1) $N_1(\xi) = a_1 \xi^2 + a_2 \xi + a_3$

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

For N_2, left vector is $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, for N_3, $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$N_1 = \frac{1}{2}(\xi^2 - \xi)$, $N_2 = 1 - \xi^2$, $N_3 = \frac{1}{2}(\xi + \xi^2)$

$\dfrac{dN_1}{d\xi} = \xi - \frac{1}{2}$, $\dfrac{dN_2}{d\xi} = -2\xi$, $\dfrac{dN_3}{d\xi} = \frac{1}{2} + \xi$

$$K = \int_{-1}^{1} \frac{\partial N}{\partial \xi} A E \frac{\partial N}{\partial \xi}^T \det(J) \, d\xi \qquad \det(J) = \frac{\ell}{2}$$

Order of polynomial is 2, 2 point integration.

$$K = \frac{\ell}{2} \cdot \begin{bmatrix} 1.167 & -1.333 & 0.1667 \\ & 2.667 & -1.333 \\ \text{sym} & & 1.167 \end{bmatrix}$$

2) Polynomial is 4th order, 3 points necessary

$$M = \int_{-1}^{1} N \, A \, \rho \, N^T \, \det(J) \, d\xi$$

$$M = \frac{\ell}{2} \begin{bmatrix} 0.48 & 0.24 & -0.12 \\ 0.24 & 1.32 & 0.24 \\ -0.12 & 0.24 & 0.48 \end{bmatrix}$$

3) No BC on nodes 5-8

For node 1

$$\begin{bmatrix} 1 & 0 & 0 & & & & & & & & & \\ 0 & 1 & 0 & & & & & & & & & \\ 0 & 0 & 1 & & & & & & & & & \\ & & & 1 & & & & & & & & \\ & & & & 1 & & & & & & & \\ & & & & & 0 & & & & & & \\ & & & & & & 1 & & & & & \\ & & & & & & & 0 & & & & \\ & & & & & & & & 0 & & & \\ & & & & & & & & & 1 & & \\ & & & & & & & & & & 0 & \\ & & & & & & & & & & & 0 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \\ d_7 \\ d_8 \\ d_9 \\ d_{10} \\ d_{11} \\ d_{12} \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

4) See text. I'm making you look it up.