Computational Methods in Structural Dynamics, Final Winter 2004

One 8.5" by 11" cheat sheet. Problems are 10 points each.

1. The equation of motion of a fixed-fixed string is given by: $\tau \frac{\partial^2 w(x,t)}{\partial x^2} = \rho(x) A \frac{\partial^2 w(x,t)}{\partial t^2}$. If $\rho(x) = \frac{M}{l} + M\delta\left(x - \frac{l}{2}\right)$, obtain a best estimate of the first natural frequency using a two term series using the Rayleigh Ritz method. How much better is a two term series at estimating the first natural frequency than a one term series?

2. A rope of length l, area A, and density ρ is hanging from a tall building. Assuming constant gravity, obtain the equation of motion and the boundary conditions.

3. Apply Given's method to the following matrix:

$$\begin{bmatrix} 4 & 1 & 1 \\ 1 & 9 & 2 \\ 1 & 2 & 16 \end{bmatrix}$$

4. Apply Jacobi's method through one complete cycle (trying to get rid of each of the off-diagonals only once) to the following matrix:

$$\begin{bmatrix} 4 & 1 & 0 \\ 1 & 9 & 2 \\ 0 & 2 & 16 \end{bmatrix}$$

5. The equation of motion of a system is given by

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \ddot{\mathbf{x}} + \begin{bmatrix} k_1 - m\Omega^2 & 0 \\ 0 & k_2 - m\Omega^2 \end{bmatrix} \mathbf{x} = \mathbf{0}$$

   Discuss the stability of the system over the range of values for Ω.