Computational Methods in Structural Dynamics, Exam 1 Winter 2005

One 8.5" by 11" cheat sheet.

1. Determine the mode shapes and natural frequencies of a simply supported (pinned-pinned) beam given the homogenous equation of a beam

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + \rho A \frac{\partial^2 w(x,t)}{\partial t^2} = 0$$

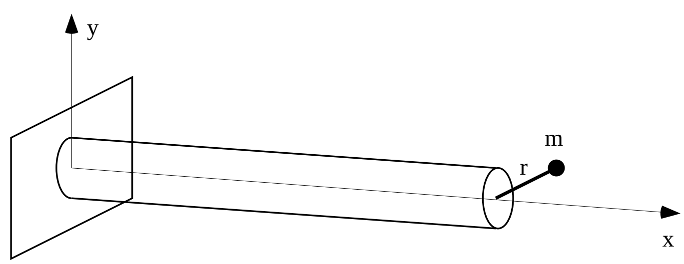
2. Determine the response of the system

$$EI\frac{\partial^4 w}{\partial x^4} + c\frac{\partial w^3}{\partial x^2 \partial t} + \rho A\frac{\partial^2 w}{\partial t^2} = \delta(t)\delta\left(x - \frac{l}{2}\right)$$

for a pinned-pinned (simply supported) boundary condition.

3. (double point value) Use Hamilton's principal (or continuous Lagrange's equation) to derive the equations of motion for the following system. A uniform cantilever beam has torsional stiffness GJ, vertical bending stiffness EI, and mass per unit length ρA, and rotational inertia (twisting) per unit length ρI_p (I_p being the polar moment of inertia for the twisting beam). The beam is cantilevered at the left end, and a massless rigid bar BC is attached at the right end. A concentrated mass is located at the end of the rigid/massless beam. Assume that bending takes place only in the $y - x$ plane with deflection $v(x,t)$ and that rotation takes place about the x axis ($\theta(x,t)$). Neglect gravity. State the equation/s of motion and boundary conditions. The potential energies are given by

$$V_{twist} = \frac{1}{2}\int_0^L GJ\left(\frac{\partial \theta}{\partial x}\right)^2 dx, \qquad V_{bending} = \frac{1}{2}\int_0^L EI\left(\frac{\partial^2 v}{\partial x^2}\right)^2 dx$$



Hint: *Include kinetic energies of torsion rod, bending beam, and point mass, then plow through Hamilton's principle.*

4. Non-dimensionalize the following equation of motion completely (so that no dimensioned terms remain in the non-dimensionalize equation). Assume EI is constant.

$$\frac{d^2}{dx^2}\left(EI\frac{d^2 w}{dx^2}\right) + \rho A\frac{d^2 w}{dt^2} = 0$$