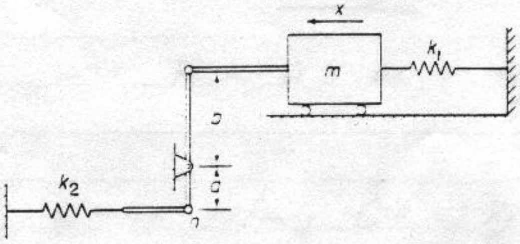
ME 460/660 Exam 1, Spring ‘9~~5~~6

1) An unknown mass (m kg) is attached to an unknown spring (k N/m). The system has a natural frequency of 5 rad/sec. After a 1 kg mass is added, the natural frequency is identified to be 4 rad/sec. Determine the unknown mass m and stiffness k.

2) A single degree of freedom (SDOF) system with a mass of 2 kg, a stiffness of 10 N/m, and a damping factor of 1 kg-s has initial conditions x_0=0 m, and v_0=.01 m. Find the natural frequency, damping ratio, damped natural frequency, and the free response.

3) Derive the equation of motion for the following system.



4) Design the suspension system for an 1000 kg automobile (choose the stiffness and damping value) subject to the following constraints: four wheels (four identical springs/dashpots), a maximum additional static displacement of 1 cm for each additional 80 kg passenger entering the car, and a displacement transmissibility of less than 2.

Vibrations Exam 1, Fa 1995 Solutions

1) $\omega_1 = 5 \text{ rad/s} = \sqrt{\frac{k}{m}}$

$\omega_2 = 4 \text{ rad/s} = \sqrt{\frac{k}{m+1}}$

$25m = k$
$16(m+1) = k$

$25m - 16m - 16 = 0$
$9m = 16$
$m = \underline{\frac{16}{9} \text{ kg}} = 1.78 \text{ kg}$

$k = 25m = \underline{44.4 \text{ N/m}}$

2) $m = 2 \text{ kg}, \; k = 10 \text{ N/m}, \; c = 1 \text{ kg/s}$

$\omega = \sqrt{\frac{k}{m}} = \sqrt{5} = \underline{2.236 \text{ rad/s}}$ *

$\zeta = \frac{c}{2m\omega} = \frac{1}{4\sqrt{5}} = \underline{.112}$ *

$\omega_d = \omega\sqrt{1-\zeta^2} = \underline{2.22 \text{ rad/s}}$ *

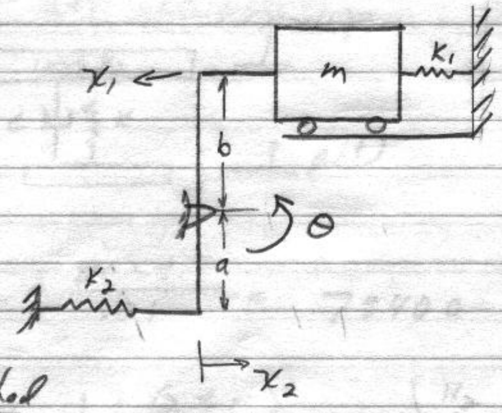
$x(t) = A e^{-\zeta\omega t} \sin(\omega_d t + \phi)$

where ζ, ω, and ω_d are given above and

$A = \frac{v_0}{\omega_d} = .0045 \text{ m}, \quad \phi = 0^\circ$

$x(t) = .0045 e^{-.250t} \sin 2.22t$ (m)

3)



Energy Method

$$U = \frac{1}{2} k_2 x_2^2 + \frac{1}{2} k_1 x_1^2$$ 12

$$T = \frac{1}{2} m \dot{x}_1^2$$ 7

$$x_1 = b\,\theta, \quad x_2 = a\,\theta$$

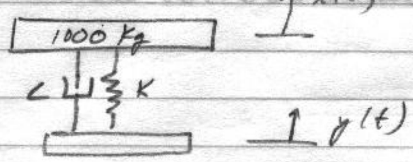
$$T + U = \frac{1}{2} k_2 a^2 \theta^2 + \frac{1}{2} k_1 b^2 \theta^2 + \frac{1}{2} m b^2 \dot{\theta}^2$$

$$\frac{d(T+U)}{dt} = 0 = k_2 a^2 \theta \dot{\theta} + k_1 b^2 \theta \dot{\theta} + m b^2 \dot{\theta} \ddot{\theta}$$ 6

Factoring out $\dot{\theta}$

$$m b^2 \ddot{\theta} + (k_2 a^2 + k_1 b^2)\theta = 0 \quad *$$

4)



$k = \frac{80 \cdot 9.8}{.01} = 78400\ N/m$, $\omega = \sqrt{\frac{k}{m}} = 8.8\ rad/s$

+15

$\frac{X}{Y} = \left(\frac{1 + (2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2} \right)^{1/2} < 2$ for all r

The peak will be very near $r = 1$.

$$\frac{1 + (2\zeta)^2}{(2\zeta)^2} < 4$$

$$1 + (2\zeta)^2 < 4(2\zeta)^2$$

$$1 < 3(2\zeta)^2$$

$$\sqrt{\frac{1}{12}} < \zeta$$

$$\zeta > .289$$

For $\zeta \approx .3$ the curve is fairly flat. Choosing $\underline{\zeta = .31}$ should compensate adequately for this. Increasing ζ too much will raise the transmissibility too much, destroying the benefits of isolation. *

$$C = 2\zeta\omega m = 5489\ kg/s$$

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4 springs, each $k = \frac{78400}{4} = 19600\ N/m$

4 dashpots, each $c = \frac{5489}{4} = 1372\ kg/s$