

Threaded Binary Tree & Heaps

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Outline

1 Threaded Binary Trees (引線二元樹)

2 Heaps

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Threaded Binary Trees

Issue

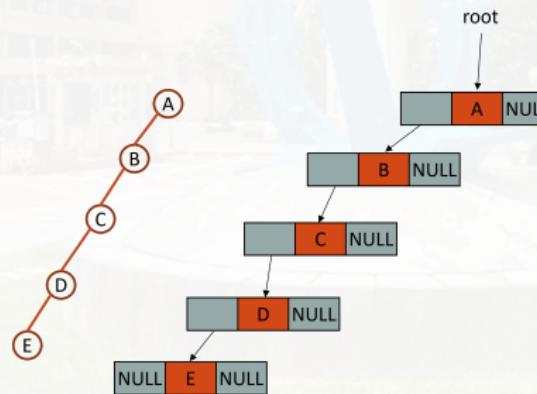
There are more null links than actual points.

Threaded Binary Trees

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- Number of nodes: n .
- Number of null non-null links: $n - 1$.
- Number of null links: $n + 1$.

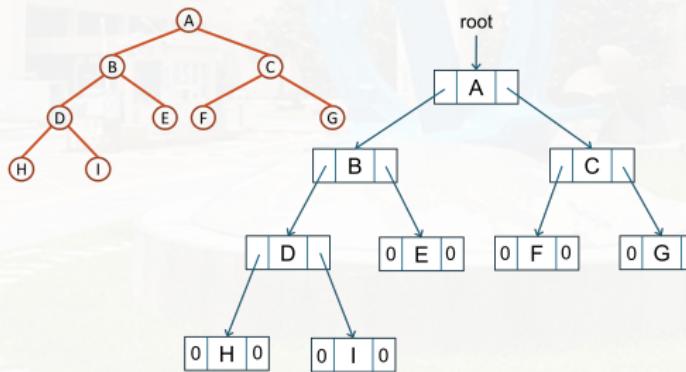


Threaded Binary Trees

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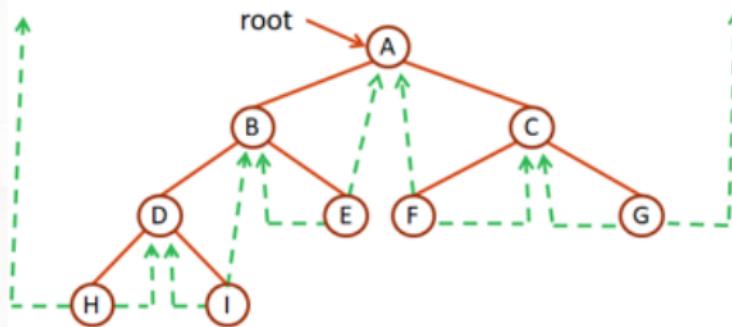
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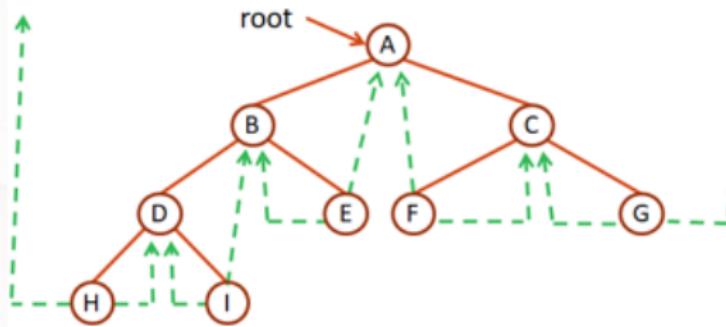
Solution

Replace the NULL links by pointers, **threads**, pointing to other nodes.



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Threading Rules

- if $\text{ptr} \rightarrow \text{leftChild}$ is NULL, then $\text{ptr} \rightarrow \text{leftChild} = \text{inorder predecessor}$ (中序前行者) of ptr .
- if $\text{ptr} \rightarrow \text{rightChild}$ is NULL, then $\text{ptr} \rightarrow \text{rightChild} = \text{inorder successor}$ (中序後續者) of ptr .

