

Algorithmic Game Theory

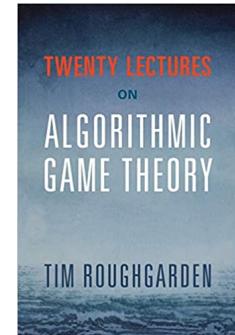
Introduction

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Textbooks and Materials

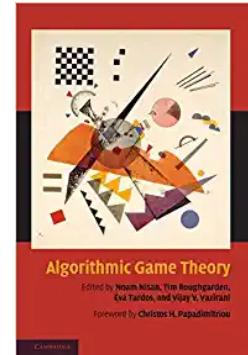
- Textbooks/Lectures:

- *Twenty Lectures on Algorithmic Game Theory*. Tim Roughgarden. Cambridge University Press. 2016.
- *Algorithmic Game Theory*. Noam Nisan, Tim Roughgarden, Eva Tardos, and Vijay V. Vazirani. Cambridge University Press. 2011.



- Other materials:

- *Essentials of Game Theory: A Concise, Multidisciplinary Introduction* (Synthesis Lectures on Artificial Intelligence and Machine Learning). Leyton- Brown and Kevin, Shoham. Cambridge University Press. 2008.



Prerequisites

- Basic CS undergraduate courses in
 - Algorithms
 - Probability theory
 - Discrete mathematics
 - Coding
- Motivation.
- Curiosity.

Topics (adjusted accordingly)

- Examples for Warm-Up
- Basic Solution Concepts
- Matrix Form and Extensive Form (Dynamic Games)
- Equilibria
- Inefficiency of Equilibria
- Network Formation Games
- Social Choice + Easy Project

Grading Policy

- Attendance (10%)
- Course-interaction (20%)
- Assignments \times 4–5 (20%)
- Project* \times 1 (20%)
- Final Presentation[†] (30%)

* Project demo: in TKU's midterm week.

† Final Presentation: in TKU's final exam week.

Grading Policy

- Attendance (10%) ✓
- Course-interaction (20%) ✓
- Assignments x 4–5 (20%) 😊 → Most of them can be finished in the class.
- Project* x 1 (20%) 😊 → 80% of the source codes will be given.
- Final Presentation[†] (30%) ❤ → Find a paper (related to game theory) to study by yourself.

* Project demo: in TKU's midterm week.

† Final Presentation: in TKU's final exam week.

Where to find the paper? (suggestions)

*Please look for papers in 2015—present.

- **International conferences:**

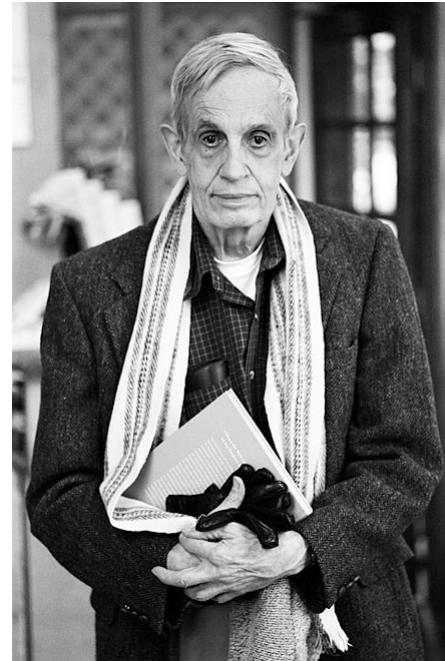
- EC: Proceedings of the ACM conference on Electronic commerce.
- NeurIPS: Conference on Neural Information Processing Systems.
- AAAI: Association for the Advancement of Artificial Intelligence.
- IJCAI: International Joint Conferences on Artificial Intelligence.
- AAMAS: International Conference on Autonomous Agents and Multiagent Systems.
- SAGT: Symposium on Algorithmic Game Theory.
- WINE: International Conference on Web and Internet Economics.
- ...

- **Journals:**

- **Games and Economic Behavior**
- **International Journal of Game Theory**
- ...

John Forbes Nash Jr. (1928–2015)

- American mathematician.
- Fundamental contributions to game theory.
- **Nobel Memorial Prize** in Economic Sciences with game theorists Reinhard Selten and John Harsanyi.
- **Abel Prize** with Louis Nirenberg for his work on nonlinear partial differential equations.

