ME3210 Fall 2013 Exam 1 Solutions

1)

F	X	Δy
0	4.7	0
0.47	7.2	2.5
1.15	10.6	5.9
1.64	12.9	8.2

$$f = m X$$

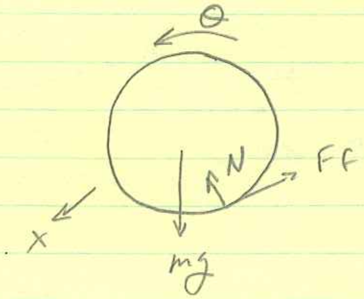
$$m = \frac{\Sigma y_i}{\Sigma x_i} \quad (\text{for } b = 0) \qquad \text{eqn } (C.1.2)$$

In our case, $y_i = F_i$

$\Sigma y_i = 3.26 \qquad \Sigma x_i = 16.6$

$m = \frac{3.26}{16.6} = 0.196$ which rounds to
0.20 lbs/inch

2) a)



$\Sigma F_y = 0 = N - mg\cos\theta$, ① $N = mg\cos\theta$

$\Sigma F_x = m\ddot{x} = -F_f + mg\sin\theta$ ② $F_f = -m\ddot{x} + mg\sin\theta$

$\Sigma M = I\ddot{\theta} = \frac{1}{2}mr^2\ddot{\theta} = +rF_f$ ③

sub ② into ③

$$\frac{1}{2}mr^2\ddot{\theta} = r(-m\ddot{x} + mg\sin\theta)$$

Since $\ddot{x} = r\ddot{\theta}$

$$\frac{1}{2}mr\ddot{x} = -rm\ddot{x} + rmg\sin\theta$$

$$\frac{3}{2}\ddot{x} = g\sin\theta$$

$$\ddot{x} = \frac{2}{3}g\sin\theta$$

Must always check $F_f < \mu_s N$

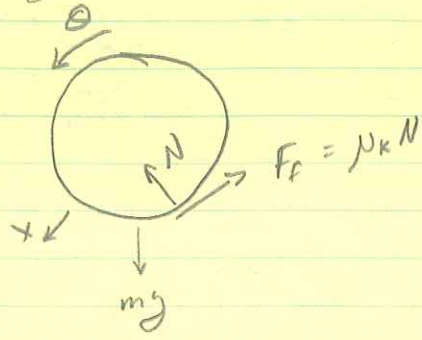
$$F_f = -m\frac{2}{3}g\sin\theta + mg\sin\theta$$

$$= \frac{1}{3}mg\sin\theta$$

$$\frac{1}{2}mg\sin\theta \overset{?}{<} \mu_s mg\cos\theta$$

$$\frac{1}{2}\tan\theta \overset{?}{<} \mu_s$$ must be satisfied for rolling without slipping

2b) Rolling with slipping



In this case, x and θ are independent.

$\Sigma F_y = 0 = N - mg\cos\theta \qquad N = mg\cos\theta$

$\Sigma F_x = m\ddot{x} = -F_f + mg\sin\theta$
$= -\mu_k mg\cos\theta + mg\sin\theta$

$\ddot{x} = g(\sin\theta - \mu_k\cos\theta)$

$\Sigma M = \frac{1}{2}mr^2\ddot{\theta} = rF_f$
$= r\mu_k mg\cos\theta$

$\ddot{\theta} = \frac{2g\mu_k}{r}\cos\theta$

For no friction, there is no rotation. This makes sense

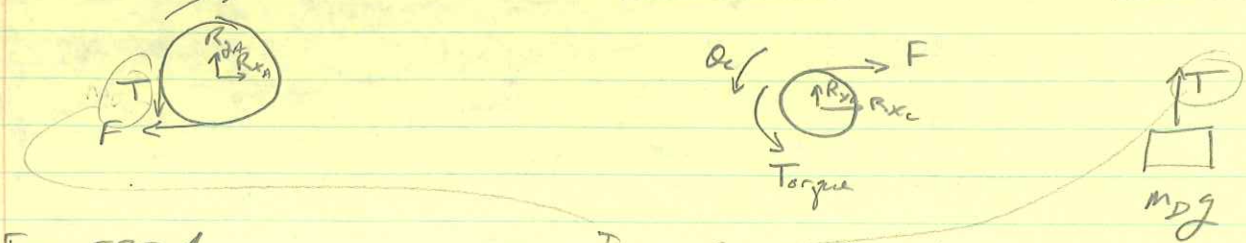
For this scenario, $\dot{x} > r\dot{\theta}$ to be rolling with slipping
Presuming start from rest, this must hold for $\ddot{x} > r\ddot{\theta}$

