VOLUME BY THE METHOD OF DISKS

THOMAS PAGE 230

1. You are given the function $y = \sqrt{x}$ from (0,0) to (4,2). It is rotated about the x axis. Use the method of disks to find the volume.

2. You are given the circle $x^2 + y^2 = R^2$. It is revolved about the x axis to generate a sphere. Find its volume.

3. Given the function $x + y = 2$ rotated about the x axis. X goes from 0 to 2. Find the volume.

4. You are given the function y = sin x as x goes from 0 to pi. It is rotated about the x axis. Find the volume of the solid.

5. You are given the region bounded by the function $y = x^2 - x$ and the line y = 0 (the x axis). Find the volume when the region is rotated about the x axis.

6. You are given the region bounded by the function $y = x^2$ and the line $y = 4$. The axis of rotation is y =4. Find the volume.

7. You are given the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. It is rotated about the x axis. Find the volume of the region generated (an ellipsoid).

8. You are given the region bounded by $y = x^2$ and the line y = 1. It is rotated about the y axis. Find the volume of the solid.

9. You are given the function $y = \ln x$ as x goes from 1 to e. The region between the function and the y axis is rotated about the y axis. Find the volume generated.

10. You are given the function $y = \sqrt{x}$ as x goes from 0 to 1. The region between the function and the y axis is rotated about the y axis. Find the volume generated.

11. You are given the function $y = \sqrt{x}$ as x goes from 0 to 1. The region between the function and the x axis is rotated about the line x = 1. Find the volume generated.

12. You are given the function $y = x^3$ as x goes from 0 to 1. The curve is rotated about the line y = 2. Find the volume generated by the region between $y = x^3$ and y = 2.

13. You are given the straight line $\frac{y}{h} + \frac{x}{b} = 1$. The region is in the first quadrant between the line and the coordinate axes. The region is rotated about the y axis. Find the volume. This is finding the volume of a cone.

14. Given the function $y = x^2$ as x goes from 0 to 1. It is rotated about the horizontal axis y = 3. Find the volume generated by the region in between the function and the axis of rotation.

DIFFICULT QUESTION

15. Given the function $y = e^{-x}$ as x goes from 0 to infinity. The curve is rotated about the x axis. Find the volume generated by the region in between the function and the x axis.

DIFFICULT QUESTION:

16. You are given the function y = sin x as x goes from 0 to pi/2. The region between the curve and the y axis is rotated (axis of rotation y axis). Find the volume generated.

DIFFICULT QUESTION

17. You are given the function y = cot x as x goes from 0 to pi/2. The region under the curve and the x axis is rotated about the x axis. Find the volume.

DIFFICULT QUESTION:

18. You are given the function $y = e^{x}$ as x goes from 0 to 1. The region between the function and the y axis is rotated about the y axis. Find the volume generated.

DIFFICULT QUESTION

19. You are given the function y = tan x where x is in the half open interval $\left[0, \frac{\pi}{2}\right)$. The curve is rotated about the x axis. Find the volume generated in the space between y = tan x and the x axis.

DIFFICULT QUESTION

20. You are given the function y = ln x where x is on the half open interval $(0,1]$. The function is rotated about the y axis. Find the volume generated between the function and the y axis.

DIFFICULT QUESTION

21. You are given the function $y = \frac{1}{x}$ where x is on the open interval $(0,1]$. The function is rotated about the y axis. Find the volume generated by the region in between the function and the y axis.

DIFFICULT QUESTION

22. You are given the function $y = \frac{1}{x}$ where x is on the open interval $[1, \infty)$. The region between the hyperbola and the x axis is rotated (about the x axis). Find the volume generated.

VOLUME BY THE METHOD OF DISKS

THOMAS PAGE 230

1. You are given the function $y = \sqrt{x}$ from (0,0) to (4,2). It is rotated about the x axis. Use the method of disks to find the volume.

