REFERENCES

[Rayleigh, 1945] J. W. S. Rayleigh, *The Theory of Sound*, Vols. 1 and 2, Dover Publications, New York, 1945.

[Young, 2011] W. C. Young, R. G. Budynas, and A. Sadegh *Roark's Formulas for Stress and Strain*, 8th ed., McGraw-Hill, New York, 2011.

PROBLEMS

Section 4.1 Spring Elements

4.1 Compute the translational spring constant of a particular steel helical coil spring, of the type used in automotive suspensions. The coil has six turns. The coil diameter is 4 in., and the wire diameter is 0.5 in. For the shear modulus, use $G = 1.7 \times 10^9$ lb/ft^2.

4.2 In the spring arrangement shown in Figure P4.2, the displacement x is caused by the applied force f. Assuming the system is in static equilibrium, sketch the plot of f versus x. Determine the equivalent spring constant k_e for this arrangement, where $f = k_e x$.

4.3 In the arrangement shown in Figure P4.3, a cable is attached to the end of a cantilever beam. We will model the cable as a rod. Denote the translational spring constant of the beam by k_b, and the translational spring constant of the cable by k_c. The displacement x is caused by the applied force f.

a. Are the two springs in series or in parallel?

b. What is the equivalent spring constant for this arrangement?

Figure P4.2

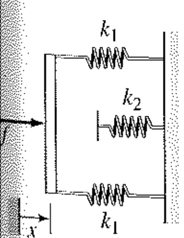


Figure P4.3

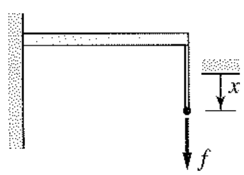
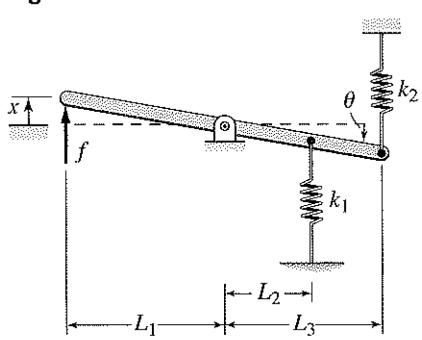


Figure P4.4



4.4 In the spring arrangement shown in Figure P4.4, the displacement x is caused by the applied force f. Assuming the system is in static equilibrium when $x = 0$ and that the angle θ is small, determine the equivalent spring constant k_e for this arrangement, where $f = k_e x$.

4.5 For the system shown in Figure P4.5, assume that the resulting motion is small enough to be only horizontal, and determine the expression for the equivalent k_e that relates the applied force f to the resulting displacement x.

4.6 The two stepped solid cylinders in Figure P4.6 consist of the same material and have an axial force f applied to them. Determine the equivalent translational spring constant for this arrangement. (Hint: Are the two springs in series or in parallel?)

Figure P4.5

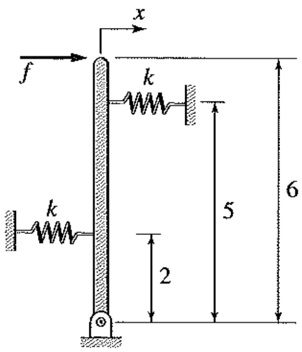
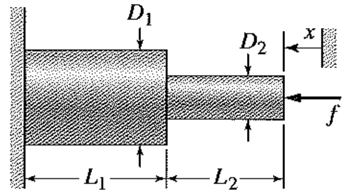


Figure P4.6



4.7 A table with four identical legs supports a vertical force. The solid cylindrical legs are made of metal with $E = 2 \times 10^{11}$ N/m^2. The legs are 1 m in length and 0.03 m in diameter. Compute the equivalent spring constant due to the legs, assuming the table top is rigid.

4.8 The beam shown in Figure P4.8 has been stiffened by the addition of a spring support. The steel beam is 3 ft long, 1 in thick, and 1 ft wide, and its mass is 3.8 slugs. The mass m is 40 slugs. Neglecting the mass of the beam,

a. Compute the spring constant k necessary to reduce the static deflection to one-half its original value before the spring k was added.

b. Compute the natural frequency ω_n of the combined system.

Figure P4.8

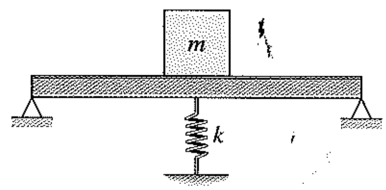
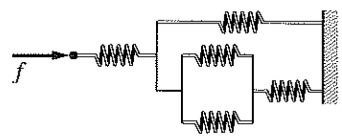


Figure P4.9



4.9 Determine the equivalent spring constant of the arrangement shown in Figure P4.9. All the springs have the same spring constant k.

4.10 Compute the equivalent torsional spring constant of the stepped shaft arrangement shown in Figure P4.10. For the shaft material, $G = 8 \times 10^{10}$ N/m^2.

4.11 Plot the spring force felt by the mass shown in Figure P4.11 as a function of the displacement x. When $x = 0$, spring 1 is at its free length. Spring 2 is at its free length in the configuration shown.

Figure P4.10

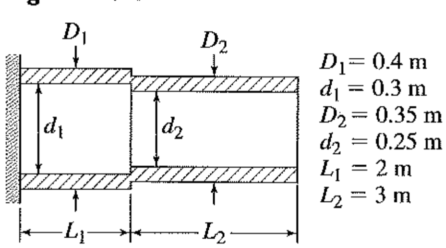
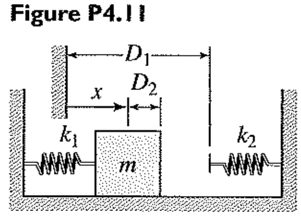


Figure P4.11



Section 4.2 Modeling Mass-Spring Systems

Note: see also the problems for Section 4.5: Additional Modeling Examples.

4.12 Calculate the expression for the natural frequency of the system shown in Figure P4.12. Disregard the pulley mass.

4.13 Obtain the expression for the natural frequency of the system shown in Figure P4.13. Assume small motions and disregard the pulley mass.

4.14 Obtain the expression for the natural frequency of the system shown in Figure P4.14. Discount the mass of the L-shaped arm.

Figure P4.12

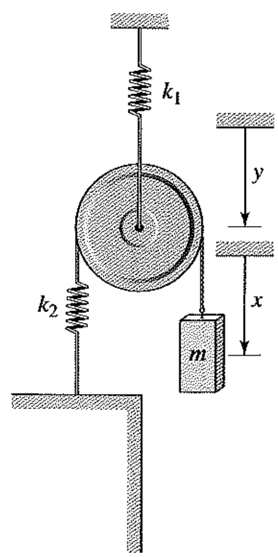


Figure P4.13

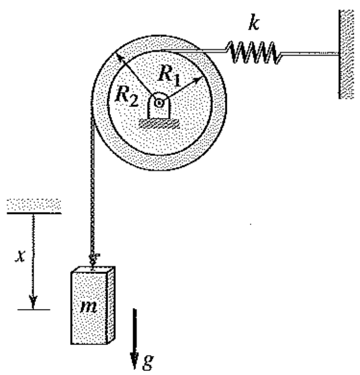
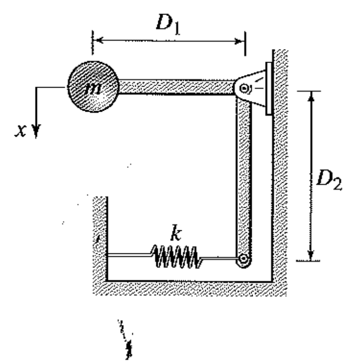


Figure P4.14



4.15 A connecting rod having a mass of 3.6 kg is shown in Figure P4.15. It oscillates with a frequency of 40 cycles per minute when supported on a knife edge, as shown. Its mass center is located 0.15 m below the support. Calculate the moment of inertia about the mass center.

