ME 360 System Dynamics SP'07 Final

1) $3\tau \approx 6$ sec

$\tau = 2$ sec

$$\boxed{\begin{array}{l} 2\dot{y} + y = u(t) \\ \text{or} \quad \dot{y} + \frac{1}{2}y = \frac{1}{2}u(t) \end{array}}$$

Note that $y(t) \approx 1 - e^{-\frac{t}{2}}$

$= y_p + y_h$

$y_h = -e^{-t/2}$

$\dot{y}_h = \frac{-1}{2} e^{-t/2}$

2) a) $y(t) = \frac{5}{2}t + C_1 + C_2 e^{-t/\tau}$

$\dot{y}(t) = \frac{5}{2} - \frac{C_2}{\tau} e^{-t/\tau}$

substituting

$$10\,\frac{5}{2} - 10\,\frac{C_2}{\tau} e^{-t/\tau} + \cancelto{0}{5t} + 2C_1 + 2C_2 e^{-t/\tau} = \cancelto{0}{5t}$$

$25 + 2C_1 = 0$, $C_1 = \frac{-25}{2}$

$\frac{-10C_2}{\tau} + 2C_2 = 0$

$2\tau = 10$, $\tau = 5$

$y(0) = 1 = C_1 + C_2$

$C_2 = 1 - C_1 = \frac{27}{2}$

$$y(t) = \frac{5}{2}t + \frac{-25}{2} + \frac{27}{2} e^{-t/5}$$

b) $y(t) = \frac{5}{2} + Ce^{-t/5}$ (reusing the time constant)

$y(0) = 1 = \frac{5}{2} + C$

$C = \frac{-3}{2}$

$y(t) = \frac{5}{2} - \frac{3}{2} e^{-t/5}$

c) $2\ddot{y} + 2\dot{y} + y = 2u(t)$, $\ddot{y} + \dot{y} + \frac{1}{2}y = u(t)$

$2r^2 + 2r + 1 = 0$

$r = \frac{-2 \pm \sqrt{4-4\cdot 2}}{4} = -.5 \pm .5j$

$\omega_d = .5$ $\tau = \frac{1}{.5} = 2$

$y(t) = 2 + Ae^{-t/2} \sin\left(\frac{t}{2} + \theta\right)$

$y(0) = 0 = 2 + A\sin\theta$

$\dot{y}(0) = 0 = -\frac{1}{2}A\sin\theta + \frac{1}{2}A\cos\theta$

$\theta = \frac{\pi}{4}$

$A = \frac{-2}{\sin\frac{\pi}{4}} = \frac{-2}{\frac{\sqrt{2}}{2}} = \frac{-4}{\sqrt{2}} = -2\sqrt{2}$

$y(t) = 2 + -2\sqrt{2}\, e^{-t/2} \sin\left(\frac{t}{2} + \theta\right)$

3) sys= tf([1 1],[10 0.1 1400])
bode(sys)

a) 0.0617 (abs((11*j+1)/(10*(11j)^2+0.1*11j+1400))

b) -24.2 dB , 84.5° (1.4748 rad)

c) $0.617 \sin\left(11t + \frac{\pi}{4} + 1.4748\right)$

d) see top (see print at end

4) For $h > D$

$$\rho A \dot{h} = q_{mi} - q_{mo} + q_1, \quad q_1 = \frac{(P_s - \rho g h)}{R_1}$$

$$q_{mo} = \frac{\rho g(h-D)}{R_2}$$

$$\rho A \dot{h} = \left(-\frac{\rho g}{R_1} - \frac{\rho g}{R_2}\right) h + q_{mi} + \frac{\rho g D}{R_2} + \frac{P_s}{R_1}$$

For $h < D$

$$\rho A \dot{h} = \frac{-\rho g}{R_1} h + q_{mi} + \frac{P_s}{R_1}$$

In state space

$$\dot{h} = \underbrace{\begin{bmatrix} -\frac{g}{AR_1} \end{bmatrix}}_{A} h + \underbrace{\begin{bmatrix} \frac{1}{\rho A} & \frac{1}{\rho A R_1} \end{bmatrix}}_{B} \begin{bmatrix} q_{mi} \\ P_s \end{bmatrix}$$

