

Multi-Parameter Mechanism Design

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- ▶ In previous lectures, we only consider single-parameter mechanism design problems.
 - ▶ The only private parameter of an agent is her valuation.
- ▶ In this lecture, we:
 - ▶ Introduce multi-parameter environments, where each agent has multiple private parameters.
 - ▶ Introduce the Vickrey-Clarke-Groves (VCG) mechanisms.
 - ▶ It shows that **DSIC welfare maximization** is possible in principle in **every multi-parameter environments**.

Outline

General Mechanism Design Environments

The VCG Mechanism

Remarks

Practical Implementation of Combinatorial Auctions
Indirect Mechanisms

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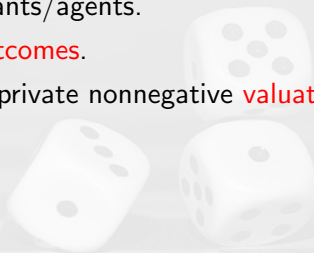
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General Multi-Parameter Design Environment

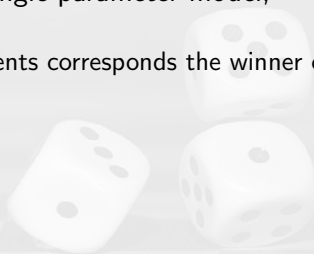
- ▶ n strategic participants/agents.
- ▶ a finite set Ω of **outcomes**.
- ▶ each agent i has a private nonnegative **valuation** $v_i(\omega)$ **for each outcome** $\omega \in \Omega$.



Single-Item Auction Revisited

Consider the single-item auction.

- ▶ In a the standard single parameter model,
 - ▶ $|\Omega| = n + 1$.
 - ▶ The $n + 1$ elements corresponds the winner of the item (if any).



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- ▶ In a the standard single parameter model,
 - ▶ $|\Omega| = n + 1$.
 - ▶ The $n + 1$ elements corresponds the winner of the item (if any).
 - ▶ The valuation of a bidder is 0 in all of the n outcomes (in which she doesn't win), leaving only one unknown parameter per bidder.
- ▶ In the general multi-parameter framework, a bidder can have a different valuation for each possible outcome (i.e., winner; competitor).

Combinatorial Auctions

- ▶ Multiple indivisible items are for sale.
- ▶ We have n bidders and a set M of m items.
- ▶ Bidders can have **preferences** between different **subsets** (bundles) of items.
- ▶ The outcome space Ω corresponds to n -vectors (S_1, \dots, S_n) , where $S_i \subseteq M$ is the bundle allocated to bidder i (no item is allocated twice).
- ★ How many different outcomes in Ω ?

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 - ▶ 2^m .

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The General Theorem

Theorem 7.3 (Multi-Parameter Welfare Maximization)

In every general mechanism design environment, there is a DSIC welfare-maximizing mechanism.

- Note that the computational efficiency is not asserted here.

The General Theorem

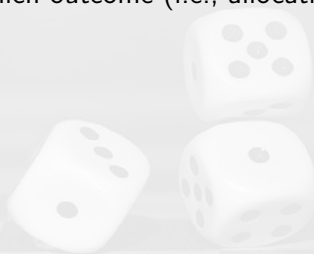
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In every general mechanism design environment, there is a DSIC welfare-maximizing mechanism.

- ▶ Note that the computational efficiency is not asserted here.
- ▶ Let's discuss the main ideas behind the theorem before proving it.

Consider the Two-Step Approach as Usual

- ▶ First, assume that the agents truthfully report their private information (i.e., $\mathbf{b} = \mathbf{v}$).
- ▶ Then, figure out which outcome (i.e., allocation) to pick.



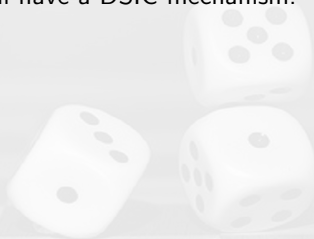
Consider the Two-Step Approach as Usual

- ▶ First, assume that the agents truthfully report their private information (i.e., $\mathbf{b} = \mathbf{v}$).
- ▶ Then, figure out which outcome (i.e., allocation) to pick.
- ▶ Pick a welfare-maximizing outcome using bids as proxies for the unknown valuations.
- ▶ We define the allocation rule

$$\omega^* := \mathbf{x}(\mathbf{b}) = \arg \max_{\omega \in \Omega} \sum_{i=1}^n b_i(\omega).$$

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- ▶ Define the payment rule to incentivize the agents!
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The Second Step

- ▶ Define the payment rule to incentivize the agents!
 - ▶ Hopefully we will have a DSIC mechanism.
- ▶ However, unlike the single-parameter environments, here:
 - ▶ The report is multi-dimensional.
 - ▶ Myerson's lemma seems not to hold beyond single-parameter environments.
 - ▶ It's unclear how to define “monotonicity”, “critical bid”, etc.

