Threaded Binary Tree & Heaps

Joseph Chuang-Chieh Lin (林莊傑)

Department of Computer Science & Engineering,
National Taiwan Ocean University

Fall 2024

Outline

Outline

Threaded Binary Trees

Issue

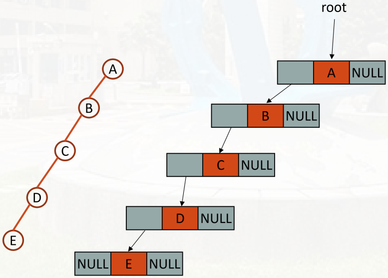
There are more null links than actual points.

Threaded Binary Trees

Issue

There are more null links than actual points.

- Number of nodes: n.
- Number of null non-null links: $n - 1$.
- Number of null links: $n + 1$.

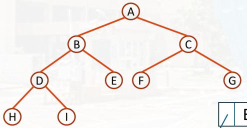


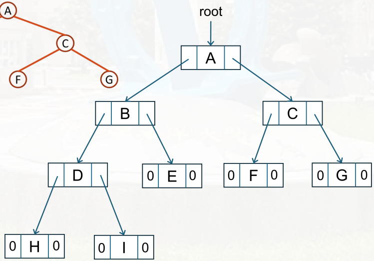
Threaded Binary Trees

Issue

There are more null links than actual points.

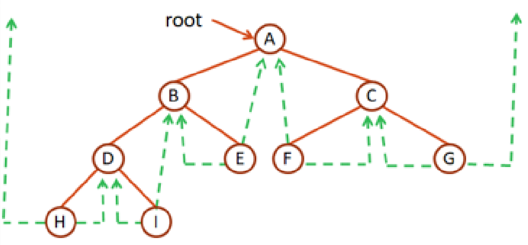
- Number of nodes: n.
- Number of null non-null links: $n-1$.
- Number of null links: $n+1$.





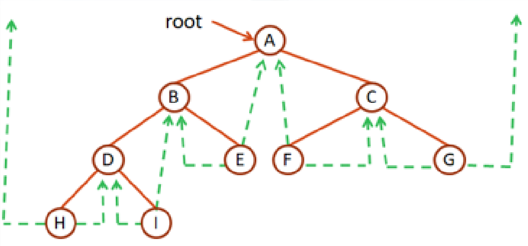
Solution

Replace the NULL links by pointers, threads, pointing to other nodes.



Solution

Replace the NULL links by pointers, threads, pointing to other nodes.



Threading Rules

- if `ptr->leftChild` is NULL, then `ptr->leftChild` = inorder predecessor (中序前行者) of `ptr`.
- if `ptr->rightChild` is NULL, then `ptr->rightChild` = inorder successor (中序後續者) of `ptr`.

To distinguish between normal pointers and threads

- Two additional fields of the node structure: left-thread, right-thread.

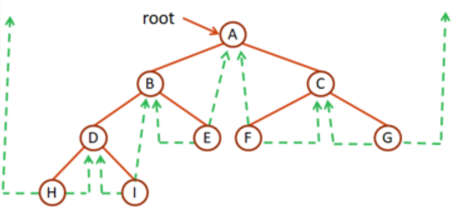
```
typedef struct threadedTree *threadedPointer;

typedef struct threadedTree {
    bool leftThread;
    threadedPointer leftChild;
    char data;
    threadedPointer rightChild;
    bool rightThread;
};
```

leftThread	leftChild	data	rightChild	rightThread

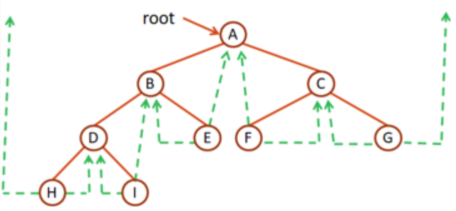
Rules of the Threading Fields

- If `ptr->leftThread == true`, `ptr->leftChild` contains a thread; Otherwise, the node contains a pointer to the left child.
- If `ptr->rightThread == true`, `ptr->righChild` contains a thread; Otherwise, the node contains a pointer to the right child.



