Assignment 4
Due date: 17 November 2023
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1. (20%) Diagonalize $\begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix} = \mathbf{PDP}^\top$ such that $\mathbf{P}$ consists of orthonormal column vectors.
2. (20%) Given $\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$. Find a singular value decomposition for $\mathbf{A}$.
3. (20%) Compute $\frac{d}{d\mathbf{x}} f(\mathbf{x}, \mathbf{y})$, where $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ and $f(\mathbf{x}, \mathbf{y}) = \mathbf{x}^\top \mathbf{y}$.
4. (20%) Given the formula $\frac{\partial \mathbf{x}^\top \mathbf{B} \mathbf{x}}{\partial \mathbf{x}} = \mathbf{x}^\top (\mathbf{B} + \mathbf{B}^\top)$ for a square matrix $\mathbf{B}$, compute the gradient $\frac{\partial}{\partial \mathbf{s}} \left((\mathbf{x} - \mathbf{A}\mathbf{s})^\top \mathbf{A}\mathbf{A}^\top (\mathbf{x} - \mathbf{A}\mathbf{s}) + \|\mathbf{s}\|^2 \right)$.
5. (20%) Compute the derivatives $d\,f/d\,\mathbf{x}$, where $f(z) = \ln(1+z)$, and $z = \mathbf{x}^\top \mathbf{x}$, for $\mathbf{x} \in \mathbb{R}^D$.