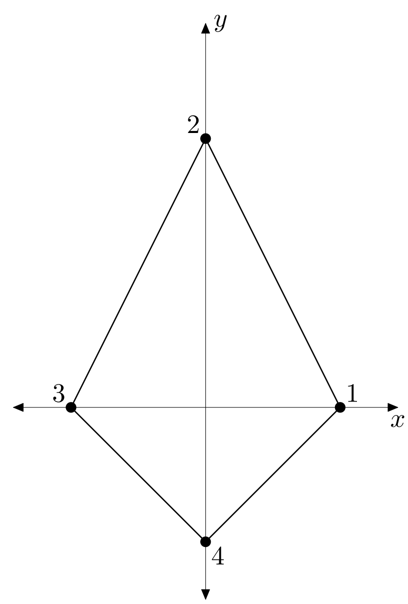
ME 712, Finite Element Method Applications — Exam 2, Spring 2012

Formula sheet, closed notes. Test books will be provided.

1. Derive the shape functions in natural coordinates for a one-dimensional (no rotation to 2-D or 3-D) 3-noded rod element with the center node at 0 in natural coordinates.

2. Determine the stiffness matrix of the one dimensional (no rotation to 2-D or 3-D) 3-noded *rod* element with the center node at 3/4 ℓ from the left in global unrotated coordinates using 3-point Gauss quadrature. Presume constant area, modulus and density and a length of 1m.

3. Plot the determinant of the Jacobian of a bilinear quadrilateral (Q4) element with nodes 1-4 at (1,0), (0,a), (-1,0), and (0,-1) along the y axis. Perform this for value of $a = 0, 1, 2, 4$. Turn in all code and plots.



4. Given the mass matrix of a beam in 2-D, find the mass moment of inertia about the left end by applying appropriate constraints. You may use the "elementary" method from basic finite elements for the part of your solution where it is appropriate, but keep in mind that it is less than half of the solution process.

$$M = \frac{\rho A \ell}{420}\begin{bmatrix} 156 & 22\ell & 54 & -13\ell \\ 22\ell & 4\ell^2 & 13\ell & -3\ell^2 \\ 54 & 13\ell & 156 & -22\ell \\ -13\ell & -3\ell^2 & -22\ell & 4\ell^2 \end{bmatrix}$$

Bonus: In problem 2, how many Gauss points should have been used?