EXAM ONE SOLUTIONS
MATH 142 CALCULUS II
SPRING 2020
INSTRUCTOR: FRIEDBERG

Instructions:

Show all formulas you are using.

Show all work.

Write NEATLY and clearly.

Please write your answers are to be written as equations. If you do not write an equation as your answer, you might lose credit.

Please refrain from talking. There is absolutely no talking during the test. This is part of the honor code – please do not violate the honor code.

Please put away all cell phones. This is also part of the honor code. Please respect it.

The exam has 13 questions total.

REVIEW QUESTIONS

1. State the **exact** values. Decimal approximations will be marked wrong.

$\sin 0 = \ 0$ $\cos 0 = \ 1$

$\sin 30 = \frac{1}{2}$ $\cos 30 = \frac{\sqrt{3}}{2}$

$\sin 45 = \frac{\sqrt{2}}{2}$ $\cos 45 = \frac{\sqrt{2}}{2}$

$\sin 60 = \frac{\sqrt{3}}{2}$ $\cos 60 = \frac{1}{2}$

$\sin 90 = \ 1$ $\cos 90 = \ 0$

2. State the following derivatives

$\frac{d\,(x^n)}{dx} = \ n\,x^{n-1}$ $\frac{d\,(\sin\ x\ )}{dx} = \cos x$

$\frac{d\,(e^x)}{dx} = \ e\hat{}x$ $\frac{d\,(\cos x)}{dx} = -\sin x$

$\frac{d\,(a^x)}{dx} = \ a^x \ln a \ \backslash$ $\frac{d\,(\tan x)}{dx} = \sec^2 x$

$\frac{d\,(\ln x)}{dx} = \frac{1}{x}$ $\frac{d\,(\sec x)}{dx} = \sec x \tan x$

$\frac{d\,(\csc x)}{dx} = -\csc x \cot x$ $\frac{d\,(\sin^{-1} x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

$\frac{d\,(\cos^{-1} x)}{dx} = -\frac{1}{\sqrt{1-x^2}}$ $\frac{d\,(\tan^{-1} x)}{dx} = \frac{1}{1+x^2}$

$\frac{d\,(\sec^{-1} x)}{dx} = \frac{1}{|x|\,\sqrt{x^2-1}}$ $\frac{d\,(\csc^{-1} x)}{dx} = -\frac{1}{\sqrt{x^2-1}}$

3. State the following integrals:

$$\int x^n \; dx = \frac{x^{n+1}}{n+1} + c \qquad \int e^x \, dx = e^x + C$$

$$\int a^x \, dx = \frac{a^x}{\ln a} + c$$

$$\int \frac{1}{x} \; dx = \int x^{-1} \, dx = \int \frac{dx}{x} = \ln|x| + C$$

$$\int \sin x \;\; dx = -\cos x + C \qquad \int \cos x \;\; dx = \sin x + c$$

$$\int \sec^2 x \;\; dx = \tan x + c \qquad \int \csc^2 x \;\; dx = -\cot x + c$$

$$\int \frac{1}{\sqrt{1-x^2}} \; dx = \int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c$$

$$\int \frac{1}{1+\,x^2} \;\; dx = \int \frac{dx}{1+x^2} = \tan^{-1} x + c$$

$$\int \frac{1}{|x|\,\sqrt{x^2-1}} \;\; dx = \sec^{-1} x + c$$

CALCULUS II QUESTIONS:

4. Evaluate the following summations:

$$\sum_{i=1}^{20} i = \frac{n\,(n+1)}{2} = \frac{(20)(21)}{2} = 210$$

$$\sum_{i=0}^{15} i^2 = \sum_{i=1}^{15} i^2 = \frac{n\,(n+1)\,(2n+1)}{6}$$

$$= \frac{15(16)\,(31)}{6} = 5(8)\,(31) = 1240$$

Adding zero for the first term does nothing

$$\sum_{i\,=\,0}^{20} i^3 = \sum_{i\,=\,1}^{20} i^3 = \left[\frac{n\,(n+1)}{2}\right]^2 = (210)^2 = 44100$$

5. You are given the function $y = f(x) = 2x + 5$. You want to find an approximate value for the area under this function on the closed interval from 0 to 6. Using a regular partition with n = 3, find the left handed sum. Clearly state delta x, the points of partition/evaluation and the formula for the Riemann sum.

