

Simulink and we will introduce more in later chapters. In addition, some blocks have additional properties that we have not mentioned. However, the examples given here will help you get started in exploring the other features of Simulink. Consult the online help for information about these items.

5.7 CHAPTER REVIEW

Part I of the chapter covers block diagrams and state-variable models. The Laplace transform enables an algebraic description of a dynamic system model to be developed. This model form, the transfer function, is the basis of a graphical description of the system, called the block diagram (Section 5.1). Thus a block diagram is a "picture" of the algebraic description of the system that shows how the subsystem elements interact with the other subsystems. Block diagrams can be developed either from the differential equation model or from a schematic diagram that shows how the system components are connected.

The state-variable model form, which can be expressed as a vector-matrix equation, is a concise representation that is useful for analytical purposes and for writing general-purpose computer programs. Section 5.2 shows how to convert models into state-variable form.

In theory it is possible to use the Laplace transform to obtain the closed-form solution of a linear constant-coefficient differential equation if the input function is not too complicated. However, the Laplace transform method cannot be used when the Laplace transform or inverse transform either does not exist or cannot be found easily, as is the case with higher-order models. The reasons include the large amount of algebra required and the need to solve the characteristic equation numerically (closed-form solutions for polynomial roots do not exist for polynomials of order five and higher). Therefore, in Part II of this chapter we introduced several types of numerical methods for solving differential equations.

Section 5.3 focuses on MATLAB functions for solving linear state-variable models. These are the `ss`, `ssdata`, `tfdata`, `step`, `impulse`, `lsim`, `initial`, and `eig` functions. Section 5.4 treats the MATLAB `ode` functions, which are useful for solving both linear and nonlinear equations.

Part III covers Simulink, which provides a graphical user interface for solving differential equations. It includes many program blocks and features that enable you to create simulations that are otherwise difficult to program in MATLAB. Section 5.5 treats applications to linear systems, while Section 5.6 treats nonlinear system applications.

Now that you have finished this chapter, you should be able to

1. Draw a block diagram either from the differential equation model or from a schematic diagram.
2. Obtain the differential equation model from a block diagram.
3. Convert a differential equation model into state-variable form.
4. Express a linear state-variable model in the standard vector-matrix form.
5. Apply the `ss`, `ssdata`, `tfdata`, `eig`, and `initial` functions to analyze linear models.
6. Use the MATLAB `ode` functions to solve linear and nonlinear differential equations.
7. Use Simulink to create simulations of linear and nonlinear models expressed either as differential equations or, if linear, as transfer functions.

REFERENCES

- [Palm, 1986] Palm, W. J. III, *Control Systems Engineering*, John Wiley & Sons, New York, 1986.
- [Palm, 2009] Palm, W. J. III, *A Concise Introduction to MATLAB*, McGraw-Hill, New York, 2009.
- [Palm, 2011] Palm, W. J. III, *Introduction to MATLAB for Engineers*, 3rd ed., McGraw-Hill, New York, 2011.

PROBLEMS

Section 5.1 Transfer Functions and Block Diagram Models

- 5.1 Obtain the transfer function $X(s)/F(s)$ from the block diagram shown in Figure P5.1.

Figure P5.1

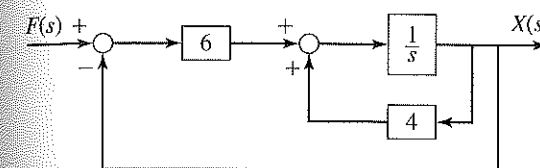
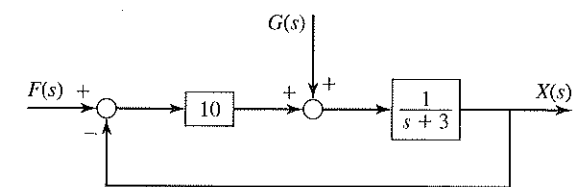


Figure P5.2



- 5.2 Obtain the transfer function $X(s)/F(s)$ from the block diagram shown in Figure P5.2.

- 5.3 Obtain the transfer function $X(s)/F(s)$ from the block diagram shown in Figure P5.3.

