Authors' Response (w.r.t. CIM-SI-2023-0045.R1)

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We thank the associated editor and the three reviewers for their feedback and comments. These comments definitely polish our manuscript. Below, we give responses to the comments.

From Editor-in-Chief

Dear Dr. Chuang-Chieh Lin:

I am sorry to inform you that your paper cannot be accepted in its current form. The manuscript needs revision to address the review comments and improve presentation quality.

As my previous email stated, proper English usage is critical to Magazine articles.

I and my editorial team provide some editorial comments on your paper CIM-SI-2023-0045.R1. Please find the comments in the attached PDF file. These comments would be useful (albeit with potential misunderstanding) for you to revise your manuscript. You can ignore some comments if you think they are incorrect or irrelevant.

In addition to our comments, please do improve the writing quality and edit/proofread your manuscript carefully. Professional English editing service also helps. If you use it, please provide the certificate of editing service with your revised manuscript.

I would be happy to receive a revision as soon as possible within 7 days (by 19-Jul-2023).

Thank you for your interest in the IEEE Computational Intelligence Magazine.

Sincerely,

Prof. Chuan-Kang Ting
Editor-in-Chief, IEEE Computational Intelligence Magazine

[*]Corresponding author.

Response: Thank you. In the previous revision, we have already sent the manuscript to a professional academic English editing service and submitted the receipt as a certificate to the ScholarOne system. Nevertheless, we still find the comments from the editorial team very helpful. In this revision, we revise our manuscript according to the suggestions, such as rephrasing some paragraphs, sparingly using first-person pronoun (e.g., we), etc. We have also proofread the manuscript carefully.

From Associate Editor

- The authors have revised the work comprehensively. Please check the technical problem raised by reviewer#1 before preparing for the publication. Thank you.

  Response: Thank you. We have already checked the technical problem raised by reviewer#1. Please refer to the paragraph below.

From Reviewer 1

1. First of all, I want to thank the authors for the comprehensive response to my previous comments. I learned a lot from their explanations. While the technical details of the article have improved a lot, I still picked up a problem with font awesome that I explain below.

   When loading the article both Firefox and Chrome complain that the mime-type of font awesome is not matching and therefore won't be loaded and that the font files cannot be found. The problem seems to be in line 20 of index.html. While it seems weird, following the browser's recommendation to change the mime type to "text/html" gets rid of the error. However, the file itself is non-existent and the line can probably be removed entirely.

   before
   <link rel="stylesheet" href="path/to/font-awesome/css/font-awesome.min.css" \>
   after
   <link rel="text/html" href="path/to/font-awesome/css/font-awesome.min.css" \>
   Looking through index.html, we can find another occurrence of loading font awesome (line 27). This call seems to be working, however, it cannot find the font files, since they are not part of the current project. Downloading the webkit (https://fontawesome.com/download) and extracting the folder webfonts to the root of this project, resolves the error.

   Response: Thank you for your valuable comments. We appreciate your input. We have carefully addressed the issue with font awesome in our revised version. Regarding line 20 of index.html, we have modified it, as it was causing the font awesome to fail loading. Specifically, we have modified line 20 from

   <link rel="stylesheet" href="path/to/font-awesome/css/font-awesome.min.css" \>

   to

   <link rel="text/html" href="path/to/font-awesome/css/font-awesome.min.css" \>.

   In addition, we have made the necessary changes to include the fonts from the webkit Font Awesome 6.

From Reviewer 2

1. The authors have addressed in detail the reviewers' comments. This reviewer has no further concerns regarding this paper.

   Response: Thank you. We appreciate your comments and feedback.

From Reviewer 3

1. All my comments have been addressed, and the contents of this manuscript are mature enough to merit publication in this journal.

   Response: Thank you. We appreciate your comments and feedback.

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Group Formation by Group Joining and Opinion Updates via Multi-Agent Online Gradient Ascent

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***Abstract*—This article aims to exemplify best-response dynamics and multi-agent online learning by group formation. This extended abstract provides a summary of the full paper in IEEE Computational Intelligence Magazine on the special issue *AI-eXplained* (AI-X). The full paper includes interactive components to facilitate interested readers to grasp the idea of pure-strategy Nash equilibria and how the system of strategic agents converges to a stable state by the decentralized online gradient ascent with and without regularization.**

I. Introduction

GAME theory has been applied in a variety of situations due to its predictability of outcomes in the real world. It can also be used in solving problems, such as saddle-point optimization that has been used extensively in generative adversarial network models [1]. In general, a *game* consists of strategic agents, each of which acts rationally to maximize its own reward (or utility) or minimize its cost. A *Nash equilibrium* is a stable state composed of the strategies of all agents such that none of the agents wants to change its own strategy unilaterally. Therefore, such a stable state is possibly achievable or even predictable. However, how to achieve a Nash equilibrium in a game may not be quite straightforward, especially when agents behave in a "decentralized" way. Indeed, when an agent's reward function depends on the strategies of the other agents, the maximizer of one agent's reward function is not necessarily a maximizer for any other agent, and it may change whenever any other agent changes its strategy.

