U SUBSTITUTIONS DIFFICULT PURCELL PG 393

1. Evaluate $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\ dx$

2. Evaluate $\int \sin^2 x \cos x\ dx$

3. Evaluate $\int x\sqrt{x^2+11}\ dx$

4. Evaluate $\int \frac{6\ e^{1/x}}{x^2}\ dx$

5. Evaluate $\int \frac{\sec^2(\sin x)}{\sec x}\ dx$

6. Evaluate $\int \frac{4e^x}{\sqrt{1-e^{2x}}}\ dx$

7. Evaluate $\int \frac{a^{\tan t}}{\cos^2 t}\ dt$

8. Evaluate $\int\ (7x-1)^{12}\ dx$

9. Evaluate $\int\ (4x^3+3x-1)^4\ (4x^2+1)\ dx$

10. Evaluate $\int\ (\sin 6x)^3\ \cos(6x)\ \ dx$

11. Evaluate $\int \cos\frac{x}{3}\ \sin\frac{x}{3}\ dx$

12. Evaluate $\int x^2\sqrt{7x^3 + 5}\ dx$

13. Evaluate $\int\ x\sin 2x^2\ \ dx$

14. Evaluate $\int\ \frac{\cos\sqrt{1-x}}{\sqrt{1-x}}\ dx$

15. Evaluate $\int e^{(x^2+2x-1)}\ (x+1)\ dx$

16. Evaluate $\int \frac{dt}{e^{3t}}$

17. Evaluate $\int\ \frac{e^{\sqrt{2x+1}}}{\sqrt{2x+1}}\ dx$

18. Evaluate $\int\ \frac{e^{\csc t}\cot t}{\sin t}\ \ dt$

19. Evaluate $\int\ \frac{(\ln t^2)^9}{t}\ dt$

20. Evaluate $\int\ \frac{(\cos^{-1} 2w)^7}{\sqrt{1-4w^2}}\ dw$

21. Evaluate $\int\ \frac{\sin x}{\cos^5 x}\ dx$

22. Evaluate $\int (\cos^2 x \ - \ \sin^2 x)^3 \ \ (\sin x \ \cos x) \ dx$

23. Evaluate $\int \frac{x}{6x^2-19} \ dx$

24. Evaluate $\int \ \frac{\sin t}{\cos t} \ \ dt$

25. Evaluate $\int \ (e^t - e^{-t})^2 \ (e^t + e^{-t}) \ dt$

26. Evaluate $\int \ \ \sin^2 y \ \ \cos y \ \ \sqrt{\sin^3 y \ + \ \ 4} \ \ dy$

27. Evaluate $\int \frac{\sqrt{\tan x}}{1 - \sin^2 x} \ dx$

28. Evaluate $\int x \ 5^{x^2-1} \ \ dx$

U SUBSTITUTIONS DIFFICULT
PURCELL PG 393

1. Evaluate $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\ dx$

   $let\ u = \sqrt{x}$ $\qquad du = \frac{1}{2\sqrt{x}}\ dx \quad or \quad 2\ du = \frac{dx}{\sqrt{x}}$

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\ dx = \int \sin u \quad 2\ du$$

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\ dx = 2\int \sin u \quad du$$

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\ dx = 2\,(-\cos u) + C$$

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\ dx = -\,2\cos\sqrt{x} + C$$

