Tree Traversals

Joseph Chuang-Chieh Lin (林莊傑)

Department of Computer Science & Engineering,
National Taiwan Ocean University

Fall 2024

Outline

1. Binary Tree Traversals (Using a Stack)
2. Level-Order Traversal
3. Additional Binary Tree Operations

Outline

1. Binary Tree Traversals (Using a Stack)
2. Level-Order Traversal
3. Additional Binary Tree Operations

Binary Tree Traversals

Question

How to visit each node of a tree exactly once?

Binary Tree Traversals

Question

How to visit each node of a tree exactly once?

- Let V, L, R stand for visiting the node, moving left, and moving right, resp.

Binary Tree Traversals

Question

How to visit each node of a tree exactly once?

- Let V, L, R stand for visiting the node, moving left, and moving right, resp.
  - Six possible combinations: LVR, LRV, VLR, VRL, RVL, RLV.

Binary Tree Traversals

Question

How to visit each node of a tree exactly once?

- Let V, L, R stand for visiting the node, moving left, and moving right, resp.
  - Six possible combinations: LVR, LRV, VLR, VRL, RVL, RLV.
- Adopting the convention that we traverse left before right, only three combinations of VLR remains:

Binary Tree Traversals

Question

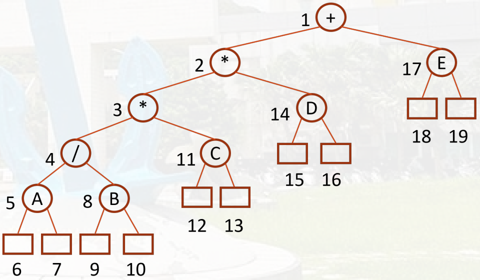
How to visit each node of a tree exactly once?

- Let V, L, R stand for visiting the node, moving left, and moving right, resp.
  - Six possible combinations: LVR, LRV, VLR, VRL, RVL, RLV.
- Adopting the convention that we traverse left before right, only three combinations of VLR remains:
  - inorder (中序走訪法)
  - postorder (後序走訪法)
  - preorder (先序走訪法)

Tree Traversals

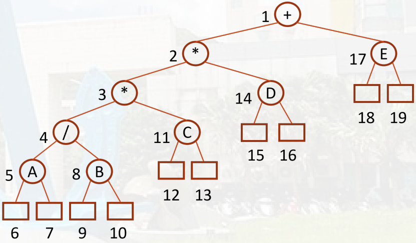
The tree represents the arithmetic expression (in infix form).

- Inorder traversal (*LVR*):
  `A/B*C*D+E`
- Preorder traversal (*VLR*):
  `+**/ABCDE`
- Postorder traversal (*LRV*):
  `AB/C*D*E+`



Code for Inorder Traversal

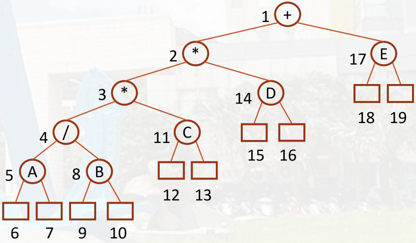
```
void inorder (treePointer ptr) {
/* inorder tree traversal */
    if (ptr) {
        inorder (ptr->leftChild);
        printf ("%d", ptr->data);
        inorder (ptr->rightChild);
    }
}
```



A/B*C*D+E

Code for Preorder Traversal

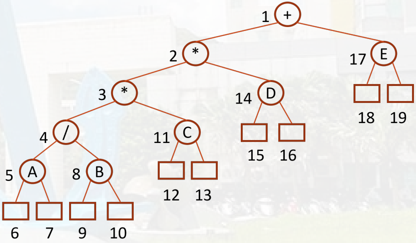
```
void Preorder (treePointer ptr) {
/* inorder tree traversal */
    if (ptr) {
        printf ("%d", ptr->data);
        inorder (ptr->leftChild);
        inorder (ptr->rightChild);
    }
}
```



+**/ABCDE

Code for Postorder Traversal

```
void Postorder (treePointer ptr) {
/* inorder tree traversal */
    if (ptr) {
        inorder (ptr->leftChild);
        inorder (ptr->rightChild);
        printf ("%d", ptr->data);
    }
}
```



AB/C*D*E+

Iterative Inorder Traversal

```
void iter_inorder (treePointer node) {
    int top = -1; // initialize the stack
    tree_pointer stack[MAX_STACK_SIZE];
    while (1) {
        while (node != NULL) {
            push(&top, node); // add node to the stack
            node = node->left_child;
        }
        node = stack[top]; // pop from stack
        pop();
        if (!node) break; // empty stack, so break!
        printf("%d", node->data);
        node = node->right_child;
    }
}
```

Analysis of Iterative Inorder Traversal

- Let n be the number of nodes in the tree.

Analysis of Iterative Inorder Traversal

- Let n be the number of nodes in the tree.
- Time complexity: $O(n)$.
  - Every node of the tree is pushed and popped from the stack exactly once.

