VOLUMES METHOD OF CYLINDRICAL SHELLS

1. You are given the region bounded below by the function $y = x^2$ bounded above by y = 4. This region is rotated about the y axis. Find the volume by the method of cylindrical shells.

2. You are given the region bounded by the x axis, the function $y = x^2$ and the vertical line x = 2. The region is rotated about the y axis. Find the volume of the solid generated using cylindrical shells.

3. You are given the region bounded by the function $= x^2$, the y axis and the horizontal line $y = 1$. The region is rotated about the line y = 1. Find the volume of the solid generated using the method of cylindrical shells.

4. You are given the region bounded by the x axis, the line y = x and the vertical line x = 1. The region is a right triangle. The region is rotated about the line x = 2. Find the volume of the solid generated using the method of cylindrical shells.

5. You are given the functions $y = x$ and $y = x^2$. The region between them is rotated about the horizontal line y = 2. Find the volume using the method of cylindrical shells.

6. You are given the region bounded by $y = \frac{4}{x}$, x = 1, x = 4 and the x axis. This region is rotated about the y axis. Find the volume using cylindrical shells.

7. You are given the region bounded by $y = \sqrt{x}$, the x axis and the line x = 4. The region is rotated about the y axis. Find the volume using shells.

8. You are given a region – it is bounded above by the curve $y = \frac{1}{4}x^3 + 2$. It is bounded below by the line $y = 2 - x$. And it is bounded on the right by the vertical line x = 2. The region is rotated about the y axis. Find the volume using shells.

9. You are given the region bounded by $x = \sqrt{2y} + 1$, the x axis, the y axis and by the horizontal line y = 2. The region is rotated about the x axis. Find the volume.

10. You are given the region bounded above by the function $y = x - x^2$ and below by the x axis. The region is rotated about the line x = 5. Find the volume of the solid.

11. The region bounded by $x = y^2$, the line y = 2 and the y axis. Region is rotated about the x axis. Find the volume.

VOLUMES METHOD OF CYLINDRICAL SHELLS

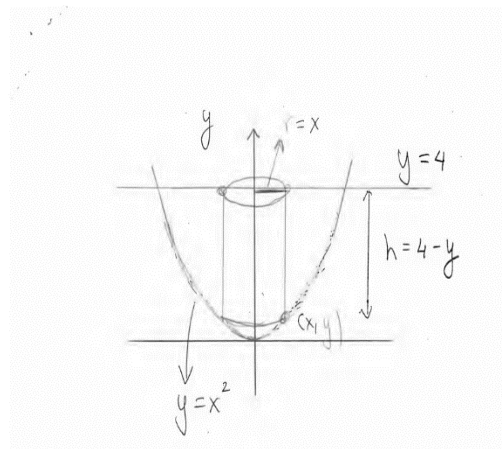
1. You are given the region bounded below by the function $y = x^2$ bounded above by y = 4. This region is rotated about the y axis. Find the volume by the method of cylindrical shells.

   STATE YOUR VALUES:

   $h = 4 - y$ $\qquad$ $r = x$ $\qquad$ $dt = dx$ $\qquad$ $dV = 2\pi r\, h\, dt$

$$V = \int_{t1}^{t2} 2\pi\, r\, h\, dt$$

DRAW A DIAGRAM TO HELP VISUALIZE!



The bounds on the integral are x = 0 and x = 2

$$V = \int_{x=0}^{x=2} 2\pi\, x\, (4 - y)\, dx$$

Replace y:

$$V = \int_{x=0}^{x=2} 2\pi\, x\, (4 - x^2)\, dx$$

$$V = 2\pi \int_{x=0}^{x=2} (4x - x^3)\, dx$$

$$V = 2\pi \left(2x^2 - \frac{x^4}{4}\right)\Bigg|_0^2$$

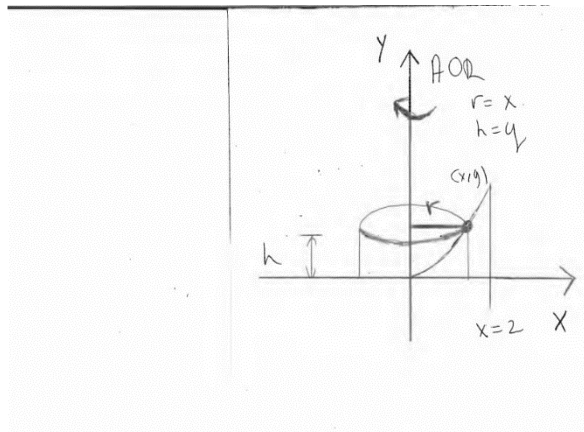
$$V = 2\pi \left[\left(8 - \frac{16}{4}\right) - (0)\right] = 8\,\pi$$

2. You are given the region bounded by the x axis, the function $y = x^2$ and the vertical line x = 2. The region is rotated about the y axis. Find the volume of the solid generated using cylindrical shells.

