ME 460/660 Final Exam 2007, Fall Solutions
Short Problems

1) Piezo

2) They can be used at very low frequencies

3) $m, c\ \omega_n$

4) $\omega_d = 0.9 \qquad \zeta\omega_n = 0.01 \qquad \zeta \simeq \frac{0.01}{.9} = .011$

$\omega_d = \omega_n\sqrt{1-\zeta^2} = \omega_n \quad (\sqrt{1-\zeta^2} = 1)$

$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0$

$\ddot{x} + 0.02\dot{x} + 0.81x = 0$

5) Can handle a) External forces, b) Damping, c) multiple DOF (get 2 of 3)

6) Through Mass and Stiffness matrices.
No in the direct sense

7) Orthogonality of the eigenvectors of F.

8) Numerical integration

1) a)

F	f_{dc}	$\|X\|$	$\angle X (°)$
10	0.1	25.1	-3
2	0.15	0.45	-177
1	0.2	3.55	-170

Exaple: @ $f_{dc} = 0.1$, $|H(j\omega_{dc})| = 8\ dB$ $\angle H(j\omega_{dc}) = -3°$

$8\ dB = 20 \log_{10}(H(j\omega_{dc}))$

$H(j\omega_{dc}) = 10^{\frac{8}{20}} = 2.51$

$|X| = |H||F| = 25.1$, $\angle X = \angle H - \angle F$

b) $X(t) = 25.1 \sin(0.2\pi t + (-3° + 0.3\ rad))$

$= 25.1 \sin(0.2\pi t + 0.248)$

2) Ce bx form

$f_0 = \frac{1}{2}$ (average) $\quad \omega_n = \frac{2\pi}{T} = \pi$

$$f = \frac{1}{2}\int_0^2 \frac{t}{2} e^{-j\pi n t}\,dt$$

$$u = \frac{t}{2} \qquad dv = e^{-j\pi n t}\,dt$$
$$du = \frac{1}{2}\,dt \qquad v = \frac{1}{-j\pi n} e^{-j\pi n t}$$

$$f_n = \frac{1}{2}\left[\frac{t}{2}\frac{1}{-j\pi n} e^{-j\pi n t}\Big|_0^2 - \int_0^2 \frac{1}{2}\frac{1}{-j\pi n} e^{-j\pi n t}\,dt\right]$$

$$= \frac{1}{2}\left[\frac{2}{2}\frac{j}{\pi n} e^{-2\pi n j} - \frac{1}{2}\frac{1}{(-j\pi n)^2} e^{-j\pi n t}\Big|_0^2\right]$$

$$= \frac{1}{2}\left[\frac{j}{n\pi} - \underbrace{\left(\frac{1}{2}\frac{-1}{n^2\pi^2} e^{-j\pi n t}\right)\Big|_0^2}_{=0}\right]$$

$$= \frac{j}{2n\pi}$$

$a_n = 0 \qquad b_n = \frac{-1}{n\pi}$

$$F(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{-1}{n\pi} \sin n\pi t$$

Bonus $h(t)$: Impulse Response Function
$H(j\omega)$: Frequency Response Function
$H(j\omega)$ is the Fourier transform of $h(t)$

3) $\det(K - M\omega^2) = 0$

$$(3000 - 100\omega^2)(4000 - 200\omega^2) - 1E6 = 0$$

$$20000\omega^4 - 1000000\omega^2 + 12000000 = 0$$

$$\omega^2 = \frac{1000000 \pm \sqrt{1000000^2 - 4 \cdot 20000 \cdot 12E6}}{2 \cdot 20E3}$$

$$\omega^2 = 16.34,\ 33.66$$

$$\omega = 4.04,\ 5.80 \text{ rad/s}$$

$\underline{U_1,\ \omega_1 = 4.04:}$

$$\begin{bmatrix} 3000 - 1634 & -1000 \\ -1000 & 4000 - 3268 \end{bmatrix} \begin{bmatrix} U_{11} \\ U_{12} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

① $1366\, U_{11} = 1000\, U_{12}$ $\quad \underline{U_1} = \begin{bmatrix} 1 \\ 1.366 \end{bmatrix}$

② $1000\, U_{11} = 732\, U_{12}$ $\quad$ or $\underline{U_1} = \begin{bmatrix} 0.732 \\ 1 \end{bmatrix}$