   $r = y \qquad dt = dx$

$$V = \int_{t1}^{t2} A\,dt = \int_{t1}^{t2} \pi r^2\,dt$$

$$V = \int_{x=0}^{x=4} \pi\,y^2\,dx$$

$$V = \int_{x=0}^{x=4} \pi\left(\sqrt{x}\right)^2 dx$$

$$V = \int_{x=0}^{x=4} \pi\,x\,dx = \pi \frac{x^2}{2}\Big|_0^4$$

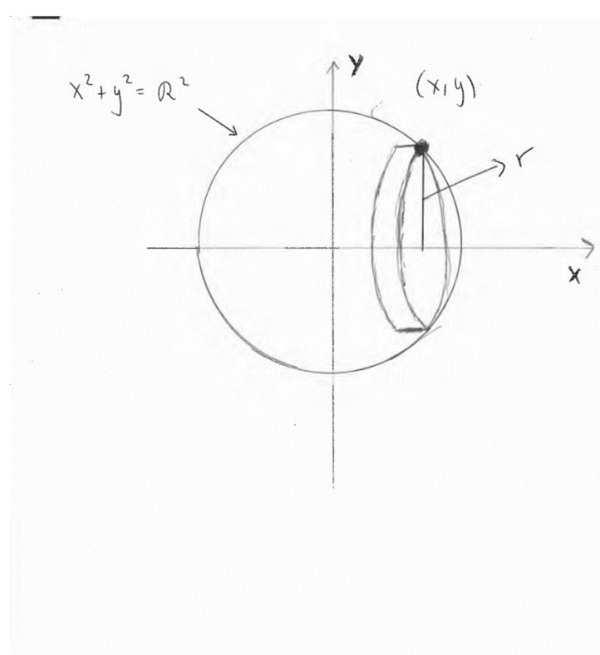
$$V = \frac{16\pi}{2} - 0 = 8\,\pi$$

2. You are given the circle $x^2 + y^2 = R^2$. It is revolved about the x axis to generate a sphere. Find its volume.

   $r = y \qquad dt = dx$

$$V = \int_{t1}^{t2} A\,dt = \int_{t1}^{t2} \pi r^2\,dt$$

$$V = \int_{x=-R}^{x=R} \pi\,y^2\,dx$$



$$V = \int_{x=-R}^{x=R} \pi\,(R^2 - x^2)\,dx$$

$$V = 2\int_{x=0}^{x=R} \pi\,(R^2 - x^2)\,dx$$

$$V = 2\pi\left(R^2x - \frac{x^3}{3}\right)\Bigg|_0^R$$

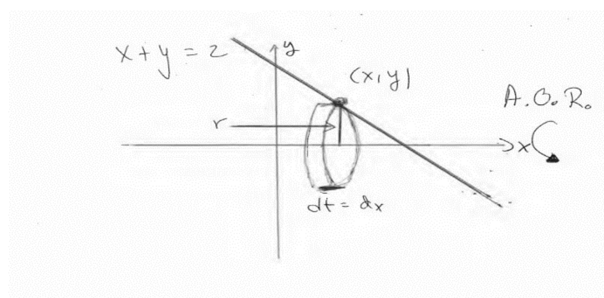
$$V = 2\pi\left(R^3 - \frac{R^3}{3}\right) - 0$$

$$V = 2\pi\left(\frac{2R^3}{3}\right) = \frac{4\pi R^3}{3}$$

3. Given the function $x + y = 2$ rotated about the x axis. X goes from 0 to 2. Find the volume.
   $r = y \qquad dt = dx$

$$V = \int_{t1}^{t2} A\,dt = \int_{t1}^{t2} \pi r^2\,dt$$

$$V = \int_{x=0}^{x=2} \pi\,y^2\,dx$$



$$V = \int_{x=0}^{x=2} \pi\,(2-x)^2\,dx$$

We can rewrite the integrand:

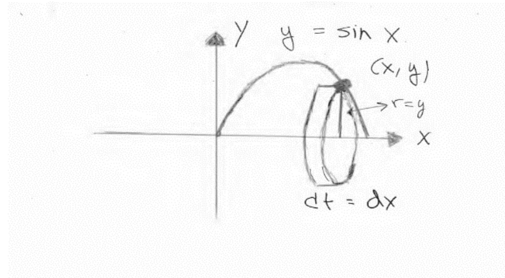
$$V = \int_{x=0}^{x=2} \pi\,(x-2)^2\,dx$$

$$V = \frac{\pi\,(x-2)^3}{3}\Bigg|_0^2$$

$$V = 0 - \frac{\pi\,(-2)^3}{3} = \frac{8\pi}{3}$$

4. You are given the function y = sin x as x goes from 0 to pi. It is rotated about the x axis. Find the volume of the solid.