$$\Delta x = \frac{b-a}{n} = \frac{6-0}{3} = 2$$

$$x_i = a + i\,\Delta x = 0 + 2i = 2i$$

$x_0 = a = 0$ $\quad\quad f(x_0) = f(0) = 5$

$x_1 = 2$ $\quad\quad f(x_1) = f(2) = 9$

$x_2 = 4$ $\quad\quad f(x_2) = f(4) = 13$

$x_3 = b = 6$ $\quad\quad f(x_3) = f(6) = 17$

$$lhs = \sum_{i=0}^{n-1} f(x_i)\ \Delta x = [\,f(x_0) + f(x_1) + f(x_2)]\,\Delta x$$

$$lhs = [5 + 9 + 13]\,(2) = 54$$

6. Repeat the previous question using right handed sum.

$$rhs = \sum_{i=1}^{n} f(x_i)\ \Delta x = [\,f(x_1) + f(x_2) + f(x_3)]\,\Delta x$$

$$rhs = [9 + 13 + 17]\,(2) = 78$$

7. Repeat the previous question using midpoints for the Riemann sum.

$x_1^* = 1$ $\quad f(x_1^*) = f(1) = 7$

$x_2^* = 3$ $\quad f(x_2^*) = f(3) = 11$

$x_3^* = 5$ $\quad f(x_3^*) = f(5) = 15$

$$mp = \sum_{i=1}^{n} f(x_i)\ \Delta x = [\, f(x_1^*) + f(x_2^*) + f(x_3^*)]\, \Delta x$$

$$mp = [7 + 11 + 15]\,(2) = 66$$

8. You are given the function $y = f(x) = x^2$. The interval is x from 0 to 1. Using a regular partition and a Riemann sum, find the exact area under the curve. Do not use integrals nor the fundamental theorem.

$$\Delta x = \frac{b-a}{n} = \frac{1}{n}$$

$$x_i = a + i\,\Delta x = \frac{i}{n}$$

$$A = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\ \Delta x$$

$$A = \lim_{n \to \infty} \sum_{i=1}^{n} x_i^2\ \Delta x$$

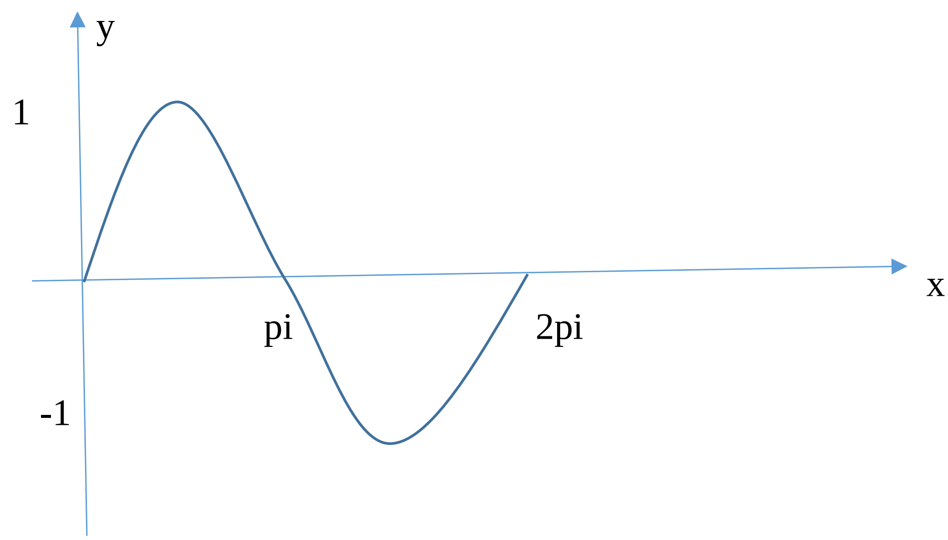
$$A = \lim_{n \to \infty} \sum_{i=1}^{n} \frac{i^2}{n^2}\,\frac{1}{n}$$

$$A = \lim_{n \to \infty} \frac{1}{n^3} \sum_{i=1}^{n} i^2 = \lim_{n \to \infty} \frac{1}{n^3}\ \frac{n\,(n+1)\,(2n+1)}{6}$$

$$A = \lim_{n \to \infty} \frac{(n+1)\,(2n+1)}{6\,n^2}$$

$$A = \lim_{n \to \infty} \frac{2n^2 + 3n + 1}{6\,n^2} = \frac{1}{3}$$

9. You are given the function y = sin x on the interval from 0 to 2pi. First draw the function on this interval. Then find two distinct things. First find the definite integral for the function on this interval. Second, find the area between the function and the x axis on this interval.



$$\int_0^{2\pi} \sin x \;\; dx \;=\; -\cos x|_0^{2\pi} \;=\; -\cos 2\pi \;+\; \cos 0 \;=\; -1 + 1 = 0$$

$$A_1 \;=\; \int_0^{\pi} \sin x \;\; dx \;=\; -\cos x|_0^{\pi} \;=\; -\cos \pi \;+ \cos 0 \;=\; 1 + 1 = 2$$

$$A_2 \;=\; -\int_{\pi}^{2\pi} \sin x \;\; dx \;=\; \cos x|_{\pi}^{2\pi} \;=\; \cos 2\pi - \;\cos \pi \;=\; 1 + 1 \;=\; 2$$

$$A \;= A_1 + A_2 \;=\; 2 + 2 = 4$$

10. You are given the function $y = f(x) = \frac{1}{1+x^2}$. Find the exact area under the function on the closed interval from x=0 to x=1. Use the fundamental theorem of calculus (i.e. use a definite integral).