Similarly $U_2,\ \omega_2 = 5.80$

$$\begin{bmatrix} 3000 - 3366 & -1000 \\ -1000 & 4000 - 6732 \end{bmatrix} \begin{bmatrix} U_{21} \\ U_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{or } \underline{U_1} = \begin{bmatrix} 0.591 \\ 0.807 \end{bmatrix}$$

$366\, U_{21} = -1000\, U_{22}$

$1000\, U_{21} = -2732\, U_{22}$ $\quad \underline{U_2} = \begin{bmatrix} 1 \\ -0.366 \end{bmatrix}$ or $U_2 = \begin{bmatrix} 2.732 \\ -1 \end{bmatrix}$

$$\underline{U_2} = \begin{bmatrix} 0.939 \\ -0.344 \end{bmatrix}$$

4) S is the matrix of mass normalized mode shapes.

The initial condition is the 1st mode shape.

Thus

$$\begin{bmatrix}1\\1\end{bmatrix} = S\, r(0),\quad r(0) = \begin{bmatrix}\frac{2}{\sqrt{2}}\\0\end{bmatrix},\quad \dot{r}(0) = \begin{bmatrix}0\\0\end{bmatrix}$$

$$\therefore r_1(t) = \frac{2}{\sqrt{2}}\cos 3t,\quad r_2(t) = 0$$

$$S\, r(t) = \begin{bmatrix}\cos 3t\\ \cos 3t\end{bmatrix}$$

$$x(t) = \begin{bmatrix}1\\1\end{bmatrix}\cos 3t$$

This problem is in class notes

5) See midterm

$$\left(m + \frac{I}{r^2}\right)\ddot{x} + \left(k_2 + \frac{k_1}{r^2}\right)x = 0$$

6) $X_n(x) = \sin\left(\frac{n\pi x}{\ell}\right)$ (to satisfy BC and

Substituting into homogeneous EOM $X'' + \beta^2 X = 0$)

$$-\omega_n^2 X_n(x) + c^2\frac{n^2\pi^2}{\ell^2}X_n(x) = 0$$

$$\omega_n = \frac{n\pi}{\ell}\sqrt{\frac{\tau}{\rho}}$$

Subst all into EOM

$$\sum_{n=1}^{\infty} \ddot{T}_n X_n + \frac{\tau}{\rho}\frac{n^2\pi^2}{\ell^2}T_n X_n = 100\,\delta\left(x - \frac{\ell}{2}\right)\sin 3t$$

multiply by X_m, integrate from $0<x<\ell$

$$\int_0^\ell X_n X_m\, dx = \frac{\ell}{2}\delta_{nm} \qquad \delta_{nm} = \begin{cases}0, & n\neq m\\ 1, & n=m\end{cases}$$

$$\ddot{T}_m + \omega_n^2 T_m = \frac{2}{\ell} \cdot 100 \sin\frac{m\pi}{2} \sin 3t$$
$$= \frac{200}{\ell} \sin\frac{m\pi}{2} \sin 3t$$

$$\sin\frac{m\pi}{2} = \begin{cases} -1^{\left(\frac{m-1}{2}\right)}, & m \text{ odd} \\ 0, & m \text{ even} \end{cases}$$

$m =$	1	2	3	4	5
$\sin\frac{m\pi}{2}$	1	0	-1	0	1

$$\ddot{T}_m + \omega_n^2 T_m = F_m \sin 3t = F(t)$$

$$\frac{T(j\omega)}{F(j\omega)} = \frac{1}{\omega_m^2 - 3^2}$$

$$T_m = \frac{\frac{200}{\ell}}{\omega_m^2 - 3^2} \sin 3t$$

$$X_m = \sin\frac{m\pi x}{\ell}$$

$$w(x,t) = \left(\sum_{n=1}^{\infty} \frac{\frac{200}{\ell}(-1)^{\frac{m-1}{2}}}{\omega_n^2 - 3^2} \sin\frac{m\pi x}{\ell} \right) \sin 3t, \ n \text{ odd}$$

where $\omega_n = \frac{n\pi}{\ell}\sqrt{\frac{T}{\rho}}$

Bonus $h(t)$: Impulse Responce Function
$H(j\omega)$: Frequency Response Function
$H(j\omega)$ is the Fourier transform of $h(t)$