

# Tree Traversals

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# Outline

- 1 Binary Tree Traversals (Using a Stack)
- 2 Level-Order Traversal
- 3 Additional Binary Tree Operations

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# Binary Tree Traversals

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  - Six possible combinations:  $LVR$ ,  $LRV$ ,  $VLR$ ,  $VRL$ ,  $RVL$ ,  $RLV$ .



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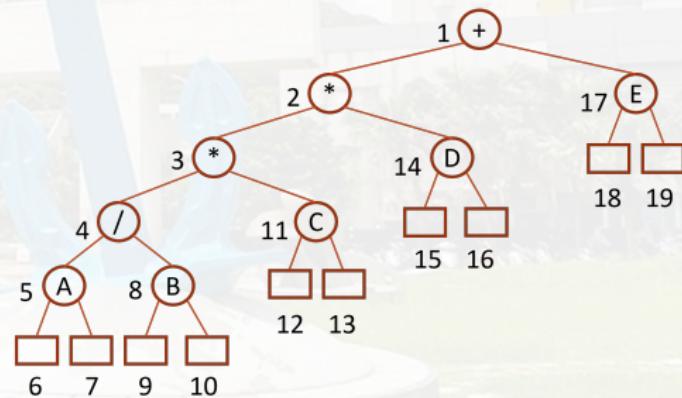
- Let  $V$ ,  $L$ ,  $R$  stand for visiting the node, moving left, and moving right, resp.
  - Six possible combinations:  $LVR$ ,  $LRV$ ,  $VLR$ ,  $VRL$ ,  $RVL$ ,  $RLV$ .
- Adopting the convention that we traverse left before right, only three combinations of  $VLR$  remains:
  - **inorder** (中序走訪法)
  - **postorder** (後序走訪法)
  - **preorder** (先序走訪法)



# Tree Traversals

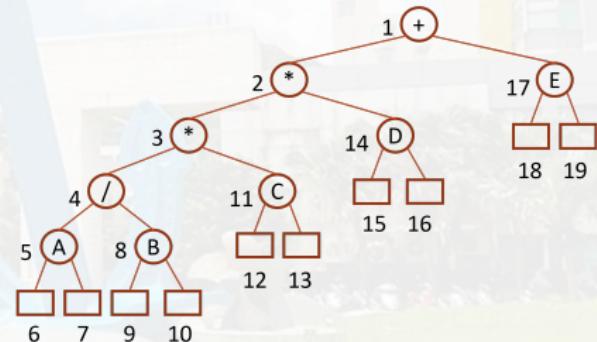
The tree represents the arithmetic expression (in infix form).

- Inorder traversal (*LVR*):  
 $A/B*C*D+E$
- Preorder traversal (*VLR*):  
 $+**/ABCDE$
- Postorder traversal (*LRV*):  
 $AB/C*D*E+$



# Code for Inorder Traversal

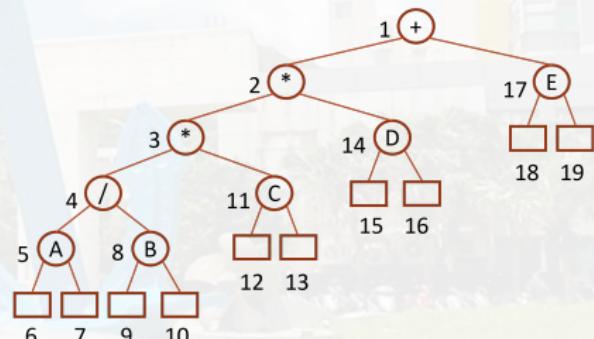
```
void inorder (treePointer ptr) {
    /* inorder tree traversal */
    if (ptr) {
        inorder (ptr->leftChild);
        printf ("%d", ptr->data);
        inorder (ptr->rightChild);
    }
}
```



A/B\*C\*D+E

# Code for Preorder Traversal

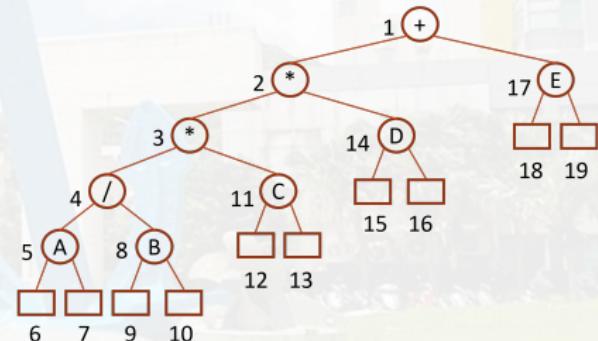
```
void preorder (treePointer ptr) {
/* inorder tree traversal */
    if (ptr) {
        printf ("%d", ptr->data);
        preorder (ptr->leftChild);
        preorder (ptr->rightChild);
    }
}
```