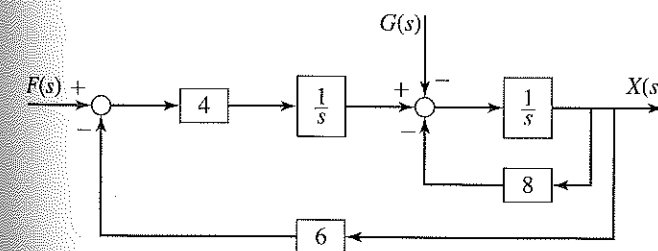


Figure P5.3

- 5.4 Draw a block diagram for the following equation. The output is $X(s)$; the inputs are $F(s)$ and $G(s)$.

$$5\ddot{x} + 3\dot{x} + 7x = 10f(t) - 4g(t)$$

- 5.5 Draw a block diagram for the following model. The output is $X(s)$; the inputs are $F(s)$ and $G(s)$. Indicate the location of $Y(s)$ on the diagram.

$$\dot{x} = y - 5x + g(t) \quad \dot{y} = 10f(t) - 30x$$

- 5.6 Referring to Figure P5.6, derive the expressions for the variables $C(s)$, $E(s)$, and $M(s)$ in terms of $R(s)$ and $D(s)$.

- 5.7 Referring to Figure P5.7, derive the expressions for the variables $C(s)$, $E(s)$, and $M(s)$ in terms of $R(s)$ and $D(s)$.

Figure P5.6

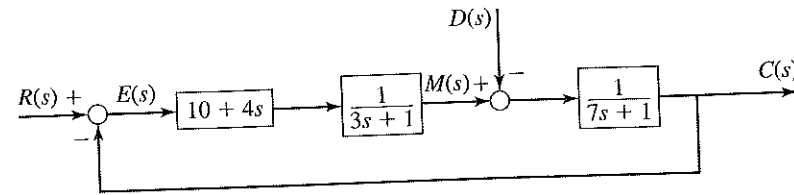
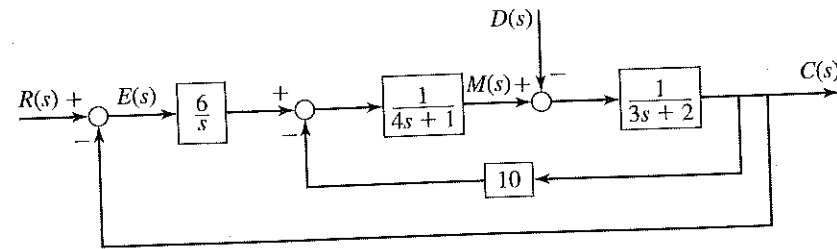


Figure P5.7



- 5.8 Use the MATLAB `series` and `feedback` functions to obtain the transfer functions $X(s)/F(s)$ and $X(s)/G(s)$ for the block diagram shown in Figure P5.3.
- 5.9 Use the MATLAB `series` and `feedback` functions to obtain the transfer functions $C(s)/R(s)$ and $C(s)/D(s)$ for the block diagram shown in Figure P5.6.
- 5.10 Use the MATLAB `series` and `feedback` functions to obtain the transfer functions $C(s)/R(s)$ and $C(s)/D(s)$ for the block diagram shown in Figure P5.7.

Section 5.2 State-Variable Models

- 5.11 Obtain the state model for the reduced-form model $5\ddot{x} + 7\dot{x} + 4x = f(t)$.
- 5.12 Obtain the state model for the reduced-form model
- $$2\frac{d^3y}{dt^3} + 5\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 7y = f(t)$$
- 5.13 Obtain the state model for the reduced-form model $2\ddot{x} + 5\dot{x} + 4x = 4y(t)$.
- 5.14 Obtain the state model for the transfer-function model
- $$\frac{Y(s)}{F(s)} = \frac{6}{3s^2 + 6s + 10}$$
- 5.15 Obtain the state model for the two-mass system whose equations of motion are
- $$m_1\ddot{x}_1 + k_1(x_1 - x_2) = f(t)$$
- $$m_2\ddot{x}_2 - k_1(x_1 - x_2) + k_2x_2 = 0$$
- 5.16 Obtain the state model for the two-mass system whose equations of motion for specific values of the spring and damping constants are
- $$10\ddot{x}_1 + 8\dot{x}_1 - 5\dot{x}_2 + 40x_1 - 25x_2 = 0$$
- $$5\ddot{x}_2 - 25x_1 + 25x_2 - 5\dot{x}_1 + 5\dot{x}_2 = f(t)$$
- 5.17 Put the following model in standard state-variable form and obtain the expressions for the matrices **A**, **B**, **C**, and **D**. The output is x .
- $$2\ddot{x} + 5\dot{x} + 4x = 4y(t)$$

