ME 712, Finite Element Method Applications Final Exam, Spring 2004

Closed book, closed notes. One formula sheet to be turned in. Problems must be in order in the blue books.

1. Itemize every *stage* from an actual mechanical part to the actual stress prediction using finite elements where errors are incurred, slight or not. One or two sentences/statements for each assumption in the process, maximum! *I am looking for* ***generic*** *statements, not a list of all possible specific cases where an error could occur in the modeling process.* **Nothing should be specific to a case or an element type!**

2. Obtain the stiffness matrix of a rod (extension: 1-D) using 1 quadratic element with a mid-node located at $L/4$. Use Gauss integration to derive the element matrices. Assume a length l, density ρ, cross sectional area of A, and a modulus of E. Print out any code that you may write to solve this problem.

3. A single standard Euler-Bernoulli beam has constraints of $v_1 = 0$, $v_2 = \theta_1 L$, and $\theta_2 = \theta_1$. Generate the reduced governing equations using Lagrange multipliers.

$$K = \frac{EI_z}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \tag{1}$$

,

$$M = \frac{m}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^3 & -22L & 4L \end{bmatrix} \tag{2}$$

Hint: Switch to using coordinates of θL in place of θ by appropriate 'transformation'.

4. Some of the following matrices have fundamental flaws that violate certain conditions. For each matrix, identify whether the matrix is flawed, and what the flaw/s is/are.

   (a)
$$K = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

   (b)
$$M = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

   (c)
$$K = \begin{bmatrix} 24 & -12 & 0 & 6 \\ -12 & 12 & -6 & -6 \\ 0 & -6 & 2 & 4 \\ 6 & -6 & 4 & 4 \end{bmatrix}$$

(d)

$$M = \begin{bmatrix} 24 & -12 & 0 & -6 \\ -12 & 12 & -6 & -6 \\ 6 & -6 & 12 & 4 \\ -6 & -6 & 4 & 12 \end{bmatrix}$$

5. Use area coordinates to determine the first moment of area of a triangular element about the x axis in terms of A and the nodal coordinates, i.e.

$$Q_x = \int_A y \; dA \tag{3}$$

given

$$\int_A \xi_1^k \xi_2^l \xi_3^m = 2A \frac{k! \; l! \; m!}{(2+k+l+m)!} \tag{4}$$

6. Find the strain at (x, y)=(0,0) of a bilinear quadrilateral (Q4) element with nodes 1-4 at (0,0), (1,0), (1,2), and (0,2) in terms of u_3 and v_3 (presume all other nodal displacements are zero).