

Vesting Exam 1, 2004

$$1) \frac{X(\omega)}{F(\omega)} = \frac{1}{-10\omega^2 + 101j\omega + 1000}$$

$$2) f(t) = \sum_{-\infty}^{\infty} F_n e^{j2\pi n t/T}$$

$$T=3$$

$$F_n = \frac{1}{3} \int_2^3 e^{-j\frac{2\pi n}{3}t} dt$$

$$F_n = \frac{1}{3} \frac{-3}{2\pi n j} e^{-j\frac{2\pi n}{3}t} \bigg|_2^3$$

$$F_n = \frac{-1}{2\pi n j} \left(e^{-j2\pi n} - e^{-j\frac{4\pi n}{3}} \right)$$

$$= \frac{1}{2\pi n j} \left(1 - e^{-j\frac{4\pi n}{3}} \right)$$

$$F_0 = \frac{1}{3} \int_2^3 1 dt = \frac{1}{3}$$

$$f(t) = \frac{1}{3} + \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \frac{1 - e^{-j\frac{4\pi n}{3}}}{2\pi n j} e^{j2\pi n t/3}$$

$$= \frac{1}{3} + \frac{1}{2\pi j} \sum_{n=1}^{\infty} \frac{1}{n} \left(e^{j\frac{2\pi n}{3}t} - e^{-j\frac{2\pi n}{3}t} \right)$$

$$+ \frac{1}{2\pi j} \sum_{n=1}^{\infty} \frac{1}{n} \left(e^{j\frac{2\pi n}{3}(2+t)} - e^{-j\frac{2\pi n}{3}(2+t)} \right)$$

$$= \frac{1}{3} + \frac{1}{2\pi j} \sum_{n=1}^{\infty} \frac{1}{n} 2j \sin \frac{2\pi n t}{3}$$

$$+ \frac{1}{2\pi j} \sum_{n=1}^{\infty} \frac{1}{n} 2j \sin \left(\frac{2\pi n}{3} (2+t) \right)$$

$$= \frac{1}{3} + \sum_{n=1}^{\infty} \frac{1}{n\pi} \left[\sin \left(\frac{2\pi n}{3} (2+t) \right) - \sin \frac{2\pi n}{3} \right]$$

$$3) \quad \psi_1 = \frac{1}{\sqrt{.0841}} \begin{bmatrix} .08450 \\ .00357 \\ .27811 \end{bmatrix} \quad \text{(not mass normalized)} \\ \text{without } \frac{1}{\sqrt{.0841}}$$

4) No. There is no information in that matrix.

$$5) \quad H(j\omega) = \frac{.35282}{1.58 - \omega^2} + \frac{-.3333}{4 - \omega^2} + \frac{-.01978}{11.42 - \omega^2}$$

$$6) \quad \underline{z}(t) = S(t) \underline{z}(0) = \begin{bmatrix} \frac{\sin \omega_n t}{\omega_n} \\ \cos \omega_n t \end{bmatrix}$$

$$x(t) = \frac{\sin \omega_n t}{\omega_n}$$

$$x(t) = \cos \omega_n t$$