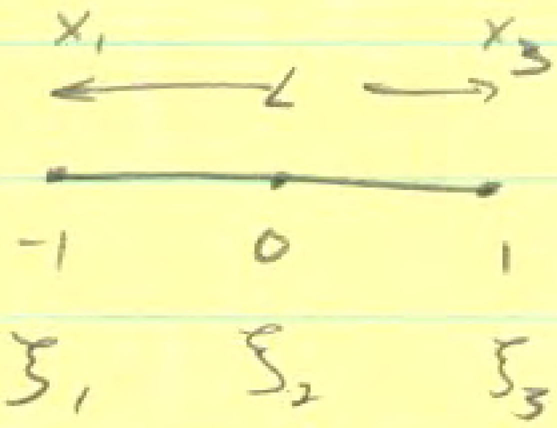
ME 712 Exam 2 Solutions, SP '05



1) $N_1 = -\frac{1}{2}\xi(\xi-1)$
$N_2 = 1-\xi^2$
$N_3 = \frac{1}{2}\xi(\xi+1)$

$J = \frac{L}{2}$ (equidistant means constant)

$$M = \rho A \int_{-1}^{1} \underbrace{\begin{bmatrix} \frac{1}{2}\xi(\xi-1) \\ 1-\xi^2 \\ \frac{1}{2}\xi(\xi+1) \end{bmatrix}}_{[N]^T} \begin{bmatrix} \frac{1}{2}\xi(\xi-1) & 1-\xi^2 & \frac{1}{2}\xi(\xi+1) \end{bmatrix} J\, d\xi$$

Polynomial is 4th order. Need 3 Gauss points.
Sample at $-\sqrt{6},\ 0,\ \sqrt{6}$
$\frac{5}{9}\quad \frac{8}{9}\quad \frac{5}{9}$

$[N_1] = [\ 0.6873 \quad .4 \quad -0.0873\ ]$ $(N @ -\sqrt{6})$

$[N_2] = [\ 0 \quad 1 \quad 0\ ]$ $[N @ 0]$

$[N_3] = [\ -.0873 \quad .4 \quad 0.6873\ ]$ $[N @ \sqrt{6}]$

$$M = \frac{\rho A L}{2}\sum_{i=1}^{3} [N_i]^T[N_i]\, w_i = \frac{\rho A L}{2}\begin{bmatrix} 0.2667 & 0.1333 & -0.06667 \\ & 1.0667 & 0.1333 \\ \text{Sym} & & 0.2667 \end{bmatrix}$$

```
%Matlab Code
a=[-sqrt(.6) 0 sqrt(.6)];
for i=1:3;
    z=a(i);
    N(i,1:3)=[1/2*z*(z-1) 1-z^2 1/2*z*(z+1)];
end;N
w=[5 8 5]/9;
m=zeros(3,3);
for i=1:3;
    m=m+w(i)*N(i,:)'*N(i,:);N(i,:)'*N(i,:)
end;
m
```

```
n = {{-1 / 2 * z * (z - 1) , 1 - z^2, 1 / 2 * z * (z + 1)}}
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$$\left\{\left\{-\frac{1}{2}\,(-1+z)\,z,\ 1-z^2,\ \frac{1}{2}\,z\,(1+z)\right\}\right\}$$

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Integrate[Transpose[n].n, {z, -1, 1}]
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$$\left\{\left\{\frac{4}{15},\ -\frac{2}{15},\ \frac{1}{15}\right\},\ \left\{-\frac{2}{15},\ \frac{16}{15},\ \frac{2}{15}\right\},\ \left\{\frac{1}{15},\ \frac{2}{15},\ \frac{4}{15}\right\}\right\}$$

