

Expression Evaluation

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Outline

1 Expressions

2 Infix to Postfix



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Expressions

- Example: $a = (3 * (5 - 2));$
 - Operators (運算子): $=, *, -$
 - Operands (運算元): $a, 3, 5, 2$
 - Parenthesis (括號): $(,)$



Expressions

- Example:

`((rear+1 == front) || ((rear == MAX_QUEUE_SIZE-1) && !front))`

- Operators (運算子): `==`, `+`, `-`, `||`, `&&`, `!`
- Operands (運算元): `rear`, `front`, `MAX_QUEUE_SIZE`
- Parenthesis (括號): `(,)`



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Within any programming language, there is a precedence hierarchy that determines the order in which we evaluate operators.



Precedence Hierarchy in C

Token	Operator	Precedence ¹	Associativity
()	function call	17	left-to-right
[]	array element		
-> .	struct or union member		
-- ++	increment, decrement ²	16	left-to-right
-- ++	decrement, increment ³	15	right-to-left
!	logical not		
-	one's complement		
- +	unary minus or plus		
& *	address or indirection		
sizeof	size (in bytes)		
(type)	type cast	14	right-to-left
* / %	multiplicative	13	left-to-right
+ -	binary add or subtract	12	left-to-right
<< >>	shift	11	left-to-right
> >=	relational	10	left-to-right
< <=			
== !=	equality	9	left-to-right
&	bitwise and	8	left-to-right
^	bitwise exclusive or	7	left-to-right
	bitwise or	6	left-to-right
&&	logical and	5	left-to-right
	logical or	4	left-to-right
?:	conditional	3	right-to-left
= += -= /= *= %=	assignment	2	right-to-left
<<= >>= &= ^= =			
,	comma	1	left-to-right

1. The precedence column is taken from Harbison and Steele.

2. Postfix form

3. Prefix form

- The associativity column indicates how we evaluate operators with the same precedence.



Infix & Postfix

Infix	Postfix
$2 + 3 * 4$	$234*+$
$a * b + 5$	$ab*5+$
$(1 + 2) * 7$	$12+7*$
$a * b / c$	$ab*c/$
$((a / (b - c + d)) * (e - a) * c$	$abc-d+ / ea-*c*$
$a / b - c + d * e - a * c$	$ab/c-de*+ac*-$

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- Infix: the standard way we are used to.
- The compilers typically use **postfix**!



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- Evaluation of an expression can be done by using a **stack**.

Token	Stack			Top
	[0]	[1]	[2]	
6	6			0
2	6	2		1
/	6/2			0
3	6/2	3		1
-	6/2-3			0
4	6/2-3	4		1
2	6/2-3	4	2	2
*	6/2-3	4*2		1
+	6/2-3+4*2			0



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Postfix Evaluation

- Expression is represented as a character array.
 - Operators: +, -, *, / and %.
 - Operands: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.
 - The operands are stored on a stack of type int.
 - The stack is represented by a global array accessed only through top.
- The declarations are:

```
#define MAX_STACK_SIZE 100 // maximum stack size
#define MAX_EXPR_SIZE 100 // max size of expression
typedef enum {
    lparen, rparen, plus, minus, times,
    divide, mod, eos, operand
} precedence;
int stack[MAX_STACK_SIZE]; // global stack
char expr[MAX_EXPR_SIZE]; // input string
```

To Get Tokens

```
precedence get_token(char *symbol, int *n) { // get the next token,  
    *symbol = expr[(*n)++];  
    switch (*symbol) {  
        case '(': return lparen;  
        case ')': return rparen;  
        case '+': return plus;  
        case '-': return minus;  
        case '/': return divide;  
        case '*': return times;  
        case '%': return mod;  
        case '\\0': return eos; // end of string  
        default: return operand; /* no error checking,  
                                default: operand */  
    }  
}
```



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- Consider the following example:

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 - $((((a / b) - c) + (d * e)) - (a * c))$



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- move all binary operators so that they replace their corresponding right parentheses. (將運算符號取代其相對應的右括號)
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 - $((((a b / c - (d e * + a c * -$
- delete all parentheses.
 - $a b / c - d e * + a c * -$



Discussions

