Exam 1: Math 142 Professor Friedberg

Student Name: Joseph Scarpa

Question 1

$\sin 0 = 0$ $\cos 0 = 1$

$\sin 30 = \frac{1}{2}$ $\cos 30 = \frac{\sqrt{3}}{2}$

$\sin 45 = \frac{\sqrt{2}}{2}$ $\cos 45 = \frac{\sqrt{2}}{2}$

$\sin 60 = \frac{\sqrt{3}}{2}$ $\cos 60 = \frac{1}{2}$

$\sin 90 = 1$ $\cos 90 = 0$

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Question 2

$$\frac{d(x^n)}{dx} = nx^{n-1}$$

$$\frac{d(\sin x)}{dx} = \cos x$$

$$\frac{d(e^x)}{dx} = e^x$$

$$\frac{d(\cos x)}{dx} = -\sin x$$

$$\frac{d(a^x)}{dx} = a^x \ln a$$

$$\frac{d(\tan x)}{dx} = \sec^2 x$$

$$\frac{d(\ln x)}{dx} = \frac{1}{x}$$

$$\frac{d(\sec x)}{dx} = \sec x \tan x$$

$$\frac{d(\csc x)}{dx} = -\csc x \cot x$$

$$\frac{d(\sin^{-1} x)}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d(\cos^{-1} x)}{dx} = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d(\tan^{-1} x)}{dx} = \frac{1}{1+x^2}$$

$$\frac{d(\sec^{-1} x)}{dx} = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d(\csc^{-1} x)}{dx} = -\frac{1}{\sqrt{x^2-1}}$$

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Question 3

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C \qquad \int e^x \, dx = e^x + C$$

$$\int a^x \, dx = \frac{a^x}{\ln a} + C$$

$$\int \frac{1}{x} \, dx = \int x^{-1} \, dx = \int \frac{dx}{x} = \ln|x| + C$$

$$\int \sin x \, dx = -\cos x + C \qquad \int \cos x \, dx = \sin x + C$$

$$\int \sec^2 x \, dx = \tan x + C \qquad \int \csc^2 x \, dx = -\cot x + C$$

$$\int \frac{1}{\sqrt{1-x^2}} \, dx = \int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$$

$$\int \frac{1}{1+x^2} \, dx = \int \frac{dx}{1+x^2} = \tan^{-1} x + C$$

$$\int \frac{1}{|x|\sqrt{x^2-1}} \, dx = \sec^{-1} x + C$$

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Question 4: Evaluate the following summations

$$\sum_{i=1}^{20} i = \frac{n(n+1)}{2} = \frac{(20)(21)}{2} = 210$$

$$\sum_{i=0}^{15} i^2 = \sum_{i=1}^{15} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{15(16)(31)}{6} = 5(8)(31) = 1240$$

$$\sum_{i=0}^{20} i^3 = \sum_{i=1}^{20} i^3 = \left[\frac{n(n+1)}{2}\right]^2 = (210)^2 = 44100$$

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Question 5: You are given the function $y = f(x) = 2x + 5$. You want to find an approximate value for the area under this function on the closed interval from 0 to 6, Using a regular partition with $n = 3$, find the left-handed sum, Clearly state Δx, the points of partition/evaluation, and the formula for the Riemann Sum.

$$\Delta x = \frac{b-a}{n} = \frac{6-0}{3} = \underline{2} \quad x_i = a + i\Delta x = 0 + 2i = \underline{2i}$$

$x_0 = a = 0$ $\qquad f(x_0) = f(0) = 5$

$x_1 = 2$ $\qquad f(x_1) = f(2) = 9$

$x_2 = 4$ $\qquad f(x_2) = f(4) = 13$

$x_3 = b = 6$ $\qquad f(x_3) = f(6) = 17$

$$\text{lhs} = \sum_{i=0}^{n-1} f(x_i)\Delta x = [f(x_0) + f(x_1) + f(x_2)]\Delta x$$

$$\text{lhs} = [5 + 9 + 13](2) = 54$$

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Question 6: Repeat previous question using right handed sum.

$rhs = \sum_{i=1}^{n} f(x_i)\Delta x = [f(x_1) + f(x_2) + f(x_3)]\Delta x$

$rhs = [9 + 13 + 17](2) = 78$

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Question 7: Repeat the previous question using midpoints for the Riemann Sum

$x_1^* = 1$ $f(x_1) = f(1) = 7$

$x_2^* = 3$ $f(x_2) = f(3) = 11$

$x_3^* = 5$ $f(x_3) = f(5) = 15$

$$mp = \sum_{i=1}^{n} f(x_i)\Delta x = [f(x_1^*) + f(x_2^*) + f(x_3^*)]\Delta x$$

$$mp = [7 + 11 + 15](2) = 66$$

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Question 8: You are given the function $y = f(x) = x^2$, the interval is x from 0 to 1. Using a regular partition and a Riemann sum, find the exact area under the curve. Do not use integrals nor the FTC.

$$\Delta x = \frac{b-a}{n} = \frac{1}{n} \qquad x_i = a + i\Delta x = \frac{i}{n}$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} x_i^2 \Delta x$$

