ME 460/660, Mechanical Vibration — Test 1, Spring 2011

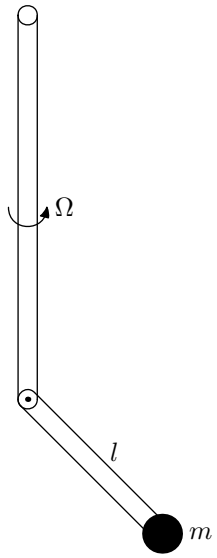
Closed book, closed notes. Test booklets will be provided. Formula sheet must be turned in with the exam. Formula sheet must be exactly the same as that posted on the web site. Calculators allowed. Problem 4 is required for graduate students.

1. (a) Find the forced response ($x(t)$, particular solution) of $\ddot{x} + 0.1\dot{x} + 3600x = 100y(t) + 0.1\dot{y}(t)$ if $y(t) = Y \sin \omega t$ for

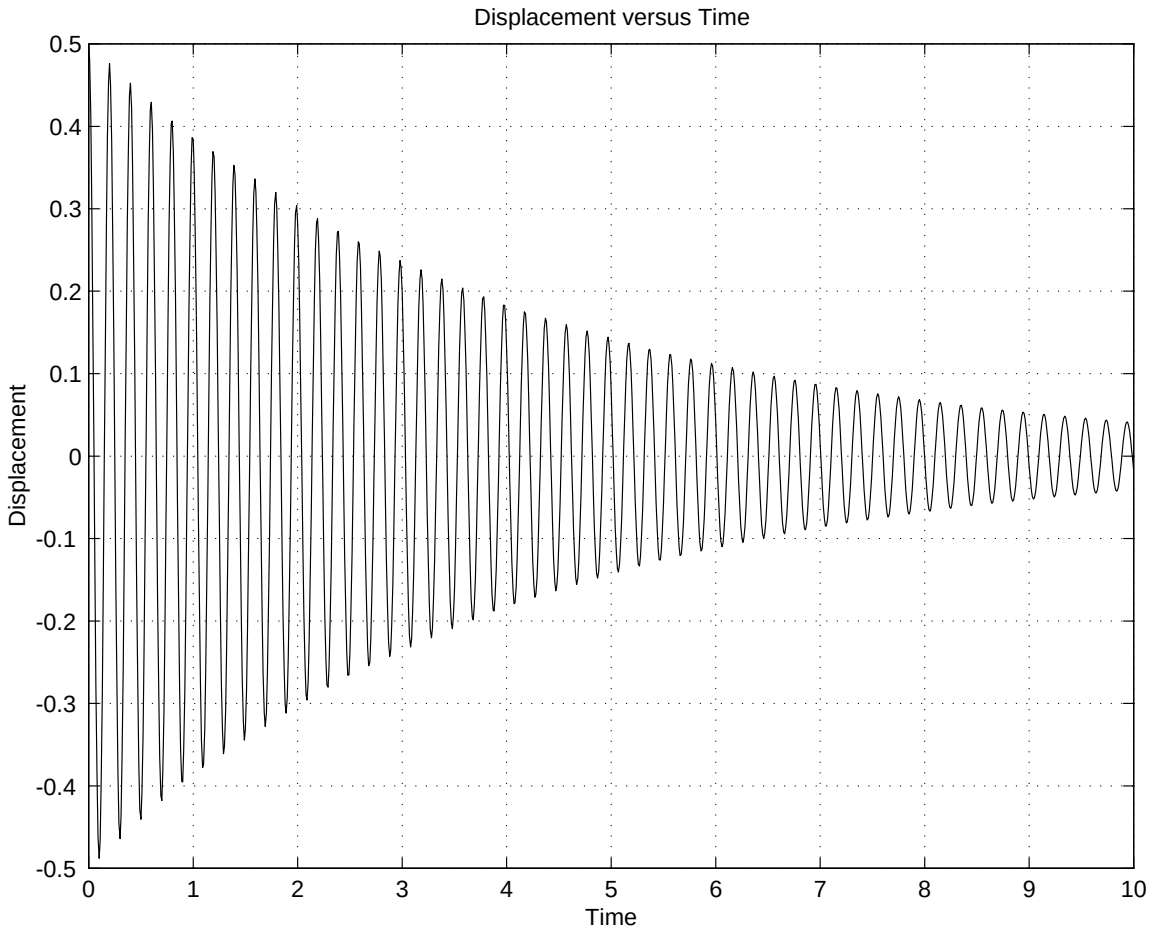
Y	f (Hz)
10	1
0.1	10
0.001	100

   (b) Sketch the magnitude (*using 1/2 of a page for scale!*) of $X(i\omega)/Y(i\omega)$ for the system above for $0 < \omega < 10\omega$ on a dB scale ($20 \times \log_{10}(\frac{X}{Y})$) labeling values (not in dB) at $f = (0, f_n, 10f_n)$ where f_n is the natural frequency in Hz.

2. A pendulum, length l with a point mass m at the end, hangs from the end of a vertical shaft by a horizontal pin. The vertical shaft starts to spin and reaches an angular velocity of Ω. Derive the governing equation. *Hint: When $\Omega = 0$, your answer should match the equation of a pendulum*



3. The free response of a system is shown below. The stiffness is given by $k = 100$ N/m. Find m and c. The time listed is in seconds.



4. *Grad student/bonus:* Determine the natural frequencies and mode shapes for a clamped-free (fixed on the left end) circular cross section extension rod. The equation of motion of a rod is $\left(\frac{E}{\rho}\right)\frac{\partial^2 w(x,t)}{\partial x^2} = \frac{\partial^2 w(x,t)}{\partial t^2}$. (20% of other points)