Basic Concepts

Performance Analysis & Measurement

Joseph Chuang-Chieh Lin (林莊傑)

Department of Computer Science & Engineering, National Taiwan Ocean University



Outline

Performance Analysis

2 Performance Measurement



Outline

Performance Analysis

2 Performance Measurement



Criteria for judging a program:

- Meet the original specification?
- Work correctly?
- The documentation.
- Does the program effectively use functions to create logic units?
- Code readability.
- Efficient usage of storage?
- Acceptable running time?



Performance Analysis

machine independent

- Space complexity
 - The amount of memory that it needs to run to completion.
- Time complexity
 - Computing time



Space complexity

$$S(P) = c + \frac{S_P(I)}{I},$$

P: the program; I: the input.

* $S_P(I)$ can be represented by $S_P(n)$ if n is the only instance characteristic.



Space complexity

$$S(P)=c+S_P(I),$$

P: the program; I: the input.

- * $S_P(I)$ can be represented by $S_P(n)$ if n is the only instance characteristic.
- Fixed space requirement: c.
 - Independent of the characteristics of the inputs and outputs.
 - Instruction space.
 - Space for simple variables, fixed-size structured variable and constants.
- Variable Space Requirement $(S_P(I))$
 - depend on the instance characteristic I.
 - For instance, additional space when the program uses recursion.
 - values of inputs and outputs associated with I.



Example

 Assume that the integers are stored in an array 'list', such that the ith integer is stored in the ith position list[i].

```
float abc(float a, float b, float c) {
    return a + b + b * c + (a + b - c) / (a + b) + 4.00;
}
```

- Fixed space requirement (c): 16.
 - Three float numbers: a, b, c and one return float number.
- $S_{abc}(I) = 0$. (for only fixed space requirements)



Example

```
float sum(float list[], int n) {
    float temp = 0;
    int i;
    for (i=0; i < n; i++)
        temp += list[i];
    return temp;
}</pre>
```

- In this program, $S_{\text{sum}}(I) = 0$.
- C Programming Language: passing the address of the first element of list[] (instead of copying).

Example (recursive)

```
float rsum(float list[], int n) {
   if (n) return list[n] + rsum(list, n-1);
   return list[0];
}
```

- Total variable space: $S_{\text{rsum}}(I) = 12n$.
 - parameter list[]: array pointer: 4 bytes.
 - parameter n: integer: 4 bytes
 - return address (internally used): 4 bytes.
- The recursive version has a far greater overhead than its iterative counterpart.



Time Complexity: $T(P) = c + T_P(I)$

- Compile time: c
 - Independent of the characteristics of the input and output.
 - Once the correctness of the program is verified, it can run without recompilation.
- Run time: $T_P(I)$ (what we are really concerned about)
 - E.g., $T_P(n) = c_a \cdot \text{ADD}(n) + c_s \cdot \text{SUB}(n) + c_l \cdot \text{LDA}(n) + c_{st} \cdot \text{STA}(n)$.
 - ADD, SUB, LDA, STA: the number of additions, subtractions, loads and stores.
 - c_a , c_s , c_l , c_{st} : the time needed to perform each operation (constants).



Time Complexity - Program Step (1/2)

* machine independent

Program Step

a syntactically or semantically meaningful program segment whose execution time is independent of the instance characteristics.

- Example of ONE program step
 - a = 2;
 - a = 2*b + 3*c/d e + f/g/a/b/c;
- In complexity analysis, we do not break down a statement into individual machine-level operations.



Time Complexity - Program Step (1/2)

Methods to compute the number of program steps

- Creating a global variable, say, count.
- Tabular method:
 - Compute the contribution of a statement:
 # program steps per execution × frequency.
 - Add up the contribution of all statements.



Example

```
float sum(float list[], int n) {
   float tempSum = 0; count++; /* for assignment */
   int i;
   for (i = 0; i < n; i++) {
      count++; /* for the "for" loop */
      tempSum += list[i]; count++; /* for assignment */
   }
   count++; /* last execution of "for" */
   count++; /* for return */
   return tempSum;
}</pre>
```

• count = 2n + 3 (steps).



Example (Tabular Method)

s/e: steps per execution.