Figure P4.15

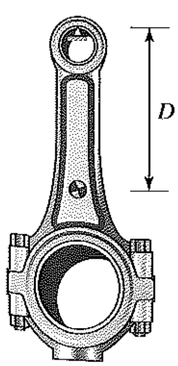
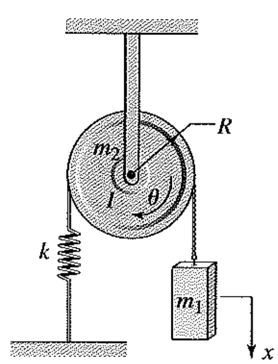


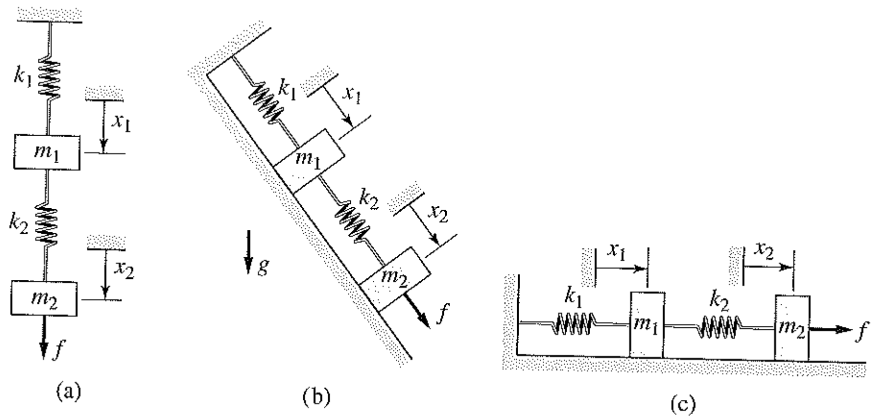
Figure P4.16



4.16 Calculate the expression for the natural frequency of the system shown in Figure P4.16.

4.17 For each of the systems shown in Figure P4.17, the input is the force f and the outputs are the displacements x_1 and x_2 of the masses. The equilibrium positions with $f = 0$ correspond to $x_1 = x_2 = 0$. Neglect any friction between the masses and the surface. Derive the equations of motion of the systems.

Figure P4.17



4.18 The mass m in Figure P4.18 is attached to a rigid lever having negligible mass and negligible friction in the pivot. The input is the displacement x. When x and θ are zero, the springs are at their free length. Assuming that θ is small, derive the equation of motion for θ with x as the input.

Figure P4.18

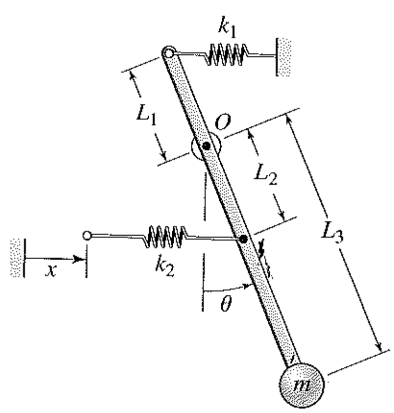
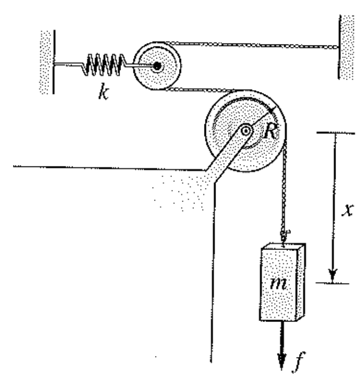


Figure P4.19



4.19 In the pulley system shown in Figure P4.19, the input is the applied force f, and the output is the displacement x. Assume the pulley masses are negligible and derive the equation of motion.

4.20 Figure P4.20 illustrates a cylindrical buoy floating in water with a mass density ρ. Assume that the center of mass of the buoy is deep enough so that the buoy motion is primarily vertical. The buoy mass is m and the diameter is D. Archimedes' principle states that the buoyancy force acting on a floating object equals the weight of the liquid displaced by the object. (a) Derive the equation of motion in terms of the variable x, which is the displacement from the equilibrium position. (b) Obtain the expression for the buoy's natural frequency. (c) Compute the period of oscillation if the buoy diameter is 2 ft and the buoy weighs 1000 lb. Take the mass density of fresh water to be $\rho = 1.94$ slug/ft^3.

4.21 Figure P4.21 shows the cross-sectional view of a ship undergoing rolling motion. Archimedes' principle states that the buoyancy force B acting on a floating object equals the weight of the liquid displaced by the object. The metacenter M is the intersection point of the line of action of the buoyancy

Figure P4.20

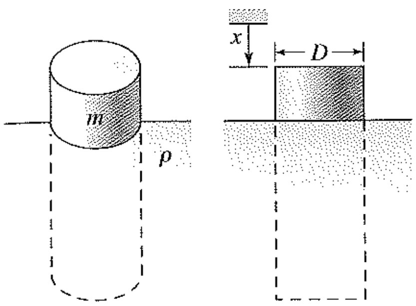
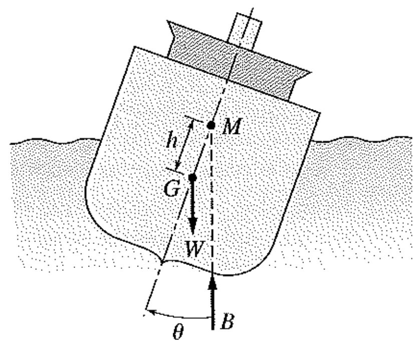


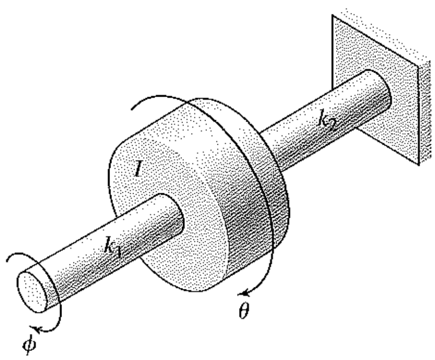
Figure P4.21



force and the ship's centerline. The distance h of M from the mass center G is called the metacentric height. (a) Obtain the equation of motion describing the ship's rolling motion in terms of the angle θ. (b) The given parameters are the ship's weight W, its metacentric height h, and its moment of inertia I about the center of gravity. Obtain an expression for the natural frequency of the rolling motion.

4.22 In the system shown in Figure P4.22, the input is the angular displacement ϕ of the end of the shaft, and the output is the angular displacement θ of the inertia I. The shafts have torsional stiffnesses k_1 and k_2. The equilibrium position corresponds to $\phi = \theta = 0$. Derive the equation of motion and find the transfer function $\Theta(s)/\Phi(s)$.

Figure P4.22



4.23 In Figure P4.23, assume that the cylinder rolls without slipping. The spring is at its free length when x and y are zero. (a) Derive the equation of motion in terms of x, with $y(t)$ as the input. (b) Suppose that $m = 10$ kg, $R = 0.3$ m, $k = 1000$ N/m, and that $y(t)$ is a unit-step function. Solve for $x(t)$ if $x(0) = \dot{x}(0) = 0$.

4.24 In Figure P4.24 when $x_1 = x_2 = 0$ the springs are at their free lengths. Derive the equations of motion.

Figure P4.23

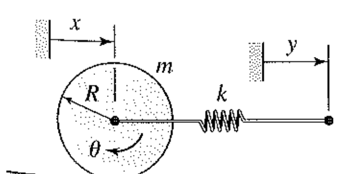
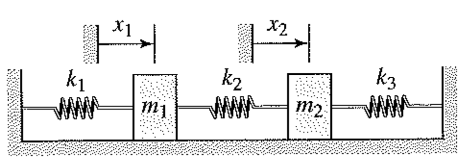


Figure P4.24



4.25 In Figure P4.25 model the three shafts as massless torsional springs. When $\theta_1 = \theta_2 = 0$ the springs are at their free lengths. Derive the equations of motion with the torque T_2 as the input.

4.26 In Figure P4.26 when $\theta_1 = \theta_2 = 0$ the spring is at its free length. Derive the equations of motion, assuming small angles.

