## 1 Final Exam

Short answers (2 points each). Answer in blue book only.

- 1. The type of material (or the effect) that makes converting mechanical stresses into voltages in load cells and accelerometers is:
- 2. Which of the following parameter(s) has(ve) no effect on the steady-state frequency of response to harmonic excitation for a linear system: mass, damping, natural frequency, driving frequency.
- 3. Why should you use Lagrange's equations instead of the Energy Method to derive equations of motion? (no more than two sentences)
- 4. Are mode shapes orthogonal to one another? If so, how?
- 5. What are the properties of the mass matrix?

Long problems. 10 points each.

1. Find the response, x(t), for all times t > 0 of the underdamped system modeled by the following equation:

$$m\ddot{x} + c\dot{x} + kx = \delta(t) + e^{\frac{-2\pi\zeta}{\sqrt{1-\zeta^2}}}\delta(t - \frac{2\pi}{\omega_d})$$

2. READ the equation very carefully. Sketch the non-dimensionalized magnitude of the response and phase versus r for multiple values of  $\zeta$  and label the values in simplest terms at r = 0, r = 1, and  $r = \infty$ .

$$m\ddot{x} + c\dot{x} + 2kx = ky$$

where  $y(t) = Y \cos(\omega_b t)$ .

3. Given

$$S = \begin{bmatrix} 1 & 1 \\ 3 & -3 \end{bmatrix}$$

and

$$\mathbf{r}(t) = \begin{bmatrix} \sin(4t) \\ 0.1\sin(4t) \end{bmatrix}$$

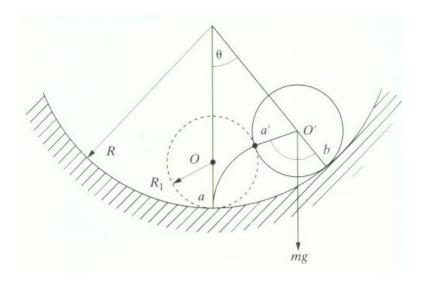
find  $\mathbf{x}(t)$ .

4. Given

$$M = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$
, and  $K = \begin{bmatrix} 65 & -35 \\ -35 & 65 \end{bmatrix}$ 

find the natural frequencies and mode shapes of the system. The mode shapes do not need to be mass normalized.

5. Obtain the equations of motion and natural frequency(s) for the system below. Assume rolling without slipping. The smaller pipe has radius  $R_1$ , and the larger pipe has a radius of R. The mass of the inner pipe is  $M_1$ , and the mass of the outer pipe is  $M_2$ . Assume a small thicknesses for the pipes.



6. Graduate Students/Undergraduate Bonus (20%): Solve for the steady-state (particular) response of the following system if the boundary conditions are presumed to be clamped-clamped where  $c = \sqrt{\tau/\rho}$ .

$$w_{tt}(x,t) - c^2 w_{xx}(x,t) = 100\delta(t)\delta(x - l/2)$$

Recall that the integral of a Dirac delta function times another function is equal to the "another function" evaluated when the argument of the argument of the Dirac delta function is zero.

BONUS: What is h(t) called? What is  $H(j\omega)$  called? What is the relationship between them? (4 points)