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Fa '04 FE Soln.

1) $m\ddot{x} + c\dot{x} + kx = \delta(t) + e^{\frac{-2\zeta\pi}{\sqrt{1-\zeta^2}}}\,\delta\left(t - \frac{2\pi}{\omega_d}\right)$

For $0 < t < \frac{2\pi}{\omega_d}$

$$x_1(t) = \frac{1}{m\omega_d} e^{-\zeta\omega_n t} \sin \omega_d t$$

For $\frac{2\pi}{\omega_d} < t$

$$x_2(t) = x_1(t) + \frac{e^{-\zeta\omega_n t}}{m\omega_d}\int_0^t e^{\frac{-2\pi\zeta}{\sqrt{1-\zeta^2}}}\,\delta\left(\tau - \frac{2\pi}{\omega_d}\right) e^{\zeta\omega_n\tau} \sin\omega_d(t-\tau)\, d\tau$$

$$= x_1(t) + \frac{e^{-\zeta\omega_n t}}{m\omega_d} e^{\frac{-2\pi\zeta}{\sqrt{1-\zeta^2}}} e^{\frac{\zeta\omega_n 2\pi}{\omega_d}} \sin\omega_d(t-\tau)$$

$$= x_1(t) + \frac{e^{-\zeta\omega_n t}}{m\omega_d} \sin\omega_d(t-\tau)$$

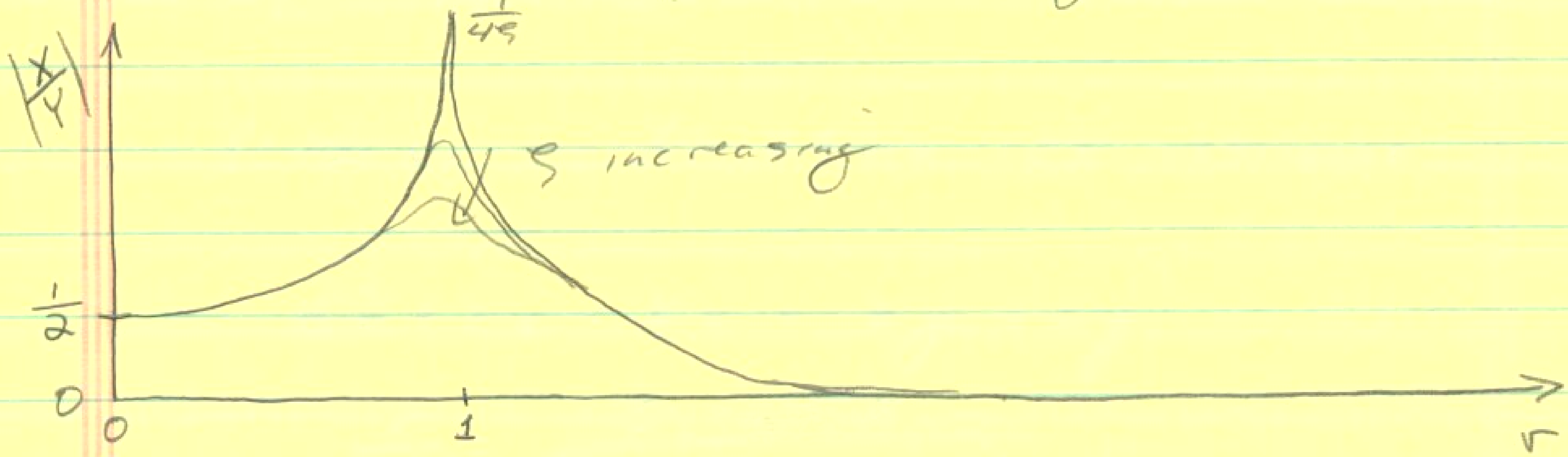
$$= \frac{2}{m\omega_d} e^{-\zeta\omega_n t} \sin\omega_d(t-\tau)$$

Note that $x_2 = 2x_1$

2)

