ME 712, Finite Element Method Applications Exam 2, Spring 2004

Open book, closed notes. Test books will be provided.

1. A beam rests on a compliant foundation. The strain energy in the foundation due to deformation of the beam is $\frac{1}{2}kv(x)^2$. Derive the change to the beam stiffness matrix elements K_{11} and K_{21} due to the addition of this foundation *using Gauss point integration*. The shape functions are

$$N = \begin{bmatrix} 1 - 3\frac{x^2}{l^2} + 2\frac{x^3}{l^3} & x - 2\frac{x^2}{l} + \frac{x^3}{l^2} & 3\frac{x^2}{l^2} - 2\frac{x^3}{l^3} & -\frac{x^2}{l} + \frac{x^3}{l^2} \end{bmatrix} \tag{1}$$

2. Determine the consistent nodal loading on the beam of problem 1 for an applied distributed load of $f(x) = a\frac{x}{l}$ *using Gauss point integration.*

3. Use area coordinates to determine the first moment of area of a triangular element about the y axis in terms of A and the nodal coordinates, i.e.

$$Q_y = \int_A x \, dA \tag{2}$$

given

$$\int_A \xi_1^k \xi_2^l \xi_3^m = 2A\frac{k!\ l!\ m!}{(2+k+l+m)!} \tag{3}$$

4. Find the stress at (x, y)=(2,2) of a bilinear quadrilateral (Q4) element with nodes 1-4 at (0,0), (1,0), (2,2), and (0,1) in terms of u_3 and v_3 (presume all other nodal displacements are zero).