Authors' Response

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We thank the associated editor and the three reviewers for the constructive comments and feedback. These comments, from the three reviewers, brought significant value to our manuscript. Below, we give responses to explain how we address all the comments.

Reviewer 1

1. This article makes use of several external libraries. Those need to be downloaded and embedded in the local files of the article, to ensure that this includes:

   - https://cdn.jsdelivr.net/npm/bootstrap@5.0.2/dist/css/bootstrap.min.css
   - https://cdn.jsdelivr.net/npm/bootstrap@5.0.2/dist/js/bootstrap.bundle.min.js

   Response: The mentioned external libraries are sourced from Bootstrap. In order to ensure the optimal functionality of all the features in this article, we have replaced all Bootstrap-supported classes with native JavaScript implementations. For example, the *mb-0* class in Bootstrap has been replaced with a custom-defined class, *my-auto lh-sm*. We believe that this adjustment can guarantee the smooth operation of various features.

2. <section id="introduction"> is never closed. Please add </section> before the start of the next section

   Response: Thank you for bringing this to our attention. We have addressed and resolved the issue.

3. lines 1241 and beyond: the transition property is marked with an error. I am not an expert of transitions myself, but "left" is not a valid property. You might have meant "linear" instead. Furthermore the ticks need to be removed. So consider replacing it with style="transition: linear .3s ease-in; height: 100px;"

   Response: Thanks for your comment. The `left` property is commonly used for positioning elements in CSS, determining the horizontal position relative to its containing element. As it is not directly related to the `transition` property, you cannot use it as a standalone property within `transition`. However, you can transition the `left` property indirectly by including it as one of the properties affected by the `transition` property. For example, `style="transition: left .3s ease-in; height: 100px;"`

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sets inline CSS styles for an image element. Here, `transition: left .3s ease-in` is a shorthand property for defining the transition effect. It specifies that any changes to the `left` property should have a duration of 0.3 seconds and use an ease-in timing function. The use of the `left` property within the `transition` property is not invalid in CSS. It is a common technique to achieve smooth transitions. Some examples and discussions related to this technique can be found on technical forums (e.g., Stack Overflow). You can refer to the following links for more information: 1) Using CSS transition with "left" property: `https://stackoverflow.com/questions/65100863/using-css-transition-with-left-property-doesnt-transition-smoothly` and 2) CSS transition with "top," "bottom," "left," and "right": `https://stackoverflow.com/questions/20383393/css-transition-doesnt-work- with-top-bottom-left-right`.

4. the ids "best_response", "best-response-discussion", "flexRadioDefault5", "flexRadioDefault6", "flexRadioDefault7", "flexRadioDefault8", "gaa_no_regularizers" appear multiple times.

   Response: We have reviewed the entire HTML file to ensure that each ID is unique and only appears once throughout the file. We believe the overall integrity of the HTML structure has been improved.

5. line 1410 and beyond: the svg width has a small typo. replace 250xp with 250px. The same typo appears in a later figure again.

   Response: These typos have been corrected in our revised file.

Reviewer 2

The authors focus their study on the best response dynamics and multi agent online learning by introducing a group formation approach. Specifically, the authors study how the online learning algorithms can help stabilize the entire system composed by strategic agents that perform their best response. The manuscript is overall well written and easy to follow and the authors have well thought out their main contributions. The provided theoretical analysis concrete, complete, and correct and the authors have provided all the intermediate steps in order to enable the average reader to easily follow it. Furthermore, the limited provided numerical results show the success of the proposed online gradient ascent approach with regularization. The authors are highly encouraged to consider the following suggestions provided by the reviewer in order to improve the scientific depth of their manuscript, as well as they need to address the following minor comments in order to improve the quality of presentation of the manuscript. Initially, the concept of satisfaction games has been recently proposed in the literature, A Paradigm Shift Toward Satisfaction, Realism and Efficiency in Wireless Networks Resource Sharing, doi: 10.1109/MNET.011.2000368, and provides a new perspective on the multi agent systems in terms of decision making. In Section 1, the authors need to discuss this aspect and clarify why the authors focus their research on maximizing the payoff rather than satisfying minimum constraints which is the current approach in the state-of-the-art. In Section 2, the physical meaning of the public preference vector and the private preference vector needs to be provided and tied with some indicative applications to enable the average reader to follow the rest of the analysis. In Section 4, the authors need to definitely provide the computational complexity of the multi agent online gradient descent algorithm and also provide some indicative numerical results in order to quantify its complexity towards realizing the pure strategy Nash equilibrium. Finally, the overall manuscript needs to be checked for typos, syntax, and grammar errors in order to improve the quality of its presentation.

