Arrays and Structures

Pattern Matching

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Outline

String ADT

- A string is:
  - objects: a finite set of zero or more characters.
- functions: for all strings s, t, and $i, j, m \in \mathbb{Z}^+$:
  - String Null(m) ::= return a string whose maximum length is m characters, but is initially set to NULL We write NULL as "".
  - Integer Compare(s, t) ::= if s equals t return 0 else if s precedes t return -1 else return $+1$
  - Boolean IsNull(s) ::= if (Compare(s, NULL)) return FALSE else return TRUE
  - Integer Length(s) ::= if (Compare(s, NULL)) return the number of characters in s else return 0.

String ADT (functions contd.)

- String Concat(s, t) $::=$ if (Compare(t, NULL)) return a string whose elements are those of s followed by those of t else return s.
- String Substr(s, i, j) $::=$ if $((j > 0)$ && $(i + j–1) <$ Length(s)) return the string containing the characters of s at positions i, $i + 1, \ldots,$ $i + j–1$. else return NULL.
- void StringInsert(s, t, i)$::=$ insert string t at position i

String Insertion

```
void strnins(char *s, char *t, int i) {
    /* insert string t into string s at position i */
    char string[MAX_SIZE], *temp = string;

    if (i < 0 && i > strlen(s)) {
        fprintf(stderr, "Position is out of bounds \n");
        exit(1);
    }
    if (!strlen(s))
        strcpy(s, t);
    else if (strlen(t)) {
        strncpy(temp, s, i); // Copy at most i characters from s to temp
        strcat(temp, t);
        strcat(temp, (s + i));
        strcpy(s, temp);
    }
}
```

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- Substring: a contiguous sequence of characters within a string.
- For example,

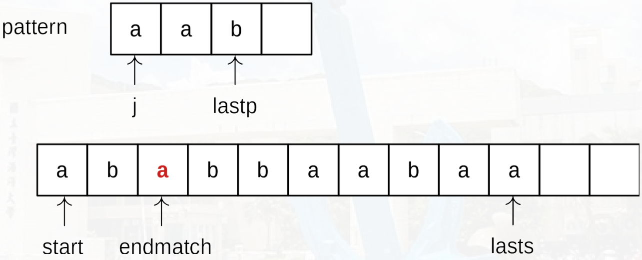
  `cgta` is a substring of `acgtacct`.

  `acgg` is NOT a substring of `acgtacct`.

