ME 460/660, Mechanical Vibration — Final, Fall 2010

Closed book, closed notes. Use $8\frac{1}{2} \times 11$ formula sheet from web and turn in with exam (nothing else may be written on the formula sheet). Test books will be provided. Calculators allowed. Knowing how to use them well is highly recommended.
Problem 5 is required for graduate students, bonus for undergraduates (worth 20% of exam points).

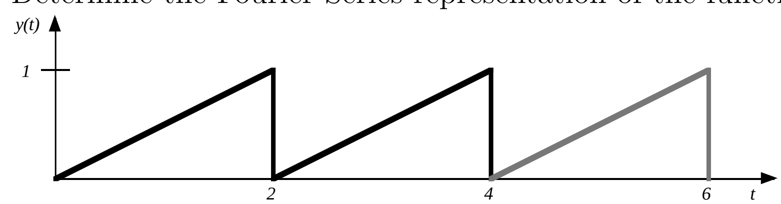
1. Parts of answers (statements within and answers) to prior exam questions are listed below each question. For each answer, without doing any calculations, identify:

   a) if the answer is definitely wrong, or possibly right,

   b) if definitely wrong/bad, **what makes the answer impossible.**

   Identifying the errors requires no calculation *and is necessary to earn a point.* They are all identifiable by observation. 1 point each, no partial credit. Be careful, some of them are right!

1.) Determine the Fourier Series representation of the function shown below.



Statements:

a. $f(t) = 2 + \ldots$

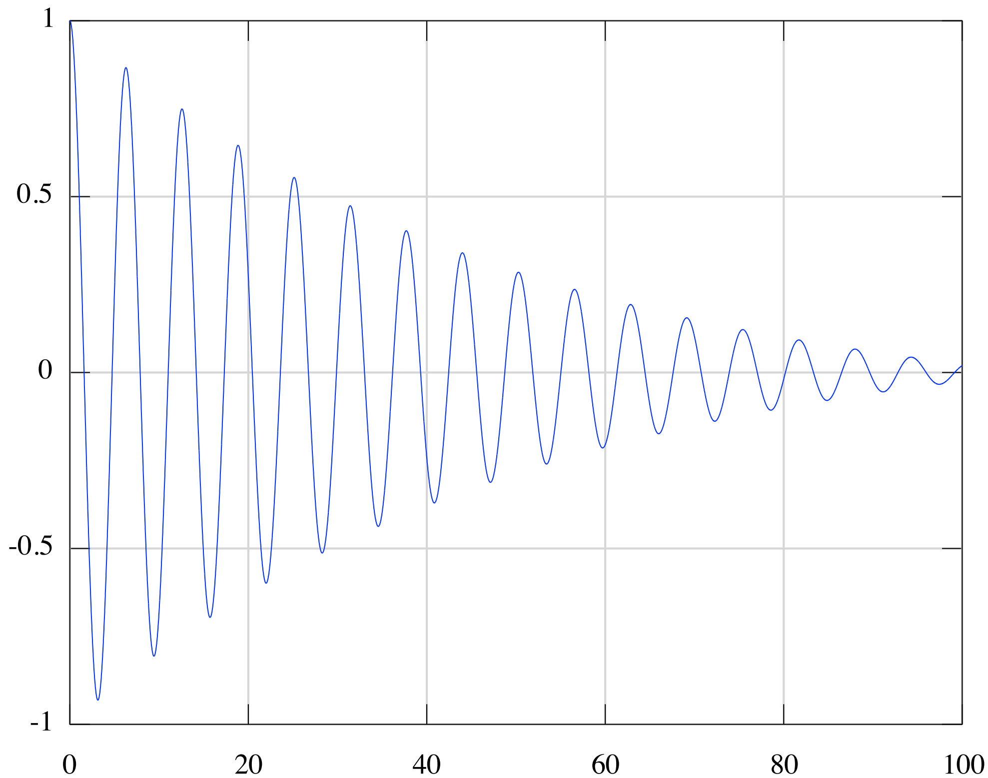
b. $a_0 = \frac{t^2}{4}$

c. $f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \frac{\cos n\pi t}{2n^2\pi^2} \cos n\pi t$

d. $a_n = \int_0^2 \cos n\pi t dt$

e. $b_n = \int_0^2 (1-t) \sin n\pi t dt$

2.) Given air damping, viscous damping, and Coulomb damping, determine which (may be one OR two) is apparent in the following response. **Prove it.** You answer will be graded on the merit of your explanation. No points will be given for a guess without sufficient explanation.



