# Linked List

## Polynomials

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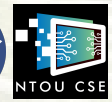
Fall 2024



# Outline



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## Goal

Represent the polynomial:

$$a_{m-1}x^{e_{m-1}} + a_{m-2}x^{e_{m-2}} + \cdots + a_0x^{e_0}.$$

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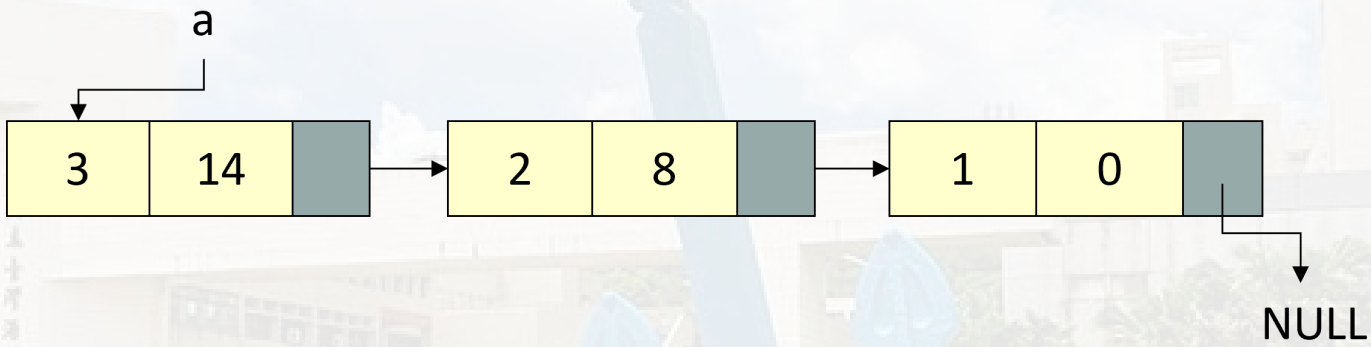
- **Idea:** Represent each term as a node containing
  - coefficient field
  - exponent field
  - pointer to the next term

# Declaration

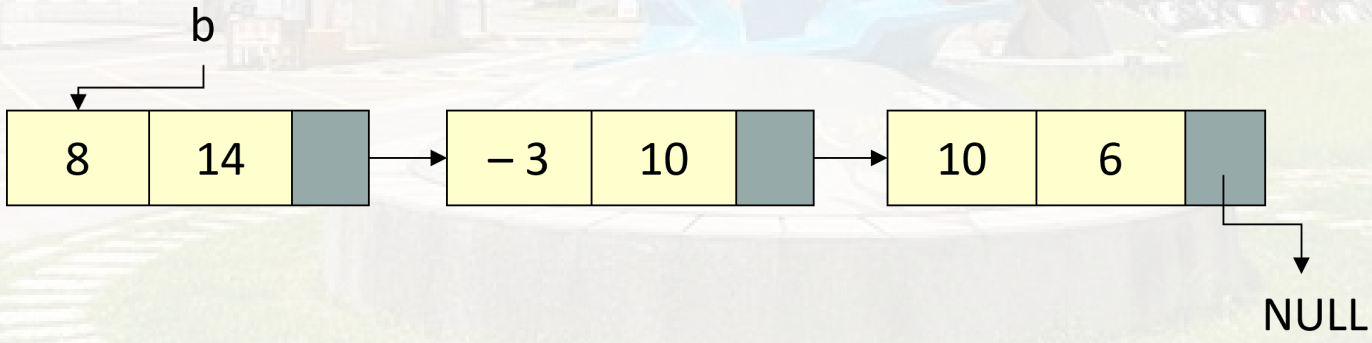
```
typedef struct polyNode *polyPointer;
struct polyNode {
    int coef;
    int expon;
    polyPointer link;
};
polyPointer a, b;
```

