TAYLOR SERIES BASIC FORMULAS

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \cdots$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} \cdots$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

$$\ln|1+x| = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} \cdots$$

$$\ln|1+x| = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \cdots$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$$

TAYLOR SERIES FOR THE EXPONENTIAL FUNCTION

$f(x) = e^x$

$f(x) = f(0) + f'(0)\ x + \frac{f''(0)}{2!}\ x^2 + \frac{f'''(0)}{3!}\ x^3 + \cdots$

$f(x) = e^x$	$f(0) = 1$
$f'(x) = e^x$	$f'(0) = 1$
$f''(x) = e^x$	$f''(0) = 1$
$f'''(x) = e^x$	$f'''(0) = 1$

$f(x) = 1 + 1 \cdot x + \frac{1}{2!}\ x^2 + \frac{1}{3!}\ x^3 + \cdots$

TAYLOR SERIES FOR SINE FUNCTION

$f(x) = \sin x$

$f(x) = f(0) + f'(0)\ x + \frac{f''(0)}{2!}\ x^2 + \frac{f'''(0)}{3!}\ x^3 + \frac{f^{iv}(0)}{4!}\ x^4 + \frac{f^{v}(0)}{5!}\ x^5 \ldots$

$f(x) = \sin x$	$f(0) = 0$
$f'(x) = \cos x$	$f'(0) = 1$
$f''(x) = -\sin x$	$f''(0) = 0$
$f'''(x) = -\cos x$	$f'''(0) = -1$
$f^{iv}(x) = \sin x$	$f^{iv}(0) = 0$

And it repeats after this. All even terms are zero. Odd terms alternate.

$f(x) = 0 + 1\ x + \frac{0}{2!}\ x^2 + \frac{-1}{3!}\ x^3 + \frac{0}{4!}\ x^4 + \frac{1}{5!}\ x^5$

$f(x) = x - \frac{1}{3!}\ x^3 + \frac{1}{5!}\ x^5 \ldots$

TAYLOR SERIES FOR COSINE

$f(x) = \cos x$

$$f(x) = f(0) + f'(0)\ x + \frac{f''(0)}{2!}\ x^2 + \frac{f'''(0)}{3!}\ x^3 + \frac{f^{iv}(0)}{4!}\ x^4 + \frac{f^{v}(0)}{5!}\ x^5 \ldots$$

$f(x) = \cos x$	$f(0) = 1$
$f'(x) = -\sin x$	$f'(0) = 0$
$f''(x) = -\cos x$	$f''(0) = -1$
$f'''(x) = \sin x$	$f'''(0) = 0$
$f^{iv}(x) = \cos x$	$f^{iv}(0) = 1$
$f^{v}(x) = -\sin x$	$f^{v}(0) = 0$
$f^{vi}(x) = -\cos x$	$f^{vi}(0) = -1$

$$f(x) = f(0) + f'(0)\ x + \frac{f''(0)}{2!}\ x^2 + \frac{f'''(0)}{3!}\ x^3 + \frac{f^{iv}(0)}{4!}\ x^4 + \frac{f^{v}(0)}{5!}\ x^5$$

$$f(x) = 1 + 0\ x + \frac{-1}{2!}\ x^2 + 0\ x^3 + \frac{1}{4!}\ x^4 + 0\ x^5$$

$$f(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \ldots$$

TAYLOR SERIES FOR NATURAL LOGARITHM

$$f(x) = \ln|1+x|$$

$$f(x) = f(0) + f'(0)\,x + \frac{f''(0)}{2!}\,x^2 + \frac{f'''(0)}{3!}\,x^3 + \frac{f^{iv}(0)}{4!}\,x^4 + \frac{f^{v}(0)}{5!}\,x^5 \ldots$$

$f(x) = \ln\|1+x\|$	$f(0) = 0$
$f'(x) = \frac{1}{1+x}$	$f'(0) = 1$
$f''(x) = -\frac{1}{(1+x)^2}$	$f''(0) = -1$
$f'''(x) = \frac{2}{(1+x)^3}$	$f'''(0) = 2$
$f^{iv}(x) = -\frac{3!}{(1+x)^4}$	$f^{iv}(0) = -3!$
$f^{v}(x) = \frac{4!}{(1+x)^5}$	$f^{v}(0) = 4!$
$f^{vi}(x) = -\frac{5!}{(1+x)^6}$	$f^{vi}(0) = -5!$

$$f(x) = f(0) + f'(0)\,x + \frac{f''(0)}{2!}\,x^2 + \frac{f'''(0)}{3!}\,x^3 + \frac{f^{iv}(0)}{4!}\,x^4 + \frac{f^{v}(0)}{5!}\,x^5 \ldots$$

$$f(x) = 0 + 1\,x + \frac{-1}{2!}\,x^2 + \frac{2}{3!}\,x^3 - \frac{3!}{4!}\,x^4 + \frac{4!}{5!}\,x^5 \ldots$$

$$f(x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5}$$

TAYLOR SERIES FOR INVERSE TANGENT

$f(x) = \tan^{-1} x$

$f(x) = f(0) + f'(0)\ x + \frac{f''(0)}{2!}\ x^2 + \frac{f'''(0)}{3!}\ x^3 + \frac{f^{iv}(0)}{4!} x^4 + \frac{f^{v}(0)}{5!}\ x^5 \ldots$

$f(x) = \tan^{-1} x$	$f(0) = 0$
$f'(x) = \frac{1}{1+x^2}$	$f'(0) = 1$
$f''(x) = -\frac{2x}{(1+x^2)^2}$	$f''(0) = 0$
$f'''(x) = \frac{6x^2 - 2}{(1 + x^2)^3}$	$f'''(0) = -2$
$f^{iv}(x) = \frac{-24x^3 + 24x}{(1+x^2)^4}$	$f^{iv}(0) = 0$
$f^{v} = \frac{120x^4 - 48x^2 - 192x + 24}{(1+x^2)^5}$	$f^{v}(0) = 24$

$f(x) = f(0) + f'(0)\ x + \frac{f''(0)}{2!}\ x^2 + \frac{f'''(0)}{3!}\ x^3 + \frac{f^{iv}(0)}{4!} x^4 + \frac{f^{v}(0)}{5!}\ x^5 \ldots$

$f(x) = 0 + 1\ x + 0\,x^2 + \frac{-2}{3!}\ x^3 + 0\,x^4 + \frac{24}{5!}\ x^5 \ldots$

$f(x) = x - \frac{1}{3} x^3 + \frac{1}{5}\ x^5 \ldots$

This is the answer to the first three terms.

I have a few things to say on this last problem. The derivatives are tedious and difficult. They do not definitely establish the pattern for f’’(0) but we do see the pattern start to form. The original method for arctangent (integrating a geometric series) is much better and much easier to get the Taylor series. By the original method, there is no doubt to the pattern established. The original method proves the formula – definitively – without ambiguity and without doubt. The current method to get the Taylor series, by taking derivatives, will work but it is slow, opaque and fails to give absolute evidence that the coefficients in the series will continue to be an alternating odd, harmonic sequence.