2. Evaluate $\int \sin^2 x \cos x\ dx$

   $let\ u = \sin x$ $\qquad du = \cos x\ \ dx$

$$\int \sin^2 x \cos x\ dx = \int u^2\ du$$

$$\int \sin^2 x \cos x\ dx = \frac{u^3}{3} + C$$

$$\int \sin^2 x \cos x\ dx = \frac{\sin^3 x}{3} + C$$

3. Evaluate $\int x\sqrt{x^2+11}\ dx$

$let\ u = x^2 + 11 \qquad du = 2x\,dx \quad or\ \frac{1}{2}\ du = x\,dx$

$$\int x\sqrt{x^2+11}\ dx = \int \sqrt{x^2+11}\ x\,dx$$

$$\int x\sqrt{x^2+11}\ dx = \int \sqrt{u}\ \frac{1}{2}\ du$$

$$\int x\sqrt{x^2+11}\ dx = \frac{1}{2}\int \sqrt{u}\ du$$

$$\int x\sqrt{x^2+11}\ dx = \frac{1}{2}\cdot\frac{2}{3}\ u^{\frac{3}{2}} + C$$

$$\int x\sqrt{x^2+11}\ dx = \frac{1}{3}\ (x^2 + 11)^{3/2} + C$$

4. Evaluate $\int \frac{6\ e^{1/x}}{x^2}\ dx$

$let\ u = \frac{1}{x} \qquad du = -\frac{1}{x^2}\ dx \quad or\ \ -du = \frac{1}{x^2}\ dx$

$$\int \frac{6\ e^{1/x}}{x^2}\ dx = 6\int e^{1/x}\ \frac{1}{x^2}\ dx$$

$$\int \frac{6\ e^{1/x}}{x^2}\ dx = 6\int e^{u}\ (-du)$$

$$\int \frac{6\ e^{1/x}}{x^2}\ dx = -6\int e^{u}\ du$$

$$\int \frac{6\ e^{1/x}}{x^2}\ dx = -6\,e^{u} + C$$

$$\int \frac{6\ e^{1/x}}{x^2}\ dx = -6\,e^{1/x} + C$$

5. Evaluate $\int \frac{\sec^2(\sin x)}{\sec x}\ dx$

$$\int \frac{\sec^2(\sin x)}{\sec x}\ dx \ = \int \sec^2(\sin x)\ \cos x\ \ dx$$

$let\ u = \sin x \quad du = \cos x\ dx$

$$\int \frac{\sec^2(\sin x)}{\sec x}\ dx \ = \int \sec^2 u\ \ du$$

$$\int \frac{\sec^2(\sin x)}{\sec x}\ dx \ = \tan u\ +\ C$$

$$\int \frac{\sec^2(\sin x)}{\sec x}\ dx \ = \tan(\sin x)\ +\ C$$

6. Evaluate $\int \frac{4e^x}{\sqrt{1-e^{2x}}}\ dx$

$let\ u\ =\ e^x \qquad du = e^x dx$

$$\int \frac{4e^x}{\sqrt{1-e^{2x}}}\ dx\ =\ 4\int \frac{e^x\ dx}{\sqrt{1\ -\ (e^x)^2}}$$

$$\int \frac{4e^x}{\sqrt{1-e^{2x}}}\ dx\ =\ 4\int \frac{du}{\sqrt{1\ -\ (u)^2}}$$

$$\int \frac{4e^x}{\sqrt{1-e^{2x}}}\ dx\ =\ 4\ arc\sin u\ +\ C$$

$$\int \frac{4e^x}{\sqrt{1-e^{2x}}}\ dx\ =\ 4\ arc\sin e^x\ +\ C$$

7. Evaluate $\int \frac{a^{\tan t}}{\cos^2 t} \, dt$

$let\ u = \tan t \qquad du = \sec^2 t \;\; dt$

$$\int \frac{a^{\tan t}}{\cos^2 t} \, dt \;=\; \int a^{\tan t} \sec^2 t \;\; dt$$

$$\int \frac{a^{\tan t}}{\cos^2 t} \, dt \;=\; \int a^u \;\, du$$

$$\int \frac{a^{\tan t}}{\cos^2 t} \, dt \;=\; \frac{a^u}{\ln a} \;+\; C$$

$$\int \frac{a^{\tan t}}{\cos^2 t} \, dt \;=\; \frac{a^{\tan t}}{\ln a} \;+\; C$$