Figure P4.25

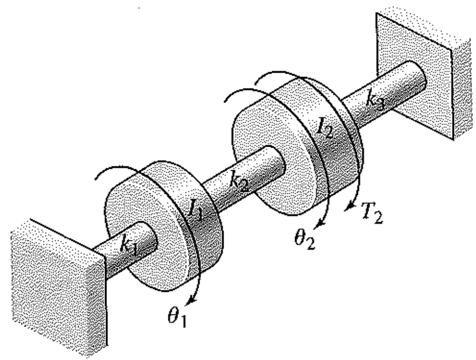
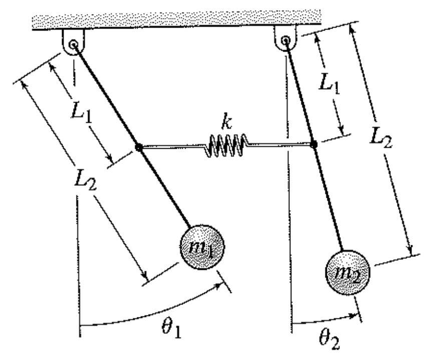


Figure P4.26



4.27 Consider the torsion-bar suspension shown in Figure 4.1.6. Assume that the torsion bar is a steel rod with a length of 4 ft and diameter 1.5 in. The wheel weighs 40 lb and the suspension arm is 2 ft long. Neglect the masses of the torsion bar and the suspension arm, and calculate the natural frequency of the system. Use $G = 1.7 \times 10^9$ lb/ft^2.

4.28 For the system shown in Figure P4.28, suppose that $k_1 = k$, $k_2 = k_3 = 2k$, and $m_1 = m_2 = m$. Obtain the equations of motion in terms of x_1 and x_2.

Figure P4.28

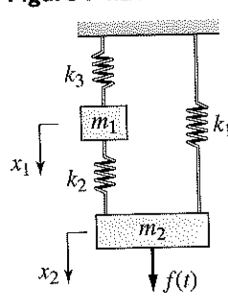
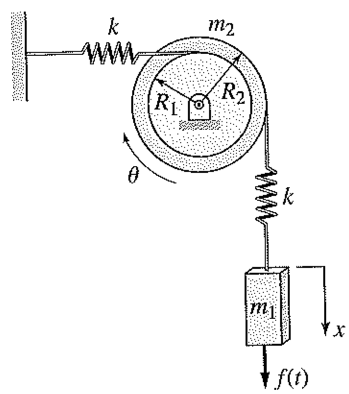


Figure P4.29



4.29 For the system shown in Figure P4.29, suppose that $R_2 = 2R_1$, $m_1 = m$, and $m_2 = 2m$. The two pulleys share a common hub and are welded together. Their total mass is m_2 and total inertia is I_2. Obtain the equations of motion in terms of x and θ.

Section 4.3 Energy Methods

4.30 For Figure P4.30, assume that the cylinder rolls without slipping and use conservation of energy to derive the equation of motion in terms of x.

4.31 For Figure P4.31, the equilibrium position corresponds to $x = 0$. Neglect the masses of the pulleys and assume that the cable is inextensible, and use conservation of energy to derive the equation of motion in terms of x.

4.32 For Figure P4.32, the equilibrium position corresponds to $x = 0$. Neglect the masses of the pulleys and assume that the cable is inextensible, and use conservation of energy to derive the equation of motion in terms of x.

Figure P4.30

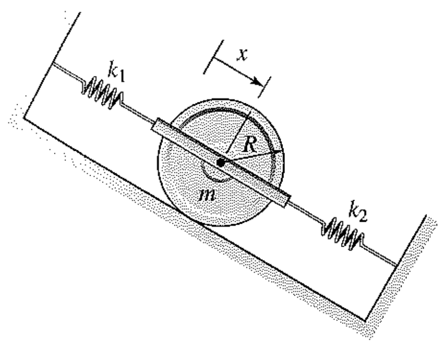


Figure P4.31

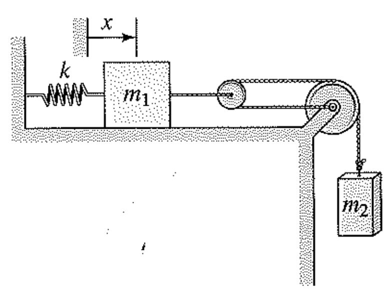
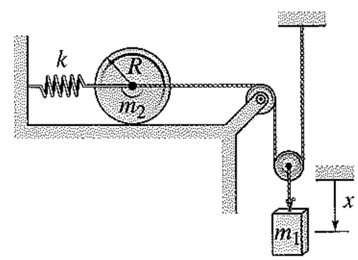


Figure P4.32



4.33 Use the Rayleigh method to obtain an expression for the natural frequency of the system shown in Figure P4.33. The equilibrium position corresponds to $x = 0$.

4.34 For Figure P4.34, assume that the cylinder rolls without slipping and use the Rayleigh method to obtain an expression for the natural frequency of the system. The equilibrium position corresponds to $x = 0$.

Figure P4.33

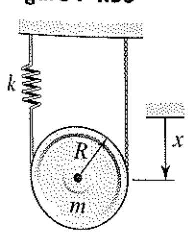


Figure P4.34

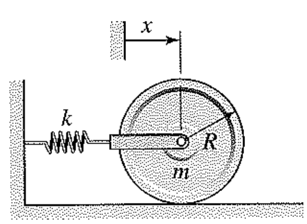
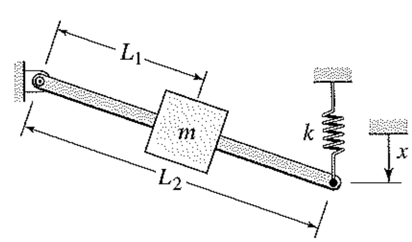


Figure P4.35



4.35 Use the Rayleigh method to obtain an expression for the natural frequency of the system shown in Figure P4.35. The equilibrium position corresponds to $x = 0$.

4.36 Use an energy method to obtain the expression for the natural frequency of the system shown in Figure P4.36.

4.37 Determine the natural frequency of the system shown in Figure P4.37 using Rayleigh's method. Assume small angles of oscillation.

4.38 Determine the natural frequency of the system shown in Figure P4.38 using an energy method. The disk is a solid cylinder. Assume small angles of oscillation.

Figure P4.36

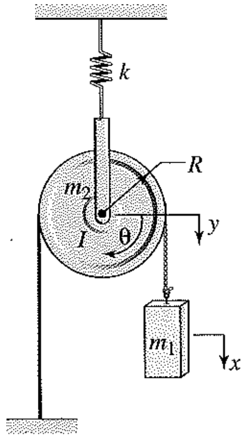


Figure P4.37

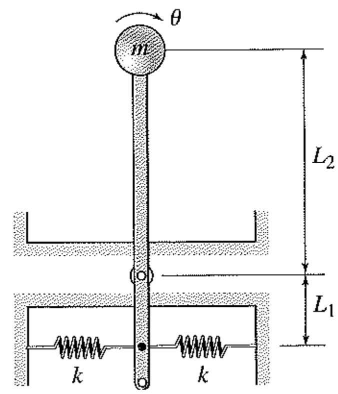
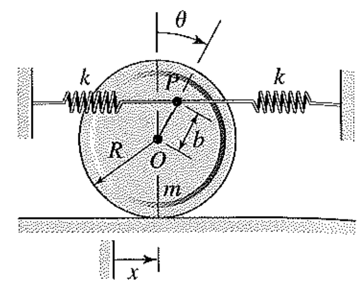


Figure P4.38



4.39 Use Rayleigh's method to calculate the expression for the natural frequency of the system shown in Figure P4.13. Assume small motions and neglect the pulley mass.

4.40 Use Rayleigh's method to obtain the expression for the natural frequency of the system shown in Figure P4.14. Disregard the mass of the L-shaped arm.

4.41 Determine the natural frequency of the system shown in Figure P4.41 using Rayleigh's method. Assume small angles of oscillation.

