ME 712, Finite Element Method Applications Final Exam, Spring 2000

Note: Due date is noon, June 7.

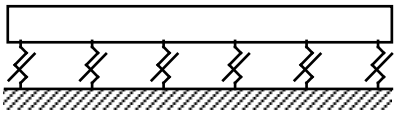
Take home exam. You may use any resource available with the exception of people other then myself. This sheet must be signed and turned in with the exam as the cover sheet.

I, _____, have not received nor given aid with respect to this exam.

_____

(signature)

1. Derive the single finite element equations for an Euler-Bernoulli beam resting on a cubic nonlinear foundation of stiffness γw^3 where γ is a constant and w represents displacement of the beam. Note: There will not be a stiffness matrix per se. Only a set of 4 nonlinear algebraic equations.



2. *Determine* the mass matrix of a beam element in two dimensions *including rotation of cross sections*. Assume the beam lies along the x axis in the $x-y$ plane.

3. Some of the following matrices each have fundamental flaws that violate certain conditions. For each matrix, identify whether the matrix is flawed, and what the flaw/s is/are.

   (a)
   $$K = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

   (b)
   $$M = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

   (c)
   $$K = \begin{bmatrix} 24 & -12 & 0 & 6 \\ -12 & 12 & -6 & -6 \\ 0 & -6 & 2 & 4 \\ 6 & -6 & 4 & 4 \end{bmatrix}$$

(d)

$$M = \begin{bmatrix} 24 & -12 & 0 & -6 \\ -12 & 12 & -6 & -6 \\ 6 & -6 & 12 & 4 \\ -6 & -6 & 4 & 12 \end{bmatrix}$$

4. Obtain the first 35 natural frequencies and mode shapes for the matrices M and K. List only the first 35 eigenvalues in your results along with your solution method. You may not use canned eigenvalue routines, except to check your answers. The matrices can be downloaded from the ME712 page in multiple formats. The ASCII files are in sparse array format so that only the lower triangular (plus diagonal) part of the matrix is given. The first column designates row number, the second column designates column number, and the third element designates the value of the entry. A .Z extension means the file has been UNIX compressed. This will speed up download times if you are using a modem. The matrices in the .mat file are also in sparse format. Type help sparse and help full in MATLAB for help. If you are using MATLAB, you will find that using sparse functions for your calculations will save significant time. Sparse arrays are not understood by older versions of MATLAB, so you may need to use MATLAB on gamma or any other machine with MATLAB 5.x.