STATE YOUR VALUES

$r = x \qquad h = y \qquad dt = dx \qquad\qquad dV = 2\pi r\, h\, dt$

$$V = \int_{t1}^{t2} 2\pi r\, h\, dt$$



DRAW A DIAGRAM TO VISULAZE THE REGION OF ROTATION!

The bounds on the integral are x = 0 to x = 2.

$$V = 2\pi \int_{x=0}^{x=2} x\ y\ dx$$

$$V = 2\pi \int_{x=0}^{x=2} x\ x^2\ dx$$

$$V = 2\pi \int_{x=0}^{x=2} x^3\ dx$$

$$V = 2\pi \frac{x^4}{4}\Bigg|_0^2 = 8\pi$$

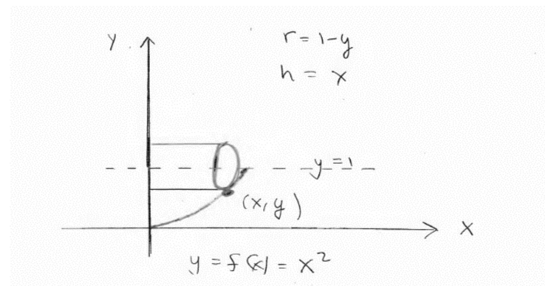
3. You are given the region bounded by the function $y = x^2$, the y axis and the horizontal line $y = 1$. The region is rotated about the line y = 1. Find the volume of the solid generated using the method of cylindrical shells.

STATE YOUR VALUES

$r = 1 - y$ $\quad h = x$ $\quad dt = dy$ $\quad dV = 2\pi r\, h\, dt$

$$V = \int_{t1}^{t2} 2\pi r\, h\, dt$$

$$V = \int_{y=0}^{y=1} 2\pi\,(1-y)\,x\,dy$$



DRAW A DIAGRAM TO HELP YOU!

$$V = 2\pi \int_{y=0}^{y=1} (1-y)\sqrt{y}\,dy$$

$$V = 2\pi \int_{y=0}^{y=1} \left(y^{1/2} - y^{3/2}\right)\,dy$$

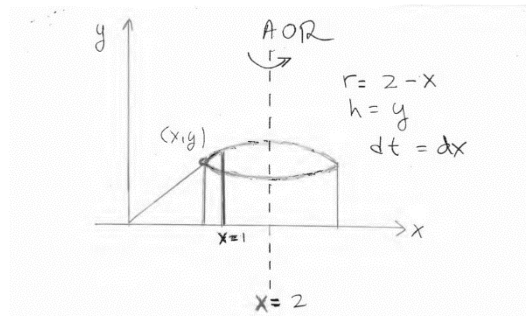
$$V = 2\pi \left(\frac{2}{3}\,y^{3/2} - \frac{2}{5}\,y^{5/2}\right)\Big|_0^1$$

$$V = 2\pi\left[\left(\frac{2}{3} - \frac{2}{5}\right) - (0)\right] = \frac{8\pi}{15}$$

4. You are given the region bounded by the x axis, the line y = x and the vertical line x = 1. The region is a right triangle. The region is rotated about the line x = 2. Find the volume of the solid generated using the method of cylindrical shells.

STATE YOUR VALUES: $r = 2 - x$ $\quad h = y$ $\quad dt = dx$ $\quad dV = 2\pi r\, h\, dt$

$$V = \int_{t1}^{t2} 2\pi r\, h\, dt$$



The bounds on the integral are x = 0 and x = 1:

$$V = \int_{x=0}^{x=1} 2\pi\,(2-x)\, y\;\, dx$$

$$V = \int_{x=0}^{x=1} 2\pi\,(2-x)\, x\;\, dx$$

$$V = 2\pi \int_{x=0}^{x=1} (2x - x^2)\;\, dx$$

$$V = 2\pi\left(x^2 - \frac{x^3}{3}\right)\Bigg|_0^1$$

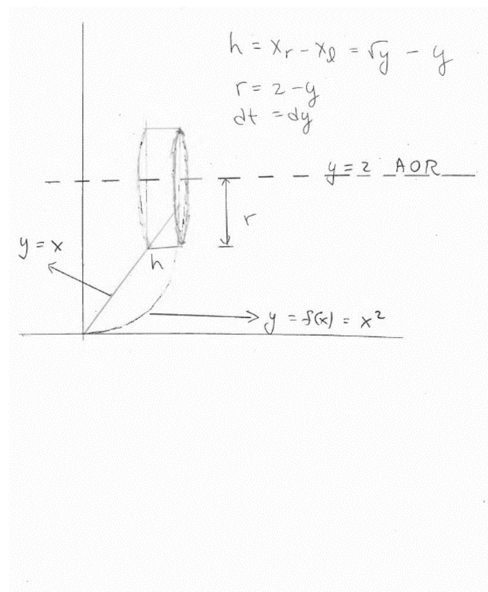
$$V = 2\pi\left[\left(1 - \frac{1}{3}\right) - (0)\right]$$