Figure P4.41

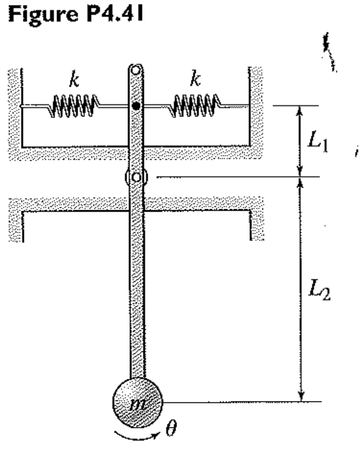
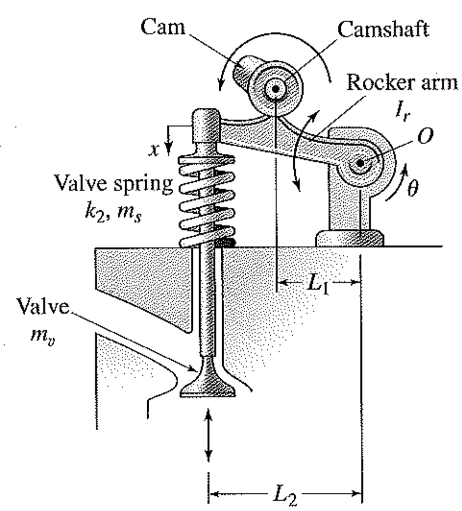


Figure P4.42



4.42 Figure P4.42 shows an engine valve driven by an overhead camshaft. The rocker arm pivots about the fixed point O and the inertia of the arm about this point is I_r. The valve mass is m_v and the spring mass is m_s; its spring constant is k_s. Let f_c denote the force exerted on the rocker arm by the camshaft. Assuming that $\theta(t)$ and its time derivatives are known (from the cam profile and the cam speed), derive a dynamic model that can be used to solve for the cam force $f_c(t)$. (This information is needed to predict the amount of wear on the cam surface.)

4.43 The vibration of a motor mounted on the end of a cantilever beam can be modeled as a mass-spring system. The motor weighs 30 lb, and the beam weighs 7 lb. When the motor is placed on the beam, it causes an additional static deflection of 0.8 in. Find the equivalent mass m and equivalent spring constant k.

4.44 The vibration of a motor mounted in the middle of a fixed-end beam can be modeled as a mass-spring system. The motor mass is 40 kg, and the beam mass is 13 kg. When the motor is placed on the beam, it causes an additional static deflection of 3 mm. Find the equivalent mass m and equivalent spring constant k.

4.45 The vibration of a motor mounted in the middle of a simply-supported beam can be modeled as a mass-spring system. The motor mass is 30 kg, and the beam mass is 10 kg. When the motor is placed on the beam, it causes an additional static deflection of 2 mm. Find the equivalent mass m and equivalent spring constant k.

4.46 A certain cantilever beam vibrates at a frequency of 5 Hz when a 30 lb motor is placed on the beam. The beam weighs 7 lb. Estimate the beam stiffness k.

4.47 A 10-kg mass is attached to a 2 kg spring. The mass vibrates at a frequency of 20 Hz when disturbed. Estimate the spring stiffness k.

4.48 The static deflection of a cantilever beam is described by

$$x_y = \frac{P}{6EI_A} y^2(3L - y)$$

where P is the load applied at the end of the beam, and x_y is the vertical deflection at a point a distance y from the support (Figure P4.48). Obtain an expression for an equivalent mass located at the end of the beam.

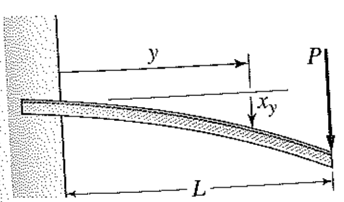


Figure P4.48

4.49 Figure P4.49 shows a winch supported by a cantilever beam at the stern of a ship. The mass of the winch is m_w, the mass of the beam plus winch bracket and motor is m_b. The object hoisted by the winch has a mass m_h; the wire rope mass m_r is assumed to be negligible compared to the other masses. Find the equation of motion for the vertical motion x_1 of the winch: (a) assuming that the rope does not stretch and (b) assuming that the rope stretches and has a spring constant k_r.

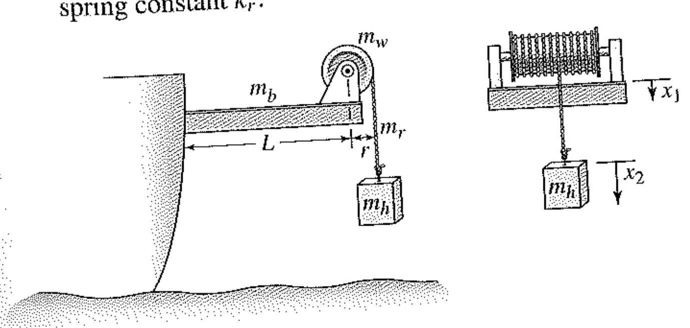


Figure P4.49

Section 4.4 Damping Elements

4.50 A 50-kg block is placed on an inclined plane whose angle with the horizontal is 25°. The viscous friction coefficient between the block and the plane is $c = 6$ N · s/m. (a) Derive the equation of motion. (b) Solve the equation of motion for the speed $v(t)$ of the block, assuming that the block is initially given a speed of 5 m/s. (c) Find the steady-state speed of the block and estimate the time required to reach that speed. (d) Discuss the effect of the initial velocity of the steady-state speed.

4.51 A certain mass-spring-damper system has the following equation of motion.

$$40\ddot{x} + c\dot{x} + 1200x = f(t)$$

Suppose that the initial conditions are zero and that the applied force $f(t)$ is a step function of magnitude 5000. Solve for $x(t)$ for the following two cases: (a) $c = 680$ and (b) $c = 400$.

4.52 For each of the systems shown in Figure P4.52, the input is the force f and the outputs are the displacements x_1 and x_2 of the masses. The equilibrium positions with $f = 0$ correspond to $x_1 = x_2 = 0$. Neglect any friction between the masses and the surface. Derive the equations of motion of the systems.

Figure P4.52

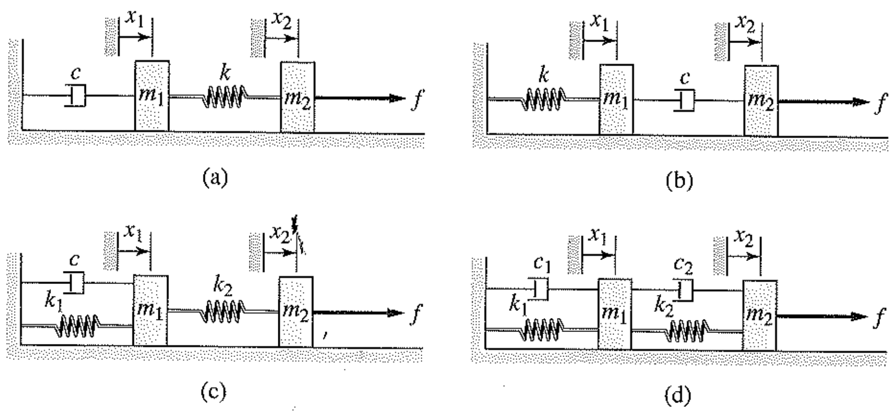
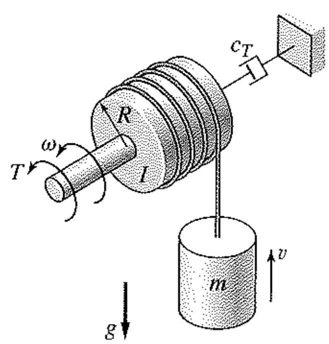


Figure P4.53



4.53 In Figure P4.53 a motor supplies a torque T to turn a drum of radius R and inertia I about its axis of rotation. The rotating drum lifts a mass m by means of a cable that wraps around the drum. The drum's speed is ω. Viscous torsional damping c_T exists in the drum shaft. Neglect the mass of the cable. (a) Obtain the equation of motion with the torque T as the input and the vertical speed v of the mass as the output. (b) Suppose that $m = 40$ kg, $R = 0.2$ m, $I = 0.8$ kg · m^2, and $c_T = 0.1$ N · m · s. Find the speed $v(t)$ if the system is initially at rest and the torque T is a step function of magnitude 300 N · m.

