ME 712, Finite Element Method Applications Midterm Exam, Spring 2008

One formula sheet, closed notes, open book. Test books will be provided. 1 hour, 15 min. *Problems must be done in order in the test books.* 10 points each.

1. Estimate the following integral using 1, 2 and 3 Gauss point integrations.

$$I = \int_{-1}^{1}\int_{-1}^{1} \cos(\frac{\pi\xi}{2})\cos(\frac{\pi\eta}{2}) d\xi d\eta$$

2. Determine the 1,4 element of the stiffness matrix for a beam in local coordinates (the beam being between $x = 0$ and $x = l$) presuming E is constant, but $I(x) = I_1 + (I_2 - I_1)\frac{x}{l}$. *Set up all math in matrix form without multiplying out the matrices, but solve only for* K_{14}.

$$N = \begin{bmatrix} 1 - 3\frac{x^2}{l^2} + 2\frac{x^3}{l^3} & x - 2\frac{x^2}{l} + \frac{x^3}{l^2} & 3\frac{x^2}{l^2} - 2\frac{x^3}{l^3} & -\frac{x^2}{l} + \frac{x^3}{l^2} \end{bmatrix}$$

3. See Figure 2.5-4 on page 67. For the frame shown, write equilibrium equations $[\mathbf{K}]\{\mathbf{D}\} = \{\mathbf{R}\}$ using DOF
$\{\mathbf{D}\} = [u_1 \quad v_1 \quad \theta_{z1} \quad \theta_{z2}]^T$. Both members are slender and have the same E, I, A, and L.

4. Consider a 3-noded beam element. Determine the shape function corresponding to the middle degree of freedom in **natural coordinates**.