

Tree Traversals

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Outline

- 1 Binary Tree Traversals (Using a Stack)
- 2 Level-Order Traversal
- 3 Additional Binary Tree Operations

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Binary Tree Traversals

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 - Six possible combinations: LVR , LRV , VLR , VRL , RVL , RLV .



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- Adopting the convention that we traverse left before right, only three combinations of VLR remains:



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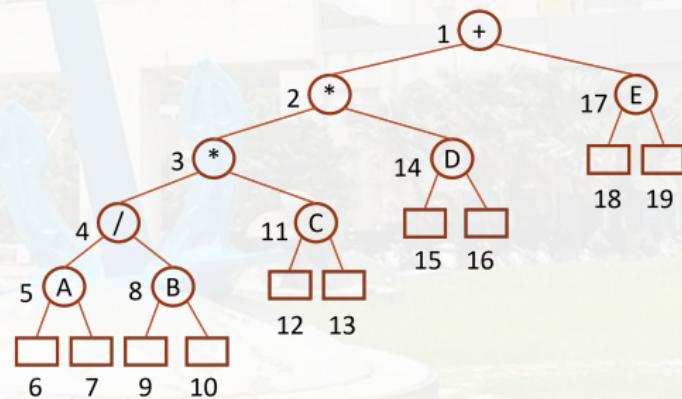
- Let V, L, R stand for visiting the node, moving left, and moving right, resp.
 - Six possible combinations: $LVR, LRV, VLR, VRL, RVL, RLV$.
- Adopting the convention that we traverse left before right, only three combinations of VLR remains:
 - **inorder** (中序走訪法)
 - **postorder** (後序走訪法)
 - **preorder** (先序走訪法)



Tree Traversals

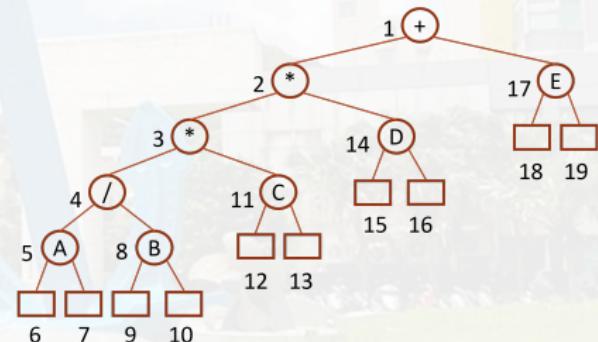
The tree represents the arithmetic expression (in infix form).

- Inorder traversal (*LVR*):
 $A/B*C*D+E$
- Preorder traversal (*VLR*):
 $+**/ABCDE$
- Postorder traversal (*LRV*):
 $AB/C*D*E+$



Code for Inorder Traversal

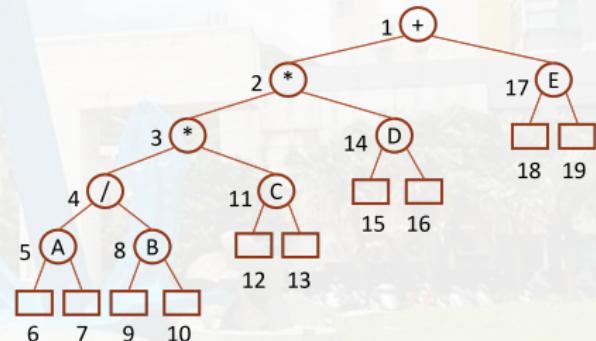
```
void inorder (treePointer ptr) {
    /* inorder tree traversal */
    if (ptr) {
        inorder (ptr->leftChild);
        printf ("%d", ptr->data);
        inorder (ptr->rightChild);
    }
}
```



A/B*C*D+E

Code for Preorder Traversal

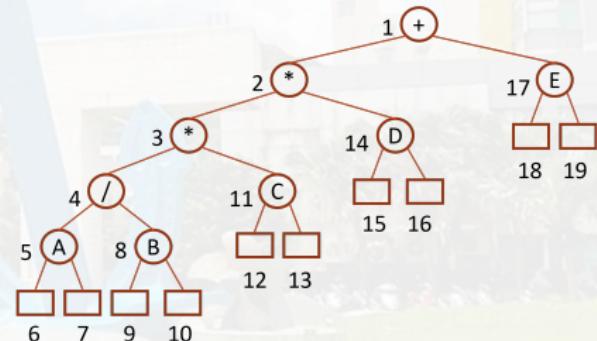
```
void Preorder (treePointer ptr) {
    /* inorder tree traversal */
    if (ptr) {
        printf ("%d", ptr->data);
        inorder (ptr->leftChild);
        inorder (ptr->rightChild);
    }
}
```



