

A Game-Theoretic Exploration on Group Competition and Formation through Online Learning Algorithms

Joseph Chuang-Chieh Lin (林莊傑)
– <https://josephcclin.github.io>

Assistant Professor

National Taiwan Ocean University
Department of Computer Science & Engineering

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Outline

- 1 Short Self-Introduction
- 2 Motivations
- 3 Election Game
- 4 Group Formation & Opinion Updating
- 5 Conclusion

Education

- BS.: Mathematics (2002),
National Cheng Kung University
- MS.: CSIE (2004),
National Chi Nan University
 - Supervisor: R. C. T. Lee
Algorithms
- Ph.D.: CSIE (2011),
National Chung Cheng University
 - Supervisors: Maw-Shang Chang
& Peter Rossmanith
FPT + Randomized Algorithms



DAAD-NSC Sandwich Program (2007–2008)

RWTH Aachen University (Funding: DAAD + NSC 96-2911-I-194-008-2.)



Postdoc in Academia Sinica (2011–2018)

研發替代役 (2011–2014)

@Genomics Research Center, Academia Sinica

- Bioinformatics,
 - Comparative Genomics
- PI: Trees-Juen Chuang



Academia Sinica
Genomics Research Center



@Institute of Information Science, Academia Sinica

- Machine Learning,
 - Game Theory
- PI: Chi-Jen Lu



Industrial Experience (2018–2020)

- Quantitative Analyst (intern) of Point72/Cubist Systematic Strategies (2018–2020).
 - US Hedge Fund; Fintech; Data Science.
 - Taipei Branch (started since in 2019).
 - CEO & Chairman: Steven A. Cohen.
 - AUM: US\$27.2 billion (Jan. 2023).
- Quantitative Analyst of Seth Technologies Inc. (2020–2021).
 - High-Frequency Trading; Hedge Fund; Fintech; Data Science.
 - Taiwan based.



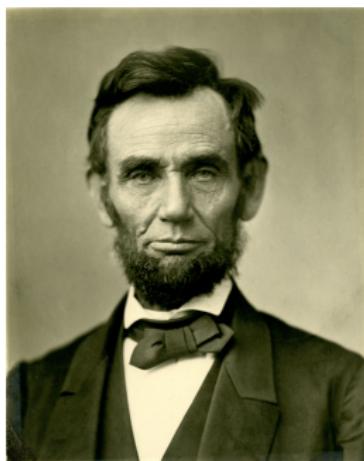
Photo: REUTERS/Lucy Nicholson
<https://www.calcalistech.com/cnchnews/article/h1td1t3s>



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The Inspiration



[...] and that government of the people, by the people, for the people, shall not perish from the earth."

— Abraham Lincoln, 1863.

Motivations (I): Why The Two-Party System?



“The simple-majority single-ballot system favours the two-party system.”
— Maurice Duverger, 1964.

Motivations (II): Social Choice Rules

Example:

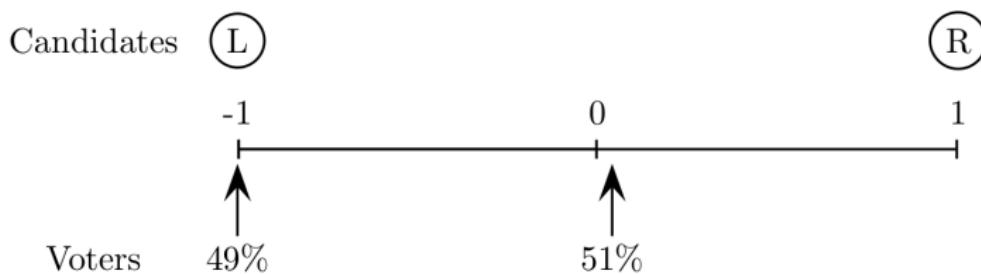
- Each voter provides an ordinal ranking of the candidates,
- Aggregate these rankings to produce either a single winner or a consensus ranking of all (or some) candidates.