# Examples

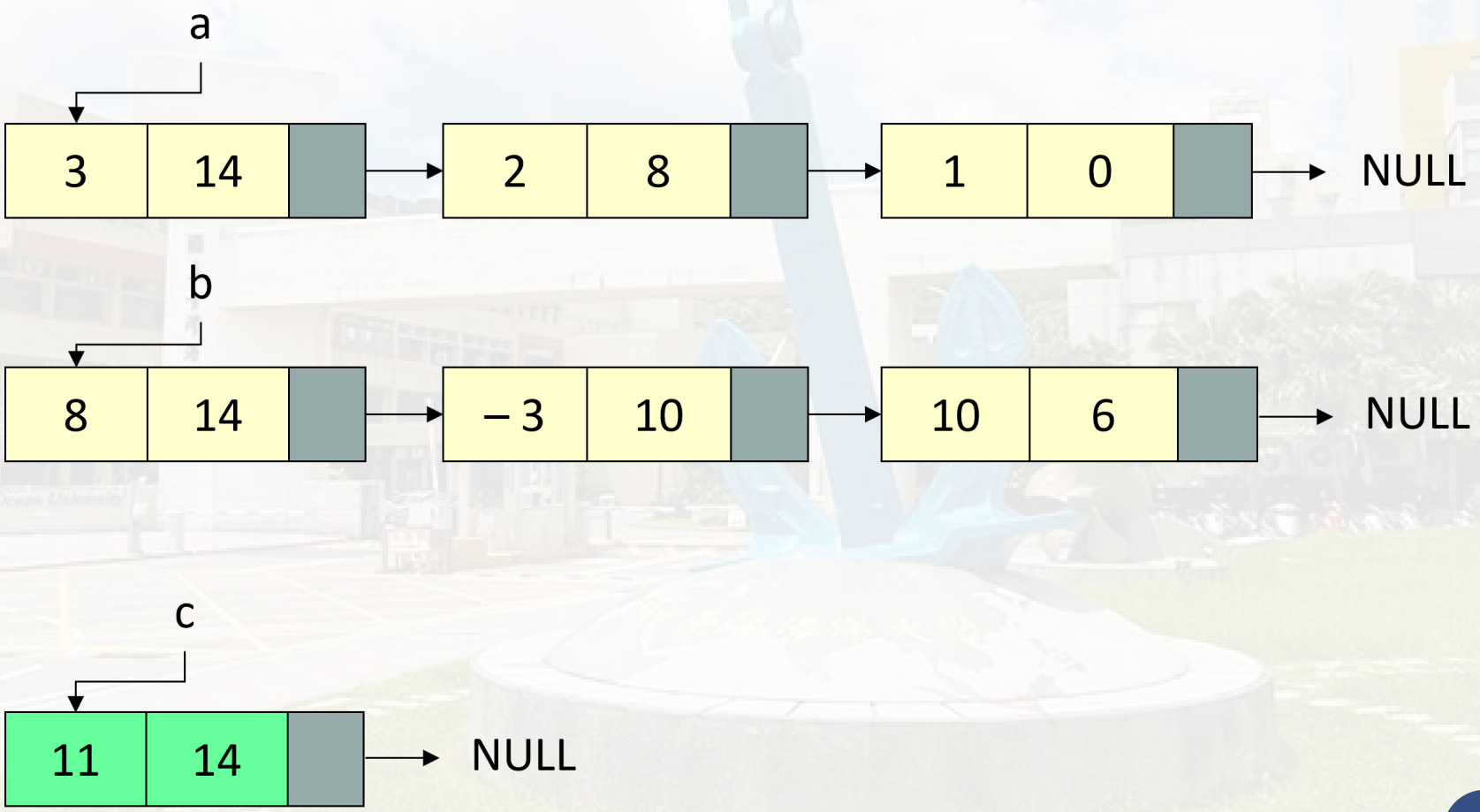
- $a = 3x^{14} + 2x^8 + 1$




- $b = 8x^{14} - 3x^{10} + 10x^6$



