Greedy Selfish Network Creation

Pascal Lenzner

The 8th Workshop on Internet & Network Economics (WINE 2012) 142–155.

Speaker: Joseph Chuang-Chieh Lin

Institute of Information Science
Academia Sinica
Taiwan

11 September 2015

- Researcher at the “Institut für Informatik, Friedrich-Schiller-Universität (Jena)”.
- *On Selfish Network Creation*. Dissertation, Humboldt-University Berlin, 2014.

Outline

Models and definitions

Motivations

- Computing a best response strategy of an agent in the network creation game: NP-hard ([Fabrikant *et al.* 2003]).
- Internet Service Providers (ISPs) prefer greedy refinements of their current strategy over radical re-design of the infrastructure.

Network Creation Games (NCG)

- A set of n agents V.
- The price of buying an edge: $\alpha > 0$.
- $S_v \subseteq V \setminus \{v\}$: the strategy of agent v.
- G: the induced network.

The sum model

$$c_v(G, \alpha) = \alpha |S_v| + \sum_{w \in V(G)} d_G(v, w).$$

- The total cost is $c(G, \alpha) = \sum_{v \in V(G)} c_v(G, \alpha)$.

- The considered operations for an agent to improve her current strategy:
  1. *greedy augmentation*: creating **one** new own link;
  2. *greedy deletion*: removing **one** own link;
  3. *greedy swap*: swapping of **one** own link.

⋆ Computing the best greedy strategy refinement for one agent:
  - $O(n^2(n+m))$ time.

A useful property

Lemma 1

In the SUM network creation game:

Agent v can NOT decrease her cost by buying **one** edge
$\Rightarrow$ buying $k > 1$ edges can NOT decrease agent v's cost.

Proof of Lemma 1:

- Assume that agent v:
  - owns q edges in (G, α);
  - can strictly decrease her cost by purchasing $k > 1$ edges $e_1, \ldots, e_k$;
    - $(G, \alpha) \xrightarrow{\text{augmented by } e_1, \ldots, e_k} (G^k, \alpha)$.
  - has distance-cost D & D^k in (G, α) & (G^k, α), resp.
- $c_v(G^k, \alpha) < c_v(G, \alpha) \Leftrightarrow k\alpha < D - D^k$.
  - $c_v(G, \alpha) = q\alpha + D$ and $c_v(G^k, \alpha) = q\alpha + k\alpha + D^k$.
- (G^{1^*}, α): agent v has built the best possible additional edge e^*.
  - $c_v(G^{1^*}, \alpha) = q\alpha + \alpha + D^{1^*} \geq c_v(G, \alpha) \Leftrightarrow \alpha \geq D - D^{1^*}$.

Proof of Lemma 1:

- Assume that agent v:
  - owns q edges in (G, α);
  - can strictly decrease her cost by purchasing $k > 1$ edges $e_1, \ldots, e_k$;
    - $(G, \alpha) \xrightarrow{\text{augmented by } e_1, \ldots, e_k} (G^k, \alpha)$.
  - has distance-cost D & D^k in (G, α) & (G^k, α), resp.
- $c_v(G^k, \alpha) < c_v(G, \alpha) \Leftrightarrow k\alpha < D - D^k$.
  - $c_v(G, \alpha) = q\alpha + D$ and $c_v(G^k, \alpha) = q\alpha + k\alpha + D^k$.
- (G^{1^*}, α): agent v has built the best possible additional edge e^*.
  - $c_v(G^{1^*}, \alpha) = q\alpha + \alpha + D^{1^*} \geq c_v(G, \alpha) \Leftrightarrow \alpha \geq D - D^{1^*}$.

Proof of Lemma 1:

- Assume that agent v:
  - owns q edges in (G, α);
  - can strictly decrease her cost by purchasing $k > 1$ edges $e_1, \ldots, e_k$;
    - $(G, \alpha) \xrightarrow{\text{augmented by } e_1, \ldots, e_k} (G^k, \alpha)$.
  - has distance-cost D & D^k in (G, α) & (G^k, α), resp.
- $c_v(G^k, \alpha) < c_v(G, \alpha) \Leftrightarrow k\alpha < D - D^k$.
  - $c_v(G, \alpha) = q\alpha + D$ and $c_v(G^k, \alpha) = q\alpha + k\alpha + D^k$.
- (G^{1^*}, α): agent v has built the best possible additional edge e^*.
  - $c_v(G^{1^*}, \alpha) = q\alpha + \alpha + D^{1^*} \geq c_v(G, \alpha) \Leftrightarrow \alpha \geq D - D^{1^*}$.

Proof of Lemma 1:

- Assume that agent v:
  - owns q edges in (G, α);
  - can strictly decrease her cost by purchasing $k > 1$ edges $e_1, \ldots, e_k$;
    - $(G, \alpha) \xrightarrow{\text{augmented by } e_1, \ldots, e_k} (G^k, \alpha)$.
  - has distance-cost D & D^k in (G, α) & (G^k, α), resp.
- $c_v(G^k, \alpha) < c_v(G, \alpha) \Leftrightarrow k\alpha < D - D^k$.
  - $c_v(G, \alpha) = q\alpha + D$ and $c_v(G^k, \alpha) = q\alpha + k\alpha + D^k$.
- (G^{1^*}, α): agent v has built the best possible additional edge e^*.
  - $c_v(G^{1^*}, \alpha) = q\alpha + \alpha + D^{1^*} \geq c_v(G, \alpha) \Leftrightarrow \alpha \geq D - D^{1^*}$.