8. Evaluate $\int \;(7x - 1)^{12} \, dx$

$let\ u = 7x - 12 \qquad du = 7\, dx \quad or \;\; \frac{1}{7}\, du = dx$

$$\int (7x-1)^{12}\, dx \;=\; \int u^{12}\, \frac{1}{7}\, du$$

$$\int (7x-1)^{12}\, dx \;=\; \frac{1}{7} \int u^{12} \;\; du$$

$$\int (7x-1)^{12}\, dx \;=\; \frac{1}{7} \cdot \frac{1}{13}\, u^{13} \;+\; C$$

$$\int (7x-1)^{12}\, dx \;=\; \frac{1}{91}\, (7x-1)^{13} \;+\; C$$

9. Evaluate $\int (4x^3+3x-1)^4\,(4x^2+1)\,dx$

$let\ u = \ 4x^3+3x-1 \qquad du = (12\,x^2+3)\ dx\ \ or\ \ \frac{1}{3}\,du = \ (4x^2+1)\,dx$

$$\int (4x^3+3x-1)^4\,(4x^2+1)\,dx \ = \int u^4 \ \frac{1}{3}\ du$$

$$\int (4x^3+3x-1)^4\,(4x^2+1)\,dx \ = \frac{1}{3}\int u^4 \ du$$

$$\int (4x^3+3x-1)^4\,(4x^2+1)\,dx \ = \frac{1}{15}\ u^5 \ + \ C$$

$$\int (4x^3+3x-1)^4\,(4x^2+1)\,dx \ = \frac{1}{15}\ (4x^3+3x-1)^5 \ + \ C$$

10. Evaluate $\int (\sin 6x)^3\ \cos(6x)\ \ dx$

$let\ u = \sin 6x \qquad du = 6\cos 6x\ \ dx\ \ \ or\ \ \frac{1}{6}du = \cos 6x\ \ dx$

$$\int (\sin 6x)^3\ \cos(6x)\ \ dx \ = \int u^3\ \frac{1}{6}du$$

$$\int (\sin 6x)^3\ \cos(6x)\ \ dx \ = \frac{1}{6}\int u^3\ du$$

$$\int (\sin 6x)^3\ \cos(6x)\ \ dx \ = \frac{1}{24}\,u^4 \ + \ C$$

$$\int (\sin 6x)^3\ \cos(6x)\ \ dx \ = \frac{1}{24}\,(\sin 6x)^4 \ + \ C$$

11. Evaluate $\int \cos\frac{x}{3} \sin\frac{x}{3}\, dx$

$let\ u = \sin\frac{x}{3} \qquad du = \frac{1}{3}\cos\frac{x}{3}\, dx \qquad or \quad 3du = \cos\frac{x}{3}\, dx$

$$\int \cos\frac{x}{3} \sin\frac{x}{3}\, dx = \int \sin\frac{x}{3} \cos\frac{x}{3}\, dx$$

$$\int \cos\frac{x}{3} \sin\frac{x}{3}\, dx = \int u \quad 3du$$

$$\int \cos\frac{x}{3} \sin\frac{x}{3}\, dx = 3\int u\, du$$

$$\int \cos\frac{x}{3} \sin\frac{x}{3}\, dx = \frac{3}{2}\, u^2 + C$$

$$\int \cos\frac{x}{3} \sin\frac{x}{3}\, dx = \frac{3}{2}\left(\sin\frac{x}{3}\right)^2 + C$$

