

ME 464/664 SP '06 Final Exam Solutions

1) Unknown magnitude of step.  
Clear 1<sup>st</sup> order response

$$ay + by = bu(t)$$

$\tau$  appears to be  $\sim 2$  sec (63% of final response)

$$y(t) = 1 - e^{-\frac{t}{\tau}}, \dot{y} = \frac{1}{\tau}$$

Subst into EOM

$$a \frac{1}{\tau} e^{-\frac{t}{\tau}} + b - b e^{-\frac{t}{\tau}} = bu(t)$$

$$a = 2b$$

EOM  $\rightarrow$

$$2b\dot{y} + by = bu(t)$$

2) a)  $10y + 2y = 5t$

$$y(t) = a + bt + ce^{-t/5}$$

Sub into EOM

$$10b - \frac{10}{5}ce^{-t/5} + 2a + 2bt + 2ce^{-t/5} = 5t$$

$$\textcircled{1} \quad 10b + 2a = 0 \quad a = -5b$$

$$\textcircled{2} \quad \frac{-10}{5} + 2 = 0 \quad t = 5$$

$$\textcircled{3} \quad 2b = 5 \quad b = 2.5$$

Thus  $a = -5 \cdot 2.5 = -12.5$

$$y(t) = -12.5 + 2.5t + ce^{-t/5}$$

If  $y(0) = 1$ ,  $1 = -12.5 + c$

$$c = 13.5$$

$$y(t) = -12.5 + 2.5t + 13.5e^{-t/5}$$

b)  $10y + 2y = 5u(t)$

From the previous problem,  $y_h = c e^{-st}$ .

$$y(t) = a + be^{-t/5}$$

Subst into EOM

$$-2b e^{-t/5} + 2a + 2be^{-t/5} = 5u(t)$$

$$a = 2.5$$

$$y(t) = 2.5 + be^{-t/5}$$

Since  $y(0) = 1$

$$1 = 2.5 + b$$

$$b = -1.5$$

$$y(t) = 2.5 - 1.5e^{-t/5}$$

$$2c) \quad \xi = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} = \sqrt{2} \approx 1.4 \quad \omega = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}} \approx 0.707$$

System is under damped

$$\omega_d = \omega_0 \sqrt{1 - \xi^2} = \frac{1}{\sqrt{2}} \sqrt{1 - \frac{1}{2}} = \frac{1}{\sqrt{2}} \sqrt{\frac{1}{2}} = \frac{1}{2} \approx 0.5$$

$$x_h = A e^{-\xi\omega t} \sin(\omega t + \phi)$$

$$x_p = 2$$

$$x(t) = 2 + A e^{-t} \sin\left(\frac{1}{2}t + \phi\right)$$

$$x(0) = 0 = 2 + A \sin \phi \quad A = 0, \text{ or } \sin \phi = 0$$

$$\dot{x}(0) = 0 = -A \sin \phi + \frac{1}{2}A \cos \phi$$

Clearly  $A = 0$ , so

$$\sin \phi = \frac{1}{2} \cos \phi$$

$$\tan \phi = \frac{1}{2} = 0.464 \quad \text{or } 3.605 \quad (26.6^\circ, 206.6^\circ)$$

Take  $\phi = 26.6^\circ$ . From 1st IC

$$A = \frac{-2}{\sin 26.6} = -4.472$$

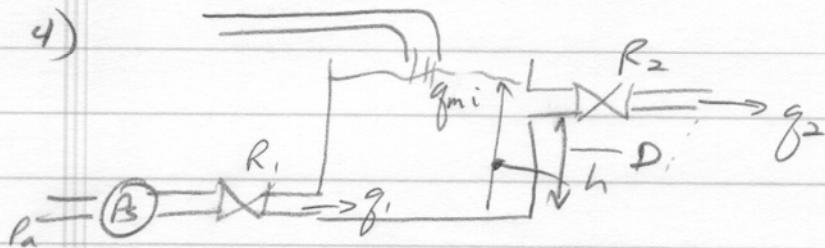
$$x(t) = 2 - 4.472 e^{-t} \sin\left(\frac{1}{2}t + 26.6\right)$$

3) a)  $|T(j\omega)| = 0.0994$   
 $\angle(j\omega) = -3.02 \text{ rad } (-173.7^\circ)$

b)  $-20.05 \text{ dB}$

c)  $x(t) = 0.994 \sin(11t - 2.25)$

d)  $s_{\text{sys}} = \text{tf}(1, [10, 0.1, 1200])$   
 $\text{bode}(\text{sys})$



Across  $R_1$    ①  $R_1 g_1 = p_s - \rho g h$ ,  $g_1 = \frac{p_s - \rho g h}{R_1}$

Across  $R_2$    ②  $R_2 g_2 = \rho g (h - D)$ ,  $g_2 = \frac{\rho g (h - D)}{R_2}$