Proof of Lemma 1 (contd.):

- g^e: the decrease in distance cost for agent v if only the edge $e \in \{e_1, \ldots, e_k, e^*\}$ is inserted into (G, α).
  - $D - D^k \leq g^{e_1} + g^{e_2} + \ldots + g^{e_k}$.
- $k\alpha < D - D^k \Rightarrow \alpha < \frac{D-D^k}{k} \leq \frac{g^{e_1}+g^{e_2}+\ldots+g^{e_k}}{k}$.
  - $\therefore \alpha < g^{e_j}$ for some $j \in [1, k]$.
- Yet $g^{e_j} \leq g^{e^*}$.

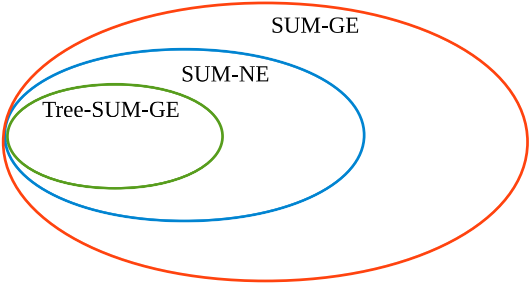
  $\triangleright$ $\alpha < g^{e^*} = D - D^{1^*}$ (contradiction).

Tree Networks in SUM Greedy Equilibria

SUM-GE $\subseteq$ SUM-NE

Theorem 2

$(G, \alpha) \in$ SUM-GE and G is a tree $\Rightarrow$ $(G, \alpha) \in$ SUM-NE.

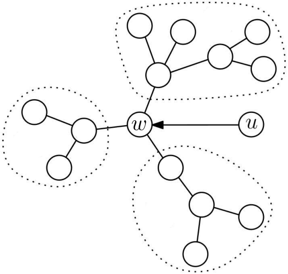


1-median [Kariv & Hakimi 1979]

$$\arg\min_{u \in V(G)} \sum_{w \in V(G)} d_G(u, w).$$

Lemma 2

Let (T, α) be a tree network in SUM-GE. If agent u owns edge $\{u, w\}$ in (T, α), then w must be a 1-median of its tree in the forest $T - \{u\}$.

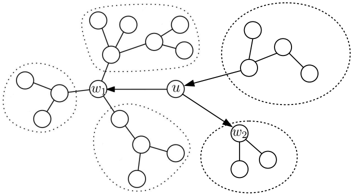


Corollary 1

Let $(T, \alpha) \in$ SUM-GE, and let

- T^u: the forest induced by removing all edges owned by u;
- F^u: T^u withOUT the tree containing u.

Then agent u's strategy in (T, α) is the **optimal** strategy among all strategies that *buy exactly one edge into each tree of* F^u.

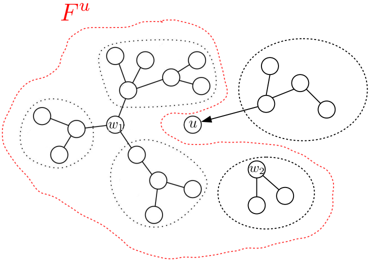


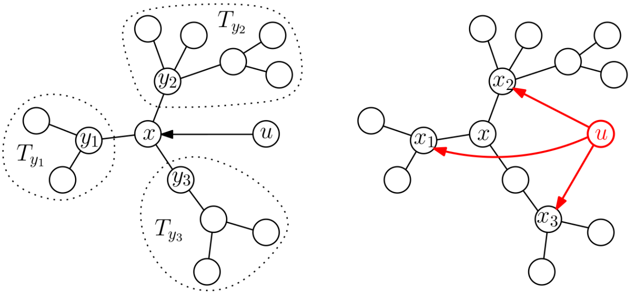
Corollary 1

Let $(T, \alpha) \in$ SUM-GE, and let

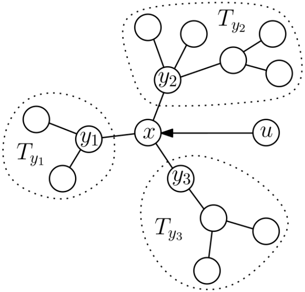
- T^u: the forest induced by removing all edges owned by u;
- F^u: T^u withOUT the tree containing u.

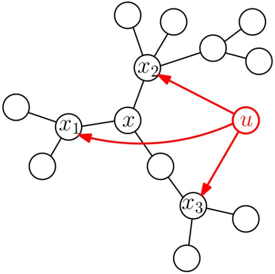
Then agent u's strategy in (T, α) is the **optimal** strategy among all strategies that *buy exactly one edge into each tree of F^u*.





- $x \in V(T)$: 1-median of T.
- (G_T^u, α): obtained by adding u and inserting edge $\{u, x\}$, where $u \notin V(T)$.
- $y_1, \ldots, y_\ell$: the neighbors of x in T.
- T_{y_i}: the maximal subtree of T rooted at y_i (withOUT x).
- $S^* = \{x_1, \ldots, x_k\}$: the best strategy of u buying ≥ 2 edges.



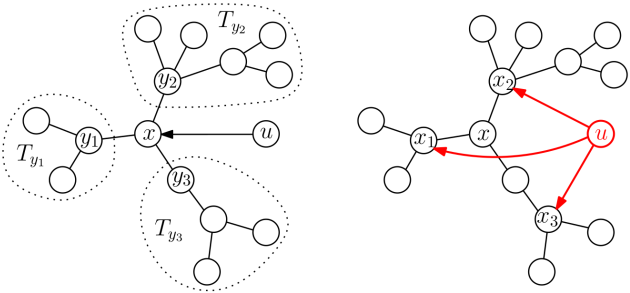


Lemma 3

There is no subtree T_{y_i}, for $1 \leq i \leq \ell$, which contains ALL vertices $x_1, \ldots, x_k$.

Key observation [Kariv & Hakimi 1979]

$x \in V(T)$ is a 1-median of tree T iff $|V(T_{y_i})| \leq n/2$ for all i.