```
MatrixForm[% // N]
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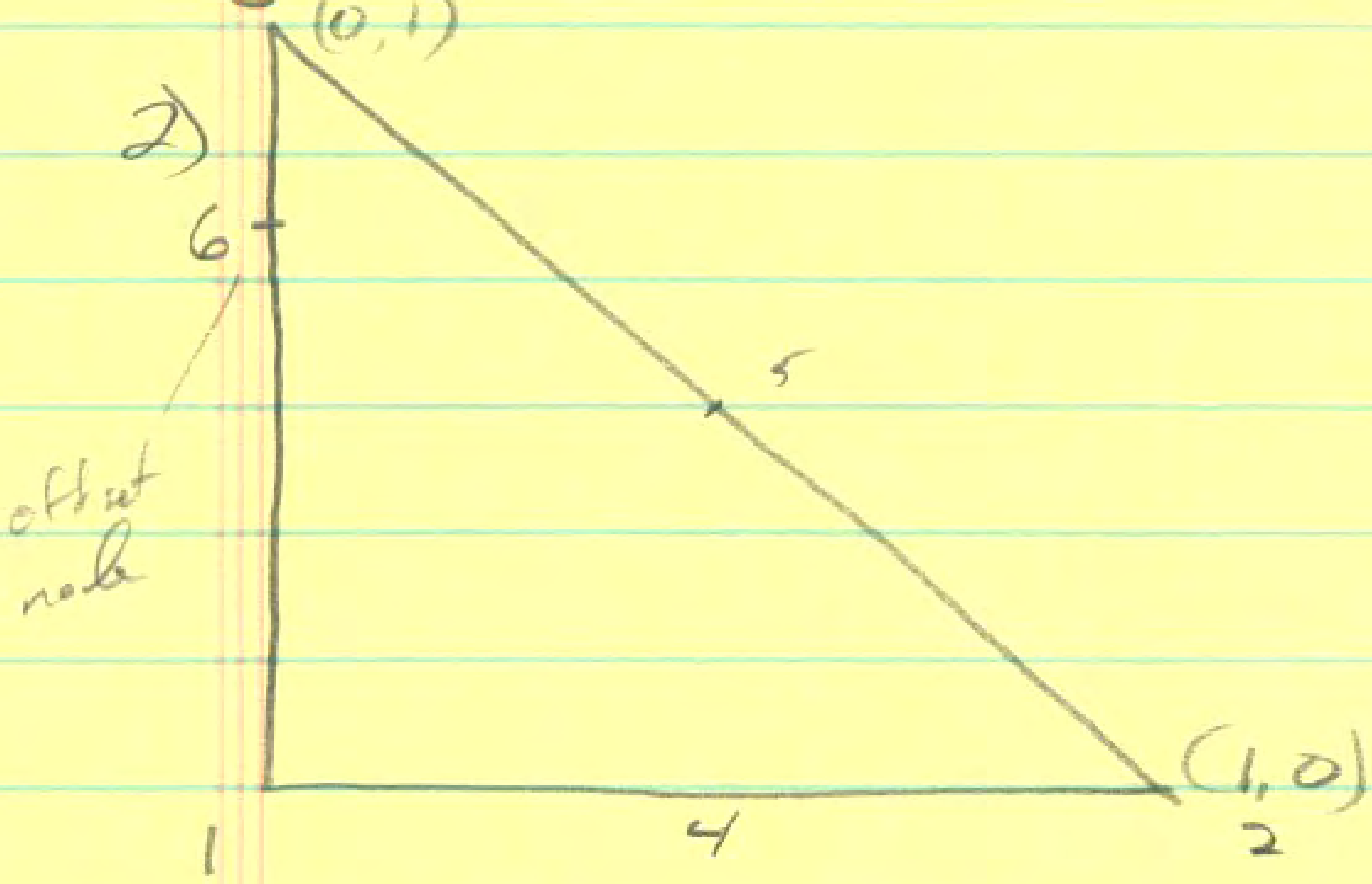
$$\begin{pmatrix} 0.266667 & -0.133333 & 0.0666667 \\ -0.133333 & 1.06667 & 0.133333 \\ 0.0666667 & 0.133333 & 0.266667 \end{pmatrix}$$

2)



see eqn 7.2-2

$J_{11} = x_{21} = 1$, $J_{21} = x_{31} = 0$ because observing 7.2-2, none of the x nodal values have shifted relative to the 'ideal' triangle.

J_{12} would be zero if $y_6 = \frac{1}{2}$. All we need is the change in the 6^{th} term