$$A = \int_0^1 \frac{1}{1+x^2}\,dx = \tan^{-1} x \,\Big|_0^1 = \tan^{-1} 1 - \tan^{-1} 0$$

$$A = \frac{\pi}{4} - 0 = \frac{\pi}{4}$$

11. Evaluate the following:

$$\frac{d}{dx}\int_2^x \ln(t+2)\,dt = \ln(x+2)$$

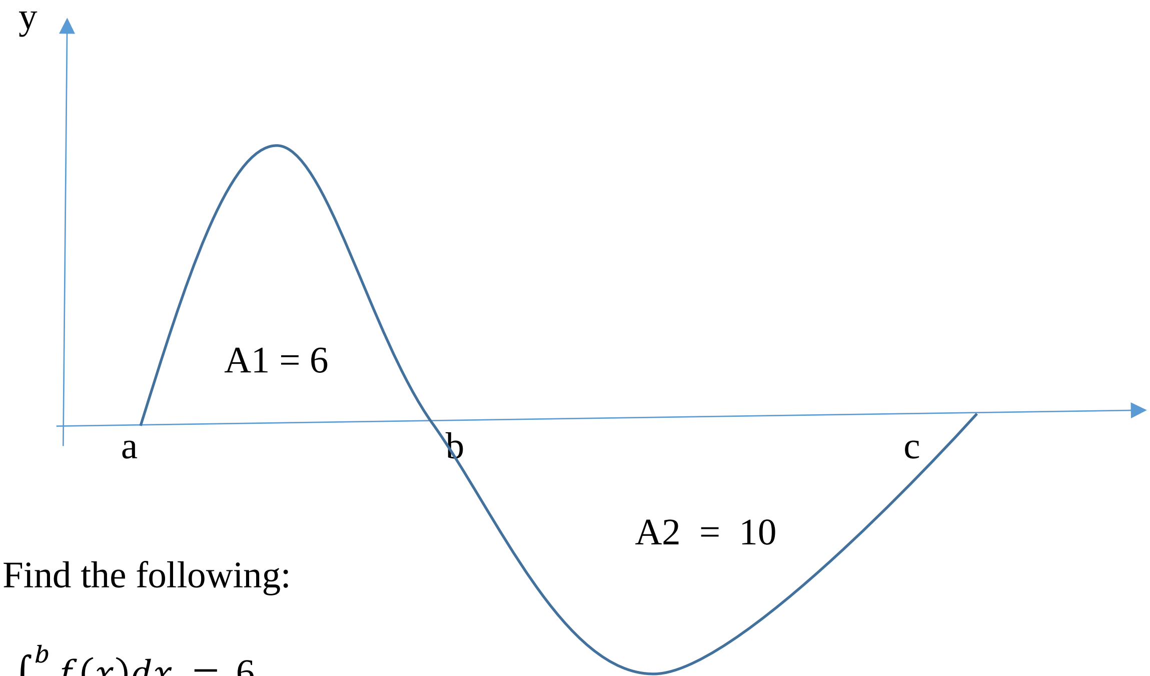
$$\frac{d}{dx}\int_x^1 \csc t\,dt = -\csc x$$

$$\frac{d}{dx}\int_5^{x^2} \frac{1}{t+\sin t}\,dt = \frac{1}{x^2+\sin x^2}\cdot 2x$$

$$\frac{d}{dx}\int_{2x}^{3x} \frac{1}{4+\cos t}\,dt = \frac{1}{4+\cos 3x}\cdot 3 - \frac{1}{4+\cos 2x}\cdot 2$$

The second part of the fundamental theorem states that $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$. If the bound is at the bottom, flip the integral and introduce a minus sign. If the bound is not x, but some function of x, use the chain rule.

12. You are given the following figure showing the function y = f(x) on the x&y axes.



Find the following:

$\int_a^b f(x)dx = 6$

$\int_b^c f(x)\, dx = -10$

$\int_a^c |f(x)|\, dx = 16$ the absolute value sign around the function means that the function is positive everywhere.

$\int_c^b |f(x)|\, dx = -10$ the integral goes from c to b, not from b to c

13. State the equation for the mean value theorem of integral calculus. Given the function $y = x^2$ on the interval from 0 to 1, find the average value of the function. Find the value of x where the function equals its average.

$f(c) = f_{avg} = \frac{1}{b-a} \int_a^b f(x)\, dx$ where a < c < b

$f_{avg} = \frac{1}{1-0} \int_0^1 x^2 \; dx$

$f_{avg} = 1 \cdot \frac{x^3}{3}\Big|_0^1 = \frac{1}{3}$

$f(c) = c^2 = f_{avg} = \frac{1}{3}$

$c = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$