Mass in tank   ③  $p_A h = g_1 - g_2 + g_{m1}$

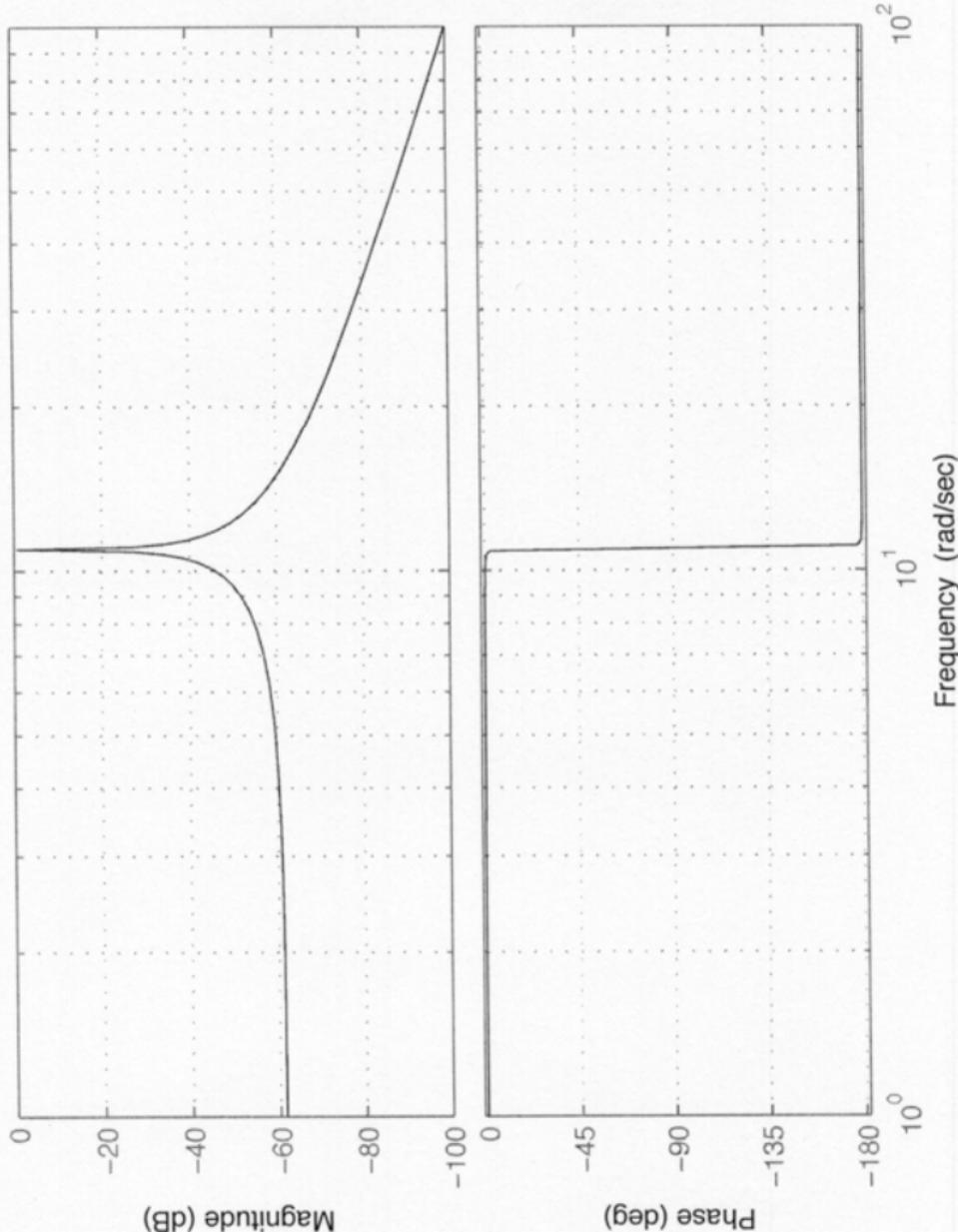
Subst ① and ② into ③

$$p_A h = \frac{p_s}{R_1} - \frac{\rho g h}{R_1} - \frac{\rho g h}{R_2} + \frac{\rho g D}{R_2} + g_{m1}$$

$$p_A h = -\rho g \left( \frac{1}{R_1} + \frac{1}{R_2} \right) h + \frac{\rho g D}{R_2} + \frac{p_s}{R_1} + g_{m1}$$

