ME 710 Exam 1 Solutions

1) Soft in translation would be beyond flimsy. So, rotation is happening. Further this is common in higher modes... rotation is greater relative to displacement than in the 1st mode. So

$$\omega_{n_{pin}} < \omega_n < \omega_{n_{clamped}}$$ by the max-min principle / inclusion

$$\left(\frac{13\pi}{4\ell}\right)^2\sqrt{\frac{EI}{\rho A}} < \omega_n < \left(\frac{7\pi}{2\ell}\right)^2\sqrt{\frac{EI}{\rho A}}$$

$\sim \frac{104}{\ell^2}$ $\qquad\qquad\qquad\qquad \sim \frac{121}{\ell^2}$

2) EOM given in problem 4. ($c=0$)

$$X(x) = A\sin\beta_n x + B\cos\beta_n x + C\sinh\beta_n x + D\cosh\beta_n x$$

$$X'(x) = A\beta_n\cos\beta_n x - B\beta_n\sin\beta_n x + C\beta_n\cosh\beta_n x + D\beta_n\sinh\beta_n x$$

@ $x=0$, $X(x)=0$, $X'(x)=0$ $\therefore$ $D=-B$
$C=-A$

$$X(x) = A(\sin\beta_n x - \sinh\beta_n x) + B(\cos\beta_n x - \cosh\beta_n x)$$

At right end

$$X(l) = 0 = A(\sin\beta_n l - \sinh\beta_n l) + B(\cos\beta_n l - \cosh\beta_n l)$$

$EI X''' = -K X'$ (Units are torsion spring)

$$EI\beta_n^3\left[A(-\cos\beta_n l - \cosh\beta_n l) + B(\sin\beta_n l - \sinh\beta_n l)\right]$$
$$= -K\beta_n\left[A(\cos\beta_n l - \cosh\beta_n l) + B(-\sin\beta_n l - \sinh\beta_n l)\right]$$

$$\begin{bmatrix} \sin\beta_n l - \sinh\beta_n l & \cos\beta_n l - \cosh\beta_n l \\ EI\beta_n^{\cancel{3}\,2}(-\cos\beta_n l - \cosh\beta_n l) + K\cancel{\beta_n}(\cos\beta_n l - \cosh\beta_n l) & EI\beta_n^{\cancel{3}\,2}(\sin\beta_n l - \sinh\beta_n l) + K\cancel{\beta_n}(-\sin\beta_n l - \sinh\beta_n l) \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} = 0$$

Take determinate of matrix to obtain characteristic equation.

$$EI\beta_n^2(\sin^2\beta_n l - 2\sin\beta_n l\sinh\beta_n l + \sinh^2\beta_n l) + K(-\sin^2\beta_n l + \sinh^2\beta_n l)$$

$$- EI\beta_n^2(-\cos^2\beta_n l + \cosh^2\beta_n l) - K(\cos^2\beta_n l - 2\cos\beta_n l\cosh\beta_n l + \cosh^2\beta_n l) = 0$$

for solving, this becomes $\frac{(\beta_n l)^2}{l^2}$ — unknown in ( ), l^2 ← known

$$-EI\beta_n^2\, 2\sin\beta_n l \sinh\beta_n l - 2K(1-\cos\beta_n l\cosh\beta_n l) = 0$$

This must be solved numerically for $\beta_n l$.
Guesses using inclusion principle.

$$\frac{A}{B} = \sigma_n = \frac{-\cos\beta_n l + \cosh\beta_n l}{\sin\beta_n l - \sinh\beta_n l}$$

$$X(x) = B\left(\sigma_n(\sin\beta_n x - \sinh\beta_n x) + (\cos\beta_n x - \cosh\beta_n x)\right)$$

$$\omega_n = \beta_n^2\sqrt{\frac{EI}{\rho A}}$$

Of course, $I = \frac{1}{12} b h^3 = 2\times10^{-8}\ m^4$

4) $X_n = \frac{\sqrt{2}}{\sqrt{\rho A}} \sin\left(n \pi \frac{x}{\ell}\right)$ (mass normalized)

$\therefore$

(78) $$W(x,t) = \sum_{n=1}^{\infty} \frac{1}{\omega_n^2 + 2\varsigma_n \omega_n i - 1} \frac{2}{\rho A} \left(\sin \frac{n\pi}{3} - \sin \frac{2n\pi}{3}\right) \sin\left(n\pi \frac{x}{\ell}\right) \sin t$$

where $\omega_n = \frac{(n\pi)^2}{\ell^2} \sqrt{\frac{EI}{\rho A}}$

$$\varsigma_n = \frac{c}{\rho A 2 \omega_n}$$

Using equs (78) as labelled. Also see example 6.2.2