

Closed book, closed notes. Use  $8\frac{1}{2} \times 11$  formula sheet from web and turn in with exam (nothing else may be written on the formula sheet). Test books will be provided. Calculators allowed. Knowing how to use them well is highly recommended.

Problems are 10 points each. Problem ?? is required for graduate students, bonus for undergraduates (worth 6 points).

L<sup>A</sup>T<sub>E</sub>X

1. The Fourier series representation of a force is given by

$$f(t) = -\frac{8}{\pi^2} \left[ \cos \frac{2\pi}{5}t + \frac{1}{9} \cos \frac{6\pi}{5}t + \frac{1}{25} \cos 2\pi t + \cdots \right]$$

Find the force response (particular solution) for the system define by

$$100\ddot{x} + 1\dot{x} + 1000x = f(t)$$

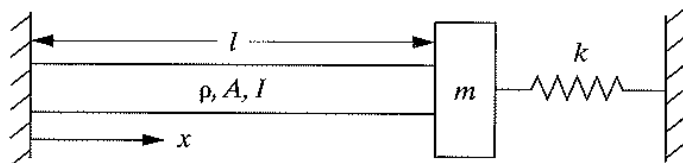
2. An undamped system is excited by a brief excitation defined by

$$f(t) = \begin{cases} 1 & t < .02 \\ 0 & .02 < t \end{cases} \quad (1)$$

Set up the integral/s to find  $x(t)$  during and after the pulse presuming  $10\ddot{x} + 100x = f(t)$ .

3. A vibrating mass of 300 kg is mounted on a massless support by a spring of stiffness 40 kN/m and a damper of unknown damping coefficient. At resonance it is observed to vibrate with a 100-mm amplitude while the support vibration has a maximum amplitude of only 2.5 mm. The governing differential equation is  $m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky$ .
- Calculate the damping coefficient approximately.
  - Calculate the force on the base at resonance due to
    - the spring
    - the dashpot
4. *Grad student/bonus*(20% of other points)

Calculate the boundary conditions of a bar fixed at  $x = 0$  and connected to ground through a mass and a spring as illustrated in Figure P6.21.



**Figure P6.21** Beam with a tip mass connected to ground via a spring.

(20% of other points)