Analysis of Iterative Inorder Traversal

- Let n be the number of nodes in the tree.
- Time complexity: $O(n)$.
  - Every node of the tree is pushed and popped from the stack exactly once.
- Space complexity: $O(n)$.
  - The stack size $\leq$ the depth of the tree.

Outline

Level-Order Traversal

- When written recursively, the inorder, preorder, and postorder traversals all require a stack.

Level-Order Traversal

- When written recursively, the inorder, preorder, and postorder traversals all require a stack.
- We now turn to a traversal that requires a queue. This traversal called level-order traversal.

Level-Order Traversal

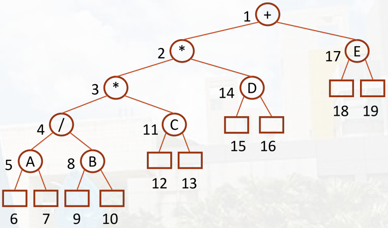
- When written recursively, the inorder, preorder, and postorder traversals all require a stack.
- We now turn to a traversal that requires a queue. This traversal called level-order traversal.

Steps of a Level-Order Traversal

1. visit the root first.
2. then the root's left child followed by the right child.
3. visit next level from leftmost node to right most node.

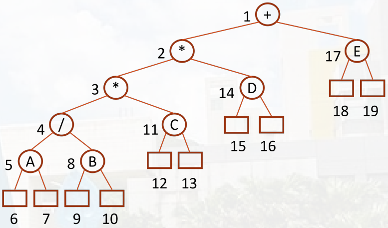
Code for the Level-Order Traversal

```
void levelOrder(treePointer ptr) {
    int front = rear = 0;
    treePointer queue[MAX_QUEUE_SIZE];
    if (!ptr) return; /* empty tree */
    add(ptr); // enqueue
    while (1) {
        ptr = delete(); // dequeue
        if (ptr) {
            printf("%d",ptr->data);
            if (ptr->leftChild)
            // leftChild exists
                add(ptr->leftChild);
                // enqueue
            if (ptr->rightChild)
            // rightChild exists
                add(ptr->rightChild);
                // enqueue
        } else break;
    }
}
```



Code for the Level-Order Traversal

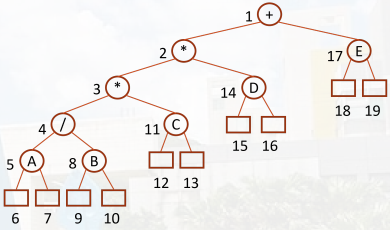
```
void levelOrder(treePointer ptr) {
    int front = rear = 0;
    treePointer queue[MAX_QUEUE_SIZE];
    if (!ptr) return; /* empty tree */
    add(ptr); // enqueue
    while (1) {
        ptr = delete(); // dequeue
        if (ptr) {
            printf("%d",ptr->data);
            if (ptr->leftChild)
            // leftChild exists
                add(ptr->leftChild);
                // enqueue
            if (ptr->rightChild)
            // rightChild exists
                add(ptr->rightChild);
                // enqueue
        } else break;
    }
}
```



+*

Code for the Level-Order Traversal

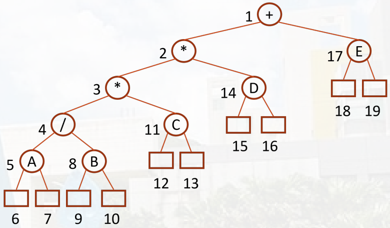
```
void levelOrder(treePointer ptr) {
    int front = rear = 0;
    treePointer queue[MAX_QUEUE_SIZE];
    if (!ptr) return; /* empty tree */
    add(ptr); // enqueue
    while (1) {
        ptr = delete(); // dequeue
        if (ptr) {
            printf("%d",ptr->data);
            if (ptr->leftChild)
            // leftChild exists
                add(ptr->leftChild);
                // enqueue
            if (ptr->rightChild)
            // rightChild exists
                add(ptr->rightChild);
                // enqueue
        } else break;
    }
}
```



+*E

Code for the Level-Order Traversal

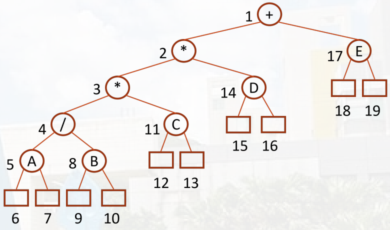
```
void levelOrder(treePointer ptr) {
    int front = rear = 0;
    treePointer queue[MAX_QUEUE_SIZE];
    if (!ptr) return; /* empty tree */
    add(ptr); // enqueue
    while (1) {
        ptr = delete(); // dequeue
        if (ptr) {
            printf("%d",ptr->data);
            if (ptr->leftChild)
            // leftChild exists
                add(ptr->leftChild);
                // enqueue
            if (ptr->rightChild)
            // rightChild exists
                add(ptr->rightChild);
                // enqueue
        } else break;
    }
}
```



