ME 712, Finite Element Method Applications   Final Exam, Spring 2006

One formula sheet to be turned in. Problems must be in order in the blue books.

1. A single standard Euler-Bernoulli beam has constraints of $v_1 = -v_2 = \theta_1 L/2$, and $\theta_2 = \theta_1$. Generate the reduced governing equations using the method of your choice.

$$K = \frac{EI_z}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}, \tag{1}$$

$$M = \frac{m}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix} \tag{2}$$

Hint: Switch to using coordinates of θL in place of θ by appropriate 'transformation'.

What have you just derived as the reduced mass matrix?

2. Obtain the mass and stiffness matrices of a bilinear quadrilateral (Q4) element with nodes 1-4 at (0,0), (1,0), (1,2), and (0,2). Use Gauss integration to derive the element matrices. Assume $E = 7.3084 \times 10^{10}$ N/m^2, $\nu = .3$, and $\rho = 2770$ kg/m^3 and presume a state of plane stress, e.g.

$$[E] = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix}$$

3. Find the strain at (x, y)=(0,1) of a bilinear quadrilateral (Q4) element with nodes 1-4 at (0,0), (1,0), (1,2), and (0,2) in terms of u_2 and v_2 (presume all other nodal displacements are zero).

4. Download the file mk.mat. Load it into Matlab. Apply Guyan reduction to reduce the matrix sizes to $n \times n$ where $\frac{m_{ii}}{k_{ii}} < 2.5 \times 10^7$.

   Turn on a diary to log the following.

   (a) What degrees of freedom did you keep (in sorted order)?

   (b) What are the first 10 natural frequencies in Hz *after* Guyan reduction.

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Hints:
To get indices of a sorted list a where
a=1:-1:-10
[y,i]=sort(a)
gives
y =
   -10     -9     -8     -7     -6     -5     -4     -3     -2     -1      0      1
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i =
    12    11    10     9     8     7     6     5     4     3     2     1
y are your sorted values, and i are their indices in a.
So your 3 highest values of x are y(10:12), and their indices in the
original list, x, are
i(10:12)
giving
     3     2     1
so
x(i(12:12)) are the largest values of x .

To select specified columns and rows of a matrix):
a=[1 2 3 4; 5 6 7 8; 9 10 11 12;13 14 15 16]
a =
     1     2     3     4
     5     6     7     8
     9    10    11    12
    13    14    15    16
    b=a([1 3:4],[1 3:4])
returns
b =
     1     3     4
     9    11    12
    13    15    16
```