$$\frac{\partial N_6}{\partial r} = -4s$$

should be $-2s$, is $-3s$, so $\underline{J_{12} = -s}$

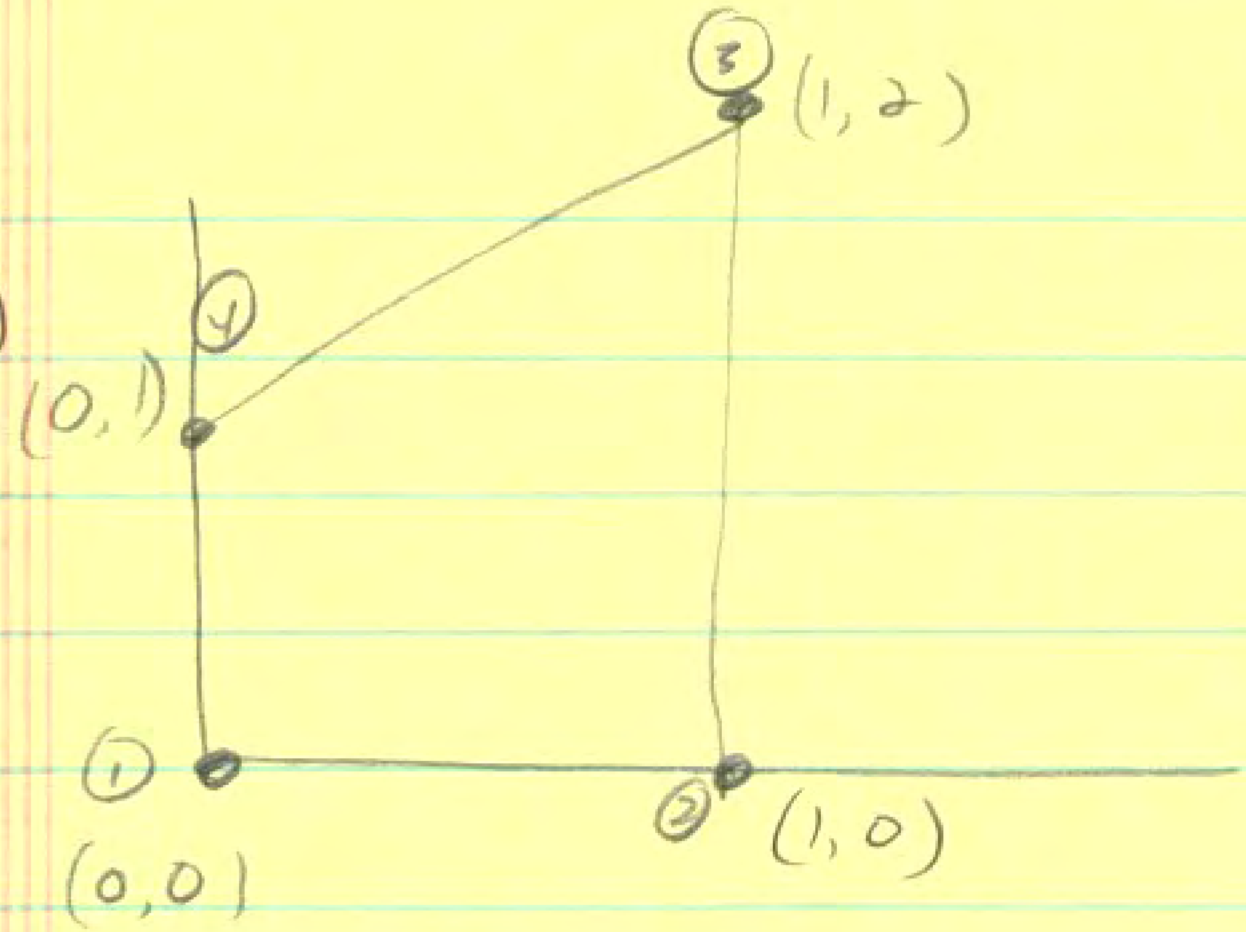
J_{22} would be 1 if $y_6 = \frac{1}{2}$. We need change in 6^{th} term, add it to 1.

$$\frac{\partial N_6}{\partial s} = 4 - 4r - 8s$$

should be $2 - 2r - 4s$, is $3 - 3r - 6s$, so $J_{22} = 2 - r - 2s$

$$J = \begin{bmatrix} 1 & -s \\ 0 & 2 - r - 2s \end{bmatrix}$$

3)



From 6.2-11

$$\begin{bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \\ \frac{\partial v}{\partial \xi} \\ \frac{\partial v}{\partial \eta} \end{bmatrix} = \left.\begin{bmatrix} \frac{\partial N_4}{\partial \xi} u_4 \\ \frac{\partial N_4}{\partial \eta} u_4 \\ \frac{\partial N_4}{\partial \xi} v_4 \\ \frac{\partial N_4}{\partial \eta} v_4 \end{bmatrix}\right|_{\xi=-1,\ \eta=1} = \frac{1}{4}\left.\begin{bmatrix} -(1+\eta) u_4 \\ (1-\xi) u_4 \\ -(1+\eta) v_4 \\ (1-\xi) v_4 \end{bmatrix}\right|_{\xi=-1,\ \eta=1} = \frac{1}{4}\begin{bmatrix} -2 u_4 \\ 2 u_4 \\ -2 v_4 \\ 2 v_4 \end{bmatrix}$$

We need Jacobian @ $(\xi, \eta) = (-1, 1)$

$$J = \frac{1}{4}\begin{bmatrix} 0 & 0 & 2 & -2 \\ -2 & 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \end{bmatrix} = J^{-1}\begin{bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \end{bmatrix} = \begin{bmatrix} -2 u_4 \\ u_4 \end{bmatrix}, \quad \begin{bmatrix} \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial y} \end{bmatrix} = J^{-1}\begin{bmatrix} \frac{\partial v}{\partial \xi} \\ \frac{\partial v}{\partial \xi} \end{bmatrix} = \begin{bmatrix} -2 v_4 \\ v_4 \end{bmatrix}$$

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} -2 u_4 \\ v_4 \\ u_4 - 2 v_4 \end{bmatrix}$$

$\underline{\sigma} = [E]\underline{\epsilon}$, but $[E]$ not given