Lemma 3

There is no subtree T_{y_i}, for $1 \leq i \leq \ell$, which contains ALL vertices $x_1, \ldots, x_k$.

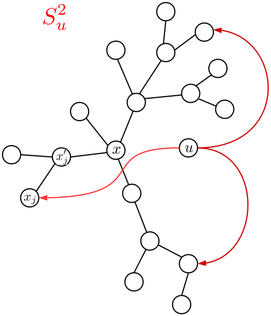
Key observation [Kariv & Hakimi 1979]

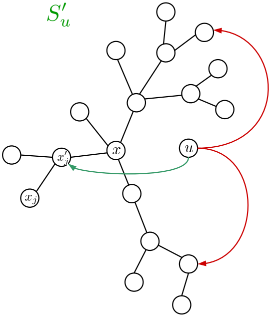
$x \in V(T)$ is a 1-median of tree T iff $|V(T_{y_i})| \leq n/2$ for all i.

- ⋆ S_u^1: agent u's best strategy that buys ≥ 2 edges including one edge towards x.
- ⋆ S_u^2: agent u's best strategy that buys ≥ 2 edges, but none of them towards x.

Lemma 4

Let $x_j := \arg\min_{v \in S_u^2} d_T(v, x)$.
If S_u^2 yields less cost for agent u than strategy S_u^1, then x_j can NOT be a leaf of G_T^u.

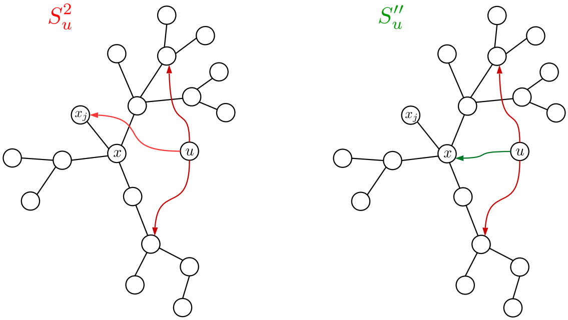




- ⋆ S_u^1: agent u's best strategy that buys ≥ 2 edges including one edge towards x.
- ⋆ S_u^2: agent u's best strategy that buys ≥ 2 edges, but none of them towards x.

Lemma 4

Let $x_j := \arg\min_{v \in S_u^2} d_T(v, x)$.
If S_u^2 yields less cost for agent u than strategy S_u^1, then x_j can NOT be a leaf of G_T^u.



Proof of Theorem 2

Theorem 2

$(G, \alpha) \in$ SUM-GE and G is a tree $\Rightarrow$ $(G, \alpha) \in$ SUM-NE.

We show that:

- an agent u can decrease her cost by a strategy-change in $(T, \alpha) \in$ SUM-GE

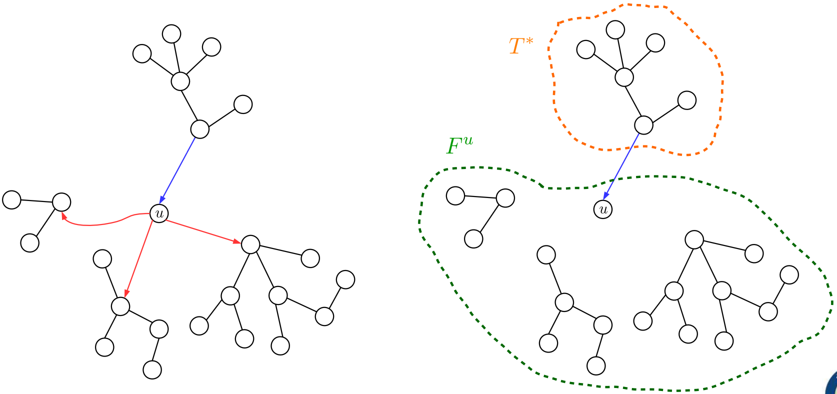
  $\Rightarrow \exists$ an agent z who can decrease her cost by a greedy strategy-change.

Assume that u cannot decrease her cost by a greedy strategy-change, but by an arbitrary one.

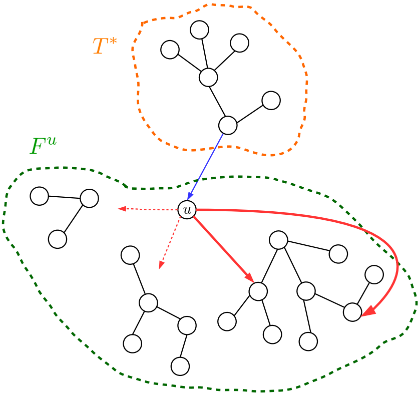
Proof of Theorem 2 (contd.)

- S^*: u's best possible *arbitrary* strategy.
  - Choose the one which buys the least number of edges.
- The only way u can possibly decrease her cost:
  removing j owned edges and building k edges simultaneously ($k > j$ by Corollary 1).
  - u can NOT remove any owned edge withOUT purchasing edges.
  - u can NOT decrease her cost by purchasing $k > 0$ *additional* edges.

Proof of Theorem 2 (contd.)

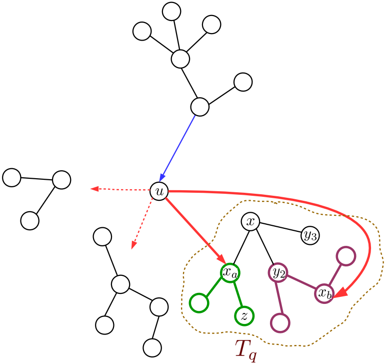


Proof of Theorem 2 (contd.)



