ME 460/660, Mechanical Vibration Final Exam, Fall 2005

Closed book, closed notes. Use one provided $8\frac{1}{2} \times 11$ formula sheet and turn in with exam. Test books will be provided. Problems are to be done in the test book.

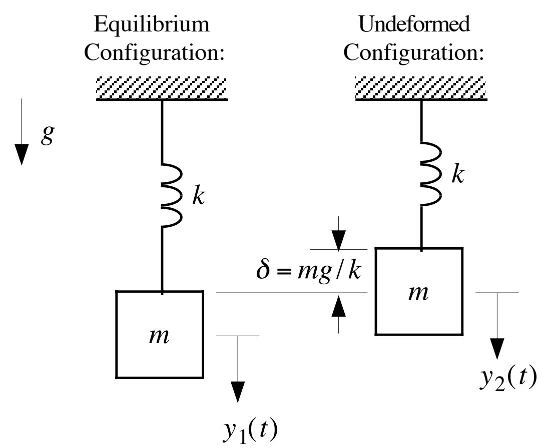
Short answers (2 points each). **Answer in blue book only**.

1. What advantage does a seismometer have over piezoceramic type accelerometer when used as an accelerometer?
2. What are the advantages of the symmetric eigenvalue problem?
3. The impulse response of system is
$$x(t) = 10e^{-.01t}\sin(.9t)$$
write the homogeneous equation of motion.
4. Why should you use Lagrange's equations instead of the Energy Method to derive equations of motion? (no more than two sentences)
5. What are the advantages of using Lagrange's equations instead of Newton's Laws.
6. Are mode shapes orthogonal to one another? If so, how?
7. What are the properties of the stiffness and mass matrices?
8. What are the properties of the modes?
9. The modal summation method takes advantage of **what property** of **what items** to significantly reduce computational effort.
10. In practice, what method is used in place of the convolution integral for solving for system responses.

Long problems. 10 points each.

1. From an EGR 101 Exam:

The deflection of a spring-mass system can be measured from either the equilibrium configuration of the spring, $y_1(t)$, or the undeformed configuration of the spring, $y_2(t)$. As illustrated in the figure below, the difference between the two is the static deflection, $\delta = mg/k$:



If the mass is displaced from the *equilibrium* configuration, the deflection $y_1(t)$ satisfies the following second order differential equation, where the right hand side is zero:

$$m\ddot{y}_1 + ky_1(t) = 0. \tag{1}$$

However, if the mass is applied suddenly to the *undeformed* configuration of the spring, the deflection $y_2(t)$ satisfies the following differential equation, where the right hand side is nonzero:

$$m\ddot{y}_2 + ky_2(t) = mg. \tag{2}$$

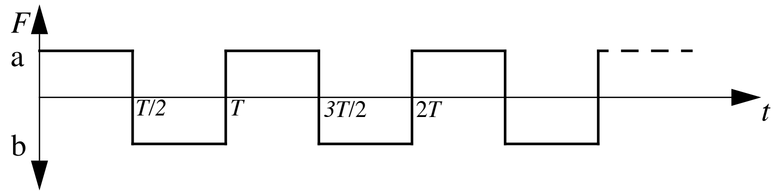
a) Determine the *transient* solution to equation (2). How does this differ from the transient solution to equation (1)?

b) Determine the *steady-state* solution to equation (2). How does this differ from the steady-state solution to equation (1)?

c) Determine the *total* solution to equation (2), subject to the initial conditions $y_2(0) = \dot{y}_2(0) = 0$.

d) Given your solution to part c), determine both the maximum and minimum values of the deflection $y_2(t)$. How does the maximum deflection compare to the static deflection δ?

2. Find the Fourier series of $F(t)$ shown below



where $a = 2$ and $b = -1$.

3. *READ the equation very carefully.* Sketch the non-dimensionalized magnitude of the response and phase versus r for multiple values of ζ and label the values in simplest terms at $r = 0$, $r = 1$, and $r = \infty$.

$$m\ddot{x} + c\dot{x} + 2kx = ky$$

where $y(t) = Y\cos(\omega_b t)$.

4. A MDOF system with

$$S = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

no damping, and natural frequencies of 3 and 10 is excited by an force of

$$\mathbf{F}(t) = \begin{bmatrix} \delta(t) \\ 0 \end{bmatrix}$$

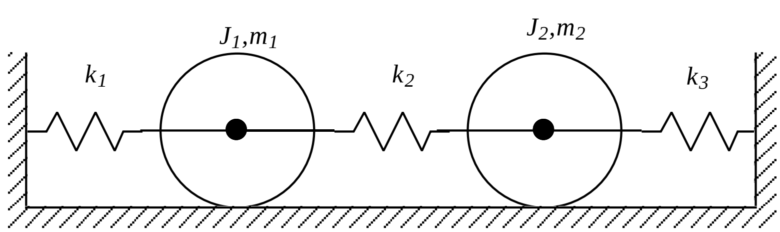
Find $\mathbf{x}(t)$

5. Given

$$M = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}, \text{ and } \qquad K = \begin{bmatrix} 35 & -35 \\ -35 & 35 \end{bmatrix}$$

find the natural frequencies and mode shapes of the system. The mode shapes do not need to be mass normalized. Describe the natural motions of the two modes/frequency pairs.

6. Obtain the equation**s** of motion the following system. Assume rolling without slipping. Use values as given. Do not assume uniform disk, or other non-available information.



7. Graduate Students/Undergraduate Bonus (20%): Solve for the steady-state (particular) response of the following system if the boundary conditions are presumed to be free-free where $c = \sqrt{\tau/\rho}$.

$$w_{tt}(x,t) - c^2 w_{xx}(x,t) = 100\delta(t)\delta(x - l/2)$$

Recall that the integral of a Dirac delta function times another function is equal to the "another function" evaluated when the argument of the argument of the Dirac delta function is zero.

BONUS: What is $h(t)$ called? What is $H(j\omega)$ called? What is the relationship between them? (4 points)