To distinguish between normal pointers and threads

- Two additional fields of the node structure: left-thread, right-thread.

```
typedef struct threadedTree *threadedPointer;

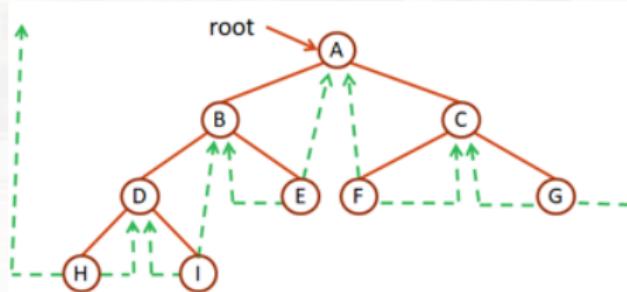
typedef struct threadedTree {
    bool leftThread;
    threadedPointer leftChild;
    char data;
    threadedPointer rightChild;
    bool rightThread;
};
```

leftThread	leftChild	data	rightChild	right Thread
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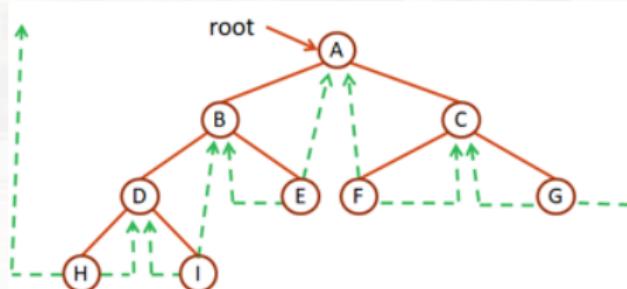
Rules of the Threading Fields

- If $\text{ptr} \rightarrow \text{leftThread} == \text{true}$, $\text{ptr} \rightarrow \text{leftChild}$ contains a thread; Otherwise, the node contains a pointer to the left child.
- If $\text{ptr} \rightarrow \text{rightThread} == \text{true}$, $\text{ptr} \rightarrow \text{rightChild}$ contains a thread; Otherwise, the node contains a pointer to the right child.



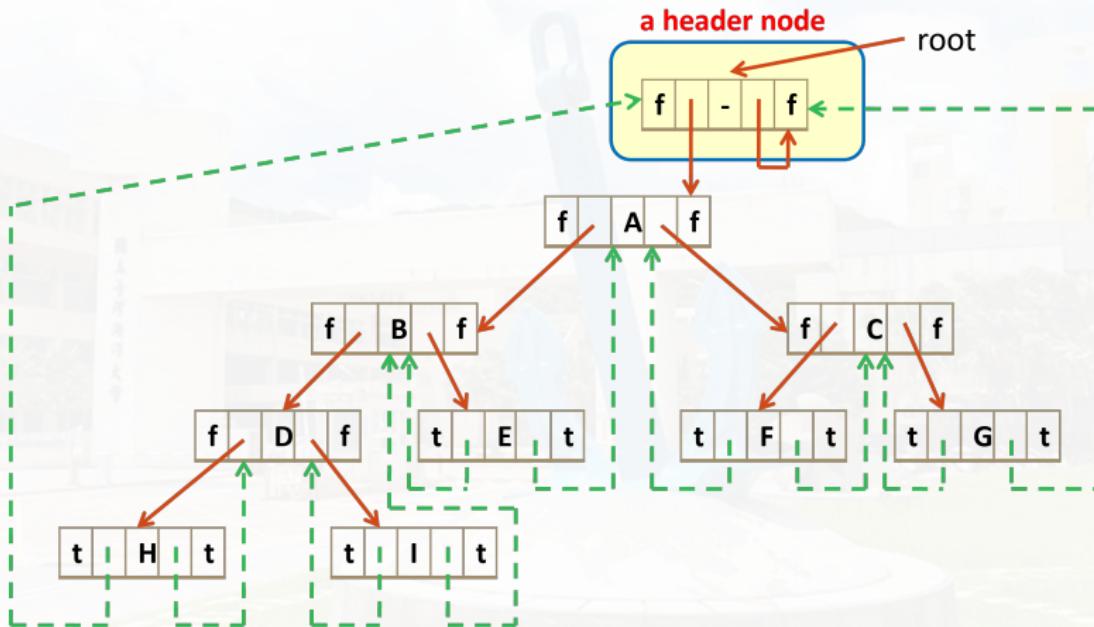
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- Two **dangling** threads at node H and G .
 ⇒ Use a header node to collect them!

- The original tree becomes the left subtree of the head node.



