MORE U SUBSTITUTIONS PURCELL 390

1. Evaluate $\int (x-2)^2\, dx$

2. Evaluate $\int \frac{1}{3x+5}\, dx$

3. Evaluate $\int 6\, e^{2x-7}\, dx$

4. Evaluate $\int \frac{x\ dx}{\cos^2(x^2)}$

5. Evaluate $\int \frac{3}{\sqrt{1-4x^2}}\, dx$

6. Evaluate $\int \cot(3x+5)\ dx$

7. Evaluate $\int (x-1)^4\, dx$

8. $\int \sqrt{2x}\, dx$

9. Evaluate $\int \frac{dx}{x+1}$

10. Evaluate $\int \frac{e^x}{1+2\,e^x}\, dx$

11. Evaluate $\int 3t\sqrt{2+t^2}\ dt$

12. Evaluate $\int \frac{dt}{\sqrt{1+t}}$

13. Evaluate $\int \sec^2\ (x+3)\ dx$

14. Evaluate $\int x\csc^2(x^2)\ dx$

15. Evaluate $\int \frac{\tan x}{\cos x}\ dx$

16. Evaluate $\int 5\sec 5x\ \tan 5x\ dx$

17. Evaluate $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\ dx$

18. Evaluate $\int e^{2x+1}\ dx$

19. Evaluate $\int \frac{e^{\sin x}}{\sec x}\ dx$

20. Evaluate $\int \frac{x}{\sqrt{1-x^4}}\ dx$

21. Evaluate $\int \frac{\cos x}{1+\sin^2 x}\ dx$

22. Evaluate $\int x \csc x^2 \cot x^2 \, dx$

23. Evaluate $\int \frac{\csc t}{\tan t} \, dt$

24. Evaluate $\int \tan(5x - 1) \, dx$

25. Evaluate $\int e^x \cot(e^x) \, dx$

SOLUTIONS

1. Evaluate $\int (x-2)^2\, dx$

   Let $u = x - 2 \qquad du = dx$

$$\int (x-2)^2\, dx \;=\; \int u^2\, du \;=\; \frac{u^3}{3} + C$$

$$\int (x-2)^2\, dx \;=\; \frac{(x-2)^3}{3} + C$$

2. Evaluate $\int \frac{1}{3x+5}\, dx$

   $let\ u = 3x + 5 \qquad du = 3dx \qquad or\ \ dx = \frac{1}{3}\, du$

$$\int \frac{1}{3x\,+\,5}\, dx \;=\; \int \frac{1}{u}\,\frac{du}{3}$$

$$\int \frac{1}{3x\,+\,5}\, dx \;= \frac{1}{3}\int \frac{1}{u}\, du \;=\; \frac{1}{3}\,\ln|u| + C$$

$$\int \frac{1}{3x\,+\,5}\, dx \;=\; \frac{1}{3}\ln|3x\,+\,5| + C$$

3. Evaluate $\int 6\, e^{2x-7}\, dx$

$$\int 6\, e^{2x-7}\, dx \;=\; 6\int e^{2x-7}\, dx$$

$let\ u = 2x\ -\ 7 \qquad du = 2\, dx \qquad or\ \ dx = \frac{du}{2}$

$$\int 6\, e^{2x-7}\, dx \;=\; 6\int e^{u}\,\frac{du}{2}$$

$$\int 6\,e^{2x-7}\,dx \;=\; 3\int e^{u}\,du$$

$$\int 6\,e^{2x-7}\,dx \;=\; 3\;e^{u} + \;C$$

$$\int 6\,e^{2x-7}\,dx \;=\; 3\;e^{2x-7} \;+\; C$$

4. Evaluate $\int \frac{x\;dx}{\cos^2(x^2)}$

$$let\; u = x^2 \qquad du = 2x\,dx \qquad\qquad or\;\; x\,dx \;= \frac{du}{2}$$

$$\int \frac{x\;dx}{\cos^2(x^2)} = \int \frac{1}{\cos^2 u}\,\frac{du}{2}$$

$$\int \frac{x\;dx}{\cos^2(x^2)} \;=\; \frac{1}{2}\int \frac{1}{\cos^2 u}\,du$$

$$\int \frac{x\;dx}{\cos^2(x^2)} \;=\; \frac{1}{2}\int \sec^2 u\;du$$

$$\int \frac{x\;dx}{\cos^2(x^2)} \;=\; \frac{1}{2}\tan u \;+\; C$$

$$\int \frac{x\;dx}{\cos^2(x^2)} \;=\; \frac{1}{2}\tan(x^2) \;+\; C$$

5. Evaluate $\int \frac{3}{\sqrt{1-4x^2}}\ dx$

$$\int \frac{3}{\sqrt{1-4x^2}}\ dx = 3\int \frac{1}{\sqrt{1-4x^2}}\ dx$$

$let\ u = 2x \qquad du = 2dx \qquad dx = \frac{1}{2}\ du$

$$\int \frac{3}{\sqrt{1-4x^2}}\ dx = 3\int \frac{1}{\sqrt{1-u^2}}\ \frac{1}{2}\ du$$

