Sp 2011 ME 460 Final Exam.

Short solutions follow.

1) $H(j\omega) = \dfrac{1}{81000 - 1000\omega^2 + 100j\omega}$

n	ω	$\frac{1}{n}$	$\frac{1}{n}\lvert H(j\omega)\rvert$	$\measuredangle H(j\omega)$ (rad)	Term
1	3	1	1.39×10^{-5}	$-0.004 \sim 0$	$1.39\times10^{-5}\sin 3t$
2	6	$\frac{1}{2}$	1.11×10^{-5}	$-0.01 \sim 0$	$1.11\times10^{-5}\sin 6t$
3	9	$\frac{1}{3}$	3.70×10^{-4}	$-1.57\ (-\frac{\pi}{2})$	$3.66\times10^{-4}\sin 9t - \frac{\pi}{2}$
4	12	$\frac{1}{4}$	3.97×10^{-6}	$-3.12\ (\sim -\pi)$	$3.97\times10^{-6}\sin 12t - \pi$
5	15	$\frac{1}{5}$	1.39×10^{-6}	$-3.13\ (\sim -\pi)$	$1.39\times10^{-6}\sin 15t - \pi$

-0.2°

-0.8°

-90.0000°

-178.9°

-179.4°

Dominant, by far

$$x(t) = 1.39\times10^{-5}\sin 3t + 1.11\times10^{-5}\sin 6t + 3.70\times10^{-4}\sin(9t - \tfrac{\pi}{2})$$
$$+ 3.97\times10^{-6}\sin 12t - \pi + 1.39\times10^{-6}\sin 15t - \pi$$

2) Since S is the matrix of mass normalized mode shapes

a) $\Lambda = S^T K S$

$$= \begin{bmatrix} -0.0269 & -0.0166 \\ -0.0166 & 0.0269 \end{bmatrix} \begin{bmatrix} 2\times10^{25} & -1\times10^{25} \\ -1\times10^{25} & 3\times10^{25} \end{bmatrix} \begin{bmatrix} -0.0269 & -0.0166 \\ -0.0166 & 0.0269 \end{bmatrix}$$

$$= \begin{bmatrix} 1.38\times10^{22} & 0 \\ 0 & 3.62\times10^{22} \end{bmatrix}$$

$\omega_n = 1.18\times10^{11},\ 1.90\times10^{11}$ rad/s

$f_n = 1.87\times10^{10},\ 3.03\times10^{10}$ Hz

b) $r_1(t) = \frac{1}{1.18\times10^{11}} \sin 1.18\times10^{11} t$ $(8.5\times10^{-12} \ldots$

$r_2(t) = \frac{1.2}{1.90\times10^{11}} \sin 1.90\times10^{11} t$ $(5.25\times10^{-12} \ldots$

$$\underline{x}(t) = S\underline{r} = \begin{bmatrix} -2.29\times10^{-13} \sin 1.18\times10^{11}t - 1.75\times10^{-13} \sin 1.90\times10^{11}t \\ -1.41\times10^{-13} \sin 1.18\times10^{11}t + 2.83\times10^{-13} \sin 1.90\times10^{11}t \end{bmatrix}$$

3) See Fall 2009 Final, problem 3.

4) See text

Key

ME 460/660, Mechanical Vibration Final, Spring 2011

Closed book, closed notes. Use $8\frac{1}{2} \times 11$ formula sheet from web and turn in with exam (nothing else may be written on the formula sheet). Test books will be provided. Calculators allowed. Knowing how to use them well is highly recommended.

Problem 4 is required for graduate students, bonus for undergraduates (worth 20% of exam points).

Short stuff

Circle the letters of the correct answers. Each question may have multiple correct answers. Circle each correct answer for each problem. *Read all questions very carefully.* One point for each correctly circled or not circled item. (30 points total)

