ME 460/660, Mechanical Vibration — Exam 2, Fall 2008

Closed book, closed notes. Use $8\frac{1}{2} \times 11$ formula sheet from web and turn in with exam (nothing else may be written on the formula sheet). Test books will be provided. Calculators allowed.

Problems are 10 points each. Problem 4 is required for graduate students, bonus for undergraduates.

1. A system governed by the differential equation $10\ddot{x} + 1\dot{x} + 8.8826 \times 10^4 x = f(t)$ is excited by the force $f(t) = \sum_{n=1}^{\infty} a_n \cos \frac{2\pi n}{T} t$ where $a_n = \begin{cases} 0 & \text{n odd} \\ \frac{-8}{n^2\pi^2} & \text{n even} \end{cases}$ and $T = 0.2$. Obtain the best representation of $x(t)$ using a single harmonic (not a summation). *Hint: Solving for the first few terms and eliminating the insignificant ones is a good bet.* ————— The following was accidentally left undeleted from a previous exam and shouldn't have been here. The Proctor explicitly stated that it wasn't part of problem 1, and it makes no sense for problem 2 (since it's before the problem statement, and the statement is that the system is undamped). Nevertheless, people who used it as part of their solution, or all of their solution, for problem 2 were given credit for doing so. All that needed to be done is add 2 impulse response functions times their corresponding $\hat{F}$ values (1 and $e^{\frac{-2\pi\zeta}{\sqrt{1-\zeta^2}}}$) to the remaining solution to problem to for the more complicated problem (as "detailed" in the solutions).

$$m\ddot{x} + c\dot{x} + kx = \delta(t) + e^{\frac{-2\pi\zeta}{\sqrt{1-\zeta^2}}}\delta(t - \frac{2\pi}{\omega_d})$$

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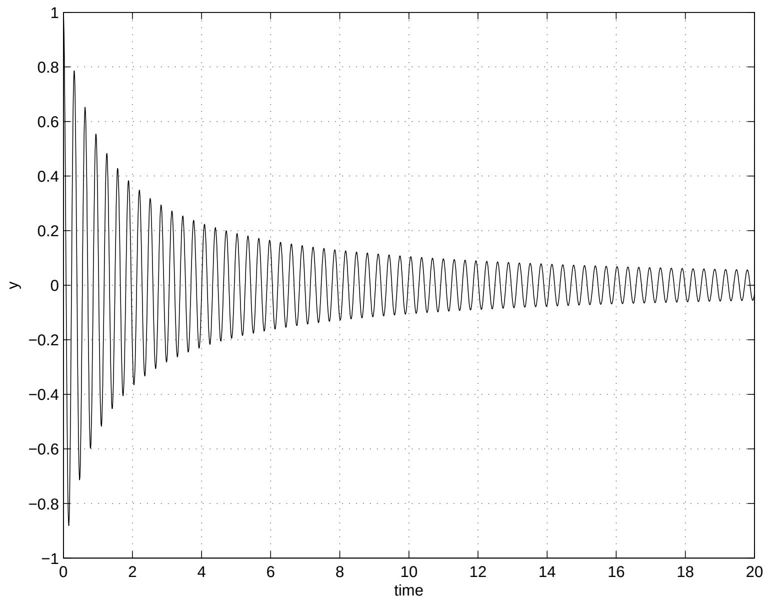
2. An undamped system is excited by a pulse (force of amplitude F) of finite (**but not infinitesimal!** duration. Find $x(t)$ during and after the pulse presuming $10\ddot{x} + 8.8826 \times 10^4 x = f(t)$ and the pulse lasts for $\frac{1}{15}$ s.

3. Given air damping, viscous damping, and Coulomb damping, determine which is apparent in the following response. **Prove it.** You answer will be graded on the merit of your explanation. No points will be given for a guess without sufficient explanation.



4. *Grad student/bonus* Determine the natural frequencies and mode shapes for a clamped-free bar. The equation of motion of a bar is $\left(\frac{E}{\rho}\right)\frac{\partial^2 w(x,t)}{\partial x^2} = \frac{\partial^2 w(x,t)}{\partial t^2}$. (20% of other points)