$g(\sin\theta - \mu_k\cos\theta) \overset{?}{>} 2g\mu_k\cos\theta$

$\sin\theta > 3\mu_k\cos\theta$

Condition is $\mu_k < \frac{\sin\theta}{3\cos\theta}$

3) FBD A: FBD C: FBD D



From FBD A

I used T for tension

$$\sum M = I\alpha$$

$$-T \cdot \underbrace{0.075}_{\text{drum radius}} + F \cdot \underbrace{0.15}_{\text{gear radius}} = 0.3^2 \cdot 100\,\alpha_A$$

$$-0.075\,T + 0.15\,F = 9\alpha_A \qquad (1)$$

From FBD C:

$$\sum M = I\alpha$$

$$-\underbrace{0.05}_{\text{small gear radius}} F + \text{Torque} = 0.08^2 \cdot 10\,\alpha_c$$

$$= 0.064\,\alpha_c \qquad (2)$$

$$0.15\,F = 3\,\text{Torque} - 0.192\,\alpha_c$$ ← rearranged for later

From FBD D

$$\sum F = m\ddot{x}$$

$$T - mg = 800\ddot{x}$$

$$T - 7840 = 800\ddot{x} \qquad (3)$$

$$T = 800\ddot{x} + 7840$$

Sub (3) into (1)

$$-0.075(800\ddot{x} + 7840) + 0.15\,F = 9\alpha_A \qquad (4)$$

Sub (2) into (4)

$$\underbrace{-60\ddot{x} + -588}_{\text{simplified terms}} + \underbrace{3\,\text{Torque} - 0.192\,\alpha_c}_{\text{substituted part}} = 9\alpha_A \qquad (5)$$

From Kinematics

$\alpha_A = \frac{1}{3}\alpha_C$, $\alpha_C = 3\alpha_A$

⑤ becomes (using $\alpha_C = 3\alpha_A$)

$$-60\ddot{x} - 588 + 3\,\text{Torque} = 9.576\,\alpha_A$$

Also from kinematics,

$\ddot{x} = \alpha_A \cdot 0.075$, $\alpha_A = \frac{\ddot{x}}{0.075}$

$$-60\ddot{x} - 588 + 3\,\text{Torque} = 127.7\,\ddot{x}$$

$$187.7\ddot{x} = 3\,\text{Torque} - 588$$

$$\text{Torque} = 300 + 15t$$

$$\ddot{x} = 0.2398t + 1.66$$

3) Using energy.

$$T = \frac{1}{2} I_{A_{eff}} \omega_{AB}^2 + \frac{1}{2} I_{c_{eff}} \omega_c^2 + \frac{1}{2} m_D V_D^2$$

$$I_{A_{eff}} = 100 \cdot 0.3^2 = 9 \text{ kg m}^2$$
$$I_{c_{eff}} = 10 \cdot 0.08^2 = 0.064 \text{ kg m}^2$$

$$\omega_{AB} = \frac{1}{3} \omega_c$$
$$v_D = 0.075\, \omega_{AB} = 0.075 \frac{1}{3} \omega_c = 0.025\, \omega_c$$

Substituting

$$T = \frac{1}{2}\, 9 \left(\frac{1}{3}\omega_c\right)^2 + \frac{1}{2}\, 0.064\, \omega_c^2 + \frac{1}{2}\, 800\, (0.025\, \omega_c)^2$$

$$= \frac{1}{2} \underbrace{1.564}_{I_{eff}} \omega_c^2$$

$$\Sigma M = I_{eff}\, \alpha$$

$$T - mg \cdot 0.075 \cdot \frac{1}{3} = 1.564\, \dot{\omega}_c$$
$$300 + 15t - 800 \cdot 9.8 \cdot 0.025 = 1.564 \left(\frac{\ddot{x}_D}{0.025}\right)$$

$$\ddot{x}_D = 0.240t + 1.67$$