FALL 98

ME 460/660 Exam 2 Solutions

■ Problem 1

The Frequency of the Base Excitation is:

In[16]:= $\omega_b = v \dfrac{2\pi}{L};$

The amplitude of the base motion is:

In[4]:= $Y = A;$

The damping ratio is:

In[6]:= $\zeta = \dfrac{c}{2\sqrt{k\,m}};$

The natural frequency is:

In[8]:= $\omega = \sqrt{\dfrac{k}{m}};$

From (2.40):

In[18]:= $X = Y\omega\sqrt{\dfrac{\omega^2 - (2\,\zeta\,\omega_b)^2}{(\omega^2 - \omega_b{}^2)^2 + (2\,\zeta\,\omega\,\omega_b)^2}}$

Out[18]= $\sqrt{\dfrac{\frac{k}{m} - \frac{4\,c^2\,\pi^2\,v^2}{k\,L^2\,m}}{\frac{4\,c^2\,\pi^2\,v^2}{L^2\,m^2} + \left(\frac{k}{m} - \frac{4\,\pi^2\,v^2}{L^2}\right)^2}}\;Y\omega$

or

In[20]:= $X = \mathrm{Simplify}[X]$

Out[20]= $\sqrt{\dfrac{k^2 - \frac{4\,c^2\,\pi^2\,v^2}{L^2}}{k\,m\left(\frac{4\,c^2\,\pi^2\,v^2}{L^2\,m^2} + \left(\frac{k}{m} - \frac{4\,\pi^2\,v^2}{L^2}\right)^2\right)}}\;Y\omega$

This is sufficient for full credit.

Phase info is not so important here, but the phase lag is a total of:

In[19]:= **ϕ = ArcTan[ω, 2 $\zeta\,\omega_b$] + ArcTan[2 $\zeta\,\omega\,\omega_b$, $\omega^2 - (\omega_b)^2$]**

Out[19]= $\mathrm{ArcTan}\left[\sqrt{\frac{k}{m}},\ \frac{2\,c\,\pi\,v}{L\sqrt{k\,m}}\right] + \mathrm{ArcTan}\left[\frac{2\,c\,\sqrt{\frac{k}{m}}\,\pi\,v}{L\sqrt{k\,m}},\ \frac{k}{m} - \frac{4\,\pi^2\,v^2}{L^2}\right]$

or

In[21]:= **ϕ = Simplify[ϕ]**

Out[21]= $\mathrm{ArcTan}\left[\sqrt{\frac{k}{m}},\ \frac{2\,c\,\pi\,v}{L\sqrt{k\,m}}\right] + \mathrm{ArcTan}\left[\frac{2\,c\,\sqrt{\frac{k}{m}}\,\pi\,v}{L\sqrt{k\,m}},\ \frac{k}{m} - \frac{4\,\pi^2\,v^2}{L^2}\right]$

The total solution is then (note that only X was required):

In[23]:= **x[t_] = X Cos[ω_b t - ϕ]**

Out[23]= $\sqrt{\frac{k^2 - \frac{4\,c^2\,\pi^2\,v^2}{L^2}}{k\,m\left(\frac{4\,c^2\,\pi^2\,v^2}{L^2\,m^2} + \left(\frac{k}{m} - \frac{4\,\pi^2\,v^2}{L^2}\right)^2\right)}}\ Y\omega$

$\mathrm{Cos}\left[\frac{2\,\pi\,t\,v}{L} - \mathrm{ArcTan}\left[\sqrt{\frac{k}{m}},\ \frac{2\,c\,\pi\,v}{L\sqrt{k\,m}}\right] - \mathrm{ArcTan}\left[\frac{2\,c\,\sqrt{\frac{k}{m}}\,\pi\,v}{L\sqrt{k\,m}},\ \frac{k}{m} - \frac{4\,\pi^2\,v^2}{L^2}\right]\right]$

■ Problem 2

In[67]:= **m = 1;**

In[68]:= **c = 0;**

In[69]:= **k = 100;**

In[70]:= **$\zeta = \frac{c}{2\sqrt{k\,m}}$;**

In[71]:= **$\omega = \sqrt{\frac{k}{m}}$;**

$\omega_d = \omega\sqrt{1 - \zeta^2}$;

Out[72]= 10

In[126]:= **tf = 10 ^ (-4) // N;**

In[111]:= **h[t_] = $\frac{\mathrm{Exp}[-\zeta\,\omega\,t]\ \mathrm{Sin}[\omega_d\,t]}{m\,\omega_d}$;**

The answer is obtained using the Convolution Integral

In[133]:= **x[t_] = $\int_0^{tf} 1\,h[t - \tau]\,d\tau$**

Out[133]= $\frac{1}{100}\,\mathrm{Cos}[10\,(-0.0001 + t)] - \frac{1}{100}\,\mathrm{Cos}[10\,t]$

