INTEGRATING POWERS OF SINE, COSINE, TANGENT AND SECANT BY DOUBLE ANGLE FORMULAS AND PYTHAGOREAN IDENTITIES

ODD POWERS OF SINE

$\int \sin x \ dx = -\cos x + c$

$\int \sin^3 x \ dx = \int \sin^2 x \sin x \ dx$ peel off a sine

$\int \sin^3 x \ dx = \int (1 - \cos^2 x) \sin x \ dx$ use Pythagorean identity on sine squared

$\int \sin^3 x \ dx = \int (\sin x - \cos^2 x \sin x) \ dx$

$\int \sin^3 x \ dx = \int (\sin x) \ dx - \int (\cos^2 x \sin x) \ dx$ let u = cos x

$\int \sin^3 x \ dx = -\cos x + \frac{1}{3} \cos^3 x + c$

$\int \sin^5 x \ dx = \int \sin^4 x \sin x \ dx$ peel off a sine

$\int \sin^5 x \ dx = \int (1 - \cos^2 x)^2 \ \sin x \ dx$ use Pythagorean identity on sine to the 4th

$\int \sin^5 x \ dx = \int (1 - 2\cos^2 x + \cos^4 x) \ \sin x \ dx$ expand the parentheses

Let u = cos x

$\int \sin^5 x \ dx = -\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + c$

$\int \sin^7 x \ dx = \int \sin^6 x \sin x \ dx$ peel off a sine

$\int \sin^7 x \ dx = \int (1 - \cos^2 x)^3 \ \sin x \ dx$ use Pythagorean identity on sine to the 6th

$\int \sin^7 x \ dx = \int (1 - 3\cos^2 x + 3\cos^4 x - \cos^6 x) \ \sin x \ dx$ expand the parentheses

Let u = cos x

$\int \sin^7 x \ dx = -\cos x + \cos^3 x - \frac{3}{5}\cos^5 x + \frac{1}{7}\cos^7 x + c$

EVEN POWERS OF SINE

$\int \ sin^2 x \ \ dx \ = \ \int \frac{1}{2} \ (1 - cos\ 2x\ )\ dx$ use double angle formula $cos\ 2x \ = \ 1 - \ 2\, sin^2 x$

$\int \ sin^2 x \ \ dx \ = \ \frac{x}{2} \ - \ \frac{sin(2x)}{4} \ + \ c$

$\int \ sin^4 x \ dx \ = \ \frac{1}{4} \ \int (1 - cos\ 2x)^2\ dx$ use double angle formula $cos\ 2x \ = \ 1 - \ 2\, sin^2 x$

$\int \ sin^4 x \ dx \ = \ \frac{1}{4} \ \int (1 - \ 2\, cos \ 2x \ + cos^2 2x) \ dx$ expand parentheses

$\int \ sin^4 x \ dx \ = \frac{1}{4}\ x \ - \frac{1}{4} sin\ 2x \ + \ \frac{1}{4} \ \int cos^2 2x \ \ dx$ integrate what you can

Let u = 2x

$\int \ sin^4 x \ dx \ = \frac{1}{4}\ x \ - \frac{1}{4} sin\ 2x \ + \ \frac{1}{4} \ \int cos^2 u \ \frac{du}{2}$

$\int \ sin^4 x \ dx \ = \frac{1}{4}\ x \ - \frac{1}{4} sin\ 2x \ + \ \frac{1}{8} \ \int cos^2 u \ du$

Use double angle formula $cos\ 2u = \ 2\, cos^2 u \ - \ 1$

$\int \ sin^4 x \ dx \ = \frac{1}{4}\ x \ - \frac{1}{4} sin\ 2x \ + \ \frac{1}{16} \ \int (1 \ + \ cos\ 2u\ )\ du$

$\int \ sin^4 x \ dx \ = \frac{1}{4}\ x \ - \frac{1}{4} sin\ 2x \ + \frac{1}{16}\ u \ + \frac{1}{32} sin\ 2u \ + \ c$

