ME 460/660, Mechanical Vibration — Exam 1, Spring 2000

Closed book, closed notes. Use one $8\frac{1}{2} \times 11$ formula sheet, front and back, no examples, derivations, or solutions. **The formula sheet must be turned in with the exam or 25 points will be deducted from your score.** Test booklets will be provided.

1. What do the following devices do with energy in a vibrating system? If a device stores energy, what form does it store energy in? (15 points)

   (a) Spring

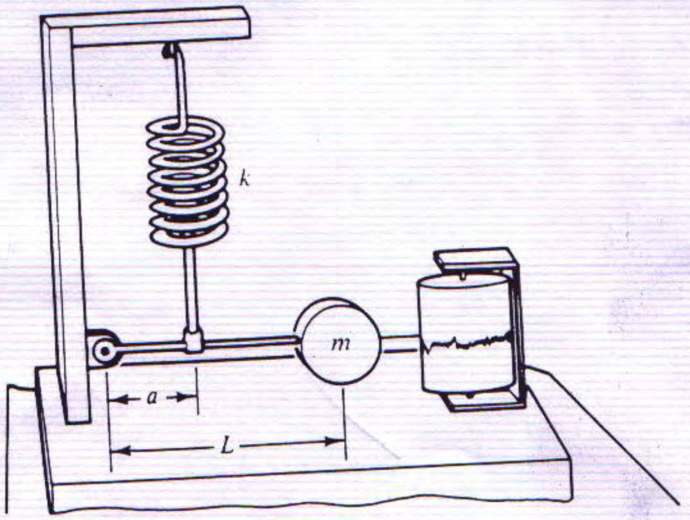
   (b) Dashpot

   (c) Mass

2. Obtain the particular solution (forced response) for the following equation of motion: $100\ddot{x} + 5\dot{x} + 10000x = 5e^{50jt}$. (15 points)

3. Prolonged exposure to 5×10^{-4} g floor oscillation (or above) is deemed intolerable for comfort. At 7 Hz, what displacement amplitude would cause discomfort? (15 points)

4. Derive the equation of motion of the system shown below. (25 points)



5. *Grad student/bonus* Determine the natural frequencies and mode shapes for a clamped-free bar. The equation of motion of a bar is $\left(\frac{E}{\rho}\right)\frac{\partial^2 w(x,t)}{\partial x^2} = \frac{\partial^2 w(x,t)}{\partial t^2}$. (20% of other points)

1a) Store potential

b) Dissipate

c) Store potential

2) $x(t) = X e^{j50t}$

$\dot{x} = 50j\, X e^{j50t}$

$\ddot{x} = -2500\, X e^{j50t}$

subst into EOM

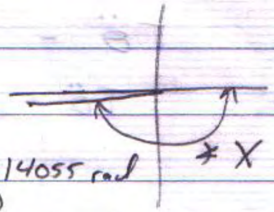
$(-250{,}000 + 250j + 10{,}000)\, X e^{j50t} = 5 e^{j50t}$

$X = \dfrac{5}{-240{,}000 + 250j}$

$= \dfrac{1}{-48{,}000 + 50j}$

$|X| = \dfrac{1}{\sqrt{48000^2 + 2500}} = 2.1 \times 10^{-5}$

$\angle X = \text{atan2}\ \dfrac{-250}{-240{,}000}$



$= -179.94°$, $.00104 - \pi$ rad $= -3.14055$ rad

$x(t) = 2.08 \times 10^{-5}\, e^{(50tj - 3.1406)}$

3) $|a| = 4.9 \times 10^{-3}\ m/s^2$

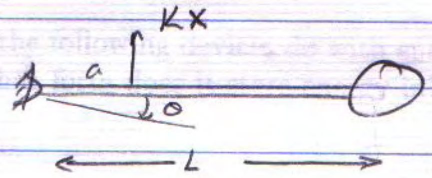
$\omega = 7 \cdot 2 \cdot \pi = 14\pi$ rad/s

$A = \dfrac{|a|}{\omega^2} = \dfrac{4.9 \times 10^{-3}}{(14\pi)^2} = 2.5 \times 10^{-3}$ mm

or 2.5×10^{-6} m

4) System is in equilibrium as shown.

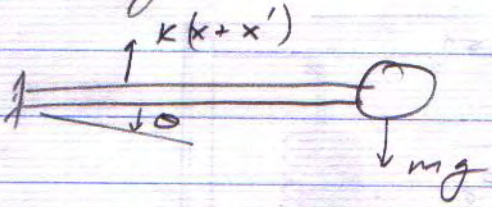
a)



$$\Sigma M = J\ddot{\theta} = (L^2 m)\ddot{\theta} = -a\,kx = -a\,k\,a\theta = -a^2 k\theta$$

$$L^2 m\ddot{\theta} + a^2 k\theta = 0$$

b) Considering all forces



$$\Sigma M = J\ddot{\theta} = -ak(x+x') + mgL$$

$$J\ddot{\theta} - mgL + a^2 k(\theta + \theta') = 0$$

$$J\ddot{\theta} + a^2 k\theta = -a^2 k\theta' + mgL$$

If $akx' = mgL$, the system will be in equilibrium at the position shown.

c) $T = \frac{1}{2} J \dot{\theta}^2$

$U = \frac{1}{2} k (a\theta)^2$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) + \frac{\partial U}{\partial \theta} = 0$$ (other terms are zero)

$$mL^2 \ddot{\theta} + a^2 k \theta = 0$$

If damping in the pen is included

$$R = \frac{1}{2} c \dot{x}^2 = \frac{1}{2} c (L' \dot{\theta})^2$$

$$\frac{\partial R}{\partial \dot{\theta}} = L'^2 c \dot{\theta}$$

So

$$mL^2 \ddot{\theta} + L'^2 c \dot{\theta} + a^2 k \theta = 0$$ is

the damped EOM.

5) $w(x,t) = X(x)\,T(t)$

$$\frac{E}{\rho} X'' T = X T''$$

$$\frac{E}{\rho}\frac{X''}{X} = \frac{T''}{T} = -\sigma^2$$

$$X'' + \sigma^2 \frac{\rho}{E} X = 0$$

$$X'' + \beta^2 X = 0 \qquad \beta = \sigma\sqrt{\frac{\rho}{E}}$$

$$X = A \sin\beta x + B \cos\beta x$$

$$X(0) = 0$$

$$\therefore B = 0$$

$$X'(L) = 0 = \beta A \cos\beta L$$

$$\beta_n L = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots \frac{(2n-1)\pi}{2}$$

The mode shapes are

$$\boxed{X_n(x) = A \sin\frac{(2n-1)\pi}{2}\frac{x}{L}}$$

$$T'' + \sigma_n^2 T = 0$$

σ_n are ω_n

$$\boxed{\omega_n = \beta_n\sqrt{\frac{E}{\rho}} = \frac{(2n-1)\pi}{2L}\sqrt{\frac{E}{\rho}}}$$