+**/ABCDE

Code for Postorder Traversal

```
void Postorder (treePointer ptr) {
    /* inorder tree traversal */
    if (ptr) {
        inorder (ptr->leftChild);
        inorder (ptr->rightChild);
        printf ("%d", ptr->data);
    }
}
```



AB/C*D*E+

Iterative Inorder Traversal

```
void iter_inorder (treePointer node) {  
    int top = -1; // initialize the stack  
    tree_pointer stack[MAX_STACK_SIZE];  
    while (1) {  
        while (node != NULL) {  
            push(&top, node); // add node to the stack  
            node = node->left_child;  
        }  
        node = stack[top]; // pop from stack  
        pop();  
        if (!node) break; // empty stack, so break!  
        printf("%d", node->data);  
        node = node->right_child;  
    }  
}
```



Analysis of Iterative Inorder Traversal

- Let n be the number of nodes in the tree.



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- Time complexity: $O(n)$.
 - Every node of the tree is pushed and popped from the stack **exactly once**.
- Space complexity: $O(n)$.
 - The stack size \leq the depth of the tree.

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2 Level-Order Traversal

3 Additional Binary Tree Operations

Level-Order Traversal

- When written recursively, the inorder, preorder, and postorder traversals all require a **stack**.



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Steps of a Level-Order Traversal

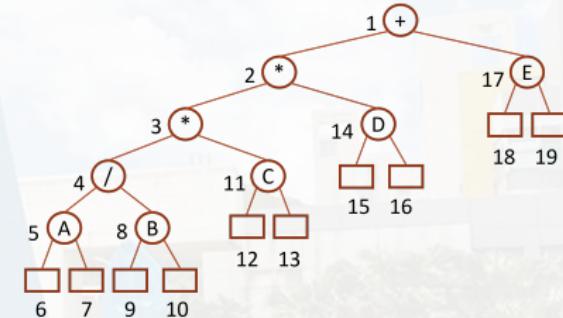
- visit the root first.
- then the root's left child followed by the right child.
- visit next level from leftmost node to right most node.

Code for the Level-Order Traversal

```

void levelOrder(treePointer ptr) {
    int front = rear = 0;
    treePointer queue[MAX_QUEUE_SIZE];
    if (!ptr) return; /* empty tree */
    add(ptr); // enqueue
    while (1) {
        ptr = delete(); // dequeue
        if (ptr) {
            printf("%d", ptr->data);
            if (ptr->leftChild)
                // leftChild exists
                add(ptr->leftChild);
                // enqueue
            if (ptr->rightChild)
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                // enqueue
        } else break;
    }
}

```

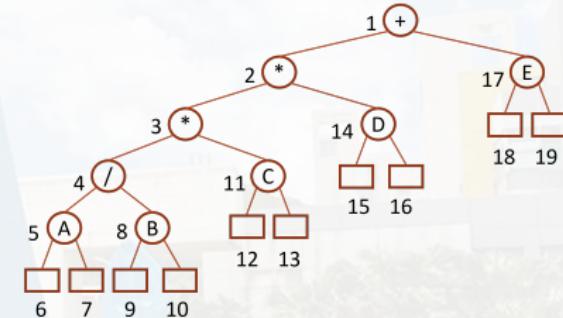


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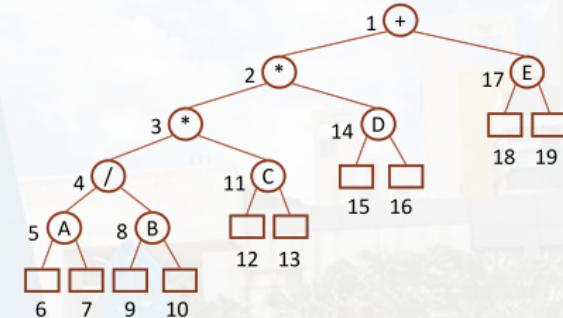
+*

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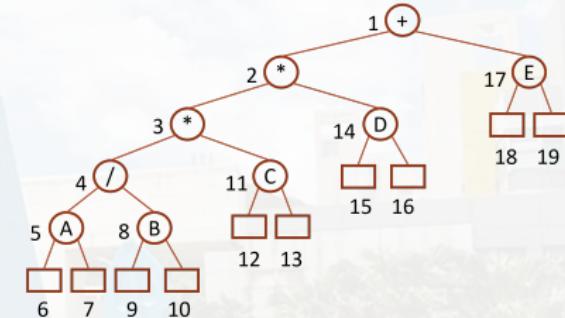
+*E