12. Evaluate $\int x^2\sqrt{7x^3 + 5}\, dx$

$let\ u = 7x^3 + 5 \qquad du = 21x^2\, dx \qquad or\ \frac{1}{21}\, du = x^2\, dx$

$$\int x^2\sqrt{7x^3 + 5}\, dx = \int \sqrt{7x^3 + 5}\, x^2\, dx$$

$$\int x^2\sqrt{7x^3 + 5}\, dx = \int \sqrt{u}\, \frac{1}{21}\, du$$

$$\int x^2\sqrt{7x^3 + 5}\, dx = \frac{1}{21}\int \sqrt{u}\, du$$

$$\int x^2\sqrt{7x^3 + 5}\, dx = \frac{1}{21}\cdot\frac{2}{3}\, u^{\frac{3}{2}} + C$$

$$\int x^2\sqrt{7x^3 + 5}\, dx = \frac{2}{63}\,(7x^3 + 5)^{\frac{3}{2}} + C$$

13. Evaluate $\int \; x \sin 2x^2 \;\; dx$

$let\ u = 2\,x^2 \qquad du = 4x\ \ dx\ \ or\ \ \frac{1}{4}\ du = x\,dx$

$$\int \; x \sin 2x^2 \;\; dx \;\; = \;\; \int \; \sin 2x^2 \;\; x\,dx$$

$$\int \; x \sin 2x^2 \;\; dx \;\; = \;\; \int \; \sin u \;\; \frac{1}{4}\ du$$

$$\int \; x \sin 2x^2 \;\; dx \;\; = \frac{1}{4} \int \; \sin u \;\; du$$

$$\int \; x \sin 2x^2 \;\; dx \;\; = \frac{1}{4}\ (-\cos u) \; + \; C$$

$$\int \; x \sin 2x^2 \;\; dx \;\; = -\frac{1}{4} \cos 2x^2 \; + \; C$$

14. Evaluate $\int \; \frac{\cos\sqrt{1-x}}{\sqrt{1-x}} \;\; dx$

$let\ u = \sqrt{1-x} \qquad\qquad du = \frac{-1}{2\ \sqrt{1-x}}\ \ dx \quad or \quad -2\ du = \frac{1}{\sqrt{1-x}}\ \ dx$

$$\int \; \frac{\cos\sqrt{1-x}}{\sqrt{1-x}} \;\; dx \;\; = \;\; \int \; \cos\sqrt{1-x} \;\; \frac{1}{\sqrt{1-x}} \;\; dx$$

$$\int \; \frac{\cos\sqrt{1-x}}{\sqrt{1-x}} \;\; dx \;\; = \;\; \int \; \cos u \;\; (-du)$$

$$\int \; \frac{\cos\sqrt{1-x}}{\sqrt{1-x}} \;\; dx \;\; = -\int \; \cos u \;\; du$$

$$\int \; \frac{\cos\sqrt{1-x}}{\sqrt{1-x}} \;\; dx \;\; = -\sin u \; + \; C$$

$$\int \; \frac{\cos\sqrt{1-x}}{\sqrt{1-x}} \;\; dx \;\; = -\sin\sqrt{1-x} \; + \; C$$

15. Evaluate $\int e^{(x^2+2x-1)}\,(x+1)\,dx$

$let\ u = x^2 + \ 2x\ + 1 \qquad\qquad du = (2x+2)\,dx \quad or \quad \frac{1}{2}\,du = (x+1)dx$

$$\int e^u \frac{1}{2}\,du \ = \frac{1}{2}\int\ e^u\ du$$

$$\int e^u \frac{1}{2}\,du \ = \frac{1}{2}\,e^u + \ C$$

$$\int e^u \frac{1}{2}\,du \ = \frac{1}{2}\,e^{(x^2+2x-1)} + \ C$$

16. Evaluate $\int \frac{dt}{e^{3t}}$

$let\ u = -3t \qquad du = \ -3dt \ \ or \ \ -\frac{1}{3}\,du = dt$

$$\int \frac{dt}{e^{3t}} \ = \ \int\ e^{-3t}\ dt$$

$$\int \frac{dt}{e^{3t}} \ = \ \int e^u\ \left(-\frac{1}{3}du\right)$$

$$\int \frac{dt}{e^{3t}} \ = \ -\frac{1}{3}\int e^u\ du$$

$$\int \frac{dt}{e^{3t}} \ = \ -\frac{1}{3}\,e^u + C$$

$$\int \frac{dt}{e^{3t}} \ = \ -\frac{1}{3}\,e^{\,-3t} + C$$

17. Evaluate $\int \frac{e^{\sqrt{2x+1}}}{\sqrt{2x+1}}\ dx$

$let\ u = \sqrt{2x+1}$ $\qquad du = \frac{1}{\sqrt{2x+1}}\ dx$

$$\int \frac{e^{\sqrt{2x+1}}}{\sqrt{2x+1}}\ dx = \int e^{\sqrt{2x+1}} \frac{1}{\sqrt{2x+1}}\ dx$$

