Computational Methods in Structural Dynamics, Exam 1 Winter 2004

One 8.5" by 11" cheat sheet.

1. Determine the response of the system

$$EI\frac{\partial^4 w}{\partial x^4} + \rho A\frac{\partial^2 w}{\partial t^2} = \delta(t)\delta\left(x - \frac{l}{2}\right)$$

for a pinned-pinned (simply supported) boundary condition.

2. Use first order perturbation methods to estimate the eigenvalues of $B$ given

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 4 \end{bmatrix}, \quad \Lambda = \begin{bmatrix} 1.2679 & 0 & 0 \\ 0 & 3.0000 & 0 \\ 0 & 0 & 4.7321 \end{bmatrix}, \quad \Phi = \begin{bmatrix} 0.7887 & 0.5774 & 0.2113 \\ 0.5774 & -0.5774 & -0.5774 \\ 0.2113 & -0.5774 & 0.7887 \end{bmatrix}$$

when

$$B = \begin{bmatrix} 2.01 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 4 \end{bmatrix}$$

3. Consider that the mass matrix for a system is the identity matrix, and the stiffness matrix is the matrix $A$ from problem 2.

   (a) If the system is constrained such that $x_1 = x_3$, place appropriate bounds on the **natural frequencies** using the min-max principle.

   (b) If the system is constrained such that $x_1 = x_2$, place appropriate bounds on the **natural frequencies** using the min-max principle.

4. Determine/prove the stability characteristics defined by

$$M\ddot{\mathbf{x}} + (G + D)\dot{\mathbf{x}} + K\mathbf{x} = \mathbf{0}$$

where $M$ and $K$ are positive definite and symmetric, $G$ is skew-symmetric, and $D$ is symmetric for the cases of

   (a) $D = 0$, $G = 0$

   (b) $G = 0$, $D$ is positive definite

   (c) $G \neq 0$, $D$ is positive definite