INTEGRATING POWERS OF TANGENT BY PYTHAGOREAN IDENTITY AND U SUBSTITUTIONS – HANDOUT

$\int tan\,x \;\; dx \;=\; -\,ln|cos\,x| \;+\; c \;\;=\; ln|sec\,x| \;\;+\;\; c$

$\int \; tan^2\,x\;\, dx \;\;=\;\; \int \;(sec^2\,x - 1)dx \;\;=\;\; tan\,x \;+\;\; x \;+\;\; c$

Use $1 + \tan^2 x = \sec^2 x$ and $d(\tan x) = \sec^2 x\, dx$

$\int tan^3\,x\;\, dx \;\;=\;\; \int tan\,x\;\, tan^2\,x \;\; dx$

$\int tan^3\,x\;\, dx \;\;=\;\; \int tan\,x \;\; (sec^2\,x - 1)dx$

$\int tan^3\,x\;\, dx \;\;=\;\; \int tan\,x\;\, sec^2\,x\;\, dx \;-\; \int\; tan\,x\;\, dx$ let $u = \tan x$ for first integral

$\int tan^3\,x\;\, dx \;=\frac{1}{2}tan^2\,x \;-\, ln|sec\,x| \;+\;\; c$

ALTERNATE: use $1 + \tan^2 x = \sec^2 x$ and $d(\sec x) = \sec x \tan x\, dx$

$\int tan^3\,x\;\, dx \;\;=\;\; \int tan\;\,x\;\, tan^2\,x \;\; dx$

$\int tan^3\,x\;\, dx \;\;=\;\; \int tan\,x \;\; (sec^2\,x - 1)dx$

$\int tan^3\,x\;\, dx \;\;=\;\; \int tan\,x\;\, sec^2\,x\;\, dx \;-\; \int\; tan\,x\;\, dx$

$\int tan^3\,x\;\, dx \;\;=\;\; \int\;\; sec\;\,x \;\;\; (sec\,x\;\, tan\,x) \;\; dx \;-\; \int\; tan\,x\;\, dx$ let $u = \sec x$ for first integral

$\int tan^3\,x\;\, dx \;\;=\; \frac{1}{2}sec^2\,x \;-\, ln|sec\,x| \;+\;\; c$

Repeatedly use $1 + \tan^2 x = \sec^2 x$ along with $d(\tan x) = \sec^2 x\, dx$

$\int tan^4 x\ dx = \int tan^2 x\ tan^2 x\ dx$

$\int tan^4 x\ dx = \int tan^2 x\ (sec^2 x - 1)\ dx$

$\int tan^4 x\ dx = \int tan^2 x\ sec^2 x\ dx - \int tan^2 x\ dx$ let u = tan x for both integrals

$\int tan^4 x\ dx = \frac{1}{3} tan^3 x - \int tan^2 x\ dx$

$\int tan^4 x\ dx = \frac{1}{3} tan^3 x - \int (sec^2 x - 1)\ dx$

$\int tan^4 x\ dx = \frac{1}{3} tan^3 x - tan\, x + x + c$

Use $1 + \tan^2 x = \sec^2 x$ along with $d(\sec x) = \sec x \tan x\, dx$

$\int tan^5 x\ dx = \int tan\, x\ tan^4 x\ dx$

$\int tan^5 x\ dx = \int tan\, x\ (sec^2 x - 1)^2\ dx$

$\int tan^5 x\ dx = \int tan\, x\ (sec^4 x - 2\, sec^2 x + 1)\ dx$

$\int tan^5 x\ dx = \int sec^4 x\, tan\, x\ dx - 2 \int sec^2 x\, tan\, x\ dx + \int tan\, x\ dx$

$\int tan^5 x\ dx = \int sec^3 x\ sec\, x\ tan\, x\ dx - 2 \int sec\ x\ sec\, x\ tan\, x\ dx + \int tan\, x\ dx$

Let u = sec x for first two integrals

$\int tan^5 x\ dx = \frac{1}{4} sec^4 x - sec^2 x + ln|sec\, x| + c$

Repeatedly use $1 + \tan^2 x = \sec^2 x$ along with $d(\tan x) = \sec^2 x\, dx$

$\int tan^6 x\ dx = \int tan^4 x\ tan^2 x\ dx$

$\int tan^6 x\ dx = \int tan^4 x\ (sec^2 x - 1)\ dx$

$\int tan^6 x\ dx = \int (\, tan^4 x\ sec^2 x - tan^4 x)\ dx$

$\int tan^6 x\ dx = \int tan^4 x\ sec^2 x\ dx - \int tan^4 x\ dx$ let u = tan x for first integral

$\int tan^6 x\ dx = \frac{1}{5} tan^5 x - \int tan^4 x\ dx$

$\int tan^6 x\ dx = \frac{1}{5} tan^5 x - \int tan^2 x\ tan^2 x\ dx$

$$\int tan^6 x\ dx = \frac{1}{5}tan^5 x - \int tan^2 x\ (sec^2 x - 1)\, dx$$

$$\int tan^6 x\ dx = \frac{1}{5}tan^5 x - \int (\,tan^2 x\ sec^2 x - tan^2 x\,)\, dx$$

$$\int tan^6 x\ dx = \frac{1}{5}tan^5 x - \int tan^2 x\ sec^2 x\ dx + \int tan^2 x\ dx$$ let u = tan x for middle term

$$\int tan^6 x\ dx = \frac{1}{5}tan^5 x - \frac{1}{3}tan^3 x + tan\, x - x + c$$

Use 1 + $\tan^2$x = $\sec^2$x and also use d(sec x) = sec x tan x dx

$$\int tan^7 x\ \ dx = \int tan^6 x\ tan\, x\ \ dx$$

$$\int tan^7 x\ \ dx = \int (sec^2 x - 1)^3\ tan\, x\ \ dx$$

$$\int tan^7 x\ \ dx = \int (sec^6 x - 3\,sec^4 x + 3\,sec^2 x - 1)\ \ tan\, x\ \ dx$$

$$\int tan^7 x\ \ dx = \int (sec^6 x - 3\,sec^4 x + 3\,sec^2 x\,)\ \ tan\, x\ \ dx - \int tan\, x\ dx$$

$$\int tan^7 x\ \ dx = \int (sec^5 x - 3\,sec^3 x + 3\,sec^1 x\,)\ sec\, x\ tan\, x\ \ dx - \int tan\, x\ dx$$

Let u = sec x for first integral

$$\int tan^7 x\ \ dx = \frac{1}{6}sec^6 x - \frac{3}{4}\,sec^4 x + \frac{3}{2}sec^2 x - ln|sec\, x| + c$$