+\*\*\*/ABCDE

# Code for Postorder Traversal

```
void postorder (treePointer ptr) {
    /* inorder tree traversal */
    if (ptr) {
        postorder (ptr->leftChild);
        postorder (ptr->rightChild);
        printf ("%d", ptr->data);
    }
}
```



AB/C\*D\*E+

# Iterative Inorder Traversal

```
void iter_inorder (treePointer node) {
    int top = -1; // initialize the stack
    tree_pointer stack[MAX_STACK_SIZE];
    while (1) {
        while (node != NULL) {
            push(&top, node); // add node to the stack
            node = node->left_child;
        }
        node = stack[top]; // pop from stack
        pop();
        if (!node) break; // empty stack, so break!
        printf("%d", node->data);
        node = node->right_child;
    }
}
```

# Analysis of Iterative Inorder Traversal

- Let  $n$  be the number of nodes in the tree.



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  - Every node of the tree is pushed and popped from the stack exactly once.
- Space complexity:  $O(n)$ .
  - The stack size  $\leq$  the depth of the tree.

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2 Level-Order Traversal

3 Additional Binary Tree Operations

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## Steps of a Level-Order Traversal

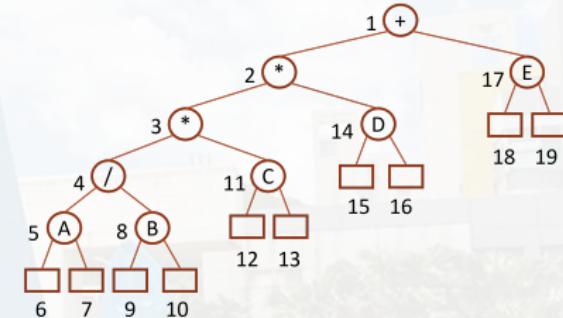
- visit the root first.
- then the root's left child followed by the right child.
- visit next level from leftmost node to right most node.

# Code for the Level-Order Traversal

```

void levelOrder(treePointer ptr) {
    int front = rear = 0;
    treePointer queue[MAX_QUEUE_SIZE];
    if (!ptr) return; /* empty tree */
    add(ptr); // enqueue
    while (1) {
        ptr = delete(); // dequeue
        if (ptr) {
            printf("%d", ptr->data);
            if (ptr->leftChild)
                // leftChild exists
                add(ptr->leftChild);
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        } else break;
    }
}