Statements	s/e	Frequency	Total Steps
float sum(float list[], int n) {	0	0	0
float tempsum = 0;	1	1	1
int i;	0	0	0
for(i=0; i <n; i++)<="" td=""><td>1</td><td>n+1</td><td>n+1</td></n;>	1	n+1	n+1
tempsum += list[i];	1	n	n
return tempsum;	1	1	1
}	0	0	0
Total			2n + 3



 Issues: determining the "exact" step count of a program could be difficult or complicated.



- Issues: determining the "exact" step count of a program could be difficult or complicated.
- ightharpoonup Using asymptotic notations: O, Ω, Θ, \dots



- Issues: determining the "exact" step count of a program could be difficult or complicated.
- \triangleright Using asymptotic notations: O, Ω, Θ, \dots
- A motivating example:



- Issues: determining the "exact" step count of a program could be difficult or complicated.
- ightharpoonup Using asymptotic notations: O, Ω, Θ, \dots
- A motivating example:
- $c_3 n < c_1 n^2 + c_2 n$ when n is sufficiently large.
 - For $c_1 = 1$, $c_2 = 2$, $c_3 = 100$, $c_1 n^2 + c_2 n \le c_3 n$ for $n \le 98$.
 - For $c_1 = 1$, $c_2 = 2$, $c_3 = 1000$, $c_1 n^2 + c_2 n \le c_3 n$ for $n \le 998$.



- Issues: determining the "exact" step count of a program could be difficult or complicated.
- ightharpoonup Using asymptotic notations: O, Ω, Θ, \dots
- A motivating example:
- $c_3 n < c_1 n^2 + c_2 n$ when n is sufficiently large.
 - For $c_1 = 1$, $c_2 = 2$, $c_3 = 100$, $c_1 n^2 + c_2 n \le c_3 n$ for $n \le 98$.
 - For $c_1 = 1$, $c_2 = 2$, $c_3 = 1000$, $c_1 n^2 + c_2 n \le c_3 n$ for $n \le 998$.
 - \star For small values of n, either one could be faster.



Big-O Notation

Definition $(O(\cdot))$

f(n) = O(g(n)) (or write $f(n) \in O(g(n))$) iff there exist positive constants c and $n_0 \in \mathbb{N}$ such that $f(n) \le cg(n)$ for all $n \ge n_0$.

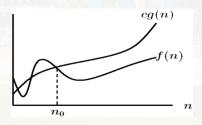


Big-O Notation

Definition $(O(\cdot))$

f(n) = O(g(n)) (or write $f(n) \in O(g(n))$) iff there exist positive constants c and $n_0 \in \mathbb{N}$ such that $f(n) \le cg(n)$ for all $n \ge n_0$.

- g(n) serves as an upper bound on f(n).
 - The smaller g(n) is, the more informative it would be!





$\mathsf{Big} ext{-}\Omega$ Notation

Definition $(\Omega(\cdot))$

 $f(n) = \Omega(g(n))$ (or write $f(n) \in \Omega(g(n))$) iff there exist positive constants c and $n_0 \in \mathbb{N}$ such that $f(n) \geq cg(n)$ for all $n \geq n_0$.

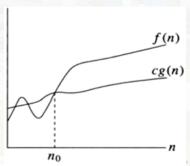


Big- Ω Notation

Definition $(\Omega(\cdot))$

 $f(n) = \Omega(g(n))$ (or write $f(n) \in \Omega(g(n))$) iff there exist positive constants c and $n_0 \in \mathbb{N}$ such that $f(n) \geq cg(n)$ for all $n \geq n_0$.

- g(n) serves as an lower bound on f(n).
 - The larger g(n) is, the more informative it would be!





•
$$3n + 2 = O(n)$$
.



- 3n + 2 = O(n).
 - $3n + 2 \le 4n$ for $n \ge 2$.



- 3n + 2 = O(n).
 - $3n + 2 \le 4n$ for $n \ge 2$.
- $3n + 3 = O(n^2)$.



- 3n + 2 = O(n).
 - $3n + 2 \le 4n$ for $n \ge 2$.
- $3n + 3 = O(n^2)$.
 - $3n + 3 \le 4n^2$ for $n \ge 2$.



- 3n + 2 = O(n).
 - 3n + 2 < 4n for n > 2.
- $3n + 3 = O(n^2)$.
 - $3n + 3 \le 4n^2$ for $n \ge 2$.
 - Why and how can we make sure this?



