

Economics and Computation

Preliminaries in Game Theory

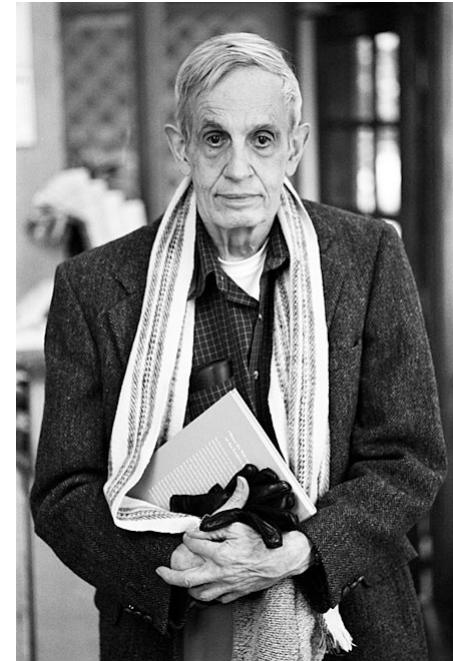
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John Forbes Nash Jr. (1928–2015)

- American mathematician.
- Fundamental contributions to game theory.
- **Nobel Memorial Prize** in Economic Sciences with game theorists Reinhard Selten and John Harsanyi.
- **Abel Prize** with Louis Nirenberg for his work on nonlinear partial differential equations.

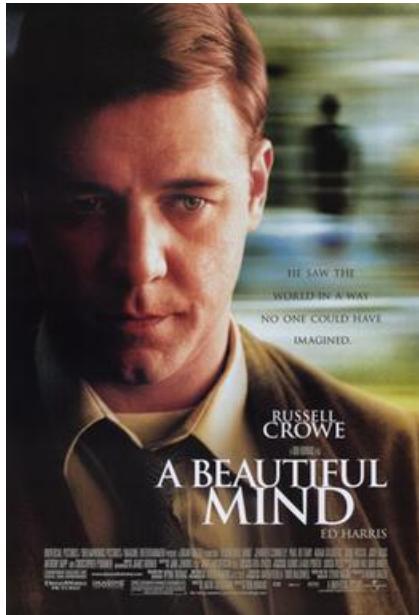


Nash in 2006.

Reference: https://en.wikipedia.org/wiki/John_Forbes_Nash_Jr.

A classic scene of “A Beautiful Mind”

- https://www.youtube.com/watch?v=2d_dtTZQyUM



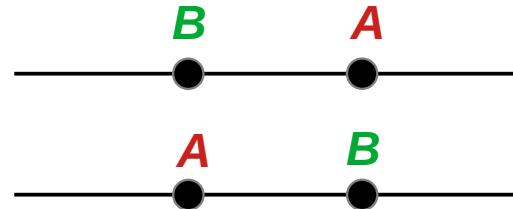
Starring: Russel Crowe

Before introducing Nash Equilibria...

- Let's play around several “games” first.

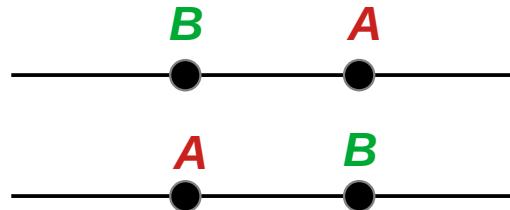
Number Guessing

- Let's say I have chosen a secret number A in my mind, which is among 1 and 100.
- Please guess it by a number B .
- If $B < A$, I will tell you “Larger, please”.
- If $B > A$, I will tell you “Smaller, please”.
- How many times do you think you can find out this secret number?