$$y = \underbrace{1}_{C} h + \underbrace{0}_{D} \underline{u}$$

5) $v_a = R_a i + L_a \frac{di}{dt} + K_b \dot{\theta}$

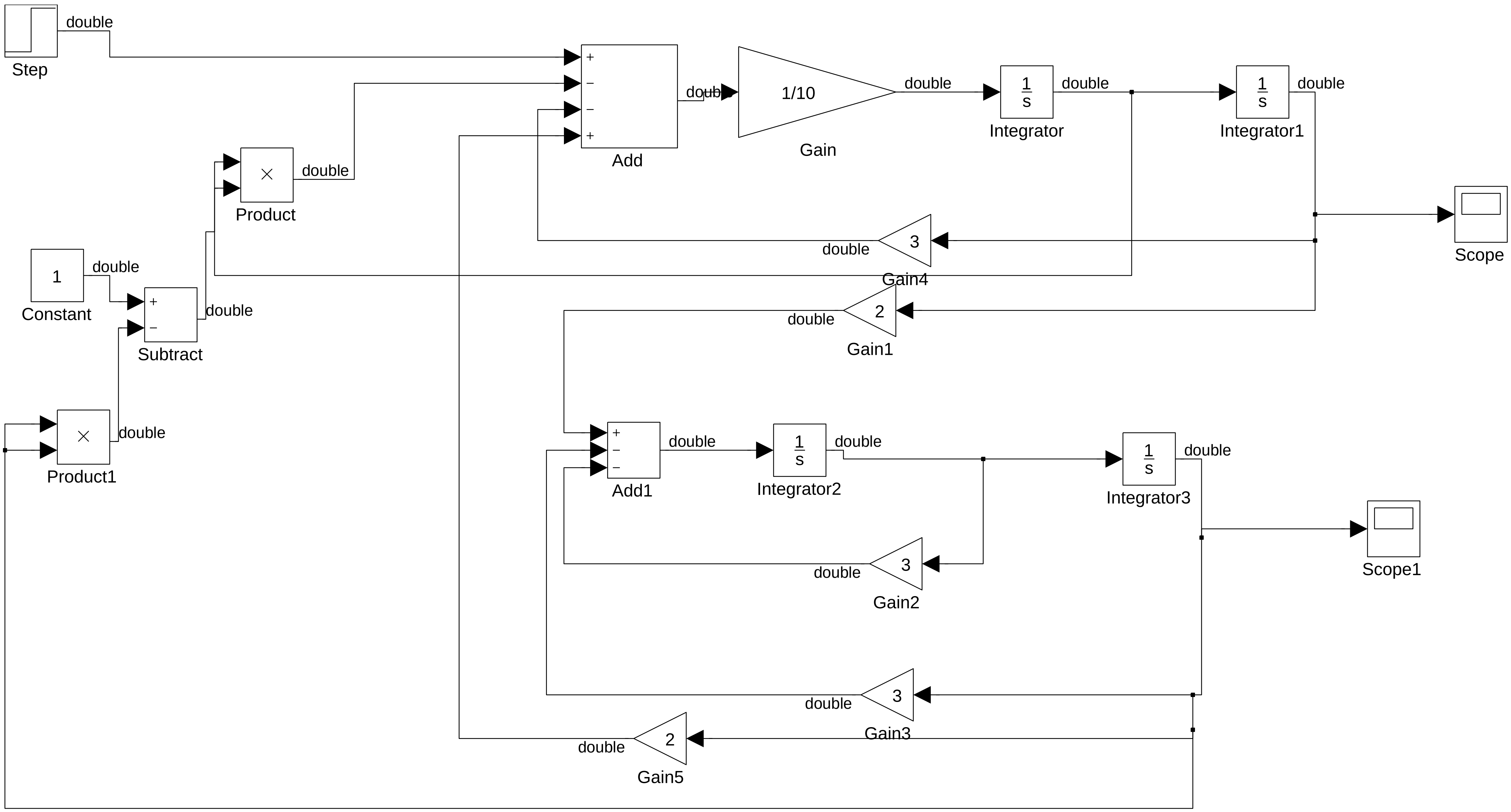
$$i K_T = (I_{eff}) \ddot{\theta} + C R^2 \dot{\theta} + K R^2 \theta$$

$$I_{eff} = I_m + I_p + m_r R^2$$

In state space

$$\begin{bmatrix} \frac{di}{dt} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{-R_a}{L_a} & \frac{-K_b}{L_a} & 0 \\ 0 & 0 & 1 \\ \frac{K_T}{I_{eff}} & \frac{-KR^2}{I_{eff}} & \frac{-CR^2}{I_{eff}} \end{bmatrix} \begin{bmatrix} i \\ \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} \\ 0 \\ 0 \end{bmatrix} v_a$$

6)



Step
Constant
Subtract
Product
Product1
Add
Gain
1/10
Integrator
Integrator1
Scope
Gain4
Gain1
Add1
Integrator2
Integrator3
Scope1
Gain2
Gain3
Gain5