# ME 460/660, Mechanical Vibration — Final Exam, Fall 2001

Closed book, closed notes. Use one provided $8\frac{1}{2} \times 11$ formula sheet and turn in with exam. Test books will be provided. **Do all work on the exam pages** with the exception of the full length problems. Full length problems are to be done in the test book.

## 1 Lab Final: 15% of course grade

Fill in the blank with the appropriate letter from the list below. One point each. **Use each answer only once!**

In the first experimental lab, we observed the ___ of a ___ measured using an ___ (type of sensor). By observing the period we were able to ascertain the ___ , and by calculating the ___ , we were able to estimate the ___ . In the second lab, we obtained the ___ by forcing the beam with an ___ . The quality of the test data as a function of frequency can be rated by observing the ___ . The damping ratio was estimated using ___ .

The objective of a vibration absorber design is to set the natural frequency of the absorber to be equal to the ___ of the excited system.

Observing the figure of the accelerometer on the next page, a ___ hoop holds the masses against the ___ material. The force that the masses apply to that material is read as a ___ . Knowing that force, and the value of the mass, one can estimate the acceleration by applying ___ .

**Use each answer only once!**

a. natural frequency

b. damped natural frequency

c. driving frequency

d. damping ratio

e. quadrature peak picking

f. log decrement

g. beam

h. truss

i. shape memory alloy

j. free response

k. frequency response function

l. coherence

m. phase

n. accelerometer

o. pressure gauge

p. displacement sensor

q. load cell

r. impulse hammer

s. shaker

t. displacement

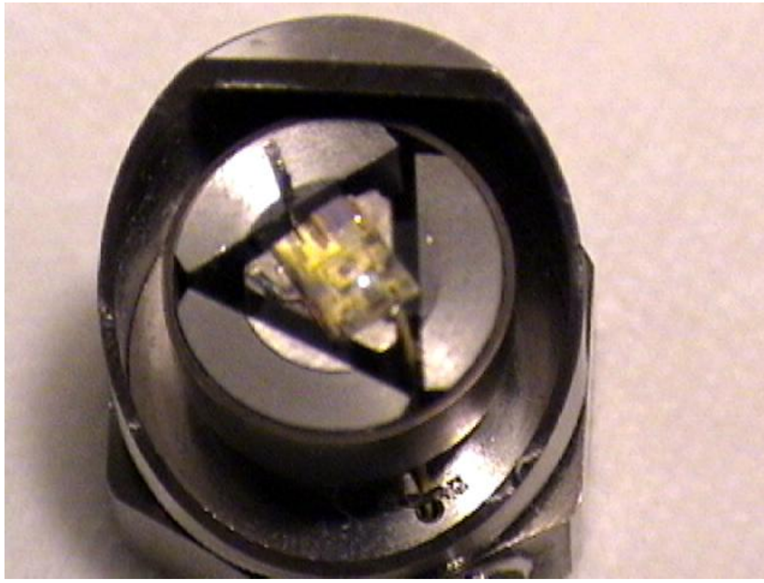
u. acceleration

v. voltage

w. piezoelectric

x. Newton's Law

y. Coulomb's Law



## 2 Formulae

| | |
|---|---|
| Euler Relations | $e^{j\beta} = \cos(\beta) + j\sin(\beta)$<br>$\sin(\beta) = \frac{e^{j\beta} - e^{-j\beta}}{2j}$<br>$\cos(\beta) = \frac{e^{j\beta} + e^{-j\beta}}{2}$ |
| Lagrange's Equation | $\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} + \frac{\partial R}{\partial \dot{q}_i} = Q_i$ |