- 5.18 Given the state-variable model

$$\dot{x}_1 = -5x_1 + 3x_2 + 2u_1$$

$$\dot{x}_2 = -4x_2 + 6u_2$$

and the output equations

$$y_1 = x_1 + 3x_2 + 2u_1$$

$$y_2 = x_2$$

obtain the expressions for the matrices **A**, **B**, **C**, and **D**.

- 5.19 Given the following state-variable models, obtain the expressions for the matrices **A**, **B**, **C**, and **D** for the given inputs and outputs.
- a. The outputs are x_1 and x_2 ; the input is u .

$$\dot{x}_1 = -5x_1 + 3x_2$$

$$\dot{x}_2 = x_1 - 4x_2 + 5u$$

- b. The output is x_1 ; the inputs are u_1 and u_2 .

$$\dot{x}_1 = -5x_1 + 3x_2 + 4u_1$$

$$\dot{x}_2 = x_1 - 4x_2 + 5u_2$$

- 5.20 Obtain the expressions for the matrices **A**, **B**, **C**, and **D** for the state-variable model you obtained in Problem 5.16. The outputs are x_1 and x_2 .

- 5.21 The transfer function of a certain system is

$$\frac{Y(s)}{F(s)} = \frac{6s + 7}{s + 3}$$

Use two methods to obtain a state-variable model in standard form. For each model, relate the initial value of the state-variable to the given initial value $y(0)$.

- 5.22 The transfer function of a certain system is

$$\frac{Y(s)}{F(s)} = \frac{s + 2}{s^2 + 4s + 3}$$

Use two methods to obtain a state-variable model in standard form. For each model, relate the initial values of the state variables to the given initial values $y(0)$ and $\dot{y}(0)$.

Section 5.3 State-Variable Methods with MATLAB

- 5.23 Use MATLAB to create a state-variable model; obtain the expressions for the matrices **A**, **B**, **C**, and **D**, and then find the transfer functions of the following models, for the given inputs and outputs.

- a. The outputs are x_1 and x_2 ; the input is u .

$$\dot{x}_1 = -5x_1 + 3x_2$$

$$\dot{x}_2 = x_1 - 4x_2 + 5u$$

- b. The output is x_1 ; the inputs are u_1 and u_2 .

$$\dot{x}_1 = -5x_1 + 3x_2 + 4u_1$$

$$\dot{x}_2 = x_1 - 4x_2 + 5u_2$$

- 5.24 Use MATLAB to obtain a state model for the following equations; obtain the expressions for the matrices **A**, **B**, **C**, and **D**. In both cases, the input is $f(t)$; the output is y .

a.

$$2\frac{d^3y}{dt^3} + 5\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 7y = f(t)$$

b.

$$\frac{Y(s)}{F(s)} = \frac{6}{3s^2 + 6s + 10}$$

- 5.25 Use MATLAB to obtain a state-variable model for the following transfer functions.

a.

$$\frac{Y(s)}{F(s)} = \frac{6s + 7}{s + 3}$$

b.

$$\frac{Y(s)}{F(s)} = \frac{s + 2}{s^2 + 4s + 3}$$

- 5.26 For the following model the output is x_1 and the input is $f(t)$.

$$\dot{x}_1 = -5x_1 + 3x_2$$

$$\dot{x}_2 = x_1 - 4x_2 + 5f(t)$$

- Use MATLAB to compute and plot the free response for $x_1(0) = 3$, and $x_2(0) = 5$.
- Use MATLAB to compute and plot the unit-step response for zero initial conditions.
- Use MATLAB to compute and plot the response for zero initial conditions with the input $f(t) = 3 \sin 10\pi t$, for $0 \leq t \leq 2$.
- Use MATLAB to compute and plot the total response using the initial conditions given in Part (a) and the forcing function given in part (c).