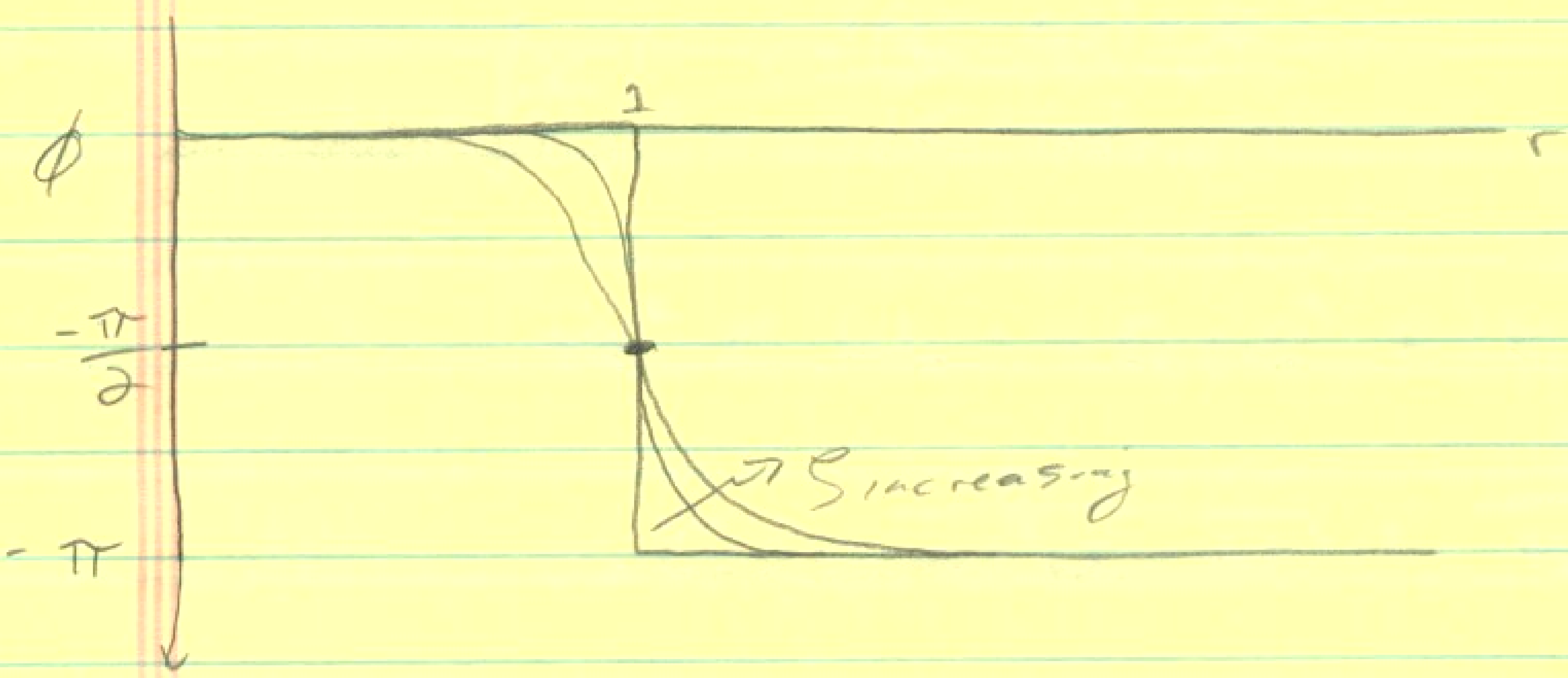
$$(2k - m\omega_{dr}^2 + cj\omega_{dr})X = kY$$

$$\frac{X}{Y} = \frac{k}{(2k - m\omega_{dr}^2) + cj\omega_{dr}}$$



$$@\ \omega_{dr} = \sqrt{\frac{2k}{m}},\ \left|\frac{X}{Y}\right| = \left|\frac{k}{cj\sqrt{\frac{2k}{m}}}\right| = \frac{\frac{1}{2}k_{eff}}{c\sqrt{\frac{k_{eff}}{m}}}$$

$$= \frac{1}{2}\frac{1}{c\sqrt{k_{eff}m}} = \frac{1}{2}\frac{1}{2\zeta} = \frac{1}{4\zeta}$$