Gibbard–Satterthwaite Theorem (1973)

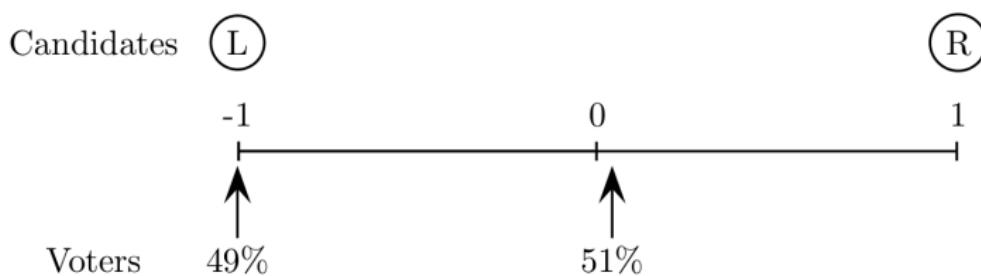
Given a deterministic electoral system that choose a single winner. For every voting rule, one of the following three things must hold:

- The rule is dictatorial.
- The rule limits the possible outcomes to two alternatives only.
- The rule is susceptible to tactical voting.

Motivations (III): Distortion of Social Choice Rules



Motivations (III): Distortion of Social Choice Rules



- The average distance from the population to candidate L: ≈ 0.5 .
- The average distance from the population to candidate R: ≈ 1.5 .
- But R will be elected as the winner in the election.

Issues of Previous Studies

- Voters' behavior on a **micro-level**.
 - Voters are strategic;
 - Voters have different preferences for the candidates.
 - Various election rules result in different winner(s).
- ⋮

Our Contribution (I)

- First, we consider an intuitive **macro** perspective instead.
 - Parties are players;
 - The strategies can be their nominated candidates (or policies);

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 - The point is:
 - who is **more likely to win** the election campaign?
 - Is the game “stable” in some sense?
 - What’s the price for stability which resembles “the distortion”?

Our Contribution (II)

- Second, we simulate the behaviors of myopic strategic agents.
 - group-joining strategies
 - opinion updates
 - opinion updates with regularization
 - not willing to deviate from their beliefs too much.

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Two-Party Election Game

Coauthors



Chi-Jen Lu
IIS, Academia Sinica
TW



Po-An Chen
IIM, NYCU
TW

Party A**Party B**

Two-Party Election Game: Formal Setting

- Party A : m candidates, party B : n candidates.
- Candidate A_i can bring social utility $u(A_i) = u_A(A_i) + u_B(A_i) \in [0, \beta]$ for some real $\beta \geq 1$.
- $p_{i,j}$: $\Pr[A_i \text{ wins over } B_j]$.
 - **Linear link**: $p_{i,j} := (1 + (u(A_i) - u(B_j))/\beta)/2$
 - **Natural**: $p_{i,j} := u(A_i)/(u(A_i) + u(B_j))$
 - **Softmax**: $p_{i,j} := e^{u(A_i)/\beta} / (e^{u(A_i)/\beta} + e^{u(B_j)/\beta})$
- Reward $r_A = p_{i,j}u_A(A_i) + (1 - p_{i,j})u_A(B_j)$.