1. An anti-resonance means

   a. the system response can be treated as quasi-static

   b. the phase is 180° from where it should be

   c. the amplitude of the response is near zero

   d. this is the frequency at which the vibration absorber works best

2. The fundamental frequency is

   a. what ever frequency a system is responding at

   b. highest natural frequency of a system

   c. the lowest natural frequency of a system

   d. an idealized parameter that doesn't exist in the real world

3. A linear-*damped* SDOF system is excited at 9 Hz ($F = 10\cos(18\pi t)$). It has a natural frequency of 10 Hz. Assuming zero initial conditions, its frequency/ies of motion will be:

   a. 9 Hz

   b. 10 Hz

   c. 9.5 Hz

   d. 19.5 Hz

4. The settling time of a system can be changed by modifying...

   a. m

   b. c

   c. k

d. Q
e. The forcing function

5. Viscous damping has the greatest impact on the response of most systems system at frequencies
   a. near $r = 0$
   b. near $r = 1$
   c. near $r = \infty$
   d. When the forcing function is random

6. The convolution integral
   a. can be used to obtain the response to an excitation of any form
   b. will not give the correct answer for a harmonically excited system
   c. applies only for impulse excitations
   d. only applies to free response

7. Coulomb damping
   a. works best at resonance
   b. works worst at resonance
   c. is a precise model of friction damping
   d. is equivalent to (the same as) viscous damping

8. The number of natural frequencies of interest in a model is
   a. The number considered before becoming bored with the model
   b. The number of frequencies for which the wave length (distance between nodes) is less than the distance between the molecules in the object being modeled
   c. The number of frequencies for which the wave length (distance between nodes) is greater than the distance between the molecules in the object being modeled
   d. The number of frequencies for which the modal forces are relatively large
   e. As many natural frequencies as can be obtained from the model, regardless of the accuracy of those natural frequencies

9. In a linear system, the velocity is
   a. 90° behind the displacement
   b. 90° behind the acceleration
   c. 90° ahead of the force when the response is quasi-static

More short stuff

Fill in the blank with the appropriate letter from the list below. One point each. Use **each answer only once!** either u

In the first experimental lab, we observed the J of a G measured using an N (type of sensor). By observing the period we were able to ascertain the B , and by calculating the F , we were able to estimate the D . In the second lab, we obtained the K by forcing the beam with an S . The quality of the test data as a function of frequency can be rated by observing the L . The damping ratio was estimated using E and I Two answers

The objective of a vibration absorber design is to set the natural frequency of the absorber to be equal to the C of the excited system.

Use each answer only once!

a. natural frequency

b. damped natural frequency

c. driving frequency

d. damping ratio

e. quadrature peak picking

f. log decrement

g. beam

h. flux capacitor

i. curve fitting

j. free response

k. frequency response function

l. coherence

m. phase

n. accelerometer

o. tachyon field

p. pressure gauge

q. displacement sensor

r. load cell

s. impulse hammer

t. displacement

u. acceleration

v. voltage

w. piezoelectric

x. Newton's Law

y. Coulomb's Law

z. Taylor series

Long problems

1. (20 points) The Fourier Series of an excitation is given by $F(t) = \sum_{1}^{\infty} \frac{1}{n} \sin(3nt)$. Given

$$m\ddot{x} + c\dot{x} + kx = F(t)$$

where $k = 81,000$ N, $c = 100$ kg/s, and $m = 1000$ kg, find the first 5 terms of the solution, $x(t)$. What is the most important term in the solution?

2. (20 points) Given

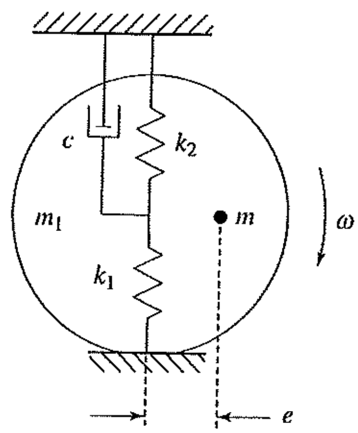
$$S = \begin{bmatrix} -0.0269 & -0.0166 \\ -0.0166 & 0.0269 \end{bmatrix}$$

$$K = \begin{bmatrix} 2 \times 10^{25} & -1 \times 10^{25} \\ -1 \times 10^{25} & 3 \times 10^{25} \end{bmatrix}$$

   (a) Find the natural frequencies in Hz showing 3 places of accuracy (confidence within 0.05%).

   (b) If the modal forces are $f_1(t) = \delta(t)$ and $f_2(t) = 2\delta(t)$, what is $x(t)$?

3. (20 points) Derive the equation of motion for the unbalanced tire shown below.



4. **Graduate Student Problem:** Derive the equation of motion of a string fixed at each end.