- 3n + 2 = O(n).
 - 3n + 2 < 4n for n > 2.
- $3n + 3 = O(n^2)$.
 - $3n + 3 \le 4n^2$ for $n \ge 2$.
 - Why and how can we make sure this?
- 100n + 6 = O(n).



- 3n + 2 = O(n).
 - 3n + 2 < 4n for n > 2.
- $3n + 3 = O(n^2)$.
 - $3n + 3 \le 4n^2$ for $n \ge 2$.
 - Why and how can we make sure this?
- 100n + 6 = O(n).
 - $100n + 6 \le 101n$ for $n \ge 10$.



- 3n + 2 = O(n).
 - $3n + 2 \le 4n$ for $n \ge 2$.
- $3n + 3 = O(n^2)$.
 - $3n + 3 < 4n^2$ for n > 2.
 - Why and how can we make sure this?
- 100n + 6 = O(n).
 - $100n + 6 \le 101n$ for $n \ge 10$.
- $10n^2 + 4n + 2 = O(n^2)$.



- 3n + 2 = O(n).
 - 3n + 2 < 4n for n > 2.
- $3n + 3 = O(n^2)$.
 - $3n + 3 < 4n^2$ for n > 2.
 - Why and how can we make sure this?
- 100n + 6 = O(n).
 - $100n + 6 \le 101n$ for $n \ge 10$.
- $10n^2 + 4n + 2 = O(n^2)$.
 - $10n^2 + 4n + 2 \le 11n^2$ for $n \ge 5$.



- 3n + 2 = O(n).
 - $3n + 2 \le 4n$ for $n \ge 2$.
- $3n + 3 = O(n^2)$.
 - $3n + 3 < 4n^2$ for n > 2.
 - Why and how can we make sure this?
- 100n + 6 = O(n).
 - $100n + 6 \le 101n$ for $n \ge 10$.
- $10n^2 + 4n + 2 = O(n^2)$.
 - $10n^2 + 4n + 2 \le 11n^2$ for $n \ge 5$.
- \bullet 6 · 2ⁿ + n² = $O(2^n)$.



- 3n + 2 = O(n).
 - $3n + 2 \le 4n$ for $n \ge 2$.
- $3n + 3 = O(n^2)$.
 - $3n + 3 \le 4n^2$ for $n \ge 2$.
 - Why and how can we make sure this?
- 100n + 6 = O(n).
 - $100n + 6 \le 101n$ for $n \ge 10$.
- $10n^2 + 4n + 2 = O(n^2)$.
 - $10n^2 + 4n + 2 \le 11n^2$ for $n \ge 5$.
- $\bullet \ 6 \cdot 2^n + n^2 = O(2^n).$
 - $6 \cdot 2^n + n^2 \le 7 \cdot 2^n$ for $n \ge 4$.



Examples (Big- Ω)

•
$$3n + 2 = \Omega(n)$$
.



- $3n + 2 = \Omega(n)$.
 - $3n+2 \ge 3n$ for $n \ge 1$.



Fall 2025

- $3n + 2 = \Omega(n)$.
 - $3n + 2 \ge 3n$ for $n \ge 1$.
- $3n + 3 = \Omega(n)$.



- $3n + 2 = \Omega(n)$.
 - $3n + 2 \ge 3n$ for $n \ge 1$.
- $3n + 3 = \Omega(n)$.
 - $3n + 2 \ge 3n$ for $n \ge 1$.



- $3n + 2 = \Omega(n)$.
 - $3n + 2 \ge 3n$ for $n \ge 1$.
- $3n + 3 = \Omega(n)$.
 - $3n + 2 \ge 3n$ for $n \ge 1$.
- $100n + 6 = \Omega(n)$.



- $3n + 2 = \Omega(n)$.
 - $3n + 2 \ge 3n$ for $n \ge 1$.
- $3n + 3 = \Omega(n)$.
 - $3n + 2 \ge 3n$ for $n \ge 1$.
- $100n + 6 = \Omega(n)$.
 - $100n + 6 \ge 100n$ for $n \ge 1$.



- $3n + 2 = \Omega(n)$.
 - $3n + 2 \ge 3n$ for $n \ge 1$.
- $3n + 3 = \Omega(n)$.
 - $3n + 2 \ge 3n$ for $n \ge 1$.
- $100n + 6 = \Omega(n)$.
 - 100n + 6 > 100n for n > 1.
- $10n^2 + 4n + 2 = \Omega(n^2)$.