Winning prob.=0.55

Expected utility for A:
 $0.55*7+0.45*3 = 5.2$

Party A



$$u(A_1) = 7 + 2 = 9$$



Winning prob.=0.45

Expected utility for B:
 $0.45*5+0.55*2 = 3.35$

Party B



$$u(B_1) = 5 + 3 = 8$$





Winning prob.=0.55

Expected utility for A:
 $0.55*7+0.45*3 = 5.2$

Party A

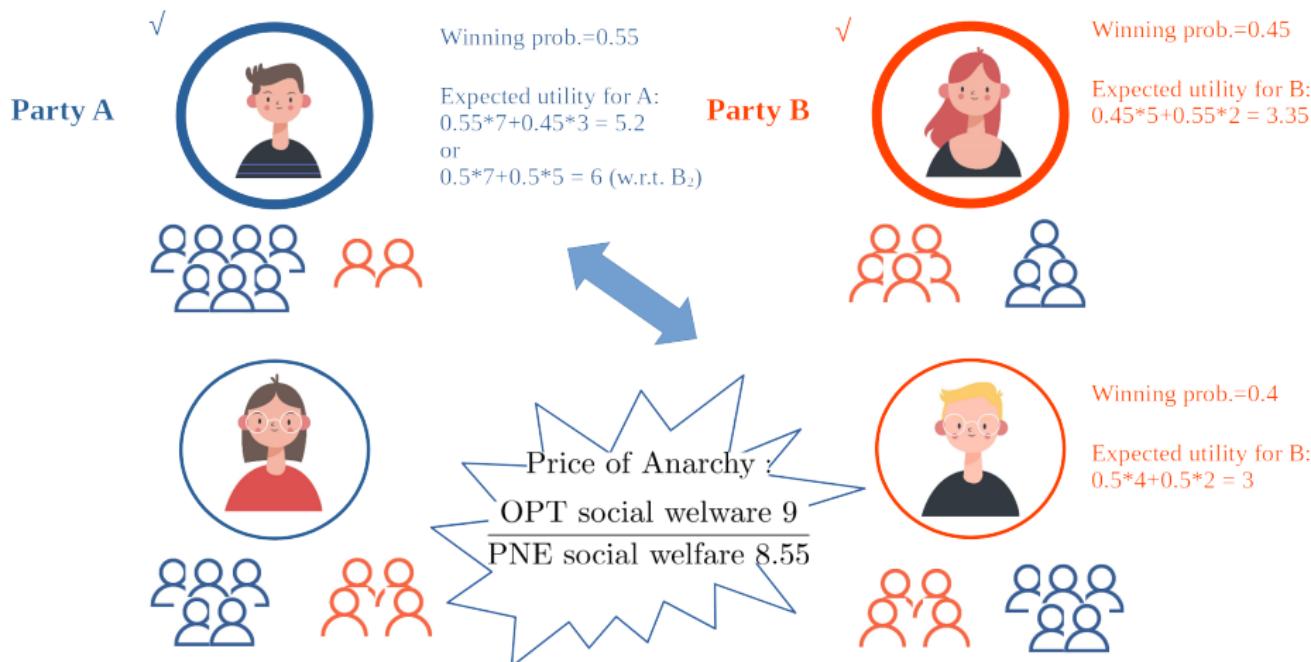


Winning prob.=0.45

Expected utility for B:
 $0.45*5+0.55*2 = 3.35$

Party B



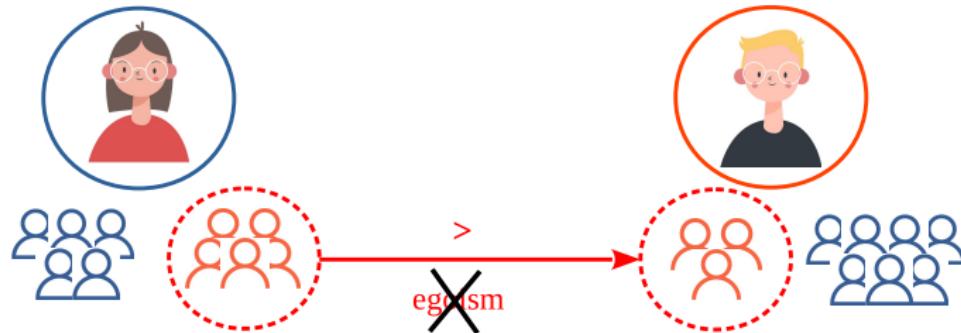
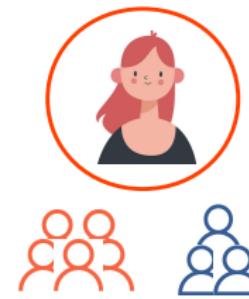


