ME 460/660, Mechanical Vibration — Final Exam, Fall 2006

Closed book, closed notes. Use one provided $8\frac{1}{2} \times 11$ formula sheet and turn in with exam. Test books will be provided. Problems are to be done in the test book.

Fill in the blank with the appropriate letter from the list below. One point each. **Use each answer only once!**

In the first experimental lab, we observed the ___ of a ___ measured using an ___ (type of sensor). By observing the period we were able to ascertain the ___ , and by calculating the ___ , we were able to estimate the ___ . In the second lab, we obtained the ___ by forcing the beam with an ___ . The quality of the test data as a function of frequency can be rated by observing the ___ . The damping ratio was estimated using ___ .

The objective of a vibration absorber design is to set the natural frequency of the absorber to be equal to the ___ of the excited system.

Observing the figure of the accelerometer on the next page, a ___ hoop holds the masses against the ___ material. The force that the masses apply to that material is read as a ___ . Knowing that force, and the value of the mass, one can estimate the acceleration by applying ___ .

Use each answer only once (or not at all)!

a. natural frequency

b. damped natural frequency

c. driving frequency

d. damping ratio

e. quadrature peak picking

f. log decrement

g. beam

h. truss

i. shape memory alloy

j. free response

k. frequency response function

l. coherence

m. phase

n. accelerometer

o. pressure gauge

p. displacement sensor

q. load cell

r. impulse hammer

s. shaker

t. displacement

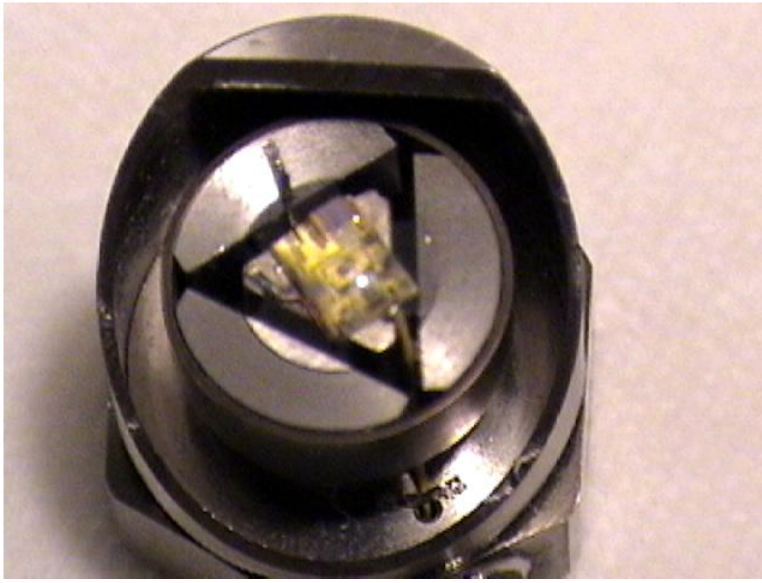
u. acceleration

v. voltage

w. piezoelectric

x. Newton's Law

y. Coulomb's Law



Long problems. 10 points each.

1. A linear system is governed by the following equation of motion:

$$m\ddot{x} + c\dot{x} + kx = c\dot{y}$$

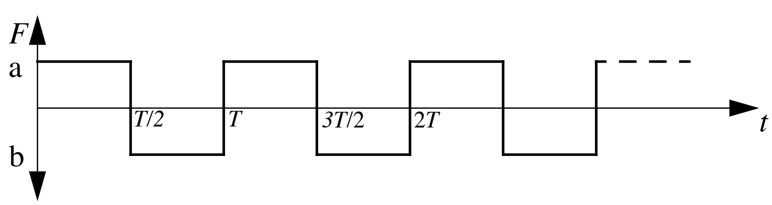
where $y(t) = Y\sin(\omega_b t)$. Given $m = 10$kg, $c = 1.0$kg/s, and $k = 10,000$kg/s^2.

(a) Sketch the FRF $\frac{X(j\omega)}{Y(j\omega)}$ for $0 < \frac{\omega_b}{\omega_n} < 60$. Use decibels ($20\log_{10}$) for the magnitude. *label limit and key values*

(b) Fill in the following table (*beware of units!*):

Y (m)	f_b (Hz)	$\lvert X\rvert$ (m)	$\measuredangle X$ (deg)
10	0		
2	5		
1	10		

2. Find the Fourier series of $F(t)$ shown below

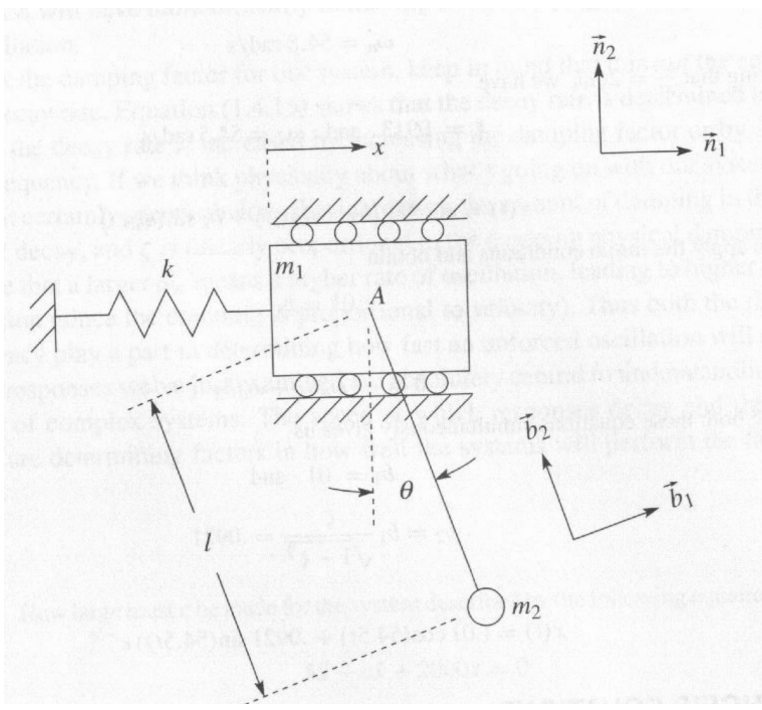


where $a = 1$ and $b = -2$.

3. Given
$$M = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}, \text{ and } \qquad K = \begin{bmatrix} 35 & -35 \\ -35 & 35 \end{bmatrix}$$
find the natural frequencies and mode shapes of the system. The mode shapes do not need to be mass normalized. Describe the natural motions of the two modes/frequency pairs.

4. A MDOF system at rest with
$$S = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$
no damping, and natural frequencies of 3 rad/sec and 10 rad/sec is excited by an force of
$$\mathbf{F}(t) = \begin{bmatrix} \frac{1}{\sqrt{2}}\delta(t) \\ \frac{1}{\sqrt{2}}\delta(t) \end{bmatrix}$$
Find $\mathbf{x}(t)$

5. Obtain the equations of motion the following system. Assume a mass-less rod.



6. Graduate Students/Undergraduate Bonus (20%): Solve for the steady-state (particular) response of the following system if the boundary conditions are presumed to be fixed-fixed ($0 < x < l$) where $c = \sqrt{\tau/\rho}$.

$$w_{tt}(x,t) - c^2 w_{xx}(x,t) = 100\delta(t)\sin\left(\frac{3\pi x}{l}\right)$$

Recall that the integral of a Dirac delta function times another function is equal to the "another function" evaluated when the argument of the argument of the Dirac delta function is zero.

BONUS: What is $h(t)$ called? What is $H(j\omega)$ called? What is the relationship between them? (4 points)