Outline

```c
int naiveFind(char *string, char *pat) {
    /* match the last character of pattern first,
       and then match from the beginning */
    int i, j, start = 0;
    int lasts  = strlen(string) - 1;
    int lastp  = strlen(pat)    - 1;
    int endmatch = lastp;

    for (i = 0; endmatch <= lasts; endmatch++, start++) {
        if (string[endmatch] == pat[lastp])
            for (j = 0, i = start; j < lastp && string[i] == pat[j]; i++, j++)
                ;  // empty statement: advance while matching
        if (j == lastp)
            return start; /* successful */
    }
    return -1;
}
```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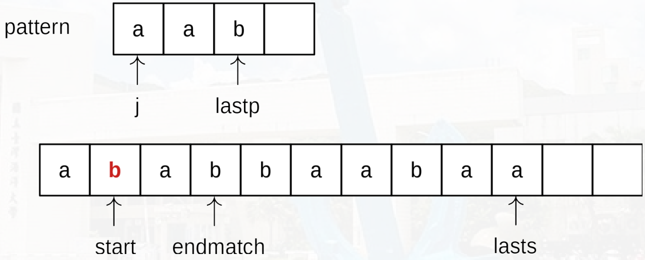
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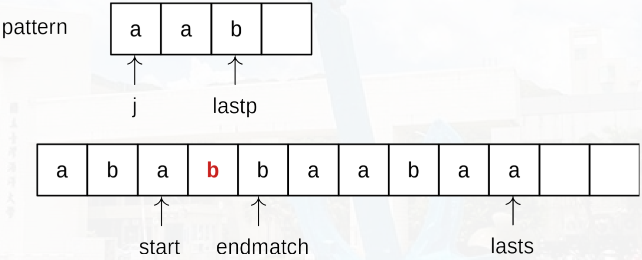
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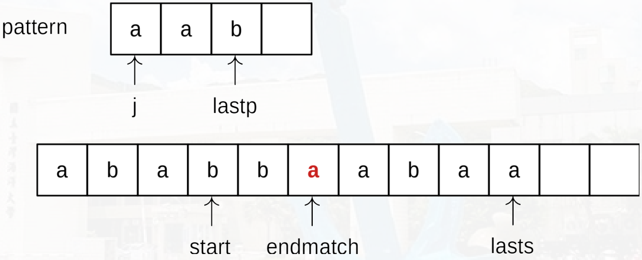
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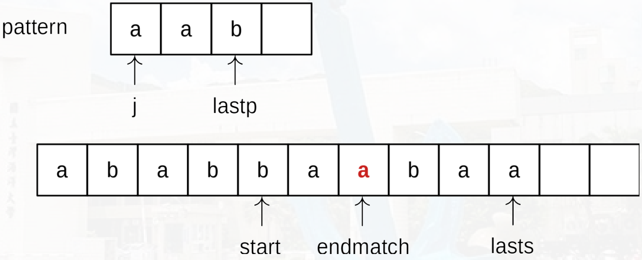
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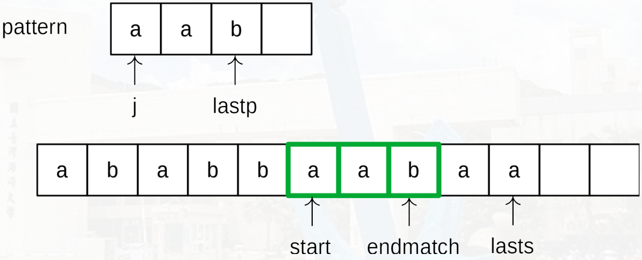
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Time Complexity

- Worst case: $O(n \cdot m)$.

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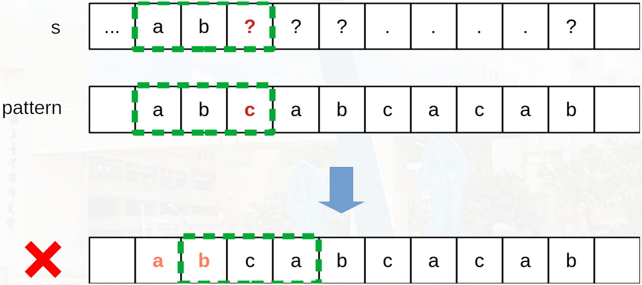
- Actually, we can do much better for the pattern matching.

Outline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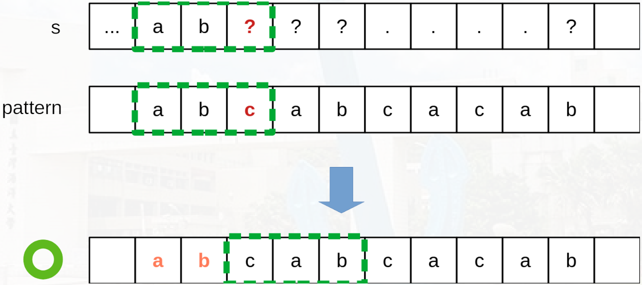
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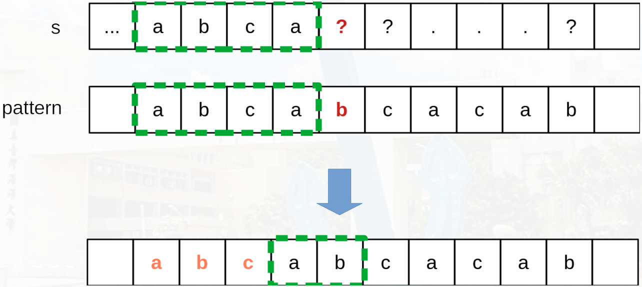
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The failure function

If $p = p_0 p_1 \cdots p_{n-1}$ is a pattern, then the failure function f is defined as $f(j) =$

- largest $k < j$ such that

$$p_0 p_1 \cdots p_k = p_{j-k} p_{j-k+1} \cdots p_j$$

  if such $k \geq 0$ exists
- -1 otherwise.

