ME 712, Finite Element Method Applications   Final Exam, Spring 2007

One formula sheet to be turned in. Problems must be in order in the blue books.

1. Define:
   (a) Isoparametric element
   (b) Subparametric element

2. Determine the stiffness matrix of a 3-noded rod element with the center node equidistant from the ends using Gauss quadrature in 1-D. Presume constant area and density.

3. Determine the centroid $\bar{x}$ of a quadratic (LST) triangular element assuming nodes at (0,0), (1,0), (0,1), (0.5,0), (0.55,0.55), (0,0.5) using Gauss point integration. Note that $\bar{x} = \frac{\int_A y dx dy}{\int_A dx dy}$[1]. Explain any assumptions. Show all work. No credit will be given for integration carried out other than as specified.

	Linear	Quadratic
$N_1 =$	$1-r-s$	$(1-r-s)(1-2r-2s)$
$N_2 =$	r	$r(2r-1)$
$N_3 =$	s	$s(2s-1)$
$N_4 =$		$4r(1-r-s)$
$N_5 =$		$4rs$
$N_6 =$		$4s(1-r-s)$

4. A single standard Euler-Bernoulli beam has constraints of $v_1 = -v_2 = \theta_1 L/2$[2], and $\theta_2 = \theta_1$. Generate the reduced governing equations using the method of your choice. Presume a length $L = 2$.

$$K = \frac{EI_z}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}, \tag{1}$$

$$M = \frac{m}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix} \tag{2}$$

Hint: Switch to using coordinates of θL in place of θ by appropriate 'transformation'.

What have you just derived as the reduced mass matrix?

---

[1] Note that this formula is wrong. Either x or y should be swapped for the other.

[2] There was an undiscovered typo in the problem. This expression should have read $v_1 = -v_2 = -\theta_1 L/2$.

5. Obtain the stiffness matrix of a bilinear quadrilateral (Q4) element with nodes 1-4 at (0,0), (1,0), (1,1), and (0,2). Use Gauss integration to derive the element matrices. Assume $E = 7.3084 \times 10^{10}$ N/m^2, $\nu = .3$, and $\rho = 2770$ kg/m^3 and presume a state of plane stress, i.e.

$$[E] = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix}$$

6. Find the strain at (x, y)=(0,1) of a bilinear quadrilateral (Q4) element with nodes 1-4 at (0,0), (1,0), (1,2), and (0,2) in terms of u_2 and v_2 (presume all other nodal displacements are zero).

7. Download the file http://www.cs.wright.edu/~jslater/classes/fem2/Exams/mk2.mat (This is a hyperlink. Selecting it will download the file to your hard drive.). Load it into Matlab. Use the power method and shifting to obtain the first two natural frequencies to two places of accuracy.

   Email me your code and a diary of the results
   Turn on a diary to log the following. (type "diary filename" and everything that displays in the Matlab window will be logged to "filename").

   (a) The initial vector guessed

   (b) Each iteration number

   (c) The corresponding natural frequency in rad/sec at each iteration.