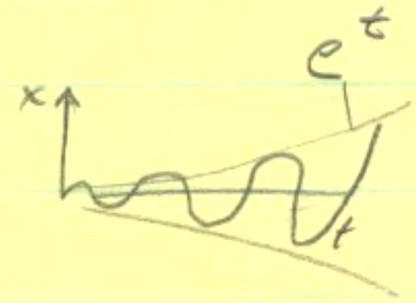
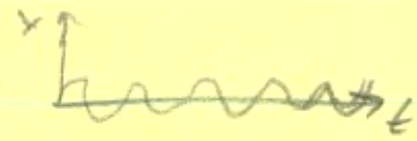
ME 464 Spring 2005 Exam 2 solns

1) a) $1 \pm 1j$ Unstable oscillation

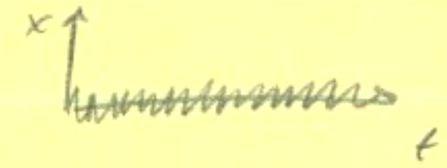


b) 0 Drift (rigid body motion)

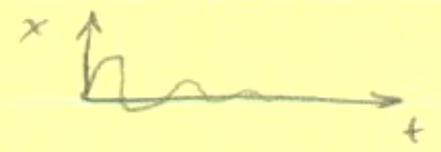
c) $\pm 1j$ Undamped oscillation



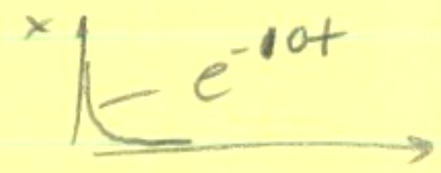
d) $\pm 10j$ High frequency undamped oscillation (buzz)



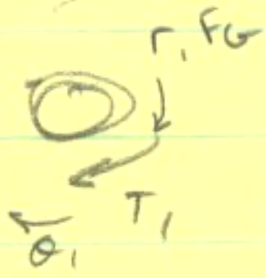
e) $-1 \pm 1j$ (could also be d)) Decay oscillation



f) -10 Fast exponential decay

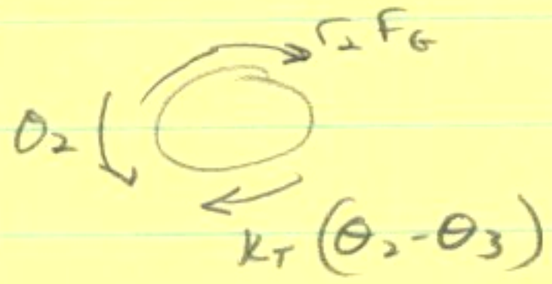


a) FBD 1 (includes small gear)



