Mathematics for Machine Learning
— Introduction

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Credits for the resource

- The slides are based on the textbook:

  - *Marc Peter Deisenroth, A. Aldo Faisal, and Cheng Soon Ong: Mathematics for Machine Learning. Cambridge University Press. 2020.*
  - *Howard Anton, Chris Rorres, Anton Kaul: Elementary Linear Algebra. Wiley. 2019.*

- We could partially refer to the monograph: *Francesco Orabona: A Modern Introduction to Online Learning. https://arxiv.org/abs/1912.13213*

Grading Policy

- Attendance (10%)

- Assignments & Quizzes (30%)

- Midterm Exam (30%)
  - 3 Nov. 2025.

- Final Exam (30%)
  - 29 Dec. 2025.

Outline

1 Introduction

Three Core Concepts of Machine Learning

- Data
- Model
- Learning

Remark on the Data

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> **Mitchell 1997**
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- **Goal:** Find good models that generalize well to yet unseen data

Four pillars of ML

The four pillars of ML:

- Regression
- Dimensionality Reduction
- Density Estimation
- Classification

Fundamentals:

- Calculus
- Linear Algebra
- Vector Algebra
- Analytic Geometry
- Matrix Decomposition
- Probability & Distributions
- Optimization

- Why are the mathematical foundations of machine learning important?

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  - To understand fundamental principles upon which more complicated machine learning systems are built.

  - To facilitate creating new machine learning solutions, understanding and debugging existing approaches.

  - To learn about the inherent assumptions and limitations of the methodologies we are working with.

What's a machine learning *algorithm*?

- Predictor: a system that makes predictions based on input data.

- Training: a system that adapts some internal parameters of the predictor so that it performs well on future unseen input data.

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- Numerical representation of the data: vectors.
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    - An arrow with a direction and magnitude (physics view)
    - An object that obeys addition and scaling (mathematical view; OOP view).

An Intuition of Learning/Training a Model

- Assume that we are given a dataset and a suitable model.

- Training a model: use the data to optimize parameters of the model w.r.t. some loss/utility function.

- The training process can be viewed as either climbing a hill to reach its peak moving downwards to the valley.

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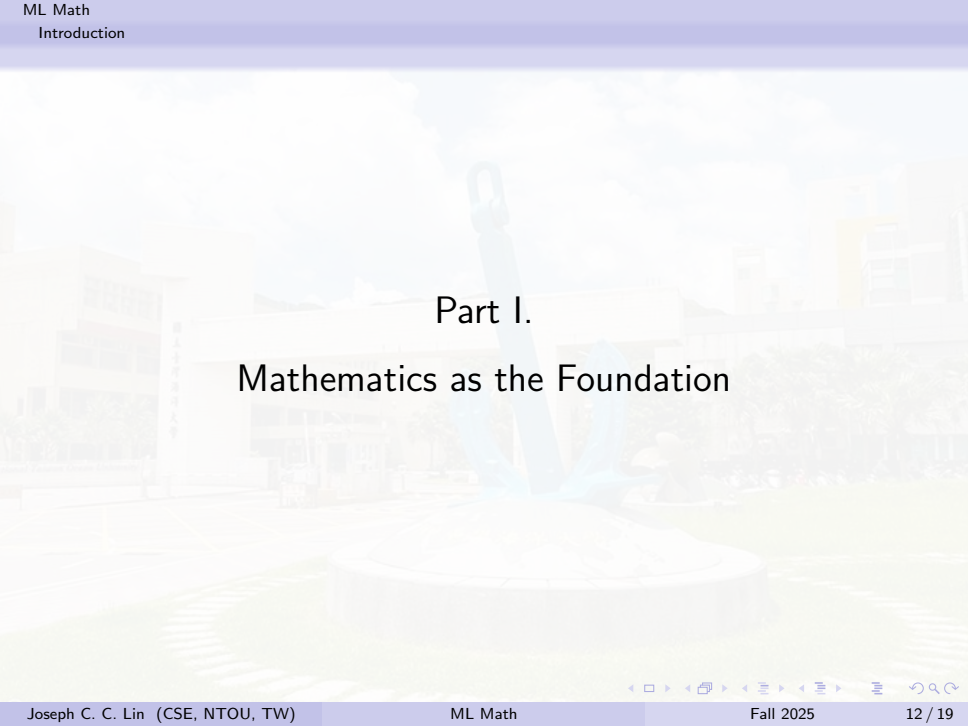- However, at the same time, we are interested in the model which performs well on unseen data.