+*E*

Code for the Level-Order Traversal

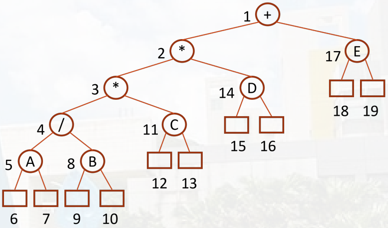
```
void levelOrder(treePointer ptr) {
    int front = rear = 0;
    treePointer queue[MAX_QUEUE_SIZE];
    if (!ptr) return; /* empty tree */
    add(ptr); // enqueue
    while (1) {
        ptr = delete(); // dequeue
        if (ptr) {
            printf("%d",ptr->data);
            if (ptr->leftChild)
            // leftChild exists
                add(ptr->leftChild);
                // enqueue
            if (ptr->rightChild)
            // rightChild exists
                add(ptr->rightChild);
                // enqueue
        } else break;
    }
}
```



+*E*D

Code for the Level-Order Traversal

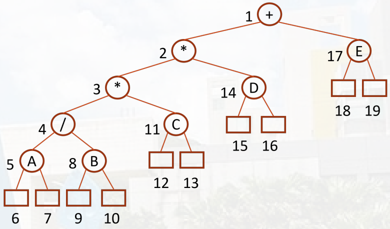
```
void levelOrder(treePointer ptr) {
    int front = rear = 0;
    treePointer queue[MAX_QUEUE_SIZE];
    if (!ptr) return; /* empty tree */
    add(ptr); // enqueue
    while (1) {
        ptr = delete(); // dequeue
        if (ptr) {
            printf("%d",ptr->data);
            if (ptr->leftChild)
            // leftChild exists
                add(ptr->leftChild);
                // enqueue
            if (ptr->rightChild)
            // rightChild exists
                add(ptr->rightChild);
                // enqueue
        } else break;
    }
}
```



+*E*D/

Code for the Level-Order Traversal

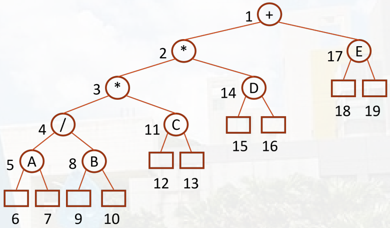
```c
void levelOrder(treePointer ptr) {
    int front = rear = 0;
    treePointer queue[MAX_QUEUE_SIZE];
    if (!ptr) return; /* empty tree */
    add(ptr); // enqueue
    while (1) {
        ptr = delete(); // dequeue
        if (ptr) {
            printf("%d",ptr->data);
            if (ptr->leftChild)
            // leftChild exists
                add(ptr->leftChild);
                // enqueue
            if (ptr->rightChild)
            // rightChild exists
                add(ptr->rightChild);
                // enqueue
        } else break;
    }
}
```



+**E*D/C

Code for the Level-Order Traversal

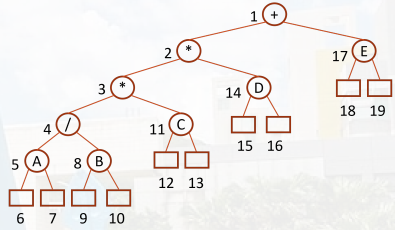
```
void levelOrder(treePointer ptr) {
    int front = rear = 0;
    treePointer queue[MAX_QUEUE_SIZE];
    if (!ptr) return; /* empty tree */
    add(ptr); // enqueue
    while (1) {
        ptr = delete(); // dequeue
        if (ptr) {
            printf("%d",ptr->data);
            if (ptr->leftChild)
            // leftChild exists
                add(ptr->leftChild);
                // enqueue
            if (ptr->rightChild)
            // rightChild exists
                add(ptr->rightChild);
                // enqueue
        } else break;
    }
}
```



+**E*D/CA

Code for the Level-Order Traversal

```c
void levelOrder(treePointer ptr) {
    int front = rear = 0;
    treePointer queue[MAX_QUEUE_SIZE];
    if (!ptr) return; /* empty tree */
    add(ptr); // enqueue
    while (1) {
        ptr = delete(); // dequeue
        if (ptr) {
            printf("%d",ptr->data);
            if (ptr->leftChild)
            // leftChild exists
                add(ptr->leftChild);
                // enqueue
            if (ptr->rightChild)
            // rightChild exists
                add(ptr->rightChild);
                // enqueue
        } else break;
    }
}
```



+**E*D/CAB

Outline

Copying a Binary Tree

```
treePointer copy(treePointer original) {
/* return a tree_pointer to an exact copy of the original tree */
    treePointer temp;
    if (original) {
        MALLOC(temp, sizeof(*temp));
        temp->leftChild = copy(original->leftChild);
        temp->rightChild = copy(original->rightChild);
        temp->data = original->data;
        return temp;
    }
    return NULL;
}
```

Testing for Equality of Binary Trees

```
int equal(treePointer first, treePointer second) {
 /* function returns FALSE if the binary trees first and second are not equ
 Otherwise it returns TRUE */
    return (
        (!first && !second) || (first && second &&
        (first->data == second->data) &&
        equal(first->leftChild, second->leftChild) &&
        equal(first->rightChild, second->rightChild))
    }
}
```

equal(treePointer first, treePointer second)

First->data == second->data

equal(treePointer first->leftChild, treePointer second->leftChild)

equal(treePointer first->rightChild, treePointer second->rightChild)

Discussions