$$V = \frac{4\pi}{3}$$

5. You are given the functions $y = x$ and $y = x^2$. The region between them is rotated about the horizontal line y = 2. Find the volume using the method of cylindrical shells.

Right function	Left function
$y = x^2$	$y = x$
$x_r = \sqrt{y}$	$x_l = y$

$r = 2 - y \qquad h = x_r - x_l = \sqrt{y} - y \qquad dt = dy$



$$V = \int_{t1}^{t2} 2\pi r\, h\, dt$$

$$V = 2\pi \int_{y=0}^{y=1} (2-y)\left(\sqrt{y} - y\right) dy$$

$$V = 2\pi \int_{y=0}^{y=1} \left(2\sqrt{y} - 2y - y^{\frac{3}{2}} + y^2\right) dy$$

$$V = 2\pi \left(\frac{4}{3} y^{\frac{2}{3}} - y^2 - \frac{2}{5} y^{\frac{5}{2}} + \frac{y^3}{3}\right)\Bigg|_0^1$$

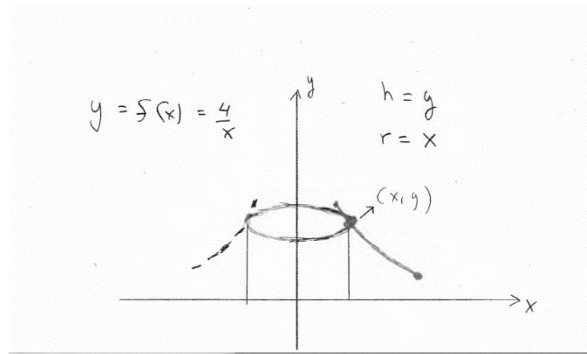
$$V = 2\pi \left(\frac{4}{3} - 1 - \frac{2}{5} + \frac{1}{3}\right)$$

$$V = \frac{8\pi}{15}$$

6. You are given the region bounded by $y = \frac{4}{x}$, x = 1, x = 4 and the x axis. This region is rotated about the y axis. Find the volume using cylindrical shells.

$h = y \qquad r = x \qquad dt = dx \qquad\qquad dV = 2\pi r\, h\, dt$

$$V = \int_{t1}^{t2} 2\pi r\, h\, dt$$



$$V = \int_{x=1}^{x=4} 2\pi\, x\, y\, dx$$

$$V = \int_{x=1}^{x=4} 2\pi\, x\, \frac{4}{x}\, dx$$

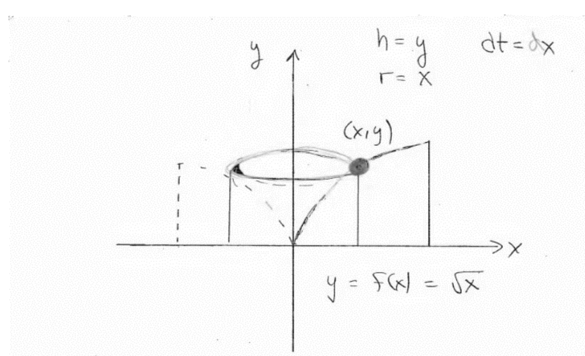
$$V = 8\pi \int_{x=1}^{x=4} dx$$

$$V = 24\,\pi$$

7. You are given the region bounded by $y = \sqrt{x}$, the x axis and the line x = 4. The region is rotated about the y axis. Find the volume using shells.