Rules of the Threading Fields

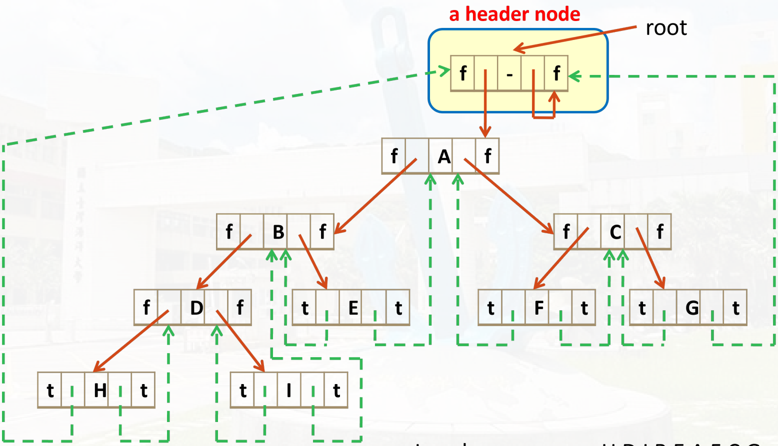
- If `ptr->leftThread == true`, `ptr->leftChild` contains a thread; Otherwise, the node contains a pointer to the left child.
- If `ptr->rightThread == true`, `ptr->righChild` contains a thread; Otherwise, the node contains a pointer to the right child.



- Two dangling threads at node *H* and *G*.

  $\Rightarrow$ Use a header node to collect them!

- The original tree becomes the left subtree of the head node.



Inorder sequence: H D I B E A F C G

Representing an Empty Binary Tree

leftThread	leftChild	data	rightChild	rightThread
true	•	—	•	**false**

Finding the Inorder Successor of Node

```
threadedPointer insucc(threadedPointer tree) {
/* find the inorder sucessor of tree in a threaded
   binary tree */
    threadedPointer temp;
    temp = tree->rightChild;
    if (!tree->rightThread) // rightChild exists!
        while (!temp->leftThread)
            temp = temp->leftChild;
    return temp;
}
```

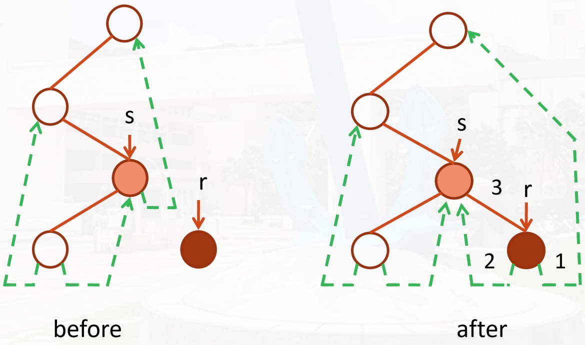
To perform an inorder traversal, we can simply make repeated calls to insucc!

Inorder Traversal of a Threaded Binary Tree

```c
void traverseInorder(threadedPointer tree) {
/* traverse the threaded binary tree inorder */
    threadedPointer temp = tree;
    while (1) {
        temp = insucc(temp);
        if (temp == tree)
            break;
        printf("%3c", temp->data);
    }
}
```

Inserting *r* as the rightChild of a node *s*

- Case I: `s->rightThread == false`

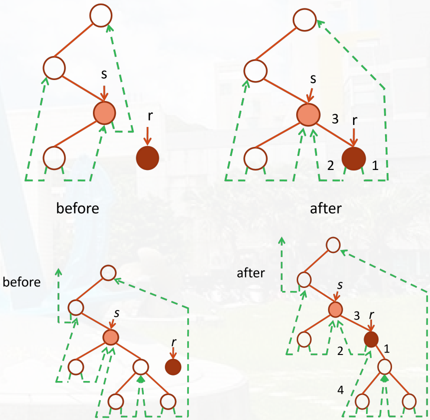


Inserting *r* as the rightChild of a node *s*

- Case II: `s->rightThread != false`

The Code for the Insertion

```
void insertRight (threadedPointer s,
                  threadedPointer r) {
/* insert r as the right child of s */
    threadedPointer temp;
    r->rightChild = s->rightChild;
    r->rightThread = s->rightThread;
    r->leftChild = s;
    r->leftThread = true;
    s->rightChild = r;
    s->rightThread = false;
    if (!r->rightThread){ // step 4
        temp = insucc(r);
        temp->leftChild = r;
    }
}
```



Outline

Heaps

Max Tree

A max tree is a tree in which

- the key value in each node $\geq$ the key values in its children.

Heaps

Max Tree

A max tree is a tree in which

- the key value in each node $\geq$ the key values in its children.