# Generating the first three terms of $c = a + b$



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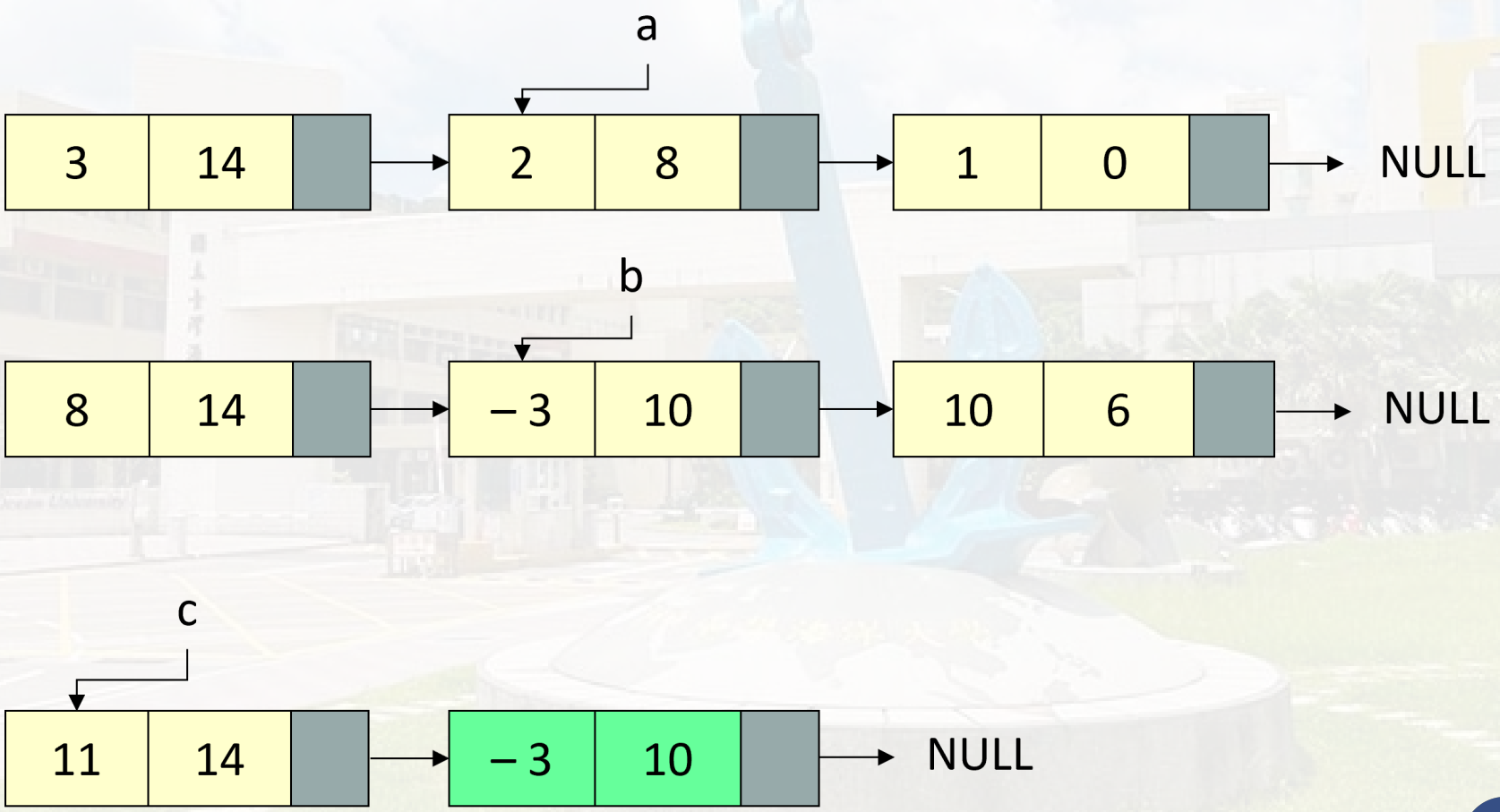