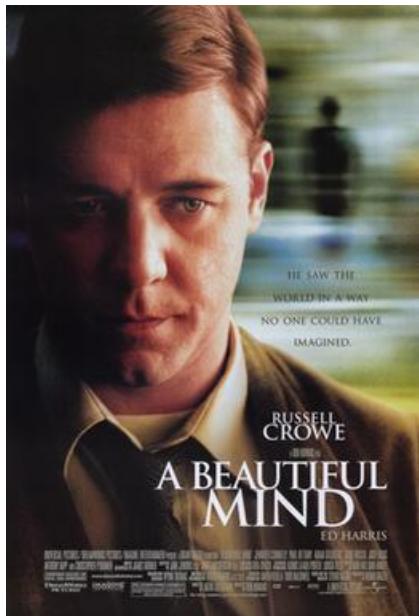


Nash in 2006.

Reference:
https://en.wikipedia.org/wiki/John_Forbes_Nash_Jr.

A classic scene of “A Beautiful Mind”

- https://www.youtube.com/watch?v=2d_dtTZQyUM



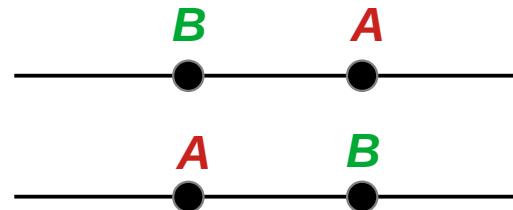
Starring: Russel Crowe

Before introducing Nash Equilibria...

- Let's play around several “games” first.

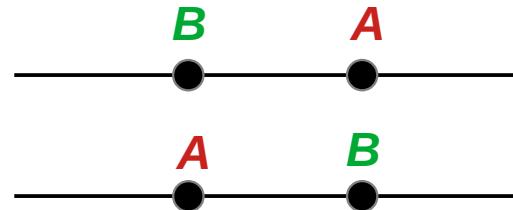
Number Guessing

- Let's say I have chosen a secret number A in my mind, which is among 1 and 100.
- Please guess it by a number B .
- If $B < A$, I will tell you “Larger, please”.
- If $B > A$, I will tell you “Smaller, please”.
- How many times do you think you can find out this secret number?



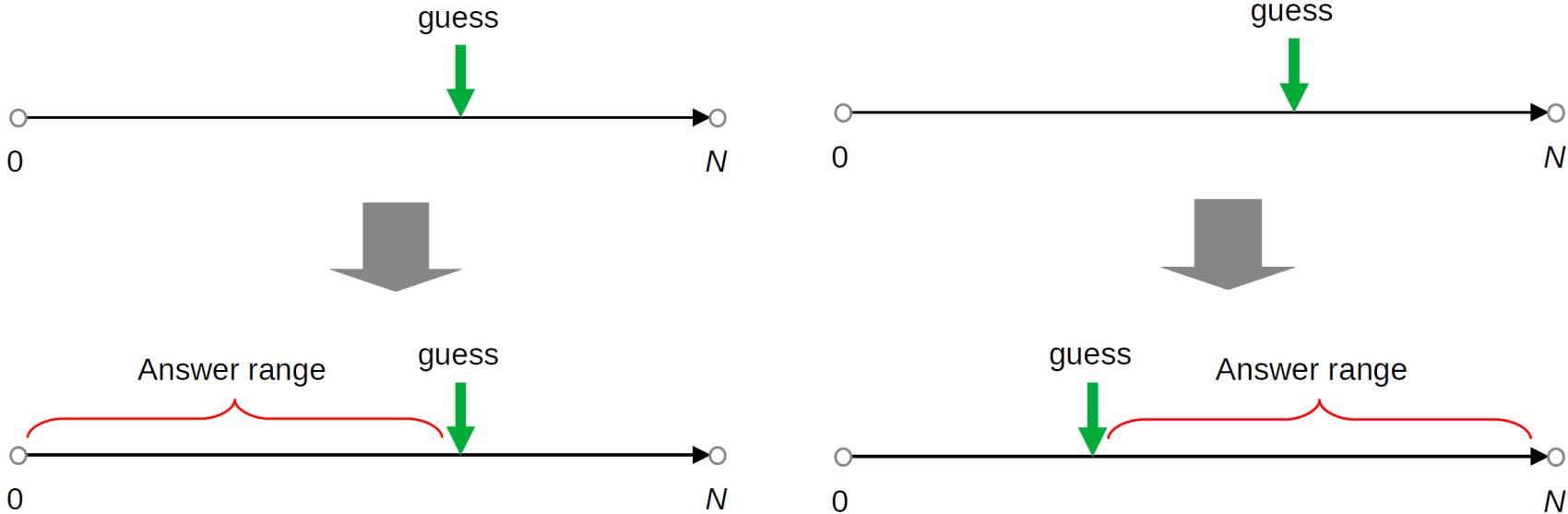
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- How many times do you think you can find out this secret number? **Let's play to feel the strategic behaviors.**