① $\sum M = I_1 \ddot{\theta}_1 = T_1 + r_1 F_G$

FBD 2
Big gear

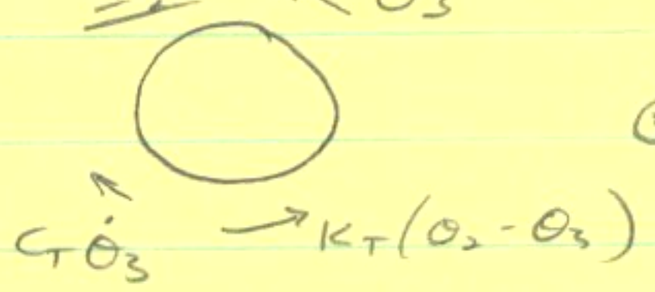


$(I_G = 0)$

② $\sum M = 0 = -r_2 F_G - K_T(\theta_2 - \theta_3)$

$F_G = \frac{-1}{r_2} K_T(\theta_2 - \theta_3)$

FBD 3



③ $\sum M = I_2 \ddot{\theta}_3 = K_T(\theta_2 - \theta_3) - C_T \dot{\theta}_3$

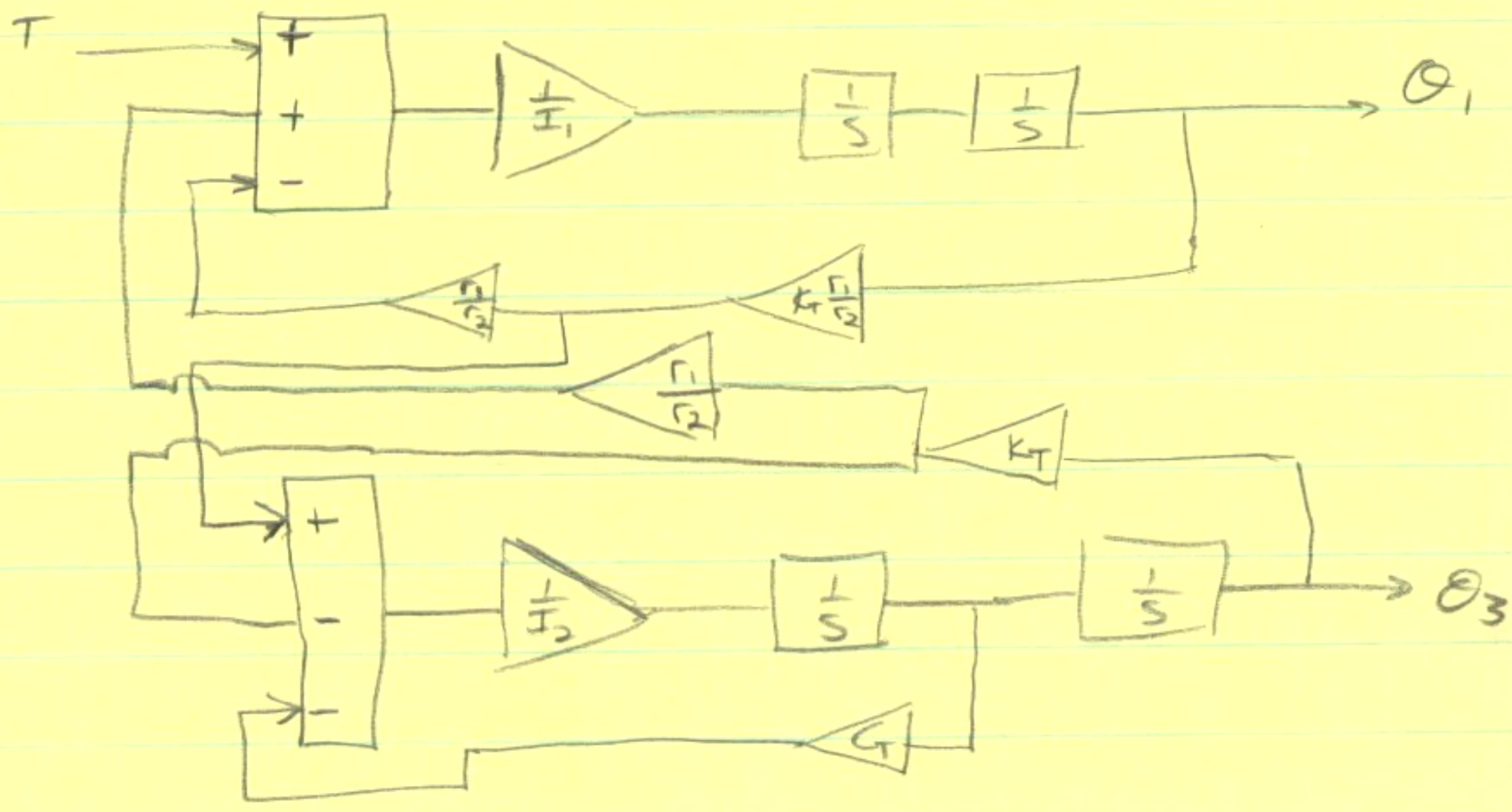
Substituting ② into ①

$$I_1 \ddot{\theta}_1 = T_1 - \frac{r_1}{r_2} K_T(\theta_2 - \theta_3)$$

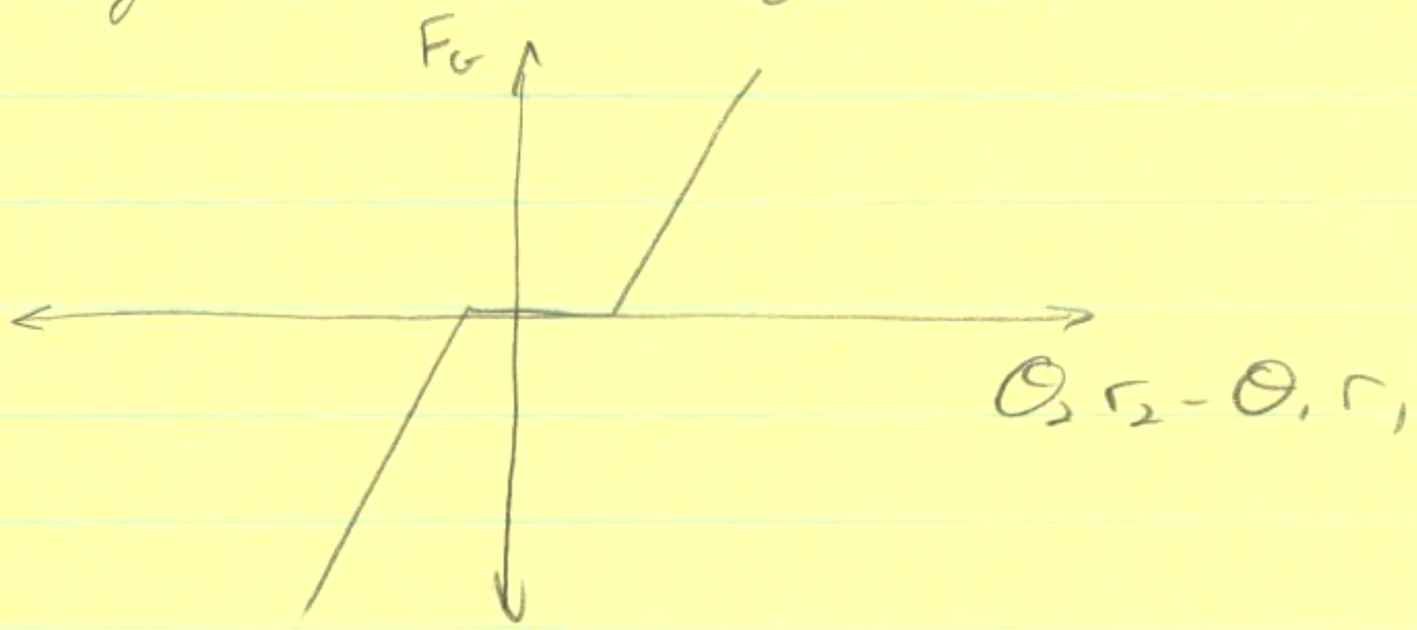
Since $\theta_1 r_1 = \theta_2 r_2$, $\theta_2 = \frac{r_1}{r_2} \theta_1$

