Comp Methods Exam 2 2008 Solns

1) 3^{rd} NF of clamped-pinned beam is

$$\frac{104}{\ell^2}\sqrt{\frac{EI}{\rho A}} \quad \text{rad/s (lower bound)}$$

3^{rd} NF of clamped-clamped beam is

$$\frac{120}{\ell^2}\sqrt{\frac{EI}{\rho A}} \quad \text{rad/s (upper bound)}$$

Other bounds are too far spread

2)

$$X(x) = \cos\beta x + B\cosh\beta x + C\sin\beta x + D\sinh\beta x$$

where I have set $A = 1$

$$X(0) = 0 = 1 + B \qquad B = -1$$
$$X'(0) = 0 = C\beta + D\beta \qquad C = -D$$

$$X(\ell) = 0 = \cos\beta\ell - \cosh\beta\ell - D\sin\beta\ell + D\sinh\beta\ell$$

$$EI\,X'' = K X'$$

I'm guessing sign, can fix later

$$12600\,X'' = s10\,X'$$

(s is for tracking the sign)

$$\ell\beta^{2}\; 12600(-\cos\beta\ell - \cosh\beta\ell + D\sin\beta\ell + D\sinh\beta\ell)$$
$$= s\underset{\ell=1}{\ell}10\cancel{\beta}(\sin\beta\ell - \sinh\beta\ell + -D\cos\beta\ell + D\cosh\beta\ell)$$

$$\underbrace{\begin{bmatrix} \cos\beta\ell - \cosh\beta\ell & -\sin\beta\ell + \sinh\beta\ell \\ +12600\beta\ell(-\cos\beta\ell - \cosh\beta\ell) & 12600\beta\ell(\sin\beta\ell + \sinh\beta\ell) \\ -s10(\sin\beta\ell - \sinh\beta\ell) & -s10(-\cos\beta\ell + \cosh\beta\ell) \end{bmatrix}}_{\det A = 0} \begin{bmatrix} 1 \\ D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

See Mathematica sheet

Changing k and s, it's easy to figure out $s = -1$ (sign choice was wrong) because increasing k must increase natural frequencies.

$\beta l_5 = 16.4934$ which is the <u>same</u> to 6 places as the clamped pinned value. Thus to any reasonable accuracy

$\beta l = 3.93, 7.07, 10.2, 13.4, 16.5$

$$f_n = \frac{(\beta_n l)^2}{l^2} \sqrt{\frac{EI}{\rho A}} / 2\pi$$

$$= (\beta_n l)^2 \, 8.26$$

$$= 127., 413, 859, 1480, 2250 \quad Hz$$

$$X(x) = \cosh \beta_n x - \cos \beta_n x - \sigma_n (\sinh \beta_n x - \sin \beta_n x)$$

β_n above

$$\sigma_n = \begin{bmatrix} 1.0008 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

3) $X(x) = \sin \frac{n\pi x}{\ell}$

$EI\, w'''' + \rho A \ddot{w} = \sin t\, \delta(x - \frac{\ell}{3})$

$w'''' = \left(\frac{n\pi}{\ell}\right)^4 w$

$\ddot{w} + \left(\frac{n\pi}{\ell}\right)^4 \frac{EI}{\rho A} w = \frac{1}{\rho A} \sin t\, \delta(x - \frac{\ell}{3})$

$w_n(x,t) = X_n(x)\, T_n(t)$

$\sum_{n=1}^{\infty} \left(\ddot{T}_n + \left(\frac{n\pi}{\ell}\right)^4 \frac{EI}{\rho A} T_n\right) X_n = \frac{1}{\rho A} \sin t\, \delta(x - \frac{\ell}{3})$

mult by $X_m(x)$ and integrate over $0 < x < \ell$

$\left(\ddot{T}_n + \left(\frac{n\pi}{\ell}\right)^4 \frac{EI}{\rho A} T_n\right) \frac{\ell}{2} = \frac{1}{\rho A} \sin \frac{n\pi}{3} \sin t$

$T_n(t) = \frac{2}{\rho A \ell} \sin \frac{n\pi}{3} \left(\frac{1}{\omega_n^2 - 1}\right)$

where $\omega_n = \left(\frac{n\pi}{\ell}\right)^2 \sqrt{\frac{EI}{\rho A}}$

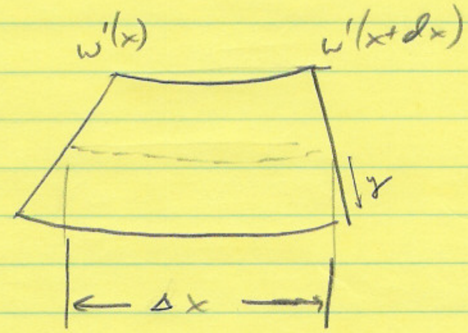
$w(x,t) = \sum_{n=1}^{\infty} \frac{2 \sin \frac{n\pi}{3}}{\rho A \ell} \frac{1}{(\omega_n^2 - 1)} \sin \frac{n\pi x}{\ell}$

4) $eg(c) = 0, 2$ null space $\underline{u}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$eg(K) = 1, 3$ $\underline{u}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\underline{u}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Mode one is undamped so system is stable, but not asymptotically.

5)



strain in bottom fiber is

$$\epsilon_f = \frac{(w'(x+\Delta x) - w'(x))}{\Delta x} \quad y$$

$$\lim_{\Delta x \to 0} \epsilon_f = \frac{d^2 w(x)}{dx^2} \quad y$$

Stress in fiber is $\sigma_f = E w'' y$

Net moment about nuetral axis is

$$M = \int_A \underbrace{y}_{\text{distance}} \underbrace{E w'' y \, dA}_{\text{Force}}$$

Presuming E and w'' are constant through cross section

$$M = E w'' \underbrace{\int y^2 \, dA}_{I_{xx}}$$

$$\boxed{M = E I_{xx} w''}$$ QED