ME 710 Final Solutions

1) Input at A causes output at B same as input at B causes output at A. Useful when access for excitation or sensing is restricted

2) $\frac{1}{2}$ the computational effort

3) Stationarity of Rayleigh Quotient

4) If the states are initially bounded by $|z| < \epsilon$, then for all time they will be bounded by $|z| < \delta$

5) Raises or no change. Varies by frequency

6) 1) Non-self-adjoint
2) Damped
3) Forced
4) Non linear
Pick 2

1) a) $EI\, w'''' + \left(\rho A + M\delta(x-\frac{\ell}{2})\right)\ddot{w} = 0$

$w(x,t) = \sin(\omega t)\, \sin(\pi \frac{x}{\ell})$

$$\left[EI\left(\frac{\pi}{\ell}\right)^4 - \left(\rho A + M\delta(x-\tfrac{\ell}{2})\right)\omega^2\right]\sin\omega t\, \sin(\pi \tfrac{x}{\ell}) = 0$$

Multiply by $\sin\frac{\pi x}{\ell}$, integrate over the domain

$$EI\left(\frac{\pi}{\ell}\right)^4 \frac{\ell}{2} - \left(\rho A \frac{\ell}{2} + M\right)\omega^2 = 0$$

$$\omega \approx \frac{\pi^2}{\ell^2}\sqrt{\frac{EI}{\rho A + M\frac{2}{\ell}}}$$

b) set $M = 0$

$$\omega \approx \frac{\pi^2}{\ell^2}\sqrt{\frac{EI}{\rho A}}$$

2) The compliance is $K^{-1} = \begin{bmatrix} 0.397 & 0.179 & 0.071 \\ 0.179 & 0.536 & 0.214 \\ 0.071 & 0.214 & 0.286 \end{bmatrix}$

Green's method is

$\lambda \underline{X}_{i+1} = K^{-1} \underline{X}_i$

3 step, from $[1\ 1\ 1]^T$

	X	λ	ω
1	$[.643\ .93\ .57]^T$	.73	1.17
2	$[.46\ .73\ .41]^T$	.76	1.15
3	$[.34\ .56\ .31]^T$	.76	1.15

Using subspace iteration, the process is the same because the reduced eigenvalue problem is a scalar problem (see notes). The rest of SSI is just Green's method.

3) $T = \frac{1}{2}\int_0^{\ell} m(x)\dot{w}^2\, dx$

compressive load

$V = \frac{1}{2}\int_0^{\ell} EI(x)\, w''^2\, dx + \int_0^{\ell} P(ds - dx)$

See helicopter problem in notes for details

$$\frac{\partial^2}{\partial x^2}\left(EI \frac{\partial^2}{\partial x^2} w(x,t)\right) + P\frac{\partial^2 w(x,t)}{\partial x^2} + m(x)\ddot{w} = 0$$

At each end

$\frac{\partial}{\partial x} EI w''$ is known OR w is known

AND

$EI \frac{\partial^2 w}{\partial x^2}$ is known OR w' is known

This end load will cause buckling if great enough.
See your strength of materials text.

d)

$$T = \int_{t_1}^{t_2} \left(\frac{1}{2} \int_0^l \rho A \dot{v}^2 \, dx + \frac{1}{2} \int_0^l \rho J_p \dot{\theta}^2 \, dx \right) + \frac{1}{2} m (\dot{v} + r\dot{\theta})^2 \Big|_{x=L} dt$$

$$\delta T = \int_{t_1}^{t_2} -\int_0^l \rho A \ddot{v} \, \delta v \, dx - \int_0^l \rho I_p \ddot{\theta} \, \delta\theta \, dx + \left[m(\dot{v} + r\dot{\theta}) \delta \dot{v} + m r (\dot{v} + r\dot{\theta}) \delta\dot{\theta} \right]_{x=L} dt$$

$$= \int_{t_1}^{t_2} \int_0^l -\rho A \ddot{v} \, \delta v - \rho I_p \ddot{\theta} \, \delta\theta \, dx \left[- m(\ddot{v} + r\ddot{\theta}) \delta v - m r (\ddot{v} + r\ddot{\theta}) \delta\theta \right]_{x=L} dt$$

$$V = \int_0^l \frac{1}{2} G J \left(\frac{\partial \theta}{\partial x} \right)^2 dx + \int_0^l \frac{1}{2} E I \left(\frac{\partial^2 v}{\partial x^2} \right)^2 dx$$

$$\delta V = \int_0^l G J \theta' \, \delta\theta' \, dx + \int_0^l E I v'' \, \delta v'' \, dx$$

$$= G J \theta' \delta\theta \Big|_0^l - \int_0^l G J \theta'' \delta\theta \, dx + E I v'' \delta v' \Big|_0^l - E I v''' \delta v \Big|_0^l + \int_0^l E I v'''' \delta v \, dx$$

$$\int_{t_1}^{t_2} \delta T - \delta V \, dt = 0$$

$$\therefore \quad \rho A \ddot{v} + E I v'''' = 0, \quad \rho J_p \ddot{\theta} - G J \theta'' = 0$$

@ $x = 0$, $\theta = 0$, $v = 0$, $\frac{\partial v}{\partial x} = 0$

@ $x = L$ $\quad m(\ddot{v} + r\ddot{\theta}) + E I v''' = 0$, $v'' = 0$

$m r(\ddot{v} + r\ddot{\theta}) - G J \theta' = 0$