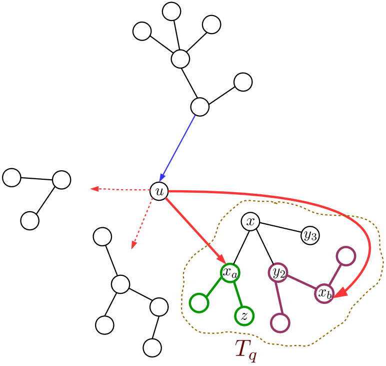
- Among the k new edges, none of them has an endpoint in T^* (by Lemma 1).
- Removing $j = 3$ owned edges and building $k = 4$ edges of u simultaneously...
- By the pigeonhole principle, there must be a tree T_q in F^u where u buys ≥ 2 edges with strategy S^*.

Proof of Theorem 2 (contd.)



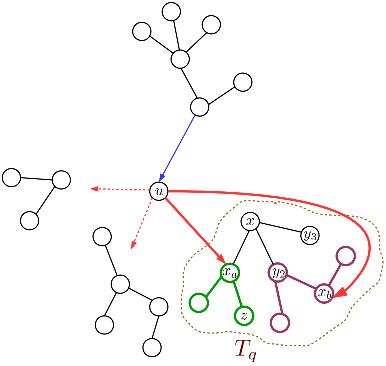
- $x \in V(T_q)$: u has a unique edge $\{u, x\}$ of T connecting u to T_q.
- x_a: the new strategy of u which has minimum distance to x.
- By Lemma 3, $\exists\ x_b$ located at a different subtree (say rooted at y_2) different from the one where x_a is.
- x_a must not be a leaf node (by Lemma 4).
- $\exists\ z$, a neighbor of x_a:
  $d_T(z, x) > d_T(x_a, x)$.
- z can buy the edge $\{z, x_b\}$ (imitating what u does).

Proof of Theorem 2 (contd.)



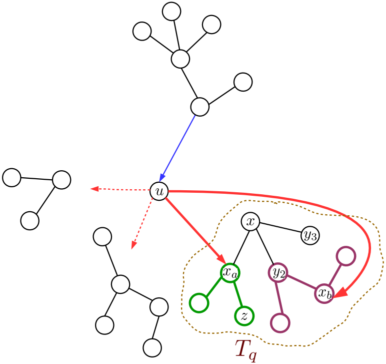
- $x \in V(T_q)$: u has a unique edge $\{u, x\}$ of T connecting u to T_q.
- x_a: the new strategy of u which has minimum distance to x.
- By Lemma 3, $\exists\, x_b$ located at a different subtree (say rooted at y_2) different from the one where x_a is.
- x_a must not be a leaf node (by Lemma 4).
- $\exists\, z$, a neighbor of x_a:
  $d_T(z, x) > d_T(x_a, x)$.
- z can buy the edge $\{z, x_b\}$ (imitating what u does).

Proof of Theorem 2 (contd.)



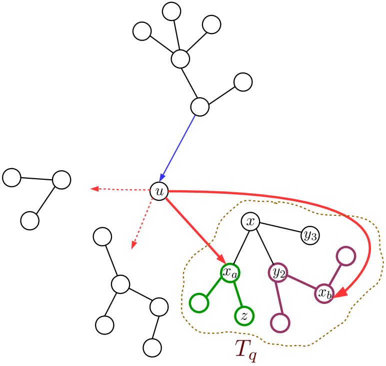
- $x \in V(T_q)$: u has a unique edge $\{u, x\}$ of T connecting u to T_q.
- x_a: the new strategy of u which has minimum distance to x.
- By Lemma 3, $\exists\ x_b$ located at a different subtree (say rooted at y_2) different from the one where x_a is.
- x_a must not be a leaf node (by Lemma 4).
- $\exists\ z$, a neighbor of x_a:
  $$d_T(z, x) > d_T(x_a, x).$$
- z can buy the edge $\{z, x_b\}$ (imitating what u does).

Proof of Theorem 2 (contd.)



- $x \in V(T_q)$: u has a unique edge $\{u, x\}$ of T connecting u to T_q.
- x_a: the new strategy of u which has minimum distance to x.
- By Lemma 3, $\exists\, x_b$ located at a different subtree (say rooted at y_2) different from the one where x_a is.
- x_a must not be a leaf node (by Lemma 4).
- $\exists\, z$, a neighbor of x_a:
  $$d_T(z, x) > d_T(x_a, x).$$
- z can buy the edge $\{z, x_b\}$ (imitating what u does).

Proof of Theorem 2 (contd.)



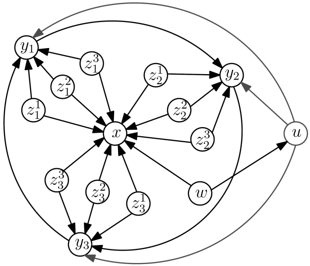
- $x \in V(T_q)$: u has a unique edge $\{u, x\}$ of T connecting u to T_q.
- x_a: the new strategy of u which has minimum distance to x.
- By Lemma 3, $\exists\, x_b$ located at a different subtree (say rooted at y_2) different from the one where x_a is.
- x_a must not be a leaf node (by Lemma 4).
- $\exists\, z$, a neighbor of x_a:
  $$d_T(z, x) > d_T(x_a, x).$$
- z can buy the edge $\{z, x_b\}$ (imitating what u does).

Non-Tree Networks in SUM Greedy Equilibria

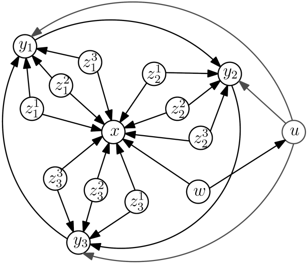
The negative result

Theorem 3

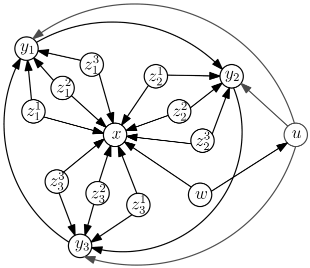
There is a network in SUM-GE which is NOT in β-approximate SUM-NE for $\beta < \frac{3}{2}$.



