ME 460/660, Mechanical Vibration — Exam 2, Spring 2011

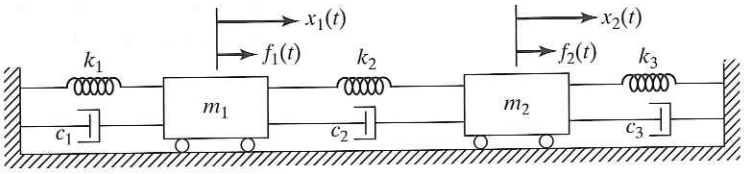
Closed book, closed notes. Use $8\frac{1}{2} \times 11$ formula sheet from web and turn in with exam (nothing else may be written on the formula sheet). Test books will be provided. Calculators allowed. Knowing how to use them well is highly recommended.

Problems are 10 points each. Problem 4 is required for graduate students, bonus for undergraduates (worth 6 points).

1. A system governed by the differential equation $10\ddot{x} + 0.1\dot{x} + 25000x = f(t)$ is excited by the force $f(t) = \sum_{n=1}^{\infty} a_n \cos\frac{2\pi n}{T}t$ where $a_n = \begin{cases} 0 & \text{n odd} \\ \frac{1}{n} & \text{n even} \end{cases}$ and $T = \frac{2\pi}{10}$. Write the 5-term (5 non-zero terms!) approximation to $x(t)$. What is the best representation of $x(t)$ using a single harmonic (not a summation)?

2. A system defined by $10\ddot{x} + 25000x = f(t)$ is excited by $f(t) = \begin{cases} 1, & 0 < t < T \\ 0, & T < t \end{cases}$. Find T so that $x(t) = 0$ for all $T < t$.

3. Derive the governing equations for the following:



4. *Grad student/bonus* (20% of other points) Determine the first natural frequency and mode shape for a clamped-clamped beam. The equation of motion of a beam is $\left(\frac{EI}{\rho A}\right)\frac{\partial^4 w(x,t)}{\partial x^4} + \frac{\partial^2 w(x,t)}{\partial t^2} = 0$. You DO NOT have to obtain all constants that can be obtained.