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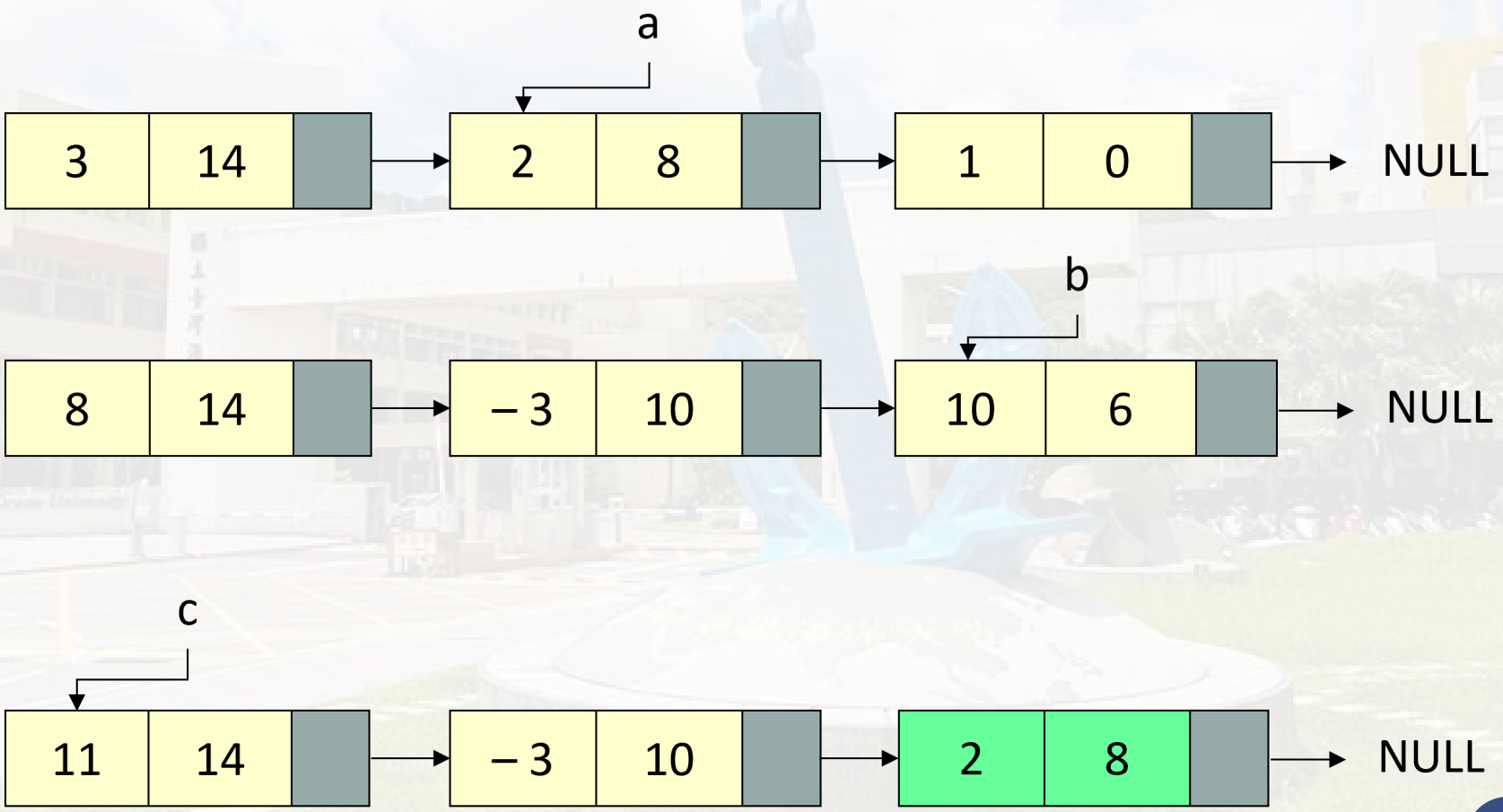

# Addition of Two Polynomials

```
polyPointer polyAdd(polyPointer a, polyPointer b) { /* return a polynomial which is the sum of a and b */
    polyPointer front, rear, temp;
    int sum; rear = (polyPointer)malloc(sizeof(*rear));
    if (IS_FULL(rear)) { printf("The memory is full\n"); exit(1) }
    front = rear;
    while (a && b) {
        switch (COMPARE(a->expon, b->expon)) {
            case -1: /* a->expon < b->expon */
                attach(b->coef, b->expon, &rear);
                b = b->link;
                break;
            case 0: /* a->expon = b->expon */
                sum = a->coef + b->coef;
                if (sum)
                    attach(sum, a->expon, &rear);
                a = a->link; b = b->link; break;
            case 1: /* a->expon > b->expon */
                attach(a->coef, a->expon, &rear);
                a = a->link;
        }
        /* copy rest of list a and then list b*/
        for (; a; a = a->link) attach(a->coef, a->expon, &rear);
        for (; b; b = b->link) attach(b->coef, b->expon, &rear);
        rear->link = NULL:
        /* delete extra initial node */
        temp = front; front = front->link; free(temp);
        return front;
    }
}
```

# Attach a new node to the end of a list

```
void attach(float coefficient, int exponent, polyPointer *ptr) {
/* create a new node with coef = coefficient and expon = exponent,
   attach it to the node pointed by ptr and update ptr to point
   to this new node */
    polyPointer temp;
    temp = (polyPointer)malloc(sizeof(*temp));
    if (IS_FULL(temp)) {
        printf("The memory is full\n");
        exit(1);
    }
    temp->coef = coefficient;
    temp->expon = exponent;
    (*ptr)->link = temp;
    *ptr = temp;
}
```

# Analysis of `polyAdd`

Consider

$$A(x) = a_{m-1}x^{e_{m-1}} + a_{m-2}x^{e_{m-2}} + \cdots + a_0 x^{e_0},$$
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- exponent comparisons:
  - In each iteration, either pointer $a$ or $b$ or both move to the next term(s).
  - The maximum number of exponent comparisons is $m + n$
- create new nodes for $C$:
  - The maximum number of terms in $C$ is $O(m + n)$.

# Erasing Polynomials

- One by one, free the nodes pointed by `ptr`.

```
void erase(polyPointer *ptr) {
/* erase the polynomial pointed to by ptr */
    polyPointer temp;
    while (*ptr) {
        temp = *ptr;
        *ptr = (*ptr)->link;
        free(temp);
    }
}
```

# A Question

Can we free all the nodes of a polynomial more efficiently?