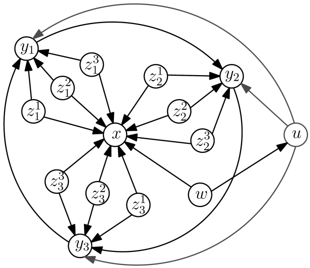
- A special family of graphs $\{G_k \mid k = 1, 2, \ldots\}$.
  - $V(G_k) = \{u, w, x, y_1, \ldots, y_k\} \cup \{z_i^j \mid 1 \leq i, j \leq k\}$.;
  - u owns edges towards $y_1, \ldots, y_k$;
  - w owns edges towards x and u;
  - each z_i^j owns an edge to x and y_i;
  - $y_1, \ldots, y_k$ form a clique (edge-ownership: arbitrary).
- First, we show that $(G_k, k+1) \in$ SUM-GE.



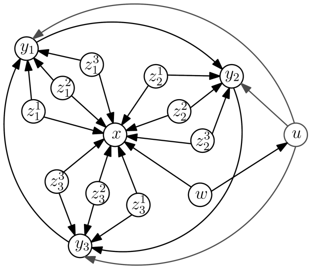
- A special family of graphs $\{G_k \mid k = 1, 2, \ldots\}$.
  - $V(G_k) = \{u, w, x, y_1, \ldots, y_k\} \cup \{z_i^j \mid 1 \le i, j \le k\}$.;
  - u owns edges towards $y_1, \ldots, y_k$;
  - w owns edges towards x and u;
  - each z_i^j owns an edge to x and y_i;
  - $y_1, \ldots, y_k$ form a clique (edge-ownership: arbitrary).
- First, we show that $(G_k, k+1) \in$ SUM-GE.



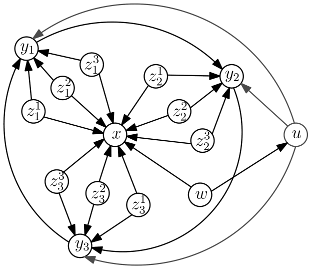
- No agent can buy an edge to decrease her cost.
  - G_k has diameter 2 & $\alpha = k + 1 > 1$.
- Swapping any own edge cannot decrease one's cost either.
  - The number of neighbors stays the same.
- How about edge removals?



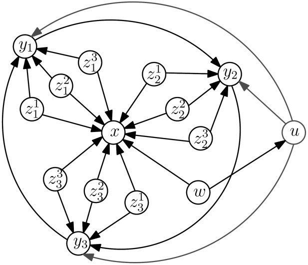
- No agent can buy an edge to decrease her cost.
  - G_k has diameter 2 & $\alpha = k + 1 > 1$.
- Swapping any own edge cannot decrease one's cost either.
  - The number of neighbors stays the same.
- How about edge removals?



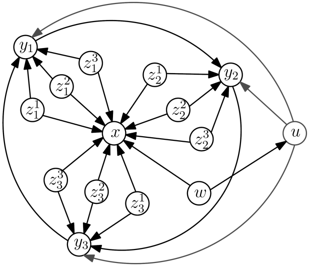
- No agent can buy an edge to decrease her cost.
  - G_k has diameter 2 & $\alpha = k + 1 > 1$.
- Swapping any own edge cannot decrease one's cost either.
  - The number of neighbors stays the same.
- How about edge removals?



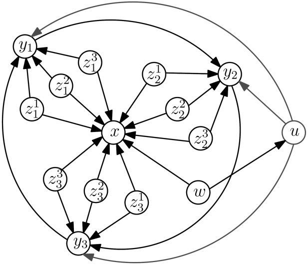
- No agent can buy an edge to decrease her cost.
  - G_k has diameter 2 & $\alpha = k + 1 > 1$.
- Swapping any own edge cannot decrease one's cost either.
  - The number of neighbors stays the same.
- How about edge removals?



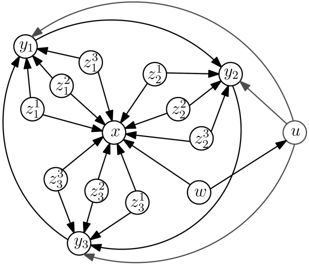
- No agent can buy an edge to decrease her cost.
  - G_k has diameter 2 & $\alpha = k + 1 > 1$.
- Swapping any own edge cannot decrease one's cost either.
  - The number of neighbors stays the same.
- How about edge removals?



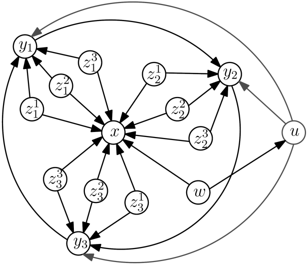
- For u:
  - Deleting $\{u, y_i\}$ increases u's distance to $y_i, z_i^1, \ldots, z_i^k$ by one.
- For y_i's:
  - Deleting $\{y_i, y_j\}$ increases y_i's distance to $y_j, z_j^1, \ldots, z_j^k$ by one.
- Similarly we can prove the case for w and all z_i^j's.



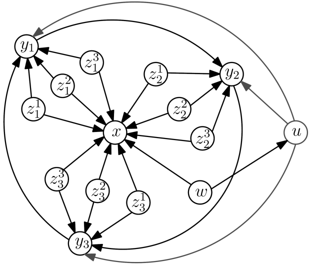
- For u:
  - Deleting $\{u, y_i\}$ increases u's distance to $y_i, z_i^1, \ldots, z_i^k$ by one.
- For y_i's:
  - Deleting $\{y_i, y_j\}$ increases y_i's distance to $y_j, z_j^1, \ldots, z_j^k$ by one.
- Similarly we can prove the case for w and all z_i^j's.