Representing an Empty Binary Tree

leftThread	leftChild	data	rightChild	rightThread
true	:	-	:	false

```
graph LR; LT1[true] --- LC1[::]; LC1 --- D1[-]; D1 --- RC1[::]; RC1 --- RT1[false];
```

Finding the Inorder Successor of Node

```
threadedPointer insucc(threadedPointer tree) {  
    /* find the inorder successor of tree in a threaded  
       binary tree */  
    threadedPointer temp;  
    temp = tree->rightChild;  
    if (!tree->rightThread) // rightChild exists!  
        while (!temp->leftThread)  
            temp = temp->leftChild;  
    return temp;  
}
```

To perform an inorder traversal, we can simply make repeated calls to insucc!



Inorder Traversal of a Threaded Binary Tree

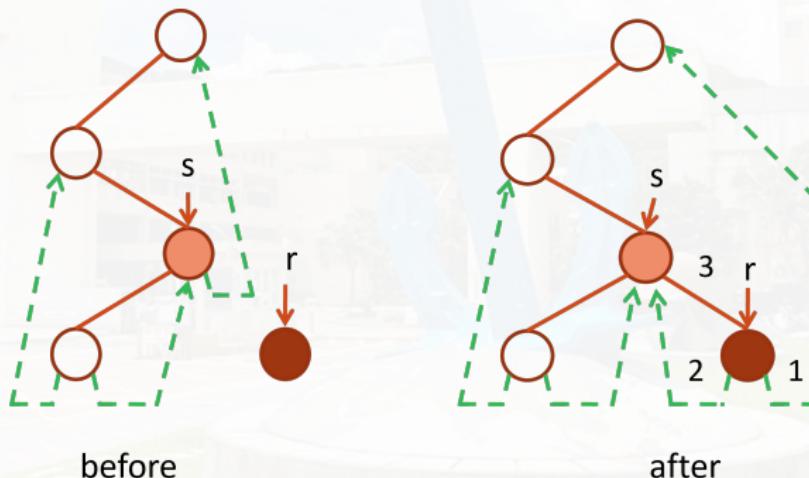
```
void traverseInorder(threadedPointer tree) {
    /* traverse the threaded binary tree inorder */
    threadedPointer temp = tree;
    while (1) {
        temp = insucc(temp);
        if (temp == tree)
            break;
        printf("%3c", temp->data);
    }
}
```

- **Note:** `temp == tree` happens when the last node is visited (then the successor becomes the header node).



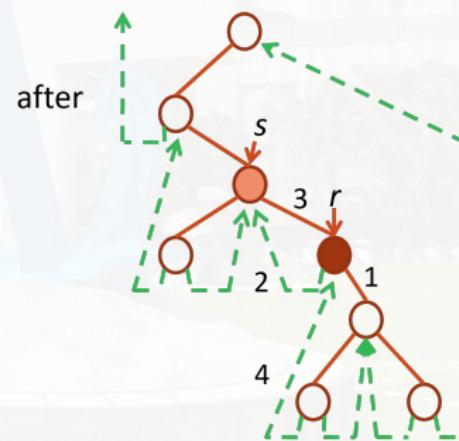
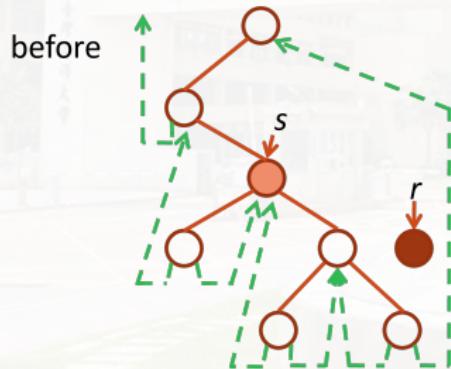
Inserting r as the rightChild of a node s

- Case I: $s \rightarrow \text{rightThread} == \text{true}$ (s has an empty subtree)



Inserting r as the rightChild of a node s

- Case II: $s \rightarrow \text{rightThread} == \text{false}$
(the right subtree of s is not empty)