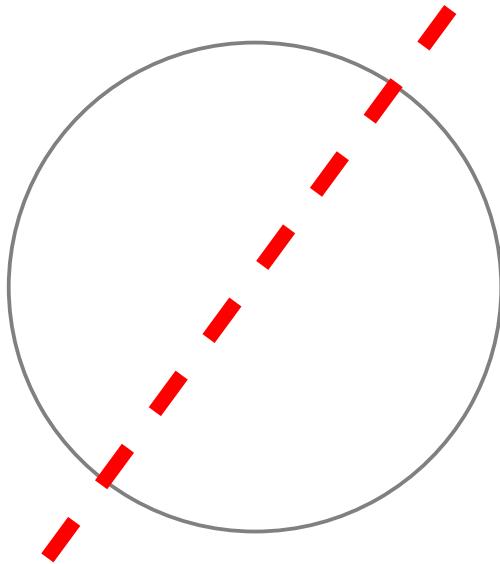
Adversarial Number Guessing

- The **demo** code.



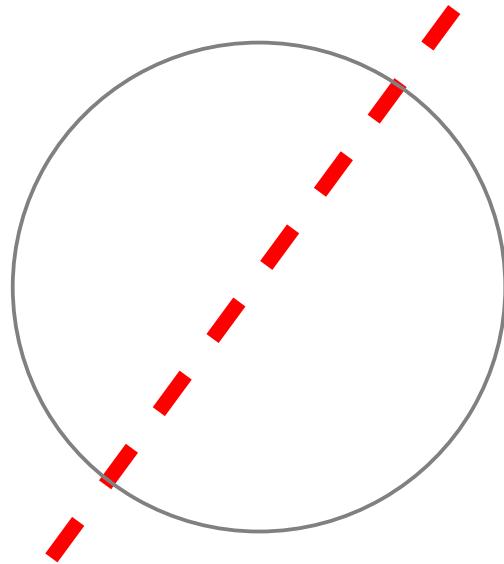
Envy-Free Cake-Cutting

- Children wants everything to be FAIR.
- Actually, in their world, nothing is FAIR..... lol



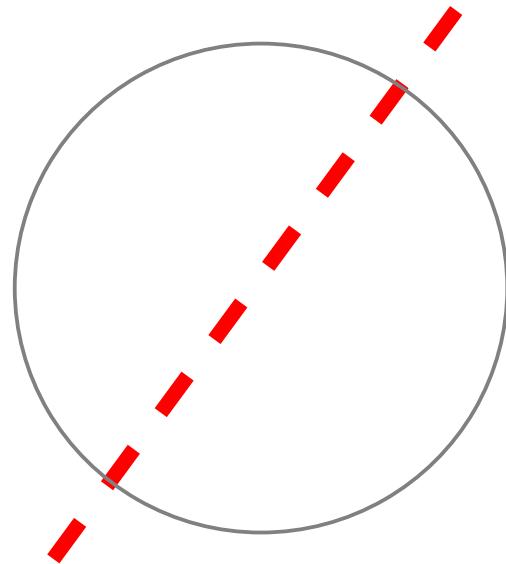
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Envy-Free Cake-Cutting

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- Actually, in their world, nothing is FAIR..... lol
- Let's say we want two kids to share a cake.
- Can you propose a way of cutting a cake so that two kids share a cake so that **no one envies the other?**



Prisoners' Dilemma

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- They had hidied the treasure before the police caught them.



source

Prisoners' Dilemma

- It was originally framed by Merrill Flood and Melvin Dresher in 1950.
- Let's say there are two guys, **A** and **B**, who broke into a luxury store and stole a treasure.
- They had hidied the treasure before the police caught them.
- They were kept in two separated rooms.
 - That means, they cannot communicate with each other...
- Each of them was offered two choices: **Denial** or **confession**.