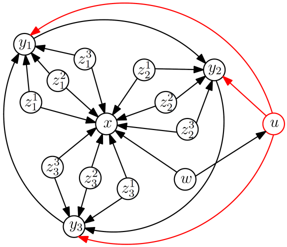
- For u:
  - Deleting $\{u, y_i\}$ increases u's distance to $y_i, z_i^1, \ldots, z_i^k$ by one.
- For y_i's:
  - Deleting $\{y_i, y_j\}$ increases y_i's distance to $y_j, z_j^1, \ldots, z_j^k$ by one.
- Similarly we can prove the case for w and all z_i^j's.



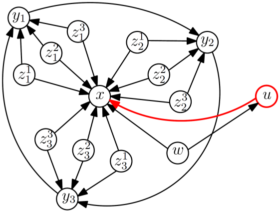
- For u:
  - Deleting $\{u, y_i\}$ increases u's distance to $y_i, z_i^1, \ldots, z_i^k$ by one.
- For y_i's:
  - Deleting $\{y_i, y_j\}$ increases y_i's distance to $y_j, z_j^1, \ldots, z_j^k$ by one.
- Similarly we can prove the case for w and all z_i^j's.



- For u:
  - Deleting $\{u, y_i\}$ increases u's distance to $y_i, z_i^1, \ldots, z_i^k$ by one.
- For y_i's:
  - Deleting $\{y_i, y_j\}$ increases y_i's distance to $y_j, z_j^1, \ldots, z_j^k$ by one.
- Similarly we can prove the case for w and all z_i^j's.

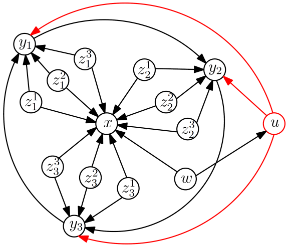


$(G_k, k+1)$

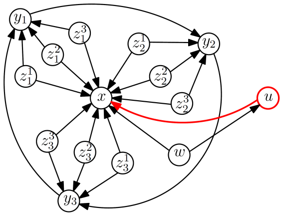


$(G_k^*, k+1)$

- Now, consider a strategy-change of u: $S_u = \{y_1, \ldots, y_k\} \rightsquigarrow S_u^* = \{x\}$.
- Easy to check that no other S_u' iwth $|S_u'| \leq 1$ outperforms S_u^*.
- Adding any edge $\{u, y_i\}$ decreases the cost by $k+1$ $(= \alpha)$.
  - Adding an edge towards z_i^j is even worse...
- By Lemma 1, even more edges cannot decrease u's cost.

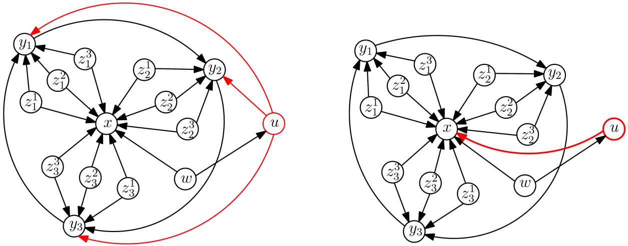


$(G_k, k+1)$



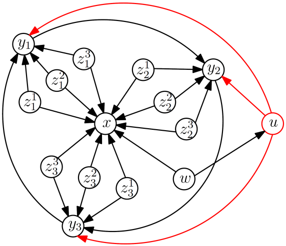
$(G_k^*, k+1)$

- Now, consider a strategy-change of u: $S_u = \{y_1, \ldots, y_k\} \rightsquigarrow S_u^* = \{x\}$.
- Easy to check that no other S_u' iwth $|S_u'| \leq 1$ outperforms S_u^*.
- Adding any edge $\{u, y_i\}$ decreases the cost by $k + 1$ $(= \alpha)$.
  - Adding an edge towards z_i^j is even worse...
- By Lemma 1, even more edges cannot decrease u's cost.

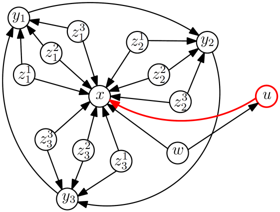


$(G_k, k+1)$ $(G_k^*, k+1)$

- Now, consider a strategy-change of u: $S_u = \{y_1, \ldots, y_k\} \rightsquigarrow S_u^* = \{x\}$.
- Easy to check that no other S'_u iwth $|S'_u| \leq 1$ outperforms S_u^*.
- Adding any edge $\{u, y_i\}$ decreases the cost by $k+1$ $(= \alpha)$.
    - Adding an edge towards z_i^j is even worse...
- By Lemma 1, even more edges cannot decrease u's cost.

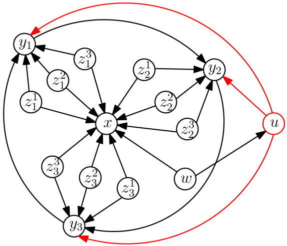


$(G_k, k+1)$

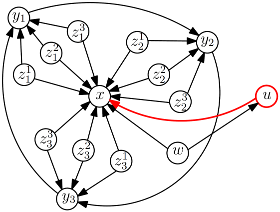


$(G_k^*, k+1)$

- Now, consider a strategy-change of u: $S_u = \{y_1, \ldots, y_k\} \rightsquigarrow S_u^* = \{x\}$.
- Easy to check that no other S_u' iwth $|S_u'| \leq 1$ outperforms S_u^*.
- Adding any edge $\{u, y_i\}$ decreases the cost by $k + 1$ $(= \alpha)$.
  - Adding an edge towards z_i^j is even worse...
- By Lemma 1, even more edges cannot decrease u's cost.

