Solution techniques

$$\det|K - \lambda M| = 0$$

Obtain polynomial called characteristic eqn.
Does not work well for 1000s of DOF

Rayleigh-Ritz method

Rayleigh's Quotient: $R = \dfrac{\underline{x}^T K \underline{x}}{\underline{x}^T M \underline{x}}$ } consider energy method

Minimize R for all vectors $\underline{x}$ will give approximation of 1st eigenvalue. System can be reduced to leave n-1 eigenvalues, repeat.

Works well when no zero eigenvalues and 1st mode shape can be guessed.

In Ritz method, problem is reduced by assuming

$$\underline{x}_n = \sum a_{in} \underline{\psi}_i$$

a_i are Ritz coordinates, $\underline{\psi}_i$ are Ritz basis vectors.

$$\underset{n\times p}{S'} = \underset{n\times p}{\Psi}\ \underset{p\times p}{[a]}$$

Reduces size of eigenvalue problem to P.

<u>Example</u>

$$(A - \lambda I)\,\underline{x} = 0$$

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2.5 & -1.5 \\ 0 & -1.5 & 3 \end{bmatrix}$$

Assume $\underline{\Psi_1} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\frac{1}{\sqrt{2}}$ $\underline{\Psi_2} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$$(A - \Lambda)\,\underline{S} = 0$$

$$A S = \Lambda S$$

$$S^T A S = \Lambda$$

Assume (guess) $S' = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 \\ 0 & 1 \\ \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{22} & a_{21} \end{bmatrix}$

$$S'^T A S' = \Lambda'$$

$$\begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \end{bmatrix} A \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 \\ 0 & 1 \\ \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \Lambda'$$

$$[a]^T \underbrace{\begin{bmatrix} 2.5 & -1.7678 \\ -1.7678 & 2.5 \end{bmatrix}}_{A_p} [a] = \Lambda'$$

$$\Lambda' = \begin{bmatrix} .7322 & 0 \\ 0 & 4.2678 \end{bmatrix}$$

$$[a] = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$S' = \begin{bmatrix} .5 & -.5 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ .5 & .5 \end{bmatrix}$$

ans

$$\begin{bmatrix} .7258 & & \\ & 2.3198 & \\ & & 4.4544 \end{bmatrix}$$

$$\begin{bmatrix} .548 & .7907 & .2729 \\ .6983 & -.2528 & -.6697 \\ .4606 & -.5575 & .6907 \end{bmatrix}$$