Closed book, closed notes. Use one $8\frac{1}{2} \times 11$ formula sheet, front and back. Test books will be provided.

Problems are 10 points each. Problem 4 is required for graduate students, bonus for undergraduates

1. Find the response, x(t), for all times t > 0 of the underdamped system modeled by the following equation:

$$m\ddot{x} + c\dot{x} + kx = \delta(t) + e^{\frac{-2\pi\zeta}{\sqrt{1-\zeta^2}}}\delta(t - \frac{2\pi}{\omega_d})$$

2. Find the response of the the system governed by the following equation of motion. *READ the equation very carefully.* Sketch its nondimensionalized magnitude (*recall that this is output relative to a constant input*) and phase versus r for multiple values of ζ , and label the values in simplest terms at r = 0, r = 1, and $r = \infty$.

$$m\ddot{x} + c\dot{x} + 2kx = ky$$

where
$$y(t) = Y \cos(\omega_b t)$$
.

- 3. A vibrating mass of 300 kg, mounted on a massless support by a spring of stiffness 40 kN/m and a damper of unknown damping coefficient, is observed to vibrate with a 100-mm amplitude while the support vibration has a maximum amplitude of only 2.5 mm at resonance. (25 points)
 - (a) Calculate the damping coefficient approximately.
 - (b) Calculate the force on the base.
- 4. Grad student/bonus Determine the natural frequencies and mode shapes for a clamped-clamped bar. The equation of motion of a bar is $\left(\frac{E}{\rho}\right) \frac{\partial^2 w(x,t)}{\partial x^2} = \frac{\partial^2 w(x,t)}{\partial t^2}$. (20% of other points)