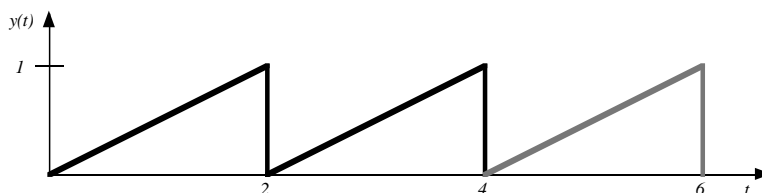


Closed book, closed notes. Use one  $8\frac{1}{2} \times 11$  formula sheet (brought by student) and turn in with exam. Test books will be provided. All problems are to be done in the test book.

1. A linear system is governed by the following equation of motion:

$$m\ddot{x} + c\dot{x} + kx = ky$$

where  $m = 10\text{kg}$ ,  $c = 1.0\text{kg/s}$ , and  $k = 10,000\text{kg/s}^2$ . Given  $y(t)$  as



where  $y(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{-1}{n\pi} \sin \pi n t$ , find the first 4 terms of  $x(t)$ .

2. An undamped system is excited by a pulse (force of amplitude  $F$ ) of finite (**but not infinitesimal!**) duration. Find  $x(t)$  during and after the pulse presuming  $10\ddot{x} + 8.8826 \times 10^4 x = f(t)$  and the pulse lasts for  $\frac{1}{30}$  s.
3. A MDOF system is released from rest has modes of

$$S = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

no damping, and natural frequencies of 3 rad/sec and 10 rad/sec. It has initial conditions of

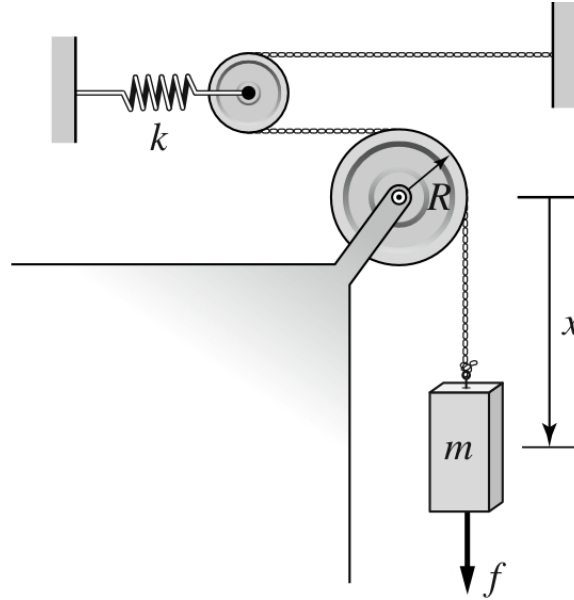
$$\mathbf{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

and

$$\mathbf{v}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Find  $\mathbf{x}(t)$ .

4. Obtain the equations of motion the following system. Include the effect of gravity. Presume:
- (a) the pulley at the end of the spring is massless
  - (b) the pulley with the radius  $R$  has a uniform mass distribution.



5. Graduate Students/Undergraduate Bonus (20% of other points): Solve for the steady-state (particular) response of the following system if the boundary conditions are presumed to be fixed-fixed ( $0 < x < l$ ) where  $c = \sqrt{\tau/\rho}$ .

$$w_{tt}(x, t) - c^2 w_{xx}(x, t) = 100\delta(x - \frac{l}{3})\delta(t)$$

Recall that the integral of a Dirac delta function times another function is equal to the “another function” evaluated when the argument of the Dirac delta function is zero.

BONUS (3 point, all or nothing): List the mass moments of inertia of

1. a uniform disk
2. a hoop
3. a rod about its center
4. a rod about its end