**Assignment 1**
**Due date: 15 March 2023**
**TA: 盧禹丞 E814**

1. Find two different row echelon forms of

$$\begin{bmatrix} 1 & 3 \\ 3 & 7 \end{bmatrix}$$

2. Solve the following system for $x, y$ and $z$:

$$\frac{1}{x} \quad +\frac{2}{y} \quad -\frac{4}{z} = 1$$

$$\frac{2}{x} \quad +\frac{3}{y} \quad +\frac{8}{z} = 0$$

$$-\frac{3}{x} \quad +\frac{9}{y} \quad +\frac{10}{z} = 5$$

3. Prove that if $ad - bc \neq 0$, then the reduced row echelon form of

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ is } \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

4. For the matrix $A = \begin{bmatrix} 1 & 0 & 0 & 0 & 1/4 & 1/4 & 1/4 & 1/4 \\ 0 & 1 & 0 & 0 & 1/4 & 1/4 & 1/4 & 1/4 \\ 0 & 0 & 1 & 0 & 1/4 & 1/4 & 1/4 & 1/4 \\ 0 & 0 & 0 & 1 & 1/4 & 1/4 & 1/4 & 1/4 \\ 0 & 0 & 0 & 0 & 1/4 & 1/4 & 1/4 & 1/4 \\ 0 & 0 & 0 & 0 & 1/4 & 1/4 & 1/4 & 1/4 \\ 0 & 0 & 0 & 0 & 1/4 & 1/4 & 1/4 & 1/4 \\ 0 & 0 & 0 & 0 & 1/4 & 1/4 & 1/4 & 1/4 \end{bmatrix}$, compute $A^{400}$.

5. If $\begin{bmatrix} a & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ b & 1 & 0 \\ 0 & b & 1 \end{bmatrix} \begin{bmatrix} a & 1 & 0 \\ 0 & a & 1 \\ 0 & 0 & a \end{bmatrix}$ and $a > 1$, find $(a, b)$.