- 5.27 Given the state-variable model

$$\dot{x}_1 = -5x_1 + 3x_2 + 2u_1$$

$$\dot{x}_2 = -4x_2 + 6u_2$$

and the output equations

$$y_1 = x_1 + 3x_2 + 2u_1$$

$$y_2 = x_2$$

Use MATLAB to find the characteristic polynomial and the characteristic roots.

- 5.28 The equations of motion for a two-mass, quarter-car model of a suspension system are

$$m_1\ddot{x}_1 = c_1(\dot{x}_2 - \dot{x}_1) + k_1(x_2 - x_1)$$

$$m_2\ddot{x}_2 = -c_1(\dot{x}_2 - \dot{x}_1) - k_1(x_2 - x_1) + k_2(y - x_2)$$

Suppose the coefficient values are: $m_1 = 240$ kg, $m_2 = 36$ kg, $k_1 = 1.6 \times 10^4$ N/m, $k_2 = 1.6 \times 10^5$ N/m, $c_1 = 98$ N·s/m.

- Use MATLAB to create a state model. The input is $y(t)$; the outputs are x_1 and x_2 .

- Use MATLAB to compute and plot the response of x_1 and x_2 if the input $y(t)$ is a unit impulse and the initial conditions are zero.
- Use MATLAB to find the characteristic polynomial and the characteristic roots.
- Use MATLAB to obtain the transfer functions $X_1(s)/Y(s)$ and $X_2(s)/Y(s)$.

- 5.29 A representation of a car's suspension suitable for modeling the bounce and pitch motions is shown in Figure P5.29, which is a side view of the vehicle's body showing the front and rear suspensions. Assume that the car's motion is constrained to a vertical translation x of the mass center and rotation θ about a single axis which is perpendicular to the page. The body's mass is m and its moment of inertia about the mass center is I_G . As usual, x and θ are the displacements from the equilibrium position corresponding to $y_1 = y_2 = 0$. The displacements $y_1(t)$ and $y_2(t)$ can be found knowing the vehicle's speed and the road surface profile.

- Assume that x and θ are small, and derive the equations of motion for the bounce motion x and pitch motion θ .
- For the values $k_1 = 1100$ lb/ft, $k_2 = 1525$ lb/ft, $c_1 = c_2 = 4$ lb-sec/ft, $L_1 = 4.8$ ft, $L_2 = 3.6$ ft, $m = 50$ slugs, and $I_G = 1000$ slug-ft², use MATLAB to obtain a state-variable model in standard form.
- Use MATLAB to obtain and plot the solution for $x(t)$ and $\theta(t)$ when $y_1 = 0$ and y_2 is a unit impulse. The initial conditions are zero.

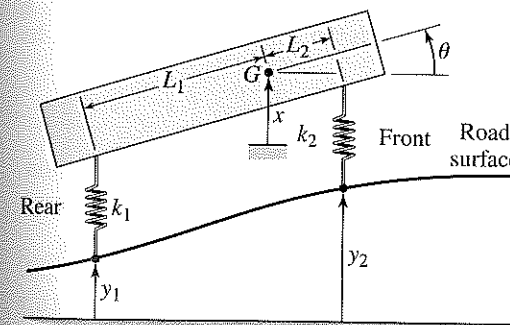


Figure P5.29

Section 5.4 The MATLAB ode Functions

- 5.30 a. Use a MATLAB ode function to solve the following equation for $0 \leq t \leq 12$. Plot the solution.

$$\dot{y} = \cos t \quad y(0) = 6$$

- Use the closed-form solution to check the accuracy of the numerical method.

- 5.31 a. Use a MATLAB ode function to solve the following equation for $0 \leq t \leq 1$. Plot the solution.

$$\dot{y} = 5e^{-4t} \quad y(0) = 2$$

- Use the closed-form solution to check the accuracy of the numerical method.

