ME 468 Fall 2007

1) $1000\ddot{x} + 81000x = \sum_{n=1}^{\infty} \frac{1}{n} \sin 3nt$

The particular solution is

$$X_n(t) = \frac{\frac{1}{n}}{81000 - 1000(3n)^2} \sin 3nt$$

The total solution is

$$x(t) = \sum_{n=1}^{\infty} X_n(t)$$

The most important term is $n=3$. That term will result in resonance.

2) $10\ddot{x} + 4000x = \delta(t - \frac{10}{\pi})$,
$\omega_n = 20$ rad/s
For $0 < t < \frac{10}{\pi}$

$$x(t) = A \sin(\omega t + \phi)$$
$$\dot{x}(t) = A\omega \cos(\omega t + \phi)$$

From the I.C., $\phi = 0$, $A = \frac{-.1}{20} = -.005$.

$$x(t) = -.005 \sin 20t$$

@ $t = \frac{10}{\pi}$, $\Delta v = 0.1 \left(\frac{\hat{F}}{m}\right)$

$$x(t)\Big|_{t=\frac{10}{\pi}} = -.005 \sin \frac{200}{\pi} = -3.69 \times 10^{-3}$$

$$\dot{x}(t)\Big|_{t=\frac{10}{\pi}^-} = -.005 \cdot 20 \cos\left(20 \cdot \frac{10}{\pi}\right) = -6.748 \times 10^{-2}$$

just after $t = \frac{10}{\pi}$

$$\dot{x}(t)\Big|_{t=\frac{10}{\pi}^+} = 3.252 \times 10^{-2}$$

$$x\left(\frac{10}{\pi}\right) = A \sin\left(20\frac{10}{\pi} + \phi\right) = -3.69 \times 10^{-3}$$

$$\dot{x}\left(\frac{10}{\pi}\right) = A\,20\cos\left(20\frac{10}{\pi} + \phi\right) = 3.252 \times 10^{-2}$$

3) $M = 9\ I$

$$K = \begin{bmatrix} 65 & -35 \\ -35 & 65 \end{bmatrix}$$

$$\det\left(\begin{bmatrix} 65-9\omega^2 & -35 \\ -35 & 65-9\omega^2 \end{bmatrix}\right) = 0$$

$\omega_1 = 1.826$ rad/s,

$\omega_2 = 3.333$ rad/s

$$\underline{u}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \underline{u}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

For mass normalized
mass normalized $\underline{u}_1$

$$\alpha_1 \underline{u}_1^T M \alpha_1 \underline{u}_1 = 1$$

$$\alpha = \frac{1}{\sqrt{\underline{u}_1^T M \underline{u}_1}} = \frac{1}{\sqrt{18}}$$

$$\underline{u}_1 = \frac{1}{3\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

For $\underline{u}_2$

$$\alpha = \frac{1}{\sqrt{\underline{u}_2^T M \underline{u}_2}} = \frac{1}{\sqrt{18}}$$

$$\underline{u}_2 = \frac{1}{3\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

4) $T = \frac{1}{2}\left(J_1\dot{\theta}_1^2 + J_2\dot{\theta}_2^2 + m_1\dot{x}_1^2 + m_2\dot{x}_2^2\right)$

Since $\dot{\theta}r = \dot{x}$

$T = \frac{1}{2}\left(\frac{J_1}{r^2} + m_1\right)\dot{x}_1^2 + \frac{1}{2}\left(\frac{J_2}{r^2} + m_2\right)\dot{x}_2^2$

$U = \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2(x_2 - x_1)^2 + \frac{1}{2}k_3(x_2)^2$

$\frac{d}{dt}\frac{\partial T}{\partial \dot{x}_1} = \left(m_1 + \frac{J_1}{r^2}\right)\ddot{x}_1$

$\frac{d}{dt}\frac{\partial T}{\partial \dot{x}_2} = \left(m_2 + \frac{J_2}{r^2}\right)\ddot{x}_2$

$\frac{\partial U}{\partial x_1} = (k_1 + k_2)x_1 - k_2x_2$

$\frac{\partial U}{\partial x_2} = (k_3 + k_2)x_2 - k_2x_1$

$\therefore$

1 $\left(m_1 + \frac{J_1}{r^2}\right)\ddot{x}_1 + (k_1 + k_2)x_1 - k_2x_2 = 0$

2 $\left(m_2 + \frac{J_2}{r^2}\right)\ddot{x}_2 + (k_2 + k_3)x_2 - k_2x_1 = 0$

5) Since the mode shapes aren't given, call the mode shapes $X_n(x)$. We know that $X_n''(x) = -\sigma_n^2 X_n(x)$. $X_n(x)$ are presumed normalized. $1 = \int_0^l X_n^2\, dx$

$\therefore$

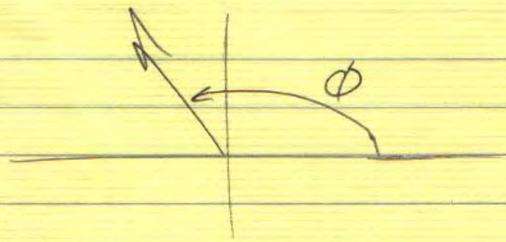
$$\left[\sum_{n=1}^{\infty} \left(X_n(x)\left(\ddot{T}_n + c^2 \sigma_n^2 T_n\right)\right) = 100\, \delta(t)\, \delta\left(x - \frac{l}{2}\right)\right] X_m$$

Integrating both sides from $0<x<l$,

$$\ddot{T}_m + \overbrace{c^2 \sigma_n^2}^{\omega_n^2} T_m = 100\, X_n\left(\frac{l}{2}\right) \delta(t)$$

$$\therefore \quad T_m(t) = \frac{100\, X_n\left(\frac{l}{2}\right)}{c\sigma_n} \sin c\sigma_n t$$

$$\boxed{w(x,t) = \sum_{n=1}^{\infty} \frac{100\, X_n\left(\frac{l}{2}\right)}{c\sigma_n} \sin c\sigma_n t\; X_n(x)}$$



$$\tan\left(\frac{200}{\pi} + \phi\right) = \frac{-3.69\times10^{-3}\cdot 20}{3.252\times10^{-2}}$$

$$\frac{200}{\pi} + \phi = 1.9859$$

$$\phi = 1.9859 + \frac{200}{\pi} = 2.816 \text{ rad}$$

$$A = \frac{-3.69\times10^{-3}}{\sin\left(\frac{200}{\pi} + 2.816\right)} = 7.633\times10^{-2}$$

$$\therefore\ x_2(t) = 7.633\times10^{-2}\sin(20t + 2.816)$$

Alternatively

$$x_2(t) = -.005\sin 20t + \frac{1}{200}\int_0^t \delta\left(\tau - \frac{10}{\pi}\right)\sin 20(t-\tau)\,d\tau$$

$$= -.005\sin 20t + \frac{1}{200}\sin 20\left(t - \frac{10}{\pi}\right)$$

which <u>may</u> be simplified to x_2 as above.