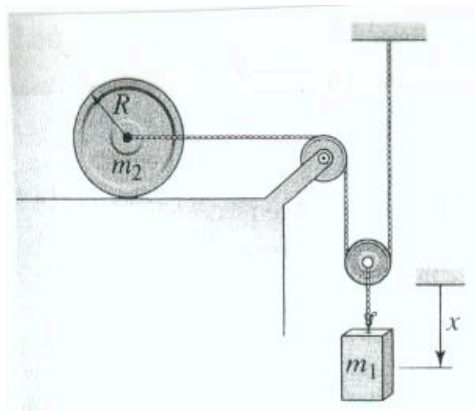


The FE reference book may be used during this exam. 4:10-5:50, 152 B.

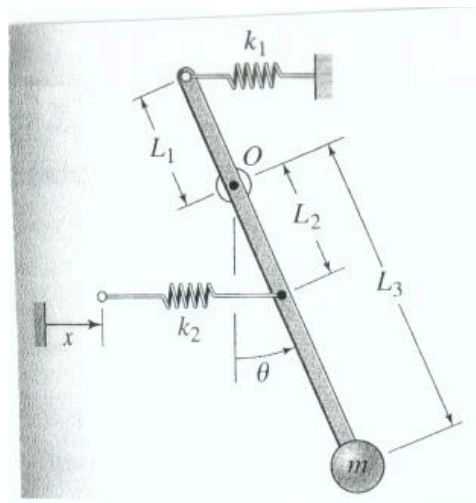
1. Quenching is the process of immersing a hot metal object in a bath for a specified time to improve properties such as hardness. A copper sphere 25 m in diameter, initially at 300C, is immersed in a bath at 0C. Measurements of a sphere's temperature versus time are shown here. Plot the data and find a functional description of the data.

Time (s)	0	0.1	0.2	0.3	0.4	0.5	0.6
temp (C)	300	150	75	35	12	5	0

2. Assume the cylinder rolls without slipping. Neglect the mass of the pulleys and derive the equation of motion of the system in terms of the displacement x .



3. The mass M is attached to a rigid lever having negligible mass and negligible friction in the pivot. The input is the displacement x . When x and θ are zero, the springs are at their free length. Assuming θ is small, derive the equation of motion for θ with x as the input.



4. For the equations (or set of equations) below, assuming all greek characters are known constants, list the numbers of the equations that are:

(a) linear

(b) time invariant (autonomous)

$$\begin{aligned}\dot{x} &= \sigma x + \beta y \\ \dot{y} &= \gamma y + \alpha xy\end{aligned}\tag{1}$$

$$\begin{aligned}\dot{x} &= \sigma x + \beta y \\ \dot{y} &= \gamma y + \alpha x\end{aligned}\tag{2}$$

$$\begin{aligned}\dot{x} &= \sigma x + t\beta y \\ \dot{y} &= \sin(\gamma)y + \alpha x\end{aligned}\tag{3}$$

$$\begin{aligned}\dot{x} &= \sigma x + \beta y \\ \dot{y} &= \sin(t)y + \alpha \sin(x)\end{aligned}\tag{4}$$