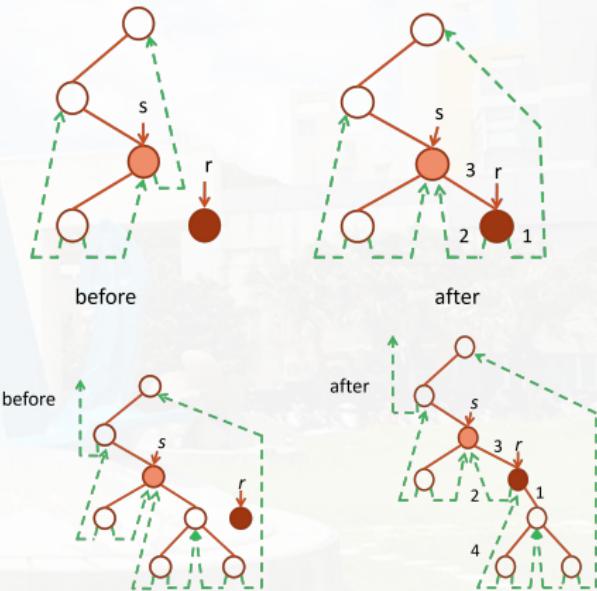


The Code for the Insertion

```

void insertRight (threadedPointer s,
                  threadedPointer r) {
/* insert r as the right child of s */
    threadedPointer temp;
    r->rightChild = s->rightChild;
    r->rightThread = s->rightThread; // (*)
    r->leftChild = s;
    r->leftThread = true;
    s->rightChild = r;
    s->rightThread = false;
    if (!r->rightThread){ // step 4 (*)
        temp = insucc(r);
        temp->leftChild = r;
    }
}

```



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Heaps

Max Tree

A **max tree** is a tree in which

- the key value in each node \geq the key values in its children.

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Max Tree

A **max tree** is a tree in which

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Min Tree

A **min tree** is a tree in which

- the key value in each node \leq the key values in its children.

Max Heap

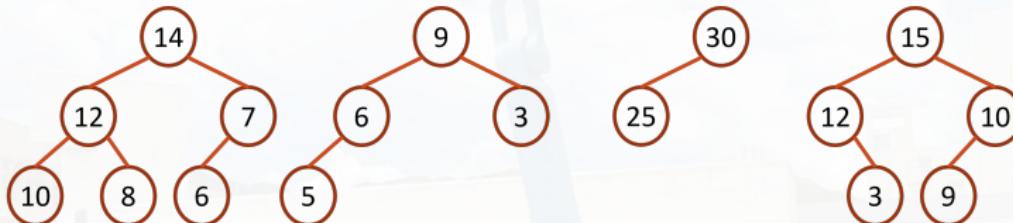
A complete binary tree that is also a max tree.

Min Heap

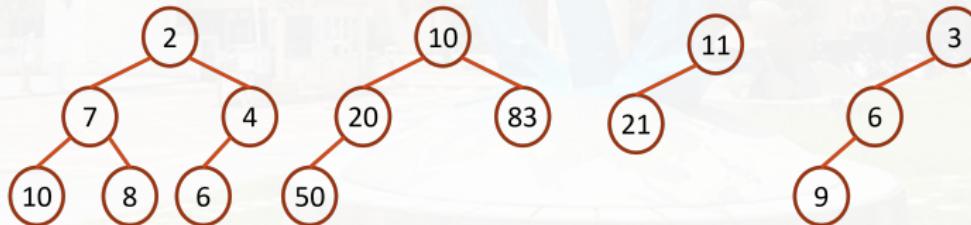
A complete binary tree that is also a min tree.



Examples: Max & Min Trees

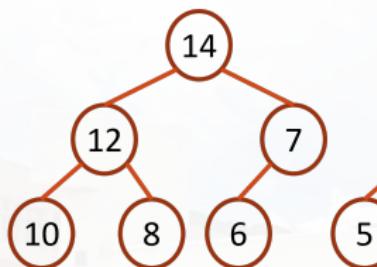


Max Trees

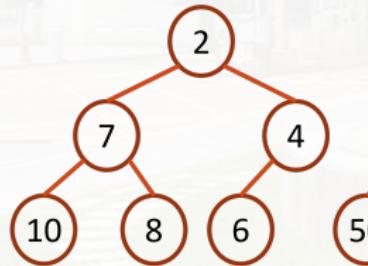


Min Trees

Examples: Max & Min Heaps



Max Heaps



Min Heaps

The Key Application: Priority Queues

- Heaps are frequently used to implement **priority queues**.
- In this kind of queue,
 - the element to be **deleted** is the one with **highest** (or **lowest**) priority.
 - at **any time**, an element with **arbitrary priority** can be **inserted** into the queue.

