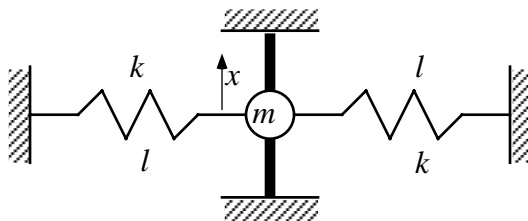


Closed book, closed notes. Use one $8\frac{1}{2} \times 11$ formula sheet, front and back. Test books will be provided. Problem ?? is required for graduate students.

1. On what law is the energy method based and when is it valid to apply the energy method? Does the same restriction apply to Lagrange's equation? (20 points)
2. Determine the natural frequency, damped natural frequency, critical damping, and damping ratio of the following system in terms of the variables in the equation (25 points):

$$(J/r^2 + m)\ddot{x} + (c_t/r^2 + c)\dot{x} + (mgl + k)x = 0$$

3. The dynamic range of operation of a system is $1 \text{ mm/s} < |v| < 10 \text{ mm/s}$ and $1 \text{ Hz} < f < 10 \text{ Hz}$. What is the maximum deflection and acceleration the system can experience, and at what frequencies do they occur? (25 points)
4. A point mass is held in place by two springs and a slider. Motion up and down on the slider is restricted only by the ability of the springs to expand and contract (they rotate freely). Assume that the springs are already stretched a length Δl beyond their unstretched length. The length of each stretched spring is l at system equilibrium. Derive the equation of motion for the system shown below. Ignore gravity. (30 points)



5. *Graduate/Extra Credit:* Consider a clamped string (both ends fixed). Determine the response of the string to an initial condition of $w(x, 0) = 0$, $w_t(x, 0) = \delta(x - l/2)$. Hint: Note that $\int_0^l f(x)\delta(x - l/2)dx = f(l/2)$. (20 points)