$r = y \qquad dt = dx$



$$V = \int_{t1}^{t2} A\,dt = \int_{t1}^{t2} \pi r^2\,dt$$

$$V = \int_{x=0}^{x=\pi} \pi\, y^2\,dx$$

$$V = \int_{x=0}^{x=\pi} \pi \sin^2 x\,dx$$

Use the double angle formula cos (2x) = 1 – 2 $\sin^2$ x

So $\sin^2$ x = ½ - ½ cos (2x)

$$V = \pi \int_{x=0}^{x=\pi} \left(\frac{1}{2} - \frac{\cos 2x}{2}\right) dx$$

$$V = \pi \left(\frac{x}{2} - \frac{\sin 2x}{4}\right)\Big|_0^{\pi}$$

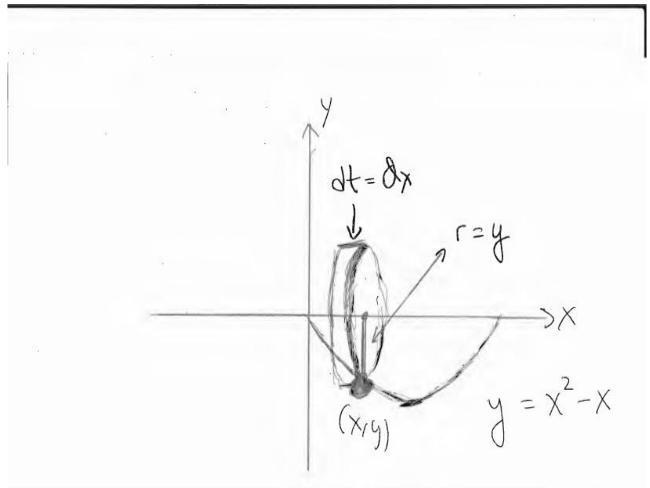
$$V = \pi \left(\frac{\pi}{2} - \frac{\sin 2\pi}{4}\right) - \pi\,(0)$$

$$V = \frac{\pi^2}{2}$$

5. You are given the region bounded by the function $y = x^2 - x$ and the line y = 0 (the x axis). Find the volume when the region is rotated about the x axis.
$r = y \qquad dt = dx$

$$V = \int_{t1}^{t2} A\,dt = \int_{t1}^{t2} \pi r^2\,dt$$

The bounds of the integral will be x=0 and x=1.



$$V = \int_{x=0}^{x=1} \pi\, y^2\,dx$$

$$V = \int_{x=0}^{x=1} \pi\,(x^2 - x)^2\,dx$$

$$V = \int_{x=0}^{x=1} \pi\,(x^2 - 2x^3 + x^4)\,dx$$

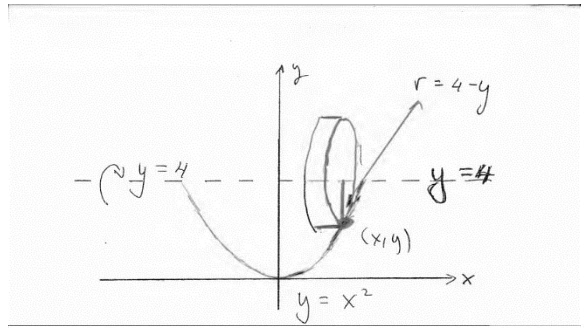
$$V = \pi\left(\frac{x^3}{3} - \frac{2\,x^4}{4} + \frac{x^5}{5}\right)\Bigg|_0^1$$

$$V = \pi\left(\frac{x^3}{3} - \frac{x^4}{2} + \frac{x^5}{5}\right)\Bigg|_0^1$$

$$V = \pi\left(\frac{1}{3} - \frac{1}{2} + \frac{1}{5}\right)$$

$$V = \frac{\pi}{30}$$

6. You are given the region bounded by the function $y = x^2$ and the line $y = 4$. The axis of rotation is y =4. Find the volume.