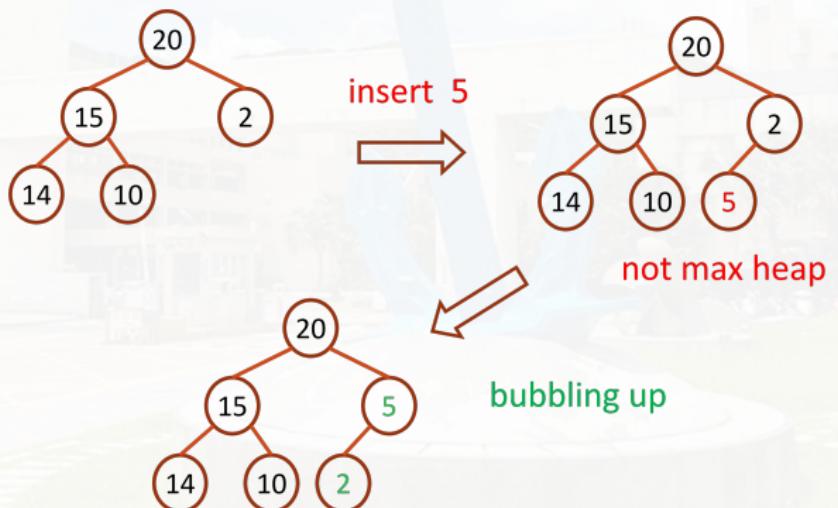
Insertion into a Max Heap

- The **bubbling process**.
 - It begins at the new node of the tree and moves toward the root.



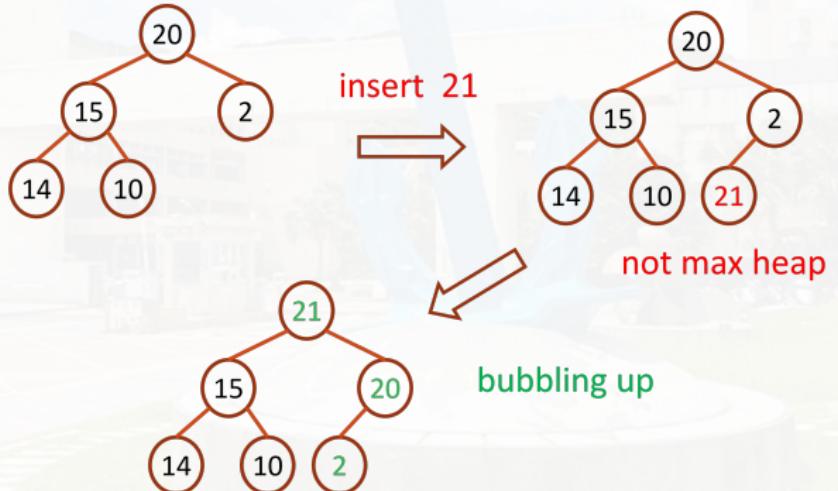
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The Code for Insertion into a Max Heap

- Consider the following declarations:

```
#define MAX_ELEMENTS 200 /* maximum heap size+1 */  
#define HEAP_FULL (n) (n == MAX_ELEMENTS -1)  
#define HEAP_EMPTY (n) (!n)  
typedef struct {  
    int key;  
    /* other fields */  
} element;  
element heap[MAX_ELEMENTS];  
int n = 0;
```

The Code for Insertion into a Max Heap

```
void push (element item, int *n) {
    /* insert item into a max heap of current size *n */
    int i;
    if (HEAP_FULL(*n)) {
        printf("The heap is full.\n");
        exit(EXIT_FAILURE);
    } // O(1) time
    i = ++(*n);
    while ((i != 1) && (item.key > heap[i/2].key)) {
        heap[i] = heap[i/2];
        i /= 2;
    } // O(lg n) time
    heap[i] = item; // O(1) time
}
```

- The time complexity of the insertion: $O(\lg n)$.



Deletion from a Max Heap

- When an element is to be deleted from a max heap, it is **ALWAYS** taken from the root of the heap.

Deletion from a Max Heap

- When an element is to be deleted from a max heap, it is **ALWAYS** taken from the root of the heap.
- The steps of deletion from a Max heap:
 - delete the root node.
 - insert the last node into the root.
 - use the **bubbling up process** to ensure that the resulting heap remains a max heap.