Use of the other form of the convolution integral requires more tact. You must use two step functions. The next line just tells *Mathematica* what a unit step function is.

```
In[104]:= << Calculus`DiracDelta`
```

This defines the F(t) formally

```
In[107]:= F[t_] = UnitStep[t] - UnitStep[t - 10^(-4)];
```

and here the second form of the convolution integral is performed

```
In[114]:= Simplify[Integrate[F[t - τ] h[τ], {τ, 0, t}]]
```

Out[114]= $\frac{1}{50}\left(-\mathrm{Sin}\left[\frac{1}{2000}-5\,t\right]^2\,\mathrm{UnitStep}\left[-\frac{1}{10000}+t\right]+\mathrm{Sin}[5\,t]^2\,\mathrm{UnitStep}[t]\right)$

Some trig functions are needed to simplify this. This is clearly not the best way to do the integral

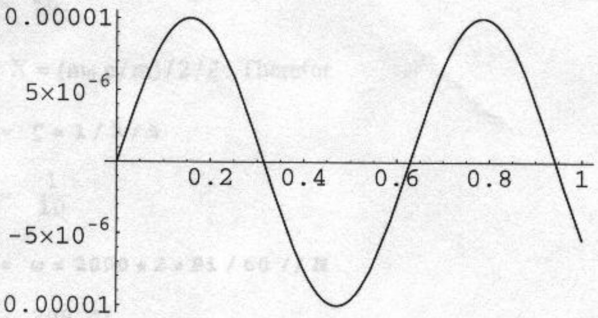
We could try breaking the integral into two parts. Here the limits on τ are difficult to figure out. We are actually integrating from the time of the negative step to the current time. The easiest way to do this is to shift τ so it starts at zero (so you see a unit step at τ=0) as is shown below. Done incorrectly, this should have yielded real junk that was obviously wrong.

```
In[134]:= Integrate[h[τ], {τ, 0, t}] + Integrate[-1 h[τ], {τ, 0, t - tf}]
```

Out[134]= $\frac{1}{100}\,\mathrm{Cos}[10\,(-0.0001+t)]-\frac{1}{100}\,\mathrm{Cos}[10\,t]$

This yields ths plotted result:

```
In[135]:= Plot[x[t], {t, 0, 1}]
```



Out[135]= - Graphics -

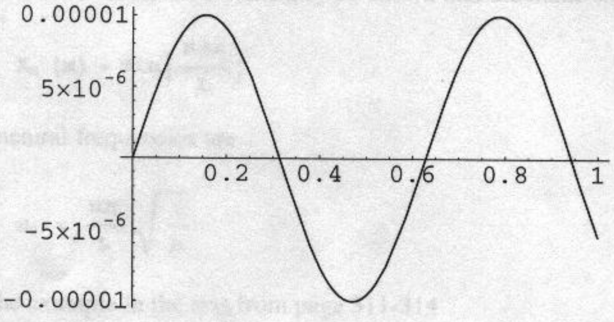
If you were paying close attention, you would have noticed that the impluse lasts for .0001 seconds, and that the period is around .63 seconds. Thus the easiest way to solve this problem is to simply state that v_0 = .0001 / *m*. Then from equation 1.10, the solution is simply:

In[138]:= $\mathbf{x[t\_] = \frac{tf}{\omega}\,Sin[\omega\ t]}$

Out[138]= 0.00001 Sin[10 t]

Plotted below, there clearly isn't much difference:

```
Plot[tf / ω Sin[ω t], {t, 0, 1}]
```



Out[136]= - Graphics -

■ Problem 3

Considering eqn. 2.51

At r = infinity, $X = m_0\, e/m$. Therefor

In[148]:= **m = 1000**

Out[148]= 1000

In[149]:= **unbalance = .05 m**

Out[149]= 50.

At r = 1, $X = (m_0\, e/m)/2/\zeta$. Therefor

In[146]:= **ζ = 1 / 2 / 5**

Out[146]= $\frac{1}{10}$

In[147]:= **ω = 2000 * 2 * Pi / 60 // N**

Out[147]= 209.44

In[151]:= **k = ω² m**

Out[151]= 4.38649×10^{7}

In[152]:= **c = 2 ζ ω m**

Out[152]= 41887.9

This is a lot more than was asked for

■ Problem 4:

By separation of variables, it can easily be shown that the mode shapes are:

$$X_n(x) = \mathrm{Sin}\left[\frac{n\pi x}{L}\right]$$

and the natural frequencies are

$$\omega_n = \frac{n\pi}{L}\sqrt{\frac{\tau}{\rho}}$$

This is the example in the text from page 311-314