ME 460/660 Exam 2, Fall 2008 Solns

1) $H(j\omega) = \frac{X(j\omega)}{F(j\omega)} = \frac{1}{8.8826\times10^4 - 10\omega^2 + 1\omega j}$

a) For $n=2$ ($n=0$ gives $a_n=0$)

$\omega = \frac{4\pi}{.2} = 62.8$, $a_n = -0.206$

$H(j\omega) = 2.026\times10^{-5} - 2.58\times10^{-8} j$

$|H(j\omega)| = 2.03\times10^{-5}$, $\measuredangle H(j\omega) = -0.07°$ (-0.00127 rad)

Contribution to $x(t)$ is

$x_2(t) = \underbrace{-4.11\times10^{-6}}_{a_n \times |H(j\omega)|} \cos\left(\underbrace{62.8}_{\omega} t \underbrace{- 0.00127}_{\measuredangle H(j\omega)}\right)$

b) Same calcs

$x_4(t) = -7.3\times10^{-7} \cos 125t - 3.14$

Since the phase passed through resonance (dropped past $-\frac{\pi}{2}$) all responses at higher frequencies will be smaller.

$X_2(t)$ is the best 1-term approximation. All other terms are almost an order of magnitude smaller or more.

2) If the accidental expression above 2 was used,

$$x(t) = \frac{1}{m\omega_d} e^{-\zeta\omega_n t} \sin\omega_d t + \frac{1}{m\omega_d} e^{-\zeta\omega_n\left(t-\frac{2\pi}{\omega_d}\right)} \sin\omega_d\left(t-\frac{2\pi}{\omega_d}\right)\left(e^{\frac{-2\pi\zeta}{\sqrt{1-\zeta^2}}}\right)$$

This can be simplified, like in class, or like the previous exam this was on.

The problem stated an "undamped system..." So using the convolution integral

For $0 < t < \frac{1}{15}$:

$$x_1(t) = \frac{1}{m\omega_n}\int_0^t F \sin\omega_n(t-\tau)\,d\tau$$

$$= \frac{F}{m\omega_n}\left(\frac{1}{\omega_n}\cos\omega_n(t-\tau)\right)\Big|_0^t$$

$$\frac{d}{d\tau}\cos\omega_n(t-\tau) = -\omega_n \times -\sin\omega_n(t-\tau)$$

$$= \frac{F}{m\omega_n^2}\left(1-\cos\omega_n t\right)$$

$$= \frac{F}{K}\left(1-\cos\omega_n t\right)$$

$$= \frac{F}{8.88\times10^4}\left(1-\cos\omega_n t\right)$$ just like in the notes.

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For $\frac{1}{15} < t$, we need to add in the negative step. Alternatively, we can use the displacement and velocity at $t = \frac{1}{15}$ sec as initial conditions for a free response.

Method a)

$$x(t) = x_1(t) + \frac{-F}{m\omega_d} \int_{\frac{1}{15}}^{t} \sin \omega_n(t-\tau)\, d\tau$$

$$= x_1(t) + \frac{-F}{m\omega_d^2} \left(\cos \omega_n(t-\tau) \right) \Big|_{\frac{1}{15}}^{t}$$

$$= \frac{F}{8.88 \times 10^4} \left(1 - \cos\omega_n t - 1 + \cos\omega_n\left(t - \tfrac{1}{15}\right) \right)$$

$$= \frac{F}{8.88 \times 10^4} \left(\cos\omega_n\left(t - \tfrac{1}{15}\right) - \cos\omega_n t \right)$$

$$= \underline{0}$$

You can simplify if you are bored.
(hint: $\omega_n \times \frac{1}{15} = 2\pi$)

Method b) More thinking, less work, but only if you see the coming result

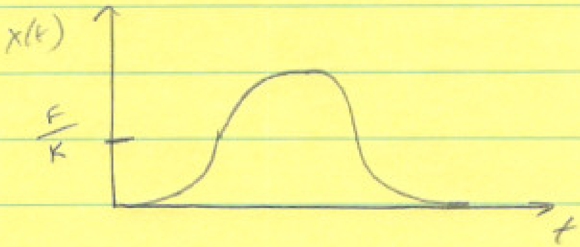
$$x_1(t)\Big|_{t=\frac{1}{15}} = \frac{F}{8.88 \times 10^4} \left(1.2087 \times 10^{-10} \right)$$

$$v_1(t)\Big|_{t=\frac{1}{15}} = \frac{F}{8.88 \times 10^4} \left(1.4 \times 10^{-3} \right)$$

hmm, what's going on?

If we stop and think (not required)
$x_1(t)$ is pretty darned close to zero considering its maximum $\left(\frac{F}{8.88 \times 10^4} \times 2\right)$. 2 is much bigger than 1.2×10^{-10}. So, why is that? Well

the period of the system is $\frac{1}{15}$ seconds.



Using $x(t) = x_0 \cos \omega_n t + \frac{V_0}{\omega_n} \sin \omega_n t$, (satisfies both IC, right?), we get $x(t) \approx 0$. That's because ~~the~~ the period of the excitation lasted one period long, and "cancels" itself out.

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3) Not Coulomb (not a linear decay envelope)

Try log decrement.

High amp $\ln\left(\frac{0.77}{.65}\right) = 0.169$ (to too many places)

Low amp $\ln\left(\frac{\text{same}}{\text{same}}\right) = 0$

Heck, the amplitude is constant at low amplitudes. This isn't viscous either

Must be air damping, since the other 2 don't fit.

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4) $\frac{E}{\rho} X'' T = \ddot{T} X$

$$\frac{X''}{X} = \frac{\ddot{T}}{T}\frac{\rho}{E} = -\sigma^2$$

$X(x) = A \sin \sigma x + B \cos \sigma x$

$X(0) = 0$, so

$X(0) = A \sin \sigma 0 + B\, 1$

$B = 0$

$X(x) = A \sin \sigma x$ (or just $\sin \sigma x$)

$X'(\ell) = 0 = \sigma \cos \sigma \ell$

$$\sigma \ell = \underset{1}{\frac{\pi}{2}},\ \underset{2}{\frac{3\pi}{2}},\ \underset{3}{\frac{5\pi}{2}},\ \underset{4}{\frac{7\pi}{2}}$$

$$\sigma_n = \frac{(2n-1)\pi}{2\ell}$$

$$\ddot{T} = -\sigma_n^2 \frac{E}{\rho} T$$

$$\omega_n = \sigma_n \sqrt{\frac{E}{\rho}} = \frac{(2n-1)\pi}{2\ell}\sqrt{\frac{E}{\rho}}$$

$X(x) = \sin \frac{(2n-1)}{2\ell} \pi x$

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