Exam 1 Sp '06 Solns

1) $\epsilon_x = \frac{\partial u}{\partial x} = a_2 + a_4 y$

ϵ_x may have $a_2 = a_4 = 0$ in one element and $a_2 \neq 0$ with $a_4 = 0$ in an adjacent element, thus continuity of strain is unlikely.

2) $$K = \int_0^l \frac{d^2}{dx^2} N^T EI \frac{d^2}{dx^2} N \, dx \longleftarrow \checkmark$$

$$K_{14} = \int_0^l \frac{d^2}{dx^2} N_1 E \left(I_1 + (I_2 - I_1)\frac{x}{l} \right) \frac{d^2}{dx^2} N_4 \, dx$$

$$= E \int_0^l \left(\frac{-6}{l^2} + \frac{12x}{l^3} \right) \left(I_1 + (I_2 - I_1)\frac{x}{l} \right) \left(\frac{-2}{l} + \frac{6x}{l^2} \right) dx$$

$$= E \int_0^l \frac{12 I_1}{l^3} + \left(\frac{-72 I_1}{l^4} + \frac{12 I_2}{l^4} \right) x + \left(\frac{132 I_1}{l^5} - \frac{60 I_2}{l^5} \right) x^2 + \left(\frac{72 I_2}{l^6} - \frac{72 I_1}{l^6} \right) x^3 \, dx$$

$$= \frac{2E}{l^2} \left(I_1 + 2 I_2 \right)$$

3) $N_1 = \left(1 - \frac{x}{\ell}\right)$ $N_2 = \frac{x}{\ell}$

$u(x) = u_1 N_1 + u_2 N_2$

$u' = u_1 N_1' + u_2 N_2' = \underbrace{\begin{bmatrix} -\frac{1}{\ell} & \frac{1}{\ell} \end{bmatrix}}_{N'} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$

There is a typo. Clearly the 1st term should be $\frac{1}{2} E A \frac{du}{dx}^2$

Also, for the potential field

$V' = \begin{bmatrix} -\frac{1}{\ell} & \frac{1}{\ell} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

Integrating over the element, taking the variation of things that can vary.

$$\delta U = \delta \underline{u} \int_0^\ell N'^T E A N' dx\, \underline{u} - \delta \underline{V} \int_0^\ell N'^T e A N' dx\, \underline{u}$$

$$- \delta \underline{u} \int_0^\ell N' e A N' dx\, \underline{V} - \delta \underline{V} \int_0^\ell N'^T \epsilon A N' dx\, \underline{V}$$

Factoring coefficients and performing integrals

$$\frac{EA}{\ell} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \underline{u} - \frac{eA}{\ell} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \underline{V} = \underline{0}$$

and

$$\frac{eA}{\ell} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \underline{u} - \frac{\epsilon A}{\ell} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \underline{V} = \underline{0}$$

4) For the left end

$$K_a = \frac{EI}{a^3}\begin{bmatrix} 12 & -6a \\ -6a & 4a^2 \end{bmatrix}$$

For the right end

$$K_b = \frac{EI}{b^3}\begin{bmatrix} 12 & 6b \\ 6b & 4b^2 \end{bmatrix}$$

Assembled

$$K = EI \begin{array}{c} \\ \\ \end{array} \begin{array}{cccc} v & \theta_a & \theta_b & \\ \end{array}$$

$$K = EI\begin{bmatrix} 12\left(\frac{1}{a^3}+\frac{1}{b^3}\right) & \frac{-6}{a^2} & \frac{6}{b^2} \\ \frac{-6}{a^2} & \frac{4}{a} & 0 \\ \frac{6}{b^2} & 0 & \frac{4}{b} \end{bmatrix} \begin{array}{c} v \\ \theta_a \\ \theta_b \end{array}$$

5) Plane strain is the state under which no strain is allowed out of the plane.
An example is any object of uniform cross section constrained between to rigid boundaries and subject to