

## Assignment 5

Due date: 6 December 2023

TA: 鄒冠勳 E814

1. (30%) Consider a mixture of two Gaussian distributions

$$0.4\mathcal{N}\left(\begin{bmatrix} 10 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) + 0.6\mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 8.4 & 2.0 \\ 2.0 & 1.7 \end{bmatrix}\right).$$

compute the marginal distributions of each dimension.

2. (30%) Consider the following example. Compute the conditional distributions  $p(x \mid Y = y_1)$  and  $p(y \mid X = x_3)$ .

|   |       |       |       |       |       |       |
|---|-------|-------|-------|-------|-------|-------|
| Y | $y_1$ | 0.01  | 0.02  | 0.03  | 0.1   | 0.1   |
|   | $y_2$ | 0.05  | 0.1   | 0.05  | 0.07  | 0.2   |
|   | $y_3$ | 0.1   | 0.05  | 0.03  | 0.05  | 0.04  |
|   |       | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ |
|   |       | X     |       |       |       |       |

3. (40%) Exercise in the slides.

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Gaussian Distribution

Sums and Linear Transformations

Exercise

Another example of *reverse transformation*.

$Y \sim \mathcal{N}(\mu_y, \Sigma)$  and  $\mathbf{y} = \mathbf{A}\mathbf{x}$  for  $\mathbf{x}, \mathbf{y} \in \mathbb{R}^M$ , and  $\mathbf{A}$  is invertible

- $p(\mathbf{y}) = \mathcal{N}(\mathbf{y} \mid \mathbf{A}\mathbf{x}, \Sigma)$ .
- Compute  $\mathbb{E}[\mathbf{x}]$ .
- Compute  $\mathbb{V}[\mathbf{x}]$ .
- Derive  $X \sim \mathcal{N}(?, ?)$ .

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