System Dynamics Exam I Sp 2008 soln

1) See code at end

$m = 114.8$

$b = -84.2$

$J = 3.685 \times 10^4$

$S = 1.69 \times 10^5$

$r^2 = 0.78$

Obviously not a great fit because the form of the function chosen was bad

2) $\omega_3 = \frac{1}{1.4^2} \omega_1$

$KE = \frac{1}{2} I_4 \omega_1^2 + \frac{1}{2} I_5 \omega_3^2$

$= \frac{1}{2}\left(I_4 + \left(\frac{1}{1.4^2}\right)^2 I_5 \right) \omega_1^2$

so I_{eff} where the torque is applied

$I_{eff} = I_4 + \frac{1}{1.4^4} I_5$

$T = I_{eff}\, \dot{\omega}_1$

Since $\omega_1^2 = (1.4)^4 \omega_3$

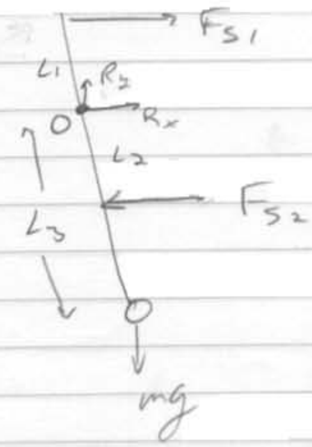
$T = I_{eff}\, (1.4)^4\, \dot{\omega}_3$

or

$T = \left(I_4 + \left(\frac{1}{1.4}\right)^4 I_5 \right) (1.4)^2\, \dot{\omega}_3$

$T = 0.09\, \dot{\omega}_3$

3) a) unstable
b) unstable
c) unstable
d) unstable, but book calls it neutrally stable
e) neutrally stable
f) unstable, but book calls it neutrally stable

4) 

$\Sigma M_0 = I\ddot{\theta}$, $I = mL_3^2$

$F_{s1} = L_1 \theta k_1$

$F_{s2} = (L_2\theta - x)k_2$

$$I\ddot{\theta} = -F_{s1}L_1 - F_{s2}L_2 - L_3 \underbrace{\sin\theta}_{\sin\theta \sim \theta \text{ for small } \theta} mg$$

$$mL_3^2\ddot{\theta} = -L_1^2 k_1\theta - L_2^2 k_2 \theta + L_2 k_2 x - L_3 mg\theta$$

$$mL_3^2\ddot{\theta} + (L_1^2 k_1 + L_2^2 k_2 + L_3 mg)\theta = L_2 k_2 x$$

```
x=[0:4]
y=[1 8 50 178 490]
a=polyfit(x,y,1)
plot(x,y,x,a(1)*x+a(2))
J=sum((a(1)*x+a(2)-y).^2)
S=sum((y-mean(y)).^2)
r2=1-J/S
```