The bounds of integration will be x = -2 and x = +2.

$r = 4 - y \qquad dt = dx$

$$V = \int_{t1}^{t2} A\,dt = \int_{t1}^{t2} \pi r^2\,dt$$

$$V = \int_{x=-2}^{x=2} \pi\,(4 - y)^2\,dx$$

We can rewrite the integrand:

$$V = \int_{x=-2}^{x=2} \pi\,(y - 4)^2\,dx$$

$$V = \int_{x=-2}^{x=2} \pi\,(x^2 - 4)^2\,dx$$

$$V = \int_{x=-2}^{x=2} \pi\,(x^4 - 8x^2 + 16)\,dx$$

Use even symmetry:

$$V = 2\int_{x=0}^{x=2} \pi\,(x^4 - 8x^2 + 16)\,dx$$

$$V = 2\pi\left(\frac{x^5}{5} - \frac{8x^3}{3} + 16x\right)\Bigg|_0^2$$

$$V = 2\pi\left(\frac{32}{5} - \frac{64}{3} + 32\right)$$

$$V = \frac{512\pi}{15}$$

7. You are given the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} \;=\; 1$. It is rotated about the x axis. Find the volume of the region generated (an ellipsoid).

$r = y \qquad\qquad dt = dx$

$$V \;=\; \int_{t1}^{t2} A\,dt \;=\; \int_{t1}^{t2} \pi r^2\,dt$$

$$V \;=\; \int_{x=-a}^{x=a} \pi\, y^2\,dx$$

$$V \;=\; \pi b^2 \int_{x=-a}^{x=a} \left(1 - \frac{x^2}{a^2}\right)\,dx$$

$$V \;=\; \pi b^2 \int_{x=-a}^{x=a} \left(1 - \;\frac{x^2}{a^2}\;\right)\,dx$$

Use even symmetry:

$$V \;= 2\,\pi b^2 \int_{x=0}^{x=a} \left(1 - \;\frac{x^2}{a^2}\;\right)\,dx$$

$$V = \;\; 2\pi b^2 \left(x - \frac{x^3}{3a^2}\;\right)\Bigg|_0^a$$

$$V \;= 2\pi\, b^2\, \left(a - \frac{a}{3}\;\right)$$

$$V \;=\; 2\pi\, b^2 \left(\frac{2a}{3}\right)$$

$$V \;=\; \frac{4\pi\, a\, b^2}{3}$$

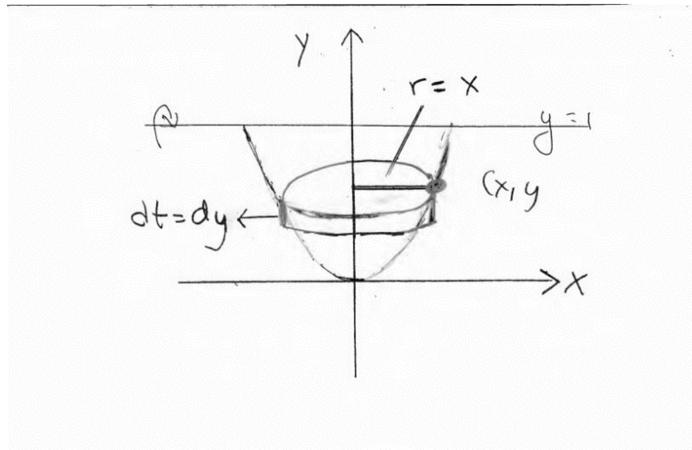
The general equation for an ellipsoid is $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ and its volume is given by

$$V = \frac{4\pi abc}{3}$$

8. You are given the region bounded by $y = x^2$ and the line y = 1. It is rotated about the y axis. Find the volume of the solid.