Prisoners' Dilemma

- They were told that:

Prisoners' Dilemma

- They were told that:
 - If both of them deny the fact of stealing the treasure, they will **BOTH** be sentenced in prison for **one** month.
 - If one of them confesses while the other one denies, the former will be set **FREE** while the latter will be sentenced in prison for **9** months.
 - If both confess, then they will both get **6** months in prison.
 - Because the police officers have got their images from the monitor...

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 - If one of them confesses while the other one denies, the former will be set **FREE** while the latter will be sentenced in prison for **9** months.
 - If both confess, then they will both get **6** months in prison.
 - Because the police officers have got their images from the monitor...
- In your opinion, what should they do?
 - They **cannot communicate**, and they must make their decisions **simultaneously**.

Prisoners' Dilemma

- We can use a “matrix” to formulate this game.
- Two **players**, two **actions** for each.
- If you are criminal *A*, what will you do?
- What's the **solution** (outcome)?

		Criminal <i>B</i>	
		Denial	Confess
Criminal <i>A</i>	Denial	-1, -1	-9, 0
	Confess	0, -9	-6, -6

Prisoners' Dilemma

- Dominant strategy?
- Socially inefficient.
 - Why is it inefficient?
- Price of Anarchy (PoA).

	Criminal B	
Criminal A	Denial	Confess
Denial	-1, -1	-9, 0
Confess	0, -9	-6, -6

Bach or Stravinsky (BoS)

- A historical two-player game.
 - The battle of sexes (in *Games and Decisions* by Luce and Raiffa, 1957).
 - Say Amy and Bob want to pick a concert to go to.

Bach or Stravinsky (BoS)

- A historical two-player game.
 - The battle of sexes (in *Games and Decisions* by Luce and Raiffa, 1957).
 - Say Amy and Bob want to pick a concert to go to.
 - Both prefer to go together than to go home.
 - However, Amy prefers Bach while Bob prefers Stravinsky.

Bach or Stravinsky (BoS)

- What are the **SOLUTIONS** of the game?
- Is there any **dominant** strategy for either Amy or Bob?

		Bob
	Bach	Stravinsky
Amy	2, 1	0, 0
Stravinsky	0, 0	1, 2



The matrix form

Battle of Sexes (BoS)

- What are the **SOLUTIONS** of the game?
- Is there any **dominant** strategy for either Amy or Bob?

		Bob	Baseball Game
	Amy	Bach	
	Movie	2, 1	0, 0
	Baseball Game	0, 0	1, 2



The matrix form

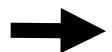
Matching Pennies

- Two players, playing a game by throwing a penny.
- Both ‘heads’ or both ‘tails’: player 1 keeps both pennies.
- Otherwise, player 2 keeps both pennies.

Matching Pennies

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The matrix form



Player 1

head

tail

Player 2

head

tail

1, -1

-1, 1

-1, 1

1, -1

Matching Pennies

- **Zero-sum?**
- Do dominant strategies exist?
- What are the solutions?

		Player 2	
		head	tail
		head	tail
Player 1	head	1, -1	-1, 1
	tail	-1, 1	1, -1

Rock-Scissors-Paper Game



rock



scissors



paper

		Player 2		
		rock	scissors	paper
Player 1	rock	0, 0	1, -1	-1, 1
	scissors	-1, 1	0, 0	1, -1
	paper	1, -1	-1, 1	0, 0

Rock-Scissors-Paper Game

- Zero-sum?
- Dominant strategies?
- Any solutions?

		Player 2		
		rock	scissors	paper
Player 1		rock	0, 0	1, -1
		scissors	-1, 1	0, 0
rock	1, -1	-1, 1	0, 0	

Mixed Strategies

- What we have discussed about are all **pure strategies**.
 - A deterministic action.

Mixed Strategies

- What we have discussed about are all **pure strategies**.
 - A deterministic action.
- What is a **mixed strategy**?