Bode Diagram

problem 3



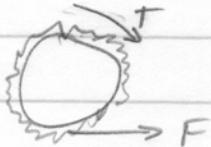
$$5) V_a = R a i + L_a \frac{di}{dt} + V_b$$

$$V_b = K_b \omega, \text{ so } V_a = R a i + L_a \frac{di}{dt} + K_b \omega$$

$$T = K_T i$$

$$I_{\text{eff}} = I_m + I_b + m_r R^2$$

$$x = R\theta$$



Resisting Force of spring is  $Kx$   
Moment is  $R_i Kx$ , so

$$T = I_{\text{eff}} \ddot{\theta} + R K x$$

$$\text{since } \theta = \frac{x}{R}$$

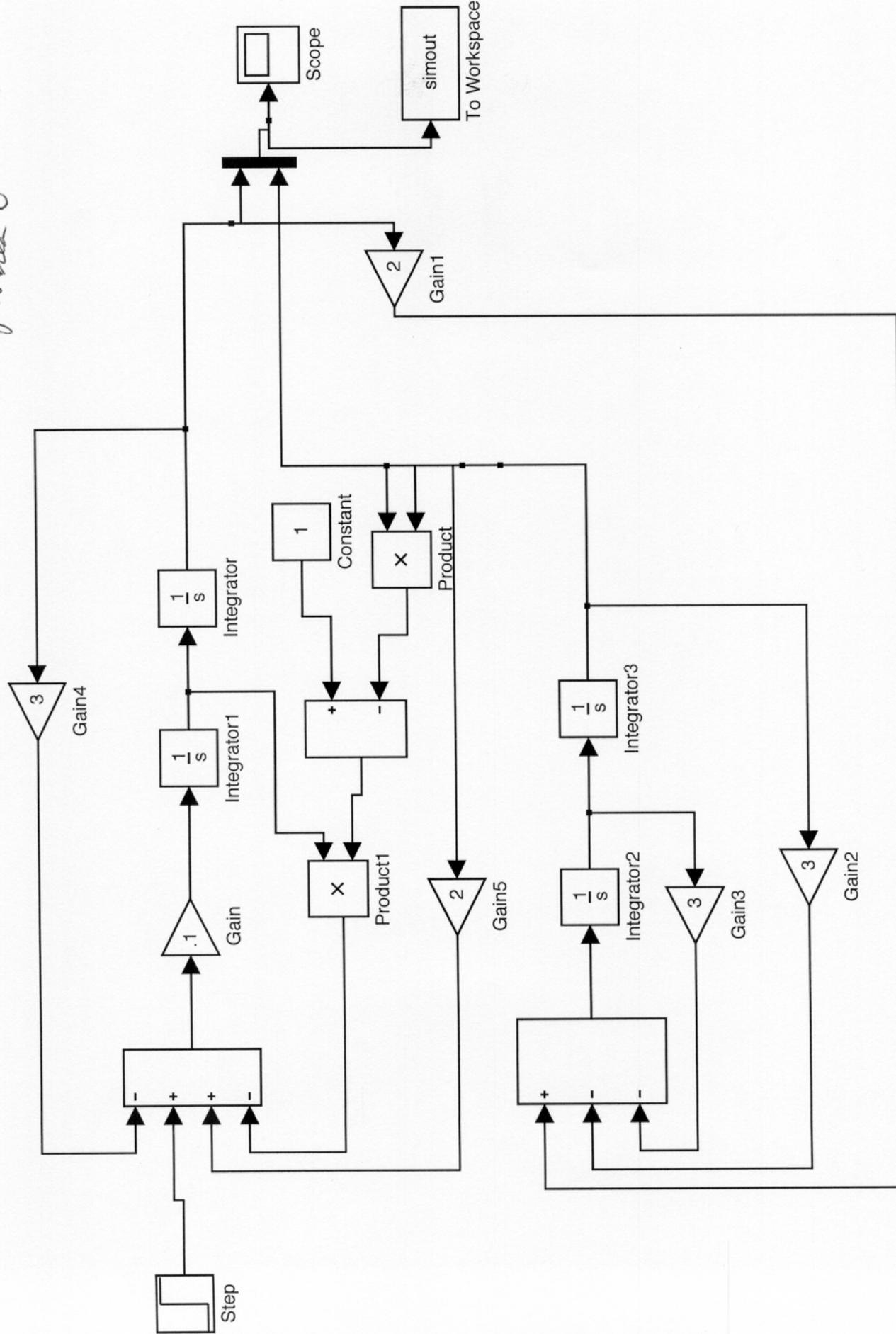
$$T = \frac{I_{\text{eff}}}{R} \ddot{x} + R K x, \quad K_T i = -\frac{I_{\text{eff}}}{R} \ddot{x} + R K x$$

States are  $i$ ,  $x$ , and  $\dot{x}$

$$\begin{bmatrix} \frac{di}{dt} \\ \dot{x} \\ \ddot{x} \end{bmatrix} = \underbrace{\begin{bmatrix} -R_a & 0 & \frac{-K_b}{L_a R} \\ 0 & 0 & 1 \\ \frac{K_T R}{I_{\text{eff}}} & \frac{-K R^2}{I_{\text{eff}}} & 0 \end{bmatrix}}_A \begin{bmatrix} i \\ x \\ \dot{x} \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{1}{L_a} \\ 0 \\ 0 \end{bmatrix}}_B V_a$$

$$y = \underbrace{\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}}_C \begin{bmatrix} i \\ x \\ \dot{x} \end{bmatrix} + \underbrace{\frac{0}{\omega} \cdot V_a}_D$$

Problem 6



Problem 6