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  Otherwise, it could be just that we find a way to memorize the data.

Part I.

Mathematics as the Foundation

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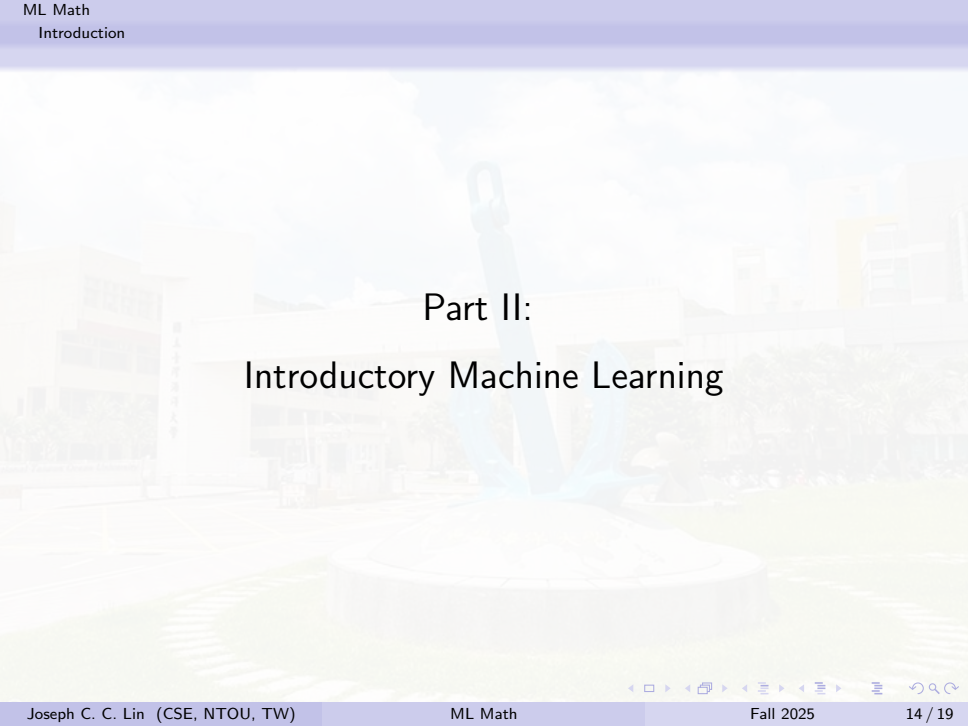
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- Intuitive interpretation of the data and better efficiency for learning: *matrix decomposition*.

Part II:

Introductory Machine Learning

Topics

- Data, model & parameter estimation.
- Continuous Optimization.
- Linear regression.
  - Map the input $\mathbf{x} \in \mathbb{R}^d$ to corresponding observed function values $y \in \mathbb{R}$.
- Density estimation.
  - Find a probability distribution that describes the data.
- Principal Component Analysis
  - Matrix decomposition.
- Classification.

Terminologies

- i.e. $\implies$ that is
- e.g. $\implies$ such as
- $\because \implies$ because
- $\therefore \implies$ therefore
- et al. $\implies$ and others
- $\forall \implies$ for any
- $\exists \implies$ there exists
- a.k.a. $\implies$ also known as
- w.r.t. $\implies$ with respect to
- i.i.d. $\implies$ identically and independently distributed
- ex ante $\implies$ based on forecasts rather than actual results
- ex post $\implies$ based on actual results rather than forecasts

Warm-up Exercise

Exercise

- Consider $\mathbf{x} = [x_1 \ x_2 \ x_3]^\top \in \mathbb{R}^3$ and $\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$.

- Compute $\mathbf{x}^\top \boldsymbol{A} \mathbf{x}$.

- Compute $\text{tr}(\boldsymbol{A}\mathbf{x}\mathbf{x}^\top)$.

Reminders

- This is a course NOT only of mathematics, but also for ENGINEERING purpose!

- This is a course which can help you build solid foundation for machine learning (for both industrial and academical tasks and jobs).

- Preview before classes and Review after classes are strongly recommended.

Discussions