Binary Search Trees & Forests

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Outline

1. Binary Search Trees
2. Forests

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Binary Search Tree (BST) (1/2)

For searching, insertions and deletions...

Binary search tree provides a better performance then any of the data structures studied so far.

Binary Search Tree (BST) (2/2)

BST

A binary search tree (BST) is a binary tree which may be empty.

If it is not empty, then it satisfies the following properties:

- Each node has exactly one key and the keys in the tree are distinct.
- The keys (if any) in the left subtree are smaller than the key in the root.
- The keys (if any) in the right subtree are larger than the key in the root.
- The left and right subtrees are also binary search tree.

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    - a flavor of recursion?

Examples of BST

- Which one of the following trees is BST? Which one is NOT?

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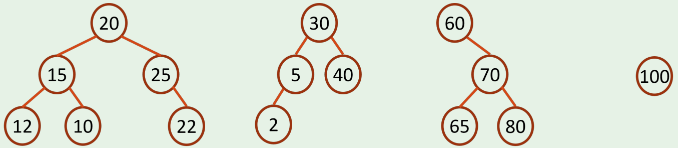


(a) (b) (c) (d)

Recursive Search of a BST

```
element* search(treePointer root, int key) {
/* return a pointer to the node that contains key,
   if there is no such node, return NULL. */
    if (!root) return NULL;
    if (k == root->data.key) return &(root->data);
    if (k < root->data.key)
        return search(root->leftChild, k);
    return search(root->rightChild, k);
}
```

- The time complexity of the search function is $O(h)$, where h is the height of the binary search tree.

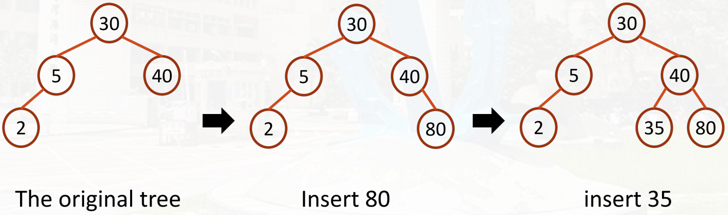
Iterative Search of a BST

```
element* iterSearch(treePointer tree, int k) {
/* return a pointer to the node that contains key,
   if there is no such node, return NULL. */
    while (tree) {
        if (k == tree->data.key) return &(tree->data);
        if (k < tree->data.key)
            tree = tree->leftChild;
        else
            tree = tree->rightChild;
    }
    return NULL;
}
```

- The time complexity of the search function is $O(h)$, where h is the height of the binary search tree.

Inserting into a BST

- Verify that the key is different from those existing elements.
- If the search is unsuccessful, insert the element at the point the search terminated.



A Modified Searching Function

`modifiedSearch`(treePointer *node, int k)

- If the BST is empty, then return NULL.
- If the key *k* exists in the BST, return NULL.
- Otherwise, return the pointer of the last node in the BST.

Inserting a Dictionary Pair into a BST

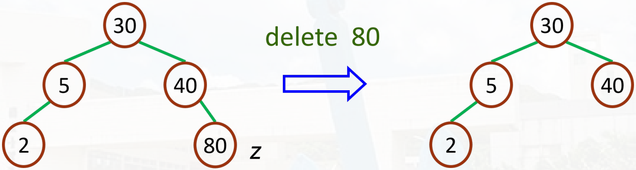
```
void insert(treePointer *node, int k, iType theItem) {
/* If k is in the tree pointed at by "node", do nothing;
   otherwise, add a new node with data = (k, theItem) */
    treePointer ptr, temp = modifiedSearch(*node, k);
    if (temp || !(*node)) { /* k is not in the tree */
        ptr = (treePointer)malloc(sizeof(node));
        ptr->data.key = k;
        ptr->data.item = theItem;
        ptr->leftChild = ptr->rightChild = NULL;
        if (*node) /* insert as child of temp */
            if (k < temp->data.key)
                temp->leftChild = ptr;
            else
                temp->rightChild = ptr;
        else // the case of empty tree
            *node = ptr;
    }
}
```

Deletion from a BST

- Case 1: **leaf**
  - delete the node and set the pointer from the parent node to NULL.
- Case 2: **having only one child**:
  - delete the node and change the pointer from the parent node to the single-child node.
- Case 3: **having two children**:
  - replaced by the largest element in its left subtree, or
    replaced by the smallest element in its right subtree.

