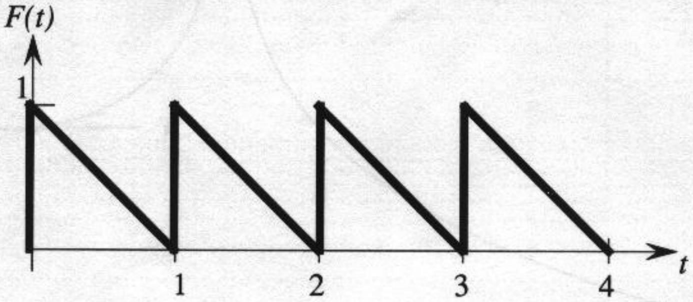
11

ME 460/660 Exam 2, Spring '96

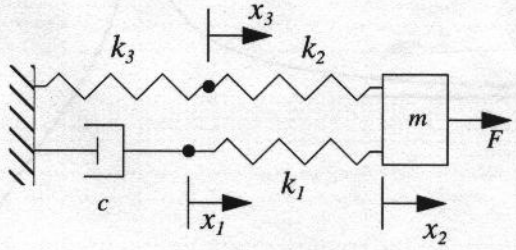
One equation sheet. Front and back. No examples. No derivations. It must be turned in with the exam.

1) Find the Fourier series representation of the following function. Write the series in the simplest form AND write the first few non-zero terms. (25 points)

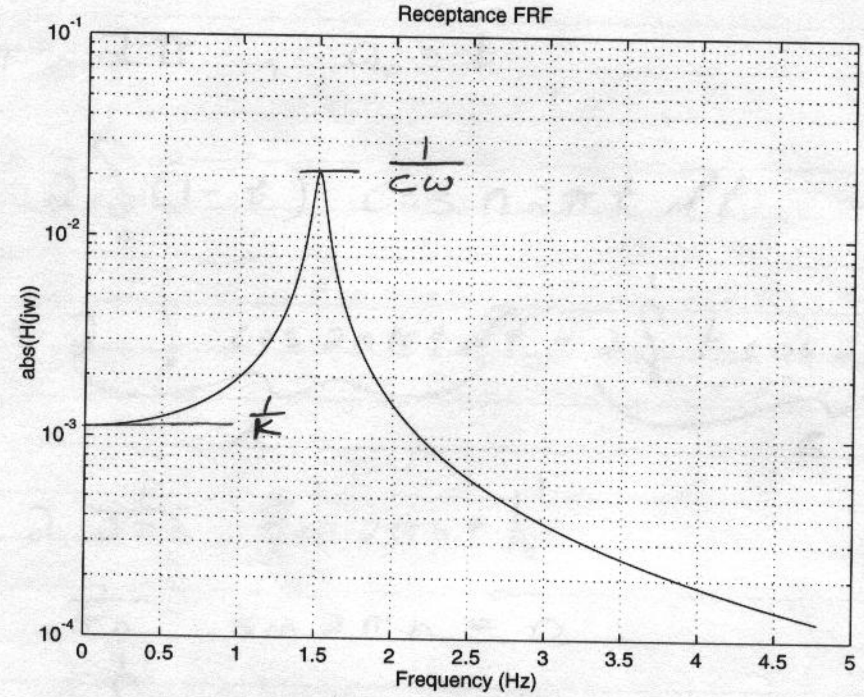


2) A 50 kg motor is attached to a 50 kg table. The specifications for the motor are such that $m_0e <$ .001 kg•m. Choose/modify *m*, *c*, and *k* such that the maximum displacement of the table is less than .02 mm for motor speeds between 0 and 3000 rpm. (25 points)

3) Derive the equations of motion (do **not** solve) for the following system: (25 points)



4) Estimate *m*, *c*, and *k* from the plots below. All units are standard SI units (kg, m, s). (25 points)