Statements:

a. Log decrement isn't constant, and the decay envelope isn't a straight line, so it must be air damping.

b. $\delta = -.3$

c. The log decrement is increasing so it must be air damping

d. Log decrement is increasing, but the decay envelope isn't a straight line, so it must be Coulomb damping plus viscous damping.

e. the decay envilope is non-liner therfor it is not coulomb damping. the log decriment of the fecency at the beginng is not equivlent to that at the end, therfor not viscus. [sic]

3.)

For the system defined by

$$M = \begin{bmatrix} 10 & 0 \\ 0 & 100 \end{bmatrix}, \qquad K = \begin{bmatrix} 10 & -10 \\ -10 & 100 \end{bmatrix} \tag{1}$$

i. Find the mass normalized mode shapes

ii. Find the natural frequencies

iii. Prove or disprove that the mass normalized mode shapes are

$$S = \begin{bmatrix} 0.953 & 0.953 \\ 0.302 & -0.302 \end{bmatrix} \tag{2}$$

Statements:

a. There is only one natural frequency

b. $\omega_1 = \begin{bmatrix} 0.987 & -0.0507 \\ -0.507 & 0.987 \end{bmatrix}$

c. $u_{11} = 0, u_{12} = 0$

d. $\omega_1 = 1.147$ rad/sec, $\omega_2 = 0.827$ rad/sec

e. $\tilde{K} = IKI$

2. (5 points: 3 points for m and k, 2 points for c) Initiate design of a vibration absorber by selecting m, c, and k (presume you can obtain any value you need, but make sure your values make sense) for the system defined by

$$10\ddot{x} + 0.1\dot{x} + 10,000x = 10\sin 150t + 3.5$$

3. A linear system is governed by the following equation of motion:

$$m\ddot{x} + c\dot{x} + kx = ky + c\dot{y}$$

where $m = 10$kg, $c = 1.0$kg/s, and $k = 250$kg/s^2. Given the first 3 terms of $y(t) = -\frac{8}{\pi^2}\left[\cos\frac{2\pi}{5}t + \frac{1}{9}\cos\frac{6\pi}{5}t + \frac{1}{25}\cos\frac{10\pi}{5}t + \cdots\right]$

a. (2 points) What is the fourth term of the Fourier series?

b. (4 points) Find $H(i\omega)$ for the system at each frequency necessary (1 point each).

c. (4 points) find the first 4 terms of $x(t)$ (1 point per term).

4. a. (3 points) An undamped system defined by

$$\begin{bmatrix} 9 & 0 \\ 0 & 27 \end{bmatrix}\ddot{\mathbf{x}} + \begin{bmatrix} 27 & -3 \\ -3 & 3 \end{bmatrix}\mathbf{x} = \begin{bmatrix} 0 \\ \sin 3t \end{bmatrix}$$

has natural frequencies of $\sqrt{2}$ rad/sec and 2 rad/sec. Given mode shapes (mass-normalized) of

$$\mathbf{u}_1 = \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{1}{3} \\ 1 \end{bmatrix}, \text{ and } \mathbf{u}_2 = \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{1}{3} \\ -1 \end{bmatrix}$$

Find the modal force vector.

b. (4 points) Considering the same system, if the modal force vector turned out to be

$$\mathbf{f}(t) = \begin{bmatrix} \cos 4t \\ 0 \end{bmatrix}$$

find $\mathbf{r}(t)$.

c. (3 points) Considering the same system, if

$$\mathbf{r}(t) = \begin{bmatrix} 10 \cos 4t \\ 5 \cos 4t \end{bmatrix}$$

find $\mathbf{x}(t)$.

5. *Grad student/bonus* (20% of other points) Determine the first natural frequency and mode shape for a clamped-free beam. The equation of motion of a beam is $\left(\frac{EI}{\rho A}\right) \frac{\partial^4 w(x,t)}{\partial x^4} + \frac{\partial^2 w(x,t)}{\partial t^2} = 0$. Yes, you have to obtain all constants that can be obtained.