4.54 Derive the equation of motion for the lever system shown in Figure P4.54, with the force f as the input and the angle θ as the output. The position $\theta = 0$ corresponds to the equilibrium position when $f = 0$. The lever has an inertia I about the pivot. Assume small displacements.

4.55 In the system shown in Figure P4.55, the input is the displacement y and the output is the displacement x of the mass m. The equilibrium position corresponds to $x = y = 0$. Neglect any friction between the mass and the surface. Derive the equation of motion and find the transfer function $X(s)/Y(s)$.

Figure P4.54

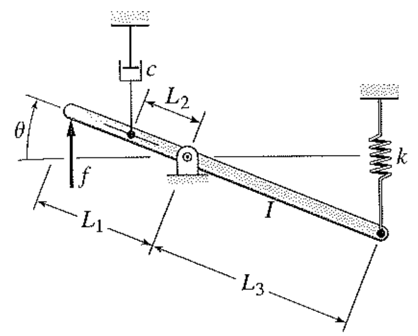
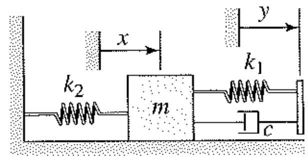


Figure P4.55



4.56 Figure P4.56a shows a Houdaille damper, which is a device attached to an engine crankshaft to reduce vibrations. The damper has an inertia I_d that is free to rotate within an enclosure filled with viscous fluid. The inertia I_p is the inertia of the fan-belt pulley. Modeling the crankshaft as a torsional spring k_T, the damper system can be modeled as shown in part (b) of the figure. Derive the equation of motion with the angular displacements θ_p and θ_d as the outputs and the crankshaft angular displacement ϕ as the input.

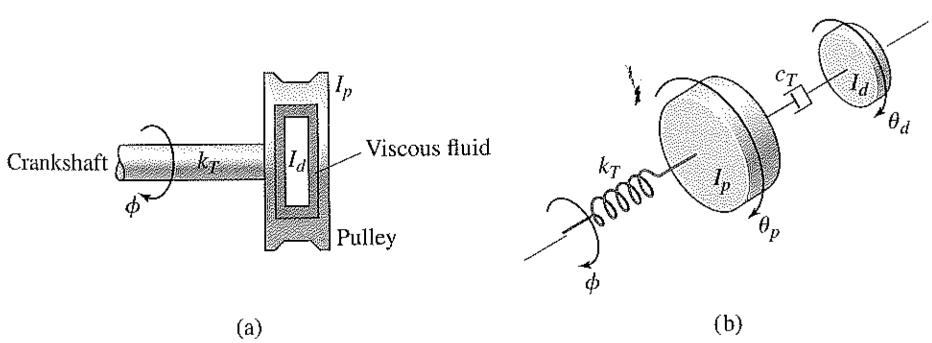


Figure P4.56

4.57 Refer to Figure P4.57. Determine the relations between c, c_1, and c_2 so that the damper shown in part (c) is equivalent to (a) the arrangement shown in part (a), and the arrangement shown in part (b).

Figure P4.57

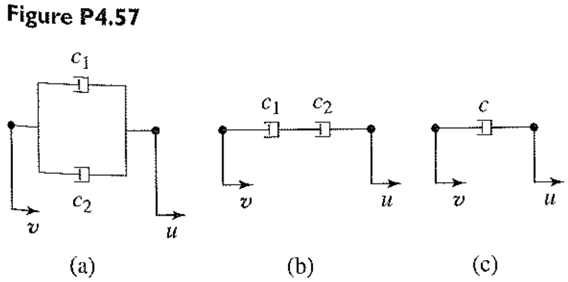
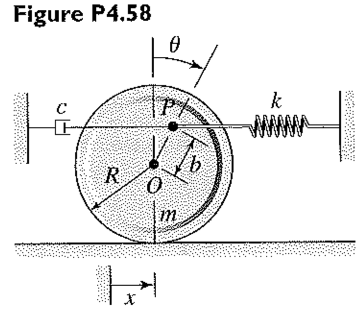


Figure P4.58



4.58 For the system shown in Figure P4.58, obtain the equation of motion in terms of θ. The disk is a solid cylinder. Assume small angles of oscillation.

4.59 Find the transfer function $Z(s)X(s)$ for the system shown in Figure P4.59.

4.60 Find the transfer function $Y(s)X(s)$ for the system shown in Figure P4.60.

4.61 Find the transfer function $Y(s)X(s)$ for the system shown in Figure P4.61.

Figure P4.59

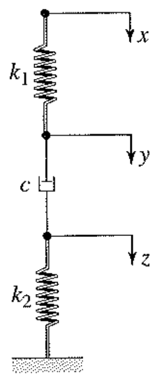


Figure P4.60

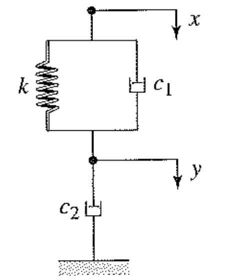


Figure P4.61

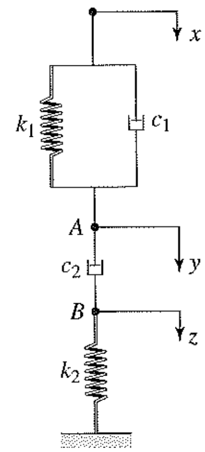
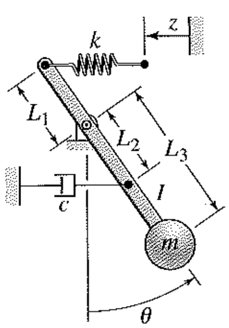


Figure P4.62



Section 4.5 Additional Modeling Examples

4.62 The mass m in Figure P4.62 is attached to a rigid rod having an inertia I about the pivot and negligible pivot friction. The input is the displacement z. When $z = \theta = 0$, the spring is at its free length. Assuming that θ is small, derive the equation of motion for θ with z as the input.

4.63 In the system shown in Figure P4.63, the input is the force f and the output is the displacement x_A of point A. When $x = x_A$ the spring is at its free length. Derive the equation of motion.

Figure P4.63

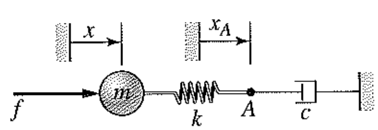
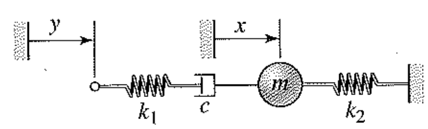


Figure P4.64



4.64 In the system shown in Figure P4.64, the input is the displacement y and the output is the displacement x. When $x = y = 0$ the springs are at their free lengths. Derive the equation of motion.

4.65 Figure P4.65 shows a rack-and-pinion gear in which a damping force and a spring force act against the rack. Develop the equivalent rotational model of the system with the applied torque T as the input variable and the angular displacement θ is the output variable. Neglect any twist in the shaft.

4.66 Figure P4.66 shows a drive train with a spur-gear pair. The first shaft turns N times faster than the second shaft. Develop a model of the system including the elasticity of the second shaft. Assume the first shaft is rigid, and neglect the gear and shaft masses. The input is the applied torque T_1. The outputs are the angles θ_1 and θ_3.

Figure P4.65

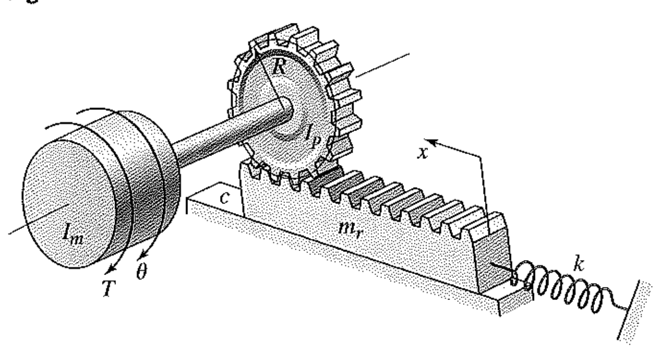
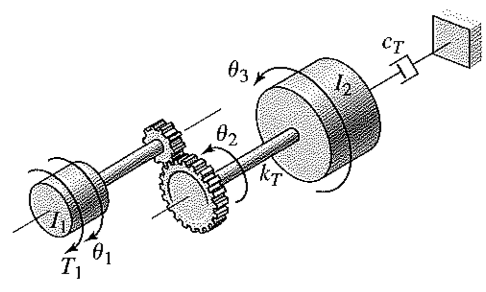


Figure P4.66



4.67 Assuming that θ is small, derive the equations of motion of the systems shown in parts (a) and (b) of Figure P4.67. When $\theta = 0$ the systems are in equilibrium. Are the systems stable, neutrally stable, or unstable?

