ME 460/660 Fall 2005 Midterm solns

1) $18\,g = 176.5\ m/s^2$. With safety

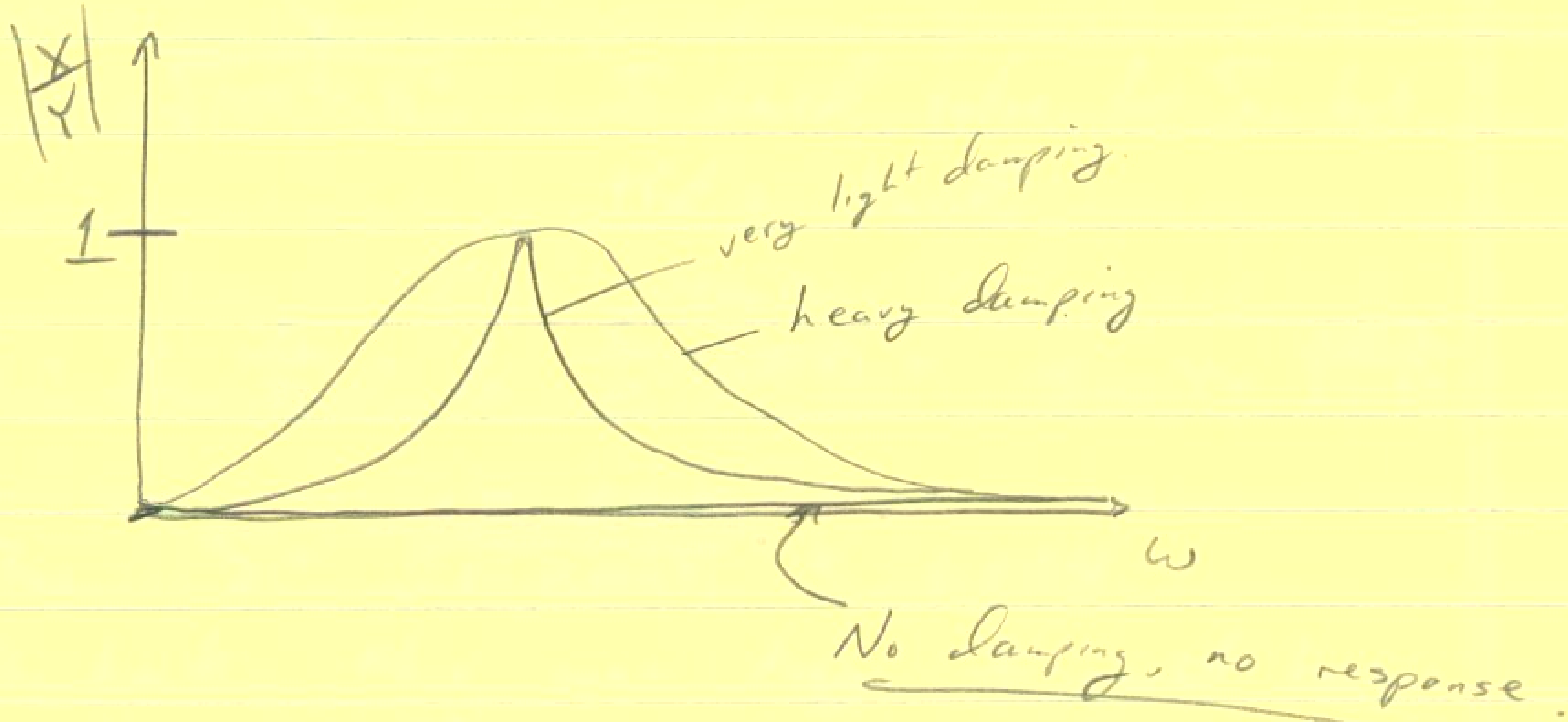
$$a_{max} = \frac{176.5}{1.1} = 160.4\ m/s^2$$

$$V_{max} = \frac{a_{max}}{\omega} = \frac{160.4}{2\pi \times 20} = \underline{1.28\ m/s}$$

$$d_{max} = \frac{a_{max}}{\omega^2} = \frac{160.4}{(2\pi \times 20)^2} \approx \underline{1.01\ cm}$$

The 18 g is for a constant acceleration. I doubt it would take very long for this to cause permanent damage. Likely a fraction of a second.

