Computational Methods in Structural Dynamics, Final Exam Winter 2008

One 8.5" by 11" cheat sheet.

Execute the following command at the *beginning of the exam*:

rm -r ~me710-in/*myaccountname*/*

Execute the following command at the *end of the exam*:

cp *.* ~me710-in/*myaccountname*

To see the list of files you have submitted: ls ~me710-in/*myaccountname*

1. Derive the equations of motion and allowable boundary conditions for a Timoshenko beam given the following.
   The bending strain energy is $V_b = \frac{1}{2}\int_0^L EI(\alpha')^2 dx$, the shear strain energy is $V_s = \frac{1}{2}\int_0^L \kappa GA(\alpha - \frac{\partial v}{\partial x})^2 dx$, the kinetic energy is $T = \frac{1}{2}\int_0^L \rho A \dot{v}^2 dx + \frac{1}{2}\int_0^L \rho I \dot{\alpha}^2 dx$, and the non-conservative variational work is $\delta W_{nc} = \int_0^L p(x,t)\delta v(x,t) dx$. Of course the total potential energy is $V = V_b + V_s$. Note that for a Timoshenko beam the rotation parameter α is independent of the slope $\frac{\partial v}{\partial x}$, both being a function of x and t. As a result, you should expect to derive two coupled differential equations– one in α, and one in v. Assume A and I are functions of x as well.

2. The partially non-dimensionalized equation of motion (it now has length 1) of a tapered beam is given by:
   $\frac{\partial^2}{\partial \xi^2}\left(\frac{2}{3}\xi^3 \frac{\partial^2 w}{\partial \xi^2}\right) + 2\xi \frac{\partial^2 w}{\partial t^2} = 0$. Assuming a deflection form of $W(\xi) = a_1\left(1-\xi\right)^2 + a_2 \xi \left(1-\xi\right)^2$, estimate the first and second natural frequencies of the beam using both one and two term representations of the mode shape/s using Galerkin's method.

3. Apply Householder's method to tri-diagonalize the matrix A.

$$A = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 4 & -3 \\ -1 & -3 & 12 \end{bmatrix}$$

4. The system below consists of two rigid links of total mass m_i and length L_i (i = 1, 2) hinged to a shaft rotating with the constant angular velocity Ω about a vertical axis. The links are hinged so as to permit motion of the links in the rotating vertical plane and their angular displacements θ and ϕ are restrained by torsional springs of stiffness $k_1$and k_2, respectively. Derive the equations of motion for arbitrarily large angles θ and ϕ.