ME 712, Finite Element Method Applications   Exam 1, Spring 2004

One formula sheet, closed notes. Test books will be provided. Two hours. *Problems must be done in order in the test books.* 10 points each.

1. A beam rests on a compliant foundation. The strain energy in the foundation due to deformation of the beam is $\frac{1}{2}k(x)v(x)^2$. Derive the change to the beam stiffness matrix elements K_{11} and K_{21} due to the addition of this foundation. The shape functions are

$$N = \begin{bmatrix} 1 - 3\frac{x^2}{l^2} + 2\frac{x^3}{l^3} & x - 2\frac{x^2}{l} + \frac{x^3}{l^2} & 3\frac{x^2}{l^2} - 2\frac{x^3}{l^3} & -\frac{x^2}{l} + \frac{x^3}{l^2} \end{bmatrix} \tag{1}$$

2. Determine the consistent nodal loading on the beam of problem 1 for an applied distributed load of $f(x) = ax$.

3. Determine the mass matrix for a rod in local coordinates (the rod being between $x = 0$ and $x = l$) presuming ρ is constant, but $A(x) = A_1 + (A_2 - A_1)\frac{x}{l}$.

4. Using the rod of problem 3, determine the full 3-D mass matrix. Derivation is not necessary if explanation for simplifications are made.

5. A piezoelectric rod of constant properties has an energy, to be treated like a potential energy, of

$$U = \frac{1}{2}EA\sigma(x)^2 - eA\frac{dV(x)}{dx}\sigma(x) - \frac{1}{2}\epsilon A\left(\frac{dV(x)}{dx}\right)^2 \tag{2}$$

where all variables are constant except those indicated above to be functions of x. Using the linear 1-D shape functions to represent the field variables $u(x)$, displacement, and $V(x)$, voltage field, in terms of nodal field quantities, u_j and V_j, determine the governing equations, in matrix form, in terms of the nodal field quantities.