Computational Methods in Structural Dynamics, Midterm Winter 2006

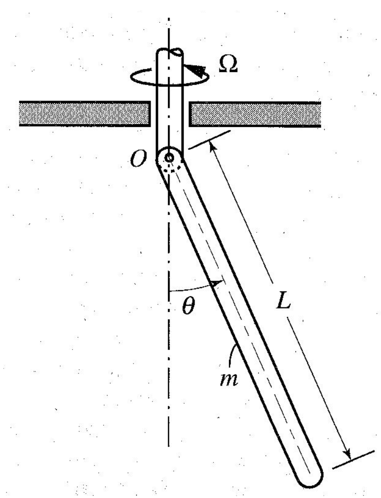
One 8.5" by 11" cheat sheet.

1. Determine the mode shapes and natural frequencies of a clamped-clamped beam given the equation of a beam
$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + \rho A\frac{\partial^2 w(x,t)}{\partial t^2} = p(x,t)$$

2. Determine the response of the system
$$EI\frac{\partial^4 w}{\partial x^4} + c\frac{\partial w^3}{\partial x^2 \partial t} + \rho A\frac{\partial^2 w}{\partial t^2} = \delta(t)\delta\left(x - \frac{l}{2}\right)$$

   for a pinned-pinned (simply supported) boundary condition.

3. (a) Derive the equation/s of motion of the following system. You may not use variables that are not listed in your final answer.
   (b) Find the equilibrium points.
   (c) Linearize about the equilibrium point that is a function of Ω.
   (d) Determine under what conditions this equilibrium point is stable.



4. Prove that the Rayleigh quotient of a symmetric 2×2 matrix is bounded by its eigenvalues.

5. The mass normalized and orthogonal mode shapes of a continuous self-adjoint system are given by $X_n(x)$, with corresponding natural frequencies of ω_n.

   Write the solution for the steady-state response at x_1 to an excitation of $F\sin(\omega_d rt)$ at x_2.