- $3n + 2 = \Omega(n)$.
 - $3n + 2 \ge 3n$ for $n \ge 1$.
- $3n + 3 = \Omega(n)$.
 - $3n + 2 \ge 3n$ for $n \ge 1$.
- $100n + 6 = \Omega(n)$.
 - 100n + 6 > 100n for n > 1.
- $10n^2 + 4n + 2 = \Omega(n^2)$.
 - $10n^2 + 4n + 2 \ge 10n^2$ for $n \ge 1$.



- $3n + 2 = \Omega(n)$.
 - $3n + 2 \ge 3n$ for $n \ge 1$.
- $3n + 3 = \Omega(n)$.
 - $3n + 2 \ge 3n$ for $n \ge 1$.
- $100n + 6 = \Omega(n)$.
 - 100n + 6 > 100n for n > 1.
- $10n^2 + 4n + 2 = \Omega(n^2)$.
 - $10n^2 + 4n + 2 \ge 10n^2$ for $n \ge 1$.
- $\bullet 6 \cdot 2^n + n^2 = \Omega(2^n).$



- $3n + 2 = \Omega(n)$.
 - $3n + 2 \ge 3n$ for $n \ge 1$.
- $3n + 3 = \Omega(n)$.
 - $3n + 2 \ge 3n$ for $n \ge 1$.
- $100n + 6 = \Omega(n)$.
 - $100n + 6 \ge 100n$ for $n \ge 1$.
- $10n^2 + 4n + 2 = \Omega(n^2)$.
 - $10n^2 + 4n + 2 \ge 10n^2$ for $n \ge 1$.
- $\bullet 6 \cdot 2^n + n^2 = \Omega(2^n).$
 - $6 \cdot 2^n + n^2 \ge 2^n$ for $n \ge 1$.



- $3n + 2 = \Omega(n)$.
 - $3n + 2 \ge 3n$ for $n \ge 1$.
- $3n + 3 = \Omega(n)$.
 - $3n + 2 \ge 3n$ for $n \ge 1$.
- $100n + 6 = \Omega(n)$.
 - $100n + 6 \ge 100n$ for $n \ge 1$.
- $10n^2 + 4n + 2 = \Omega(n^2)$.
 - $10n^2 + 4n + 2 \ge 10n^2$ for $n \ge 1$.
- $\bullet 6 \cdot 2^n + n^2 = \Omega(2^n).$
 - $6 \cdot 2^n + n^2 > 2^n$ for n > 1.
- $10n^2 + 4n + 2 = \Omega(1)$.



- $3n + 2 = \Omega(n)$.
 - $3n + 2 \ge 3n$ for $n \ge 1$.
- $3n + 3 = \Omega(n)$.
 - $3n + 2 \ge 3n$ for $n \ge 1$.
- $100n + 6 = \Omega(n)$.
 - 100n + 6 > 100n for n > 1.
- $10n^2 + 4n + 2 = \Omega(n^2)$.
 - $10n^2 + 4n + 2 \ge 10n^2$ for $n \ge 1$.
- $\bullet \ 6 \cdot 2^n + n^2 = \Omega(2^n).$
 - $6 \cdot 2^n + n^2 \ge 2^n$ for $n \ge 1$.
- $10n^2 + 4n + 2 = \Omega(1)$.
- $\bullet \ 6 \cdot 2^n + n^2 = \Omega(1).$



A Disproof Example

• Claim: 3n + 2 is NOT $\Omega(n^2)$.

Proof (by contradiction). Assume $3n + 2 \in \Omega(n^2)$. Then there exist constants c > 0 and $n_0 \in \mathbb{N}$ such that

$$\forall n \ge n_0: \quad 3n+2 \ge c n^2. \tag{*}$$

However, for all $n \ge 1$, we have 3n+2 < 3n+2n = 5n. Moreover, if $n \ge \frac{5}{c}$, then $5n \le c \, n^2$. Hence for

for all
$$n \ge \max\{1, \lceil \frac{5}{c} \rceil, n_0 \}$$
, we have $3n + 2 < c n^2$,

which contradicts (*). Therefore $3n + 2 \notin \Omega(n^2)$.



Theorem 1.2

If
$$f(n) = a_k n^k + a_{k-1} n^{k-1} + \dots + a_1 n + a_0$$
, then $f(n) = O(n^k)$.