Egoism (Selfishness)

Party A



Party B



Election Game

```

# iteration 9625:
Party A's candidates: (446, 4), (323, 372)
Party B's candidates: (503, 84), (428, 262)

    240.203,      287.682      331.92,      266.88
    216.406,      430.426      292.653,     399.86
PoA updated: 1.31186

# iteration 78207:
Party A's candidates: (530, 13), (420, 362)
Party B's candidates: (485, 58), (405, 317)

    294,          249          404.437,     244.084
    282.259,      408.802      371.59,      382.21
PoA updated: 1.38821

# iteration 1440494:
Party A's candidates: (552, 16), (517, 52)
Party B's candidates: (667, 6), (483, 508)

    250.335,      375.678      520.694,     348.27
    234.928,      391.48       510.601,     358.441
PoA updated: 1.38822

# iteration 3280308:
Party A's candidates: (361, 33), (230, 363)
Party B's candidates: (436, 7), (374, 212)

    175.327,      244.373      272.196,     236.236
    135.225,      394.025      221.063,     368.462
PoA updated: 1.40463

# iteration 5403654:
Party A's candidates: (393, 55), (332, 293)
Party B's candidates: (402, 35), (332, 312)

    215.969,      226.592      344.562,     220.646
    211.418,      337.254      321.81,      312.87
PoA updated: 1.43411

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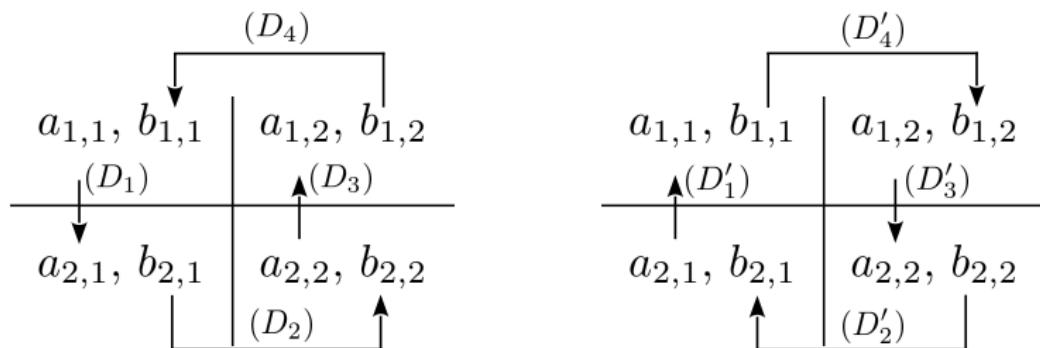
Counterexamples (Natural function)

<i>A</i>		<i>B</i>	
$u_A(A_i)$	$u_B(A_i)$	$u_B(B_j)$	$u_A(B_j)$
91	0	11	1
90	8	10	20

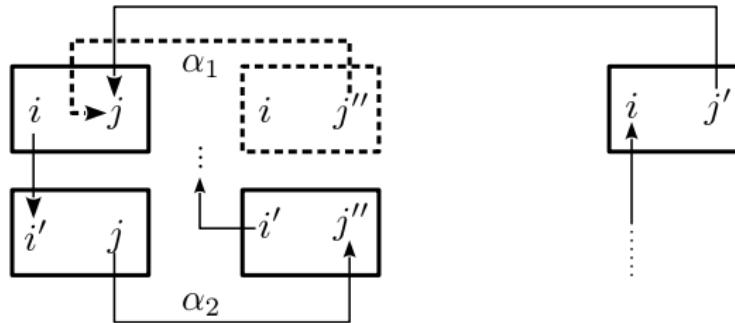
<i>A</i>		<i>B</i>	
$u_A(A_i)$	$u_B(A_i)$	$u_B(B_j)$	$u_A(B_j)$
44	10	37	17
39	55	10	5

	B_1	B_2	\approx		B_1	B_2
	$a_{1,1}, b_{1,1}$	$a_{1,2}, b_{1,2}$			$a_{2,1}, b_{2,1}$	$a_{2,2}, b_{2,2}$
A_1	$a_{1,1}, b_{1,1}$	$a_{1,2}, b_{1,2}$			80.51, 1.28	73.84, 2.17
	$a_{2,1}, b_{2,1}$	$a_{2,2}, b_{2,2}$			80.29, 8.32	74.02, 8.23
	B_1	B_2			B_1	B_2
	A_1	30.50, 23.50			35.52, 10.00	