3) $\underline{x} = S\,\underline{r} = \begin{bmatrix} \sin 4t + 0.1 \sin 4t \\ 3\sin 4t - 0.3 \sin 4t \end{bmatrix}$

4) $\det(K - M\omega^2) = 0$

$\omega = 4.375,\ 8.852$ rad/s

$\underline{u}_1 = \begin{bmatrix} 0.6067 \\ 0.7949 \end{bmatrix} \qquad \underline{u}_2 = \begin{bmatrix} 0.9343 \\ -0.3565 \end{bmatrix}$

5) $U = M_1 g h_1 = M_1 g (R - R_1)(1 - \cos\theta)$

$T = \frac{1}{2} M_1 v_1^2 + \frac{1}{2} M_1 R_1^2 \dot{\phi}_1^2$ where $\dot{\phi}_1$ is angular velocity of inner pipe

From Kinematics

$R_1 \dot{\phi}_1 = (R - R_1)\dot{\theta} = V_1$

$\therefore\ T = \frac{1}{2} M_1 (R - R_1)^2 \dot{\theta}^2 + \frac{1}{2} M_1 R_1^2 \left(\frac{R - R_1}{R_1}\right)^2 \dot{\theta}$

$= \frac{1}{2} M_1\, 2(R - R_1)^2 \dot{\theta}^2$

$\frac{d}{dt}\frac{\partial T}{\partial \dot{\theta}} = 2 M_1 (R - R_1)^2 \ddot{\theta}$

$\frac{\partial U}{\partial \theta} = M_1 g (R - R_1) \sin\theta$

Subst into Lagrange's Eqn

$2 M_1 (R - R_1)^2 \ddot{\theta} + M_1 g (R - R_1)\theta = 0$

$\omega_n = \sqrt{\frac{g}{2(R - R_1)}}$

6) This is a string (2 derivatives in space)
Applying fixed BC ($X(\ell)=0$, $X(0)=0$)

$$X(x) = \sin \frac{n\pi x}{\ell} \qquad \sigma_n = \frac{n\pi}{\ell}$$

$$\omega_n = c\,\sigma_n = \sqrt{\frac{\tau}{\ell}}\ \frac{n\pi}{\ell}$$

The solution is of the form

$$w = \sum_{n=1}^{\infty} T_n X_n$$

Subst into EOM

$$\left(\ddot{T}_n + \omega_n^2 T_n\right) X_n = 100\,\delta(t)\,\delta\!\left(x-\frac{\ell}{2}\right)$$

Mult by $X_m(x)$ and integrate over ℓ

$$\ddot{T}_n + \omega_n^2 T_n = \overbrace{\frac{2}{\ell}\cdot 100\, X_n\!\left(\frac{\ell}{2}\right)}^{\hat{F}}\ \delta(t)$$

From the impulse response

$$T_n = \frac{1}{\omega_n}\,\frac{200}{\ell}\,X_n\!\left(\frac{\ell}{2}\right)\sin\omega_n t$$

$$= \frac{200}{\ell\omega_n}\,X_n\!\left(\frac{\ell}{2}\right)\sin\omega_n t$$

$$w(x,t) = \sum_{n=1}^{\infty} \frac{200}{\ell\omega_n}\,X_n\!\left(\frac{\ell}{2}\right)\sin\omega_n t\,\sin\frac{n\pi x}{\ell}$$

OR $$w(x,t) = \sum_{n=1}^{\infty} \frac{200}{\ell\omega_n}(-1)^{n+1}\sin\sqrt{\frac{\tau}{\ell}}\,\frac{(2n-1)\pi}{\ell}t\,\sin\frac{(2n-1)\pi}{\ell}x$$