Computational Methods in Structural Dynamics, Final Winter 2002

One 8.5" by 11" cheat sheet. Problems are 10 points each.

1. The equation of motion of a fixed-fixed string is given by: $\tau\frac{\partial^2 w(x,t)}{\partial x^2} = \rho(x)A\frac{\partial^2 w(x,t)}{\partial t^2}$. If $\rho(x) = M\delta\left(x - \frac{l}{2}\right)$, obtain a best estimate of the first natural frequency using a two term series using:
   (a) Rayleigh Ritz Method
   (b) the Collocation method.

2. An rotating shaft of length 1 is modeled by the following equations:
$$\ddot{v} + \frac{\omega_n^2}{\pi^4}v'''' - \Omega^2 v - \Omega\dot{w} = 0$$
and
$$\ddot{w} + \frac{\omega_n^2}{\pi^4}w'''' - \Omega^2 w + \Omega\dot{v} = 0$$
Assume $v = y(t)\sin(\pi x)$, and $w = z(t)\sin(\pi x)$. Apply Galerkin's Method to obtain the discrete equations of motion, and discuss the relationship between Ω and $\omega_n = \pi^2\sqrt{EI/\rho A}$.

3. A rope of length l, area A, and density ρ is hanging from a tall building. Assuming constant gravity, obtain the equation of motion and the boundary conditions.

4. In less than 10 words for each response, state concisely, and precisely, what the following methods do. You will be penalized 1 point for each word over 10. (2 points each).
   (a) Matrix Deflation
   (b) Hamilton's principle.
   (c) Gauss Elimination
   (d) Cholesky decomposition
   (e) Givens' method