$r = x \qquad dt = dy$

$$V = \int_{t1}^{t2} A\,dt = \int_{t1}^{t2} \pi r^2\,dt$$



$$V = \int_{y=0}^{y=1} \pi\, x^2\, dy$$

$$V = \int_{y=0}^{y=1} \pi\, y\, dy$$

$$V = \left.\frac{\pi\, y^2}{2}\right|_0^1$$

$$V = \frac{\pi}{2}$$

9. You are given the function $y = \ln x$ as x goes from 1 to e. The region between the function and the y axis is rotated about the y axis. Find the volume generated.

$r = x \qquad dt = dy$

$$V = \int_{t1}^{t2} A\,dt = \int_{t1}^{t2} \pi r^2\,dt$$

$$V = \int_{y=0}^{y=1} \pi\, x^2\, dy$$

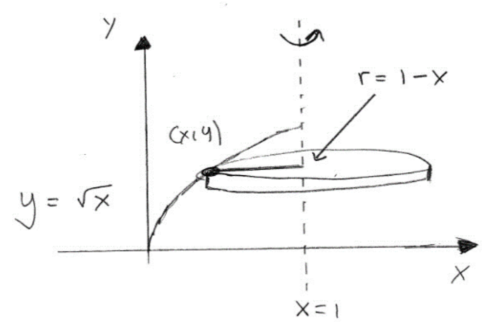
Since y = ln x this means that $x = e^y$.

$$V = \int_{y=0}^{y=1} \pi\, e^{2y}\, dy$$

$$V = \left.\frac{\pi\, e^{2y}}{2}\right|_0^1 = \frac{\pi e^2}{2} - \frac{\pi}{2}$$

10. You are given the function $y = \sqrt{x}$ as x goes from 0 to 1. The region between the function and the y axis is rotated about the y axis. Find the volume generated.

$r = x \qquad\qquad dt = dy$



$$V = \int_{t1}^{t2} A\,dt = \int_{t1}^{t2} \pi r^2\,dt$$

$$V = \int_{y=0}^{y=1} \pi\, x^2\, dy$$

$$V = \int_{y=0}^{y=1} \pi\, y\, dy = \left.\frac{\pi y^2}{2}\right|_0^1 = \frac{\pi}{2}$$

11. You are given the function $y = \sqrt{x}$ as x goes from 0 to 1. The region between the function and the x axis is rotated about the line x = 1. Find the volume generated.

$r = 1 - x \qquad\qquad dt = dy$

$$V = \int_{t1}^{t2} A\,dt = \int_{t1}^{t2} \pi r^2\,dt$$

$$V = \int_{y=0}^{y=1} \pi\,(1-x)^2\,dy$$

$$V = \int_{y=0}^{y=1} \pi\,(1-y^2)^2\,dy$$

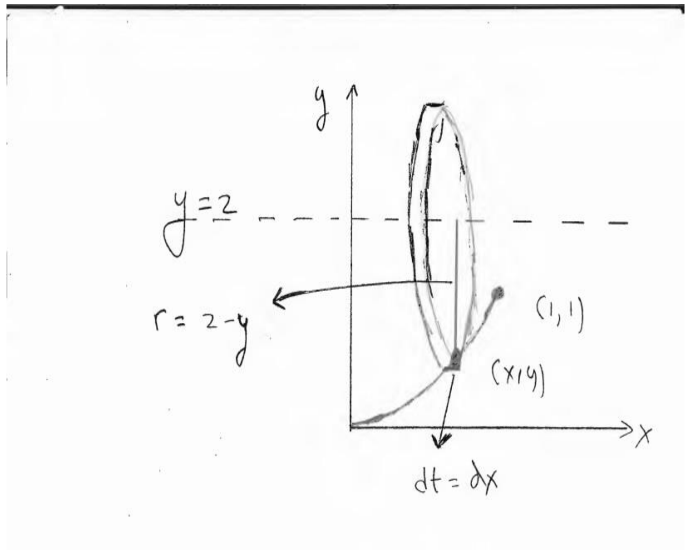
$$V = \int_{y=0}^{y=1} \pi\,(y^4 - 2y^2 + 1)\,dy$$

$$V = \pi\left.\left(\frac{y^5}{5} - \frac{2y^3}{3} + y\right)\right|_0^1$$

$$V = \pi\left(\frac{1}{5} - \frac{2}{3} + 1\right) - \pi(0) = \frac{8\pi}{15}$$

12. You are given the function y = x^3 as x goes from 0 to 1. The curve is rotated about the line y = 2. Find the volume generated by the region between y = x^3 and y = 2.

