How Bad Can an Election Game of Two or More Parties Be?

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a joint work with

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Outline

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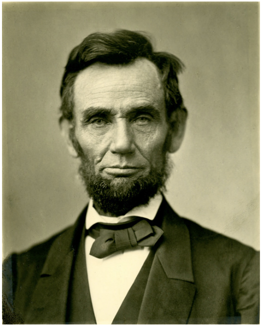
The Inspiration (an EC'17 paper)



"[. . .] and that government of the people, by the people, for the people, shall not perish from the earth."

— Abraham Lincoln, 1863.

Most Previous Studies from a Micro Perspective

- Strategic behaviors of voters.
- Design of ballots.
- Social choice function or voting rules.

  ⋮

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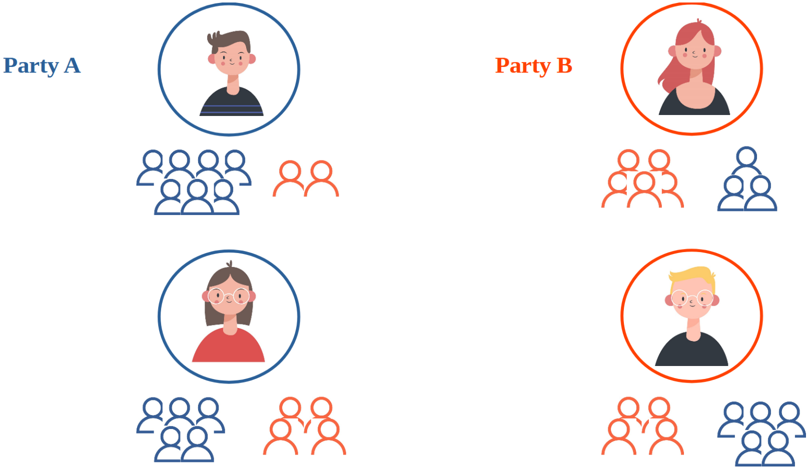
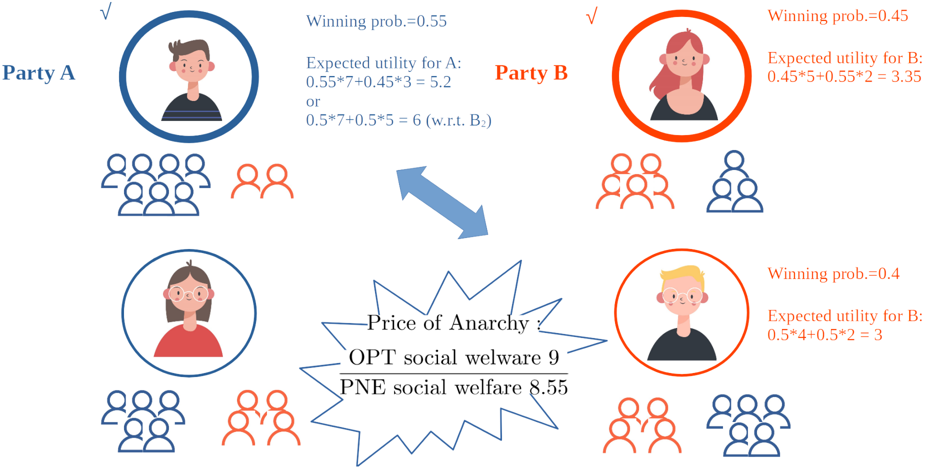
  - A candidate beats the other one from other parties with uncertainty.
  - The payoff of each party: expected utility its supporters can get.
  - The egoistic property: each candidate of party $\mathcal{P}$ brings more utility to $\mathcal{P}$'s supporters than any candidate from the other parties does to $\mathcal{P}$'s supporters.


