ME 710, Exam 1 Winter 2012

One 8.5" by 11" cheat sheet.

1. Prove that $\lambda_1 < \mathbf{x}^T A\mathbf{x} < \lambda_n$ where A is a symmetric real matrix, $\mathbf{x}$ is an arbitrary unit vector of length n, and λ_i represent the sorted eigenvalues of A.

2. A beam with constant properties E, I, ρ, A and ℓ (where ρ is mass per volume) is supported in a simply supported configuration. Find the total displacement as function of space and time if the right end of the beam vibrates with a motion $w(\ell, t) = a\sin(\omega t)$. Hint: consider a shift of coordinates where $w(x,t) = y(x,t) + a\sin(\omega t)\frac{x}{\ell}$. Solve for the contributions of the first two modes explicitly in terms of the variables given.

3. (10 points) Use first order perturbation methods to estimate the eigenvalues of B given

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 4 \end{bmatrix}, \quad \Lambda = \begin{bmatrix} 1.2679 & 0 & 0 \\ 0 & 3.0000 & 0 \\ 0 & 0 & 4.7321 \end{bmatrix}, \quad \Phi = \begin{bmatrix} 0.7887 & 0.5774 & 0.2113 \\ 0.5774 & -0.5774 & -0.5774 \\ 0.2113 & -0.5774 & 0.7887 \end{bmatrix}$$

when

$$B = \begin{bmatrix} 1.99 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 4 \end{bmatrix}$$

$$X(x) = A\cos(\beta_n x/\ell) + B\sin(\beta_n x/\ell) + Ce^{-\beta_n x/\ell} + De^{\beta_n x/\ell}$$

$$\omega_n = \frac{\beta_n^2}{\ell^2}\sqrt{\frac{EI}{\rho A}}$$

Boundary Conditions	Mode Number (n)	A	B	C	D	β_n
Free-free	1	1	0	0	0	0
	2	0[1]	0	0	0	0
	3	1	-0.983	0.991	-0.009	4.730
	4	1	-1.001	1.000	0.000	7.853
	>4	1	-1.000	1.000	0.000	$\frac{(2n-3)\pi}{2}$
Clamped-free: cantilever	1	-1	0.734	0.867	0.133	1.875
	2	-1	1.019	1.009	-0.009	4.694
	3	-1	0.999	1.000	0.000	7.855
	>3	-1	1.000	1.000	0.000	$\frac{(2n-1)\pi}{2}$
Clamped-pinned	1	-1	1.001	1.000	0.000	3.927
	>1	-1	1.000	1.000	0.000	$\frac{(4n+1)\pi}{4}$
Clamped-sliding	1	-1	0.983	0.991	0.001	2.365
	>1	-1	1.000	1.000	0.000	$\frac{(4n-1)\pi}{4}$
Clamped-clamped	1	-1	0.983	0.991	0.001	4.730
	2	-1	1.001	1.000	0.000	7.853
	>2	-1	1.000	1.000	0.000	$\frac{(2n+1)\pi}{2}$
Pinned-pinned: simply supported	0	1	0	0	0	$n\pi$