- 5.32 a. Use a MATLAB `ode` function to solve the following equation for $0 \leq t \leq 1$. Plot the solution.

$$\dot{y} + 3y = 5e^{4t} \quad y(0) = 10$$

- b. Use the closed-form solution to check the accuracy of the numerical method.

- 5.33 a. Use a MATLAB `ode` function to solve the following nonlinear equation for $0 \leq t \leq 4$. Plot the solution.

$$\dot{y} + \sin y = 0 \quad y(0) = 0.1 \quad (1)$$

- b. For small angles, $\sin y \approx y$. Use this fact to obtain a linear equation that approximates equation (1). Use the closed-form solution of this linear equation to check the output of your program.

- 5.34 Sometimes it is tedious to obtain a solution of a linear equation, especially if all we need is a plot of the solution. In such cases, a numerical method might be preferred. Use a MATLAB `ode` function to solve the following equation for $0 \leq t \leq 7$. Plot the solution.

$$\dot{y} + 2y = f(t) \quad y(0) = 2$$

where

$$f(t) = \begin{cases} 3t & \text{for } 0 \leq t \leq 2 \\ 6 & \text{for } 2 \leq t \leq 5 \\ -3(t-5) + 6 & \text{for } 5 \leq t \leq 7 \end{cases}$$

- 5.35 A certain jet-powered ground vehicle is subjected to a nonlinear drag force. Its equation of motion, in British units, is

$$50\dot{v} = f - (20v + 0.05v^2)$$

Use a numerical method to solve for and plot the vehicle's speed as a function of time if the jet's force is constant at 8000 lb and the vehicle starts from rest.

- 5.36 The following model describes a mass supported by a nonlinear, hardening spring. The units are SI. Use $g = 9.81 \text{ m/s}^2$.

$$5\ddot{y} = 5g - (900y + 1700y^3)$$

Suppose that $\dot{y}(0) = 0$. Use a numerical method to solve for and plot the solution for two different initial conditions: (1) $y(0) = 0.06$ and (2) $y(0) = 0.1$.

- 5.37 Van der Pol's equation is a nonlinear model for some oscillatory processes. It is

$$\ddot{y} - b(1 - y^2)\dot{y} + y = 0$$

Use a numerical method to solve for and plot the solution for the following cases:

1. $b = 0.1$, $y(0) = \dot{y}(0) = 1$, $0 \leq t \leq 25$
2. $b = 0.1$, $y(0) = \dot{y}(0) = 3$, $0 \leq t \leq 25$
3. $b = 3$, $y(0) = \dot{y}(0) = 1$, $0 \leq t \leq 25$

- 5.38 Van der Pol's equation is

$$\ddot{y} - b(1 - y^2)\dot{y} + y = 0$$

This equation can be difficult to solve for large values of the parameter b . Use $b = 1000$ and $0 \leq t \leq 3000$, with the initial conditions $y(0) = 2$ and $\dot{y}(0) = 0$. Use `ode45` to plot the response.

- 5.39 The equation of motion for a pendulum whose base is accelerating horizontally with an acceleration $a(t)$ is

$$L\ddot{\theta} + g \sin \theta = a(t) \cos \theta$$

Suppose that $g = 9.81 \text{ m/s}^2$, $L = 1 \text{ m}$, and $\dot{\theta}(0) = 0$. Solve for and plot $\theta(t)$ for $0 \leq t \leq 10 \text{ s}$ for the following three cases.

- a. The acceleration is constant: $a = 5 \text{ m/s}^2$, and $\theta(0) = 0.5 \text{ rad}$.
- b. The acceleration is constant: $a = 5 \text{ m/s}^2$, and $\theta(0) = 3 \text{ rad}$.
- c. The acceleration is linear with time: $a = 0.5t \text{ m/s}^2$, and $\theta(0) = 3 \text{ rad}$.

