ME 712, Finite Element Method Applications Exam 1, Spring 2006

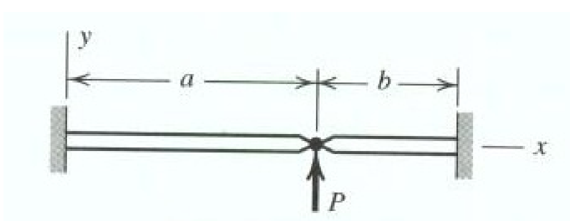
One formula sheet, closed notes. Test books will be provided. Two hours. *Problems must be done in order in the test books.* 10 points each.

1. Strain ϵ_x is given by $\epsilon_x = \frac{\partial u}{\partial x}$. What expression for ϵ_x is given when u in a four-noded element is given by $u = a_1 + a_2 x + a_3 y + a_4 xy$? Four a mesh of such elements, what can you say about the interelement continuity of ϵ_x?

2. Determine the 1,4 element of the stiffness matrix for a beam in local coordinates (the beam being between $x = 0$ and $x = l$) presuming E is constant, but $I(x) = I_1 + (I_2 - I_1)\frac{x}{l}$. *Set up all math, but solve only for K_{14}.*
$$N = \begin{bmatrix} 1 - 3\frac{x^2}{l^2} + 2\frac{x^3}{l^3} & x - 2\frac{x^2}{l} + \frac{x^3}{l^2} & 3\frac{x^2}{l^2} - 2\frac{x^3}{l^3} & -\frac{x^2}{l} + \frac{x^3}{l^2} \end{bmatrix} \tag{1}$$

3. A piezoelectric rod (truss) of constant properties has an energy, to be treated like a potential energy, of
$$U = \frac{1}{2}EA\epsilon(x)^2 - eA\frac{dV(x)}{dx}\epsilon(x) - \frac{1}{2}\bar{\varepsilon}A\left(\frac{dV(x)}{dx}\right)^2 \tag{2}$$
where all variables are constant except those indicated above to be functions of x (note that ϵ is *not* the same as $\bar{\varepsilon}$). Using the linear 1-D shape functions to represent the field variables $u(x)$, displacement, and $V(x)$, voltage field, in terms of nodal field quantities, u_j and V_j, determine the governing equations, in matrix form, in terms of the nodal field quantities. **Hint:** expand both $u(x) = \epsilon(x)$ and $V(x)$ in terms of shape functions and modal displacements.

4. Two collinear cantilever beams are connected by a frictionless hinge as shown. Flexural stiffness, EI_z, is the same for both beams. Load P and deformations are confined to the xy plane. Write the stiffness matrix that operates on the "active" DOFs. Ignore transverse shear deformation. The elemental stiffness matrix is given by
$$K = \frac{EI_x}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$



5. Define plane strain. Give an example of a problem for which the plane strain assumption is appropriate.

6. The rod mass matrix for a uniformly distributed rod is

$$M = \frac{\rho A \ell}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

What is the matrix in 3-D?