Matrix Iteration (Inman p 451) (Power Method)

Matrix Deflation

Consider a matrix A size $m \times m$

Consider a vector

$\underline{x} = \sum c_i \underline{u}_i$ $\quad$ c_i + $\underline{u}_i$ unknown
$\underline{u}_i$ are unknown eigenvectors

$\underline{x}_1 = A\underline{x} = c_1 A \underline{u}_1 + c_2 A \underline{u}_2 + \cdots$ (1st trial vector)

This can also be written $(A\underline{u} = \lambda \underline{u})$ $\quad$ λ

$\underline{x}_1 = c_1 \underbrace{\lambda_1}_{\text{eigenvalues}} \underbrace{\underline{u}_1}_{\text{eigenvectors}} + c_2 \lambda_2 \underline{u}_2 + \cdots$

Normalize $\underline{x}_1$ $\quad$ $\lambda_i = \frac{1}{\omega_i^2}$, $A = K^{-1} M$

$\underline{x}_2 = A \underline{x}_1$

$\vdots$

Normalize $\quad$ $\lambda I \underline{x} = A \underline{x}$

$\underline{x}_r = c_1 \lambda_1^r \underline{u}_1 + c_2 \lambda_2^r \underline{u}_2 + \cdots$

When this converges

$\underline{x}_{n+1} \approx c_1 \lambda_1^r \underline{x}_1$

Assuming $\lambda_m >> \lambda_{m+1}$, the m^{th} term of the summation dominates.

So $\underline{x}_{n+1} \approx c_1 \lambda_1 \underline{u}_1$

Then

$\underline{x}_{n+1} / \underline{x}_n = \lambda_1$

31.23

Example

$$K^{-1} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$\underline{x}_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad \underline{x}_1 = K^{-1}\underline{x}_0$$

$$\underline{x}_1 = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \qquad \underline{x}_1 = \begin{bmatrix} -.4472 \\ .8944 \end{bmatrix}$$

$$\underline{x}_2 = K^{-1}\underline{x}_1 = \begin{bmatrix} -.6247 \\ .7809 \end{bmatrix}$$ (after normalizing)

norm = 2.86

$$\underline{x}_3 = K^{-1}\underline{x}_2 = \begin{bmatrix} -.6805 \\ .7328 \end{bmatrix}$$ 2.98

$$\underline{x}_4 = K^{-1}\underline{x}_3 = \begin{bmatrix} -.6983 \\ .7158 \end{bmatrix}$$ 2.998

$$\underline{x}_5 = K^{-1}\underline{x}_4 = \begin{bmatrix} -.7042 \\ .7100 \end{bmatrix}$$ 2.9998

$$\underline{x}_6 = K^{-1}\underline{x}_5 = \begin{bmatrix} -.7061 \\ .7081 \end{bmatrix}$$ 3.0000

31.24

A new matrix $D\underline{1}$ is formed

$$D\underline{1} = K^{-1} - \mu_1 \underline{u}_1 \underline{u}_1^T$$

It will not have the eigenvalue μ_1

$$D\underline{1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

It has eigenvalues 0 and 1

The remaining eigenvalues can be determined by repeating this with the new matrix.