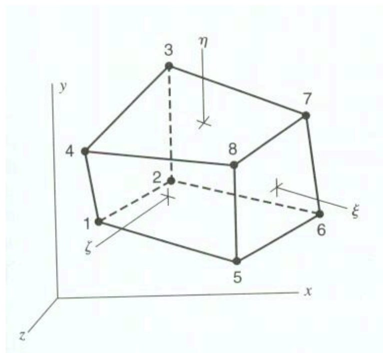
ME 712, Finite Element Method Applications — Final Exam, Spring 2012

One formula sheet to be turned in. No usage of computers other than matlab, mathematica, and printing. Not USBs or access to any files allowed. **Problems must be done in order in the blue books.**

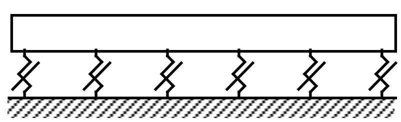
Log your commands and results to a file using the diary command. Edit out the irrelevant parts later.

1. (not for points on this exam) For each project, list the fraction of effort you performed (the remainder performed by your partner).

2. The brick element should be validated to make sure that it satisfied $k = \frac{EA}{\ell}$ (stiffness matches the closed form solution). If the load is applied to nodes 5-8, what boundary conditions should be applied to each of the 8 nodes. The drawing below is given only to illustrate order to node numbering. Presume the element is a perfect brick.



3. Consider an Euler-Bernoulli beam resting on a foundation (as if laying on the ground). The ground applied a force of γw per unit length where w is the deflection. Obtain the change in the 1,1, element of the stiffness matrix due to this applied load. Solve using shape functions in natural coordinates and Gauss point integration.

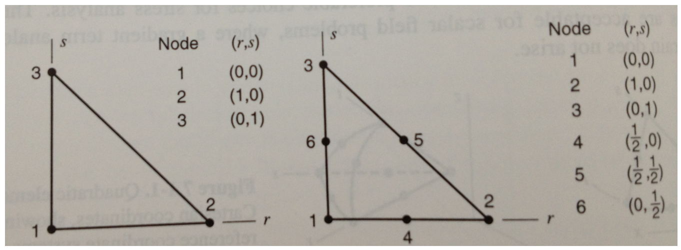


4. Apply two full steps of the subspace iteration method (for finding the first natural frequency and mode shape) to the following system:

$$K = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}, \qquad M = I$$

5. Determine the radius of gyration, k_y, of a quadratic (LST) triangular element assuming nodes at (0,0), (1,0), (0,1), (0.5,0), (0.5,0.5), (0,0.5) using Gauss point integration. Note that $k_y^2 = \frac{\int_A x^2 dxdy}{\int_A dxdy}$. Explain any assumptions. Show all work. No credit will be given for integration carried out other than as specified.

	Linear	Quadratic
$N_1 =$	$1 - r - s$	$(1 - r - s)(1 - 2r - 2s)$
$N_2 =$	r	$r(2r - 1)$
$N_3 =$	s	$s(2s - 1)$
$N_4 =$		$4r(1 - r - s)$
$N_5 =$		$4rs$
$N_6 =$		$4s(1 - r - s)$



Order	Sampling Location	Weights
1	0	2
2	$\pm\sqrt{3}$	1
3	$\pm\sqrt{0.6}$, 0	5/9, 8/9

Degree of Precision	Sampling Location	Weights
1	$\left(\frac{1}{3}, \frac{1}{3}\right)$	1
2	$\left(\frac{2}{3}, \frac{1}{6}\right), \left(\frac{1}{6}, \frac{1}{6}\right), \left(\frac{1}{6}, \frac{2}{3}\right)$	$\frac{1}{3}$