Vibration Testing — Exam 2, Winter 2004

Closed book, closed notes, one formula sheet. Test booklets will be provided. *All work must be done in the exam book. No extra paper, for scrap or not, may be used.* Formula sheet must be turned in with the exam.

1. Put the following system in state space form:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \ddot{\boldsymbol{x}} + \begin{bmatrix} .2 & -.1 \\ -.1 & .2 \end{bmatrix} \dot{\boldsymbol{x}} + \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \boldsymbol{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \sin(\omega t) \tag{1}$$

2. Determine the mode shapes and eigenvalues for the system described by equation (1), **but with a zero damping matrix**. Use these to generate the state space representation eigenvectors. *You may check your answers with a calculator, but will not receive credit unless you show how the mode shapes of the second order system relate to the eigenvectors of the first order model.*

3. Calculate $e^{A\Delta t}$ for the system defined by $\ddot{x} + 10x = 0$ with $\Delta t = .01$.

4. Data is sampled at $f = 1.5915$ Hz. You expect to use Ho-Kalman minimum realization to obtain a system with a natural frequency of approximately 19.9 rad/s. Your identified model has a natural frequency of 9.9 rad/s. Is this possible? If so, demonstrate how this can happen.

5. Consider a the state space system for which

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & 1 & 0 & 0 \\ 1 & -3 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix}, \quad \text{and} \quad D = [0]$$

   Determine whether the system is observable. Use either approach.

6. Determine whether the system of problem 5 is controllable.