Problem:
Square root convergents Problem 57
It is possible to show that the square root of two can be expressed as an infinite continued fraction. In the first one-thousand expansions, how many fractions contain a numerator with more digits than denominator?
Solution:
The $\sqrt{2}$ can be expressed as

$$\sqrt{2} = 1 + \frac{1}{1+\sqrt{2}} \approx 1 + \frac{1}{2}$$

Call this expansion level 0. This can be expanded again through recursion an arbitrary number of times

$$\sqrt{2} = 1 + \frac{1}{2+\frac{1}{\sqrt{2}}} \approx 1 + \frac{1}{2+\frac{1}{2}}$$

Call this expansion level 1. The point being that the greater the number of expansions, the closer the approximation of $\sqrt{2}$. Now let n be our expansion level, let *f(n)* be our function that returns the approximation of $\sqrt{2}$ at a given level of expansion.
Now notice that at the deepest level of our continued fraction we are simply calculating

$$\frac{1}{2+\frac{1}{2}}$$

Let's define another subroutine *g(x)*

$$g(x) = \frac{1}{2+x}$$

At the next level up $n-1, n>0$ we calculate

$$g \circ g(\frac{1}{2})$$

The insight I am trying to convey here is that for any exansion lebel n, the value approximation can be expressed as

$$f(n) = 1 + g^{\circ n}(\frac{1}{2})$$

With this intuition, coding up a solution becomes trivial.