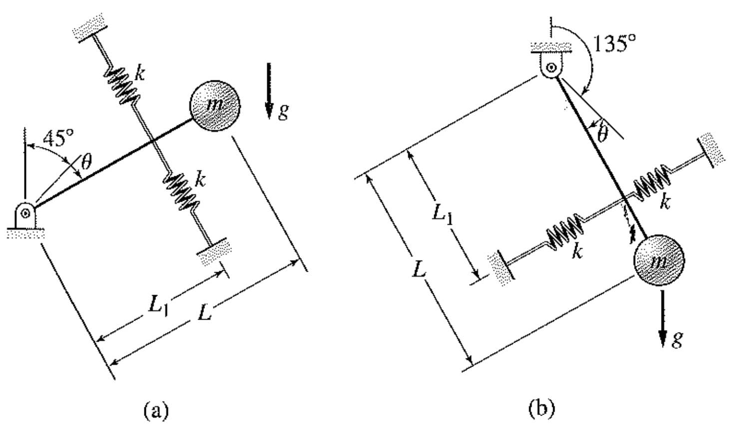


Figure P4.67

4.68 Assuming that θ is small, derive the equation of motion of the pendulum shown in Figure P4.68. The pendulum is in equilibrium when $\theta = 0$. Is the system stable, neutrally stable, or unstable?

4.69 Assuming that θ is small, derive the equation of motion of the pendulum shown in Figure P4.69. The input is $y(t)$ and the output is θ. The equilibrium

Figure P4.68

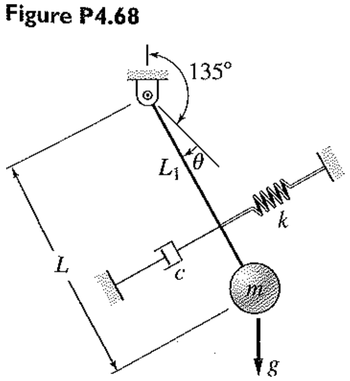


Figure P4.69

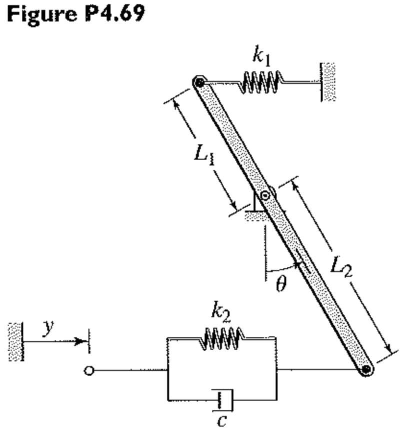
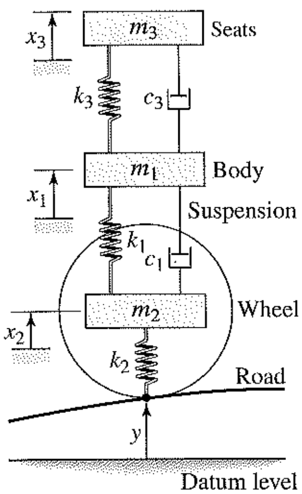


Figure P4.70



corresponds to $y = \theta = 0$, when the springs are at their free lengths. The rod inertia about the pivot is I.

4.70 Figure P4.70 shows a quarter-car model that includes the mass of the seats (including passengers). The constants k_3 and c_3 represent the stiffness and damping in the seat supports. Derive the equations of motion of this system. The input is the road displacement $y(t)$. The displacements are measured from equilibrium.

4.71 The top view of a solid door is shown in Figure P4.71. The door has a mass of 40 kg and is 2.1 m high, 1.2 m wide, and 0.05 m thick. Its door closer has a torsional spring constant of 13.6 N·m/rad. The door will close as fast as possible without oscillating if the torsional damping coefficient c in the closer is set to the critical damping value corresponding to a damping ratio of $\zeta = 1$. Determine this critical value of c.

4.72 Derive the equation of motion for the system shown in Figure P4.72. Assume small angles of oscillation, and neglect the rod mass. What relation among L_1, L_2, m, and k must be satisfied for the system to be stable?

Figure P4.71

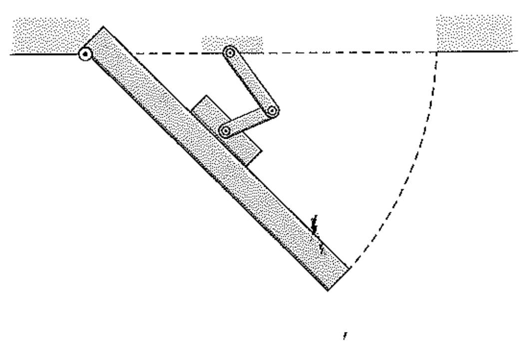
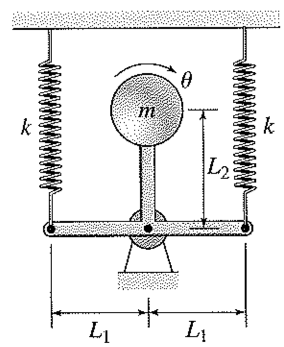
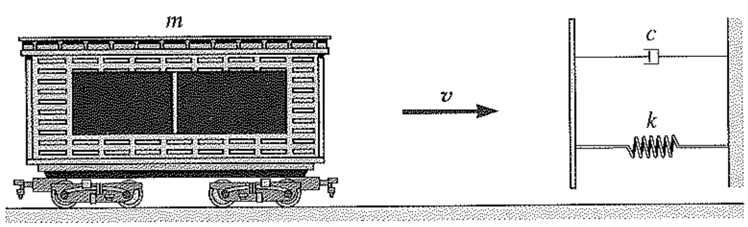


Figure P4.72



4.73 A boxcar moving at 1.3 m/s hits the shock absorber at the end of the track (Figure P4.73). The boxcar mass is 18 000 kg;. The stiffness of the absorber is $k = 73\,000$ N/m, and the damping coefficient is $c = 88\,000$ N·s/m. Determine the maximum spring compression and the time for the boxcar to stop.

Figure P4.73



4.74 For the systems shown in Figure P4.74, assume that the resulting motion is small enough to be only horizontal and determine the expression for the equivalent damping coefficient c_e that relates the applied force f to the resulting velocity v.

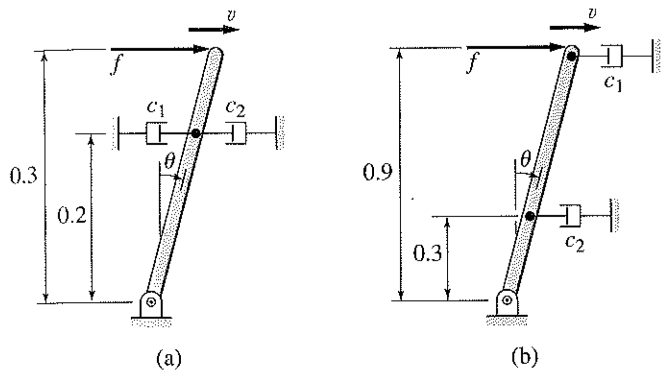


Figure P4.74

4.75 Refer to Figure P4.75a, which shows a ship's propeller, drive train, engine, and flywheel. The diameter ratio of the gears is $D1/D2 = 1.5$. The inertias in kg·m^2 of gear 1 and gear 2 are 500 and 100, respectively. The flywheel, engine, and propeller inertias are 104, 103, and 2500, respectively. The torsional stiffness of shaft 1 is 5×106 N-m/rad, and that of shaft 2 is 106 N-m/rad. Because the flywheel inertia is so much larger than the other inertias, a simpler model of the shaft vibrations can be obtained by assuming the flywheel does not rotate. In addition, because the shaft between the engine and gears is short, we will assume that it is very stiff compared to the other shafts. If we also disregard the shaft inertias, the resulting model consists of two inertias, one obtained by lumping the engine and gear inertias, and one for the propeller (Figure P4.75b). Using these assumptions, obtain the natural frequencies of the system.