This article examines the group formation of strategic agents to illustrate their strategic behaviors. A strategic agent can either join a group or change its opinion to maximize its reward. The eventual equilibrium of the game hopefully suggests predictable outcomes for the whole society. For the case in which agents apply group-joining strategies, the *pure-strategy Nash equilibrium* (PNE) is considered as the solution concept, where a pure strategy means a strategy played with a probability of 1. For the case in which agents change their opinions, each agent executes an *online gradient ascent algorithm*, which guarantees the time-average convergence to a hindsight optimum for a single agent (see [2] for the cost-minimization case), in a decentralized way, and then the possibly convergent state of the system is investigated.

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II. Group and Opinion Formation

Given a set V of n agents $v_1, v_2, \ldots, v_n$, each agent v_i is represented as a *public preference vector* z_i and a *private preference vector* s_i, such that the former (i.e., an *opinion*) corresponds to the preference revealed to all the agents while the latter corresponds to its *belief*, which is unchangeable. Consider $s_i, z_i \in \mathcal{K}$ such that $\mathcal{K} := \{x \in [-1,1]^k : \|x\|_2 \leq 1\} \subset \mathbb{R}^k$ is the feasible set. Each dimension of the domain stands for a certain social issue, such that -1 maps to the far-left politics, while 1 maps to far-right politics. The bounded 2-norm constraint is in line with the bounded rationality of a person, or the bounded budget for a group. Denote by $\mathbf{z} = (z_1, z_2, \ldots, z_n)$ and $\mathbf{s} = (s_1, s_2, \ldots, s_n)$ the two profiles that include each agent's opinion and belief, respectively. Each agent is initially regarded as a group. The *opinion of a group* is the average of the opinions of its members. Similar to the *monotone* setting in [3], a group wins with higher odds if its opinion brings more utility to all the agents. The *reward* (i.e., payoff) of an agent is the expected utility that it can get from all the groups. Specifically, assume that there are currently $m \leq n$ groups $G_1, G_2, \ldots, G_m$, and denote by $|G_i| = n_i$ the number of members in group G_i. Let $\mathcal{G} = (G_1, G_2, \ldots, G_m)$ denote the profile of the groups. To ease the notation, let $\tau = (\mathbf{z}, \mathbf{s}, \mathcal{G})$ denote the *state* of the game. The reward function of agent i is $r_i(\tau) = \sum_{j=1}^{m} p_j(\tau)\langle s_i, \bar{g}_j\rangle$, where $\bar{g}_j = \sum_{v \in G_j} z_i/|n_j|$ represents the opinion of group G_j and the winning probability $p_j(\tau)$ of group G_j is

$$p_j(\tau) = \frac{e^{n_j\langle \bar{g}_j, \sum_{v_r \in V} s_r\rangle}}{\sum_{i \in [m]; n_i > 0} e^{n_i\langle \bar{g}_i, \sum_{v_r \in V} s_r\rangle}},$$

where $[m]$ denotes $\{1, 2, \ldots, m\}$. The following strategic behaviors of an agent in such a game will be considered:

- Group Joining:
  - Each agent seeks a specific group that hopefully maximizes its reward and then joins the group.
- Opinion Updating without Regularization:
  - Each agent in a certain group tries to maximize its reward by changing its own opinion.
- Opinion Updating with Regularization:
  - Each agent in a certain group tries to maximize its reward by changing its own opinion, while the reward includes the regularization $-\|s_i - z_i\|_2^2$, which hopefully constrains an agent's strategic behavior by preventing it from moving too far from its own belief.

III. Group Joining and Pure-Strategy Nash Equilibria

A. 1D Representation

When the opinions and beliefs are assumed to be in $[-1, 1] \subset \mathbb{R}$, these vectors as well as the dynamics of changes can be illustrated in a real line. For example, consider the five agents v_1, v_2, v_3, v_4, and v_5 in Fig. 1. Keeping the opinions z_1, z_2, z_3, and z_4 of v_1, v_2, v_3, and v_4, respectively, fixed and altering the opinion z_5 from from -1 to 1, the variations in the winning probabilities and rewards of all the agents can be observed.

