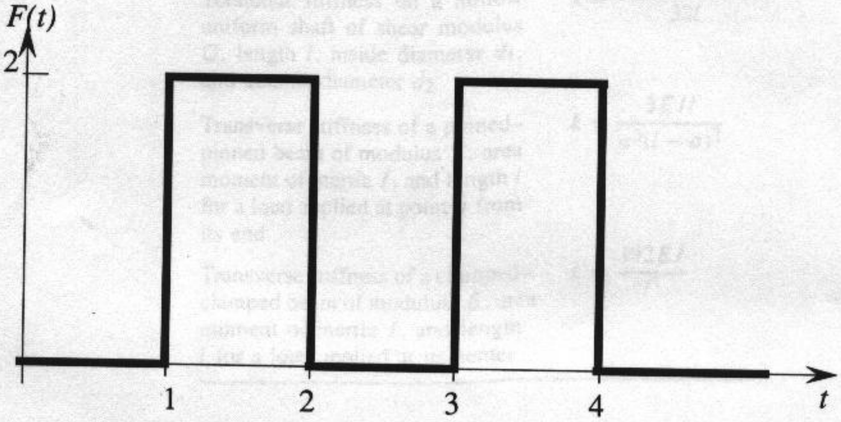
ME 460/660 Exam 2, Fall'95

1) Find the Fourier series representation of the following function. (25 points)



2) Find the natural frequencies and mode shapes of the following system: (25 points)

$$M = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$$

$$K = \begin{bmatrix} 4 & -1 \\ -1 & 4 \end{bmatrix}$$

3) A 100 kg motor is purchased by your company to drive the "Walleygag Mechanism". The manufacturer of the motor guarantees that the shaft is balanced such that $em_0 < 0.0001$ kg-m. Assuming that the coupling between the motor and the "Walleygag Mechanism" provides negligible stiffness and damping, design a table for the motor that will keep its displacement below 1 mm for motor speed between 0 and 2000 rpm under normal operation. Sketch your table design, including dimensions. Choose a proper material and maintain a low cost. Be concerned that the design criteria is fixed, but that the operation limits supplied may not be completely accurate. Neglect the mass of the table. (50 points)

Constraint (2000 rpm, 1mm) too loose.

SAMPLE SPRING CONSTANTS

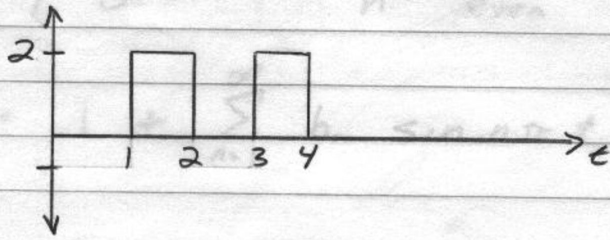
Axial stiffness of a tapered bar of length l, modulus E, and end diameters d_1 and d_2	$k = \dfrac{\pi E d_1 d_2}{4l}$
Torsional stiffness on a hollow uniform shaft of shear modulus G, length l, inside diameter d_1, and outside diameter d_2	$k = \dfrac{\pi G(d_2^4 - d_1^4)}{32l}$
Transverse stiffness of a pinned–pinned beam of modulus E, area moment of inertia I, and length l for a load applied at point a from its end	$k = \dfrac{3EIl}{a^2(l-a)^2}$
Transverse stiffness of a clamped–clamped beam of modulus, E, area moment of inertia I, and length l for a load applied at its center	$k = \dfrac{192EI}{l^3}$

PHYSICAL CONSTANTS FOR SOME COMMON MATERIALS

Material	Young's modulus, E(N/m^2)	Density, (kg/m^3)	Shear modulus, G(N/m^2)
Steel	2.0×10^{11}	7.8×10^3	8.0×10^{10}
Aluminum	7.1×10^{10}	2.7×10^3	2.67×10^{10}
Brass	10.0×10^{10}	8.5×10^3	3.68×10^{10}
Copper	6.0×10^{10}	2.4×10^3	2.22×10^{10}
Concrete	3.8×10^9	1.3×10^3	—
Rubber	2.3×10^9	1.1×10^3	8.21×10^8
Plywood	5.4×10^9	6.0×10^2	—

1)

Find the Fourier series for



$T = 2$ Note $F(t) = 0$ for $0 < t < 1$

$$a_0 = \frac{2}{T}\int_0^2 F(t)\,dt = \frac{2}{2}\; 2t\Big|^2 = 4 - 2 = 2$$ +5

$$a_n = \frac{2}{T}\int_1^2 2\cos n\pi t\,dt$$ +10

$$= 2\,\frac{1}{n\pi}\sin n\pi t\Big|_1^2$$

$$= \frac{2}{n\pi}\left(\sin \cancel{2n\pi}^{\,0} - \cancel{\sin n\pi}^{\,0}\right) = 0$$