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  if such $k \geq 0$ exists
- -1 otherwise.

- For example, suppose $p = \texttt{abcabcacab}$,

j	0	1	2	3	4	5	6	7	8	9
p	a	b	c	a	b	c	a	c	a	b
failure	-1	-1	-1	0	1	2	3	-1	0	1

The rule of using the failure function

- If a partial match is found and $j \neq 0$ such that

$$s_{i-j} \cdots s_{i-1} = p_0 p_1 \cdots p_{j-1}$$

and

$$s_i \neq p_j,$$

then matching may be resumed by comparing s_i and $p_{f(j-1)+1}$.

- If $j = 0$, then we may continue by comparing s_{i+1} and p_0.

A fast way to compute the failure function

$$f(j) = \begin{cases} -1, & j = 0, \\ f^m(j-1)+1, & \text{if } m \geq 0 \text{ is the least integer such that } p_{f^m(j-1)+1} = p_j, \\ -1, & \text{otherwise.} \end{cases}$$

- **Note:** $f^1(j) = f(j)$ and $f^m(j) = f(f^{m-1}(j))$.

Failure Function Code ($O(m)$)

```
void fail(const char* p, int* f) { // p: pattern; f: failure
    /* compute the pattern's failure function */
    int m = std::strlen(p);
    f[0] = -1;
    for (int j = 1; j < m; j++) {
        int i = f[j-1];
        while ((i >= 0) && (p[j] != p[i+1])) {
            i = f[i];
        }
        if (p[j] == p[i+1])
            f[j] = i + 1;
        else
            f[j] = -1;
    }
}
```

j	0	1	2	3	4	5	6	7	8	9
p	a	b	c	a	b	c	a	c	a	b
f	-1	-1	-1	0	1	2	3	-1	0	1

The time complexity of fail

- In each iteration of the while loop, the value of i decreases ($\because f$).
- i is reset at the beginning of each iteration of the for loop.
  - It is either reset to -1 or to a value 1 greater than its terminal value on the previous iteration (i.e., $f[j] = i + 1$).
  - Since the for loop is iterated only $m - 1$ times, the value of i is incremented for at most $m - 1$ times.
    - Hence, it cannot be decremented more than $m - 1$ times.
- The while loop is iterated $\leq m - 1$ times overall.

Declarations

```
#include <stdio.h>
#include <string.h>

#define MAX_STRING_SIZE  100
#define MAX_PATTERN_SIZE 100

int KMPmatch(void);
void fail(void);

int  f[MAX_PATTERN_SIZE]; // computed by the failure function
char s[MAX_STRING_SIZE];
char p[MAX_PATTERN_SIZE];
```

The KMP Algorithm

```
int KMPmatch(char *s, char *p) {
    int i = 0, j = 0; // i: for string s, and j: for pattern p
    int n = strlen(s); // n: length of string s
    int m = strlen(p); // m: length of pattern p

    while (i < n && j < m) {
        if (s[i] == p[j]) {
            i++; j++;
        } else if (j == 0) {
            i++;
        } else {
            j = f[j-1] + 1;
        }
    }
    return (j == m) ? (i - m) : -1;
}
```

- the pattern is not found $\Rightarrow$ the pattern index $j \neq m$ $\Rightarrow$ return -1.
- the pattern is found $\Rightarrow$ the starting position is $i - m$.

Analysis of KMPmatch

- The while loop is iterated until the end of either the string or the pattern is reached.
- i is never decreased, so the lines that increase i cannot be executed $> n$ times.
- The resetting of j to $f[j-1]+1$ decreases the value of j.
    - This cannot be done more times than j is incremented by the statement j++ (otherwise j falls off the pattern...).
    - Each time j++ is executed, i is also incremented.
    - So, j cannot be incremented $> n$ times.
- No statement in the program is executed $> n$ times, therefore the time complexity of KMTmatch is $O(n+m)$.

Discussions