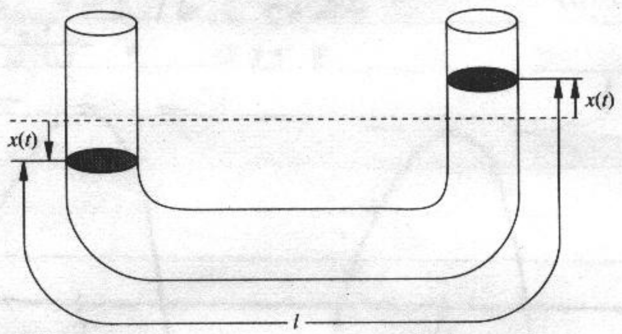
ME 460/660, Mechanical Vibration — Exam 1, Sp 1997

Closed book, closed notes. Use one formula sheet $8\frac{1}{2} \times 11$. Test books will be provided.

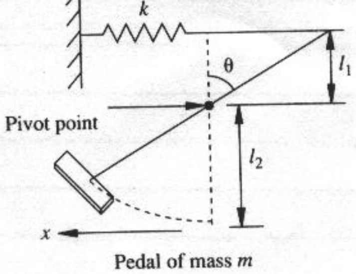
1. A system is defined as having a mass of 10 kg, a natural frequency of 1000 rad/sec, and a log decrement of .02. Find the equivalent spring stiffness and damping coefficient and **clearly define the units** of your answers.

2. Find the natural frequency and damping ratio and sketch the free response of the system defined by the differential equation $\ddot{x} + .1\dot{x} + 10x = 0$ to an initial condition of $x_0 = 5$ and $v_0 = 0$.

3. Estimate the undamped natural frequency of a column of water moving back and forth in a U tube manometer. Assume the column of water moves together as a unit, and that the radius of the pipe is small relative to the radius of the curve. Use a mass density of water ρ, a pipe radius of r (i.e. the cross sectional area of the pipe is πr^2), a length l of the pipe, and a gravitational constant of g. *Hint: Use the energy method.*



4. A control pedal of an aircraft can be modeled as the single-degree-of-freedom (SDOF) system shown below. Consider the lever to be a massless shaft and the pedal to be a lumped mass at the end of the shaft. Determine the equation of motion in θ and determine the natural frequency of the system **assuming θ remains small** even though it isn't drawn small in the figure.



ME 460/660 Sp '97 Test 1 Solutions

1.) $m = 10\ \text{Kg}$, $\omega = 1000\ \text{rad/s}$, $\delta = .02$

$\omega = \sqrt{\frac{k}{m}}$ $\quad k = \omega^2 m = 1\times10^7\ \text{N/m}$

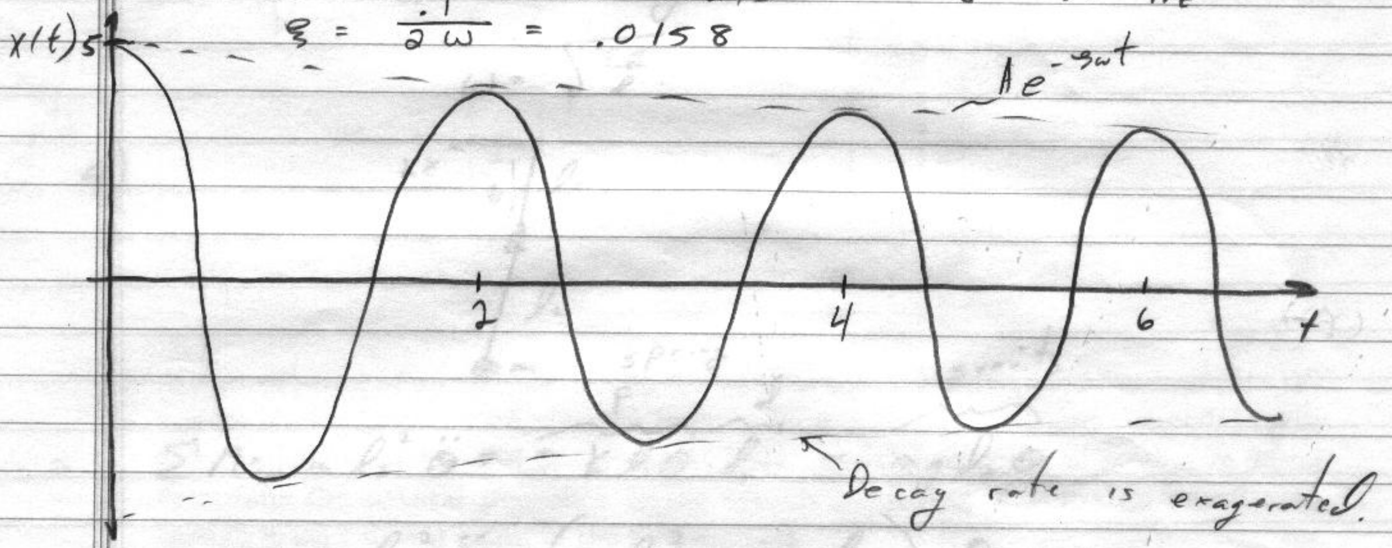
$\zeta = \dfrac{\delta}{\sqrt{4\pi^2+\delta^2}} = .0032$

$c = 2\zeta\omega m = \underline{64\ \text{Kg/s}}$

2) $\ddot{x} + .1\dot{x} + 10x = 0$

$\omega = \sqrt{10} = 3.16\ \text{rad/s}$ $\qquad f \sim .5\ \text{Hz}$

$\zeta = \dfrac{.1}{2\omega} = .0158$



3) For a motion $x(t)$, one can consider a slug of fluid on the left side to be taken away and stacked on the right side, so

+8 $U = mg\Delta h = \rho\pi r^2 x\, g\, x = x^2 \rho\pi r^2 g$

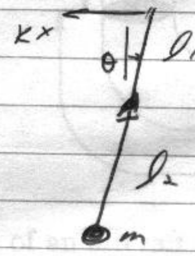
+8 $T = \frac{1}{2}(l\pi r^2\rho)\dot{x}^2$

+9 $\frac{d}{dt}(T+U) = 0 = (l\pi r^2\rho\ddot{x} + 2\rho\pi r^2 g x)\dot{x} = 0$

$$l\ddot{x} + 2gx = 0$$

$$\omega = \sqrt{\frac{2g}{l}}$$

4)



$$\Sigma M = m l_2^2\ddot{\theta} = \overbrace{-\overbrace{k l_1\theta}^{F}\,\overbrace{l_1}^{d}}^{\text{spring}} \overbrace{- m g l_2\theta}^{\text{gravity}}$$

$$m l_2^2\ddot{\theta} + (k l_1^2 + m g l_2)\theta = 0$$

$$\omega \approx \sqrt{\frac{k l_1^2 + m g l_2}{m l_2^2}}$$

Or, from energy

$$T = \frac{1}{2} m(l_2\dot{\theta})^2, \quad U = mg l_2(1-\cos\theta) + \frac{1}{2}k(\theta l_1)^2$$

$\frac{d}{dt}(T+U)$ results in the same EOM.