$$\int \frac{3}{\sqrt{1-4x^2}}\ dx = \frac{3}{2}\int \frac{1}{\sqrt{1-u^2}}\ du$$

$$\int \frac{3}{\sqrt{1-4x^2}}\ dx = \frac{3}{2}\arcsin u + C$$

$$\int \frac{3}{\sqrt{1-4x^2}}\ dx = \frac{3}{2}\arcsin(2x) + C$$

6. Evaluate $\int \cot(3x+5)\ dx$

$let\ u = 3x + 5 \qquad du = 3\,dx \quad or \quad dx = \frac{1}{3}\ du$

$$\int \cot(3x+5)\ dx = \frac{1}{3}\int \cot u\ du$$

$$\int \cot(3x+5)\ dx = \frac{1}{3}\int \frac{\cos u}{\sin u}\ du$$

$$\int \cot(3x+5)\ dx = \frac{1}{3}\ln|\sin u| + C$$

$$\int \cot(3x+5)\ dx = \frac{1}{3}\ln|\sin(3x+5)| + C$$

7. Evaluate $\int (x-1)^4\, dx$

$let\ u = x - 1 \qquad du = dx$

$$\int (x-1)^4\, dx = \int u^4\, du$$

$$\int (x-1)^4\, dx = \frac{u^5}{5} + C$$

$$\int (x-1)^4\, dx = \frac{(x-1)^5}{5} + C$$

8. $\int \sqrt{2x}\, dx$

$let\ u = 2x \qquad du = 2\, dx \quad or \quad dx = \frac{1}{2}\, du$

$$\int \sqrt{2x}\, dx = \int \sqrt{u}\, \frac{1}{2}\, du$$

$$\int \sqrt{2x}\, dx = \frac{1}{2} \int \sqrt{u}\ \ du$$

$$\int \sqrt{2x}\, dx = \frac{1}{2}\frac{2}{3}\, u^{\frac{3}{2}} + C$$

$$\int \sqrt{2x}\, dx = \frac{1}{3}\ u^{3/2} + C$$

$$\int \sqrt{2x}\, dx = \frac{1}{3}\ (2x)^{3/2} + C$$

Of course this question could have been done without a u substitution:

$$\int \sqrt{2x}\, dx = \sqrt{2} \int \sqrt{x}\ dx = \sqrt{2}\ \frac{2}{3}\ x^{3/2} + C$$

$$\int \sqrt{2x}\, dx = \frac{2\sqrt{2}}{3}\ x^{3/2} + C$$

9. Evaluate $\int \frac{dx}{x+1}$

$let\ u = x + 1 \qquad du = dx$

$$\int \frac{dx}{x+1} = \int \frac{du}{u}$$

$$\int \frac{dx}{x+1} = \ln|u| + C$$

$$\int \frac{dx}{x+1} = \ln|x+1| + C$$

10. Evaluate $\int \frac{e^x}{1 + 2e^x}\ dx$

$let\ u = 1 + 2e^x \qquad du = 2e^x\,dx \qquad or \qquad e^x dx = \frac{1}{2}\,du$