```

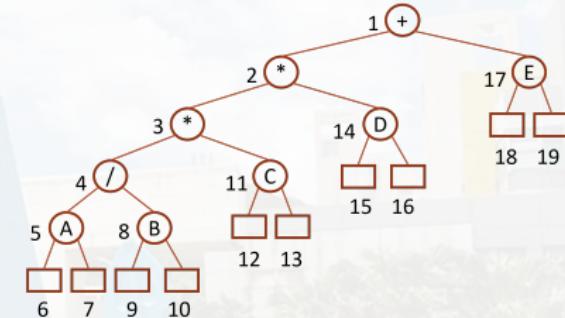


# Code for the Level-Order Traversal

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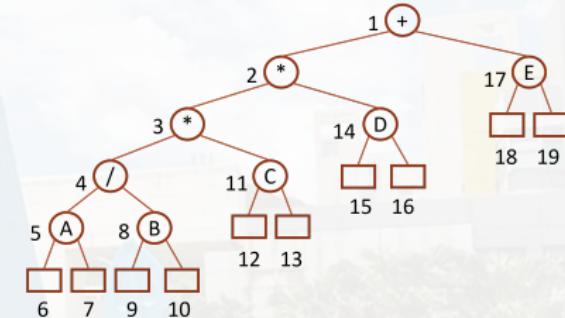
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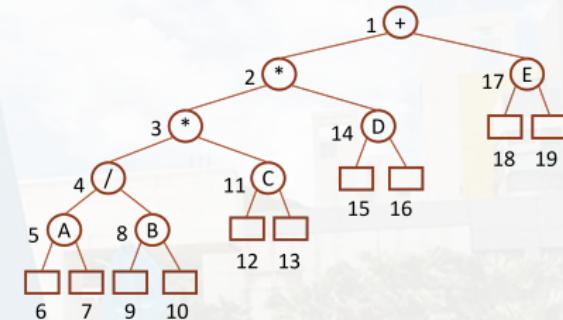
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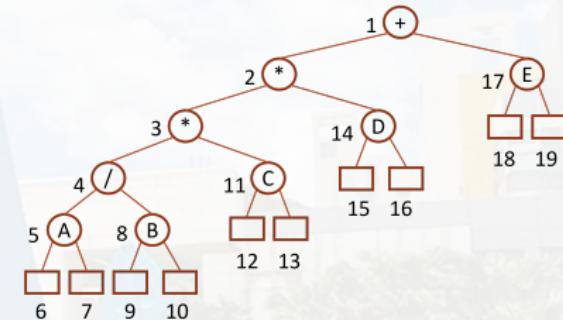
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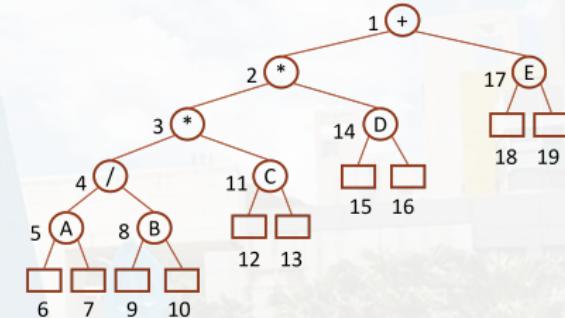
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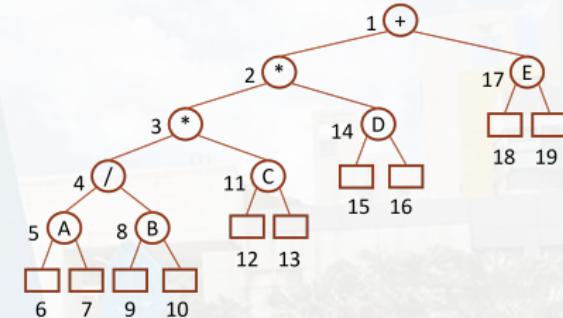
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    }
}

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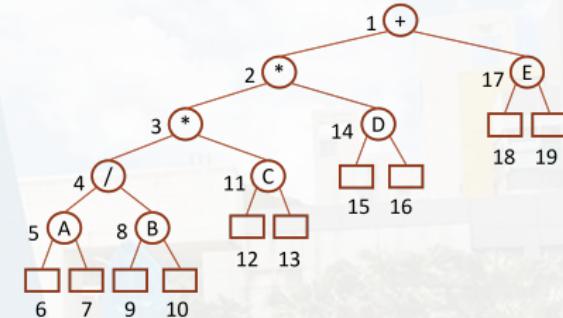
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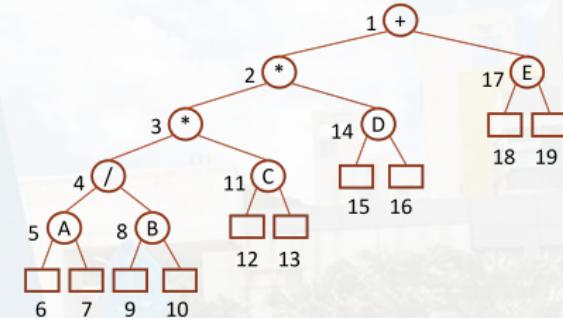
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\*\*E\*D/CAB

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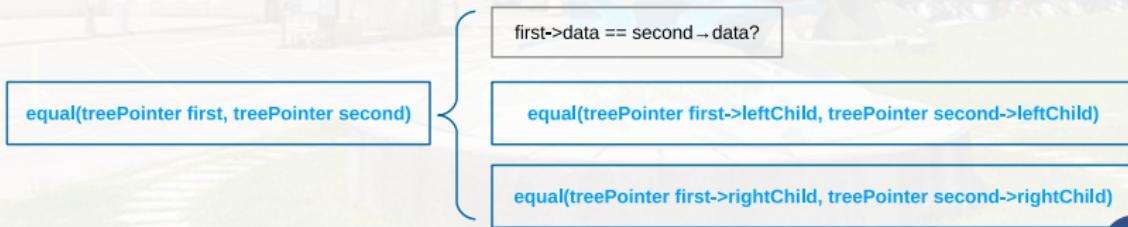
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# Copying a Binary Tree

```
treePointer copy(treePointer original) {
    /* return a tree_pointer to an exact copy of the original tree */
    treePointer temp;
    if (original) {
        temp = (treePointer)malloc(sizeof(node));
        temp->leftChild = copy(original->leftChild);
        temp->rightChild = copy(original->rightChild);
        temp->data = original->data;
        return temp;
    }
    return NULL;
}
```

# Testing for Equality of Binary Trees

```
int equal(treePointer first, treePointer second) {  
    /* return "false" if the binary trees first and second are not equal,  
    Otherwise it returns "true" */  
    return (  
        (!first && !second) || (first && second &&  
        (first->data == second->data) &&  
        equal(first->leftChild, second->leftChild) &&  
        equal(first->rightChild, second->rightChild))  
}
```



# Discussions

