ME 460/660, Mechanical Vibration — Final Exam, Fall 2009

Closed book, closed notes. Use one $8\frac{1}{2} \times 11$ formula sheet (brought by student) and turn in with exam. Test books will be provided. All problems are to be done in the test book. **Please read the problems carefully. They may not be the ones you think they are!**

1. A linear system is governed by the following equation of motion:

$$m\ddot{x} + c\dot{x} + kx = ky + c\dot{y}$$

where $m = 10\text{kg}$, $c = 1.0\text{kg/s}$, and $k = 10,000\text{kg/s}^2$. Given $y(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{-1}{n\pi} \sin \pi n t$, find the first 4 terms of $x(t)$.

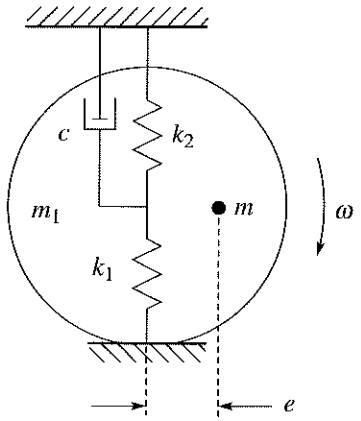
2. For the system represented by:

$$M = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}, \qquad K = \begin{bmatrix} 27 & -4 \\ -4 & 10 \end{bmatrix}$$

   (a) Find the natural frequencies, and number them accordingly

   (b) Find the mass normalized mode shapes, numbering them accordingly

3. Derive the equation/s of motion of the following unbalanced tire system.



4. Graduate Students/Undergraduate Bonus (20% of other points): Solve for the steady-state (particular) response of the following system if the boundary conditions are presumed to be fixed-free ($0 < x < l$) where $c = \sqrt{\tau/\rho}$.

$$w_{tt}(x,t) - c^2 w_{xx}(x,t) = 100\delta(x - \frac{l}{2}) \sin(3t)$$

Recall that the integral of a Dirac delta function times another function is equal to the "another function" evaluated when the argument of the argument of the Dirac delta function is zero.