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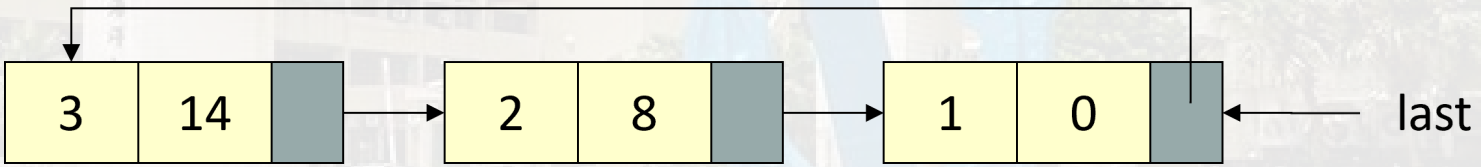
- Consider the circular list representation.

# Circular List Representation

## Circular List

A singly linked list in which the link field of the last node points to the first node is called a circular list.

$3x^{14} + 2x^8 + 1$:

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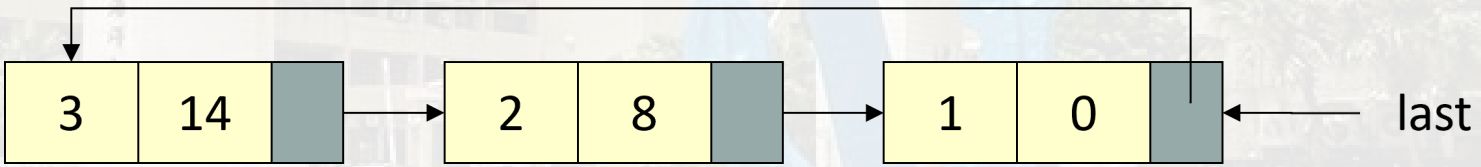


- In order to obtain an efficient erase algorithm, we maintain a list of nodes that have been freed.
- Only when the list is empty do we need to use `malloc` to create a new node.

# Functions to deal with freed nodes of polynomials

- Let `avail` be a variable of type `polyPointer` which points to the first node in the list of freed nodes.

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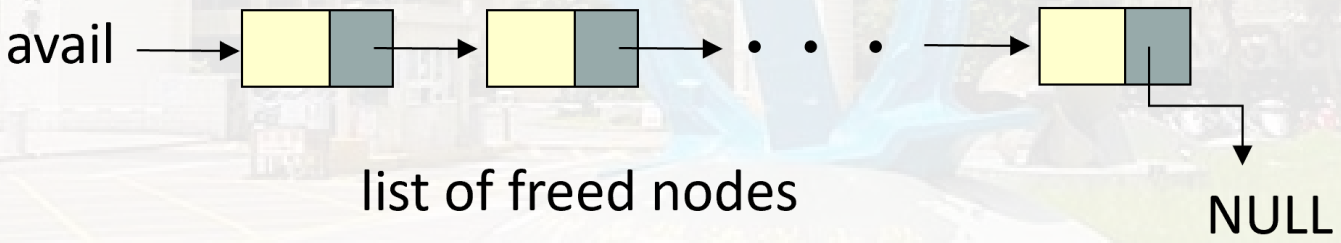
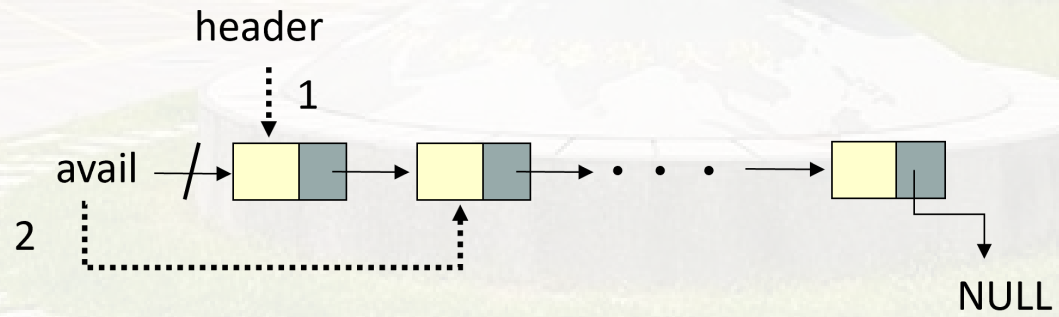
- Let `avail` be a variable of type `polyPointer` which points to the first node in the list of freed nodes.
- Instead of using `malloc()` and `free()`, we now use `getNode()` and `retNode()`.