- 5.40 Suppose the spring in Figure P5.40 is nonlinear and is described by the cubic force-displacement relation. The equation of motion is

$$m\ddot{x} = c(\dot{y} - \dot{x}) + k_1(y - x) + k_2(y - x)^3$$

where $m = 100$, $c = 600$, $k_1 = 8000$, and $k_2 = 24000$. Approximate the unit-step input $y(t)$ with $y(t) = 1 - e^{-t/\tau}$, where τ is chosen to be small compared to the period and time constant of the model when the cubic term is neglected. Use MATLAB to plot the forced response $x(t)$.

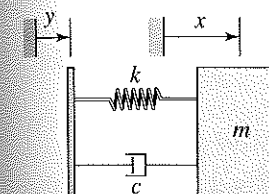


Figure P5.40

Section 5.5 Simulink and Linear Models

- 5.41 Create a Simulink model to plot the solution of the following equation for $0 \leq t \leq 6$.

$$10\ddot{y} = 7 \sin 4t + 5 \cos 3t \quad y(0) = 4 \quad \dot{y}(0) = 1$$

- 5.42 A projectile is launched with a velocity of 100 m/s at an angle of 30° above the horizontal. Create a Simulink model to solve the projectile's equations of motion, where x and y are the horizontal and vertical displacements of the projectile.

$$\begin{aligned} \ddot{x} &= 0 & x(0) &= 0 & \dot{x}(0) &= 100 \cos 30^\circ \\ \ddot{y} &= -g & y(0) &= 0 & \dot{y}(0) &= 100 \sin 30^\circ \end{aligned}$$

Use the model to plot the projectile's trajectory y versus x for $0 \leq t \leq 10 \text{ s}$.

- 5.43 In Chapter 2 we obtained an approximate solution of the following problem, which has no analytical solution even though it is linear.

$$\dot{x} + x = \tan t \quad x(0) = 0$$

The approximate solution, which is less accurate for large values of t , is

$$x(t) = \frac{1}{3}t^3 - t^2 + 3t - 3 + 3e^{-t}$$

Create a Simulink model to solve this problem and compare its solution with the approximate solution over the range $0 \leq t \leq 1$.

- 5.44 Construct a Simulink model to plot the solution of the following equation for $0 \leq t \leq 10$.

$$15\dot{x} + 5x = 4u_s(t) - 4u_s(t-2) \quad x(0) = 2$$

- 5.45 Use Simulink to solve Problem 5.18 for zero initial conditions, u_1 a unit-step input, and $u_2 = 0$.

- 5.46 Use Simulink to solve Problem 5.18 for the initial conditions $x_1(0) = 4$, $x_2(0) = 3$, and $u_1 = u_2 = 0$.

- 5.47 Use Simulink to solve Problem 5.19a for zero initial conditions and $u = 3 \sin 2t$.

- 5.48 Use Simulink to solve Problem 5.26c.

Section 5.6 Simulink and Nonlinear Models

- 5.49 Use the Transfer Function block to construct a Simulink model to plot the solution of the following equation for $0 \leq t \leq 4$.

$$2\ddot{x} + 12\dot{x} + 10x^2 = 5u_s(t) - 5u_s(t-2) \quad x(0) = \dot{x}(0) = 0$$

- 5.50 Construct a Simulink model to plot the solution of the following equation for $0 \leq t \leq 4$.

$$2\ddot{x} + 12\dot{x} + 10x^2 = 5 \sin 0.8t \quad x(0) = \dot{x}(0) = 0$$

- 5.51 Use the Saturation block to create a Simulink model to plot the solution of the following equation for $0 \leq t \leq 6$.

$$3\dot{y} + y = f(t) \quad y(0) = 2$$

where

$$f(t) = \begin{cases} 8 & \text{if } 10 \sin 3t > 8 \\ -8 & \text{if } 10 \sin 3t < -8 \\ 10 \sin 3t & \text{otherwise} \end{cases}$$

- 5.52 Construct a Simulink model of the following problem.

$$5\dot{x} + \sin x = f(t) \quad x(0) = 0$$

The forcing function is

$$f(t) = \begin{cases} -5 & \text{if } g(t) \leq -5 \\ g(t) & \text{if } -5 \leq g(t) \leq 5 \\ 5 & \text{if } g(t) \geq 5 \end{cases}$$

where $g(t) = 10 \sin 4t$.

