ME 460/660 Test 1 Solns, Fall 2008

1) $f_n = 60\ Hz$ (units not given, presume standard SI?)

a) $\omega_n = 60 \cdot 2\pi = 377\ rad/s$

b) $\frac{1}{k} \approx -103\ dB$

$\frac{1}{k} \approx 7.079 \times 10^{-6}$

$k = \boxed{1.41 \times 10^{5}\ N/m}$

Answers will vary due to difficulty reading graph.

c) $\omega_n = \sqrt{\frac{k}{m}}$

$m = \frac{k}{\omega_n^2} = \boxed{0.99\ kg}$

d) At resonance

$\frac{1}{c\omega_n} = -58\ dB$

$\frac{1}{c\omega_n} = 1.26 \times 10^{-3}$

$c = \frac{1}{1.26 \times 10^{-3} \cdot 377} = \boxed{2.1}$

e) $F_c = c\,\omega_{dr} X$

We don't have X!

However $c\,\omega_{dr} = 2.1 \times 80 \cdot 2 \cdot \pi$

$= \boxed{1.1 \times 10^{3}\ X}$

2) $x(t) = A e^{-\zeta \omega_n t} \sin(\omega_d t + \phi)$

$\omega_n = 10$

$\zeta = \frac{0.1}{2 \cdot 10} = 5 \times 10^{-3}$

$\omega_d = 10$

$x(0) = 0 = A \sin \phi \qquad (\phi) = 0 \qquad ①$

$\dot{x}(t) = -A \zeta \omega_n e^{-\zeta \omega_n t} \sin(\omega_d t + \phi) + A \omega_d e^{-\zeta \omega_n t} \cos(\omega_d t + \phi)$

$\dot{x}(0) = -A \zeta \omega_n \sin \phi + A \omega_d \cos \phi$

$1 = -A \, 5 \times 10^{-3} \sin \phi + A \omega_d \cos \phi \qquad ②$

From ①, ② becomes

$0.1 = A$

$x(t) = 0.1 \, e^{-0.05 t} \sin 10 t$

3) $T = \frac{1}{2} m_2 \dot{x}_2^2 + \frac{1}{2} J \dot{\theta}_2^2 + \frac{1}{2} m_1 \dot{x}^2$

1st, $x = \frac{1}{2} x_2$

2nd, $\dot{x}_2 = \dot{\theta}_2 R$, $\dot{\theta}_2 = \frac{\dot{x}_2}{R} = 10 \dot{x}_2$

3rd, $J = \frac{1}{2} m_2 R^2$

$= 0.05$

Why? We need to Guess, and it's clearly not a hoop

$J = 0.6 m_2 R^2$ would also be reasonable

$T = \frac{1}{2} 10 \dot{x}_2^2 + \frac{1}{2} (0.05)(10 \dot{x}_2)^2 + \frac{1}{2} (10 \frac{1}{2^2}) \dot{x}_2^2$

$m_{eff} = 10 + 0.05 \cdot 100 + \frac{10}{4} = 17.5 \text{ kg}$

$\omega_n = \sqrt{\frac{k_{eff}}{m_{eff}}}$

$k_{eff} = (5 \cdot 2 \cdot \pi)^2 \cdot 17.5 = 1727 \text{ N/m}$

Note: Using Lagrange

$T = \frac{1}{2} 17.5 \dot{x}_2^2$, $U = \frac{1}{2} k x_2^2$

Subst into Lagrange

$17.5 \ddot{x}_2 + k x_2 = 0$

$\omega^2 = 25 \cdot (2\pi)^2 = \frac{k}{m}$

$k = 1727 \text{ N/m}$

4) $\frac{G}{\rho} w'' = \ddot{w}$

$$\frac{G}{\rho} X'' T = X \ddot{T}$$

$$\frac{X''}{X} = \frac{\ddot{T} \rho}{T G} = -\sigma^2$$

$$X(x) = A \sin \sigma x + B \cos \sigma x$$

$$X'(x) = \sigma A \cos \sigma x - \sigma B \sin \sigma x$$

$$X'(0) = 0, \quad \therefore A = 0$$

$$X'(\ell) = 0 = \sin \sigma x$$

$$\sigma_n = \frac{n\pi}{\ell}$$

$$\frac{\ddot{T}}{T} = -\omega_n^2 = -\sigma_n^2 \frac{G}{\rho}$$

$$\boxed{\omega_n = \sigma_n \sqrt{\frac{G}{\rho}} = \frac{n\pi}{\ell} \sqrt{\frac{G}{\rho}}}$$

$$\boxed{X_n(x) = B_n \cos \frac{n\pi x}{\ell}}$$