$$\int \frac{e^{\sqrt{2x+1}}}{\sqrt{2x+1}}\ dx = \int e^{u}\ du$$

$$\int \frac{e^{\sqrt{2x+1}}}{\sqrt{2x+1}}\ dx = e^{u} + C$$

$$\int \frac{e^{\sqrt{2x+1}}}{\sqrt{2x+1}}\ dx = e^{\sqrt{2x+1}} + C$$

18. Evaluate $\int \frac{e^{\csc t}\cot t}{\sin t}\ dt$

$let\ u = \csc t$ $\qquad du = -\csc t \cot t\ \ dt$

$$\int \frac{e^{\csc t}\cot t}{\sin t}\ dt = \int e^{\csc t}\ \cot t \csc t\ \ dt$$

$$\int \frac{e^{\csc t}\cot t}{\sin t}\ dt = \int e^{u}\ (-du)$$

$$\int \frac{e^{\csc t}\cot t}{\sin t}\ dt = -\int e^{u}\ du$$

$$\int \frac{e^{\csc t}\cot t}{\sin t}\ dt = -e^{u} + C$$

$$\int \frac{e^{\csc t}\cot t}{\sin t}\ dt = -e^{\csc t} + C$$

19. Evaluate $\int \frac{(\ln t^2)^9}{t}\ dt$

$let\ u = \ln t$ $\qquad du = \frac{1}{t}\ dt$

$$\int \frac{(\ln t^2)^9}{t}\ dt = \int \frac{(\ 2\ \ln t)^9}{t}\ dt$$

$$\int \frac{(\ln t^2)^9}{t}\ dt = 2^9 \int \frac{(\ \ln t\ )^9}{t}\ dt$$

$$\int \frac{(\ln t^2)^9}{t}\ dt = 2^9 \int (\ \ln t\ )^9\ \frac{1}{t}\ dt$$

$$\int \frac{(\ln t^2)^9}{t}\ dt = 2^9 \int (\ u\ )^9\ du$$

$$\int \frac{(\ln t^2)^9}{t}\ dt = 2^9\ \frac{u^{10}}{10} + C$$

$$\int \frac{(\ln t^2)^9}{t}\ dt = \frac{2^9\ (\ln t)^{10}}{10} + C$$

20. Evaluate $\int \frac{(\cos^{-1} 2w)^7}{\sqrt{1-4w^2}}\ dw$

$let\ u = \cos^{-1} 2w$ $\qquad du = \frac{-2}{\sqrt{1-4w^2}}\ dw \quad or \quad -\frac{1}{2} du = \frac{1}{\sqrt{1-4\,w^2}}\ dw$

$$\int \frac{(\cos^{-1} 2w)^7}{\sqrt{1-4w^2}}\ dw = \int (\cos^{-1} 2w)^7\ \frac{1}{\sqrt{1-4w^2}}\ dw$$

$$\int \frac{(\cos^{-1} 2w)^7}{\sqrt{1-4w^2}}\ dw = \int (u)^7\ \left(-\frac{1}{2} du\right)$$

$$\int \frac{(\cos^{-1} 2w)^7}{\sqrt{1-4w^2}}\ dw = -\frac{1}{2} \int (u)^7\ du$$

$$\int \frac{(\cos^{-1} 2w)^7}{\sqrt{1-4w^2}}\ dw = -\frac{1}{16}\ u^8 + C$$

$$\int \frac{(\cos^{-1} 2w)^7}{\sqrt{1-4w^2}}\ dw = -\frac{1}{16}\ (\cos^{-1} 2w)^8 + C$$