# getNode()

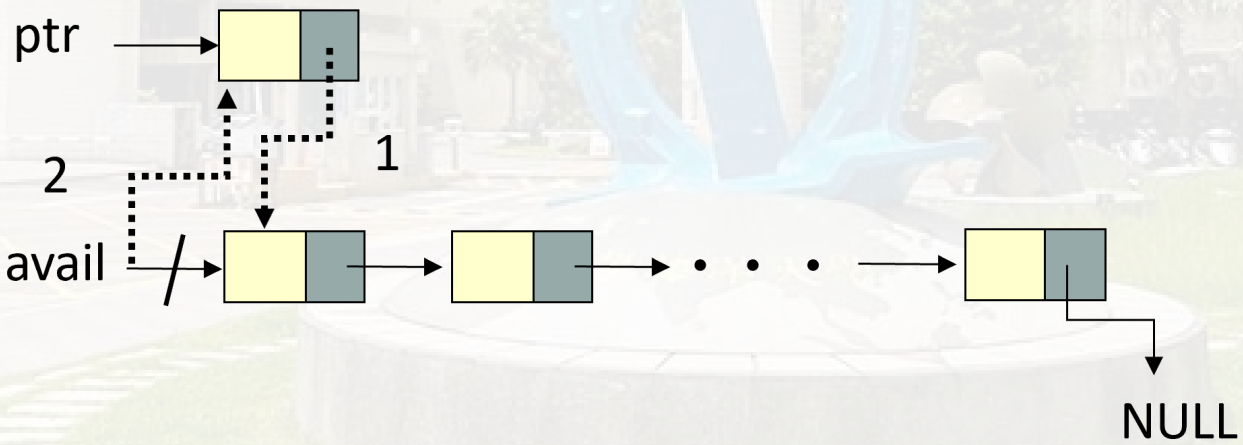
⋆ To avoid the case of zero polynomials, we introduce a header into each polynomial.

```
polyPointer getNode(void) {
/* provide a node header for use */
    polyPointer header;
    if (avail) {
        header = avail;
        avail = avail->link;
    } else
        header = malloc(sizeof(*header));
    return header;
}
```



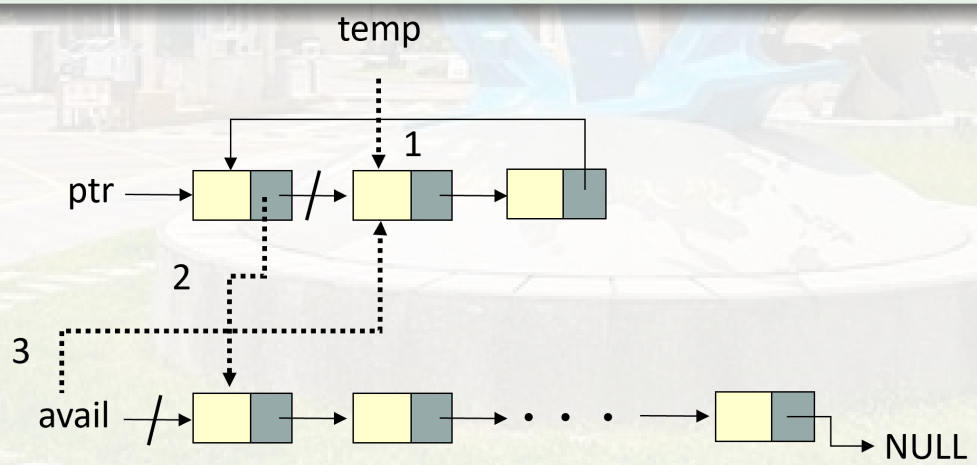
# retNode()

```
void retNode(polyPointer ptr) {
/* return a node to the available list */
     ptr->link = avail;
     avail = ptr;
}
```