A_2	30.97, 48.43	34.32, 48.81				

The Deviation Cycles



Extending to general cases $m, n \geq 2$



PoA (Linear & Softmax; tight)

<i>A</i>		<i>B</i>	
$u_A(A_i)$	$u_B(A_i)$	$u_B(B_j)$	$u_A(B_j)$
ϵ	0	ϵ	0
$\epsilon - \delta$	$\epsilon - \delta$	$\epsilon - \delta$	$\epsilon - \delta$

	B_1	B_2		B_1	B_2
A_1	$a_{1,1}, b_{1,1}$	$a_{1,2}, b_{1,2}$	\approx	$\frac{\epsilon}{2}, \frac{\epsilon}{2}$	$\epsilon - \frac{\delta}{2}, \frac{\epsilon}{2} - \frac{\delta}{2}$
A_2	$a_{2,1}, b_{2,1}$	$a_{2,2}, b_{2,2}$		$\frac{\epsilon}{2} - \frac{\delta}{2}, \epsilon - \frac{\delta}{2}$	$\epsilon - \delta, \epsilon - \delta$



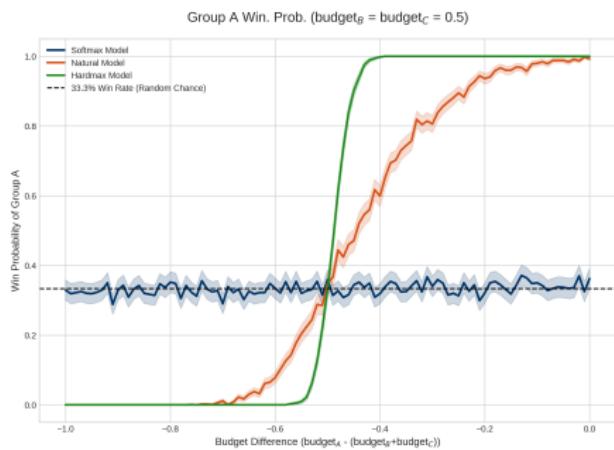
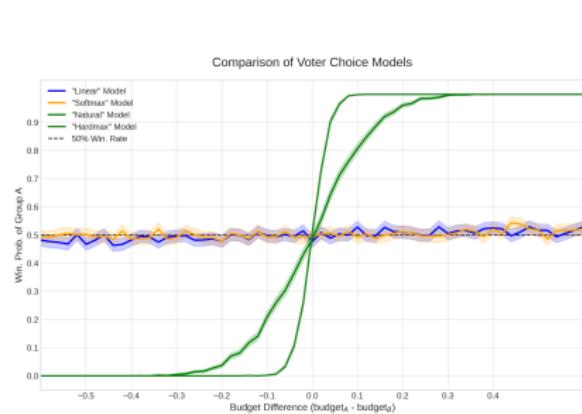
Results (Two-Party)

	Linear Link	Natural	Softmax
PNE w/ egoism	✓	✗	✓
PNE w/o egoism	✗	✗	?#
Worst PoA w/ egoism	$\leq 2^*$	≤ 2	$\leq 1 + e$
Worst PoA w/o egoism	∞	∞	∞

- Lin, Lu, Chen: *Theoret. Comput. Sci.*, 2021.

How about the Game for Two-or-More Parties?

Verifying the Monotone Property



Example: Three-Party Election Game

- Party A, B, C : with m_1, m_2, m_3 candidates, resp.
- E.g., candidate A_i can bring social utility
 $u(A_i) = u_A(A_i) + u_B(A_i) + u_C(A_i) \in [0, \beta]$ for some real $\beta \geq 1$.
- $p_{i,(j,k)}$: $\Pr[A_i \text{ wins over } B_j \text{ and } C_k]$.
 - **Natural**: $p_{i,(j,k)}^A := u(A_i) / (u(A_i) + u(B_j) + u(C_k))$
 - **Softmax**: $p_{i,(j,k)}^A := e^{u(A_i)/\beta} / (e^{u(A_i)/\beta} + e^{u(B_j)/\beta} + e^{u(C_k)/\beta})$
- Reward $r_A = p_{i,(j,k)}^A u_A(A_i) + p_{j,(i,k)}^B u_A(B_j) + p_{k,(i,j)}^C u_A(C_k)$.

k -Party Election Game, $k \geq 2$

- Party A, B, C, \dots : with m_1, m_2, m_3, \dots candidates, resp.
- E.g., candidate A_i can bring social utility
 $u(A_i) = u_A(A_i) + u_B(A_i) + u_C(A_i) + \dots \in [0, \beta]$ for some real $\beta \geq 1$.
- $p_{i,(j,k,\dots)}^A$: $\Pr[A_i \text{ wins over the other candidates}]$.