21. Evaluate $\int \frac{\sin x}{\cos^5 x}\ dx$

$let\ u = \cos x \qquad du = -\sin x\ \ dx$

$$\int \frac{\sin x}{\cos^5 x}\ dx = \int -\frac{du}{u^5}$$

$$\int \frac{\sin x}{\cos^5 x}\ dx = -\int u^{-5}\ du$$

$$\int \frac{\sin x}{\cos^5 x}\ dx = \frac{u^{-4}}{4} + C$$

$$\int \frac{\sin x}{\cos^5 x}\ dx = \frac{(\cos x)^{-4}}{4} + C$$

22. Evaluate $\int (\cos^2 x - \sin^2 x)^3\ \ (\sin x\ \cos x)\ dx$

$$\int (\cos^2 x - \sin^2 x)^3\ \ (\sin x\ \cos x)\ dx = \int (\cos 2x)^3\ \frac{1}{2}\sin 2x\ \ dx$$

$$\int (\cos^2 x - \sin^2 x)^3\ \ (\sin x\ \cos x)\ dx = \frac{1}{2}\int (\cos 2x)^3\ \ \sin 2x\ \ dx$$

$let\ u = \cos 2x \qquad du = -2\sin 2x\ \ dx \quad or \quad -\frac{1}{2}\ du = \sin 2x\ \ dx$

$$\int (\cos^2 x - \sin^2 x)^3\ \ (\sin x\ \cos x)\ dx = \frac{1}{2}\int (u)^3\ \left(-\frac{1}{2}\ du\right)$$

$$\int (\cos^2 x - \sin^2 x)^3\ \ (\sin x\ \cos x)\ dx = -\frac{1}{4}\int u^3\ du$$

$$\int (\cos^2 x \;-\; \sin^2 x)^3 \;\; (\sin x \;\cos x) \;\, dx \;\; = -\frac{1}{16} \;\, u^4 \; + \; C$$

$$\int (\cos^2 x \;-\; \sin^2 x)^3 \;\; (\sin x \;\cos x) \;\, dx \;\; = -\frac{1}{16} \;\; (\cos 2x)^4 \; + \; C$$

23. Evaluate $\int \frac{x}{6x^2-19} \; dx$

$let \; u = 6x^2 - 19 \qquad\qquad du = 12\, x\, dx \;\;\; or \; \frac{1}{12}\, du = x\, dx$

$$\int \frac{x}{6x^2 - 19} \;\; dx \; = \int \frac{1}{6x^2 - 19} \;\; x \; dx$$

$$\int \frac{x}{6x^2 - 19} \;\; dx \; = \int \frac{1}{u} \;\; \frac{1}{12} du$$

$$\int \frac{x}{6x^2 - 19} \;\; dx \; = \frac{1}{12} \;\; \int \frac{1}{u} \;\; du$$

$$\int \frac{x}{6x^2 - 19} \;\; dx \; = \frac{1}{12} \;\; \ln|u| \; + \; C$$

$$\int \frac{x}{6x^2 - 19} \;\; dx \; = \frac{1}{12} \;\; \ln|6x^2 - 19| \; + \; C$$

24. Evaluate $\int \; \frac{\sin t}{\cos t} \;\; dt$

$let \; u = \cos t \qquad\qquad du = \; - \sin t \;\; dt$

$$\int \; \frac{\sin t}{\cos t} \;\; dt \;\; \int \frac{1}{u} \; (-du) \;\;\; = - \;\; \int \frac{1}{u} \; du \;\; = -\ln|u| \; + \; C \; = -\ln|\cos t| \; + \; C$$