- 5.53 Create a Simulink model to plot the solution of the following equation for $0 \leq t \leq 3$.

$$\dot{x} + 10x^2 = 2 \sin 4t \quad x(0) = 1$$

- 5.54 Construct a Simulink model of the following problem.

$$10\dot{x} + \sin x = f(t) \quad x(0) = 0$$

The forcing function is $f(t) = \sin 2t$. The system has the dead-zone nonlinearity shown in Figure 5.6.6.

- 5.55 The following model describes a mass supported by a nonlinear, hardening spring. The units are SI. Use $g = 9.81 \text{ m/s}^2$.

$$5\ddot{y} = 5g - (900y + 1700y^3) \quad y(0) = 0.5 \quad \dot{y}(0) = 0$$

Create a Simulink model to plot the solution for $0 \leq t \leq 2$.

- 5.56 Consider the system for lifting a mast, discussed in Chapter 3 and shown again in Figure P5.56. The 70-ft-long mast weighs 500 lb. The winch applies a force $f = 380 \text{ lb}$ to the cable. The mast is supported initially at an angle of $\theta = 30^\circ$, and the cable at A is initially horizontal. The equation of motion of the mast is

$$25,400\ddot{\theta} = -17,500 \cos \theta + \frac{626,000}{Q} \sin(1.33 + \theta)$$

where

$$Q = \sqrt{2020 + 1650 \cos(1.33 + \theta)}$$

Create and run a Simulink model to solve for and plot $\theta(t)$ for $\theta(t) \leq \pi/2 \text{ rad}$.

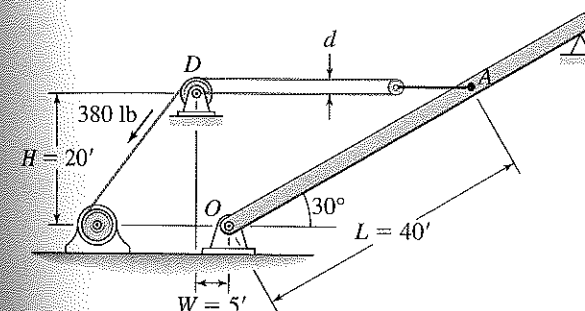


Figure P5.56

- 5.57 A certain mass, $m = 2 \text{ kg}$, moves on a surface inclined at an angle $\phi = 30^\circ$ above the horizontal. Its initial velocity is $v(0) = 3 \text{ m/s}$ up the incline. An external force of $f_1 = 5 \text{ N}$ acts on it parallel to and up the incline. The coefficient of dynamic friction is $\mu_d = 0.5$. Use the Coulomb Friction block or the Sign block and create a Simulink model to solve for the velocity of the mass until the mass comes to rest. Use the model to determine the time at which the mass comes to rest.

- 5.58 If a mass-spring system has Coulomb friction on the horizontal surface rather than viscous friction, its equation of motion is