A Key Idea to the Payment Rule

- ▶ Charge the agent the “externality” caused by agent i .
 - ▶ The welfare loss inflicted on the other $n - 1$ agents by agent i 's presence.
- ▶ This remains well defined in general mechanism design environments!
- ▶ The corresponding payment rule:

$$p_i(\mathbf{b}) = \left(\max_{\omega \in \Omega} \sum_{j \neq i} b_j(\omega) \right) - \sum_{j \neq i} b_j(\omega^*),$$

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- ▶ You may check that the payment $p_i(\mathbf{b}) \geq 0$.

The VCG Mechanism

Definition: VCG Mechanism

A mechanism (\mathbf{x}, \mathbf{p}) with allocation and payment rule as

$$\omega^* := \mathbf{x}(\mathbf{b}) = \arg \max_{\omega \in \Omega} \sum_{i=1}^n b_i(\omega).$$

and

$$p_i(\mathbf{b}) = \left(\max_{\omega \in \Omega} \sum_{j \neq i} b_j(\omega) \right) - \sum_{j \neq i} b_j(\omega^*),$$

respectively, is a Vickrey-Clarke-Groves or mechanism.

Another Form of the Payment

$$p_i(\mathbf{b}) = \underbrace{b_i(\omega^*)}_{\text{bid}} - \underbrace{\left[\sum_{j=1}^n b_j(\omega^*) - \max_{\omega \in \Omega} \sum_{j \neq i} b_j(\omega) \right]}_{\text{rebate}}$$

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 - ▶ The highest bidder pays $b_1 - (b_1 - b_2) = b_2$!
- ▶ Truthful reporting always guarantees nonnegative utility!

Proof of Theorem 7.3 (1/2)

- ▶ Fix an arbitrary general mechanism design environment.
- ▶ Let (\mathbf{x}, \mathbf{p}) denote the corresponding VCG mechanism.
- ▶ The mechanism maximizes the social welfare whenever the reports are truthful (by definition).
- ▶ Next, we have to verify the DSIC condition.
 - ▶ We need to show that for every agent i and every set \mathbf{b}_{-i} , agent i maximizes her utility $v_i(\mathbf{x}(\mathbf{b})) - p_i(\mathbf{b})$ by setting $\mathbf{b}_i = \mathbf{v}_i$.

Proof of Theorem 7.3 (2/2)

► Fix i and \mathbf{b}_{-i} . When the chosen outcome $\mathbf{x}(\mathbf{b})$ is ω^* , we have

$$\begin{aligned}
 v_i(\omega^*) - p_i(\mathbf{b}) &= v_i(\omega^*) - \left(\max_{\omega \in \Omega} \sum_{j \neq i} b_j(\omega) - \sum_{j \neq i} b_j(\omega^*) \right) \\
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- (B) can be viewed as a constant. So we focus on maximizing (A).
- For agent i , assume that ω^* is chosen, setting $b_i = v_i$ makes (A) maximized.
- $v_i(\omega^*) + \sum_{j \neq i} b_j(\omega^*) = \sum_j b_j(\omega^*)$.
 - Recall that $\omega^* = \arg \max_{\omega \in \Omega} \sum_{i=1}^n b_i(\omega)$.

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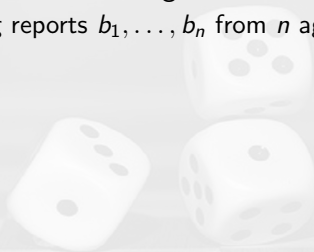
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Practical Considerations (1/3)

- ▶ Preference elicitation is a challenge.
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 - ▶ Consider getting reports b_1, \dots, b_n from n agents in a combinatorial auction with m items.
 - ▶ Each bidder has 2^m private parameters!
 - ▶ It's hard for her to figure out or write down so many numbers.
 - ▶ No seller would want to read them.

Practical Considerations (2/3)

- ▶ As in single-parameter environments, welfare maximization could be a computationally intractable.
 - ▶ Recall the knapsack auction.
- ▶ Sometimes even approximate welfare maximization is still computationally intractable.

Practical Considerations (3/3)

- ▶ VCG mechanisms can have bad revenue.

Exercise 4 (5%)

1. Consider a combinatorial auction with two bidders and two items A and B . The first bidder only wants both items, so $v_1(\{A, B\}) = 1$ and is 0 otherwise. The second bidder only wants item A , so $v_2(\{A, B\}) = v_2(\{A\}) = 1$ and is 0 otherwise. **Please show that the revenue of the VCG mechanism is 1 in this example.**
2. Now suppose that we add a third bidder who **only wants item B** , so $v_3(\{A, B\}) = v_3(\{B\}) = 1$. **Please show that the maximum welfare is 2 but the VCG revenue is 0 in this case.**

Recall the issues

- ▶ Combinatorial auction: n bidders, m items, bidder i 's valuation $v_i(S)$ for each bundle S of items.
- ▶ The number of parameters that each bidder reports in the VCG mechanism (or any other direct-revelation mechanism) grows **exponentially** with m .

