

Closed book, closed notes. Use one $8\frac{1}{2} \times 11$ formula sheet, front and back. Test books will be provided.

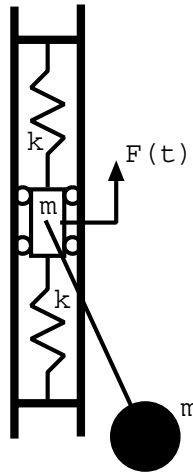
Problems are 10 points each. Problem 4 is required for graduate students.

1. Determine the response of a single degree of freedom *undamped* system to a step excitation defined by $F(t) = 0$ for $t < 0$ and $F(t) = F_0$ for $t > 0$.
2. Find the frequency response function X/Y for the system below:

$$10\ddot{x} + 1\dot{x} + 1000x = 1\dot{y}$$

Sketch the magnitude and phase of the FRF for $0 < r < 2$. Label the values at $r = 0$, $r = 1$, and $r = \infty$.

3. Derive the equation of motion of the system shown below using Lagrange's equation. Assume no damping, and don't forget to include gravity. Assume no damping or other energy loss mechanisms. Define $x = 0$ to be at the geometric center of the springs. Both springs are unstretched when $x = 0$. *Do not neglect gravity at any place in the problem.*



4. Graduate students/bonus: Determine the mode shapes and natural frequencies of a bar with constant circular cross section in torsional vibration with free-free boundary conditions. The equation of motion is given by $GJ_1 \frac{\partial^2 \theta(x,t)}{\partial x^2} = \rho J \frac{\partial^2 \theta(x,t)}{\partial t^2}$.