Theorem 1.2

If
$$f(n) = a_k n^k + a_{k-1} n^{k-1} + \dots + a_1 n + a_0$$
, then $f(n) = O(n^k)$.

Proof:

$$f(n) \leq \sum_{i=0}^{k} |a_i| n^i$$



Theorem 1.2

If
$$f(n) = a_k n^k + a_{k-1} n^{k-1} + \dots + a_1 n + a_0$$
, then $f(n) = O(n^k)$.

Proof:

$$\mathit{f(n)} \ \leq \ \sum_{i=0}^k |a_i| n^i = n^k \sum_{i=0}^k |a_i| n^{i-k} \leq n^k \sum_{i=0}^k |a_i|, \ \text{for} \ n \geq 1.$$



Theorem 1.2

If
$$f(n) = a_k n^k + a_{k-1} n^{k-1} + \dots + a_1 n + a_0$$
, then $f(n) = O(n^k)$.

Proof:

$$f(n) \le \sum_{i=0}^k |a_i| n^i = n^k \sum_{i=0}^k |a_i| n^{i-k} \le n^k \sum_{i=0}^k |a_i|, \text{ for } n \ge 1.$$

Note that $n^{i-k} \le 1$ if $i \le k$ and $\sum_{i=0}^{k} |a_i|$ is a constant.



Fall 2025

- * with respect to the input of size n.
 - O(1): constant.
 - O(n): linear.
 - $O(n^2)$: quadratic.
 - $O(n^3)$: cubic.
 - $O(2^n)$: exponential.



- * with respect to the input of size n.
 - O(1): constant.
 - O(n): linear.
 - $O(n^2)$: quadratic.
 - $O(n^3)$: cubic.
 - $O(2^n)$: exponential.
 - $O(2^{\sqrt{n}})$: sub-exponential.



- * with respect to the input of size n.
 - O(1): constant.
 - O(n): linear.
 - $O(n^2)$: quadratic.
 - $O(n^3)$: cubic.
 - $O(2^n)$: exponential.
 - $O(2^{\sqrt{n}})$: sub-exponential.
 - $O(\log n)$: logarithmic.



- * with respect to the input of size n.
 - O(1): constant.
 - O(n): linear.
 - $O(n^2)$: quadratic.
 - $O(n^3)$: cubic.
 - $O(2^n)$: exponential.
 - $O(2^{\sqrt{n}})$: sub-exponential.
 - $O(\log n)$: logarithmic.
 - $O(\lg n)$?



- * with respect to the input of size n.
 - O(1): constant.
 - O(n): linear.
 - $O(n^2)$: quadratic.
 - $O(n^3)$: cubic.
 - $O(2^n)$: exponential.
 - $O(2^{\sqrt{n}})$: sub-exponential.
 - $O(\log n)$: logarithmic.
 - $O(\lg n)$? $O(\ln n)$?
 - $O(n \log n)$: log linear.



Polynomial (Lower Bound)

Theorem 1.3

If
$$f(n) = a_k n^k + a_{k-1} n^{k-1} + \dots + a_1 n + a_0$$
 and $a_k > 0$, then $f(n) = \Omega(n^k)$.



Fall 2025

Polynomial (Lower Bound)

Theorem 1.3

If
$$\mathit{f}(\mathit{n}) = \mathit{a_k} \mathit{n^k} + \mathit{a_{k-1}} \mathit{n^{k-1}} + \cdots + \mathit{a_1} \mathit{n} + \mathit{a_0}$$
 and $\mathit{a_k} > 0$, then $\mathit{f}(\mathit{n}) = \Omega(\mathit{n^k})$.

Proof:

Skipped and left as an exercise.



Fall 2025

Polynomial (Lower Bound)

Theorem 1.3

If
$$f(n) = a_k n^k + a_{k-1} n^{k-1} + \dots + a_1 n + a_0$$
 and $a_k > 0$, then $f(n) = \Omega(n^k)$.

Proof:

- Skipped and left as an exercise.
- Note: a_i might be negative for $0 \le i < k$.



Theta Notation (Θ)

Definition (Θ)

$$f(n) = \Theta(g(n))$$
 iff $f(n) = O(g(n))$ and $f(n) = \Theta(g(n))$.

 \bullet More precise than simply using big-O or big- Ω notations.



