# ME 710 Sp '10 Final Exam Solutions

1) The motion must be prescribed by
$\vec{\theta} = \theta_x \hat{i} + \theta_y \hat{j} + \theta_z \hat{k}$.

The displacement must then be

$$\vec{d} = \vec{\theta} \times \vec{r}$$

$$= (\theta_x \hat{i} + \theta_y \hat{j} + \theta_z \hat{k}) \times (x\hat{i} + y\hat{j} + z\hat{k})$$

$$= \theta_x y \hat{k} - \theta_x z \hat{j} - \theta_y x \hat{k} + \theta_y z \hat{i} + \theta_z x \hat{j} - \theta_z y \hat{i}$$

$$d_x \hat{i} + d_y \hat{j} + d_z \hat{k} = (\theta_y z - \theta_z y)\hat{i} + (\theta_z x - \theta_x z)\hat{j} + (\theta_x y - \theta_y x)\hat{k}$$

Thus there are 3 constraint equations

$$d_x = (\theta_y z - \theta_z y), \quad d_y = (\theta_z x - \theta_x z), \quad d_z = (\theta_x y - \theta_y x)$$

$$\begin{bmatrix} 0 & z & -y & -1 & 0 & 0 \\ -z & 0 & x & 0 & -1 & 0 \\ y & -x & 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \theta_x \\ \theta_y \\ \theta_z \\ d_x \\ d_y \\ d_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

2) $V_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$V_2 = \begin{bmatrix} 0.707 \\ -0.707 \\ 0 \end{bmatrix}$

$V_3 = \begin{bmatrix} 0.667 \\ -0.667 \\ 0.333 \end{bmatrix}$

$V_4 = \begin{bmatrix} 0.566 \\ -0.707 \\ 0.424 \end{bmatrix}$

$V_5 = \begin{bmatrix} 0.529 \\ -0.706 \\ 0.471 \end{bmatrix}$

$V_6 = \begin{bmatrix} 0.512 \\ -0.707 \\ 0.488 \end{bmatrix}$

$V_7 = \begin{bmatrix} 0.505 \\ -0.707 \\ 0.495 \end{bmatrix}$

$V_8 = \begin{bmatrix} 0.502 \\ -0.707 \\ 0.498 \end{bmatrix}$

$V_9 = \begin{bmatrix} 0.501 \\ -0.707 \\ 0.499 \end{bmatrix}$

$V_{10} = \begin{pmatrix} 0.500 \\ -0.707 \\ 0.500 \end{pmatrix}$

$\lambda = \underline{2.414}$

Matlab (not even code)

```
v = [1; 0; 0]
K=[1 -1 0; -1 1 -1; 0 -1 1]
V= K*v; norm(v), v= v/norm(v)
```

Hit up arrow (↑) to repeat the last line until it converges.

```
L=1
K=zeros(3,3)

locs=[-sqrt(.6) 0 sqrt(.6)]
weights=[5 8 5]/9
for i=1:3
xi=locs(i)
w=weights(i)

J=L/2*(1-xi)
B=[(xi-.5) -2*xi (xi+.5)]/(L/2*(1-xi))
K=K+B'*B*J*w
end
```

```
L =  1
K =

   0   0   0
   0   0   0
   0   0   0

locs =

  -0.77460   0.00000   0.77460

weights =

   0.55556   0.88889   0.55556

xi = -0.77460
w =  0.55556
J =  0.88730
B =

  -1.43649   1.74597  -0.30948

K =

   1.017193  -1.236335   0.219142
  -1.236335   1.502689  -0.266354
   0.219142  -0.266354   0.047212

xi = 0
w =  0.88889
J =  0.50000
B =

  -1  -0   1

K =

   1.46164  -1.23634  -0.22530
  -1.23634   1.50269  -0.26635
  -0.22530  -0.26635   0.49166
```

```
xi =  0.77460
w =  0.55556
J =  0.11270
B =

    2.4365  -13.7460   11.3095

K =

    1.8333   -3.3333    1.5000
   -3.3333   13.3333  -10.0000
    1.5000  -10.0000    8.5000
```

3) See mid term

4) See mid term, but $(\xi, \eta) = (1, 1)$

$$J = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

$$\begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \end{bmatrix} = J^{-1} \begin{bmatrix} \frac{\partial u}{\partial \xi} \\ \frac{\partial u}{\partial \eta} \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \frac{1}{4}(1+\eta) \\ \frac{1}{4}(1+\xi) \end{bmatrix} d_1$$

$$= \begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 0 \end{bmatrix} d_1$$

$$\begin{bmatrix} \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial y} \end{bmatrix} = J^{-1} \begin{bmatrix} \frac{\partial v}{\partial \xi} \\ \frac{\partial v}{\partial \eta} \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} d_2 = \begin{bmatrix} -1 \\ 0 \end{bmatrix} d_2$$

$\epsilon_x = -d_1$

$\epsilon_y = 0$

$\gamma_{xy} = -d_2$