$r = 2 - y \qquad\qquad dt = dx$



$$V = \int_{t1}^{t2} A\,dt = \int_{t1}^{t2} \pi r^2\,dt$$

$$V = \int_{x=0}^{x=1} \pi\,(2 - y)^2\,dx$$

$$V = \int_{x=0}^{x=1} \pi\,(2 - x^3)^2\,dx$$

$$V = \int_{x=0}^{x=1} \pi\,(x^6 - 4x^3 + 4)\,dx$$

$$V = \pi\left(\frac{x^7}{7} - x^4 + 4x\right)\Bigg|_0^1 = \pi\left[\left(\frac{1}{7} - 1 + 4\right) - (0)\right]$$

$$V = \frac{22}{7}\,\pi$$

13. You are given the straight line $\frac{y}{h}+\frac{x}{b}=1$. The region is in the first quadrant between the line and the coordinate axes. The region is rotated about the y axis. Find the volume. This is finding the volume of a cone.

$r = x \qquad dt = dy$

$$V = \int_a^b \pi\, r^2\, dt$$

$$V = \int_{y=0}^{y=h} \pi\ x^2\, dy$$

$x = b\left(1-\frac{y}{h}\right)$

$$V = \pi\ b^2 \int_{y=0}^{y=h} \left(1-\frac{y}{h}\right)^2 dy$$

$$V = \pi\ b^2 \int_{y=0}^{y=h} \left(\frac{y}{h} - 1\right)^2 dy$$

$$V = \pi\, h\, b^2 \int_{y=0}^{y=h} \left(\frac{y}{h} - 1\right)^2 \frac{1}{h}\, dy$$

$$V = \frac{1}{3}\pi\ h\, b^2 \left(\frac{y}{h}-1\right)^3 \Big|_0^h$$

$$V = \frac{1}{3}\pi\, h\, b^2\left[\left(\frac{h}{h}-1\right)^3 - \left(\frac{0}{h}-1\right)^3\right]$$

$$V = \frac{1}{3}\pi\, h\, b^2[0 - -1] = \frac{1}{3}\pi\, h\, b^2$$

14. Given the function y = x^2 as x goes from 0 to 1. It is rotated about the horizontal axis y = 3. Find the volume generated by the region in between the function and the axis of rotation.

$r = 3 - y \qquad\qquad dt = dx$

$$V = \int_{t1}^{t2} A\,dt = \int_{t1}^{t2} \pi r^2\,dt$$

$$V = \int_{x=0}^{x=1} \pi\,(3 - y)^2\,dx$$

$$V = \int_{x=0}^{x=1} \pi\,(3 - x^2)^2\,dx$$

$$V = \int_{x=0}^{x=1} \pi\,(x^4 - 6x^2 + 9)\,dx$$

$$V = \pi\left(\frac{x^5}{5} - 2x^3 + 9x\right)\Bigg|_0^1$$

$$V = \pi\left(\frac{1}{5} - 2 + 9\right) = \frac{36\pi}{5}$$

DIFFICULT QUESTION

15. Given the function $y = e^{-x}$ as x goes from 0 to infinity. The curve is rotated about the x axis. Find the volume generated by the region in between the function and the x axis.

$r = y \qquad dt = dx$

$$V = \int_{t1}^{t2} A\,dt = \int_{t1}^{t2} \pi r^2\,dt$$

$$V = \int_{x=0}^{x=\infty} \pi\,y^2\,dx$$

$$V = \lim_{b \to \infty} \int_{x=0}^{x=b} \pi\,(e^{-x})^2\,dx$$

$$V = \lim_{b \to \infty} \int_{x=0}^{x=b} \pi\,e^{-2x}\,dx$$

$$V = \lim_{b \to \infty} -\left.\frac{\pi\,e^{-2x}}{2}\right|_0^b$$

$$V = \lim_{b \to \infty} \left.\frac{\pi\,e^{-2x}}{2}\right|_b^0$$

$$V = \frac{\pi}{2} \lim_{b \to \infty} \left(1 - e^{2b}\right) = \frac{\pi}{2}$$

DIFFICULT QUESTION:

16. You are given the function y = sin x as x goes from 0 to pi/2. The region between the curve and the y axis is rotated (axis of rotation y axis). Find the volume generated.