Party A
Party B

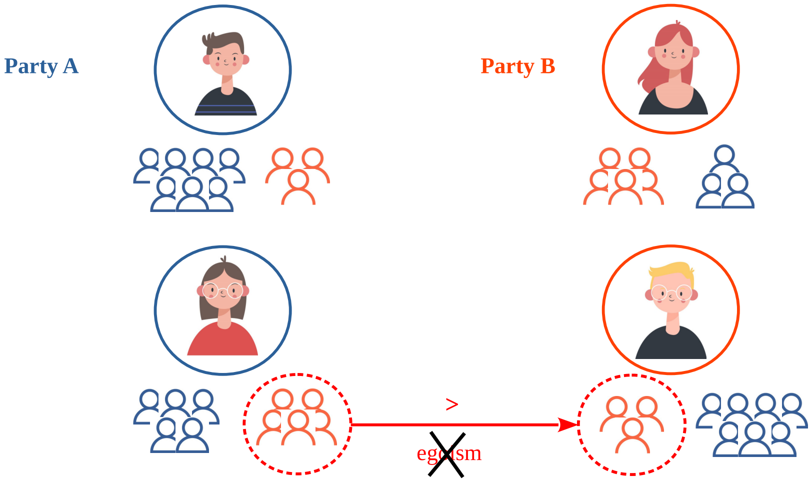
Two-Party Election Game: Formal Setting

- Party A: m candidates, party B: n candidates.
- Candidate A_i can bring social utility $u(A_i) = u_A(A_i) + u_B(A_i) \in [0, \beta]$ for some real $\beta \geq 1$.
- $p_{i,j}$: $\Pr[A_i$ wins over $B_j]$.
  - Linear: $p_{i,j} := (1 + (u(A_i) - u(B_j))/\beta)/2$
  - Natural: $p_{i,j} := u(A_i)/(u(A_i) + u(B_j))$
  - Softmax: $p_{i,j} := e^{u(A_i)/\beta}/(e^{u(A_i)/\beta} + e^{u(B_j)/\beta})$
- Reward $r_A = p_{i,j}u_A(A_i) + (1 - p_{i,j})u_A(B_j)$.

Price of Anarchy (poA)



Egoism (Selfishness)



m-Party Election Game, $m \geq 2$

- Party $\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \ldots$: with $n_1, n_2, n_3, \ldots$ candidates, resp.
- E.g., candidate s_i of party $\mathcal{P}_i$ can bring social utility $u(s_i) = u_1(s_i) + u_2(s_i) + \ldots + u_m(s_i) \in [0, \beta]$ for some $\beta \geq 0$.
- $p_{i,\mathbf{s}}$: $\Pr[s_i$ wins the campaign w.r.t. $\mathbf{s}]$.
  - $\mathbf{s}$: the strategy profile of all party players.
  - Consider all monotone winning probability functions.
    - E.g., $p_{i,\mathbf{s}_{-i}} \geq p_{i',\mathbf{s}_{-i}}$ whenever $u(s_i) \geq u(s_{i'})$.
- Reward $r_i = p_{1,\mathbf{s}_{-i}} u_i(s_1) + p_{2,\mathbf{s}_{-i}} u_i(s_2) + \cdots + p_{m,\mathbf{s}_{-i}} u_i(s_m)$.

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    - Upper bound: number of parties.
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    - Upper bound: number of parties.
    - The bound is tight for some cases.
  - Incentives of forming a coalition for each party.

Counterexamples (Natural function)

A		B	
$u_A(A_i)$	$u_B(A_i)$	$u_B(B_j)$	$u_A(B_j)$
91	0	11	1
90	8	10	20

A		B	
$u_A(A_i)$	$u_B(A_i)$	$u_B(B_j)$	$u_A(B_j)$
44	10	37	17
39	55	10	5

	B_1	B_2
A_1	$a_{1,1},\ b_{1,1}$	$a_{1,2},\ b_{1,2}$
A_2	$a_{2,1},\ b_{2,1}$	$a_{2,2},\ b_{2,2}$

$\approx$

	B_1	B_2
A_1	80.51, 1.28	73.84, 2.17
A_2	80.29, 8.32	74.02, 8.23

,

	B_1	B_2
A_1	30.50, 23.50	35.52, 10.00
A_2	30.97, 48.43	34.32, 48.81

Counterexamples (Softmax; Three parties)

