ME 3210 Exam 1 Solution, Fall 2015

1) Given $v = iR$ as the presumed solution, we are curve fitting a line with the y intercept equal to zero, so $b = 0$.

In class, the equations given were

$$y = mx + b \quad (v = Ri + 0)$$

$$m \sum x_i^2 + b \sum x_i = \sum y_i x_i$$

and $m \sum x_i + bn = \sum y_i$

or $$\begin{bmatrix} \sum x_i^2 & \sum x_i \\ \sum x_i & n \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} \sum y_i x_i \\ \sum y_i \end{bmatrix}$$

Since $b = 0$, we can simply use either equation with $b = 0$

$$m \sum x_i + \cancel{nb}^{\,0} = \sum y_i$$

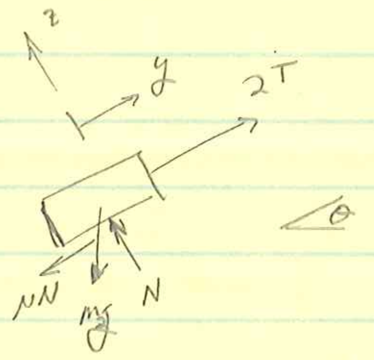
$$0.0950\, m = 10$$

$$m = \frac{10}{0.0950} = 100.5 \text{ Ohms}$$

$$\boxed{R = 100.5 \text{ Ohms}}$$

As a note, the portion after the decimal is suspect because it is tiny compared to the errors in the data (0.5% versus up to 5%)

2)

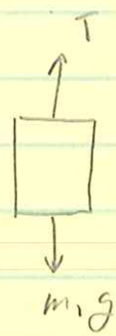


$$\Sigma F_y = m_2 \ddot{y} = 2T - \mu N - m_2 g \sin\theta$$

$$\Sigma F_z = 0 = -m_2 g \cos\theta + N$$

$$N = m_2 g \cos\theta$$

$$\therefore \quad m_2 \ddot{y} = 2T - \mu m_2 g \cos\theta - m_2 g \sin\theta \qquad (1)$$



$$\Sigma F_x : \quad m_1 \ddot{x} = m_1 g - T$$

$$T = -m_1 \ddot{x} + m_1 g \qquad (2)$$

From Kinematics, the change in the length of the rope is 0, so

$$-2y + x = 0$$

$$\ddot{x} = 2\ddot{y} \qquad (3)$$

Substituting ③ into ②

$$T = m_1 g - 2 m_1 \ddot{y} \qquad ④$$

substituting ④ into ①

$$m_2 \ddot{y} = 2(m_1 g - 2 m_1 \ddot{y}) - m_2 g(\mu \cos\theta + \sin\theta)$$

$$(m_2 + 4m_1)\ddot{y} = 2 m_1 g - m_2 g(\mu \cos\theta + \sin\theta)$$

$$\ddot{y} = \frac{g}{m_2 + 4m_1}\left(2 m_1 - m_1(\mu \cos\theta + \sin\theta)\right)$$

3) $RC\dot{v} + v = v_s\, u(t)$

Homogeneous solution

$$RC\dot{v} + v = 0$$

$$v = Ae^{-t/\tau} \quad , \quad \dot{v} = -\frac{1}{\tau}Ae^{-t/\tau}$$

$$-\frac{RC}{\tau}Ae^{-t/\tau} + Ae^{-t/\tau} = 0$$

$$\left(-\frac{RC}{\tau} + 1\right)Ae^{-t/\tau} = 0$$

$$RC = \tau$$

steady state solution is $v_{ss} = v_s$

$$v_{total} = v_s + Ae^{-t/RC}$$

$$v(0) = 0 = v_s + Ae^{0} \qquad \therefore A = -v_s$$

$$v(t) = v_s\left(1 - e^{-t/RC}\right)$$