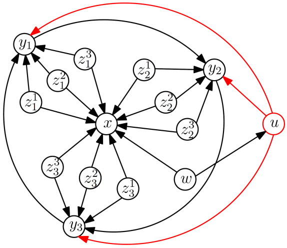


$(G_k, k+1)$

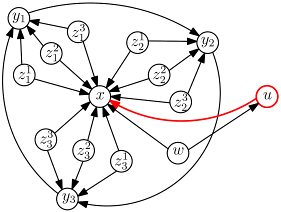


$(G_k^*, k+1)$

- Now, consider a strategy-change of u: $S_u = \{y_1, \ldots, y_k\} \rightsquigarrow S_u^* = \{x\}$.
- Easy to check that no other S_u' iwth $|S_u'| \leq 1$ outperforms S_u^*.
- Adding any edge $\{u, y_i\}$ decreases the cost by $k + 1$ $(= \alpha)$.
  - Adding an edge towards z_i^j is even worse...
- By Lemma 1, even more edges cannot decrease u's cost.

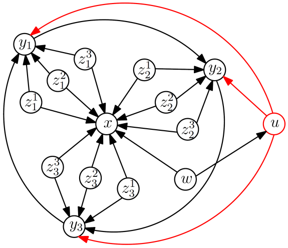


$(G_k, k+1)$

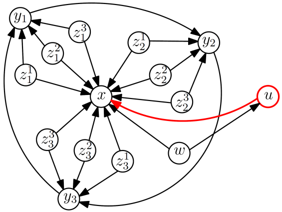


$(G_k^*, k+1)$

- S_u is agent u's best possible strategy not buying an edge towards x (proof omitted).
- $c(S_u, k)$: agent u's cost in $(G_k, k+1)$;
  $c(S_u^*, k)$: agent u's cost in $(G_k^*, k+1)$.

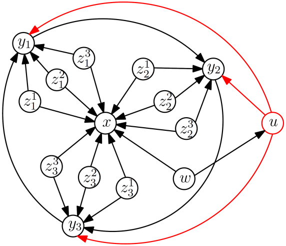


$(G_k, k+1)$

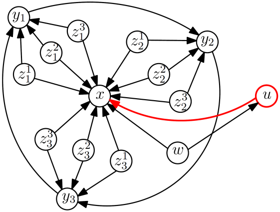


$(G_k^*, k+1)$

- S_u is agent u's best possible strategy not buying an edge towards x (proof omitted).
- $c(S_u, k)$: agent u's cost in $(G_k, k+1)$;
  $c(S_u^*, k)$: agent u's cost in $(G_k^*, k+1)$.

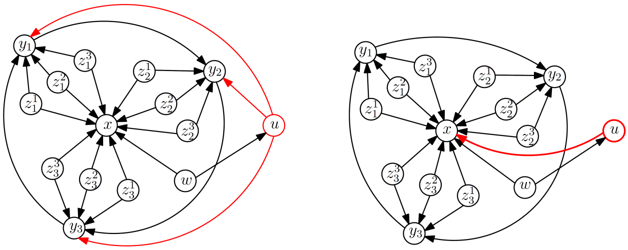


$(G_k, k+1)$



$(G_k^*, k+1)$

- $\lim_{k\to\infty} \frac{c(S_u, k)}{c(S_u^*, k)} = \lim_{k\to\infty} \frac{k\alpha + k + 1 + 2(k^2+1)}{\alpha + 2 + 2k^2 + 3k} = \lim_{k\to\infty} \frac{3k^2 + 2k + 3}{2k^2 + 4k + 3} = \frac{3}{2}.$
- For any $\beta < \frac{3}{2}$, there is a k' such that $c(S_u, k') > \beta \cdot c(S_u^*, k')$
  - ▷ SUM-GE$(G_{k'}, k'+1)$ is not a β-approximate SUM-NE for $\beta < \frac{3}{2}$.



$(G_k, k+1)$ $(G_k^*, k+1)$

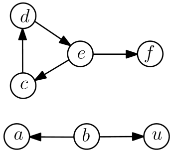
- $$\lim_{k\to\infty} \frac{c(S_u, k)}{c(S_u^*, k)} = \lim_{k\to\infty} \frac{k\alpha + k + 1 + 2(k^2+1)}{\alpha + 2 + 2k^2 + 3k} = \lim_{k\to\infty} \frac{3k^2 + 2k + 3}{2k^2 + 4k + 3} = \frac{3}{2}.$$
- For any $\beta < \frac{3}{2}$, there is a k' such that $c(S_u, k') > \beta \cdot c(S_u^*, k')$
  - ▷ SUM-GE$(G_{k'}, k'+1)$ is not a β-approximate SUM-NE for $\beta < \frac{3}{2}$.

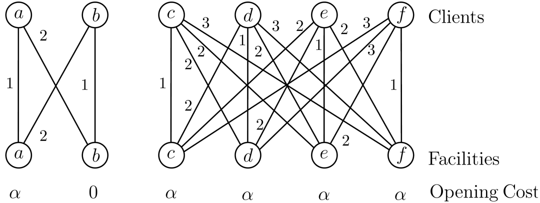
Good news

Theorem 4

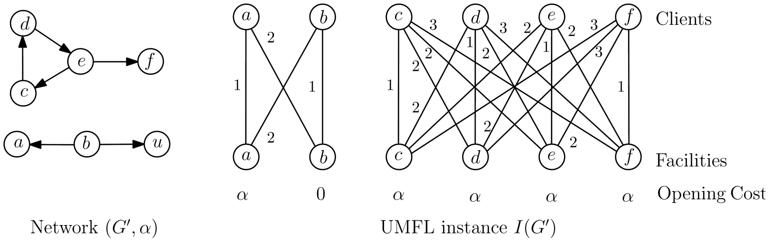
Every network in SUM-GE is in 3-approximate SUM-NE.

