ME 710, Exam 1 Winter 2011

One 8.5" by 11" cheat sheet.

1. A beam with constant properties E, I, ρ, A and l (where ρ is mass per volume) is supported in a clamped-clamped configuration. However, the right-end clamp seems to be flimsy. Find the upper and lower bounds of the 3rd natural frequency using the provided page. How would you judge flimsy, and how did you decide on your answer to the bounds?

2. Find the natural frequencies and mode shapes of the beam of the previous problem if $E = 2.1E11$ Pa, $\rho = 7.8E3$ kg/m^3, $G = 8.0E10$ Pa, $l = 1$ m. The cross section is rectangular with $h = 2$ cm, $w = 3$ cm. The spring stiffness is $k = 10$ Nm.

3. Nondimensionalize the governing equation of the preceding problem.

4. Determine the steady state (non-transient) response of the system

$$EI\frac{\partial^4 w}{\partial x^4} + c\frac{\partial w}{\partial t} + \rho A\frac{\partial^2 w}{\partial t^2} = \sin(t)\left(\delta\left(x - \frac{l}{3}\right) - \delta\left(x - \frac{2l}{3}\right)\right)$$

   for a pinned-pinned (simply supported) boundary condition.

$$X(x) = A\cos(\beta_n x/\ell) + B\sin(\beta_n x/\ell) + Ce^{-\beta_n x/\ell} + De^{\beta_n x/\ell}$$

$$\omega_n = \frac{\beta_n^2}{\ell^2}\sqrt{\frac{EI}{\rho A}}$$

Boundary Conditions	Mode Number (n)	A	B	C	D	β_n
Free-free	1	1	0	0	0	0
	2	0[1]	0	0	0	0
	3	1	-0.983	0.991	-0.009	4.730
	4	1	-1.001	1.000	0.000	7.853
	>4	1	-1.000	1.000	0.000	$\frac{(2n-3)\pi}{2}$
Clamped-free: cantilever	1	-1	0.734	0.867	0.133	1.875
	2	-1	1.019	1.009	-0.009	4.694
	3	-1	0.999	1.000	0.000	7.855
	>3	-1	1.000	1.000	0.000	$\frac{(2n-1)\pi}{2}$
Clamped-pinned	1	-1	1.001	1.000	0.000	3.927
	>1	-1	1.000	1.000	0.000	$\frac{(4n+1)\pi}{4}$
Clamped-sliding	1	-1	0.983	0.991	0.001	2.365
	>1	-1	1.000	1.000	0.000	$\frac{(4n-1)\pi}{4}$
Clamped-clamped	1	-1	0.983	0.991	0.001	4.730
	2	-1	1.001	1.000	0.000	7.853
	>2	-1	1.000	1.000	0.000	$\frac{(2n+1)\pi}{2}$
Pinned-pinned: simply supported	0	1	0	0	0	$n\pi$