Arrays and Structures:

The Sparse Matrix Data Type

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Outline

1. The Sparse Matrix ADT
2. Matrix Transpose
3. Matrix Multiplication

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An $m \times n$ Matrix

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  - Totally 15 elements with 15 nonzero entries.

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$$\begin{bmatrix} 15 & 0 & 0 & 22 & 0 & \text{-15} \\ 0 & 11 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & \text{-6} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 91 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 28 & 0 & 0 & 0 \end{bmatrix}$$

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- How to efficiently store this sparse matrix?

Remarks & The Idea

- The standard representation of a matrix is a two-dimensional array defined as `a[MAX_ROWS][MAX_COLS]`.
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- The standard representation of a matrix is a two-dimensional array defined as `a[MAX_ROWS][MAX_COLS]`.
  - We can locate quickly any element by writing `a[i][j]`.
- Let's consider alternative forms of representation for the matrix.
  - Store only nonzero elements of the matrix.

The Sparse Matrix ADT (1/2)

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▷

A[0]	6	6	8

index, # rows, # cols, # nonzeros

	Row	Col	Value
A[0]	6	6	8
A[1]	0	0	15
A[2]	0	3	22
A[3]	0	5	−15
A[4]	1	1	11
A[5]	1	2	3
A[6]	2	3	−6
A[7]	4	0	−91
A[8]	5	2	−28

The Sparse Matrix ADT (2/2)

```
#define MAX_TERMS 101
// maximum number of items + 1

typedef struct {
    int col;
    int row;
    int value;
} term;

term a[MAX_TERMS];
```

	Row	Col	Value
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A[2]	0	3	22
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Transposing a Matrix

$M \in \mathbb{R}^{6\times 6}$:

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Discussions