Trees (I)

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Fall 2024

Outline

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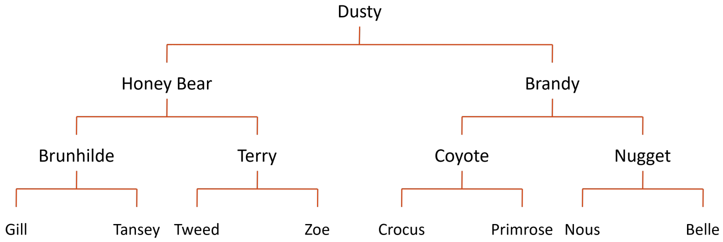
Introduction

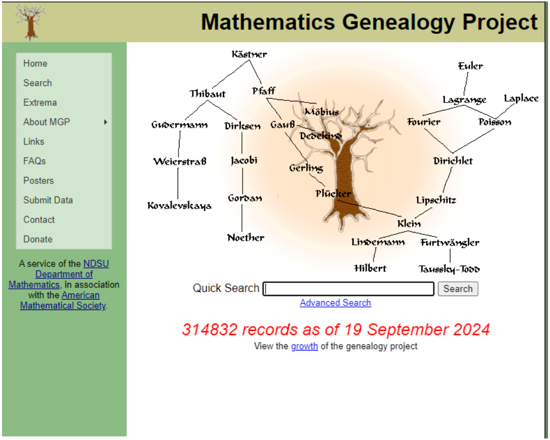
- Intuitively, a tree structure organized data in a hierarchical manner.

Example: Pedigree Chart



Example: Mathematical Genealogy Project

Figure reference: https://www.mathgenealogy.org/



Definitions

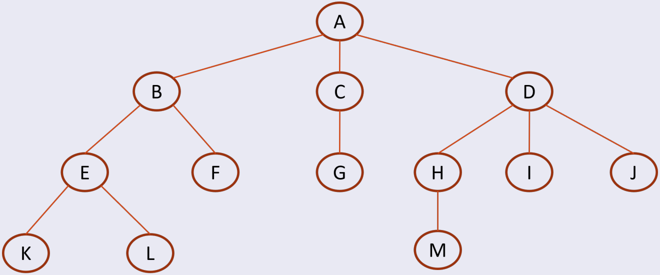
Tree

- A tree is a finite set of one or more nodes such that:
    - There is a specially designated node called root.
    - The remaining nodes are partitioned into $n \geq 0$ disjoint sets, $T_1, \ldots, T_n$, where each of these sets is a tree.
    - $T_1, \ldots, T_n$: subtrees of the root.

Definitions

Node

- A node stands for the item of information plus the branches to other nodes.



Definitions

Degree

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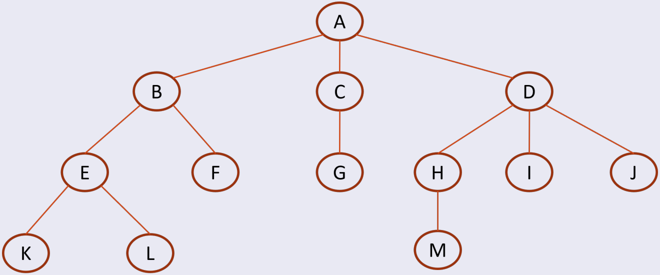
  - $\deg(A) = 3$, $\deg(C) = 1$, $\deg(F) = 0$.



Definitions

Leaf, children, parent

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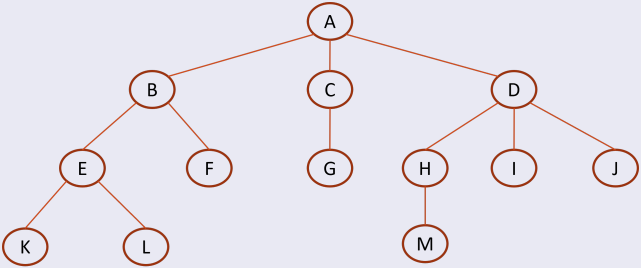
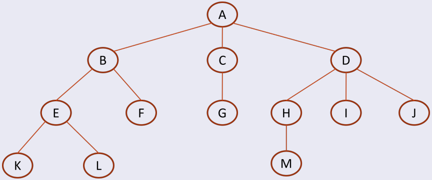
- The roots of the subtrees of a node X are the children of X. X is the parent of its children.



Definition

Siblings, degree, ancestors

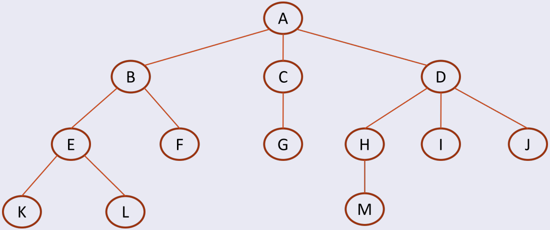
- Children of the same parent are said to be siblings.
    - Example: H, I and J are siblings; B, C and D are siblings.
- The degree of a tree is the **maximum** of the degree of the nodes in the tree.
    - The tree in this example has degree 3.
- The ancestors of a node are all the nodes along the path from the root to that node.
    - The ancestors of M are A, D, and H.



Definition

Level, height or depth

- The level of a node:
  - the root: 1.
  - if a node is at level k, then its children are at level $k+1$.
  - Example: $\text{level}(A) = 1$, $\text{level}(H) = 3$, $\text{level}(L) = 4$.
- The height or depth of a tree is defined to be the maximum level of any node in the tree.
  - The depth of the tree in this example is 4.

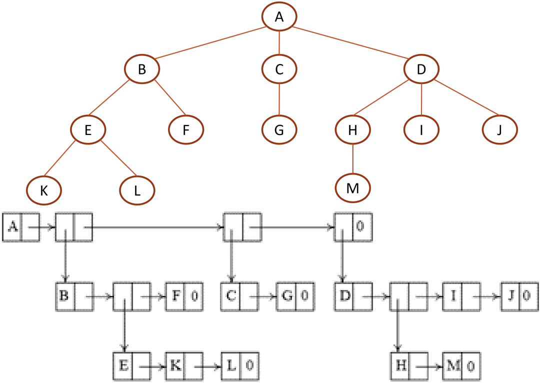


Representation of Trees

- The tree in the example can be written as

$$(A(B(E(K, L), F), C(G), D(H(M), I, J))).$$

- **Rule:** root node $\rightarrow$ list of its subtrees.



A Possible Node Structure of a Tree of Degree k

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- However, one only uses nodes of a fixed size to represent tree nodes in practice.

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- Then, how to choose such a fixed size?

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Discussions