- Proof: providing a "locality gap preserving" reduction to the Uncapacitated Metric Facility Location problem (UMFL).

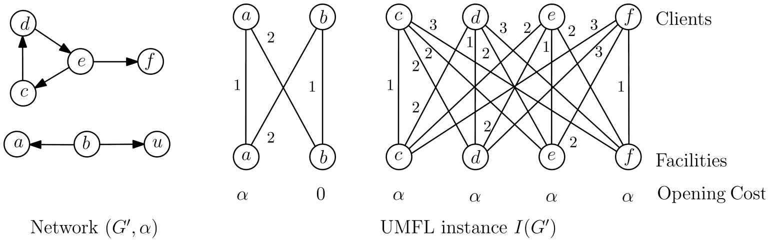




- (G', α): all edges owned by u are removed.
- Z: the set of vertices which own an edge towards u.
- $\mathcal{S} := \{U \mid U \subseteq (V(G') \setminus \{u\} \text{ and } U \cap Z = \emptyset\}$.
  - u's strategies in (G', α) (not including multi-edges or self-loop).
- $F = C = V(G') \setminus \{u\}$ (F: facilities; C: clients).
  - The *opening cost* is **0** for each $f \in Z \cap F$, others have opening cost α.
- For every $i, j \in F \cup C$, $d_{ij} = d_{G'}(i, j) + 1$.

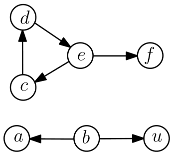


- Any strategy $S \in \mathcal{S}$ of agent u in (G', α) corresponds to the solution of the UMFL instance $I(G')$.
  - Exactly the facilities in $F_S = S \cup Z$ are opened
  - All clients are assigned to their nearest open facility.
- Every solution $F' = X \cup Z$, where $X \subseteq F \setminus Z$, for instance $I(G')$ corresponds to agent u's strategy $X \in \mathcal{S}$ in (G', α).

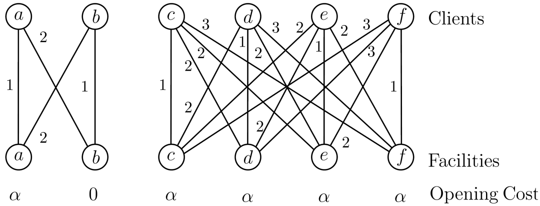


- Any strategy $S \in \mathcal{S}$ of agent u in (G', α) corresponds to the solution of the UMFL instance $I(G')$.
  - Exactly the facilities in $F_S = S \cup Z$ are opened
  - All clients are assigned to their nearest open facility.

- Every solution $F' = X \cup Z$, where $X \subseteq F \setminus Z$, for instance $I(G')$ corresponds to agent u's strategy $X \in \mathcal{S}$ in (G', α).



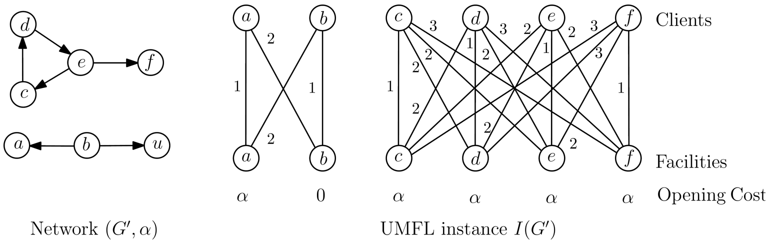
Network (G', α)



UMFL instance $I(G')$

- (G_S, α): the network (G', α) where u has bought all edges towards $v \in S$.
- $cost(F_S)$: the cost of the solution F_S to instance $I(G')$.

$$
\begin{aligned}
c_u(G_S, \alpha) &= \alpha|S| + \sum_{w \in V(G_S) \setminus \{u\}} \left(1 + \min_{x \in S \cup Z} d_{G'}(x, w)\right) \\
&= \alpha|S| + 0|Z| + \sum_{w \in V(G_S) \setminus \{u\}} \min_{x \in S \cup Z} d_{xw} \\
&= \alpha|F_S \setminus Z| + 0|Z| + \sum_{w \in C} \min_{x \in F_S} d_{xw} = cost(F_S).
\end{aligned}
$$

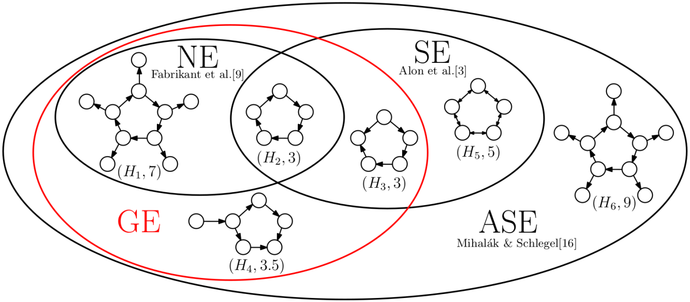


Claim

If agent u plays $S \in \mathcal{S}$ and cannot decrease her cost by buying, deleting or swapping *one* edge in (G_S, α), then the cost of the corresponding solution to instance $I(G')$ cannot be strictly decreased by opening, closing or swapping *one* facility.

- Any UMFL solution that cannot be improved by opening, closing or swapping one facility is a 3-approximation of the optimal solution. [Arya *et al.* SIAM J. Comput. 2004].

Concluding remarks



Concluding remarks

- PoA or PoS in SUM-GE?
- On dynamics in Selfish Network Creation (SPAA 2013).

Thank you.

2:46:03 (Alan Turing)

KOBE MARATHON 2015