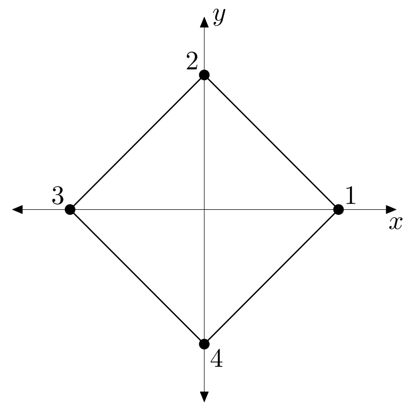
One formula sheet, closed notes, closed book. Test books will be provided. 1 hour, 15 min. *Problems must be done in order in the test books.* 10 points each.

1. Find the strain at (ξ, η)=(-1,-1) of a bilinear quadrilateral (Q4) element with nodes 1-4 at (1,0), (0,1), (-1,0), and (0,-1) in terms of u_3 and v_3 (presume all other nodal displacements are zero).



2. Write the stiffness matrix of a beam in 3D (no more than 4 rows and 4 columns should be non-zero) where the beam points in the x direction and deflects in the y direction (given below "simply"). Consider these to be local coordinates. Write the coordinate transformation matrix to rotate the matrix to the new coordinates if the local x points in the global Y direction, the local y points in the global Z direction, and the local z axis points in the global X direction. Write the transformed stiffness matrix (by any means of your choice).

$$K = \frac{EI_x}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

3. Using 2 point Gauss integration, determine and calculate the percent error of

$$\int_0^\pi \sin\theta d\theta$$

Table 1: Approximate Gauss point integration values

# points	ξ_i	w_i
1	0	2
2	$\pm\frac{1}{\sqrt{3}}$	1
3	0, $\pm\sqrt{0.6}$	$\frac{8}{9}$, $\frac{5}{9}$
4	-0.86, -0.34, 0.34, 0.86	0.35, 0.65, 0.65, 0.35