$$m\ddot{y} = -ky + f(t) - \mu mg \quad \text{if } \dot{y} \geq 0$$

$$m\ddot{y} = -ky + f(t) + \mu mg \quad \text{if } \dot{y} < 0$$

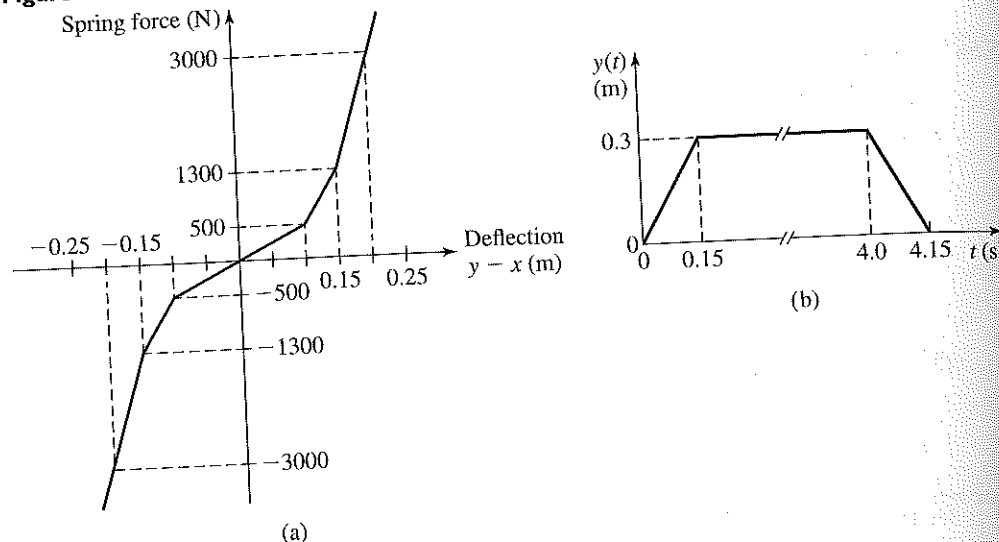
where μ is the coefficient of friction. Develop a Simulink model for the case where $m = 1 \text{ kg}$, $k = 5 \text{ N/m}$, $\mu = 0.4$, and $g = 9.8 \text{ m/s}^2$. Run the simulation for two cases: (a) the applied force $f(t)$ is a step function with a magnitude of 10 N , and (b) the applied force is sinusoidal: $f(t) = 10 \sin 2.5t$.

- 5.59 Redo the Simulink suspension model developed in subsection 5.6.2, using the spring relation and input function shown in Figure P5.59, and the following damper relation.

$$f_d(v) = \begin{cases} -500|v|^{1.2} & v \leq 0 \\ 50v^{1.2} & v > 0 \end{cases}$$

Use the simulation to plot the response. Evaluate the overshoot and undershoot.

Figure P5.59



- 5.60 Consider the system shown in Figure P5.60. The equations of motion are

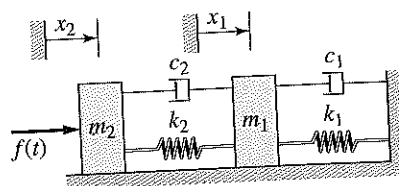
$$\begin{aligned} m_1 \ddot{x}_1 + (c_1 + c_2) \dot{x}_1 + (k_1 + k_2)x_1 - c_2 \dot{x}_2 - k_2 x_2 &= 0 \\ m_2 \ddot{x}_2 + c_2 \dot{x}_2 + k_2 x_2 - c_2 \dot{x}_1 - k_2 x_1 &= f(t) \end{aligned}$$

Suppose that $m_1 = m_2 = 1$, $c_1 = 3$, $c_2 = 1$, $k_1 = 1$, and $k_2 = 4$.

- Develop a Simulink simulation of this system. In doing this, consider whether to use a state-variable representation or a transfer function representation of the model.
- Use the Simulink program to plot the response $x_1(t)$ for the following input. The initial conditions are zero.

$$f(t) = \begin{cases} t & 0 \leq t \leq 1 \\ 2 - t & 1 < t < 2 \\ 0 & t \geq 2 \end{cases}$$

Figure P5.60



Electrical and Electromechanical Systems

CHAPTER OUTLINE

CHAPTER OBJECTIVES

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When you have finished this chapter, you should be able to

- Develop models of electrical circuits. This includes
 - Application of Kirchhoff's voltage and current laws,
 - Application of the voltage- and current-divider rules,
 - Use of loop analysis,
 - Selection of appropriate state variables, and
 - Use of transformed equations and impedance.
- Obtain circuit models in transfer-function state variable, and block-diagram form.
- Apply impedance methods to obtain models of circuits and op-amp systems.
- Apply Newton's laws, electrical circuit laws, and electromagnetic principles to develop models of electromechanical systems, including dc motors and sensors.
- Assess the performance of motors and amplifiers when used in motion control systems.
- Apply MATLAB and Simulink to analyze models of circuits and electromechanical systems in state variable, transfer function, and block diagram form.

The majority of engineering systems now have at least one electrical subsystem. This may be a power supply, sensor, motor, controller, or an acoustic device such as a speaker. So an understanding of electrical systems is essential to understanding the behavior of many systems.