

How Bad Can an Election Game of Two or More Parties Be?

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Outline

- 1 Motivations
- 2 Our Contribution
- 3 Conclusion

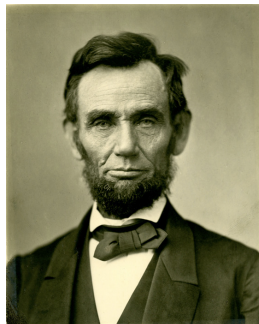
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The Inspiration (an EC'17 paper)



“[...] and that government of the people, by the people, for the people, shall not perish from the earth.”

— Abraham Lincoln, 1863.

Issues of Previous Studies

- Voters' behavior on a **micro-level**.
 - Voters are strategic;
 - Voters have different preferences for the candidates.
 - Various election rules result in different winner(s).
 -

The “Macro” Setting

- Instead, we consider an intuitive **macro** perspective instead.
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- Instead, we consider an intuitive **macro** perspective instead.
 - Parties are players.
 - Strategies: their candidates (or policies).
 - A candidate beats the other one from other parties with **uncertainty**.
 - The payoff of each party: expected utility its supporters can get.
 - The **egoistic** property: each candidate of party \mathcal{P} brings more utility to \mathcal{P} 's supporters than any candidate from the other parties does to \mathcal{P} 's supporters.

Party A



Party B



Two-Party Election Game: Formal Setting

- Party A : m candidates, party B : n candidates.
- Candidate A_i can bring social utility $u(A_i) = u_A(A_i) + u_B(A_i) \in [0, \beta]$ for some real $\beta \geq 1$.
- $p_{i,j}$: $\Pr[A_i \text{ wins over } B_j]$.
 - **Linear**: $p_{i,j} := (1 + (u(A_i) - u(B_j))/\beta)/2$
 - **Natural**: $p_{i,j} := u(A_i)/(u(A_i) + u(B_j))$
 - **Softmax**: $p_{i,j} := e^{u(A_i)/\beta} / (e^{u(A_i)/\beta} + e^{u(B_j)/\beta})$
- Reward $r_A = p_{i,j}u_A(A_i) + (1 - p_{i,j})u_A(B_j)$.

Party A



Winning prob.=0.55

Expected utility for A:
 $0.55 \cdot 7 + 0.45 \cdot 3 = 5.2$ 

$$u(A_1) = 7 + 2 = 9$$



Party B



Winning prob.=0.45

Expected utility for B:
 $0.45 \cdot 5 + 0.55 \cdot 2 = 3.35$ 

$$u(B_1) = 5 + 3 = 8$$



Party A



Winning prob.=0.55

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 $0.55 \cdot 7 + 0.45 \cdot 3 = 5.2$



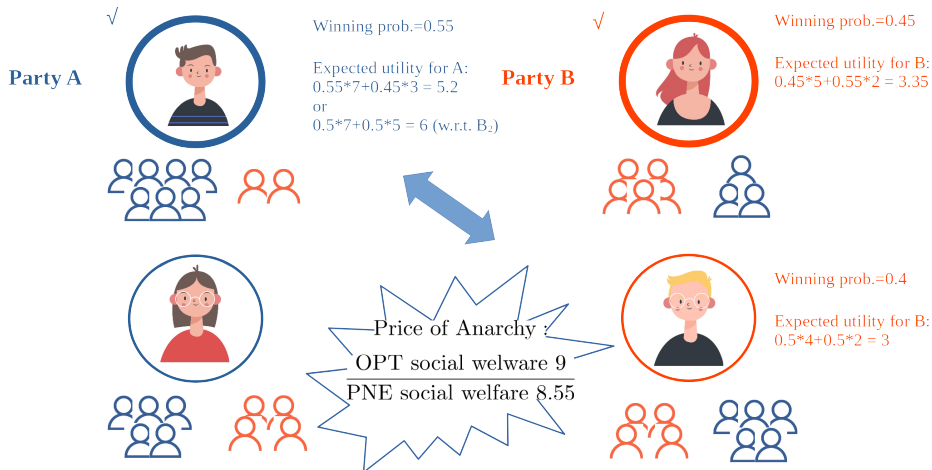
Party B



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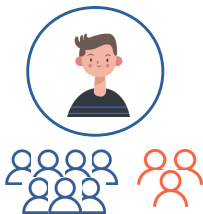
Expected utility for B:
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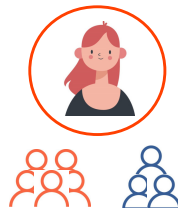


Egoism (Selfishness)

Party A



Party B



>
~~egoism~~

m -Party Election Game, $m \geq 2$

- Party $\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \dots$: with n_1, n_2, n_3, \dots candidates, resp.
- E.g., candidate s_i of party \mathcal{P}_i can bring social utility $u(s_i) = u_1(s_i) + u_2(s_i) + \dots + u_m(s_i) \in [0, \beta]$ for some $\beta \geq 0$.
- $p_{i,s}$: $\Pr[s_i \text{ wins the campaign w.r.t. } \mathbf{s}]$.
 - \mathbf{s} : the strategy profile of all party players.
 - Consider all **monotone** winning probability functions.
 - E.g., $p_{i,s_{-i}} \geq p_{i',s_{-i}}$ whenever $u(s_i) \geq u(s_{i'})$.
- Reward $r_i = p_{1,s_{-i}} u_i(s_1) + p_{2,s_{-i}} u_i(s_2) + \dots + p_{m,s_{-i}} u_i(s_m)$.

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 - The probability for a party to win the election campaign?
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 - The **complexity** of computing a PSNE (≥ 2 parties).
 - NP-completeness & an FPT algorithm.

