n \geq 5$.



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 - $10n^2 + 4n + 2 \leq 11n^2$ for $n \geq 5$.
- $6 \cdot 2^n + n^2 = O(2^n)$.
 - $6 \cdot 2^n + n^2 \leq 7 \cdot 2^n$ for $n \geq 4$.



Examples (Big- Ω)

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Polynomial

Theorem 1.2

If $f(n) = a_k n^k + a_{k-1} n^{k-1} + \dots + a_1 n + a_0$, then $f(n) = O(n^k)$



Polynomial

Theorem 1.2

If $f(n) = a_k n^k + a_{k-1} n^{k-1} + \dots + a_1 n + a_0$, then $f(n) = O(n^k)$

Proof:

$$f(n) \leq \sum_{i=0}^k |a_i| n^i = n^k \sum_{i=0}^k |a_i| n^{i-k} \leq n^k \sum_{i=0}^k |a_i|, \text{ for } n \geq 1.$$



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Note that $n^{i-k} \leq 1$ if $i \leq k$ and $\sum_{i=0}^k |a_i|$ is a constant.



Most often seen big- O complexities

* with respect to the input of size n .

- $O(1)$: constant.
- $O(n)$: linear.
- $O(n^2)$: quadratic.
- $O(n^3)$: cubic.
- $O(2^n)$: exponential.
- $O(\log n)$: logarithmic.
- $O(n \log n)$: log linear.



Polynomial (Lower Bound)

Theorem 1.3

If $f(n) = a_k n^k + a_{k-1} n^{k-1} + \dots + a_1 n + a_0$, then $f(n) = \Omega(n^k)$



Polynomial (Lower Bound)

Theorem 1.3

If $f(n) = a_k n^k + a_{k-1} n^{k-1} + \dots + a_1 n + a_0$, then $f(n) = \Omega(n^k)$

Proof:

- Skipped and left as an exercise.



Theta Notation (Θ)

Definition (Θ)

$f(n) = \Theta(g(n))$ iff $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$.

- More precise than simply using big- O or big- Ω notations.



Theta Notation (Θ)

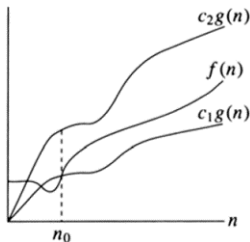
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Theorem 1.4

If $f(n) = a_k n^k + a_{k-1} n^{k-1} + \dots + a_1 n + a_0$, then $f(n) = \Theta(n^k)$



Example (Tabular Method)

Statements	s/e	Frequency	Total Steps	Asymptotic Complexity
void add (int a[][MAX_SIZE],...) {	0	0	0	0
int i, j;	0	0	0	0
for (i = 0; i < row; i++)	1	rows+1	rows+1	$\Theta(\text{rows})$
for (j=0; j< cols; j++)	1	rows*(cols+1)	rows*(cols+1)	$\Theta(\text{rows} \cdot \text{cols})$
c[i][j] = a[i][j]+b[i][j];	1	rows*cols	rows*cols	$\Theta(\text{rows} \cdot \text{cols})$
}	0	0	0	0
Total	$2 \cdot \text{rows} \cdot \text{cols} + 2 \cdot \text{rows} + 1$			$\Theta(\text{rows} \cdot \text{cols})$



Function Values & Plots

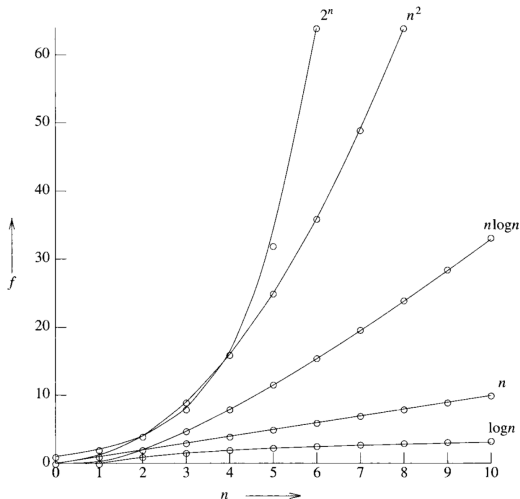
Refer to Fig. 1.7 & 1.8 in the textbook.

		Instance characteristic n					
Time	Name	1	2	4	8	16	32
1	Constant	1	1	1	1	1	1
$\log n$	Logarithmic	0	1	2	3	4	5
n	Linear	1	2	4	8	16	32
$n \log n$	Log linear	0	2	8	24	64	160
n^2	Quadratic	1	4	16	64	256	1024
n^3	Cubic	1	8	64	512	4096	32768
2^n	Exponential	2	4	16	256	65536	4294967296
$n!$	Factorial	1	2	24	40326	20922789888000	26313×10^{33}



Function Values & Plots

Refer to Fig. 1.7 & 1.8 in the textbook.



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Motivations

- Sometimes we still need to consider how long the algorithm executes on our machine.
- In order to obtain accurate times, we can repeatedly run the programs for several times (and take the average running time).



The Tricks

```
#include<time.h>
```

	1st Method	2nd Method
start timing	<code>start = clock();</code>	<code>start = time(NULL);</code>
stop timing	<code>end = clock();</code>	<code>end = time(NUL);</code>
type returned	<code>clock_t</code>	<code>time_t</code>

Result (in seconds):

- 1st Method: `duration = (double)(stop-start))/(CLOCKS_PER_SEC);`
- 2nd Method: `duration = (double)difftime(stop, start);`



The Tricks (Example)

```
... // previous code omitted
clock_t start, stop;
double duration;
printf("n time\n");
for(i=0; i< ITERATIONS; i++) {
    for(j=0; j<sizeList[i]; j++)
        list[j] = sizeList[i]-j; /* worst case */
    start = clock();
    sort(list, sizeList[i]);
    stop = clock();
    /* number of clock ticks per second */
    duration = ((double) (stop-start));
    printf("%6d %f\n", sizeList[i], duration);
}
}
```

⇒ sample code.



Discussions