2) $$\frac{X}{Y} = \frac{cj\omega}{k - m\omega^2 + cj\omega}$$



Coupling through only an energy dissipator is incapable of showing extraordinary amplitudes.

3) $\ddot{x} + 4\dot{x} + 1 = 0 \qquad x(0) = 1 \quad , \quad v(0) = 0$

$x = X e^{\lambda t}$

$\lambda^2 + 4\lambda + 1 = 0$

$\lambda = \dfrac{-4 \pm \sqrt{4^2 - 4}}{2} = -2 \pm \sqrt{3} \qquad \lambda_1 = -2 + \sqrt{3} = -0.2679$

$x(t) = X_1 e^{\lambda_1 t} + X_2 e^{\lambda_2 t} \qquad \lambda_2 = -2 - \sqrt{3} = -3.3732$

$X(0) = X_1 + X_2 = 1 \qquad (1)$

$\dot{x}(0) = -0.2679 X_1 - 3.3732 X_2 = 0 \qquad (2)$

Mult 1 by 0.2679

$0.2679 X_1 + 0.2679 X_2 = 0.2679$

$-3.105 X_2 = 0.2679$

$X_2 = -0.086$

$X_1 = 1.086$

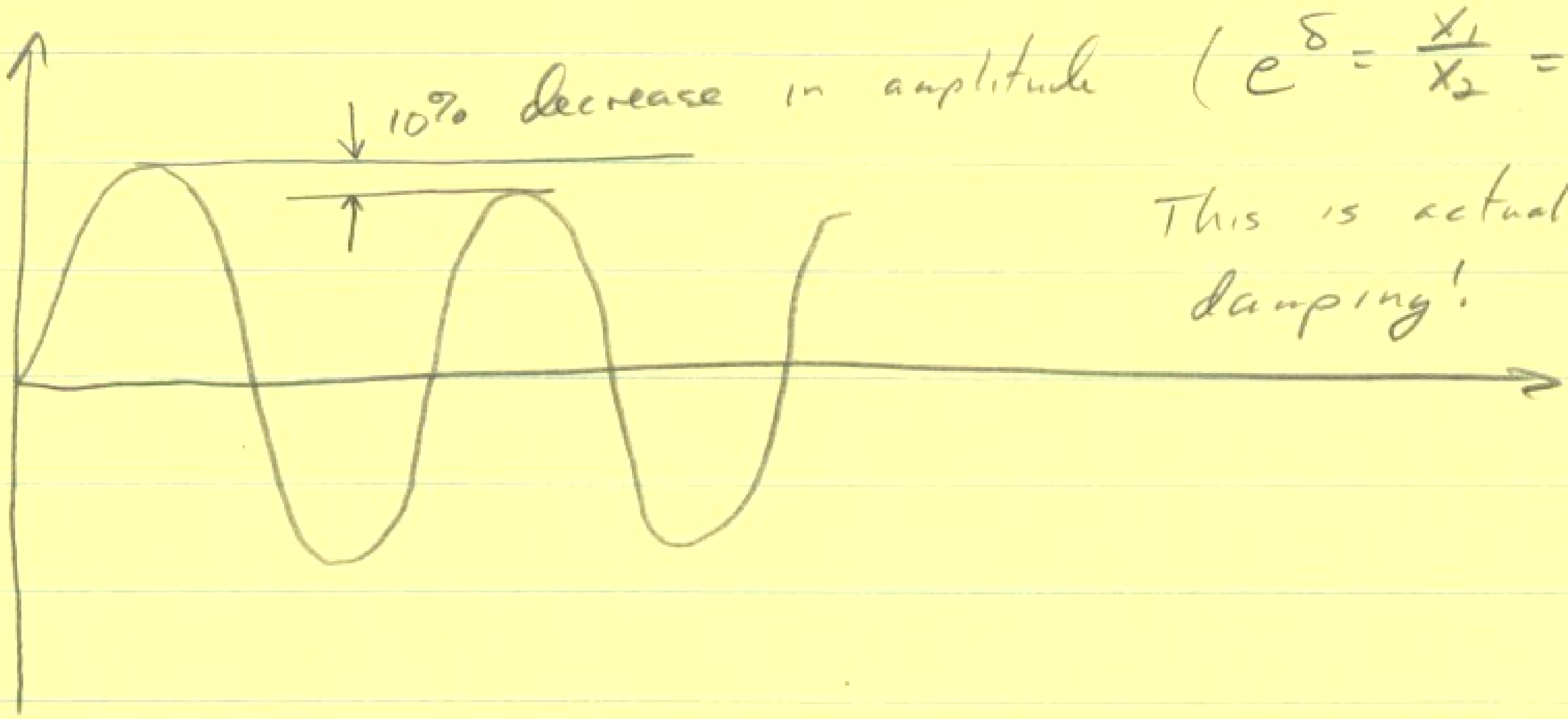
$x(t) = 1.086\, e^{-0.268 t} - 0.86\, e^{-3.373 t}$