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 - What's the **price of anarchy (PoA)**?
 - Upper bound: number of parties.
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 - The **complexity** of computing a PSNE (≥ 2 parties).
 - NP-completeness & an FPT algorithm.
 - What's the **price of anarchy (PoA)**?
 - Upper bound: number of parties.
 - The bound is tight for some cases.
 - Incentives for forming a coalition for each party.
 - Existence for **strongly egoistic** games.

Counterexamples (Natural function)

A		B	
$u_A(A_i)$	$u_B(A_i)$	$u_B(B_j)$	$u_A(B_j)$
91	0	11	1
90	8	10	20

A		B	
$u_A(A_i)$	$u_B(A_i)$	$u_B(B_j)$	$u_A(B_j)$
44	10	37	17
39	55	10	5

	B_1	B_2	
A_1	$a_{1,1}, b_{1,1}$	$a_{1,2}, b_{1,2}$	\approx
A_2	$a_{2,1}, b_{2,1}$	$a_{2,2}, b_{2,2}$	

	B_1	B_2	
A_1	80.51, 1.28	73.84, 2.17	,
A_2	80.29, 8.32	74.02, 8.23	

	B_1	B_2	
A_1	30.50, 23.50	35.52, 10.00	
A_2	30.97, 48.43	34.32, 48.81	

Counterexamples (Softmax; Three parties)

$u_1(x_{1,i})$	$u_2(x_{1,i})$	$u_3(x_{1,i})$	$u_1(x_{2,i})$	$u_2(x_{2,i})$	$u_3(x_{2,i})$	$u_1(x_{3,i})$	$u_2(x_{3,i})$	$u_3(x_{3,i})$
29	4	21	23	59	7	8	32	54
27	43	3	3	57	38	20	13	53

$r_{1,(1,1,1)}$	$r_{2,(1,1,1)}$	$r_{3,(1,1,1)}$	$r_{1,(1,1,2)}$	$r_{2,(1,1,2)}$	$r_{3,(1,1,2)}$	\approx
$r_{1,(1,2,1)}$	$r_{2,(1,2,1)}$	$r_{3,(1,2,1)}$	$r_{1,(1,2,2)}$	$r_{2,(1,2,2)}$	$r_{3,(1,2,2)}$	
18.81	34.64	28.51	23.49	27.82	27.38	
11.27	34.67	39.70	15.57	28.09	38.93	

$r_{1,(2,1,1)}$	$r_{2,(2,1,1)}$	$r_{3,(2,1,1)}$	$r_{1,(2,1,2)}$	$r_{2,(2,1,2)}$	$r_{3,(2,1,2)}$	\approx
$r_{1,(2,2,1)}$	$r_{2,(2,2,1)}$	$r_{3,(2,2,1)}$	$r_{1,(2,2,2)}$	$r_{2,(2,2,2)}$	$r_{3,(2,2,2)}$	
18.74	44.53	22.84	23.18	38.35	21.61	
11.58	44.25	33.66	15.67	38.27	32.77	

Complexity and PoA Bounds

Theorem

For any m -party election game, $m \geq 2$, we have $\text{PoA} \leq m$ if

- The winning probability function is **monotone**.
- The game is egoistic.

Theorem

To determine if a PSNE exists in the egoistic m -party election game is NP-complete but FPT (+natural parameters).

- The game instance is in a succinct representation.
- A reduction from the SAT problem.

Key Propositions

Proposition

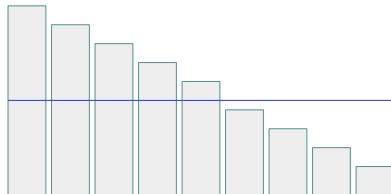
Let $\mathbf{s} = (s_i)_{i \in [m]}$ be a PNE and $\mathbf{s}^* = (s_i^*)_{i \in [m]}$ be the optimal profile. Then, $\sum_{i \in [m]} u(s_i) \geq \max_{i \in [m]} u(s_i^*)$.

Two Important Observations

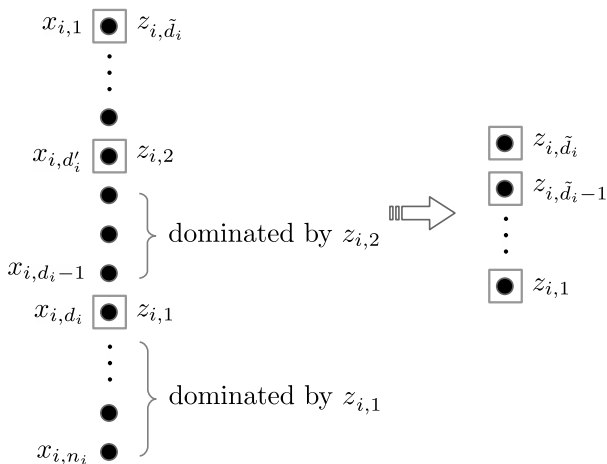
$$SW(\mathbf{s}) = \sum_{1 \leq i \leq m} p_{i,\mathbf{s}} \cdot u(s_i) \leq \max_{1 \leq i \leq m} u(s_i)$$

$$SW(\mathbf{s}) = \sum_{1 \leq i \leq m} p_{i,\mathbf{s}} \cdot u(s_i) \geq \frac{1}{m} \cdot \sum_{1 \leq i \leq m} u(s_i).$$

The idea for the lower bound



Shrinking nominating depth of a party



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Concluding Remarks

- We assume the utility is evenly distributed to the voters.
- The PoA is small in most game instances (simulations).
- We will conduct experiments to simulate voters' voting decisions to see how *monotone* the winning probability function is.

Thanks for your attention!