Computational Methods in Structural Dynamics, Midterm — Winter 2011

One 8.5" by 11" cheat sheet.

1. A system is errantly modeled as

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \ddot{\mathbf{x}} + \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \sin(\omega t) \tag{1}$$

   Using the eigensolution for the given system, by perturbation methods, determine the natural frequencies of the correct system if $m_1 = 1.1$.

   Determine the error caused by using a perturbation method instead of re-solving the eigen problem.

2. A beam with constant properties E, I, ρ, A and l (where ρ is mass per volume) is clamped at the right end and has a point mass m connected to a transverse spring, k, at the left end. Derive the governing equation and boundary conditions using variational principles.

3. Determine the steady state (non-transient) response of the system

$$EI \frac{\partial^4 w}{\partial x^4} + \rho A \frac{\partial^2 w}{\partial t^2} = 0$$

   for a pinned-pinned (simply supported) boundary condition provided that the foundation that the beam is connected to is moving up and down with a motion $y(t) = a\sin(\omega t)$. Hint: set $w(x,t) = u(x,t) + ay(t)$ and solve for $u(x,t)$.

4. Prove that the internal moment in a torsion-rod is $GJ\frac{d\theta}{dx}$, or prove me wrong.