Vibration Testing — Final Exam, Winter 2004

Closed book, closed notes, one formula sheet. Test booklets will be provided. *All work must be done shown in the exam book. No extra paper, for scrap or not, may be used.* Formula sheet must be turned in with the exam. It is suggested that you keep a diary and turn in an edited version to show your work.

1. Determine the mode shapes and eigenvalues for the system described by equation (1), **but with a zero damping matrix**. Use these to generate the state space representation eigenvectors. *You may check your answers with a calculator, but will not receive credit unless you show how the mode shapes of the second-order system relate to the eigenvectors of the first-order model.*

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \ddot{\boldsymbol{x}} + \begin{bmatrix} .2 & -.1 \\ -.1 & .2 \end{bmatrix} \dot{\boldsymbol{x}} + \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \boldsymbol{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \sin(\omega t) \tag{1}$$

2. For the system of equation (1), with $C_a = [1 \quad 0]$, and $\Delta t = 0.1$, determine the discrete state-space model matrices A_d, B_d, C, and D.

3. Determine the mode shapes of the system defined by

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \ddot{\boldsymbol{x}} + \begin{bmatrix} .2 & -.1 \\ -.1 & .2 \end{bmatrix} \dot{\boldsymbol{x}} + \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix} \boldsymbol{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \sin(\omega t) \tag{2}$$

4. The frequency response of a system is given in the file `fedat2.mat`. Determine M, C, and K as well as the mode shapes, natural frequencies, and damping ratios.

5. Prove $\sigma_x^2 = \int_{-\infty}^{\infty} G_{xx}(j\omega) d\omega$.