ME 360 Exam 2 Wi '09 Solutions

1) a) $\omega_n = 10$ rad/s $\left(\sqrt{\frac{1000}{10}}\right)$ $T = \frac{2\pi}{\omega_d}$

$2\zeta\omega_n = \frac{100}{10} = 10$

$2\zeta = 1$ // $\zeta\omega_n = 5/\text{sec}$

$\zeta = 0.5$ (must use ω_d for calculating periods)

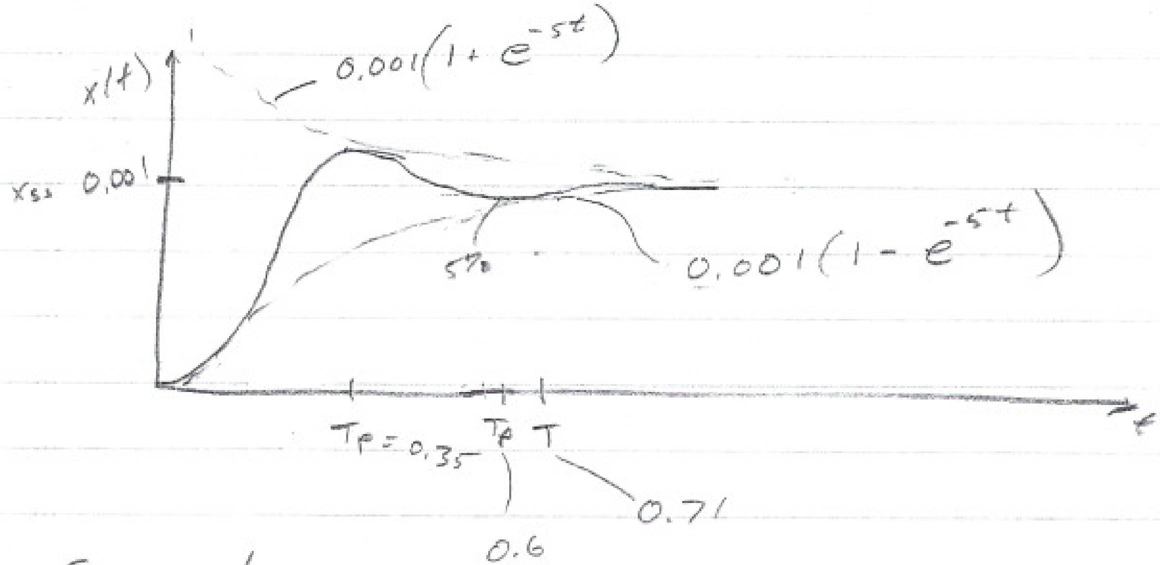
$T_s = \frac{3}{\zeta\omega_n} = 0.6$ sec , $\omega_d = 10\sqrt{1-.5^2} = 8.66$ rad/s

$T = \frac{2\pi}{\omega_d} = 0.71$ sec $T_p = \frac{1}{2}T = 0.35$ sec

$X_{ss} = \frac{1}{1000} = 0.001$

$x(T_p) = \left(e^{-\zeta\omega_n T_p} + 1\right)0.001$ (see sketch)

