ME 460/660, Mechanical Vibration Final Exam, Spring 2009

Closed book, closed notes. Use one $8\frac{1}{2} \times 11$ formula sheet (brought by student) and turn in with exam. Test books will be provided. All problems are to be done in the test book.

1. Find the natural frequencies and mode shapes for the MDOF system with

$$M = \begin{bmatrix} 100 & 0 \\ 0 & 200 \end{bmatrix} \quad \text{and} \quad K = \begin{bmatrix} 3000 & -1000 \\ -1000 & 4000 \end{bmatrix}$$

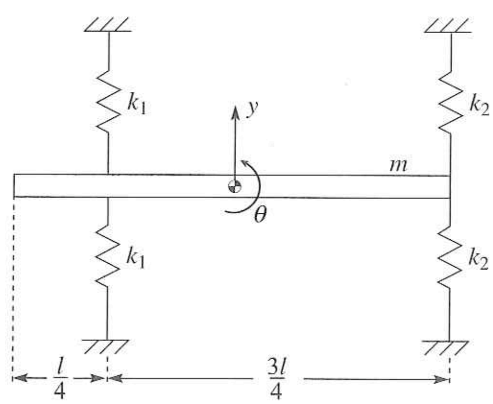
2. An undamped system is excited by a pulse (force of amplitude F) of finite (**but not infinitesimal!**) duration. Find $x(t)$ during and after the pulse presuming $10\ddot{x} + 8.8826 \times 10^4 x = f(t)$ and the pulse lasts for $\frac{1}{30}$ s.

3. The first DOF of a MDOF system is excited by $f_1(t) = 10 \sin 5t$ ($f_2(t) = 0$). It has mass normalized modes of

$$S = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

no damping, and natural frequencies of 3 rad/sec and 10 rad/sec. Find the steady state solution $\mathbf{x}_p(t)$.

4. Obtain the equations of motion the following system. Of course, you must include the effect of gravity. The mass moment of inertia of a uniform rod (which this problem has) about its *center of gravity* is $\frac{1}{12}Ml^2$. **Assume small rotations ($\sin\theta = \theta$) and no side-to-side motion. Use coordinates as labeled**.



5. Graduate Students/Undergraduate Bonus (20% of other points): Solve for the steady-state (particular) response of the following system if the boundary conditions are presumed to be free-free ($0 < x < l$) where $c = \sqrt{\tau/\rho}$.

$$w_{tt}(x,t) - c^2 w_{xx}(x,t) = 100\delta(x - \frac{l}{3}) \sin(3t)$$

Recall that the integral of a Dirac delta function times another function is equal to the "another function" evaluated when the argument of the argument of the Dirac delta function is zero.