SP '07 T1 Solutions
ME 360

Note: Students had Matlab

1)

x_i Time	Temp	y_i $\ln \Delta T$
0	207	4.92
5	182	4.71
10	167	4.58
15	155	4.44
20	143	4.29
25	135	4.17
30	128	4.06
35	123	3.97
40	118	3.87
45	114	3.78
50	109	3.66

$\Delta T = A e^{bt}$

$\ln \Delta T = \ln A + bt$ (intercept, slope)

$b \sum x_i^2 + \ln A \sum x_i = \sum x_i y_i$

$b \sum x_i + 11 \ln A = \sum y_i$

$\sum x_i = 275 \qquad \sum y_i = 46.47$

$\sum x_i^2 = 9625 \qquad \sum x_i y_i = 1095$

$9625\, b + 275 \ln A = 1095$

$275\, b + 11 \ln A = 46.47$

$b = -2.43 \times 10^{-2}$, $\ln A = 4.83$ $\qquad A = 125$

$\Delta T = 125\, e^{-0.024\, t}$

2) $U = \frac{1}{2} K (2x)^2 = \frac{1}{2} (4K) x^2$

$T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} J \dot{\theta}^2$

For uniform disk, $J = \frac{1}{2} m R^2$

$\dot{\theta} = \frac{\dot{x}}{R}$

$T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} \frac{1}{2} m R^2 \left(\frac{\dot{x}}{R}\right)^2$

$= \frac{1}{2} m \dot{x}^2 + \frac{1}{2} \left(\frac{1}{2} m\right) (\dot{x})^2$

$= \frac{1}{2} \left(\frac{3}{2} m\right) \dot{x}^2$

$\omega_n = \sqrt{\frac{K_{eff}}{m_{eff}}} = \sqrt{\frac{4K}{\frac{3}{2} m}} = \sqrt{\frac{8K}{3m}}$

3) For the 1st inertia

$\sum M = I\alpha \quad T_1 - T_{SG} = I_1 \ddot{\theta}_1$ ①

Define $N = \frac{\omega_1}{\omega_2}$

T_{SG} is torque on small gear

$T_{SG} = \frac{1}{N} T_{LG}$ ② through gears

T_{LG} is " large gear.

$T_{LG} = (\theta_2 - \theta_3) K_T$ ③

$\sum M = I\alpha$

$T_{LG} - C_T \dot{\theta}_3 = I_2 \ddot{\theta}_3$

$(\theta_2 - \theta_3) K_T - C_T \dot{\theta}_3 = I_2 \ddot{\theta}_3$ ④

Sub ② into ①

$$T_1 - \frac{1}{N}(\theta_2 - \theta_3) K_T = I_1 \ddot{\theta}_1$$

Since $\theta_2 = \frac{\theta_1}{N}$

$$T_1 - \frac{1}{N}\left(\frac{\theta_1}{N} - \theta_3\right) K_T = I_1 \ddot{\theta}_1$$

$$I_1 \ddot{\theta}_1 + \frac{1}{N^2}\theta_1 K_T - \frac{1}{N}\theta_3 = T_1 \quad \text{first eqn}$$

Sub for θ_2 in ④

$$\left(\frac{\theta_1}{N} - \theta_3\right) K_T - C_T \dot{\theta}_3 = I_2 \ddot{\theta}_3$$

$$I_2 \ddot{\theta}_3 + C_T \dot{\theta}_3 + K_T \theta_3 - \frac{K_T}{N}\theta_1 = 0 \quad \text{2nd eqn}$$

4) a) 2, 3

b) 1, 2