ME 712 Final SP '09 Solutions

1)
$$\frac{\rho AL}{6}\begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -\frac{\ell}{2} \\ 0 & 1 & 0 \\ 0 & 0 & \frac{\ell}{2} \end{bmatrix}}_{T} \begin{bmatrix} d_1 \\ d_3 \\ \theta \end{bmatrix}$$

$$M_{red} = T^T M T$$

If I use $\theta\ell$ as a coordinate, I can do numerical transformation

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -\frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} & & \\ & M & \\ & & \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

$$= \frac{\rho AL}{6}\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}$$, but using θ would give

$$M_{red} = \frac{\rho AL}{6}\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & \frac{\ell^2}{2} \end{bmatrix}$$ and $M_{red}(3,3)$ is $\frac{1}{12} M\ell^2$, I_{CG}.

2) $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - 3 \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$

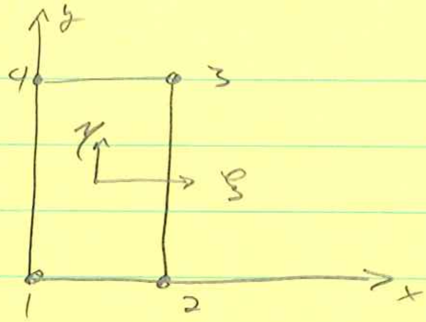
$= \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} - 3 \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$

$= \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

3) Symmetric Positive semi-definite with 6 zero eigenvalues.

③

4)



$$J = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} \quad (6.2\text{-}6)$$

you should see by observation

Using 6.2-13

$$\begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \frac{1}{4} \begin{bmatrix} -1 & 1 & 1 & -1 \\ -1 & -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ u_3 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} u_3 = \begin{bmatrix} \frac{u_3}{2} \\ \frac{u_3}{4} \end{bmatrix}$$

Similarly, $\frac{\partial v}{\partial x} = \frac{v_3}{2}$, $\frac{\partial v}{\partial y} = \frac{v_3}{4}$

$$\epsilon_x = \frac{u_3}{2}, \quad \epsilon_y = \frac{v_3}{4}, \quad \gamma_{xy} = \frac{u_3}{4} + \frac{v_3}{2}$$

5) a) 1) Reduces # DOFs approximately
2) Low mass to stiffness issues
3) a) Brick element extra dof reduction
b) Static quasi-condensation of stiff DOFs

b) 1) Modal reduction of # DOFs
2) When smaller, but still accurate model is needed

c) 1) Numerical integration in time
2) Find time histories

d) 1) Eigensolver for one vector/eigenvalue
2) Very large matrix eigensolution

e) 1) Eigensolver for large matrices
2) Very large system needing multiple mode solutions
3) Shifting - to allow inverse of K