 - Consider all **monotone** winning probability functions.
 - E.g., $p_{i,(-i)}^A \geq p_{i',(-i)}^A$ whenever $u(A_i) \geq u(A_{i'})$.
- Reward $r_A = p_{i,(j,k)}^A u_A(A_i) + p_{j,(i,k)}^B u_A(B_j) + p_{k,(i,j)}^C u_A(C_k) + \dots$

PoA in General & FPT

Bad News

Three-party election games do not always have a PNE, even it is egoistic.

Theorem

For any k -party election game, $k \geq 2$, we have $\text{PoA} \leq k$ if

- The winning probability function is monotone.
- The game is egoistic.

Theorem

To compute a PNE of the egoistic k -party election game is NP-hard but FPT (+natural parameters).

Key Propositions

Proposition

Let $\mathbf{s} = (s_i)_{i \in [m]}$ be a PNE and $\mathbf{s}^* = (s_i^*)_{i \in [m]}$ be the optimal profile. Then,
 $\sum_{i \in [m]} u(s_i) \geq \max_{i \in [m]} u(s_i^*)$.

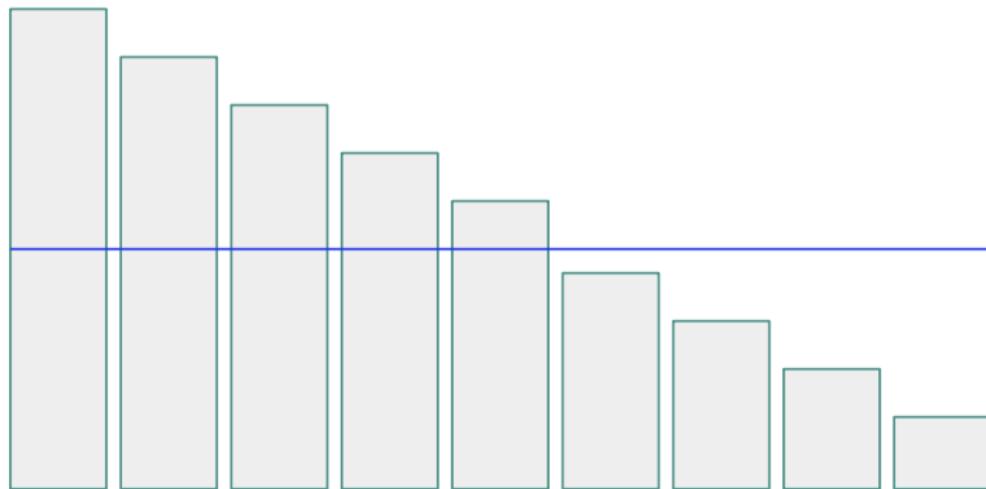
Two Important Observations

$$SW(\mathbf{s}) = \sum_{1 \leq i \leq k} p_{i,\mathbf{s}} \cdot u(s_i) \leq \max_{1 \leq i \leq k} u(s_i)$$

$$SW(\mathbf{s}) = \sum_{1 \leq i \leq k} p_{i,\mathbf{s}} \cdot u(s_i) \geq \frac{1}{m} \cdot \sum_{1 \leq i \leq k} u(s_i).$$



An idea on the train



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IIM, NYCU
TW



Chih-Chieh Hung
MIS, NCHU
TW

Group Formation

- n agents v_1, v_2, \dots, v_n with opinions $\mathbf{z} = (z_1, z_2, \dots, z_n)$ and beliefs $\mathbf{s} = (s_1, s_2, \dots, s_n)$.
- Agents in groups $\mathcal{G} = (G_1, G_2, \dots, G_m)$.
- State of the game: $\tau = (\mathbf{z}, \mathbf{s}, \mathcal{G})$
- Winning prob. of group G_j :

$$p_j(\tau) = \frac{e^{n_j \langle \bar{g}_j, \sum_{v_r \in V} s_r \rangle}}{\sum_{i \in [m]; n_i > 0} e^{n_i \langle \bar{g}_i, \sum_{v_r \in V} s_r \rangle}},$$

- Reward of agent i :

$$r_i(\tau) = \sum_{j=1}^m p_j(\tau) \langle s_i, \bar{g}_j \rangle,$$

