

Closed book, closed notes. Use one $8\frac{1}{2} \times 11$ formula sheet, front and back. Test books will be provided.

1. An experiment is run to determine the kinetic coefficient of friction between a block and a surface. The block is attached to a spring and displaced 150 mm from equilibrium. It is observed that the period of motion is 0.5 s and that the amplitude decreases by 10 mm each cycle. Determine the coefficient of friction and how many cycles of motion the block executes before motion ceases.
2. A boat and trailer are being pulled over an undulating road at a velocity v . The contour of the road is such that it can be approximated fairly accurately by a sine wave having a wave length of $l = 10$ ft and an amplitude of $Y = 0.5$ in. The total static deflection of the springs and the tires of the trailer due to the weight of the boat and trailer has been measured as 1.5 in.

Assume that the damping inherent in the system is viscous in nature and of such magnitude that $\zeta = 0.05$. Determine the speed v at which the maximum amplitude $|X|$ of the boat and trailer will be maximum and the value of $|X|$ at this speed.

3. Determine the response of a single degree of freedom undamped system to a ramp excitation $F(t) = F_0 t$.
4. **Graduate/Bonus:** Consider the vibration of a shaft that is clamped at the left end ($x = 0$) and has a disk of rotational mass moment of inertia J_p attached to the right end ($x = l$). Let G , J , and ρ all have unit values and determine the mode shapes of the system and the transcendental equation that could be used to solve for the natural frequencies of the system. The equation of motion for a shaft is $\frac{\partial^2 \theta(x,t)}{\partial t^2} = \left(\frac{GJ}{\rho J} \right) \frac{\partial^2 \theta(x,t)}{\partial x^2}$.