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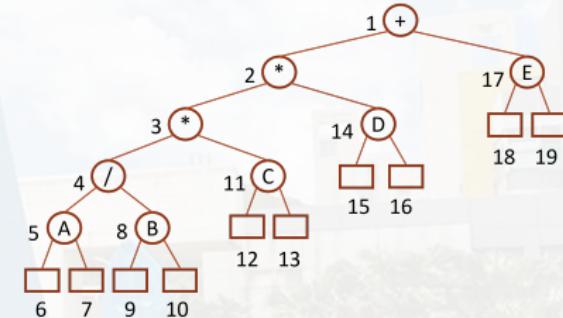
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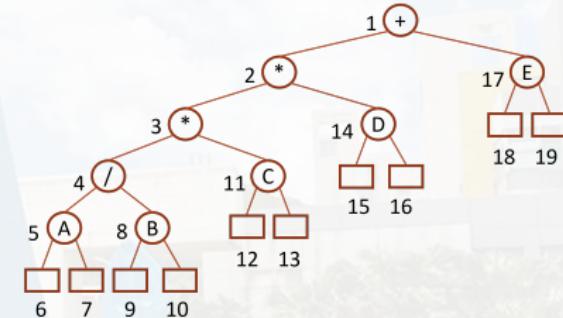
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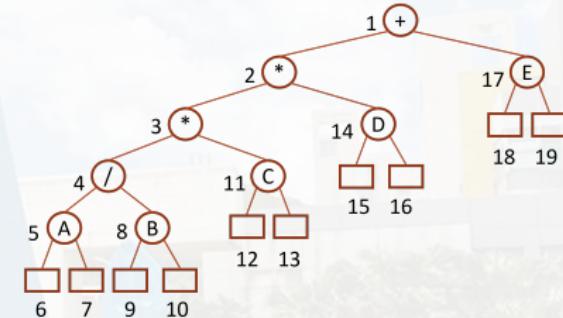
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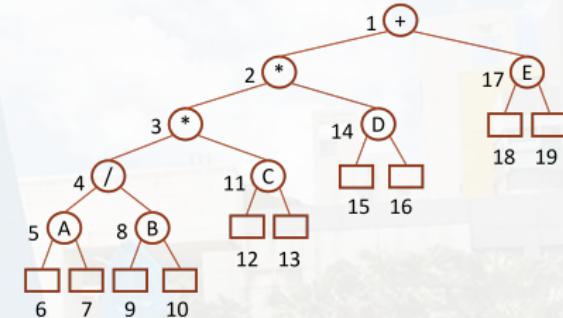
**E*D/C

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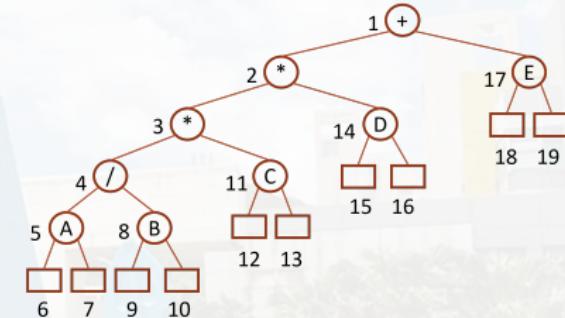
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**E*D/CAB

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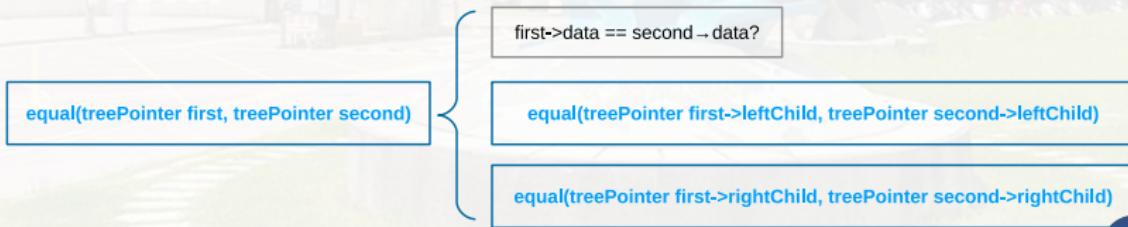
3 Additional Binary Tree Operations

Copying a Binary Tree

```
treePointer copy(treePointer original) {
    /* return a tree_pointer to an exact copy of the original tree */
    treePointer temp;
    if (original) {
        temp = (treePointer)malloc(sizeof(node));
        temp->leftChild = copy(original->leftChild);
        temp->rightChild = copy(original->rightChild);
        temp->data = original->data;
        return temp;
    }
    return NULL;
}
```

Testing for Equality of Binary Trees

```
int equal(treePointer first, treePointer second) {  
    /* return "false" if the binary trees first and second are not equal,  
    Otherwise it returns "true" */  
    return (  
        (!first && !second) || (first && second &&  
        (first->data == second->data) &&  
        equal(first->leftChild, second->leftChild) &&  
        equal(first->rightChild, second->rightChild))  
}
```



Discussions

