ME 712, Finite Element Method Applications Final Exam, Spring 2005

Open book, closed notes. One formula sheet to be turned in. Problems must be in order in the blue books.

1. Itemize every *stage* from an actual mechanical part to the actual stress prediction using finite elements where errors are incurred, slight or not. One or two sentences/statements for each assumption in the process, maximum! *I am looking for* ***generic*** *statements, not a list of all possible specific cases where an error could occur in the modeling process.* **Nothing should be specific to a case or an element type!**

2. Some of the following matrices have fundamental flaws that violate certain conditions. For each matrix, identify whether the matrix is flawed, and what the flaw/s is/are.

   (a)
   $$K = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

   (b)
   $$M = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

   (c)
   $$K = \begin{bmatrix} 24 & -12 & 0 & 6 \\ -12 & 12 & -6 & -6 \\ 0 & -6 & 2 & 4 \\ 6 & -6 & 4 & 4 \end{bmatrix}$$

   (d)
   $$M = \begin{bmatrix} 24 & -12 & 0 & -6 \\ -12 & 12 & -6 & -6 \\ 6 & -6 & 12 & 4 \\ -6 & -6 & 4 & 12 \end{bmatrix}$$

3. Obtain the stiffness matrix of a rod (extension: 1-D) using 1 quadratic element with a mid-node located at $L/4$. Use Gauss integration to derive the element matrices. Assume a length l, density ρ, cross sectional area of A, and a modulus of E. Print out any code that you may write to solve this problem.

4. Find the strain at (x, y)=(0,0) of a bilinear quadrilateral (Q4) element with nodes 1-4 at (0,0), (1,0), (1,2), and (0,2) in terms of u_2 and v_2 (presume all other nodal displacements are zero).

5. Download the file mk.mat. Load it into matlab. Type *diary on* to save a diary (to be turned in). Note that the mass and stiffness matrices are in sparse form. You can use the **full** command to put them “regular” form, and the **sparse** command to return them. You may want to look at them using the **spy** command to see where non-zero elements are.

Coordinates 385-420 are Lagrange multipliers. The command **eigs** can be used to solve the sparse eigenvalue problem.

(a) Outline your planned solution approach (treating this as a large eigenproblem), including itemizing all necessary checks and matrix adjustments that you may deem necessary to obtain a solution. Define every step, including simple math operations not embedded inside other processes.

(b) Find the first 5 non-zero natural frequencies using techniques taught in class.

Hint (to select specified columns and rows of a matrix):
a=[1 2 3 4; 5 6 7 8; 9 10 11 12;13 14 15 16]
a =
1 2 3 4
5 6 7 8
9 10 11 12
13 14 15 16
b=a([1 3:4],(1 3:4])
returns
b =
1 3 4
9 11 12
13 15 16