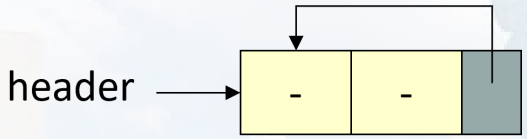


# Erasing a circular list (independent of the number of nodes)

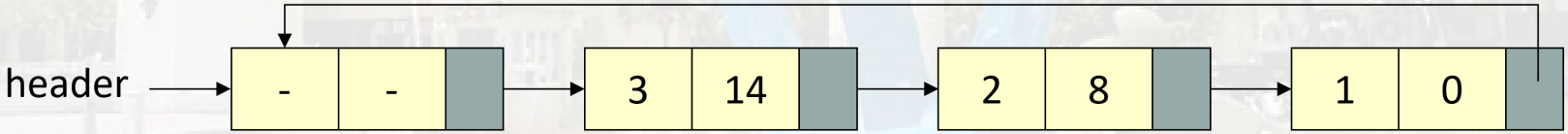
```
void cir_erase(polyPointer *ptr) {
// erase the circular list pointed by ptr
    polyPointer temp;
    if (*ptr) {
        temp = (*ptr)->link;
        (*ptr)->link = avail;
        avail = temp;
        *ptr = NULL;
    }
}
```



# Polynomials with header nodes (Examples)




(a) Zero polynomial




(b) $3x^{14} + 2x^8 + 1$

# Outline

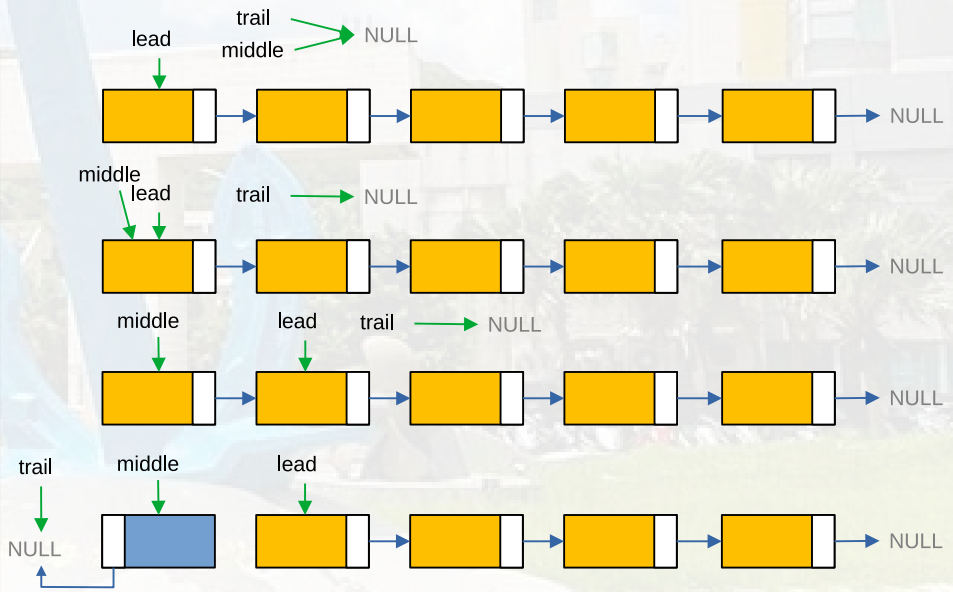


# Additional List Operations

- a chain (singly linked list in which the last node has a null link):
  - inversion (反轉)
  - concatenation (連接) of two chains
- a circular list (singly linked list in which the last node points to the first node):
  - insert a node at the front of a circular list
  - determine the length of a circular list

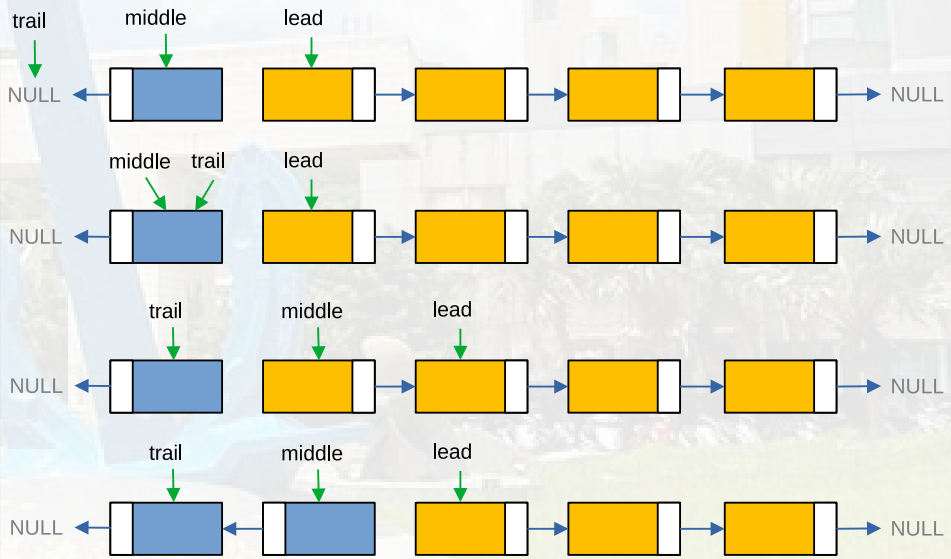
# Inversion of a chain

```
listPointer invert (listPointer lead) {
// invert the list pointed to by lead
    listPointer middle, trail;
    middle = NULL;
    while (lead) {
        trail = middle;
        middle = lead;
        lead = lead->link;
        middle->link = trail;
    }
    return middle;
}
```



