

Quiz 2

Due date: 15 May 2024

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1. (25%) Suppose that \mathbf{u} , \mathbf{v} , and \mathbf{w} are vectors such that inner product $\langle \mathbf{u}, \mathbf{v} \rangle = 2$, $\langle \mathbf{v}, \mathbf{w} \rangle = -3$ and $\langle \mathbf{u}, \mathbf{w} \rangle = 1$. Moreover, the norms $\|\mathbf{u}\| = 1$, $\|\mathbf{v}\| = 2$, $\|\mathbf{w}\| = 5$. Please compute the value of $|\langle 2\mathbf{v} - \mathbf{w}, 3\mathbf{u} + 2\mathbf{w} \rangle|$.
2. (25%) Find the adjoint of the matrix $A = \begin{bmatrix} 1 & -1 & 0 \\ 5 & -4 & 3 \\ 2 & 0 & -1 \end{bmatrix}$.
3. (25%) Find the $\|\text{proj}_{\mathbf{a}} \mathbf{u}\|$, where $\mathbf{u} = (1, -2)$, $\mathbf{a} = (-4, -3)$.
4. (25%) Show that $\|v\|_1 \leq \sqrt{n}\|v\|_2$ for any $v \in \mathbb{R}^n$. (*Hint: Using Cauchy–Schwarz inequality*)