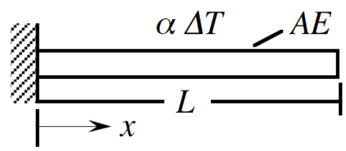
Final Exam
ME 412/612 Finite Element Analysis
Winter, 2005

Instructions: This exam consists of three problems worth a total of 60 points. All students will be graded out of 50, so that problem #3 (worth 10 points) can be considered extra credit. The only materials permitted are a calculator and a total of THREE (3) 8 1/2 x 11" crib sheets, which MUST be turned in with the exam.

1. Consider a fixed-free rod of axial rigidity AE subjected to a thermal expansion $\alpha\Delta T$:



The governing differential equation and boundary conditions for the axial displacement $u(x)$ can be written as

$$\begin{gathered}\frac{d}{dx}\left(AE\frac{du}{dx}\right)=0,\ 0 \le x \le L \\ u(0)=0,\ AE\frac{du}{dx}\bigg|_{x=L}=AE\alpha\Delta T\end{gathered}. \tag{1}$$

Note that the governing equation satisfies the 1-D model problem, and that the thermal effects are incorporated only through the derivative boundary condition at $x=L$.

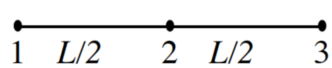
Also, the total potential energy functional for a rod under thermal loading is given by

$$\pi(u)=\int_0^L\left[\frac{AE}{2}\left(\frac{du}{dx}-\alpha\Delta T\right)^2\right]dx. \tag{2}$$

Determine the displacement at $x=L/2$ by each of the following methods:

a) The Rayleigh-Ritz Method. Assume an approximate solution of the form $u(x)=ax$. (5 points)

b) The finite element method, using two equally spaced *1-D linear elements*. Obtain the solution using the direct stiffness method in direct analogy with the 1-D model problem (10 points):

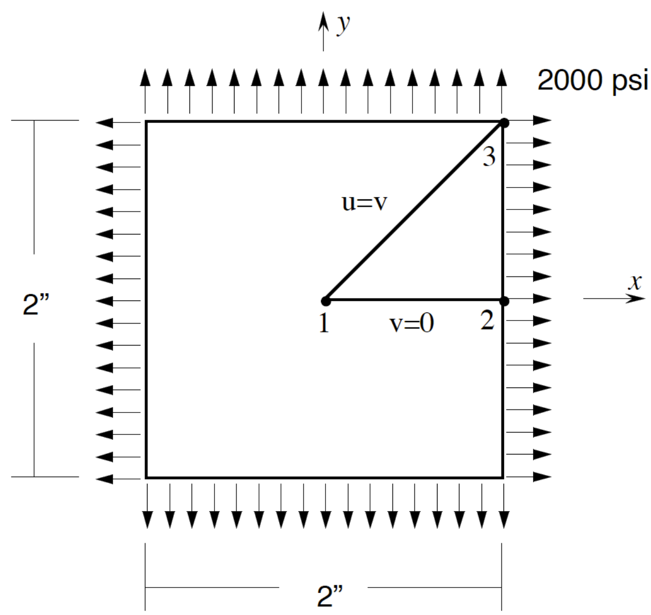


c) The finite element method, using two *axial force member* elements. (5 points)

d) How does your solution for *each* of the above methods compare to the exact solution? Why is this the case? (5 points)

<u>Total Points</u>: 25

2. Consider the plane stress elasticity problem of a 2"x2"x1" plate under a uniform biaxial tension of 2000 psi. The plate is made of a unique material having an elastic modulus of E=20x10^6 psi and a Poisson's ratio of ν=0. Obtain the finite element solution using a 1/8 model and a single linear triangular element. As indicated in the figure, the appropriate symmetry conditions for the displacements are as follows: zero y displacement along side 1-2, and equal x and y displacements along side 1-3.



a) Write down the 6x6 system of equations for the nodal displacements. Note that the symmetry conditions will result in nonzero reaction forces in both directions at nodes 1 and 3, and in the y direction at node 2. (10 points)

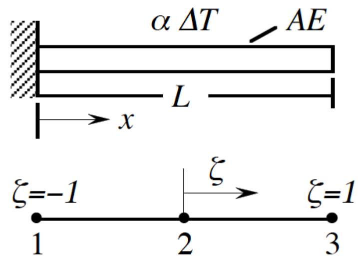
b) Incorporate the boundary conditions and determine the unknown displacements. Note that the symmetry condition requires the x and y reaction forces at node 3 to be equal and opposite, i.e., P_6=-P_5. (5 points)

c) Determine the element stresses and compare them to the exact solution. Are your results expected? (5 points)

d) Given the specified material properties, how would the plane strain solution differ from the plane stress solution obtained in parts b) and c)? (5 points)

<u>Total Points</u>: 25

3. Consider the solution to problem #1 using a single 1-D *quadratic* element:



Derive the 3x3 element stiffness matrix and write down the system of equations for the nodal displacements. <u>Hint</u>: In the natural coordinate system, the element stiffness matrix is given by

$$K_{ij}^{e} = \frac{2AE}{L}\int_{-1}^{1} \frac{dN_i}{d\zeta}\frac{dN_j}{d\zeta}\,d\zeta, \text{ for } i,j=1,2,3.$$

Is the solution to your system of equations exact? Explain why or why not.

<u>Total Points</u>: 10