4) $\zeta = \dfrac{\delta}{\sqrt{4\pi^2 + \delta^2}}$ I could solve for δ, but that's hard.

$\delta = \ln \dfrac{X_1}{X_2} = \ln \dfrac{e^{-\zeta\omega_n t}}{e^{-\zeta\omega_n (t+T)}} = \zeta\omega_n T = \dfrac{2\pi\zeta}{\sqrt{1-\zeta^2}}$

Thus $\delta = \dfrac{2\pi \cdot 0.015}{\sqrt{1 - 0.015^2}} = 0.094$

Oh, heck, this was tiny. I could have just said $\delta^2 \approx 0$ in the first place!



This is actually significant damping!

5) This was a HW problem. If you didn't solve it, you should have sought a solution. Briefly,

$U = mgh = mg(R-R_1)(1-\cos\theta)$

$T = \frac{1}{2} m v^2 + \frac{1}{2} J \Omega^2$ (using $T_{cg\,trans} + T_{cg\,rotation}$)

$v = (R-R_1)\dot{\theta}$, $J = m R_1^2$

$\Omega = v/R_1 = \left(\frac{R}{R_1} - 1\right)\dot{\theta}$

$T = \frac{1}{2} m (R-R_1)^2 \dot{\theta}^2 + \frac{1}{2} m \cancel{R_1^2} \frac{1}{\cancel{R_1^2}} (R-R_1)^2 \dot{\theta}^2$

$= m (R-R_1)^2 \dot{\theta}^2$

$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) = 2m(R-R_1)^2 \ddot{\theta}$

$\frac{\partial T}{\partial \theta} = 0$

$\frac{\partial U}{\partial \theta} = mg(R-R_1)\sin\theta$

Other terms in Lagrange's Eqn are zero

So, substituting

$$2m(R-R_1)^2\ddot{\theta} + mg(R-R_1)\sin\theta = 0$$

For small θ, $\sin\theta \sim \theta$, so

$$\omega_n = \sqrt{\frac{g}{2(R-R_1)}}$$

6) $\frac{E}{\rho} W_{xx} = W_{tt}$

$w(x,t) = X(x)\,T(t)$

$\frac{E}{\rho} X'' T = X \ddot{T}$

$\frac{E}{\rho} \frac{X''}{X} = \frac{\ddot{T}}{T} = -\omega_n^2$

$X'' + \frac{\rho}{E} \omega_n^2 X = 0$

$X = A \sin \sigma x + B \cos \sigma x$ where $\sigma = \omega_n \sqrt{\frac{\rho}{E}}$

BC state

$X(0) = 0$, $X(\ell) = 0$

From $X(0) = 0$, $B = 0$

$X(\ell) = 0 = A \sin \sigma \ell$

$\sigma \ell = n\pi$

$\therefore \sigma_n = \frac{n\pi}{\ell}$ $\boxed{\omega_n = \frac{n\pi}{\ell} \sqrt{\frac{E}{\rho}}}$

$\boxed{X(x) = A \sin \frac{n\pi x}{\ell}}$