$u_1(x_{1,i})$	$u_2(x_{1,i})$	$u_3(x_{1,i})$	$u_1(x_{2,i})$	$u_2(x_{2,i})$	$u_3(x_{2,i})$	$u_1(x_{3,i})$	$u_2(x_{3,i})$	$u_3(x_{3,i})$
29	4	21	23	59	7	8	32	54
27	43	3	3	57	38	20	13	53

$$\begin{array}{ccc|ccc} r_{1,(1,1,1)} & r_{2,(1,1,1)} & r_{3,(1,1,1)} & r_{1,(1,1,2)} & r_{2,(1,1,2)} & r_{3,(1,1,2)} \\ \hline r_{1,(1,2,1)} & r_{2,(1,2,1)} & r_{3,(1,2,1)} & r_{1,(1,2,2)} & r_{2,(1,2,2)} & r_{3,(1,2,2)} \end{array} \approx \begin{array}{ccc|ccc} 18.81 & 34.64 & 28.51 & 23.49 & 27.82 & 27.38 \\ \hline 11.27 & 34.67 & 39.70 & 15.57 & 28.09 & 38.93 \end{array}$$

$$\begin{array}{ccc|ccc} r_{1,(2,1,1)} & r_{2,(2,1,1)} & r_{3,(2,1,1)} & r_{1,(2,1,2)} & r_{2,(2,1,2)} & r_{3,(2,1,2)} \\ \hline r_{1,(2,2,1)} & r_{2,(2,2,1)} & r_{3,(2,2,1)} & r_{1,(2,2,2)} & r_{2,(2,2,2)} & r_{3,(2,2,2)} \end{array} \approx \begin{array}{ccc|ccc} 18.74 & 44.53 & 22.84 & 23.18 & 38.35 & 21.61 \\ \hline 11.58 & 44.25 & 33.66 & 15.67 & 38.27 & 32.77 \end{array}$$

Previous Results (Two-Party)

	Linear	Natural	Softmax
PNE w/ egoism	✓	$\times$	✓
PNE w/o egoism	$\times$	$\times$	$?^{\#}$
Worst PoA w/ egoism	$\leq 2^*$	≤ 2	$\leq 1 + e$
Worst PoA w/o egoism	∞	∞	∞

- Lin, Lu, Chen: *Theoret. Comput. Sci.*, 2021.

Complexity & PoA Bounds for $m \geq 2$ Parties (GAIW'2024)

Non-Existence of a Pure-Strategy Nash Equilibrium

The three-party election game does NOT always have a PSNE.

Theorem

For any m-party election game, $m \geq 2$, we have PoA $\leq m$ if

- The winning probability function is **monotone**.
- The game is egoistic.

Theorem

To determine if a PSNE exists in the egoistic m-party election game is NP-complete but FPT (+natural parameters).

- The game instance is in a succinct representation.
- A reduction from the SAT problem.

Key Propositions

For the egoistic election game:

Proposition

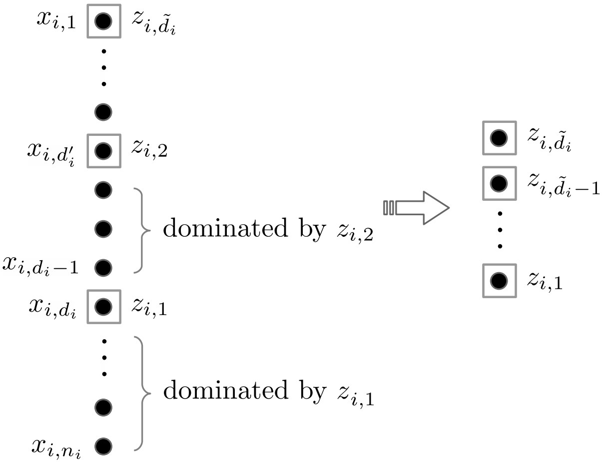
Let $\mathbf{s} = (s_i)_{i\in[m]}$ be a PSNE and $\mathbf{s}^* = (s_i^*)_{i\in[m]}$ be the optimal profile. Then, $\sum_{i\in[m]} u(s_i) \geq \max_{i\in[m]} u(s_i^*)$.

Two Important Observations

$$SW(\mathbf{s}) = \sum_{1\leq i\leq m} p_{i,\mathbf{s}} \cdot u(s_i) \leq \max_{1\leq i\leq m} u(s_i)$$

$$SW(\mathbf{s}) = \sum_{1\leq i\leq m} p_{i,\mathbf{s}} \cdot u(s_i) \geq \frac{1}{m} \cdot \sum_{1\leq i\leq m} u(s_i).$$

Shrinking nominating depth of a party



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Concluding Remarks

- We assume the utility is evenly distributed to the voters.
- The PoA is small in most game instances (simulations).
- We will conduct experiments to simulate voters' voting decisions to see how *monotone* the winning probability function is.

Thanks for your attention!