Mathematics for Machine Learning (2025 Fall)

Assignment 01*

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1. (20%) Consider $\mathbf{x} = [-2\ 1\ 1]^\top$ and the following matrix A, compute $\mathrm{tr}(A\mathbf{x}\mathbf{x}^\top)$.

$$A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & 2 \\ 2 & 2 & 3 \end{bmatrix}.$$

2. (20%) Show that $\|\mathbf{v}\|_\infty \leq \|\mathbf{v}\|_1 \leq \sqrt{n}\|\mathbf{v}\|_2$ for any $\mathbf{v} \in \mathbb{R}^n$ (*Hint*: Use Cauchy-Schwarz Inequality).

3. (20%) Consider the following two matrices $A_1 = \begin{bmatrix} 9 & 6 \\ 6 & 5 \end{bmatrix}$, $A_2 = \begin{bmatrix} 9 & 6 \\ 6 & 3 \end{bmatrix}$. Prove the following arguments:
   (a) A_1 is positive definite.
   (b) A_2 is not positive definite

4. (20%) Prove that for any $A \in \mathbb{R}^{m\times n}, B \in \mathbb{R}^{n\times m}$, $m, n \in \mathbb{N}$, we have $\mathrm{tr}(AB) = \mathrm{tr}(BA)$.

5. (20%) For the identity matrix $I_n \in \mathbb{R}^{n\times n}$,
   (a) What are its eigenvalues and the associated eigenvectors?
   (b) What are the eigenspaces?

* List the required intermediate steps next to each problem. Note that any answers generated directly by AI are invalid for this assignment.