REVIEW DIFFERENTIAL CALCULUS

1. NOTATIONS FOR DERIVATIVE

$f'(x)$ Lagrange

$\frac{dy}{dx}$ Leibnitz

$\dot{f}$ Newton

$D\,f(x)$ Cauchy - Operator notation

2. Definition of Derivative

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = f'(x)$$

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = f'(a)$$

3. Basic Formulas

$D\,x^n = n\,x^{n-1}$

$D\,e^x = e^x$

$D\,a^x = a^x \ln a$

$D \ln x = \frac{1}{x}$ x > 0

$D \sin x = \cos x$

$D \cos x = -\sin x$

$D \tan x \ = \sec^2 x$

$D \cot x \ = -\csc^2 x$

$D \sec x \ = \sec x \ \tan x$

$D \csc x \ = -\csc x \ \cot x$

$D \sin^{-1} x \ = \frac{1}{\sqrt{1 - x^2}}$

$D \cos^{-1} x \ = -\frac{1}{\sqrt{1 - x^2}}$

$D \tan^{-1} x \ = \frac{1}{1 + x^2}$

$D \cot^{-1} x \ = -\frac{1}{1 + x^2}$

$D \sec^{-1} x \ = \frac{1}{|x| \sqrt{x^2 - 1}}$

$D \csc^{-1} x \ = -\frac{1}{|x| \sqrt{x^2 - 1}}$

4. Power Rule

$D(fg) = f' g + f g'$

Extension

$D(fgh) \ = f'gh + fg'h + fgh'$ and so on

5. Quotient Rule

$$D\left(\frac{f}{g}\right) = \frac{gf' - fg\prime}{g^2}$$

6. Definition of continuity at a point

$f(a)$ exists – it equals a real number

$\lim_{x \to a} f(x)$ exists – it equals a real number

$\lim_{x \to a} f(x) = f(a)$ the value of the limit equals the value of the function

ALTERNATE – continuity can be viewed as an interchange of evaluating the function and evaluating the limit

$$\lim_{x \to a} f(x) = f\left(\lim_{x \to a} x\right)$$

CAUCHY'S DEFINITION OF CONTINUITY AT THE POINT x = a

$f(x)$ will be very close in value to $f(a)$ whenever x is close to a

Mathematically this is written $|f(x) - f(a)| < \epsilon$ whenever $|x - a| < \delta$
Where both epsilon and delta are small numbers. In general, delta is a function of both epsilon and a: $\delta = \delta(\epsilon, a)$

7. If a function is differentiable, then it is continuous. The converse is not true – functions can be continuous and not be differentiable.

8. Types of discontinuities:

Holes
Jumps
Vertical asymptotes
Essential singularities (this type of discontinuity might not be covered in calculus one but is covered in calculus of a complex variable).

9. Extreme value theorem
   If a function is continuous on a closed interval, then it will have a global maximum and a global minimum on that interval

10. Intermediate value theorem
    If a function is continuous on a closed interval $[a, b]$ then $f(x)$ will attain every value in between $f(a)$ and $f(b)$ and it will do so at least once

    Corollary
    Let $f(x)$ be continuous on [a,b]. Let $f(a)$ be negative and let $f(b)$ be positive. Then $f(x)$ will equal zero at some value x= c, where c is in the interval (a, b) and this will happen at least once.

11. Types of critical points
    a. Stationary points where the derivative equals zero
    b. Cusps where the derivative goes to positive and negative infinity on respective sides of a point. The right sided limit of the derivative goes to one infinity. The left sided limit goes to the other infinity.
    c. Corners where the derivative has different values on each side of point – the left sided limit for the derivative equals a real number – and the right sided limit of the derivative equals a number – but these numbers are not equal.