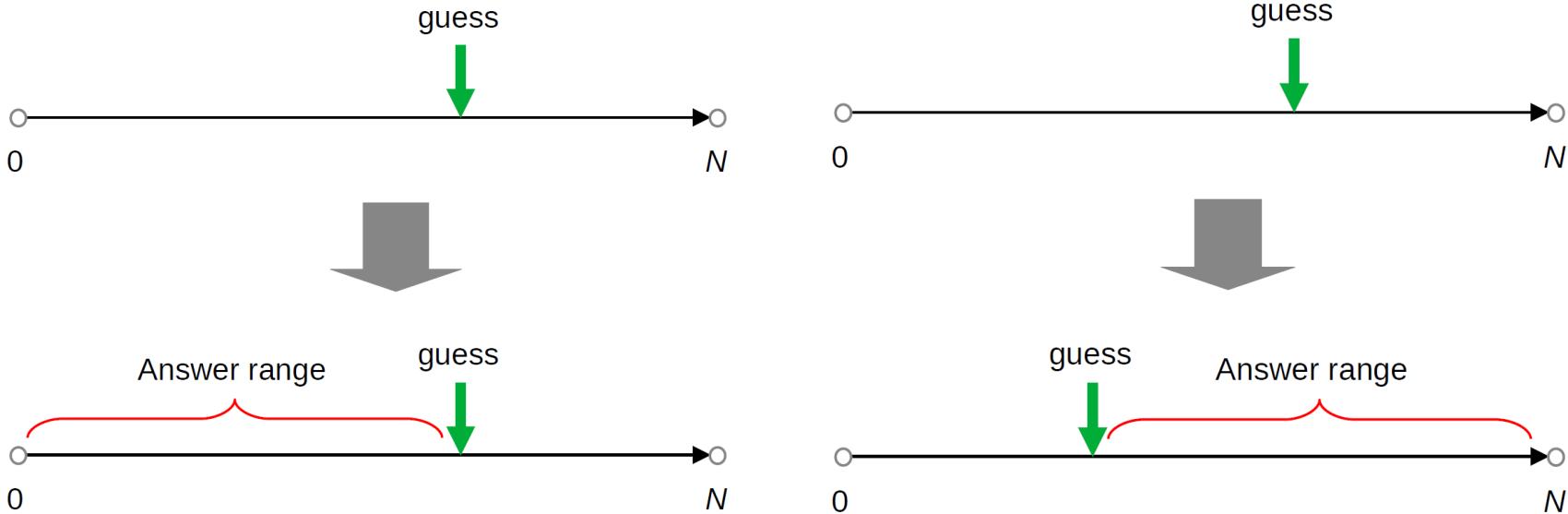
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- How many times do you think you can find out this secret number? **Let's play to feel the strategic behaviors.**



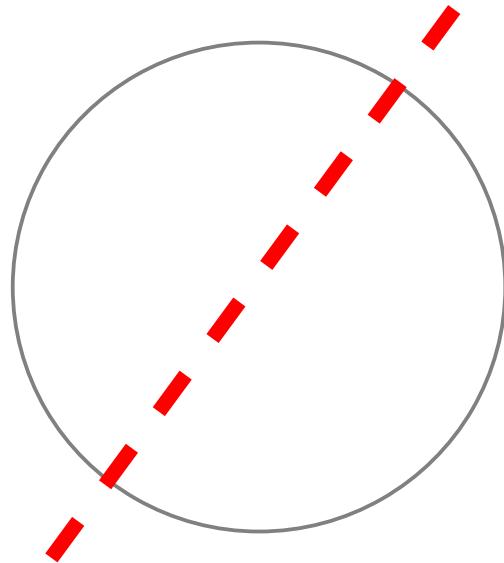
Adversarial Number Guessing

- The **demo** code.



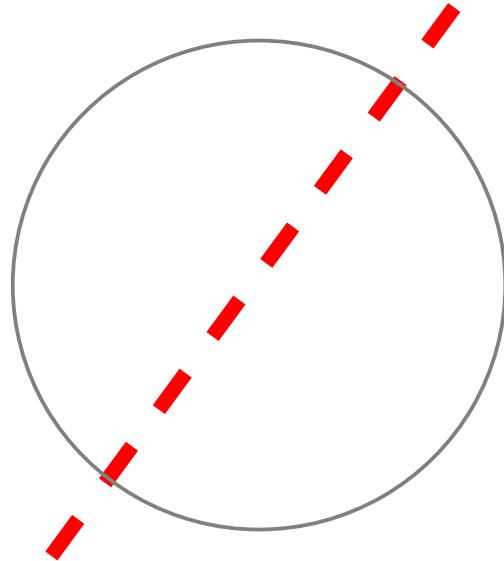
Envy-Free Cake-Cutting

- Children wants everything to be FAIR.
- Actually, in their world, nothing is FAIR..... lol



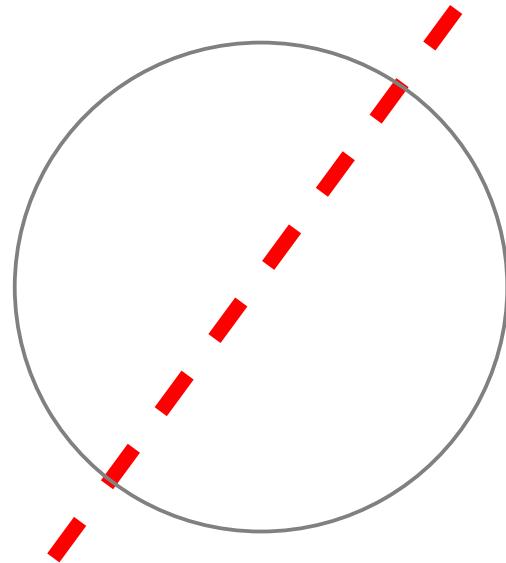
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Envy-Free Cake-Cutting