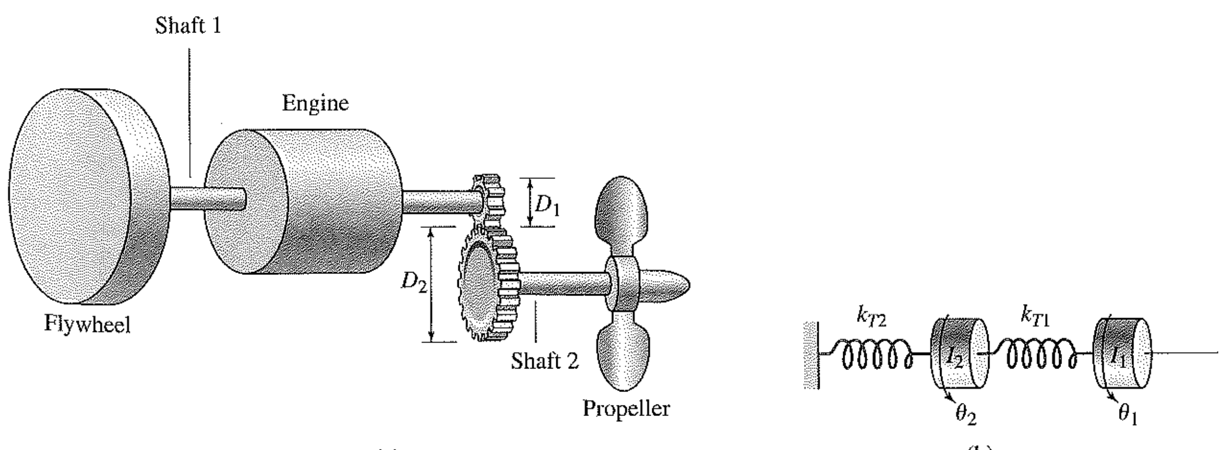


Figure P4.75

4.76 In this problem, we make all the same assumptions as in Problem 4.75, but we do not discount the flywheel inertia, so our model consists of three inertias, as shown in Figure P4.76. Obtain the natural frequencies of the system.

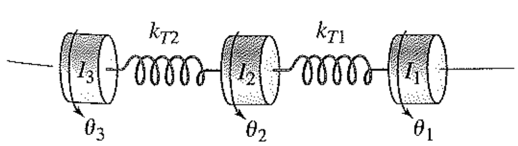


Figure P4.76

4.77 Refer to Figure P4.77, which shows a turbine driving an electrical generator through a gear pair. The diameter ratio of the gears is $D_2/D_1 = 1.5$. The inertias in kg·m^2 of gear 1 and gear 2 are 100 and 500, respectively. The turbine and generator inertias are 2000 and 1000, respectively. The torsion stiffness of shaft 1 is 3×105 N·m/rad, and that of shaft 2 is 8×104 N·m/rad. Disregard the shaft inertias and obtain the natural frequencies of the system.

Figure P4.77

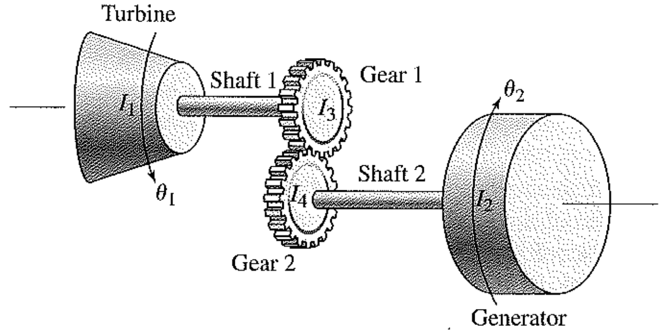
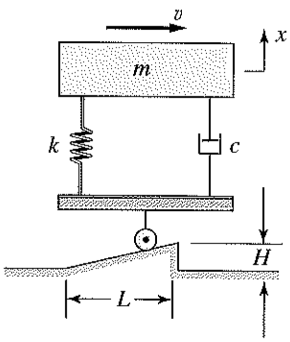


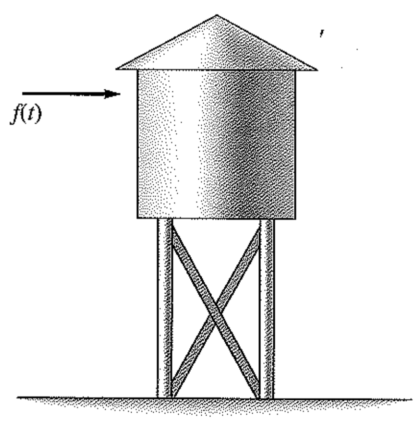
Figure P4.78



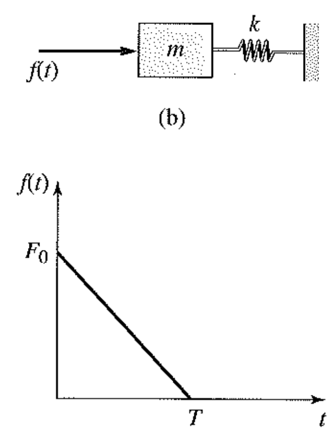
4.78 Refer to Figure P4.78, which is a simplified representation of a vehicle striking a bump. The vertical displacement x is 0 when the tire first meets the bump. Assuming that the vehicle's horizontal speed v remains constant and that the system is critically damped, obtain the expression for $x(t)$.

4.79 Refer to Figure P4.79a, which shows a water tank subjected to a blast force $f(t)$. We will model the tank and its supporting columns as the mass-spring system shown in part (b) of the figure. The blast force as a function of time is shown in part (c) of the figure. Assuming zero initial conditions, obtain the expression for $x(t)$.

Figure P4.79



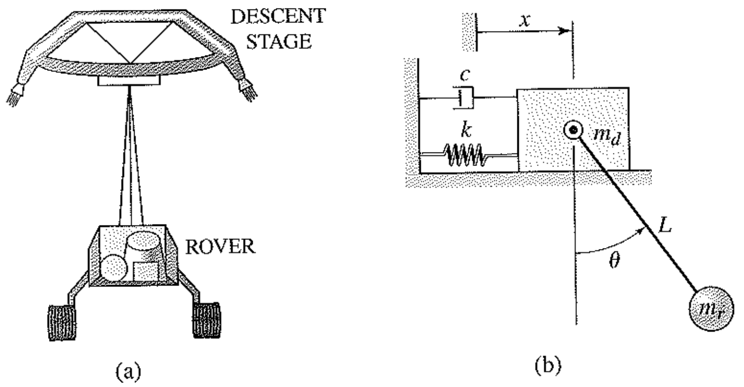
(a)



(c)

4.80 The "sky crane" shown on the text cover was a novel solution to the problem of landing the 2000 lb Curiosity rover on the surface of Mars. Curiosity hangs from the descent stage by 60-ft long nylon tethers (Figure P4.80a). The descent stage uses its thrusters to hover as the rover is lowered to the surface. Thus the rover behaves like a pendulum whose base is moving horizontally. The side

Figure P4.80



thruster force is not constant but is controlled to keep the descent stage from deviating left or right from the desired vertical path. As we will see in Chapter 10, such a control system effectively acts like a spring and a damper, as shown in Figure P4.80b. The rover mass is m_r, the descent stage mass is m_d, and the net horizontal component of the thruster forces is $f = kx + c\dot{x}$. Among other simplifications, this model neglects vertical motion and any rotational motion.

Derive the equations of motion of the system in terms of the angle θ and the displacement x.

4.81 Obtain the equations of motion for the system shown in Figure P4.81 for the case where $m_1 = m_2$ and $m_2 = 2m$. The cylinder is solid and rolls without slipping. The platform translates without friction on the horizontal surface.

