Cholesky Decomposition

Decomposition of a matrix into simpler matrices.

$$A = LL^T$$

This must exist for P.D. A.

$$L = \begin{bmatrix} l_{11} & 0 & 0 & 0 \\ l_{21} & l_{22} & 0 & 0 \\ \vdots & \vdots & \ddots & 0 \\ l_{n1} & l_{n2} & \cdots & l_{nn} \end{bmatrix}$$

By inspection

$$l_{ii} = \left(a_{ii} - \sum_{j=1}^{i-1} l_{ij}^2 \right)^{1/2}$$

$$l_{ki} = \left(a_{ik} - \sum_{j=1}^{i-1} l_{ij} l_{kj} \right) \frac{1}{l_{ii}}$$

(10, 16, 18)

Example

$$A = \begin{bmatrix} 16 & -20 & -24 \\ -20 & 89 & -50 \\ -24 & -50 & 280 \end{bmatrix}$$

$$l_{11} = a_{11}^{1/2} = 4$$

$$l_{21} = \frac{1}{l_{11}}(a_{12}) = -5$$

$$l_{22} = (a_{22} - l_{21}^2)^{1/2} = (89 - (-5)^2)^{1/2} = 8$$

$$l_{31} = \frac{1}{l_{11}} a_{13} = \frac{-24}{4} = -6$$

$$l_{32} = \frac{1}{l_{22}}(a_{23} - l_{21} l_{31}) = \frac{1}{8}[-50 - (-5)(-6)] = -10$$

$$l_{33} = (a_{33} - l_{31}^2 - l_{32}^2)^{1/2} = (280 - (-6)^2 - (-10)^2)^{1/2} = 12$$

$$L = \begin{bmatrix} 4 & 0 & 0 \\ -5 & 8 & 0 \\ -6 & -10 & 12 \end{bmatrix}$$

Consider

$$M\ddot{\underline{x}} + K\underline{x} = 0$$

$$M = LL^T, \quad \text{set } \underline{x} = L^{T^{-1}} \underline{q}$$

$$L^{-1} L L^T L^{T^{-1}} \ddot{\underline{q}} + L^{-1} K L^{T^{-1}} \underline{q} = \underline{0}$$

$$I \ddot{\underline{q}} + \hat{K} \underline{q} = \underline{0}$$

This is a very robust way of mass normalizing discrete eqns.