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- Actually, in their world, nothing is FAIR..... lol
- Let's say we want two kids to share a cake.
- Can you propose a way of cutting a cake so that two kids share a cake so that **no one envies the other?**



Prisoners' Dilemma

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- Let's say there are two guys, **A** and **B**, who broke into a luxury store and stole a treasure.

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- They had hidied the treasure before the police caught them.



source

Prisoners' Dilemma

- It was originally framed by Merrill Flood and Melvin Dresher in 1950.
- Let's say there are two guys, **A** and **B**, who broke into a luxury store and stole a treasure.
- They had hidied the treasure before the police caught them.
- They were kept in two separated rooms.
 - That means, they cannot communicate with each other...
- Each of them was offered two choices: **Denial** or **confession**.

Prisoners' Dilemma

- They were told that:

Prisoners' Dilemma

- They were told that:
 - If both of them deny the fact of stealing the treasure, they will **BOTH** be sentenced in prison for **one** month.
 - If one of them confesses while the other one denies, the former will be set **FREE** while the latter will be sentenced in prison for **9** months.
 - If both confess, then they will both get **6** months in prison.
 - Because the police officers have got their images from the monitor...

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 - If both confess, then they will both get **6** months in prison.
 - Because the police officers have got their images from the monitor...
- In your opinion, what should they do?
 - They **cannot communicate**, and they must make their decisions **simultaneously**.

Prisoners' Dilemma

- We can use a “matrix” to formulate this game.
- Two **players**, two **actions** for each.
- If you are criminal A, what will you do?
- What's the **solution** (outcome)?

	Criminal B	
Criminal A	Denial	Confess
Denial	-1, -1	-9, 0
Confess	0, -9	-6, -6

Prisoners' Dilemma

- Dominant strategy?
- Socially inefficient.
 - Why is it inefficient?
- Price of Anarchy (PoA).

	Criminal B	
Criminal A	Denial	Confess
Denial	-1, -1	-9, 0
Confess	0, -9	-6, -6

Bach or Stravinsky (BoS)

- A historical two-player game.
 - The battle of sexes (in *Games and Decisions* by Luce and Raiffa, 1957).
 - Say Amy and Bob want to pick a concert to go to.

Bach or Stravinsky (BoS)

- A historical two-player game.
 - The battle of sexes (in *Games and Decisions* by Luce and Raiffa, 1957).
 - Say Amy and Bob want to pick a concert to go to.
 - Both prefer to go together than to go home.
 - However, Amy prefers Bach while Bob prefers Stravinsky.

Bach or Stravinsky (BoS)

- What are the **SOLUTIONS** of the game?
- Is there any **dominant** strategy for either Amy or Bob?

		Bob
	Bach	Stravinsky
Amy	2, 1	0, 0
Stravinsky	0, 0	1, 2



The matrix form

Battle of Sexes (BoS)

- What are the **SOLUTIONS** of the game?
- Is there any **dominant** strategy for either Amy or Bob?

		Bob	Baseball Game
	Amy	Bach	
	Movie	2, 1	0, 0
	Baseball Game	0, 0	1, 2



The matrix form

Matching Pennies

- Two players, playing a game by throwing a penny.
- Both ‘heads’ or both ‘tails’: player 1 keeps both pennies.
- Otherwise, player 2 keeps both pennies.

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The matrix form



Player 1

head

tail

Player 2

head

tail

1, -1	-1, 1
-1, 1	1, -1

Matching Pennies

- Zero-sum?
- Do dominant strategies exist?
- What are the solutions?

		Player 2	
		head	tail
		head	tail
Player 1	head	1, -1	-1, 1
	tail	-1, 1	1, -1

Rock-Scissors-Paper Game



rock



scissors



paper

Player 1

rock

scissors

paper

Player 2

rock

scissors

paper

0, 0	1, -1	-1, 1
-1, 1	0, 0	1, -1
1, -1	-1, 1	0, 0

Rock-Scissors-Paper Game

- Zero-sum?
- Dominant strategies?
- Any solutions?