Group Formation

- By the group joining strategy:
 - $j = \arg \max_\ell p_\ell(\tau) \cdot \langle \bar{g}_\ell, s_i \rangle$, where $\bar{g}_j = \sum_{v_i \in G_j} z_i / |n_j|$: the opinion of group G_j .

Group Formation

- Multiagent Online Gradient Ascent + Regularization

$$r_i(\tau_t) = \sum_{j=1}^m p_j(\tau_t) \langle s_i, \bar{g}_j \rangle - \|z_i - s_i\|_2^2.$$

- τ_t : state at time t ; p_j : win. prob. of group j ;
- z_i, s_i : opinion and belief of agent i respectively; \bar{g}_j : avg. opinion of group j .

Group Formation (IEEE CIM - AI-eXplained)

Group Formation by Group Joining and Opinion Updates via Multi-Agent Online Gradient Ascent

An interactive article on illustrating group joining strategies and opinion updates via online gradient ascent to analyze group formation dynamics. Learn how the choices of coalition of agents lead to a pure-strategy Nash equilibrium and how updating agents' opinions eventually stabilizes group formation.

Authors

Chuang-Chieh Lin | Tamkang University, Taiwan

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Chih-Chieh Hung | National Chung Hsing University, Taiwan

Chi-Jen Lu | Academia Sinica, Taiwan

Po-An Chen | National Yang Ming Chiao Tung University, Taiwan



Indicates interactive elements

Contents:

I. INTRODUCTION | II. THE GAME SETTING | III. GROUP JOINING | IV. OPINION UPDATES BY ONLINE GRADIENT ASCENT |

V. DISCUSSION | VI. CONCLUSION

Thoughts

- Asking good questions is important.

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- Asking good questions is important.
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- Keep learning and enjoy the process.
- To teach to learn.

Mottos

“Think hard, and work smartly.” – R. C. T. Lee & Maw-Shang Chang

“Every job is a self-portrait of the person who did it. Autograph your work with quality.” - Prof. D. T. Lee