Vibration Testing — Final Exam, Winter 2004

Closed book, closed notes, one formula sheet. Test booklets will be provided. *All work must be done shown in the exam book. No extra paper, for scrap or not, may be used.* Formula sheet must be turned in with the exam.

1. Determine the mode shapes and eigenvalues for the system described by equation (1), **but with a zero damping matrix**. Use these to generate the state space representation eigenvectors. *You may check your answers with a calculator, but will not receive credit unless you show how the mode shapes of the second-order system relate to the eigenvectors of the first-order model.*

$$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\ddot{\boldsymbol{x}} + \begin{bmatrix}.2 & -.1\\ -.1 & .2\end{bmatrix}\dot{\boldsymbol{x}} + \begin{bmatrix}2 & -1\\ -1 & 2\end{bmatrix}\boldsymbol{x} = \begin{bmatrix}1\\ 0\end{bmatrix}\sin(\omega t) \tag{1}$$

2. For the system of equation (1), with $C_a = [1 \quad 0]$, and $\Delta t = 0.1$, determine the discrete state-space model matrices A_d, B_d, C, and D.

3. Determine the mode shapes of the system defined by

$$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\ddot{\boldsymbol{x}} + \begin{bmatrix}.2 & -.1\\ -.1 & .2\end{bmatrix}\dot{\boldsymbol{x}} + \begin{bmatrix}2 & -1\\ -1 & 3\end{bmatrix}\boldsymbol{x} = \begin{bmatrix}1\\ 0\end{bmatrix}\sin(\omega t) \tag{2}$$

4. The impulse response of a system is given in the file `fedat1.mat` in the VibrationTesting folder of the U drive. Using the Ho-Kalman method, **not using EZERA**, to identify the continuous state system matrices. Write out on your exam paper the first 3 rows and the first 4 columns of your Hankel matrix (a 3×4 sub-section of the Hankel matrix).

5. At a particular frequency, $S_{xx} = 1$, $S_{ff} = 2$, and $S_{fx} = 0.95459 - 0.94549j$. Calculate H_1, H_2, H_v, and γ_{xf}.