$$\int \frac{e^x}{1 + 2e^x}\ dx = \int \frac{1}{2}\frac{du}{u}$$

$$\int \frac{e^x}{1 + 2e^x}\ dx = \frac{1}{2}\int \frac{du}{u}$$

$$\int \frac{e^x}{1 + 2e^x}\ dx = \frac{1}{2}\ln|u| + C$$

$$\int \frac{e^x}{1 + 2e^x}\ dx = \frac{1}{2}\ln|1 + 2e^x| + C$$

11. Evaluate $\int 3t\sqrt{2+t^2}\ dt$

$$\int 3t\sqrt{2+t^2}\ dt = 3\int \sqrt{2+t^2}\ \ t\,dt$$

$let\ u = 2+t^2 \quad du = 2t\,dt \quad t\,dt = \frac{1}{2}\,du$

$$\int 3t\sqrt{2+t^2}\ dt = 3\int \sqrt{u}\,\frac{1}{2}\,du$$

$$\int 3t\sqrt{2+t^2}\ dt = \frac{3}{2}\int \sqrt{u}\ du$$

$$\int 3t\sqrt{2+t^2}\ dt = \frac{3}{2}\ \frac{2}{3}\ u^{3/2} + C$$

$$\int 3t\sqrt{2+t^2}\ dt = u^{3/2} + C$$

$$\int 3t\sqrt{2+t^2}\ dt = (2+t^2)^{3/2} + C$$

12. Evaluate $\int \frac{dt}{\sqrt{1+t}}$

$let\ u = 1 + t \qquad du = dt$

$$\int \frac{dt}{\sqrt{1+t}} = \int \frac{du}{\sqrt{u}}$$

$$\int \frac{dt}{\sqrt{1+t}} = 2\sqrt{u} + C$$

$$\int \frac{dt}{\sqrt{1+t}} = 2\sqrt{1+t} + C$$

13. Evaluate $\int \sec^2\ (x+3)\ dx$

$let\ u = x+3 \quad du = dx$

$$\int \sec^2\ (x+3)\ dx\ =\ \int \sec^2 u\ du$$

$$\int \sec^2\ (x+3)\ dx\ = \tan u\ +\ C$$

$$\int \sec^2\ (x+3)\ dx\ = \tan(x+3)\ +\ C$$

14. Evaluate $\int\ x \csc^2(x^2)\ dx$

$let\ u = x^2 \qquad du = 2x\ dx \quad or \quad x\ dx = \frac{1}{2}\ du$

$$\int\ x \csc^2(x^2)\ dx\ =\ \int\ \csc^2(x^2)\ x\ dx$$

$$\int\ x \csc^2(x^2)\ dx\ =\ \int\ \csc^2(u)\ \frac{1}{2}\ du$$

$$\int\ x \csc^2(x^2)\ dx\ =\ \frac{1}{2}\int\ \csc^2(u)\ \ du$$

$$\int\ x \csc^2(x^2)\ dx\ =\ -\frac{1}{2}\cot u\ +\ C$$

$$\int\ x \csc^2(x^2)\ dx\ =\ -\frac{1}{2}\cot(x^2)\ +\ C$$

15. Evaluate $\int\ \frac{\tan x}{\cos x}\ dx$

$\int\ \frac{\tan x}{\cos x}\ dx\ =\ \int\ \frac{\sin x}{\cos^2 x}\ dx$

$let\ u = \cos x \qquad du = -\sin x\ \ dx$

$\int\ \frac{\tan x}{\cos x}\ dx\ =\ \int -\frac{1}{u^2}\ du$

$\int\ \frac{\tan x}{\cos x}\ dx\ = \frac{1}{u}\ +\ C$

$\int\ \frac{\tan x}{\cos x}\ dx\ = \frac{1}{\cos x}\ +\ C$

This could have been done another way: $\int\ \frac{\tan x}{\cos x}\ dx\ =\ \int \tan x \sec x\ dx\ = \sec x + C$

16. Evaluate $\int 5 \sec 5x \ \tan 5x \ \ dx$

$let\ u = 5x \quad du = 5dx$

$$\int 5 \sec 5x \ \tan 5x \ \ dx = \int \ \sec 5x \ \tan 5x \ 5 \ dx$$

$$\int 5 \sec 5x \ \tan 5x \ \ dx = \int \ \sec u \ \tan u \ \ du$$

$$\int 5 \sec 5x \ \tan 5x \ \ dx = \sec u \ + \ C$$

$$\int 5 \sec 5x \ \tan 5x \ \ dx = \sec 5x \ + \ C$$

17. Evaluate $\int \ \frac{\cos \sqrt{x}}{\sqrt{x}} \ dx$

$$\int \ \frac{\cos \sqrt{x}}{\sqrt{x}} \ dx \ = \ \int \ \cos \sqrt{x} \ \ \frac{dx}{\sqrt{x}}$$

$let\ u \ = \ \sqrt{x} \qquad du = \frac{dx}{2\sqrt{x}} \ \ or \ \frac{dx}{\sqrt{x}} \ = \ 2 \ du$

$$\int \ \frac{\cos \sqrt{x}}{\sqrt{x}} \ dx \ = \ \int \ \cos u \ \ 2 \ du$$

$$\int \ \frac{\cos \sqrt{x}}{\sqrt{x}} \ dx \ = 2 \ \int \ \cos u \ \ du$$

$$\int \ \frac{\cos \sqrt{x}}{\sqrt{x}} \ dx \ = 2 \ \sin u \ + \ C$$

$$\int \ \frac{\cos \sqrt{x}}{\sqrt{x}} \ dx \ = 2 \ \sin \sqrt{x} \ + \ C$$

18. Evaluate $\int e^{2x+1}\, dx$

$let\ u = 2x + 1 \qquad du = 2dx \quad or \quad dx = \frac{1}{2}\, du$

$$\int e^{2x+1}\, dx = \int e^u \frac{du}{2} = \frac{1}{2}\int e^u\, du = \frac{1}{2}\, e^u + C = \frac{1}{2}\, e^{(2x+1)} + C$$

