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Name

Closed book, closed notes. Use $8\frac{1}{2} \times 11$ formula sheet from web and turn in with exam (nothing else may be written on the formula sheet). All solutions must be in this document. Calculators allowed. Knowing how to use them well is highly recommended.

Problem 5 is required for graduate students, bonus for undergraduates (worth 20% of exam points).

1. What's wrong with these statements, if anything (1 point each).

   1.) Determine the Fourier Series representation of the function shown below…

      a. The average is clearly 1/2.
      b. a_0 must be a constant
      c. The coefficient to the sine or cosine in a Fourier Series is a constant, not a function of time
      d. Function for one cycle is missing
      e. Function is there, but is wrong.

   2.) Given air damping, viscous damping, and Coulomb damping, determine which (may be one OR two) is apparent in the following response. **Prove it.** You answer will be graded on the merit of your explanation. No points will be given for a guess without sufficient explanation.

      Statements:

      a. Log decrement not being constant eliminates only purely viscous. Not being a straight line only eliminates purely Coulomb damping. Viscous+Coulomb, Viscous plus air, and air damping are still possibilities with those two facts.
      b. Log decrement can't be negative
      c. Log decrement decreases for air damping
      d. True.
      e. Incomprehensible. (and there is no such thing as a "log decrement of a frequency")

   3.) For the system defined by

$$M = \begin{bmatrix} 10 & 0 \\ 0 & 100 \end{bmatrix}, \qquad K = \begin{bmatrix} 10 & -10 \\ -10 & 100 \end{bmatrix} \tag{1}$$

      Statements:

      a. 2 DOF means 2 frequencies. It's that simple.
      b. A natural frequency isn't a matrix.
      c. A mode shape can't be zeros. (Trivial solution)
      d. ω_1 is always the lowest natural frequency
      e. Why have a formula that gives you back the same thing and have a new name for it?

2. (5 points: 3 points for m and k, 2 points for c) Initiate design of a vibration absorber by selecting

   (a) $0.05m < 0.1$
   (b) $k =$ what ever number with a) that gives a natural frequency of 150

(c) $c = 0$. Additional damping is bad, start hoping for zero.

3. A linear system is governed by the following equation of motion...

   a. (2 points) Fourth term of the Fourier series
   $\frac{-8}{\pi^2}\frac{1}{49}\cos\frac{14\pi}{5}t$

   b. (4 points) $H(i\omega)$ for the system at each frequency necessary (0.5 point each magnitude/phase).

Frequency	Amplitude	Phase (degrees)
$\frac{2\pi}{5}$	1.067	-0.019 (0)
$\frac{6\pi}{5}$	2.316	-1.14
$\frac{10\pi}{5}$	1.726	183.9
$\frac{14\pi}{5}$	0.477	183.0

   c. (4 points) find the first 4 terms of $x(t)$ (1 point per term).

      i. term 1: $-0.865\cos\left(\frac{2\pi}{5} - 0.019°\right)$ (either label degrees or use radians...)

      ii. term 2: $-0.209\cos\left(\frac{6\pi}{5} - 1.14°\right)$

      iii. term 3: $-0.056\cos\left(\frac{10\pi}{5} + 183.9°\right)$

      iv. term 4: $-0.008\cos\left(\frac{14\pi}{5} + 183.0°\right)$

4. a. (3 points) Modal force vector

$$f_m(t) = \begin{bmatrix} \frac{1}{\sqrt{2}}\sin(3t) \\ -\frac{1}{\sqrt{2}}\sin(3t) \end{bmatrix}$$

   b. (4 points) $\mathbf{r}(t)$ Note: the second modal force is zero, so $r_2(t) = 0$ very easily

$$\mathbf{r}(t) = \begin{bmatrix} \frac{\frac{1}{\sqrt{2}}}{2^2 - 4^2}\cos 4t \\ 0 \end{bmatrix} = \begin{bmatrix} -0.0714\cos 4t \\ 0 \end{bmatrix}$$

   c. (3 points) $\mathbf{x}(t)$

$$\mathbf{x}(t) = \begin{bmatrix} 3.535\cos 4t \\ 3.535\cos 4t \end{bmatrix}$$

5. *Grad student/bonus* (20% of other points) Determine the first natural frequency and mode shape for a clamped-free beam. The equation of motion of a beam is
$\left(\frac{EI}{\rho A}\right)\frac{\partial^4 w(x,t)}{\partial x^4} + \frac{\partial^2 w(x,t)}{\partial t^2} = 0$. Yes, you have to obtain all constants that can be obtained.

   Solution method shown in an example, and this answer is listed in the table.