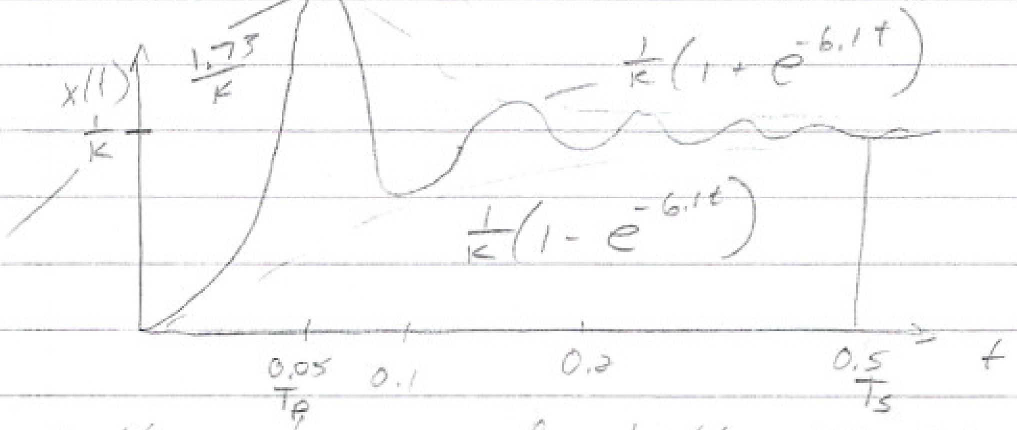
$= 1.7\times10^{-4}$



Compare to

s = tf(1, [10, 100, 1000]); step(s)

b) Parameters for sketch are almost given
We don't know the coefficient to the lowest state (x for $m\ddot{x}+c\dot{x}+kx=f(t)$)

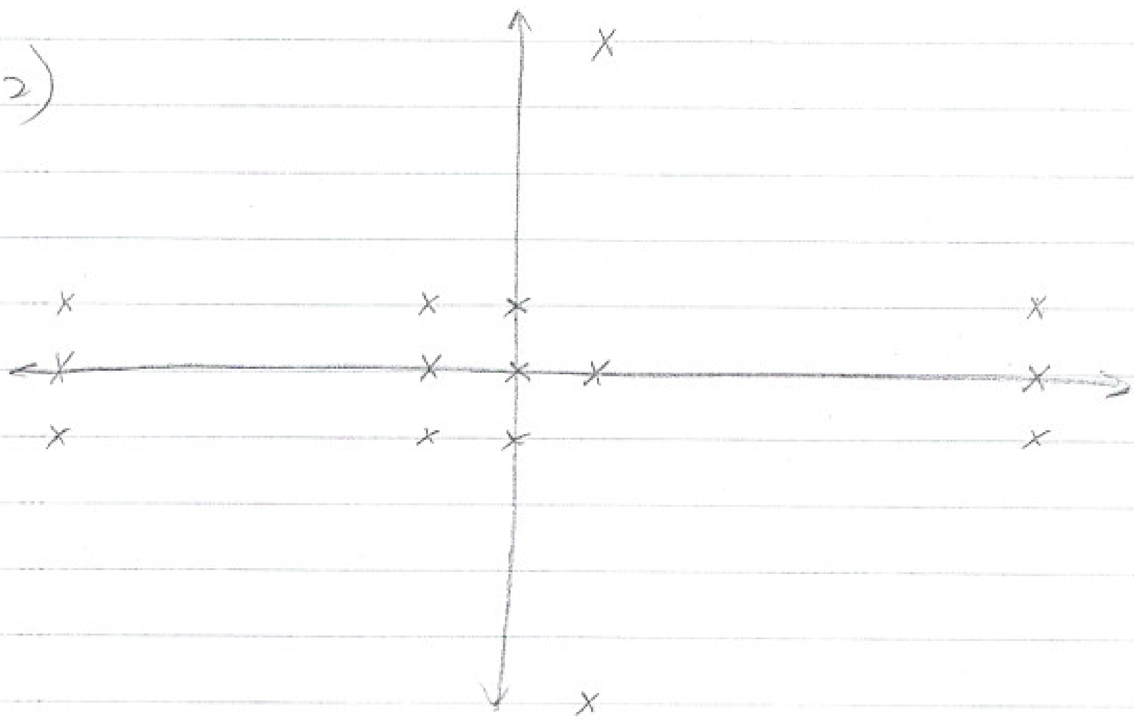


With a frequency of 10 Hz, T = 0.1 sec,
5 cycles until 5% settling time ($t_s = 0.5$ sec)