19. Evaluate $\int \frac{e^{\sin x}}{\sec x}\, dx$

$$\int \frac{e^{\sin x}}{\sec x}\, dx = \int e^{\sin x} \cos x\ dx$$

Let u = sin x so du = cos x dx

$$\int \frac{e^{\sin x}}{\sec x}\, dx = \int e^u\, du$$

$$\int \frac{e^{\sin x}}{\sec x}\, dx = e^u + C$$

$$\int \frac{e^{\sin x}}{\sec x}\, dx = e^{\sin x} + C$$

20. Evaluate $\int \frac{x}{\sqrt{1-x^4}}\, dx$

$let\ u = x^2 \quad du = 2x\, dx \quad or \quad x\, dx = \frac{1}{2}\, du$

$$\int \frac{x}{\sqrt{1-x^4}}\, dx = \int \frac{1}{\sqrt{1-x^4}}\ x\ dx$$

$$\int \frac{x}{\sqrt{1-x^4}}\, dx = \int \frac{1}{\sqrt{1-u^2}}\, \frac{1}{2}\, du$$

$$\int \frac{x}{\sqrt{1-x^4}}\, dx = \frac{1}{2}\int \frac{1}{\sqrt{1-u^2}}\ du$$

$$\int \frac{x}{\sqrt{1-x^4}}\, dx = \frac{1}{2}\ arc\sin u + C$$

$$\int \frac{x}{\sqrt{1-x^4}}\, dx = \frac{1}{2}\ arc\sin x^2 + C$$

21. Evaluate $\int \frac{\cos x}{1+\sin^2 x}\ dx$

Let u = sin x so du = cos x dx

$$\int \frac{\cos x}{1 + \sin^2 x}\ dx = \int \frac{1}{1 + \sin^2 x}\ \cos x\ dx$$

$$\int \frac{\cos x}{1 + \sin^2 x}\ dx = \int \frac{1}{1 + u^2}\ du$$

$$\int \frac{\cos x}{1 + \sin^2 x}\ dx = arc\tan u + C$$

$$\int \frac{\cos x}{1 + \sin^2 x}\ dx = arc\tan(\sin x) + C$$

22. Evaluate $\int x \csc x^2\ \cot x^2\ dx$

$$\int x \csc x^2\ \cot x^2\ dx = \int \csc x^2\ \cot x^2\ x\ dx$$

$let\ u = x^2\quad so\ du = 2x\ dx\quad or\ x\ dx = \frac{1}{2}\ du$

$$\int x \csc x^2\ \cot x^2\ dx = \int \csc u\ \cot u\ \frac{1}{2}\ du$$

$$\int x \csc x^2\ \cot x^2\ dx = \frac{1}{2} \int \csc u\ \cot u\ \ du$$

$$\int x \csc x^2\ \cot x^2\ dx = \frac{1}{2}\ (-\csc u) + C$$

$$\int x \csc x^2\ \cot x^2\ dx = -\frac{1}{2}\csc(2x) + C$$

23. Evaluate $\int \frac{\csc t}{\tan t}\ dt$

$$\int \frac{\csc t}{\tan t}\ dt = \int \frac{\cos t}{\sin^2 t}\ dt$$

$let\ u = \sin t \qquad du = \cos t\ \ dt$

$$\int \frac{\csc t}{\tan t}\ dt = \int \frac{1}{u^2}\ du$$

$$\int \frac{\csc t}{\tan t}\ dt = -\frac{1}{u} + C$$

$$\int \frac{\csc t}{\tan t}\ dt = -\frac{1}{\sin t} + C$$

This could be solve in a different way: $\int \frac{\csc t}{\tan t}\ dt = \int \csc t \cot t\ \ dt = -\csc t + C$

24. Evaluate $\int \tan(5x-1)\ \ dx$

Let u = 5x – 1 du = 5 dx or dx = du/5

$$\int \tan(5x-1)\ \ dx = \int \tan u\ \ \frac{1}{5}\ du$$

$$\int \tan(5x-1)\ \ dx = \frac{1}{5}\int \tan u\ \ du$$

$$\int \tan(5x-1)\ \ dx = \frac{1}{5}\ \ln|\sec u| + C$$

$$\int \tan(5x-1)\ \ dx = \frac{1}{5}\ \ln|\sec(5x-1)| + C$$

25. Evaluate $\int e^x \cot(e^x)\ dx$

$$\int e^x \cot(e^x)\ dx = \int \cot(e^x)\ e^x\ dx$$

$let\ u = e^x\ \ so\ \ du = e^x\ dx$

$$\int e^x \cot(e^x)\ dx = \int \cot(u)\ du$$

$$\int e^x \cot(e^x)\ dx = \ln|u| + C$$

$$\int e^x \cot(e^x)\ dx = \ln|e^x| + C$$