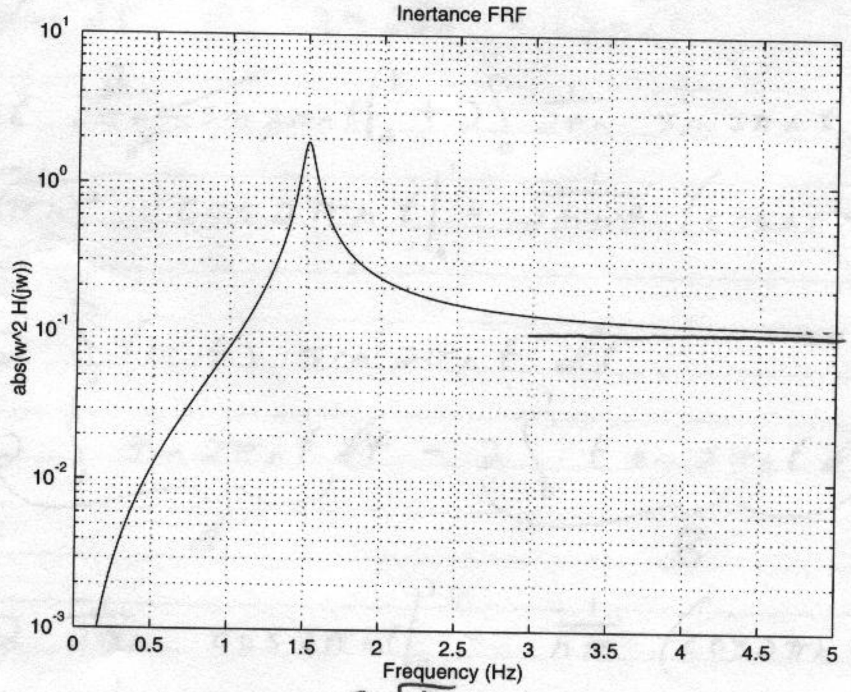


$K \approx \frac{1}{.0011} \approx \underline{909\,N/m}$

$C\omega \approx \frac{1}{.021}$

$C \approx \frac{1}{3\pi \cdot .021} \approx 60$

$\approx \underline{5\ Kg/s}$



$\frac{1}{m} = .1$

$\underline{m = 10}$

check $\omega \overset{?}{\approx} \sqrt{\frac{k}{m}}$

$3\pi \approx \sqrt{\frac{909}{10}}$

$9.4 \approx 9.53$

1) $\omega_T = 2\pi \quad , \quad a_0 = 1$

$$a_n = 2\int_0^1 (1-t)\cos n 2\pi t\, dt$$

$$= \underbrace{2\int_0^1 \cos 2n\pi t\, dt}_{A} - \underbrace{2\int_0^1 t\cos 2n\pi t\, dt}_{B}$$

$$A = 2\,\frac{1}{2\pi n}\sin 2\pi n t\Big|_0^1$$

$$= \frac{1}{\pi n}\sin 2\pi n = 0$$

$$B = -2\int_0^1 t\cos 2n\pi t\, dt$$

$u = t \qquad dv = \cos 2n\pi t\, dt$

$du = dt \qquad v = \frac{1}{2\pi n}\sin 2\pi n t$

$$B = \cancel{-t\,\frac{2}{2\pi n}\sin 2\pi n t\Big|_0^1}^{\,0} + 2\int_0^1 \frac{1}{2\pi n}\sin 2\pi n t\, dt$$

$$B = \frac{-1}{2(\pi n)^2}\cos 2\pi n t\Big|_0^1 = \frac{-1}{2(\pi n)^2}\left(\cos 2\pi n - 1\right) = 0$$

$$b_n = 2\int_0^1 (1-t)\sin 2\pi n t\, dt$$

$$= \underbrace{2\int_0^1 \sin 2\pi n t\, dt}_{A} - \underbrace{2\int_0^1 t\sin 2\pi n t\, dt}_{B}$$

$$A = 2\,\frac{-1}{2\pi n}\cos 2\pi n t\Big|_0^1 = \frac{-1}{n\pi}\left(\cos 2\pi n - \cos 0\right) = 0$$