$r = x \qquad\qquad dt = dy$

$$V = \int_{t1}^{t2} A\,dt = \int_{t1}^{t2} \pi r^2\,dt$$

$$V = \int_{y=0}^{y=1} \pi\,x^2\,dy$$

$$V = \int_{y=0}^{y=1} \pi\,(arc\sin y)^2\,dy$$

Let $u = arc\sin y \qquad y = \sin u \qquad dy = \cos u\,\,du$

$$V = \int_{u=0}^{u=\pi/2} \pi\,u^2 \cos u\,\,du$$

Use extended method of parts:

$\int f\,g' du = fg - f' \int g + f'' \iint g - f''' \iiint g + \cdots$

$f = u^2 \qquad g' = \cos u$

$f' = 2u \qquad g = \sin u$

$f'' = 2 \qquad \int g = -\cos u$

$f''' = 0 \qquad \iint g = -\sin u$

$$V = \pi \int_{u=0}^{u=\pi/2} u^2 \cos u\,\,du = \pi\left(u^2 \sin u - 2u\,(-\cos u) + 2(-\sin u)\right)\Big|_0^{\pi/2}$$

$$V = \pi\left(\frac{\pi^2}{4} - 2\right) - \pi\,(0)$$

$$V = \frac{\pi^3}{4} - 2\pi$$

DIFFICULT QUESTION

17. You are given the function y = cot x as x goes from 0 to pi/2. The region under the curve and the x axis is rotated about the x axis. Find the volume.

$$V = \int_{t1}^{t2} A\,dt = \int_{t1}^{t2} \pi r^2\,dt$$

$$V = \int_{x=0}^{x=\pi/2} \pi\, y^2\,dx$$

$$V = \int_{x=0}^{x=\pi/2} \pi\, \cot^2 x\,dx$$

$$V = \lim_{a \to 0} \int_{x=a}^{x=\pi/2} \pi\, \cot^2 x\,dx$$

$$V = \pi \lim_{a \to 0} \int_{x=a}^{x=\pi/2} (\csc^2 x - 1)\,dx$$

$$V = \pi \lim_{a \to 0} (-\cot x - x)\Big|_{a}^{\pi/2}$$

$$V = \pi \lim_{a \to 0} (\cot x + x)\Big|_{\pi/2}^{a}$$

$$V = \pi \lim_{a \to 0} (\cot a + a) - \left(\cot\frac{\pi}{2} + \frac{\pi}{2}\right)$$

Cotangent goes to infinity at zero. The integral diverges.

DIFFICULT QUESTION:

18. You are given the function $y = e^x$ as x goes from 0 to 1. The region between the function and the y axis is rotated about the y axis. Find the volume generated.

$r = x \qquad\qquad dt = dy$

$$V = \int_{t1}^{t2} A\,dt = \int_{t1}^{t2} \pi r^2\,dt$$

$$V = \int_{y=1}^{y=e} \pi\,x^2\,dy$$

We change the variable of integration to dx – this is allowable since the function is monotonic.

Since y = e^x then dy = e^x dx. The bounds go back to zero and one:

$$V = \int_{x=0}^{x=1} \pi\,x^2\,e^x\,dx$$

Use the extended method of parts:

f = pi x^2 g' = e^x

f' = 2pi x g = e^x

f''= 2pi $\int g = e^x$

f''' = 0 $\iint g = e^x$

$$\int f\,g'du = fg - f'\int g + f''\iint g - f'''\iiint g + \cdots$$

$$V = \pi\,x^2\,e^x - 2\pi\,x\,e^x + 2\pi\,e^x|_0^1$$

$$V = (\pi e - 2\pi e + 2\pi e) - (0 - 0 + 2\pi)$$

$$V = \pi(e-1)$$

DIFFICULT QUESTION

19. You are given the function y = tan x where x is in the half open interval $\left[0, \frac{\pi}{2}\right)$. The curve is rotated about the x axis. Find the volume generated in the space between y = tan x and the x axis,