$$I_1 \ddot{\theta}_1 + \left(\frac{r_1}{r_2}\right)^2 K_T \theta_1 - \frac{r_1}{r_2} K_T \theta_3 = T_1$$

$$I_2 \ddot{\theta}_3 + C_T \dot{\theta}_3 + K_T \theta_3 - K_T \frac{r_1}{r_2} \theta_1 = 0$$



b) See original EOM. F_G is now



3) $z_1 = x \quad , z_2 = \dot{x} \quad , z_3 = y \quad , z_4 = \dot{y}$

$$* \qquad \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \begin{bmatrix} z_2 \\ -\frac{1}{10} z_2 (1 - z_3^2) - .3 z_1 + .2 z_3 \\ z_4 \\ -3 z_4 - 3 z_3 + 2 z_1 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{f(t)}{10} \\ 0 \\ 0 \end{bmatrix}$$

$$** \qquad y = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}$$

Need to find nominal values

$$z_2 = \dot{z}_1 = z_4 = \dot{z}_4 = 0$$

From *

$$-.3 z_1 + .2 z_3 = .1$$

From * *

$$2 z_1 + -3 z_1 = 0$$

$$\bar{z}_1 = -.6 \qquad \bar{z}_3 = -.4$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -.3 & .084 & .2 & 0 \\ 0 & 0 & 0 & 1 \\ 2 & 0 & -3 & -3 \end{bmatrix}$$

$$\frac{\partial f_2}{\partial z_2} = -\frac{1}{10}(1 - z_3^2)$$

eval at $\bar{z}_3$ gives

$$\underline{0.084}$$

$$\left.\frac{\partial f_2}{\partial z_3}\right|_{\bar{z}} = .2$$

4) a)

① $p_1 = \rho g h_1$ $\quad p_2 = \rho g h_2$

② $q_1 = \frac{1}{R_1}(p_1 - p_2)$ $\quad q_2 = \frac{1}{R_2} p_2$

③ $A_1 \dot{h}_1 \rho = q_{in} - q_1$ $\quad A_2 \dot{h}_2 \rho = q_1 - q_2$

$A_1 \dot{h}_1 \rho = q_{in} - \frac{1}{R_1}(p_1 - p_2)$ $\quad A_2 \dot{h}_2 \rho = \frac{1}{R_1}(p_1 - p_2) - \frac{1}{R_2} p_2$

$A_1 \dot{h}_1 \rho = q_{in} - \frac{\rho g}{R_1}(h_1 - h_2)$ $\quad A_2 \dot{h}_2 \rho = \frac{\rho g}{R_1}(h_1 - h_2) - \frac{\rho g}{R_2} h_2$

$$\begin{bmatrix} \dot{h}_1 \\ \dot{h}_2 \end{bmatrix} = \begin{bmatrix} -\frac{g}{A_1 R_1} & \frac{g}{A_1 R_1} \\ \frac{g}{A_2 R_1} & -\frac{g}{A_2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{A_1 \rho} \\ 0 \end{bmatrix} q_{in}$$

b) 2 become

$q_1 = \sqrt{\frac{p_1 - p_2}{R_1}}$ $\quad q_2 = \sqrt{\frac{p_2}{R_2}}$

$$\begin{bmatrix} \dot{h}_1 \\ \dot{h}_2 \end{bmatrix} = \begin{bmatrix} \frac{-1}{A_1 \rho}\sqrt{\frac{\rho g (h_1 - h_2)}{R_1}} \\ \frac{1}{A_2 \rho}\left(\sqrt{\frac{\rho g (h_1 - h_2)}{R_1}} - \sqrt{\frac{\rho g h_2}{R_2}}\right) \end{bmatrix} + \begin{bmatrix} \frac{1}{A_1 \rho} \\ 0 \end{bmatrix} q_{in}$$