Fall 2004 Midterm Soln

1) $m\ddot{x} + c\dot{x} + kx = F(t)$

$$X(j\omega) = \frac{F}{k - m\omega^2 + cj\omega}$$

$$= \frac{6}{10 - 10(10\pi)^2 + 5 \cdot 10\pi j} = \frac{6}{-9.86 \times 10^3 + 1.571 \times 10^2}$$

$$= -6.084 \times 10^{-4} - 9.693 \times 10^{-6} j$$

$$|X(j\omega)| = 6.08 \times 10^{-4} \text{ m}$$

$$\angle X(j\omega) = -179^\circ \quad (-3.126 \text{ rad})$$

(phase lag of 179°)

2) $\frac{|V|}{|x|} = \omega_n = 10$ rad/s

$\frac{|A|}{|x|} = \omega_n^2 = 125$ rad/s^2, $\omega_n = 11.18$ rad/s

$\frac{|A|}{|V|} = \omega_n = 12.5$ rad/s

Inconsistancy of frequency calculation means that at least one measurement has the wrong calibration factor.

3) $T = \frac{1}{2} J \dot{\theta}^2 + \frac{1}{2} m \dot{x}^2 \qquad \theta = \frac{x}{r}$

$= \frac{1}{2} \frac{J}{r^2} \dot{x}^2 + \frac{1}{2} m \dot{x}^2$

$= \frac{1}{2} \left(\frac{J}{r^2} + m \right) \dot{x}^2 = \frac{1}{2} m_{eff} \dot{x}^2$

$U = \frac{1}{2} k_2 x^2 + \frac{1}{2} k_1 \theta^2$

$= \frac{1}{2} \left(k_2 + \frac{k_1}{r^2} \right) \theta^2 = \frac{1}{2} k_{eff} x^2$

$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{x}} \right) + \frac{\partial U}{\partial x} = 0$

$m_{eff} \ddot{x} + k_{eff} x = 0$

where $m_{eff} = \frac{J}{r^2} + m, \quad k_{eff} = k_2 + \frac{k_1}{r^2}$

$\omega_n = \sqrt{\frac{k_{eff}}{m_{eff}}} = \sqrt{\frac{k_2 + \frac{k_1}{r^2}}{m + \frac{J}{r^2}}} = \sqrt{\frac{k_2 r^2 + k_1}{m r^2 + J}}$

4) $X(x) = a \sin \sigma x + b \cos \sigma x$

$X'(x) = a\sigma \cos \sigma x - b\sigma \sin \sigma x$

$X(0) = 0 = b$

$X'(0) = 0 = a\sigma \cos \sigma l$

$l\sigma_n = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots = \left(\frac{2n-1}{2} \pi \right)$

$-\omega_n^2 = \frac{\ddot{T}}{T} = \frac{X''}{X} \left(\frac{E}{\rho} \right) = -\sigma_n^2 \frac{E}{\rho}$

$\omega_n = \sqrt{\frac{E}{\rho}} \left(\frac{2n-1}{2l} \right) \pi$

$X(x) = a \sin \frac{2n-1}{2l} \pi l$