

ME 360 System Dynamics SP'07 Final

1) $3\tau \approx 6 \text{ sec}$

$\tau \approx 2 \text{ sec}$

$$\boxed{\begin{aligned} 2\dot{y} + y &= u(t) \\ \text{or } \dot{y} + \frac{1}{2}y &= \frac{1}{2}u(t) \end{aligned}}$$

Note that $y(t) \approx 1 - e^{-\frac{t}{2}}$

$y_h = e^{-t/2} y_p + y_h$

$y_h = -\frac{1}{2} e^{-t/2}$

2) a) $y(t) = \frac{5}{2}t + C_1 + C_2 e^{-t/\tau}$

$\dot{y}(t) = \frac{5}{2} - \frac{C_2}{\tau} e^{-t/\tau}$

substituting

$10 \frac{5}{2} - 10 \frac{C_2}{\tau} e^{-t/\tau} + 5t + 2C_1 + 2C_2 e^{-t/\tau} = 5t$

$25 + 2C_1 = 0, C_1 = -\frac{25}{2}$

$-\frac{10C_2}{\tau} + 2C_2 = 0$

$2\tau = 10, \tau = 5$

$y(0) = 1 = C_1 + C_2$

$C_2 = 1 - C_1 = \frac{27}{2}$

$y(t) = \frac{5}{2}t + \frac{-25}{2} + \frac{27}{2} e^{-t/5}$

b) $y(t) = \frac{5}{2} + C e^{-t/5}$ (reusing the time constant)

$$y(0) = 1 = \frac{5}{2} + C$$

$$C = -\frac{3}{2}$$

$$y(t) = \frac{5}{2} - \frac{3}{2} e^{-t/5}$$

c) $2\ddot{y} + 2\dot{y} + y = 2u(t), \quad \ddot{y} + \dot{y} + \frac{1}{2}y = u(t)$

$$2r^2 + 2r + 1 = 0$$

$$r = \frac{-2 \pm \sqrt{4 - 4 \cdot 2}}{4} = -0.5 \pm 0.5j$$

$$\omega_d = 0.5$$

$$\tau = \frac{1}{0.5} = 2$$

$$y(t) = 2 + A e^{-t/2} \sin\left(\frac{t}{2} + \theta\right)$$

$$y(0) = 0 = 2 + A \sin \theta$$

$$\dot{y}(0) = 0 = -\frac{1}{2}A \sin \theta + \frac{1}{2}A \cos \theta$$

$$\theta = \frac{\pi}{4}$$

$$A = \frac{-2}{\sin \frac{\pi}{4}} = \frac{-2}{\frac{\sqrt{2}}{2}} = \frac{-4}{\sqrt{2}} = -2\sqrt{2}$$

$$y(t) = 2 + -2\sqrt{2} e^{-t/2} \sin\left(\frac{t}{2} + \theta\right)$$

$$3) \text{ sys} = \text{tf}([1 \ 1], [10 \ 0.1 \ 1400])$$

$$\text{bode}(\text{sys})$$

$$a) 0.0617 \left(\text{abs}((11 \times 1 + 1) / (10 \times (11)^2 + 0.1 \times 11 + 1400)) \right)$$

$$b) -24.2 \text{ dB}, 84.5^\circ (1.4748 \text{ rad})$$

$$c) 0.617 \sin(11t + \frac{\pi}{4} + 1.4748)$$

d) see top (see print at end)

$$4) \text{ For } h \geq D$$

$$PA \dot{h} = g_{mi} + g_{mo} + g_1, \quad g_1 = \frac{(P_s - P_g h)}{R_1}$$

$$g_{mo} = \frac{P_g(h-D)}{R_2}$$

$$PA \dot{h} = \left(-\frac{P_g}{R_1} - \frac{P_g}{R_2} \right) h + g_{mi} + \frac{P_g D}{R_2} + \frac{P_s}{R_1}$$

For $h < D$

$$PA \dot{h} = -\frac{P_g}{R_1} h + g_{mi} + \frac{P_s}{R_1}$$

In state space

$$\dot{h} = \underbrace{\left[-\frac{P_g}{PA R_1} \right]}_A h + \underbrace{\left[\frac{1}{PA} \quad \frac{1}{PA R_1} \right]}_B \begin{bmatrix} g_{mi} \\ P_s \end{bmatrix}$$

$$y = \underbrace{1}_C h + \underbrace{0}_D u$$

$$5) \quad v_a = R_a i + L_a \frac{di}{dt} + K_b \dot{\theta}$$

$$i K_T = (I_{eff}) \ddot{\theta} + c R^2 \dot{\theta} + k R^2 \theta$$

$$I_{eff} = I_m + I_p + m_r R^2$$

In state space

$$\begin{bmatrix} \frac{di}{dt} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{K_b}{L_a} & 0 \\ 0 & 0 & 1 \\ \frac{K_T}{I_{eff}} & -\frac{c R^2}{I_{eff}} & -\frac{k R^2}{I_{eff}} \end{bmatrix} \begin{bmatrix} i \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} \\ 0 \\ 0 \end{bmatrix} v_a$$

6)

