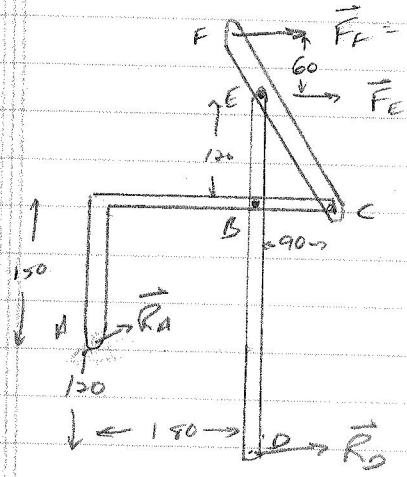
6.100: Find components of forces acting on ABC at B and C

Apply: Equilibrium (Newton's Law in 2-D) ($\Sigma \vec{F} = \vec{0}, \Sigma \vec{M} = \vec{0}$)



Position	Force
$\vec{r}_{F/A} = 135\,\hat{i} + 330\,\hat{j}$	$\vec{F}_F = 80\,\hat{i}$
$\vec{r}_{E/A} = 180\,\hat{i} + 270\,\hat{j}$	$\vec{F}_E = 192\,\hat{i}$
$\vec{r}_{D/A} = 180\,\hat{i} - 120\,\hat{j}$	$\vec{R}_D = R_D\,\hat{i}$
	$\vec{R}_A = R_{Ax}\,\hat{i} + R_{Ay}\,\hat{j}$

Units in mm and Newtons

$\Sigma F_x = 0 = 80 + 192 + R_D + R_{Ax}$

$\Sigma F_y = 0 = R_{Ay}$ $\qquad R_{Ay} = 0\ N$

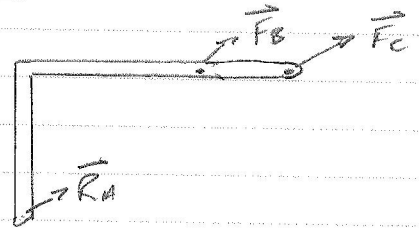
$\Sigma \vec{M}_A = \vec{0} = \vec{r}_{F/A} \times \vec{F}_F + \vec{r}_{E/A} \times \vec{F}_E + \vec{r}_{D/A} \times \vec{R}_D$

$= 330\,\hat{j} \times 80\,\hat{i} + 270\,\hat{j} \times 192\,\hat{i} - 120\,\hat{j} \times R_D\,\hat{i}$

$R_D = \dfrac{330 \times 80 + 270 \cdot 192}{120} = 652\ N$

$R_{Ax} = -80 - 192 - R_D = -924\ N$

FBD ABC



Position	Force
	$\vec{R}_A = -924\hat{i}$
$\vec{r}_{B/A} = 180\hat{i} + 150\hat{j}$	$\vec{F}_B = F_{Bx}\hat{i} + F_{By}\hat{j}$
$\vec{r}_{C/A} = 270\hat{i} + 150\hat{j}$	$\vec{F}_C = F_{Cx}\hat{i} + F_{Cy}\hat{j}$

(1) $\Sigma F_x = 0 = -924 + F_{Bx} + F_{Cx}$

(2) $\Sigma F_y = 0 = F_{By} + F_{Cy}$

$F_{Cx} = 924 - 2119 = -1195\ N$

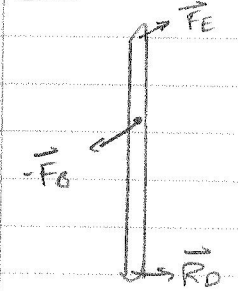
(3) $\Sigma \vec{M}_A = \vec{0} = \vec{r}_{B/A} \times \vec{F}_B + \vec{r}_{C/A} \times \vec{F}_C$

$= (180\hat{i} + 150\hat{j}) \times (F_{Bx}\hat{i} + F_{By}\hat{j}) + (270\hat{i} + 150\hat{j}) \times (F_{Cx}\hat{i} + F_{Cy}\hat{j})$

$= (180F_{By} - 150F_{Bx} + 270F_{Cy} - 150F_{Cx})\hat{k}$

Yuck

FBD EBC



Position	Force
	$\vec{F}_E = F_{Ex}\hat{i} + F_{Ey}\hat{j}$
$\vec{r}_{B/E} = -120\hat{j}$	$-\vec{F}_B = -F_{Bx}\hat{i} - F_{By}\hat{j}$
$\vec{r}_{D/E} = -390\hat{j}$	$\vec{R}_D = 652\hat{i}$

$\Sigma \vec{M}_E = \vec{0} = -120F_{Bx}\hat{k} + 390 \cdot 652\hat{k}$

$F_{Bx} = 2119\ N$

Thus $F_{Cx} = 924 - 2119 = -1195\ N$ (From (1))

Substituting knowns into (2) and (3) and solving

$$\begin{bmatrix} 1 & 1 \\ 180 & 270 \end{bmatrix} \begin{bmatrix} F_{By} \\ F_{Cy} \end{bmatrix} = \begin{bmatrix} 0 \\ 1.386 \times 10^5 \end{bmatrix}$$

$F_{By} = -1540\ N$, $F_{Cx} = 1540\ N$