Min Tree

A min tree is a tree in which

- the key value in each node $\leq$ the key values in its children.

Heaps

Max Tree

A max tree is a tree in which

- the key value in each node $\geq$ the key values in its children.

Min Tree

A min tree is a tree in which

- the key value in each node $\leq$ the key values in its children.

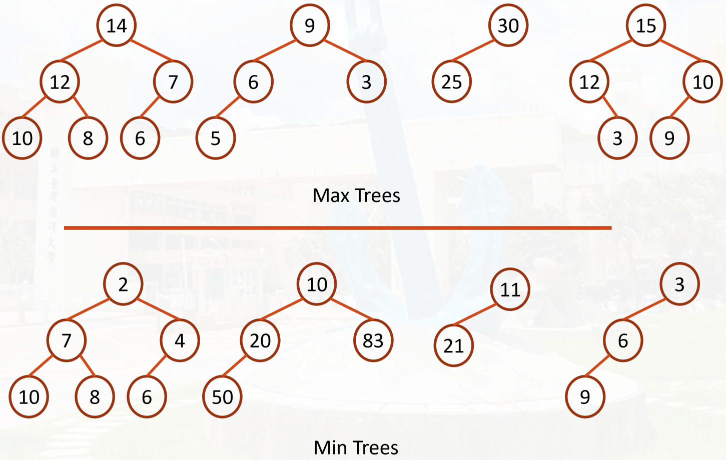
Max Heap

A complete binary tree that is also a max tree.

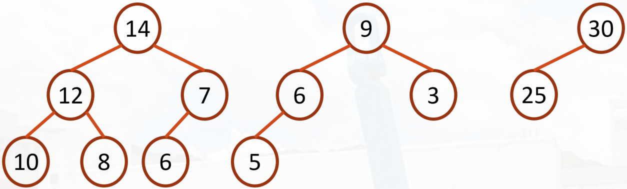
Min Heap

A complete binary tree that is also a min tree.

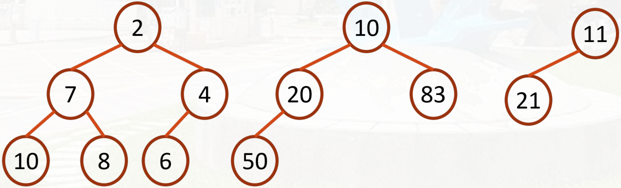
Examples: Max & Min Trees



Examples: Max & Min Heaps



Max Heaps



Min Heaps

The Key Application: Priority Queues

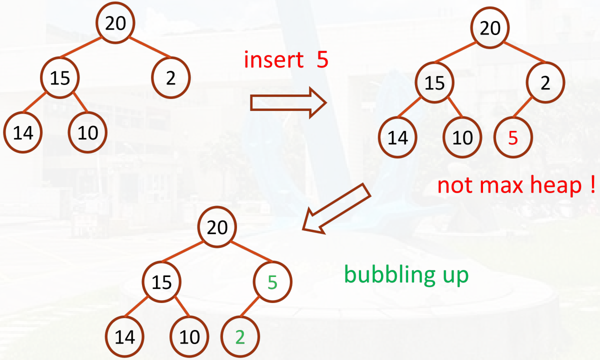
- Heaps are frequently used to implement priority queues.
- In this kind of queue,
  - the element to be deleted is the one with highest (or lowest) priority.
  - at **any time**, an element with arbitrary priority can be **inserted** into the queue.

Insertion into a Max Heap

- The bubbling process.
  - It begins at the new node of the tree and moves toward the root.

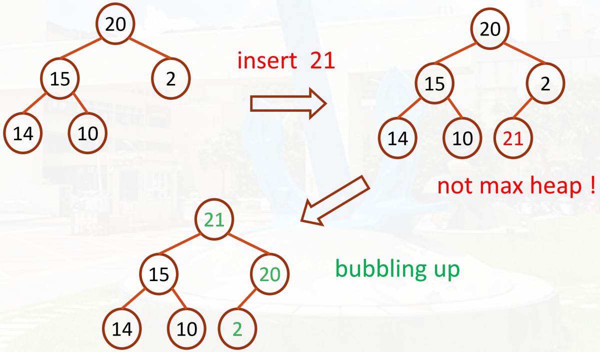
Insertion into a Max Heap

- The bubbling process.
  - It begins at the new node of the tree and moves toward the root.



Insertion into a Max Heap

- The bubbling process.
  - It begins at the new node of the tree and moves toward the root.