$$b_n = \frac{2}{T}\int_1^2 2\sin n\pi t\,dt$$ +10

$$= 2\,\frac{-1}{n\pi}\cos n\pi t\Big|_1^2$$

$$= \frac{-2}{n\pi}\left(\cos 2n\pi - \cos n\pi\right)$$

$\cos 2n\pi = 1$ for any integer n

$\cos n\pi = 1$ for even n

$\quad\quad\;\; -1$ for odd n

$$b_n = \begin{cases} \dfrac{-4}{n\pi} & n \text{ odd} \\ 0 & n \text{ even} \end{cases}$$

$$F(t) = 1 + \sum_{n=1}^{\infty} b_n \sin n\pi t$$

2) $M = 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad K = \begin{bmatrix} 4 & -1 \\ -1 & 4 \end{bmatrix}$

$$\det(K - M\omega^2) = 0$$

$$\det\begin{pmatrix} 4-9\omega^2 & -1 \\ -1 & 4-9\omega^2 \end{pmatrix} = 0$$

$$16 - 72\omega^2 + 81\omega^4 - 1 = 0$$

$$81\omega^4 - 72\omega^2 + 15 = 0$$

$$\omega^2 = \frac{72 \pm \sqrt{72^2 - 4 \cdot 15 \cdot 81}}{2 \cdot 81}$$

$$\omega^2 = \frac{1}{3}, \frac{5}{9}$$

$$\omega = \frac{1}{\sqrt{3}}, \frac{\sqrt{5}}{3}$$

$\underline{\omega^2 = \frac{1}{3}}$:

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_{11} \\ u_{12} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$u_{11} = u_{12}$ $\qquad$ $\underline{u}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\underline{\omega^2 = \frac{5}{9}}$:

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} u_{21} \\ u_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$u_{21} = -u_{22}$ $\qquad$ $\underline{u}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

2) The stiffness should be high enough such that resonance is well above 2000 rpm ($r \ll 1$). Dampers tend to be relatively expensive and are more likely to fail due to the moving parts. Thus an undamped design will be performed. The worst case scenario is $\omega_r = 2000$ rpm and $m_0 e = .0001$ kg-m. For safety, let's assume $m_0 e = .002$ kg-m. For rotating unbalance

$$X = \frac{m_0 e}{m} \frac{r^2}{1-r^2}$$

For our design

$$50 = \frac{r^2}{1-r^2},$$

$$r = \frac{\omega_{dr}}{\omega} = \frac{209.4}{\sqrt{\frac{K}{100}}}$$

$$r^2 = \frac{4.386 \times 10^6}{K}$$

$$50 - 50r^2 = r^2$$

$$r^2 = \frac{50}{51} = \frac{4.386 \times 10^6}{K}$$

$$K > 4.474 \times 10^6 \text{ N/m}$$

Let's choose $k = 1\times10^7$ N/m so that if the "Walleyggay Mechanism" fails, the motor speed won't be so likely to induce resonance.

Choose steel because of relative high modulus.

For a table, the legs are unlikely to provide significant resisting moments at the ends. Thus the table is similar to a simply supported beam.

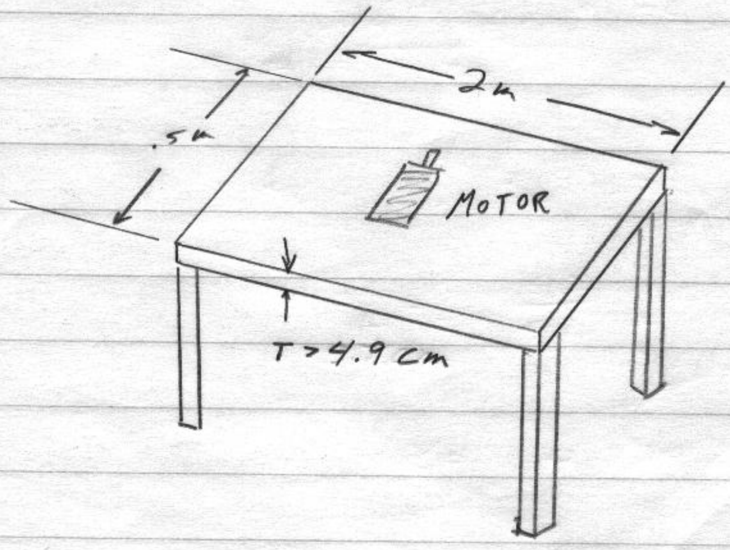
$$K = \frac{3EI\ell}{.25(\ell - .5)^2} = 1 \times 10^7 \ N/m$$

$$2.133 \times 10^{12}\, I = 1 \times 10^7 \ N/m$$

$$I = 4.688 \times 10^{-6}$$

$$I = \frac{1}{12} b h^3 = 4.688 \times 10^{-6} \ m^4$$

To use Euler-Bernoulli beam theory, b and h must be $\approx \frac{1}{10}\ell$, but the results will be close for $b \approx \frac{1}{4}\ell$. Choosing $b = .5$ m, $h = .0483$ m, or 4.9 cm.



Note: In general, the bending stiffness of the surface is much less than the stiffness of the legs.