Given's Method.

Similar to Jacobi method, but goal is to tridiagonalize the matrix, making other techniques feasible.

Rotation is not in plane of element of matrix to be annihilated.

Example

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

Annihilation of a_{13}

Rotate in $(2,3)$ plane

$$\sin \theta_1 = \frac{a_{13}}{(a_{12}^2 + a_{13}^2)^{1/2}} = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}}$$

$$\cos \theta_1 = \frac{a_{12}}{(a_{12}^2 + a_{13}^2)^{1/2}} = \frac{1}{\sqrt{2}}$$

$$R_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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$$A_1 = R_1^T A_0 R_1 = \begin{bmatrix} 1 & \sqrt{2} & 0 & 1 \\ & 4.5 & .5 & 3.53 \\ & & .5 & 1/\sqrt{2} \\ & \text{sym} & & 4 \end{bmatrix}$$

Next, Annihilate a_{14}. Rotate in 2, 4

$$\sin\theta_2 = \frac{\overset{\text{annihilation term}}{a_{14}}}{\sqrt{a_{14}^2 + a_{12}^2}} = .577$$

$$\cos\theta_2 = \frac{a_{12}}{\sqrt{a_{14}^2 + a_{12}^2}} = .816$$

$$R_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_2 & 0 & -\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & \sin\theta_2 & 0 & \cos\theta_2 \end{bmatrix}$$

$$A_2 = R_2^T A_1 R_2 = \begin{bmatrix} 1 & 1.73 & 0 & 0 \\ & 7.66 & .816 & .942 \\ & & .5 & .289 \\ & \cancel{\text{sym}} & & .833 \end{bmatrix}$$

Next, Annihilate a_{24}, Rotate in 3, 4.

$$\sin\theta_3 = \frac{a_{24}}{\sqrt{a_{24}^2 + a_{23}^2}} = .755$$

$$\cos\theta_3 = \frac{a_{23}}{\sqrt{a_{24}^2 + a_{23}^2}} = .654$$

$$R_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\theta & -\sin\theta \\ 0 & 0 & \sin\theta & \cos\theta \end{bmatrix}$$

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$$A_3 = R_3^T A_2 R_3 = \begin{bmatrix} 1 & 1.73 & 0 & 0 \\ & 7.66 & 1.25 & 0 \\ & \text{sym} & .976 & .124 \\ & & & .357 \end{bmatrix}$$

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