		Player 2		
		rock	scissors	paper
Player 1		rock	0, 0	1, -1
		scissors	-1, 1	0, 0
rock	0, 0	1, -1	-1, 1	0, 0
scissors	-1, 1	0, 0	1, -1	0, 0
paper	1, -1	-1, 1	0, 0	0, 0

Mixed Strategies

- What we have discussed about are all **pure strategies**.
 - A deterministic action.

Mixed Strategies

- What we have discussed about are all **pure strategies**.
 - A deterministic action.
- What is a **mixed strategy**?

Mixed Strategies

- Like this?
 - Nine-headed Dragon Strike.
- Or like this?
 - Man of many pitches.
- For a portfolio manager in a hedge fund:
 - Portfolio weighting.

Back to the Game of Matching Pennies

- Setting the weights?

		Player 2	
		head	tail
		head	tail
Player 1	head	1, -1	-1, 1
	tail	-1, 1	1, -1

Back to the Game of Matching Pennies

- Setting the weights? $0 < \epsilon, \rho < 1$

		Player 2	
		head	tail
		head	tail
Player 1	head	1, -1	-1, 1
	tail	-1, 1	1, -1

ϵ $1 - \epsilon$ ρ $1 - \rho$

Back to the Game of Matching Pennies

- Setting the weights? $0 < \epsilon, \rho < 1$
- The expected utility of player 1 playing ‘head’:

$$f = 1 \cdot \epsilon + (-1) \cdot (1 - \epsilon)$$

- The expected utility of player 1 playing ‘tail’:

$$g = -1 \cdot \epsilon + 1 \cdot (1 - \epsilon)$$

		Player 2	
		head	tail
		head	tail
Player 1	head	1, -1	-1, 1
	tail	-1, 1	1, -1

ϵ $1 - \epsilon$

ρ $1 - \rho$

An intuitive definition of a **Nash equilibrium**

- A state such that no player can increase her expected payoff (profit, gain, advantage, money, etc.) by a **unilateral** deviation.
- **Nash's Theorem:**
Every **finite** game (a finite number of players, each has a finite number of pure strategies) has **at least one** Nash equilibrium.

Back to the Game of Matching Pennies

- Setting the weights? $0 < \epsilon, \rho < 1$
- The expected utility of player 1 playing ‘head’:
$$f = 1 \cdot \epsilon + (-1) \cdot (1 - \epsilon)$$
- The expected utility of player 1 playing ‘tail’:
$$g = 1 \cdot \epsilon + (-1) \cdot (1 - \epsilon)$$
- What if $f \neq g$?

		Player 2	
		head	tail
		head	tail
Player 1	head	1, -1	-1, 1
	tail	-1, 1	1, -1

ρ
 ϵ
 $1 - \rho$
 $1 - \epsilon$

Back to the Game of Matching Pennies

- Setting the weights? $0 < \epsilon, \rho < 1$

- The expected utility of player 1 playing ‘head’:

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- The expected utility of player 1 playing ‘tail’:

$$g = -1 \cdot \epsilon + 1 \cdot (1 - \epsilon)$$

- What if $f \neq g$?

Consider Player 1’s expected utility: $\rho \cdot f + (1 - \rho) \cdot g$

		Player 2	
		head	tail
Player 1	head	1, -1	-1, 1
	tail	-1, 1	1, -1

ρ $1 - \rho$

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Back to the Game of Matching Pennies

- Setting the weights? $0 < \epsilon, \rho < 1$

- The expected utility of player 1 playing ‘head’:

$$f = 1 \cdot \epsilon + (-1) \cdot (1 - \epsilon)$$

- The expected utility of player 1 playing ‘tail’:

$$g = -1 \cdot \epsilon + 1 \cdot (1 - \epsilon)$$

- **Solving** $f = g \Rightarrow \epsilon = 0.5.$

Now it’s your turn to solve ρ .