25. Evaluate $\int \; (e^t - e^{-t})^2 \; (e^t + e^{-t}) \; dt$

$let \; u = e^t - \;\; e^{-t} \qquad\qquad du = (e^t + e^{-t}) \;\; dt$

$$\int \; (e^t - e^{-t})^2 \; (e^t + e^{-t}) \; dt \;\; = \; \int \; (u)^2 \; du$$

$$\int \; (e^t - e^{-t})^2 \; (e^t + e^{-t}) \; dt \;\; = \frac{u^3}{3} \; + \; C$$

$$\int \; (e^t - e^{-t})^2 \; (e^t + e^{-t}) \; dt \;\; = \frac{(e^t - e^{-t})^3}{3} \; + \; C$$

26. Evaluate $\int \sin^2 y \cos y \sqrt{\sin^3 y + 4}\, dy$

$let\ u = \sin^3 y + 4 \qquad du = 3\sin^2 y \cos y \; dy \quad or\ \frac{1}{3} du = \sin^2 y \cos y \; dy$

$$\int \sin^2 y \cos y \sqrt{\sin^3 y + 4}\, dy = \int \sqrt{\sin^3 y + 4}\, \sin^2 y \cos y \, dy$$

$$\int \sin^2 y \cos y \sqrt{\sin^3 y + 4}\, dy = \int \sqrt{u}\, \frac{1}{3} du$$

$$\int \sin^2 y \cos y \sqrt{\sin^3 y + 4}\, dy = \frac{1}{3} \int \sqrt{u}\, du$$

$$\int \sin^2 y \cos y \sqrt{\sin^3 y + 4}\, dy = \frac{1}{3} \cdot \frac{2}{3}\, u^{\frac{3}{2}} + C$$

$$\int \sin^2 y \cos y \sqrt{\sin^3 y + 4}\, dy = \frac{2}{9}\, (\sin^3 y + 4)^{\frac{3}{2}} + C$$

27. Evaluate $\int \frac{\sqrt{\tan x}}{1 - \sin^2 x}\, dx$

$$\int \frac{\sqrt{\tan x}}{1 - \sin^2 x}\, dx = \int \frac{\sqrt{\tan x}}{\cos^2 x}\, dx$$

$$\int \frac{\sqrt{\tan x}}{1 - \sin^2 x}\, dx = \int \sqrt{\tan x}\, \sec^2 x \, dx$$

$$\int \frac{\sqrt{\tan x}}{1 - \sin^2 x}\, dx = \int (\tan x)^{1/2} \sec^2 x \, dx$$

$let\ u = \tan x \qquad du = \sec^2 x \; dx$

$$\int \frac{\sqrt{\tan x}}{1 - \sin^2 x}\, dx = \int u^{1/2}\, du$$

$$\int \frac{\sqrt{\tan x}}{1 - \sin^2 x}\, dx = \frac{2}{3}\, u^{3/2} + C$$

$$\int \frac{\sqrt{\tan x}}{1 - \sin^2 x}\, dx = \frac{2}{3}\, (\tan x)^{3/2} + C$$

28. Evaluate $\int x\, 5^{x^2-1}\ dx$

$$\int x\, 5^{x^2-1}\ dx\ = \int\ 5^{x^2-1}\ x\ dx$$

$let\ u = 5x^2 - 1 \qquad du = 10\ x\ dx\ \ or\ \frac{1}{10}\ du = x\ dx$

$$\int x\, 5^{x^2-1}\ dx\ = \int\ 5^{u}\ \frac{1}{10} du$$

$$\int x\, 5^{x^2-1}\ dx\ = \frac{1}{10}\ \int\ 5^{u}\ du$$

$$\int x\, 5^{x^2-1}\ dx\ = \frac{1}{10}\ \frac{5^{u}}{\ln 5}\ +\ C$$

$$\int x\, 5^{x^2-1}\ dx\ = \frac{1}{10}\ \frac{5^{(x^2-1)}}{\ln 5}\ +\ C$$