| Fourier Series (Real Form) | $F(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos(n\omega_T t) + b_n \sin(n\omega_T t)\right)$ where $\omega_T = 2\pi/T$, and $T$ is the period of the function<br>$a_0 = \frac{2}{T}\int_0^T F(t)dt$,<br>$a_n = \frac{2}{T}\int_0^T F(t)\cos(n\omega_T t)dt$, and<br>$b_n = \frac{2}{T}\int_0^T F(t)\sin(n\omega_T t)dt$ |
| Fourier Series (Complex Form) | $F(t) = \sum_{n=0}^{\infty}\left(a_n e^{j\omega_T n t}\right)$ where $\omega_T = 2\pi/T$, and $T$ is the period of the function<br>$a_n = \frac{1}{T}\int_0^T F(t)e^{j\omega_T n t}dt$ |
| Convolution Integral | $x(t) = \frac{1}{m\omega_d}e^{-\zeta\omega_n t}\int_0^t \left[F(\tau)e^{\zeta\omega_n \tau}\sin\left(\omega_d(t-\tau)\right)\right]d\tau$<br>or $x(t) = \frac{1}{m\omega_d}\int_0^t \left[F(t-\tau)e^{\zeta\omega_n(t-\tau)}\sin\left(\omega_d \tau\right)\right]d\tau$ |
| Log Decrement | $\delta = \frac{1}{n}\ln\left(\frac{x(t)}{x(t+nT)}\right)$, $\zeta = \frac{\delta}{\sqrt{4\pi^2+\delta^2}}$ |

# 3 Final Exam

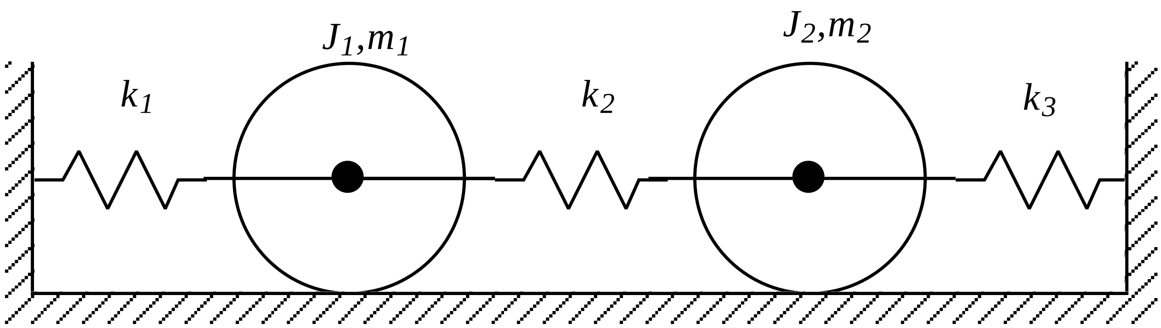
Problems are 10 points each.

1. The Fourier Series of an excitation is given by $F(t) = \sum_1^\infty \frac{1}{n}\sin(3nt)$. Given $k = 81,000$ N, and $m = 1000$, find $x(t)$. What is the most important term in the solution?

2. Find $x(t)$ for the system defined by $10\ddot{x} + 4000x = \delta\left(t - \frac{10}{\pi}\right)$, given $x(0) = 0$, and $\dot{x}(0) = -0.1$. Sketch your solution.

3. Given
$$M = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}, \text{and} \qquad K = \begin{bmatrix} 65 & -35 \\ -35 & 65 \end{bmatrix}$$
find the natural frequencies and mode shapes. *You do not need to find the mass-normalized mode shapes.*

4. Obtain the equations of motion for the system below. Assume the disks roll without slipping. *They are not solid disks! Put your answer in terms of the variables shown in the figure* (20 points):




5. Graduate Students/Undergraduate Bonus (25%): Solve the following equation for the steady state response $w(x,t)$ where $c = \sqrt{\tau/\rho}$.
$$w_{tt}(x,t) - c^2 w_{xx}(x,t) = 100\delta(t)\delta(x - l/2)$$
Recall that the integral of a Dirac delta function times another function is equal to the "another function" evaluated when the argument of the argument of the Dirac delta function is zero.

BONUS: Find the mass normalized mode shapes of problem 3, and prove that they are the mass normalized mode shapes (checking all conditions). (4 points)