Theta Notation (Θ)

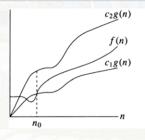
Definition (Θ)

$$f(n) = \Theta(g(n))$$
 iff $f(n) = O(g(n))$ and $f(n) = \Theta(g(n))$.

• More precise than simply using big-O or big- Ω notations.

Theorem 1.4

If
$$f(n) = a_k n^k + a_{k-1} n^{k-1} + \dots + a_1 n + a_0$$
 and $a_k > 0$, then $f(n) = \Theta(n^k)$





Little "oh" and little "omega"

Definition (o)

The function f(n) = o(g(n)) iff

$$\lim_{n\to\infty}\frac{f(n)}{g(n)}=0.$$

Definition (ω)

The function $f(n) = \omega(g(n))$ iff

$$\lim_{n\to\infty}\frac{g(n)}{f(n)}=0.$$



Examples

•
$$3n + 2 = o(n^2)$$
 (i.e., $n^2 = \omega(n)$).

- $\bullet 3n + 2 = o(n \log n).$
- $\bullet 3n + 2 = o(n \log \log n).$
- $6 \cdot 2^n + n^2 \neq o(2^n)$.
- $6 \cdot 2^n + n^2 \neq o(2^n)$.
- $6 \cdot 2^n + n^2 = o(2^n \log n)$.



Example (Tabular Method)

Statements	s/e	Frequency	Total Steps	Asymptotic Complexity
<pre>void add (int a[][MAX_SIZE],) { int i, j; for (i = 0; i < row; i++)</pre>	0 0 1 1 1 0	0 0 rows+1 rows*(cols+1) rows*cols	0 0 rows+1 rows*(cols+1) rows*cols	$\begin{matrix} 0 \\ 0 \\ \Theta(\text{rows}) \\ \Theta(\text{rows} \cdot \text{cols}) \\ \Theta(\text{rows} \cdot \text{cols}) \\ 0 \end{matrix}$
Total	$2 \cdot \text{rows} \cdot \text{cols} + 2 \cdot \text{rows} + 1$			$\Theta(\text{rows} \cdot \text{cols})$



Function Values & Plots

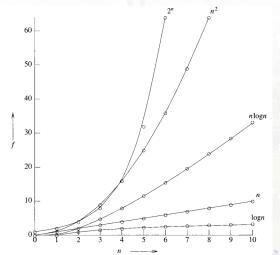
Refer to Fig. 1.7 & 1.8 in the textbook.

Instance characteristic n								
Time	Name	1	2	4	8	16	32	
1	Constant	l	1	1	1	1	1	
$\log n$	Logarithmic	0	1	2	3	4	5	
n	Linear	1	2	4	8	16	32	
$n \log n$	Log linear	0	2	8	24	64	160	
n^2	Quadratic	1	4	16	64	256	1024	
n^3	Cubic	1	8	64	512	4096	32768	
2 ⁿ	Exponential	2	4	16	256	65536	4294967296	
n!	Factorial	1	2	24	40326	20922789888000	26313×10^{33}	



Function Values & Plots

Refer to Fig. 1.7 & 1.8 in the textbook.





Outline

1 Performance Analysis

2 Performance Measurement



Motivations

- Sometimes we still need to consider how long an algorithm executes on our machine.
- In order to obtain accurate times, we can repeatedly run the programs for several times (and take the average running time).



The Tricks

#include<time.h>

	1st Method	2nd Method
start timing	<pre>start = clock();</pre>	<pre>start = time(NULL);</pre>
stop timing	<pre>end = clock();</pre>	<pre>end = time(NUL);</pre>
type returned	clock_t	time_t

Result (in seconds):

• 1st Method: duration = (double)(stop-start))/(CLOCKS_PER_SEC);

Performance Analysis

• 2nd Method: duration = (double)difftime(stop, start);



The Tricks (Example)

```
... // previous code omitted
    clock_t start, stop;
    double duration;
    printf("n time\n");
    for(i=0; i < ITERATIONS; i++) {</pre>
        for(j=0; j<sizeList[i]; j++)</pre>
            list[j] = sizeList[i]-j; /* worst case */
            start = clock();
            sort(list, sizeList[i]);
            stop = clock();
            /* number of clock ticks per second */
            duration = ((double) (stop-start));
            printf("%6d %f\n", sizeList[i], duration);
```

 \Rightarrow sample code.



Discussions