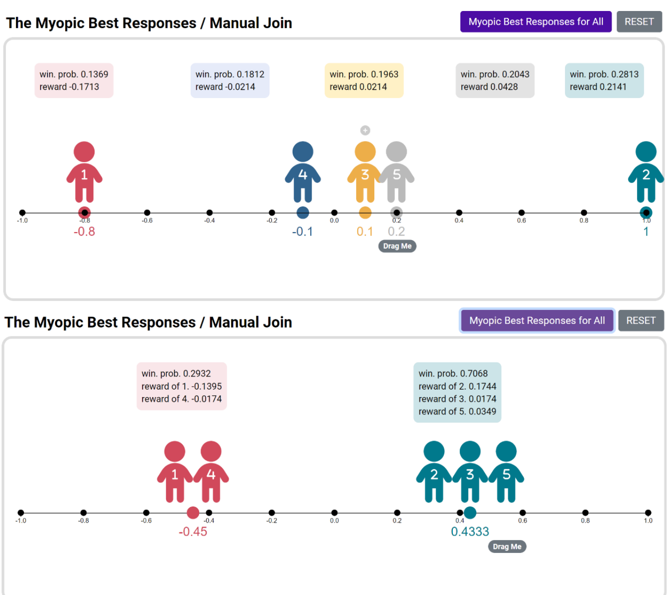

Fig. 1. 1D Representation: myopic best-responses and a PNE.

B. Group Joining: Myopic Best Responses

Assume that agent i decides to join G_j, for which $j = \arg\max_\ell p_\ell(\tau) \cdot \langle \bar{g}_\ell, s_i \rangle$. Such a strategy is called a *myopic best response*. An agent joins a group by considering not only its winning probability but also the utility that the agent can get from the group before joining. The state at the bottom of Fig. 1 is an example of a PNE.

IV. Opinion Updates by Online Learning

A. 2D Representation

The opinions and beliefs as well as the dynamics of opinion changes are illustrated in $\mathcal{K} := \{x \in [-1, 1]^2 : \|x\|_2 \leq 1\} \subset \mathbb{R}^2$. The 2-norm constraint that $\|z_i\|_2, \|s_i\|_2 \leq 1$ correlates the dimensions. A projection of the opinion is required if the constraint is not satisfied.

B. Online Gradient Ascent

Consider the setting that each agent tries to maximize its own reward by "changing its opinion" without deviating from the group to which it belongs. Each agent executes the online gradient ascent algorithm to iteratively update its opinion so as to maximize its reward. The update is done by adding a certain quantity (tuned by the learning rate η) toward the direction of the gradient. A "projection" $\Pi_{\mathcal{K}}(x)$ which projects x onto the feasible set $\mathcal{K}$ by dividing its 2-norm is performed if necessary.

Algorithm: Multi-Agent Online Gradient Ascent

Input: feasible set $\mathcal{K}$, T, learning rate η.

```
for t ← 1 to T do
    for each agent i do
        observe reward r_i(τ), where state τ = (z, s, G)
        z_{i,t+1} ← Π_K(z_{i,t} + η∇_{z_i} r_i(τ))
    end for
end for
```

C. Online Gradient Ascent with Regularization

The reward function for agent i including the regularizer, is defined as $r_i(\tau) = \sum_{j=1}^{m} p_j(\tau)\langle s_i, \bar{g}_j \rangle - \|z_i - s_i\|_2^2$. Since $-\|z_i - s_i\|_2^2$ is always non-positive, an agent will be constrained to consider "not being too far from its belief." Our experimental illustrations show that such a regularization helps the game converge to a state where agents' opinions will not be too far from their beliefs (see Fig. 2).

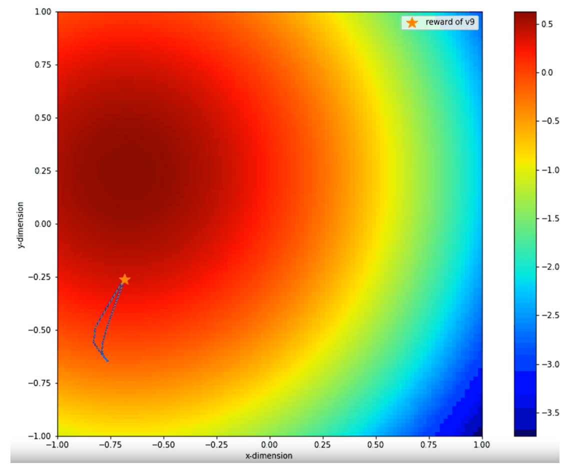

Fig. 2. Opinion updates via the online gradient ascent with regularization.

V. Conclusion

This article presents a preliminary study on the dynamics of group formation. From the illustrations, readers can have a better grasp of a pure-strategy Nash equilibrium in a system of multi-agents and also learn how an online gradient ascent algorithm as one of the dynamics can reach a stable state.

Acknowledgments

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References

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