$h = y \qquad r = x \qquad dt = dx \qquad\qquad dV = 2\pi\, r\, h\, dt$

$$V = \int_{t1}^{t2} 2\pi r\, h\, dt$$



$$V = \int_{x=0}^{x=4} 2\pi\, x\;\; y\;\; dx$$

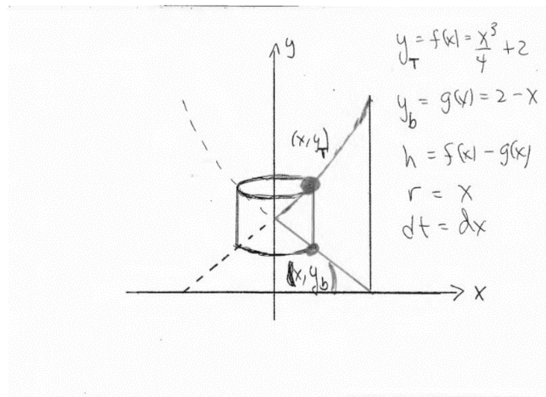
$$V = 2\pi \int_{x=0}^{x=4} x^{3/2}\;\; dx$$

$$V = \frac{4}{5}\,\pi\, x^{\frac{5}{2}}\Big|_0^4 = \frac{128\pi}{5}$$

8. You are given a region – it is bounded above by the curve $y = \frac{1}{4}\, x^3 + 2$. It is bounded below by the line $y = 2 - x$. And it is bounded on the right by the vertical line x = 2. The region is rotated about the y axis. Find the volume using shells.

$r = x \qquad dt = dx \qquad h = y_{hi} - y_{low} \; = \; \left(\frac{x^3}{4} + 2\right) - (2 - x)$

$$V = \int_{t1}^{t2} 2\pi r\, h\, dt$$



$$V = \int_{x\,=\,0}^{x\,=\,2} 2\pi\; x\; \left[\left(\frac{x^3}{4} + 2\right) - (2 - x)\right] dx$$

$$V = \int_{x\,=\,0}^{x\,=\,2} 2\pi\; x\; \left(\frac{x^3}{4} + x\right)\; dx$$

$$V = 2\pi \int_{x\,=\,0}^{x\,=\,2} \left(\frac{x^4}{4} + x^2\right)\; dx$$

$$V \;=\; 2\pi \left(\frac{x^5}{20} + \frac{x^3}{3}\right)\Bigg|_0^2$$

$$V \;=\; 2\pi \left(\frac{32}{20} + \frac{8}{3}\right) \;=\; \frac{128\,\pi}{15}$$

9. You are given the region bounded by $x = \sqrt{2y} + 1$, the x axis, the y axis and by the horizontal line y = 2. The region is rotated about the x axis. Find the volume.

$h = x \qquad r = y \qquad dt = dy$

$$V = \int_{t1}^{t2} 2\pi r\, h\, dt$$

$$V = \int_{y=0}^{y=2} 2\pi y\, x\, dy$$

$$V = \int_{y=0}^{y=2} 2\pi y \left(\sqrt{2y} + 1 \right) dy$$

$$V = 2\pi \int_{y=0}^{y=2} \left(\sqrt{2}\, y^{3/2} + y \right) dy$$

$$V = 2\pi \left(\frac{2\sqrt{2}\ y^{5/2}}{5} + \frac{y^2}{2} \right) \Bigg|_0^2$$

$$V = 2\pi \left(\frac{16}{5} + 2 \right) = \frac{52\pi}{5}$$

10. You are given the region bounded above by the function $y = x - x^2$ and below by the x axis. The region is rotated about the line x = 5. Find the volume of the solid.
$r = 5 - x$ $\qquad h = y$ $\qquad dt = dx$

$$V = \int_{t1}^{t2} 2\pi r\, h\, dt$$

$$V = \int_{x\,=\,0}^{x\,=\,1} 2\pi\,(5 - x)\, y\; dx$$

$$V = \int_{x\,=\,0}^{x\,=\,1} 2\pi\,(5 - x)\,(x - x^2)\; dx$$

$$V = 2\pi \int_{x\,=\,0}^{x\,=\,1} (x^3 - 6x^2 + 5x)\; dx$$

$$V = 2\pi \left(x^4 - 2x^3 + \frac{5x^2}{2}\right)\Bigg|_0^1$$

$$V = 2\pi\left(1 - 2 + \frac{5}{2}\right) = 3\pi$$

11. The region bounded by $x = y^2$, the line y = 2 and the y axis. Region is rotated about the x axis. Find the volume. $h = x$ $\qquad r = y$ $\qquad dt = dy$

$$V = \int_{t1}^{t2} 2\pi r\, h\, dt$$

$$V = \int_{y\,=\,0}^{y\,=\,2} 2\pi\;\; y\;\, x\;\, dy$$

$$V = \int_{y\,=\,0}^{y\,=\,2} 2\pi\;\; y\;\, y^2\;\, dy$$

$$V = 2\pi \int_{y\,=\,0}^{y\,=\,2} y^3\;\, dy$$

$$V = 2\pi \frac{y^4}{4}\Bigg|_0^2 = \frac{2\pi\,(16)}{4} = 8\,\pi$$