ME 710
Computational Methods in Structural Dynamics
Winter 1996
Exam #2
Closed Books, Closed Notes, No Cheat Sheet.

1) Given $T = T(\dot{q}_i)$ and $V = V(q_i, \dot{q}_i)$ for a discrete system, derive Lagrange's equations for a discrete system using Hamilton's principle. Don't forget to include work done by external forces.

2) The operator, L, for a beam is given by $\frac{\partial^2}{\partial x^2}\left(EI(x)\frac{\partial^2}{\partial x^2}\right)$. The boundary conditions are given by $w(0)=0$, $w'(0)=0$, $\left.\frac{\partial^2 w}{\partial x^2}\right|_{x=L} = 0$ and $\left.\frac{\partial}{\partial x}\left(EI\frac{\partial^2 w}{\partial x^2}\right)\right|_{x=L} = \left.m\frac{\partial^2 w}{\partial t^2}\right|_{x=L}$
where the final boundary condition is due to a point mass at the end of the beam. Determine if:
a) the system is self-adjoint
b) the system is positive definite.

Clearly demonstrate all results.

HINT: Assume a form for the temporal part of the solution.

3) Use Hamilton's principal to derive the equation of motion for the following system. A uniform cantilever beam has torsional stiffness GJ, vertical bending stiffness EI, and mass per unit length ρA, and rotational inertia per unit length ρI_p (I_p being the polar moment of inertia for the twisting beam). The beam is cantilevered at end A, and a massless rigid bar BC is attached at end B. A Concentrated mass is located at point C. Assume that bending takes place only in the z-x plane with deflection $w(x,t)$ and that rotation takes place about the x axis $\theta(x,t)$. Neglect gravity. State the equation of motion and boundary conditions. The potential energies are given by

$$V_{twist} = \frac{1}{2}\int_0^L GJ\left(\frac{\partial\theta}{\partial x}\right)^2 dx$$

$$V_{bending} = \frac{1}{2}\int_0^L EI\left(\frac{\partial w}{\partial x}\right)^2 dx$$