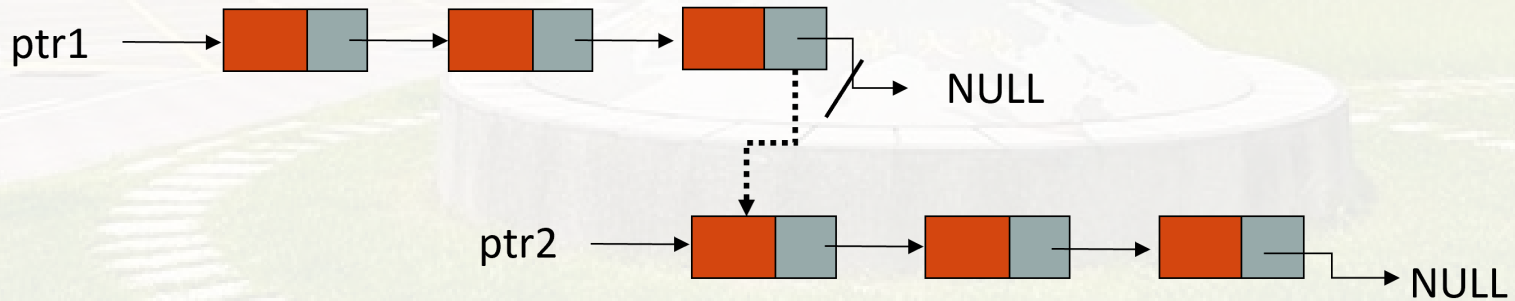
# Inversion of a chain

```
listPointer invert (listPointer lead) {
// invert the list pointed to by lead
    listPointer middle, trail;
    middle = NULL;
    while (lead) {
        trail = middle;
        middle = lead;
        lead = lead->link;
        middle->link = trail;
    }
    return middle;
}
```



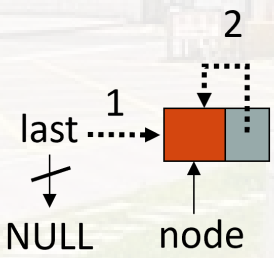
# Concatenation of two singly linked lists

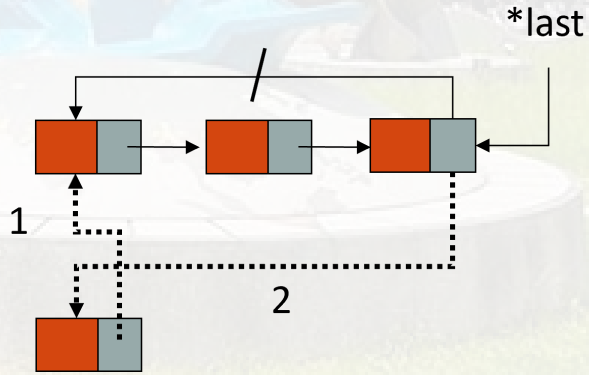
```
listPointer concatenate (listPointer ptr1, listPointer ptr2) {
/* produce a new list that contains the list
   ptr1 followed by the list ptr2. */
    listPointer temp;
    // check for empty lists
    if (!ptr1) return ptr2;
    if (!ptr2) return ptr1;
    /* neither list is empty, find end of first list */
    for (temp = ptr1; temp->link; temp = temp->link);
    // linked end of first to start of second
    temp->link = ptr2;
}
```



# Inserting at the front of a circular list

```
void insertFront (listPointer *last, listPointer node) {
/*  insert node at the front of the circular list
    whose last node is last */
    if (!(*last)){ //  list is empty, change last to point to new entry
        *last = node;
        node->link = node;
    } else { // list is not empty, add a new entry at front
        node->link = (*last)->link;
        (*last)->link = node;
    }
}
```



# Find the length of a circular list

```
int length (listPointer last) {
/* find the length of circular list last */
    listPointer temp;
    int count = 0;
    if (last) {
        temp = last;
        do {
            count++;
            temp = temp->link;
        } while (temp != last);
    }
    return count;
}
```


# Discussions