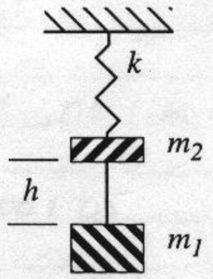
ME 460/660 Exam 1, Spring ‘96

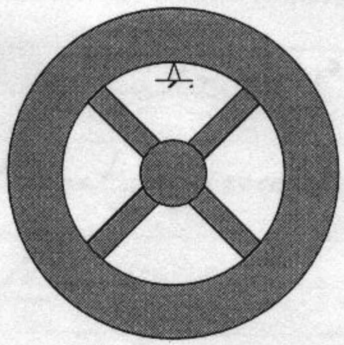
One cheat sheet. Front and back. No examples. No derivations. It must be turned in with the exam.

1) Choose m, c, and k such that a system with initial conditions of $x_0 = 0$ and $v_0 = 10$ mm/s does not exceed a displacement of 1 mm. The mass is restricted to values between 10 kg $< m <$ 15 kg.

2) A mass m_1 hangs from a spring k (N/m) and is in static equilibrium. A second mass m_2 drops through a height h from above m_1 and sticks to m_1 without rebound. Determine the subsequent motion. (Hint: Apply conservation of momentum at the instant of impact)



3) A flywheel weighing 980 N was allowed to swing as a pendulum about a knife edge at the inner side of the rim as shown below. If the measured period of oscillation was 1.28 s, determine the moment of inertia of the flywheel about its center.



4) A weight is attached to a spring of stiffness 525 N/m and a dashpot with unknown damping. When the weight is displaced and released, the period of vibration is found to be 1.8 sec, and the ratio of consecutive peak amplitudes is 4.2 to 1.0. Determine the amplitude and phase when a force $F = 2 \cos 3t$ acts on the system.

ME 460/660 Exam 1 Solutions, Sp '96

25 points

1) The amplitude of the velocity is .01 m/s
The amplitude of the displacement must not exceed .001 m.

$V_{max} = A\omega \quad , \quad x_{max} = A$

$A < .001\ m \qquad A\omega = .01\ m/s$

$A = \frac{.01}{\omega} < .001\ m$

$.01 < .001\ \omega$

$\omega > 10\ rad/s$

pick $m = \underline{12\ kg}$ (inside range)

$\omega = 15$

$\omega = \sqrt{\frac{k}{m}} \qquad k = m\omega^2 = \underline{2700\ N/m}$

Damping will not increase peak amplitude

$\underline{c = 10\ kg/s}$

2) Define positive down

1^{st}, find velocity of m_2 before collision

$$v = \sqrt{2gh} \qquad \left(\tfrac{1}{2} m_2 v_2^2 = m_2 g h\right.$$

The initial velocity of the system is then obtained from conservation of momentum.

$$m_2 \sqrt{2gh} = (m_1 + m_2)\, v_0$$

$$v_0 = \frac{m_2}{m_1 + m_2} \sqrt{2gh} \quad , \omega = \sqrt{\frac{k}{m_1 + m_2}}$$

The free response of an SDOF system is

$$x(t) = \frac{\sqrt{\omega^2 x_0^2 + v_0^2}}{\omega} \sin\left(\omega t + \operatorname{atan} \frac{\omega x_0}{v_0}\right)$$

The new equilibrium is shifted down by $\frac{m_2 g}{k}$. Setting $x = 0$ at this point,

$$x(0) = \frac{-m_2 g}{k} \qquad v_0 = \frac{m_2}{m_1 + m_2} \sqrt{2gh}$$

$$\phi = \operatorname{atan} \frac{-\omega m_2 g\, (m_1 + m_2)}{k\, m_2 \sqrt{2gh}} = \operatorname{atan} \frac{-\sqrt{g}}{\omega \sqrt{2h}}$$

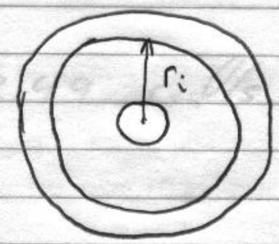
substituting yields the total response.

3) $T = 1.28\ s$

$f = .78125\ Hz$

$\omega = 4.908\ rad/s$ +5

$m = 100\ Kg$



Use energy method to derive E.O.M.

$T = \frac{1}{2} J \dot{\theta}^2$

$U = mg\, r_i (1 - \cos\theta)$

$\frac{d}{dt}(T + U) = 0 = J\ddot{\theta} + mg\, r_i \sin\theta$

For small θ, $\sin\theta \approx \theta$

$J\ddot{\theta} + mg\, r_i\, \theta = 0$ x10

$\omega = \sqrt{\frac{mg\, r_i}{J}}$

$24.1\, J = 980\, r_i$

$J = 40.67\, r_i$ x10

This is J about pivot point. Using // axis theorem

$J = J_o + m r_i^2$

$$\boxed{J_o = 40.67\, r_i - 100\, r_i^2}$$

4) $T = 1.8\,s, \quad f = .5556\ Hz, \quad \omega_d = 3.49\ rad/s$

$\omega_d = 3.49\ rad/s$

$\delta = \ln \frac{4.2}{1} = 1.435$

$\zeta = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} = .222$

$\omega = \frac{\omega_d}{\sqrt{1-\zeta^2}} = 3.57\ rad/s$

$m = \frac{k}{\omega^2} = \frac{525}{3.57^2} = 41.0\ kg$

$f_0 = \frac{2}{41} = .0488\ N/kg$

$A_0 = \frac{f_0}{\sqrt{(\omega^2 - \omega_{dr}^2)^2 + (2\zeta\omega\omega_{dr})^2}} = \frac{.0488}{\sqrt{14.02 + 22.6}}$

$= 8.06 \times 10^{-3}\ m \quad (8\ mm)$

$\phi = \operatorname{atan} \frac{2\zeta\omega\omega_{dr}}{\omega^2 - \omega_{dr}^2} = \frac{4.75}{3.74} = 51.78^\circ$

$.903\ rad$