

- 1) We are targeting 1100 rpm. Thus $\omega_a = \frac{1100 \cdot 2\pi}{60}$
 $\omega_a = 115.2 \text{ rad/s}$

Choosing $\nu = .25$, $m_a = 2268 \text{ kg}$, $k_a = \omega_a^2 m_a = 3.01 \times 10^7 \text{ N/m}$
 This will work unless there are two natural frequencies near ω_d .
 Without more information, this is the best that can be done.

2) $\xi_i = \frac{\alpha}{2\omega_i} + \frac{\beta\omega_i}{2}$

For $\omega_1 + \omega_2$, choose ξ_i at maximum

$\xi_1 = .3 = .966\alpha + .2588\beta$

$\xi_3 = .3 = .2588\alpha + .966\beta$

$\alpha = .245$

$\beta = .245$

Thus $\xi_2 = \frac{.245}{2 \cdot 1.4142} + \frac{.245 \cdot 1.4142}{2} = .26$

Will $\alpha = \beta = .245$, ξ_1 , ξ_2 + ξ_3 are inside the bounds

Choosing $\alpha = \beta = .2$

$\xi_1 = .245$

$\xi_2 = .2121$

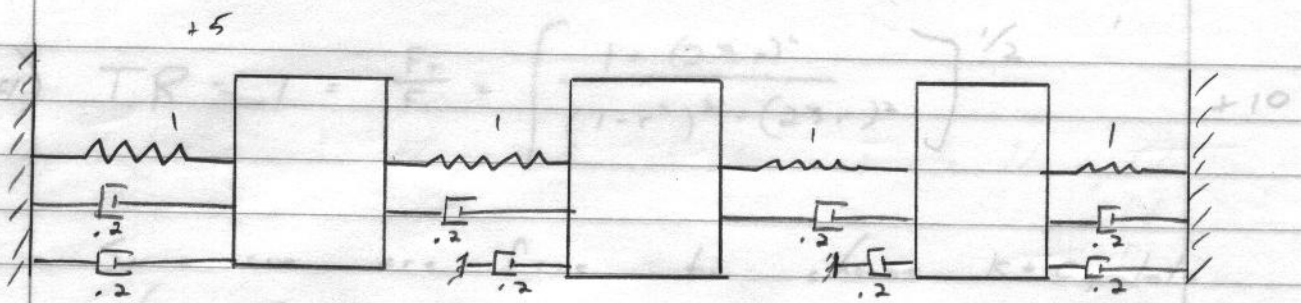
$\xi_3 = .245$

The damping ratios are now well within the bounds

From the diagram

$K = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$, $M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$C = \begin{bmatrix} .6 & -.2 & 0 \\ -.2 & .6 & -.2 \\ 0 & -.2 & .6 \end{bmatrix}$



$$3) \quad t_s = \frac{3}{5\omega}, \quad t_p = \frac{\pi}{\omega_d} \quad +6$$

$$5\omega = 3 \Rightarrow \omega = 0.6, \quad \omega_d = \pi$$

$$\omega \sqrt{1-\xi^2} = \pi$$

$$\frac{\sqrt{1-\xi^2}}{\xi} = \frac{\pi}{3}$$

$$1-\xi^2 = \left(\frac{\pi}{3}\right)^2 \xi^2$$

$$\xi^2 = \frac{1}{1 + \left(\frac{\pi}{3}\right)^2} = 9.04 \times 10^{-3}$$

$$\xi = 0.0951 \quad +3$$

$$\omega = \frac{3}{0.0951} = 3.156 \text{ rad/s} \quad +3$$

$$2\ddot{x} + 2\dot{x} + 2x = u = -g_1 x - g_2 \dot{x}$$

$$\ddot{x} + \left(1 + \frac{g_1}{2}\right)\dot{x} + \left(1 + \frac{g_2}{2}\right)x = 0 \quad +4$$

$$1 + \frac{g_1}{2} = \omega^2$$

$$g_1 = 2(\omega^2 - 1) = 17.9 \text{ N/m} \quad +2$$

$$1 + \frac{g_2}{2} = 2\xi\omega$$

$$g_2 = 2(2\xi\omega - 1) = 2(2 \cdot 0.0951 \cdot 3.156 - 1) = -0.8 \quad +2$$

$$u = -17.9x + 0.8\dot{x}$$

$$4) T.R. = .1 = \frac{F_T}{F_c} = \left[\frac{1 + (2gr)^2}{(1-r^2)^2 + (2gr)^2} \right]^{1/2} + 10$$

Since we are free to choose k & c , let's choose $g=0$

$$.1 = \frac{1}{r^2 - 1}$$

$$r^2 - 1 = 10$$

$$r^2 - 9 = 0$$

$$r = 3$$

$$\omega_{dr} = \frac{180 \cdot 2\pi}{60} = 18.85 \text{ rad/s}$$

$$\omega = \frac{1}{3} \cdot 18.85 = 6.28 \text{ rad/s}$$

$$* k = m \omega^2 = 987 \text{ N/m}$$

$$* c = 0 \text{ kg/s}$$

Thus, the eq. of motion is

$$(m \cdot b^2 + m \cdot a^2) \ddot{\theta} + (k \cdot a^2 + k \cdot b^2) \theta + m \cdot g \cdot a \cdot \sin \theta = 0$$

Or for small θ

$$(m \cdot b^2 + m \cdot a^2) \ddot{\theta} + (k \cdot a^2 + k \cdot b^2 + m \cdot g \cdot a) \theta = 0$$

5) Let x_1 be the motion of mass 1 to the right
 " x_2 " " 2 " left

$$T = \frac{1}{2} (m_1 \dot{x}_1^2 + m_2 \dot{x}_2^2) \quad +5$$

$x_1 \approx b\theta$ where θ is the clockwise rotation of the lever

$$x_2 \approx a\theta$$

$$T = \frac{1}{2} (m_1 b^2 \dot{\theta}^2 + m_2 a^2 \dot{\theta}^2)$$

$$T = \frac{1}{2} (m_1 b^2 + m_2 a^2) \dot{\theta}^2$$

$$U = \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k_2 x_2^2 + a(1 - \cos\theta) m_2 g \quad +5$$

$$U = \frac{1}{2} (k_1 a^2 + k_2 b^2) \theta^2 + a(1 - \cos\theta) m_2 g$$

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{\theta}} \right) = (m_1 b^2 + m_2 a^2) \ddot{\theta}$$

$$\frac{\partial U}{\partial \theta} = (k_1 a^2 + k_2 b^2) \theta + a \sin\theta m_2 g$$

Thus, the eqn of motion is

$$(m_1 b^2 + m_2 a^2) \ddot{\theta} + (k_1 a^2 + k_2 b^2) \theta + m_2 g a \sin\theta = 0$$

Or, for small θ

$$(m_1 b^2 + m_2 a^2) \ddot{\theta} + (k_1 a^2 + k_2 b^2 + m_2 g a) \theta = 0$$

+5

+5