$r = y \qquad\qquad dt = dx$

$$V = \int_{t1}^{t2} A\,dt = \int_{t1}^{t2} \pi r^2\,dt$$

$$V = \int_{x=0}^{\frac{\pi}{2}} \pi\ y^2\,dx$$

$$V = \int_{x=0}^{\frac{\pi}{2}} \pi\ \tan^2 x\ dx$$

$$V = \lim_{b \to \frac{\pi}{2}} \int_{x=0}^{b} \pi\ \tan^2 x\ dx$$

$$V = \pi \lim_{b \to \frac{\pi}{2}} \int_{x=0}^{b} (\sec^2 x - 1)\,dx$$

$$V = \pi \lim_{b \to \frac{\pi}{2}} \tan x - x|_0^b$$

$$V = \pi \lim_{b \to \frac{\pi}{2}} \tan b - b = \infty$$

The integral diverges.

DIFFICULT QUESTION

20. You are given the function y = ln x where x is on the half open interval $(0,1]$. The function is rotated about the y axis. Find the volume generated between the function and the y axis.
$r = x$ $\qquad dt = dy$

$$V = \int_{t1}^{t2} A\,dt = \int_{t1}^{t2} \pi r^2\,dt$$

$$V = \int_{y=-\infty}^{y=0} \pi\ x^2\,dy$$

$$V = \lim_{a\to-\infty} \int_{y=a}^{y=0} \pi\ x^2\,dy$$

$$V = \lim_{a\to-\infty} \int_{y=a}^{y=0} \pi\ (e^y)^2\,dy$$

$$V = \lim_{a\to-\infty} \int_{y=a}^{y=0} \pi\ e^{2y}\ dy$$

$$V = \lim_{a\to-\infty} \frac{\pi}{2}\ e^{2y}\Big|_a^0$$

$$V = \frac{\pi}{2} \lim_{a\to-\infty} e^0 - e^{2a} = \frac{\pi}{2} \lim_{a\to-\infty} (1 - e^{2a}) = \frac{\pi}{2}\ (1-0) = \frac{\pi}{2}$$

DIFFICULT QUESTION

21. You are given the function $y = \frac{1}{x}$ where x is on the open interval $(0,1]$. The function is rotated about the y axis. Find the volume generated by the region in between the function and the y axis.

$r = \quad x \qquad\qquad dt = dy$

$$V = \int_{t1}^{t2} A\,dt = \int_{t1}^{t2} \pi r^2\,dt$$

$$V = \int_{y=1}^{y=\infty} \pi\, x^2\,dy$$

$$V = \lim_{b \to \infty} \int_{y=1}^{y=b} \pi\, \frac{1}{y^2}\,dy$$

$$V = \pi \lim_{b \to \infty} -\frac{1}{y}\Big|_1^b = \pi \lim_{b \to \infty} \left(-\frac{1}{b} - -\frac{1}{1}\right) = \pi$$

DIFFICULT QUESTION

22. You are given the function $y = \frac{1}{x}$ where x is on the open interval $[1, \infty)$. The region between the hyperbola and the x axis is rotated (about the x axis). Find the volume generated.

$r = y \qquad dt = dx$

$$V = \int_{t1}^{t2} A\,dt = \int_{t1}^{t2} \pi r^2\,dt$$

$$V = \int_{x=1}^{x=\infty} \pi\ y^2\,dx$$

$$V = \lim_{a \to \infty} \int_{x=1}^{x=a} \frac{\pi}{x^2}\,dx$$

$$V = \lim_{a \to \infty} \left.\frac{-\pi}{x}\right|_1^a$$

$$V = \lim_{a \to \infty} \left.\frac{\pi}{x}\right|_a^1$$

$$V = \lim_{a \to \infty} \pi - \frac{\pi}{a} = \pi$$