Mixed Strategies

- Like this?
 - Nine-headed Dragon Strike.
- Or like this?
 - Man of many pitches.
- For a portfolio manager in a hedge fund:
 - Portfolio weighting.

Back to the Game of Matching Pennies

- Setting the weights?

		Player 2	
		head	tail
		1, -1	-1, 1
Player 1	head	1, -1	-1, 1
	tail	-1, 1	1, -1

Back to the Game of Matching Pennies

- Setting the weights? $0 < \epsilon, \rho < 1$

		Player 2	
		head	tail
		1, -1	-1, 1
Player 1	head	1, -1	-1, 1
	tail	-1, 1	1, -1
		ϵ	$1 - \epsilon$

Back to the Game of Matching Pennies

- Setting the weights? $0 < \epsilon, \rho < 1$
- The expected utility of player 1 playing ‘head’:

$$f = 1 \cdot \epsilon + (-1) \cdot (1 - \epsilon)$$

- The expected utility of player 1 playing ‘tail’:

$$g = -1 \cdot \epsilon + 1 \cdot (1 - \epsilon)$$

		Player 2	
		head	tail
Player 1	head	1, -1	-1, 1
	tail	-1, 1	1, -1

ρ ϵ $1 - \rho$ $1 - \epsilon$

An intuitive definition of a **Nash equilibrium**

- A state such that no player can increase her expected payoff (profit, gain, advantage, money, etc.) by a **unilateral** deviation.
- **Nash's Theorem:**
Every **finite** game (a finite number of players, each has a finite number of pure strategies) has **at least one** Nash equilibrium.

Back to the Game of Matching Pennies

- Setting the weights? $0 < \epsilon, \rho < 1$
- The expected utility of player 1 playing ‘head’:
$$f = 1 \cdot \epsilon + (-1) \cdot (1 - \epsilon)$$
- The expected utility of player 1 playing ‘tail’:
$$g = 1 \cdot \epsilon + (-1) \cdot (1 - \epsilon)$$
- What if $f \neq g$?

		Player 2	
		head	tail
		head	tail
Player 1	head	1, -1	-1, 1
	tail	-1, 1	1, -1

ρ $1 - \rho$
 ϵ $1 - \epsilon$

Back to the Game of Matching Pennies

- Setting the weights? $0 < \epsilon, \rho < 1$

- The expected utility of player 1 playing ‘head’:

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- The expected utility of player 1 playing ‘tail’:

$$g = -1 \cdot \epsilon + 1 \cdot (1 - \epsilon)$$

- What if $f \neq g$?

Consider Player 1’s expected utility: $\rho \cdot f + (1 - \rho) \cdot g$

		Player 2	
		head	tail
Player 1	head	1, -1	-1, 1
	tail	-1, 1	1, -1

ρ $1 - \rho$

ϵ $1 - \epsilon$

Back to the Game of Matching Pennies

- Setting the weights? $0 < \epsilon, \rho < 1$

- The expected utility of player 1 playing ‘head’:

$$f = 1 \cdot \epsilon + (-1) \cdot (1 - \epsilon)$$

- The expected utility of player 1 playing ‘tail’:

$$g = -1 \cdot \epsilon + 1 \cdot (1 - \epsilon)$$

- **Solving** $f = g \Rightarrow \epsilon = 0.5.$

Now it’s your turn to solve ρ .