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^2}{n^2}\frac{1}{n}$$

$$A = \lim_{n\to\infty} \frac{1}{n^3} \sum_{i=1}^{n} i^2 = \lim_{n\to\infty} \frac{1}{n^3} \frac{n(n+1)(2n+1)}{6}$$

$$A = \lim_{n\to\infty} \frac{(n+1)(2n+1)}{6n^2}$$

$$A = \lim_{n\to\infty} \frac{2n^2+3n+1}{6n^2} = \boxed{\frac{1}{3}}$$

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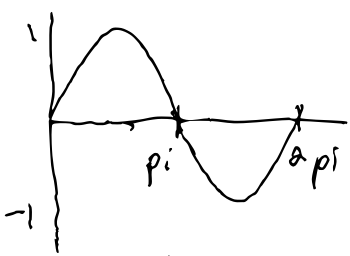
Question 9 : You are given the function $y = \sin x$ on the interval from 0 to 2pi. First draw the function on this interval, then find 2 distinct things. Find the definite integral for the function on this interval. Second, find the area between the function and x-axis on this interval



$$\int_0^{2\pi} \sin x\,dx = -\cos x \Big|_0^{2pi} = -\cos 2pi + \cos 0$$
$$-1+1=0$$

$$A_1 = \int_0^{pi} \sin x\,dx = -\cos x \Big|_0^{pi} = -\cos pi + \cos 0 = 1+1 = 2$$

$$A_2 = -\int_{pi}^{2pi} \sin x\,dx = \cos x \Big|_{pi}^{2pi} = \cos 2pi + \cos pi = 1+1 = 2$$

$$A = A_1 + A_2 = 2+2 = 4$$

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Question 10: You are given the function $y = f(x) = \frac{1}{1+x^2}$

Find the exact area under the function on the closed interval from $x=0$ to $x=1$. Use the FTC.

$$A = \int_0^1 \frac{1}{1+x^2}\,dx = \tan^{-1} x \Big|_0^1 = \tan^{-1} 1 - \tan^{-1} 0$$

$$A = \frac{\pi}{4} - 0 = \frac{\pi}{4}$$

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Question 11 : Evaluate the following :

$$\frac{d}{dx}\int_{2}^{x} \ln(t+2)\,dt = \ln(x+2)$$

$$\frac{d}{dx}\int_{x}^{1} \csc\,dt = -\csc x$$

$$\frac{d}{dx}\int_{5}^{x^2} \frac{1}{t+\sin t}\,dt = \frac{1}{x^2+\sin x^2}\cdot 2x$$

$$\frac{d}{dx}\int_{2x}^{3x} \frac{1}{4+\cos t} = \frac{1}{4+\cos 3x}\cdot 3 - \frac{1}{4+\cos 2x}\cdot 2$$

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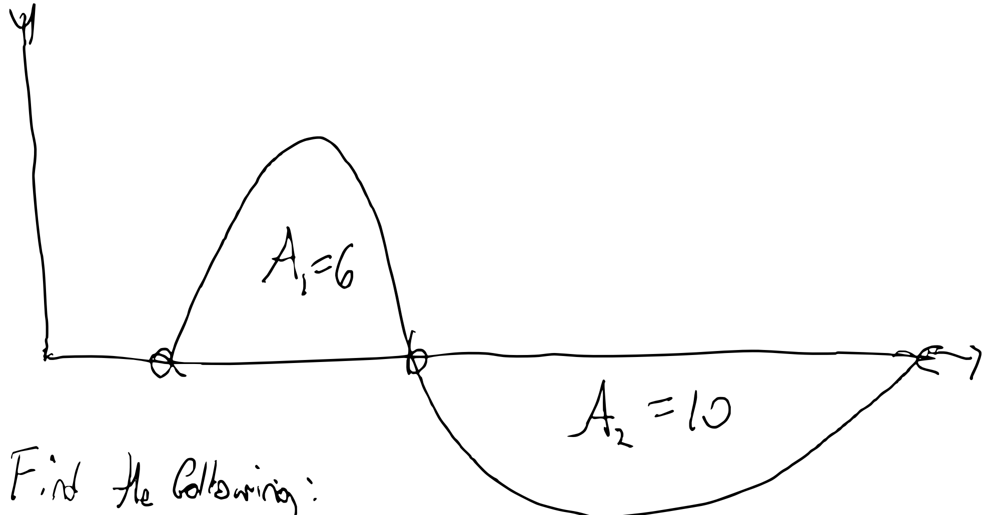
Question 12



Find the following:

$\int_a^b f(x)dx = 6$

$\int_b^c f(x)dx = -10$

$\int_a^c |f(x)|dx = 16$

$\int_c^b |f(x)|dx = -10$

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Question 13. State the equation for the MVT of integral calculus. Given the function $y=x^2$ on the interval from 0 to 1, find the average value of the function. Find the x-value where the function equals its average.

$$f(c) = f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$f_{avg} = \frac{1}{1-0}\int_0^1 x^2\,dx$$

$$f_{avg} = 1 \cdot \frac{x^3}{3}\Big|_0^1 = \frac{1}{3}$$

$$f(c) = c^2 = f_{avg} = \frac{1}{3}$$

$$c = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$