% overshoot : $e^{-\frac{\pi \zeta}{\sqrt{1-\zeta^2}}}$. We need ζ.
Since $\omega_n = 20\pi \approx 61.4$ rad/s, $\omega_d \approx 61.4$ rad/s
$t_s = \frac{3}{\zeta \omega_n}$, $\zeta = \frac{3}{0.5 \cdot 61.4} = 0.1$ which is low,
so $\omega_n \approx \omega_d$

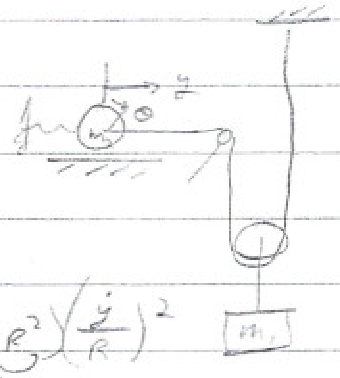
$$e^{-\frac{\pi \zeta}{\sqrt{1-\zeta^2}}} = 0.73$$

2)



a) Unstable, exponentially growing (slower than c)
b) Exponential decay, slowly (stable)
c) Fast divergent exponential growth (unstable)
d) Fast exponential convergence
e) Fast oscillating, slow growth (unstable)
f) Neutral stable, oscillating fast
g) stable oscillating/decaying (both slow)
h) slow oscillating, quick decay
i) Drifting neutral stability
j) Neutral stable oscillation.

3) Single DOF



$y = 2x$

$$U = -m_1 g x + \frac{1}{2} k y^2$$

$$= -m_1 g x + 2 k x^2$$

$$k_{eff} = 4k$$

$$T = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} m_2 \dot{y}^2 + \frac{1}{2} \underbrace{\left(\frac{1}{2} m_2 R^2\right)}_{J} \left(\frac{\dot{y}}{R}\right)^2$$

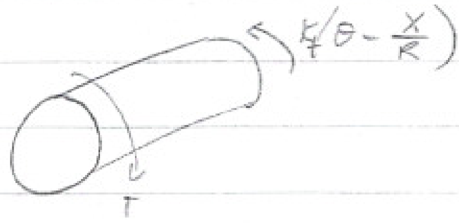
T_x

$$= \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} \left(\frac{3}{2} m_2\right) \dot{y}^2$$

$$= \frac{1}{2} \underbrace{\left(m_1 + 6 m_2\right)}_{m_{eff}} \dot{x}^2$$

$$(m_1 + 6 m_2) \ddot{x} + 4 k x = m g$$

4) For the "drum"

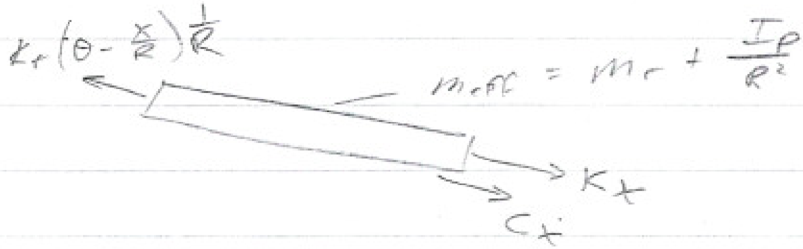


$$\Sigma M = I_m \ddot{\theta}$$

$$T - k_t\left(\theta - \frac{x}{R}\right) = I_m \ddot{\theta}$$

$$I_m \ddot{\theta} + k_t \theta - k_t \frac{x}{R} = T \qquad (1)$$

For Gear/rack



$$\Sigma F = m\ddot{x}$$

$$-kx - c\dot{x} + \frac{k_t}{R}\theta - \frac{k_t}{R^2}x = \left(m_r + \frac{I_p}{R^2}\right)\ddot{x}$$

$$\left(m_r + \frac{I_p}{R^2}\right)\ddot{x} + c\dot{x} + \left(k + \frac{k_t}{R^2}\right)x - \frac{k_t}{R}\theta = 0 \qquad (2)$$