Figure P4.81

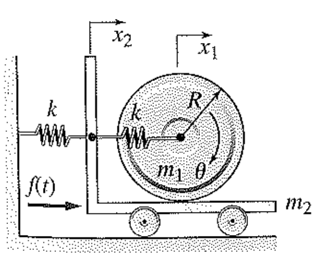
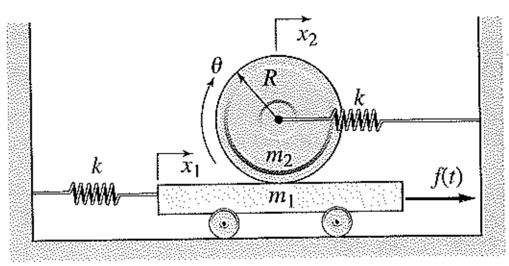


Figure P4.82



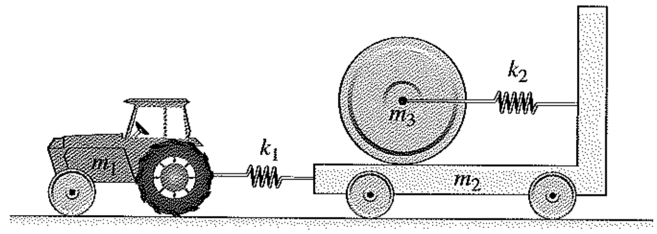
4.82 Obtain the equations of motion for the system shown in Figure P4.82 for the case where $m_1 = m_2 = m$. The cylinder is solid and rolls without slipping. The platform translates without friction on the horizontal surface.

4.83 In Figure P4.83 a tractor and a trailer is used to carry objects, such as a large paper roll or pipes. Assuming the cylindrical load m_3 rolls without slipping, obtain the equations of motion of the system.

Section 4.6 Collisions and Impulse Response

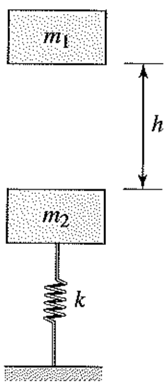
4.84 Suppose a mass m moving with a speed v_1 becomes embedded in mass m_2 after striking it (Figure 4.6.1). Suppose $m_2 = 5m$. Determine the expression for the displacement $x(t)$ after the collision.

Figure P4.83



4.85 Consider the system shown in Figure 4.6.3. Suppose that the mass m moving with a speed v_1 rebounds from the mass $m_2 = 5m$ after striking it. Assume that the collision is perfectly elastic. Determine the expression for the displacement $x(t)$ after the collision.

Figure P4.86



4.86 The mass m_1 is dropped from rest a distance h onto the mass m_2, which is initially resting on the spring support (Figure P4.86). Assume that the impact is inelastic so that m_1 sticks to m_2. Calculate the maximum spring deflection caused by the impact. The given values are $m_1 = 0.5$ kg, $m_2 = 4$ kg, $k = 400$ N/m, and $h = 2$ m.

4.87 Figure P4.87 shows a mass m with an attached stiffness, such as that due to protective packaging. The mass drops a distance h, at which time the stiffness element contacts the ground. Let x denote the displacement of m after contact with the ground. Determine the maximum required stiffness of the packaging if a 10 kg package cannot experience a deceleration greater than 8g when dropped from a height of 2 m.

4.88 Figure P4.88 represents a drop forging process. The anvil mass is $m_1 = 1000$ kg, and the hammer mass is $m_2 = 200$ kg. The support stiffness is $k = 107$ N/m, and the damping constant is $c = 1$ N-s/m. The anvil is at rest when the hammer is dropped from a height of $h = 1$ m. Obtain the expression for the displacement of the anvil as a function of time after the impact. Do this for two types of collisions: (a) an inelastic collision and (b) a perfectly elastic collision.

Figure P4.87

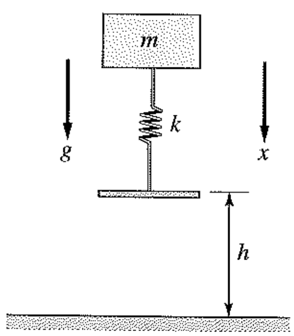


Figure P4.88

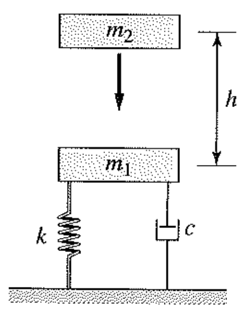
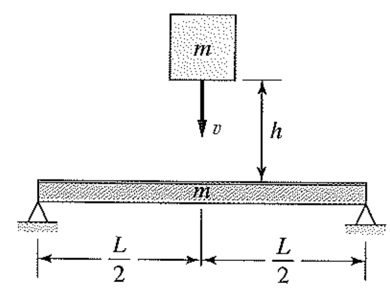


Figure P4.89



4.89 Refer to Figure P4.89. A mass m drops from a height h and hits and sticks to a simply supported beam of equal mass. Obtain an expression for the maximum deflection of the center of the beam. Your answer should be a function of h, g, m, and the beam stiffness k.

Section 4.7 MATLAB Applications

4.90 (a) Obtain the equations of motion of the system shown in Figure P4.90. The masses are $m_1 = 20$ kg and $m_2 = 60$ kg. The spring constants are $k_1 = 3 \times 10^4$ N/m and $k_2 = 6 \times 10^4$ N/m. (b) Obtain the transfer functions $X_1(s)/F(s)$ and $X_2(s)/F(s)$. (c) Obtain a plot of the unit-step response of x_1 for zero initial conditions.

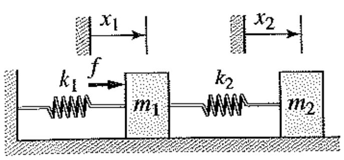


Figure P4.90

4.91 (a) Obtain the equations of motion of the system shown in Figure P4.25. (b) Suppose the inertias are $I_1 = I$ and $I_2 = 2I$ and the torsional spring constants are $k_1 = k_2 = k_3 = k$. Obtain the transfer functions $\Theta_1(s)/T_2(s)$ and $\Theta_2(s)/T_2(s)$ in terms of I and k. (c) Suppose that $I = 10$ and $k = 60$. Obtain a plot of the unit-impulse response of θ_1 for zero initial conditions.

4.92 Refer to part (a) of Problem 4.90. Use MATLAB to obtain the transfer functions $X_1(s)/F(s)$ and $X_2(s)/F(s)$. Compare your answers with those found in part (b) of Problem 4.90.

4.93 Refer to Problem 4.91. Use MATLAB to obtain the transfer functions $\Theta_1(s)/T_2(s)$ and $\Theta_2(s)/T_2(s)$ for the values $I_1 = 10$, $I_2 = 20$, and $k_1 = k_2 = k_3 = 60$. Compare your answers with those found in part (b) of Problem 4.91.

4.94 (a) Obtain the equations of motion of the system shown in Figure P4.26. Assume small angles. The spring is at its free length when $\theta_1 = \theta_2 = 0$. (b) For the values $m_1 = 1$ kg, $m_2 = 4$ kg, $k = 10$ N/m, $L_1 = 2$ m, and $L_2 = 5$ m, use MATLAB to plot the free response of θ_1 if $\theta_1(0) = 0.1$ rad and $\dot{\theta}_1(0) = \dot{\theta}_2(0) = \theta_2(0) = 0$.

4.95 (a) Obtain the equations of motion of the system shown in Figure P4.95. (b) Suppose that the masses are $m_1 = 1$ kg, $m_2 = 2$ kg, and the spring constants are $k_1 = k_2 = k_3 = 1.6 \times 10^4$ N/m. Use MATLAB to obtain the plot of the free response of x_1. Use $x_1(0) = 0.1$ m, $x_2(0) = \dot{x}_1(0) = \dot{x}_2(0) = 0$.

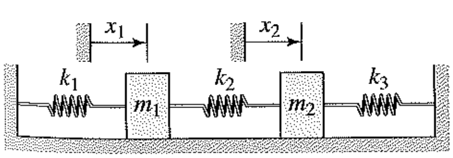


Figure P4.95