$\int \ sin^4 x \ dx \ = \frac{1}{4}\ x \ - \frac{1}{4} sin\ 2x \ + \frac{1}{16}\ (2x) \ + \frac{1}{32} sin\ 4x \ + \ c$

$\int \ sin^4 x \ dx \ = \frac{3}{8} \ x \ - \ \frac{7}{32} sin\ 2x \ + \ c$

$\int \ sin^6 x \ \ dx \ = \ \frac{1}{8} \int (1 - \ cos \ 2x)^3\ dx$ use double angle formula $cos\ 2x \ = \ 1 - \ 2\, sin^2 x$

$\int \ sin^6 x \ \ dx \ = \ \int (1 - \ 3 \ cos\ 2x + \ 3 \ cos^2 2x \ \ - cos^3 2x) \ dx$ expand parentheses

$\int \ sin^6 x \ \ dx \ = x - \frac{3}{2} sin\ 2x \ + \ \int \ 3 \ cos^2 2x \ \ dx \ - \ \int cos^3 2x \ \ dx$

Let u = 2x

$\int \ sin^6 x \ \ dx \ = x - \frac{3}{2} sin\, 2x \ + \frac{3}{2} \int \ \ cos^2 u \ \ du \ \ - \frac{1}{2} \int cos^3 u \ \ du$ integrate cosine squared

$\int \ sin^6 x \ \ dx \ = x - \frac{3}{2} sin\, 2x \ + \frac{3}{2} \left(\frac{u}{2} \ + \frac{sin\, 2u}{4}\right) - \frac{1}{2} \int cos^3 u \ \ du$

$\int \ sin^6 x \ \ dx \ = x - \frac{3}{2} sin\, 2x \ + \frac{3}{2} \left(\frac{2x}{2} \ + \frac{sin\, 2(2x)}{4}\right) - \frac{1}{2} \int cos^3 u \ \ du$

$\int \ sin^6 x \ \ dx \ = x - \frac{3}{2} sin\, 2x \ + \frac{3}{2} \left(x \ + \frac{sin\, 4x}{4}\right) - \frac{1}{2} \int cos^3 u \ \ du$ peel off a cosine

$\int \ sin^6 x \ \ dx \ = x - \frac{3}{2} sin\, 2x \ + \left(\frac{3}{2}\ x \ + \frac{3\ sin\, 4x}{8}\right) - \frac{1}{2} \int cos^2 u \ \ cos\, u \ \ du$ use Pythagorean relation on cosine squared

$\int \ sin^6 x \ \ dx \ = x - \frac{3}{2} sin\, 2x \ + \left(\frac{3}{2}\ x \ + \frac{3\ sin\, 4x}{8}\right) - \frac{1}{2} \int \ (1 - sin^2 u) \ \ cos\, u \ \ du$

Let t = sin u so dt = cos u du and integrate

$\int \ sin^6 x \ \ dx \ = x - \frac{3}{2} sin\, 2x \ + \left(\frac{3}{2}\ x \ + \frac{3\ sin\, 4x}{8}\right) - \frac{1}{2} sin\, u \ + \ \frac{1}{2} \frac{sin^3 u}{3} \ + \ c$

Recall u = 2x

$\int \ sin^6 x \ \ dx \ = x - \frac{3}{2} sin\, 2x \ + \left(\frac{3}{2}\ x \ + \frac{3\ sin\, 4x}{8}\right) - \frac{1}{2} sin\, 2x \ + \ \frac{1}{2} \frac{sin^3 2x}{3} \ + \ c$

Even powers of sine are more difficult. Odd powers of sine are easier.

Integrating powers of cosine are identical in behavior to this. Odd powers of cosine are easy and even powers of cosine are hard. Integrating powers of cosine make use of the exact same formulas – the same Pythagorean identity and the same double angle formulas.