1. Initially, the concept of satisfaction games has been recently proposed in the literature, *A Paradigm Shift Toward Satisfaction, Realism and Efficiency in Wireless Networks Re-*

source Sharing, doi: 10.1109/MNET.011.2000368, and provides a new perspective on the multi agent systems in terms of decision making. In Section 1, the authors need to discuss this aspect and clarify why the authors focus their research on maximizing the payoff rather than satisfying minimum constraints which is the current approach in the state-of-the-art.

Response: We thank the reviewer for the comment. We find that Papavassiliou et al.'s work "A Paradigm Shift Toward Satisfaction, Realism and Efficiency in Wireless Networks Resource Sharing, doi: 10.1109/MNET.011.2000368" is interesting and relevant to our work. Hence, we add a paragraph in subsection "RELATED WORK" of Section 1 and discuss the "satisfaction equilibrium" where each player targets satisfaction of quality of service requirements rather than individual utility maximization. We point out why we focus on utility maximization and why the satisfaction equilibrium is not applicable in our case.

2. In Section 2, the physical meaning of the public preference vector and the private preference vector needs to be provided and tied with some indicative applications to enable the average reader to follow the rest of the analysis.

   Response: We thank the reviewer for this helpful comment. In the first paragraph of Section 2, we add physical meaning of the preference vectors and provide examples and arguments to motivate the our discussion and facilitate the readers to follow the rest of the analysis as well.

3. In Section 4, the authors need to definitely provide the computational complexity of the multi agent online gradient descent algorithm and also provide some indicative numerical results in order to quantify its complexity towards realizing the pure strategy Nash equilibrium

   Response: We thank the reviewer for the comment. The analysis of computational complexity of our algorithm is added in Section 4. Furthermore, we've also added indicative numerical results to exemplify our proposed multi-agent online learning algorithm just before the analysis of the computational complexity.

4. Finally, the overall manuscript needs to be checked for typos, syntax, and grammar errors in order to improve the quality of its presentation.

   Response: We thank the reviewer for the suggestion. We have sent the manuscript to a professional academic English editing service (the receipt is attached as a certificate) and improved its presentation quality. In addition, The REFERENCES are listed in alphabetical order now.

Reviewer 3

This paper provides an interesting interactive application to facilitate audiences to grasp the idea of pure-strategy Nash equilibrium and how the system converges to a stable state by means of decentralized online gradient ascent. However, even though this work has some contributions, both theoretically and practically, the main contribution is not clearly described. Moreover, the details of this research cannot be found in this paper. Here are some of the issues that the authors need to address to improve the quality of the paper.

Overall, some contents of this paper are not mature enough to merit for publication in this journal. I suggest that this paper be revised to improve its quality.

1. The related work section should be improved. More related works need to be added in this paper to give a complete discussion of such system.

Response: We thank the reviewer for the comment. We have added subsection "RELATED WORK" in Section 1. Four categories of related literature, such as "On Group Formation and Collective Behavior", "On Opinion Formation", "On Satisfaction Game", and "On Multiagent Online Learning", are discussed there.

2. I also suggest that authors need to give more details about the proposed algorithm (or system) to attract the attention of researchers.

   Response: We thank the reviewer for the comment. We exemplify our algorithm by providing an illustrating example of how the algorithm computes the utilities, winning probabilities, reward, gradients, updates, etc., of agents. We've also improved the statements in Section 4 (the paragraph after Figure 7 and the second paragraph after "Regularization") to better explain and motivate our multiagent online learning algorithm.

3. More comparisons between the proposed algorithm (or system) and other recent state-of-the-art algorithms should be given to show the importance of this research.

   Response: We thank the reviewer for the comment. In the new added subsection "RELATED WORK" in Section 1, We discuss other recent state-of-the-art algorithms or approaches and point out the connection and difference between our work and the related work. We also point out the importance of our work there.