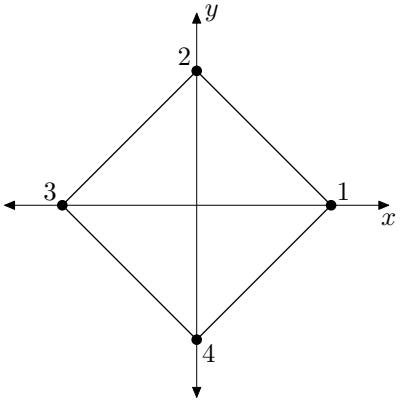
ME 712, Finite Element Method Applications Midterm Exam, Spring 2010

One formula sheet, closed notes, open book. Test books will be provided. 1 hour, 15 min. *Problems must be done in order in the test books.* 10 points each.

1. Find the strain at (ξ, η)=(0,0) of a bilinear quadrilateral (Q4) element with nodes 1-4 at (1,0), (0,1), (-1,0), and (0,-1) in terms of u_3 and v_3 (presume all other nodal displacements are zero).



2. Obtain first row of the stiffness matrix of a rod (extension: 1-D) using 1 quadratic element with a mid-node located at $3L/4$. Use Gauss integration to derive the element matrices. Assume a length l, density ρ, cross sectional area of A, and a modulus of E.

3. Determine the consistent nodal loading on a beam for an applied distributed load of $f(x) = a\frac{x}{l}$ *using Gauss point integration.*

$$N = \begin{bmatrix} 1 - 3\frac{x^2}{l^2} + 2\frac{x^3}{l^3} & x - 2\frac{x^2}{l} + \frac{x^3}{l^2} & 3\frac{x^2}{l^2} - 2\frac{x^3}{l^3} & -\frac{x^2}{l} + \frac{x^3}{l^2} \end{bmatrix}$$

Table 1: Approximate Gauss point integration values

# points	ξ_i	w_i
1	0	2
2	$\pm\frac{1}{\sqrt{3}}$	1
3	0, $\pm\sqrt{0.6}$	$\frac{8}{9}$, $\frac{5}{9}$
4	-0.86, -0.34, 0.34, 0.86	0.35, 0.65, 0.65, 0.35