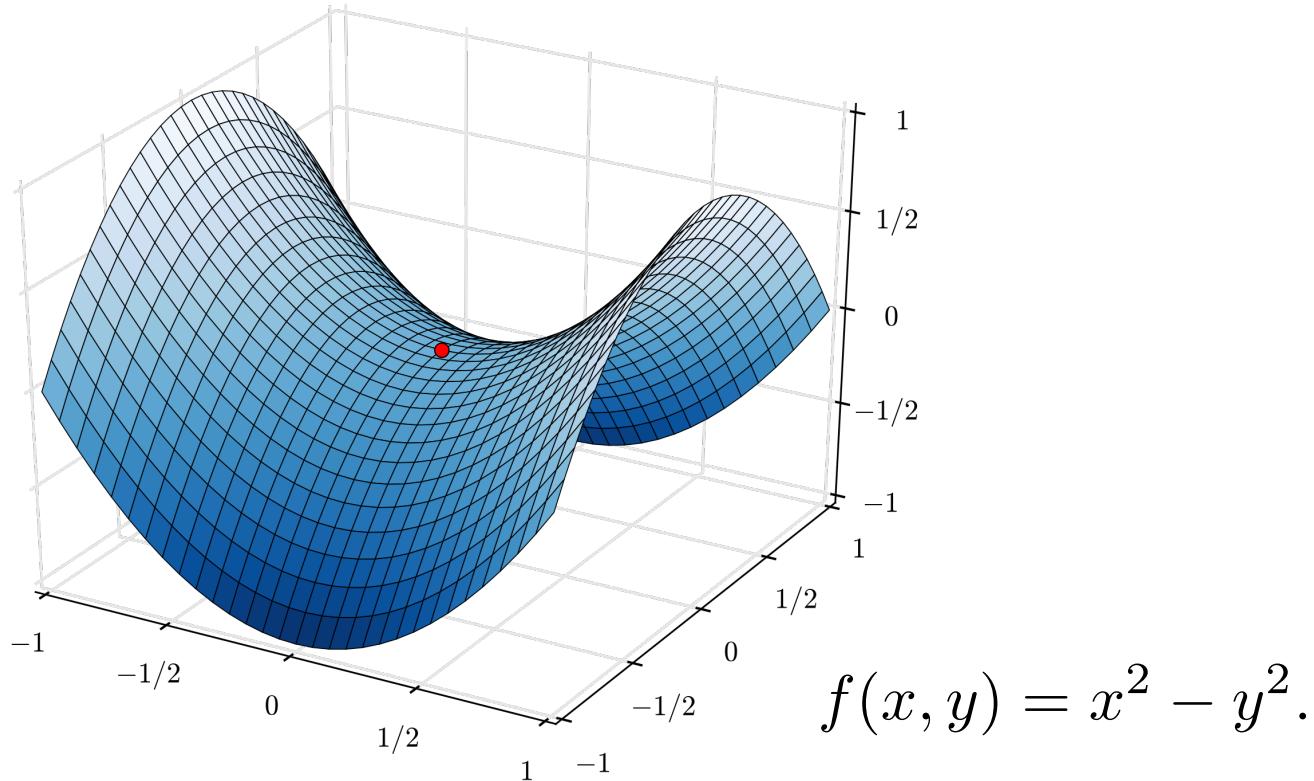
		Player 2	
		head	tail
Player 1	head	1, -1	-1, 1
	tail	-1, 1	1, -1
		ϵ	$1 - \epsilon$

Back to the Game of Matching Pennies

- Take your time.
- So we just proved that the game has a kind of solution:
“Mixed-Strategy Nash Equilibrium”.

		Player 2	
		head	tail
		1, -1	-1, 1
Player 1		-1, 1	1, -1
		ϵ	$1 - \epsilon$

Saddle point illustration



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- **Nash's Theorem:**
Every **finite** game (a finite number of players, each has a finite number of pure strategies) has **at least one** Nash equilibrium.
- The concept of **best responses & mixed strategies.**

Back to the classic scene of “A Beautiful Mind”

- https://www.youtube.com/watch?v=2d_dtTZQyUM
- Do you observe anything strange or anything wrong?
 - https://www.youtube.com/watch?v=DTcmmD_MWas

An Easy Exercise

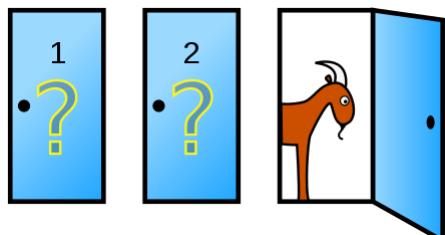
- Please find out a mixed-strategy Nash equilibrium of the rock-scissors-paper game.

		Player 2		
		rock	scissors	paper
Player 1	rock	0, 0	1, -1	-1, 1
	scissors	-1, 1	0, 0	1, -1
	paper	1, -1	-1, 1	0, 0

The Monty Hall Problem

- From an American TV show *Let's Make a Deal* hosted by Monty Hall.

Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" **Is it to your advantage to switch your choice?**



https://en.wikipedia.org/wiki/Monty_Hall_problem