Illustration of Deletion from a Max Heap

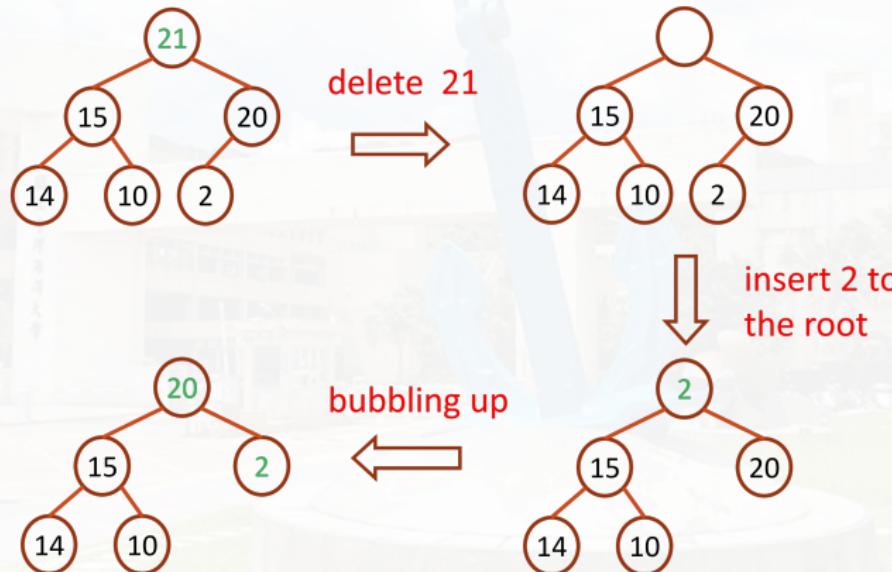
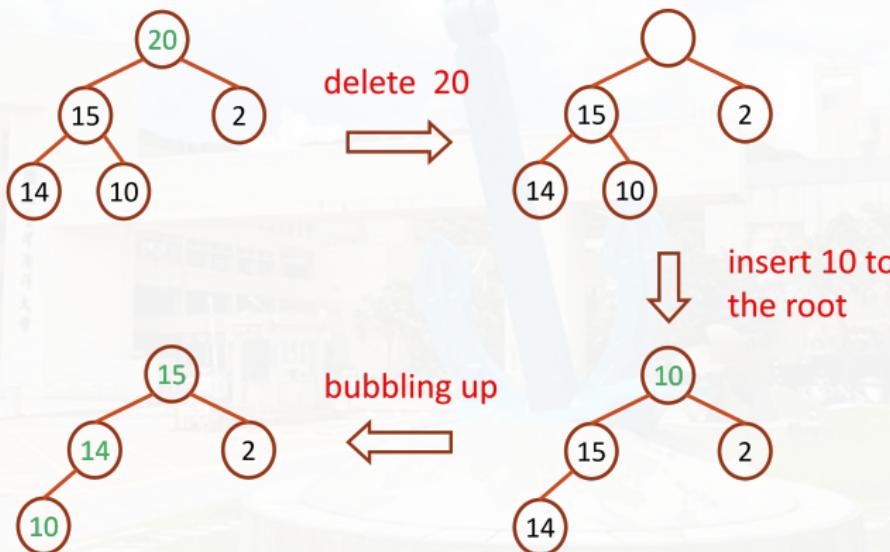


Illustration of Deletion from a Max Heap



The Code for Deletion from a Max Heap

```
element pop(int *n) {
    /* delete element with the highest key from the heap */
    int parent, child;
    element item, temp;
    if (HEAP_EMPTY(*n)) {
        fprintf(stderr, "The heap is empty\n");
        exit(EXIT_FAILURE);
    }
    /* save value of the element with the highest key */
    item = heap[1];
    /* use last element in heap to adjust heap */
    temp = heap[(*n)--];
    parent = 1;
    child = 2; // default: the left child
    while (child <= *n) { // O(lg n) time
        /* find the larger child of the current parent */
        if ((child < *n) && (heap[child].key < heap[child+1].key))
            child++; // okay, it's the right child!
        if (temp.key >= heap[child].key) break; // the new root is the maximum!
        /* move to the next lower level */
        heap[parent] = heap[child];
        parent = child;
        child *= 2;
    }
    heap[parent] = temp;
    return item;
}
```

Time Complexity of the Deletion from a Max Heap

- Delete the root node: $O(1)$.
- Insert the last node to the root: $O(1)$.
- Since the height of the heap is $\lceil \lg(n + 1) \rceil$, the while loop is iterated for $O(\lg n)$ times.
- Thus, the overall time complexity: the time complexity of the deletion: $O(\log n)$.

Discussions

