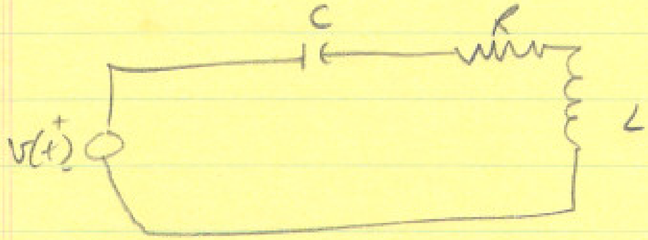
Sp '08 Final Solutions ME 360

1) Underdamped second order system



$$L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt = v(t)$$

$f_n = \frac{16}{10} \approx 1.6\ Hz$ $\omega_n = 2\pi\, 1.6 \approx 10$ rad/s

From 5% settling time $\frac{3}{\zeta\omega_n} = 6$ $\zeta\omega_n = \frac{1}{2}$

$\zeta = 0.05$

If $\underline{\frac{1}{C} = 100}$, then $\omega_n^2 = 100 = \frac{\frac{1}{C}}{L}$, $\underline{L=1}$

$R = L\,2\zeta\omega_n = \underline{1\ \Omega}$

$C = 0.01$ $R = 1\Omega$ $L = 1\ H$

To see 0.01 V steady state, split capacitor in 2

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$\frac{1}{C_1} = 1$, $\frac{1}{C_2} = 99$, $C_2 = 0.0101$

$C_1 = 1\ F$ $R = 1\Omega$

$C_2 = 0.0101\ F$ $L = 1\ H$

Voltage measured across C_1 will give the plotted response.

2a) $T(j\omega) = \dfrac{j\omega}{+1000(j\omega)^3 + 10(j\omega)^2 + 0.1(j\omega) + 1400}$

For $\omega = 10$ rad/s, $T(j\omega) = -1\times10^{-5} + 4\times10^{-9}j$

$|T(j\omega)| = 1\times10^{-5}$

$\angle T(j\omega) = 180°$

b) $20\log_{10}(1\times10^{-5}) = -100$ dB

c) $x(t) = -2\times10^{-4}\sin\left(10t + \frac{3\pi}{4}\right)$

d) $T(s) = \dfrac{s}{1000s^3 + 10s^2 + 0.1s + 1400}$

e) See result

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sys= tf([1 0],[1000 , 10 , 0.1 , 1400])
bode(sys)
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3) Using equivalent inertias, dampers, springs

$$\left(I_m + I_p + m_R R^2\right)\ddot{\theta} + cR^2\dot{\theta} + kR^2\theta = T(t)$$

4) Cons of mass tank 1

$$q_{mi} - q_{m1} = \rho A_1 \dot{h}_1 \qquad (1)$$

Tank 2

$$q_{m1} - q_{mo} = \rho A_2 \dot{h}_2 \qquad (2)$$

Resistor 1

$$R_1 q_{m1} = \rho g h_1 - \rho g h_2 \qquad (3)$$

Resistor 2

$$R_2 q_{mo} = \rho g h_2 \qquad (4)$$

From 1, using 3

$$\rho A \dot{h}_1 = -\frac{\rho g}{R} h_1 + \frac{\rho g}{R} h_2 + q_{mi} \qquad (5)$$

From 2, using 3 and 4

$$4\rho A \dot{h}_2 = \frac{\rho g}{R} h_1 - \frac{\rho g}{R} h_2 - \frac{\rho g}{3R} h_2$$

$$4\rho A \dot{h}_2 = \frac{\rho g}{R} h_1 - \frac{4}{3}\frac{\rho g}{R} h_2 \qquad (6)$$

In Laplace, 5 becomes

$$\left(\rho A s + \frac{\rho g}{R}\right) H_1(s) = \frac{\rho g}{R} H_2(s) + Q_{mi}(s) \qquad (7)$$

and 6 becomes

$$\left(4\rho A s + \frac{4}{3}\frac{\rho g}{R}\right) H_2(s) = \frac{\rho g}{R} H_1(s) \qquad (8)$$

To get part a, sub 8 into 7 to remove $H_1(s)$

$$\left(\rho A s + \frac{\rho g}{R}\right)\frac{R}{\rho g}\left(4\rho A s + \frac{4}{3}\frac{\rho g}{R}\right) H_2(s) = \frac{\rho g}{R} H_2(s) + Q_{mi}(s)$$

Solving for $\frac{H_2(s)}{Q_{mi}(s)}$

$$\frac{H_2(s)}{Q_{mi}(s)} = \frac{3gR}{\rho(g^2 + 16AgRs + 12A^2R^2s^2)}$$

FYI (not required) for $q_{mi}(t) =$ constant,

$$h_2(t) = \frac{3gR}{\rho g^2} = \frac{3R}{\rho g}$$

b) Using ⑤ and ⑥

$$\begin{bmatrix} \dot{h}_1 \\ \dot{h}_2 \end{bmatrix} = \begin{bmatrix} \frac{-g}{RA} & \frac{g}{RA} \\ \frac{g}{4RA} & \frac{-g}{3RA} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{\rho A} \\ 0 \end{bmatrix} q_{mi}(t)$$

$$\begin{bmatrix} h_1 \\ h_2 \\ q_{mo} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & \frac{\rho g}{3R} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} q_{mi}(t)$$

To get steady-state for $q_{mi}(t) =$ const, set $\dot{h}_1 = \dot{h}_2 = 0$

$$\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = -\begin{bmatrix} \frac{-g}{RA} & \frac{g}{RA} \\ \frac{g}{4RA} & \frac{-g}{3RA} \end{bmatrix}^{-1} \begin{bmatrix} \frac{1}{\rho A} \\ 0 \end{bmatrix} q_{mi}$$

$$= \begin{bmatrix} \frac{4R}{\rho g} \\ \frac{3R}{\rho g} \end{bmatrix}$$