Computational Methods in Structural Dynamics, Exam 1 Winter 1999

One 8.5" by 11" cheat sheet.

1. Given $L = L(q_i, \dot{q}_i)$ $(i = 1, 2, \dots n)$ for a discrete system, derive Lagrange's equations for a discrete system using Hamilton's principle. Dont forget to include work done by external forces. (30 points)

2. Determine the characteristic equation in β of a free-free beam given the homogenous equation of a beam
$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + \rho A\frac{\partial^2 w(x,t)}{\partial t^2} = 0$$
Note: *You cannot explicitly solve for β, I want the nonlinear equation that can be numerically solved for β* (30 points)
Hint: $\sinh^2 \beta l - \cosh^2 \beta l = -1$

3. Use Hamilton's principal to derive the equations of motion for the following system. A uniform cantilever beam has torsional stiffness GJ, vertical bending stiffness EI, and mass per unit length ρA, and rotational inertia (twisting) per unit length ρI_p (I_p being the polar moment of inertia for the twisting beam). The beam is cantilevered at end A, and a massless rigid bar BC is attached at end B. A concentrated mass is located at point C. Assume that bending takes place only in the $z - x$ plane with deflection $w(x,t)$ and that rotation takes place about the x axis $q(x,t)$. Neglect gravity. State the equation of motion and boundary conditions. The potential energies are given by
$$V_{twist} = \frac{1}{2}\int_0^L GJ\left(\frac{\partial \theta}{\partial x}\right)^2 dx, \qquad V_{bending} = \frac{1}{2}\int_0^L EI\left(\frac{\partial v}{\partial x}\right)^2 dx$$

Do not attempt to solve the equations of motion. (30 points)

Bonus: Consider a cantilevered beam fixed at the left end, and free at the right. If a point mass is attached to the right end of the beam (at $x = L$), what are the two boundary conditions at the right end? (10 points)