The Code for Insertion into a Max Heap

- Consider the following declarations:

```
#define MAX_ELEMENTS 200  /* maximum heap size+1 */
#define HEAP_FULL (n) (n == MAX_ELEMENTS -1)
#define HEAP_EMPTY (n)  (!n)
typedef struct {
    int key;
    /* other fields */
} element;
element heap[MAX_ELEMENTS];
int n = 0;
```

The Code for Insertion into a Max Heap

```
void push (element item, int *n) {
/* insert item into a max heap of current size *n */
    int i;
    if (HEAP_FULL(*n)) {
        printf("The heap is full.\n");
        exit(EXIT_FAILURE);
    } // O(1) time
    i = ++(*n);
    while ((i != 1) && (item.key > heap[i/2].key)) {
        heap[i] = heap[i/2];
        i /= 2;
    } //  O(lg n) time
    heap[i] = item; // O(1) time
}
```

- The time complexity of the insertion: $O(lgn)$.

Deletion from a Max Heap

- When an element is to be deleted from a max heap, it is ALWAYS taken from the root of the heap.

Deletion from a Max Heap

- When an element is to be deleted from a max heap, it is ALWAYS taken from the root of the heap.
- The steps of deletion from a Max heap:
  - delete the root node.
  - insert the last node into the root.
  - use the bubbling up process to ensure that the resulting heap remains a max heap.

Illustration of Deletion from a Max Heap

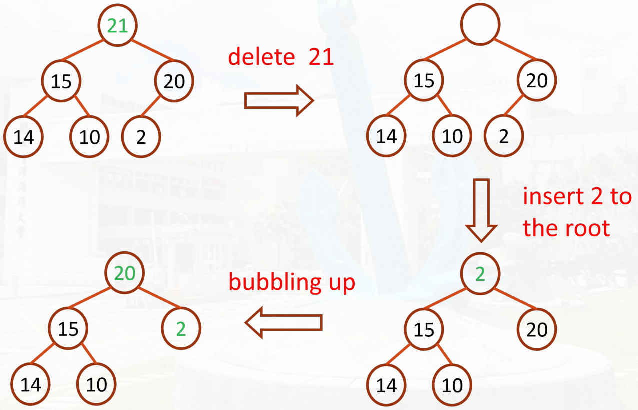
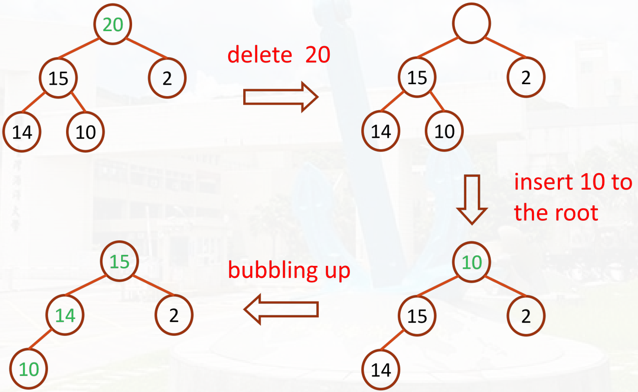


Illustration of Deletion from a Max Heap



The Code for Deletion from a Max Heap

```
element pop(int *n) {
/* delete element with the highest key from the heap */
    int parent, child;
    element item, temp;
    if (HEAP_EMPTY(*n)) {
        fprintf(stderr, "The heap is empty\n");
        exit(EXIT_FAILURE);
    }
    /* save value of the element with the highest key */
    item = heap[1];
    /* use last element in heap to adjust heap */
    temp = heap[(*n)--];
    parent = 1;
    child = 2;
    while (child <= *n) { // O(lg n) time
    /* find the larger child of the current parent */
        if ((child < *n) && (heap[child].key < heap[child+1].key))
            child++;
        if (temp.key >= heap[child].key) break;
        /* move to the next lower level */
        heap[parent] = heap[child];
        parent = child;
        child *= 2;
    }
    heap[parent] = temp;
    return item;
}
```

Time Complexity of the Deletion from a Max Heap

- Delete the root node: $O(1)$.
- Insert the last node to the root: $O(1)$.
- Since the height of the heap is $\lceil \lg(n+1)) \rceil$, the while loop is iterated for $O(\lg n)$ times.
- Thus, the overall time complexity: the time complexity of the deletion: $O(\log n)$.

Discussions