Illustraton (Case 1 & 2)

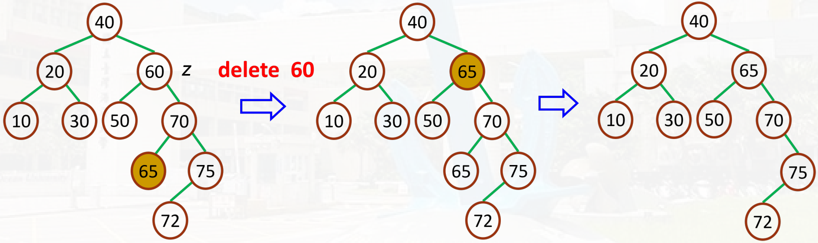
Case 1:



Case 2:

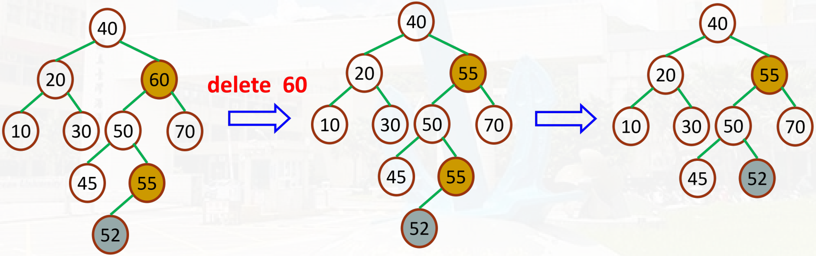
Illustraton (Case 3)

Case 3:



Illustraton (Case 3)

Case 3:



Time Complexity Analysis of Deleting a Node in a BST

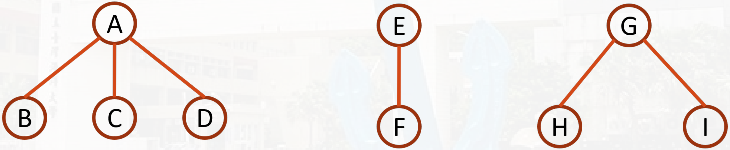
- The case: Deleting a nonleaf node that has two children.
- We can verify (Exercise) that, in both ways, it is originally in a node with a degree of at most one.
  - Check the largest and smallest elements in a subtree.
- The time complexity for case 3 is $O(h)$ (h: the height of the BST).
- A deletion can be performed in $O(h)$ time.

Outline

Forest

Forest

A forest is a set of $n \geq 0$ disjoint trees.



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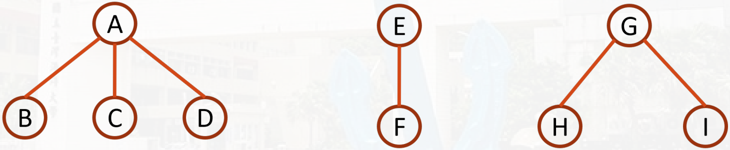


Question

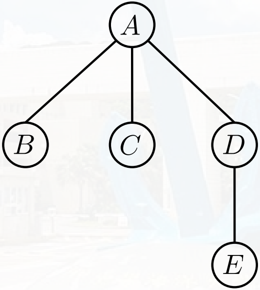
Is a binary tree a forest?

Rule of Transforming a Forest into a Binary Tree

- If $T_1, T_2, \ldots, T_n$ is a forest of disjoint trees $T_1, T_2, \ldots, T_n$, then the binary tree corresponding to this forest, denoted by $B(T_1, T_2, \ldots, T_n)$,
  - is empty if $n = 0$.
  - has root equal to root(T_1);
  - has left subtree equal to $B(T_{11}, T_{12}, \ldots, T_{1m})$, where $T_{11}, T_{12}, \ldots, T_{1m}$ are the subtrees of root(T_1); and
  - has right subtree $B(T_2, \ldots, T_n)$.

Binary Tree Representation of a Forest

How about this forest?



Discussions