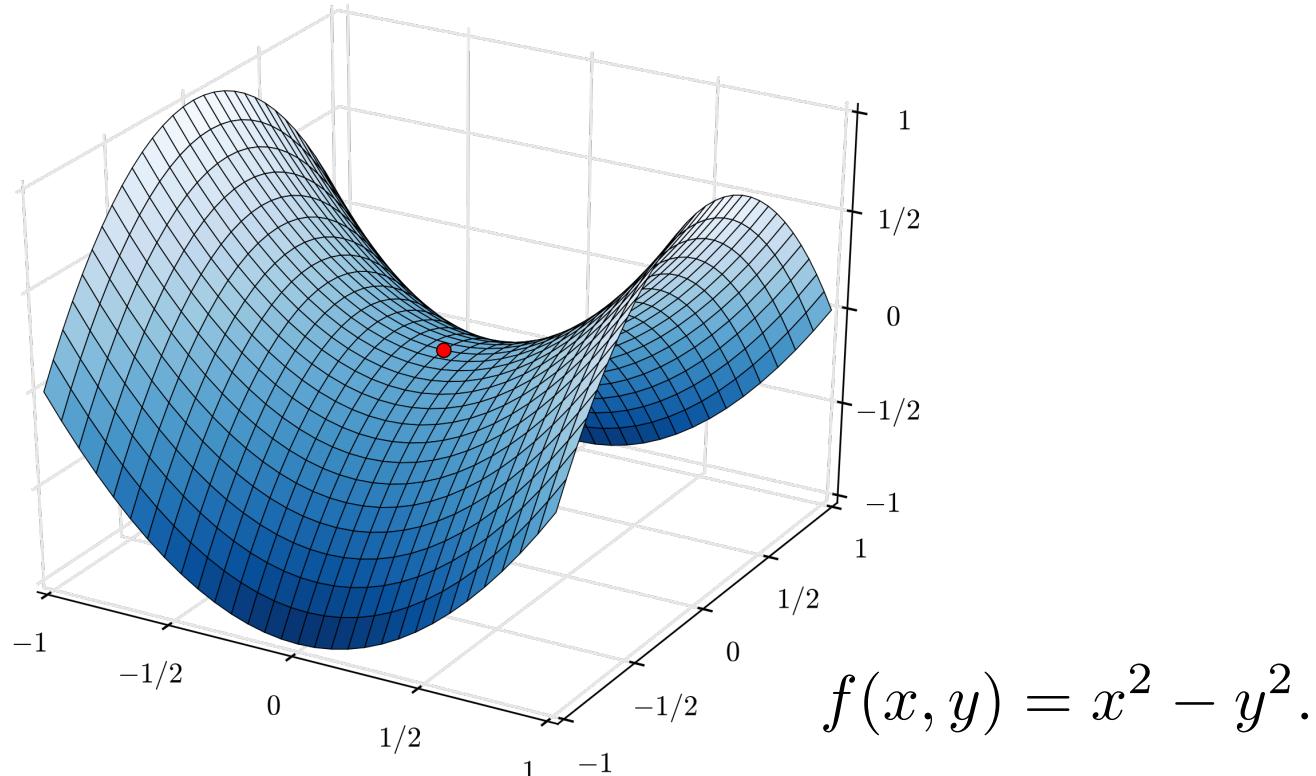
		Player 2	
		head	tail
Player 1	head	1, -1	-1, 1
	tail	-1, 1	1, -1
		ϵ	$1 - \epsilon$
		ρ	$1 - \rho$

Back to the Game of Matching Pennies

- Take your time.
- So we just proved that the game has a kind of solution:
“Mixed-Strategy Nash Equilibrium”.

		Player 2	
		head	tail
		1, -1	-1, 1
Player 1		-1, 1	1, -1
		ϵ	$1 - \epsilon$

Saddle point illustration



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- **Nash's Theorem:**
Every **finite** game (a finite number of players, each has a finite number of pure strategies) has **at least one** Nash equilibrium.
- The concept of **best responses & mixed strategies.**

Back to the classic scene of “A Beautiful Mind”

- https://www.youtube.com/watch?v=2d_dtTZQyUM
- Do you observe anything strange or anything wrong?
 - https://www.youtube.com/watch?v=DTcmmD_MWas

An Easy Exercise

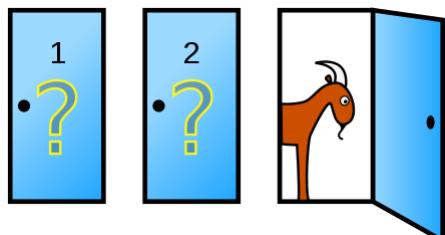
- Please find out a mixed-strategy Nash equilibrium of the rock-scissors-paper game.

		Player 2		
		rock	scissors	paper
Player 1	rock	0, 0	1, -1	-1, 1
	scissors	-1, 1	0, 0	1, -1
	paper	1, -1	-1, 1	0, 0

The Monty Hall Problem

- From an American TV show *Let's Make a Deal* hosted by Monty Hall.

Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" **Is it to your advantage to switch your choice?**



https://en.wikipedia.org/wiki/Monty_Hall_problem