$$B = -2\int_0^1 t \sin 2\pi n t \, dt$$

$u = t \qquad dv = \sin 2\pi n t \, dt$

$du = dt \qquad v = \frac{-1}{2\pi n} \cos 2\pi n t$

$$B = +2 \, t \frac{1}{2\pi n} \cos 2\pi n t \Big|_0^1 - 2\int_0^1 \frac{1}{2\pi n} \cos 2\pi n t \, dt$$

$$= \frac{1}{\pi n}(1 - 0) - 2\frac{1}{(2\pi n)^2} \sin 2\pi n t \Big|_0^1$$

$$= \frac{1}{n\pi} - \frac{2}{(2\pi n)^2}\left(\sin 2\pi n - \sin 0\right)$$

$$= \frac{1}{n\pi}$$

$$f(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{1}{n\pi} \sin 2\pi n t$$

$$= \frac{1}{2} + \frac{1}{\pi}\sin 2\pi t + \frac{1}{2\pi}\sin 4\pi t + \cdots$$

2) $m_i = 100$ kg $\quad m_o e \leq .001$ kg m

Pick $m, c + k$ so $X < .02$ mm $= .00002$ m
for 0 rpm $< f_r <$ 3000 rpm.

0 rad/s $< \omega_r <$ 314 rad/s +5 } plus understand $r<1$

First, let's try to solve this with $c = 0$.
(cheaper + more robust design) +5

$$X = \frac{m_o e}{m} \frac{r^2}{abs(1-r^2)} \quad +5$$

$$2\times 10^{-5} > \frac{.001}{m} \frac{r^2}{(1-r^2)} \quad \text{OR} \quad 2\times 10^{-5} > \frac{.001}{m} \frac{r^2}{r^2-1}$$

Since we don't want r to ever be 1, $r_{max} < 1$, then the expression on the left is the correct one.

$$.02 > \frac{1}{m} \frac{r^2}{1-r^2}$$

Let's try to leave the mass alone. $m = 100$

$$2 > \frac{r_{max}^2}{1-r_{max}^2}$$

$$2 - 2r^2 > r_{max}^2$$

$$r_{max} < \sqrt{\frac{2}{3}} = .816, \quad +4$$

$C > 62800$ at resonance ($\omega = 100$)

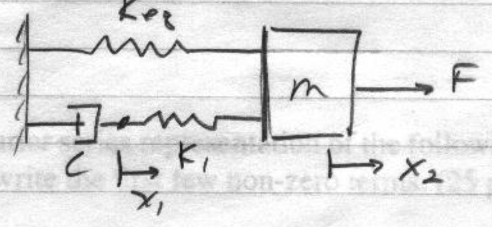
For safety, pick $r_{max} = .6 = \dfrac{\omega_r}{\omega}$ +3

$$\omega = \frac{314}{.6} \approx 525 \text{ rad/s}$$

$K = m\omega^2 \approx 2.7\times 10^7$ N/m, $m = 100$ kg, $c = 0$ kg/s +3

$\omega > 385$ works marginally. ($K > 1.5\times 10^7$ for $m = 100$)

3) Reducing the system



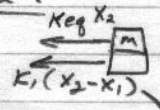
$K_{eq} = \dfrac{K_2 K_3}{K_2 + K_3}$

FBD 1:

$C\dot{x}_1 \longleftarrow \bullet \longrightarrow K_1(x_2 - x_1)$

10

FBD 2:



At point 1 — no mass at point 1

$$\Sigma F = m\ddot{x}_1 = 0 = -C\dot{x}_1 - K(x_1 - x_2)$$

7

$$C\dot{x}_1 + K(x_1 - x_2) = 0 \qquad (1)$$

At point 2

$$\Sigma F = m\ddot{x}_2 = -K_1(x_2 - x_1) - K_{eq}x_2$$

8

$$m\ddot{x}_2 + \left(K_1 + \frac{K_2 K_3}{K_2 + K_3}\right)x_2 - K_1 x_1 = 0 \qquad (2)$$