Wi '09 Final Exam Solutions

1) $t_s \approx 3 \text{ sec} = \frac{3}{\zeta\omega_n}$

① $\zeta\omega_n = 1$

$f_{os} \approx 72\% = 0.72 = e^{-\zeta\omega_n \frac{\pi}{\omega_d}}$

$= 0.72 = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$

For ζ small

$\ln 0.72 = -\pi\zeta$

$\boxed{\zeta = 0.1}$

check $e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}\Big|_{\zeta=0.1} = 0.73$, within engineering accuracy

Since $\zeta\omega_n = 1$, $\boxed{\omega_n \approx 10 \text{ rad/s}}$

From steady state response, $\frac{1}{K} = 0.01$, $\boxed{K = 100}$

Since the homogeneous equation can be written

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0$$

or $\quad m\ddot{x} + c\dot{x} + Kx = 0$

$K = m\omega_n^2$, thus $\boxed{m = 1}$

$c = 2\zeta\omega_n m = 2 \cdot 0.1 \cdot 10 \cdot 1 = \boxed{2}$

EOM is

$$\ddot{x} + 2\dot{x} + 100\,\omega_n^2 = u(t)$$

$$\boxed{m = 1,\ c = 2 \quad K = 100}$$

(No units given)

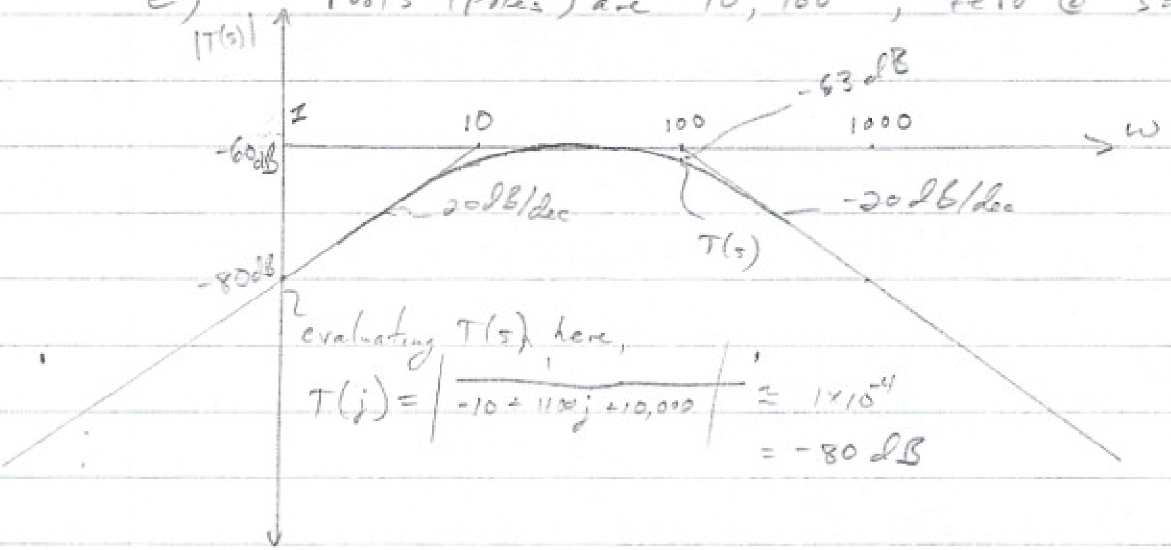
2) a) $\dfrac{X(s)}{F(s)} = \dfrac{5}{10s^2 + 1100s + 10000}$

b) 7×10^{-4}

c) 39° (0.68 rad)

d) $0.014 \sin(10t - 0.1)$

e) roots (poles) are 10, 100, zero @ $s = 0$



evaluating T(s) here,

$T(j) = \left| \dfrac{1}{-10 + 1100j + 10{,}000} \right| \approx 1 \times 10^{-4}$

$= -80$ dB

3) For tank 1 For R_1

$$q_{mi} - q_1 = \rho A_1 \dot{h}_1 \quad ① \qquad \rho g h_1 = R_1 q_1 \quad ③$$

For tank 2

$$q_1 - q_2 = \rho A_2 \dot{h}_2 \quad ② \qquad \rho g h_2 = R_2 q_2 \quad ④$$

also q_{mo}

Subst ③ and ④ into ① and ②

$$⑤ \quad q_{mi} - \frac{\rho g h_1}{R_1} = \rho A_1 \dot{h}_1 \; , \quad ⑥ \quad \frac{\rho g h_1}{R_1} - \frac{\rho g h_2}{R_2} = \rho A_2 \dot{h}_2$$

In Laplace

$$Q_{mi}(s) - \frac{\rho g}{R_1} H_1(s) = \rho A_1 s H_1(s) \quad ⑦$$

$$\frac{\rho g}{R_1} H_1(s) - \frac{\rho g}{R_2} H_2(s) = \rho A_2 s H_2(s) \quad ⑧$$

sub ⑧ into ⑦ for $H_1(s)$

$$Q_{mi}(s) = \left(\frac{\rho g}{R_1} + \rho A_1 s\right)\left(\frac{R_1}{\rho g}\right)\left(\frac{\rho g}{R_2} + \rho A_2 s\right) H_2(s)$$

$$\boxed{\frac{H_2(s)}{Q_{mi}(s)} = \frac{1}{\left(1 + \frac{A_1 s R_1}{\rho g}\right)\left(\frac{\rho g}{R_2} + \rho A_2 s\right)}}$$

b) State space

From ⑤ and ⑥

$$\begin{bmatrix} \dot{h}_1 \\ \dot{h}_2 \end{bmatrix} = \begin{bmatrix} \frac{-g}{R_1 A_1} & 0 \\ \frac{g}{R_1 A_2} & \frac{-g}{R_2 A_2} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{\rho A_1} \\ 0 \end{bmatrix} q_{mi}$$

$$\begin{bmatrix} h_1 \\ h_2 \\ q_{mo} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & \frac{\rho g}{R_2} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} \\ \\ \\ \end{bmatrix} q_{mi}$$