Indirect Mechanism

- ▶ Learn information about bidders' preferences only on a **need-to-know** basis.
- ▶ The canonical indirect auction: **ascending English auction**.
 - ★ An auctioneer asks for takers at successively higher prices.
 - ★ The auction ends when no one accepts the currently proposed price.
 - ★ The winner (if any): the bidder who accepted the previously proposed price
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 - ✓ This previous price is the final sale price.
- ▶ Empirically, bidders are more likely to play their dominant strategies in this kind of auction than a sealed-bid second-price auction.
 - ▶ Bidders: not likely to overbid;
 - ▶ Seller: only learns a lower bound on the highest bid.

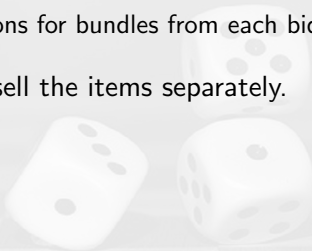
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 - ▶ Eliciting valuations for bundles from each bidder is avoided.
- ▶ The simplest way: sell the items separately.
- ▶ What's the issue or problem?

Substitutes vs. Complements

For two items A and B ,

- ▶ **substitute condition:** $v(AB) \leq v(A) + v(B)$.
 - ▶ Spectrum auction: two licenses (the same area & equal-sized frequency ranges).
 - ▶ iPhone 13 + iPhone 14 announced together?
 - ★ Welfare maximization is computationally tractable.
- ▶ **complement condition:** $v(AB) > v(A) + v(B)$.
 - ▶ Spectrum auction: a collection of licenses that are adjacent (geographically or frequency ranges).
 - ▶ Two items with additional enhancement when they are both provided.
 - ★ Welfare maximization is computationally *intractable*.
- ▶ In real world: mixture of substitutes and complements.

Typical mistake #1 of running separate single-item auctions

Hold the single-item auctions **sequentially**, one at a time.

- ▶ The scenario: A sequence of single-item auctions for two identical items.
 - ▶ Items are sold via back-to-back second-price auctions.
- ▶ Let's say you are a bidder with a VERY HIGH valuation.
- ▶ Suppose that every other bidder bid truthfully.

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- ▶ Suppose that every other bidder bid truthfully.
- ★ If you participate in the first auction, you will win and pay the second-highest valuation.
- ★ If you skip it, the bidder with the second-highest valuation wins the first auction and disappear.
 - Then you would win the second auction at a price equal to the **third-highest** valuation.

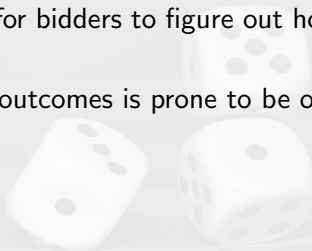
A story from the textbook

- ▶ In March 2000, Switzerland auctioned off three blocks of spectrum via a **sequence of second-price auctions**.
- ▶ The first two auctions were for identical items, 28 MHz blocks, and sold for 121 million and 134 million Swiss francs, respectively.
 - ▶ This is already more price variation than one would like for identical items.
- ▶ But in the third auction, where a larger 56 MHz block was being sold, the selling price was only 55 million francs!
- ▶ Some of the bids must have been far from optimal, and both the welfare and revenue achieved by this auction are suspect.

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Use sealed-bid single-item auctions (simultaneously).

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- ▶ Consider that there are 3 bidders and 2 identical items, and each bidder wants **only one**.
- ▶ With simultaneous second-price single-item auctions,
 - ▶ if each bidder targets only one item, one of the licenses is likely to have only one bidder and will be given away for **free** or sold at the reserve price.

A story from the textbook

- ▶ In 1990, the New Zealand government auctioned off essentially identical licenses for television broadcasting using **simultaneous (sealed-bid) second-price auctions**.
- ▶ The revenue in the 1990 New Zealand auction was only \$36 million, a paltry fraction of the projected \$250 million.
 - ▶ On one license, the high bid was \$100,000 while the second-highest bid (and selling price) was \$6!
 - ▶ On another, the high bid was \$7 million and the second-highest was \$5,000.

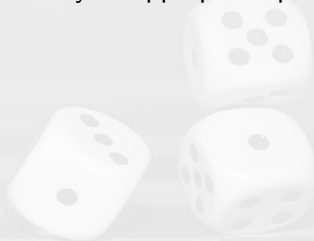
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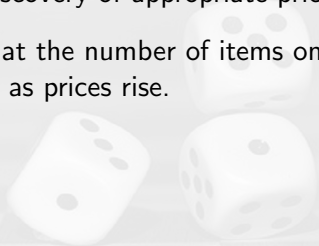
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- ▶ The primary reason that SAAs work better is **price discovery**.
 - ▶ A bidder can abandon the items in the mid-course as she acquires better information about the likely selling prices of the items.
 - ▶ Again, bidder only need to determine their valuations on a **need-to-know** basis.
- ▶ Though such kind a mechanisms still has its vulnerabilities (skipped here for further readings).