Computational Methods in Structural Dynamics, Exam 2 Winter 2004

One 8.5" by 11" cheat sheet. 10 points each. All work must be done in the exam book. No extra scrap paper allowed.

1. Derive the equations of motion and allowable boundary conditions for a Timoshenko beam given the following.
   The bending strain energy is $V_b = \frac{1}{2}\int_0^L EI(\alpha')^2 dx$, the shear strain energy is $V_s = \frac{1}{2}\int_0^L \kappa GA(\alpha - \frac{\partial v}{\partial x})^2 dx$, the kinetic energy is $T = \frac{1}{2}\int_0^L \rho A \dot{v}^2 dx + \frac{1}{2}\int_0^L \rho I \dot{\alpha}^2 dx$, and the non-conservative variational work is $W_{nc} = \int_0^L p(x,t)\delta v(x,t) dx$. Of course the total potential energy is $V = V_b + V_s$. Note that for a Timoshenko beam the rotation parameter α is independent of the slope, $\frac{\partial v}{\partial x}$, both being a function of x and t. As a result, you should expect, trust me on this, to derive two, that is **two**, coupled differential equations– one in α, and one in v. Assume A and I are functions of x as well.

2. An system is defined by the operator $L = EI\frac{\partial^4}{\partial x^4} + \beta\frac{\partial^3}{\partial x^3}$ with the boundary conditions $x(0) = 0$, $\left.\frac{\partial^2 w}{\partial x^2}\right|_{x=0} = 0$, $x(l) = 0$, and $\left.\frac{\partial^2 w}{\partial x^2}\right|_{x=l} = 0$.

   - Is the system self adjoint?
   - If so, is the system positive definite?