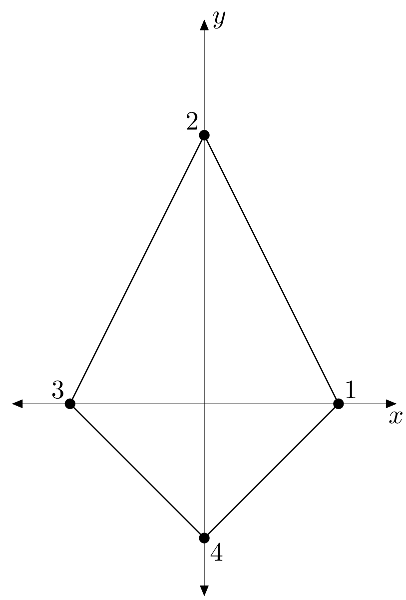
One formula sheet, closed notes, closed book. Test books will be provided. 1 hour, 40 min. *Problems must be done in order in the test books.* 10 points each.

1. Find the strain at (ξ, η)=(-1,-1) of a bilinear quadrilateral (Q4) element with nodes 1-4 at (1,0), (0,2), (-1,0), and (0,-1) in terms of u_3 and v_3 (presume all other nodal displacements are zero).

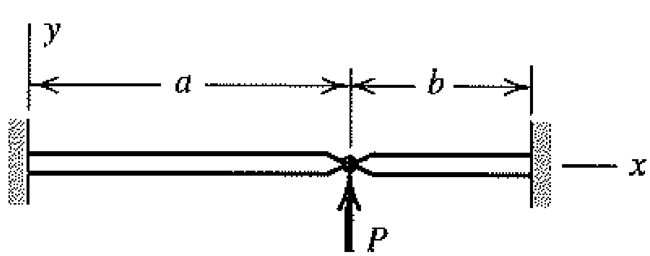


2. Node 2 of the preceding problem significantly skews the element, perhaps beyond its ability to provide reliable results.

   (a) Identify minimum and maximum values of y for node 2 using a quantitative metric that can be calculated within computer code to produce a warning. You will have to take a rough guess at what are reasonable values. Provide justification (write up why you chose these values).

   (b) Explain a method you would use to determine what are "good" and "bad" values of the quantitative metric

3. Two collinear cantilever beams are connected by a frictionless hinge as shown. Flexural stiffness EI_z is the same for both beams. Load P and the deflections are confined to the xy plane. Write the stiffness matrix that operates on the "active" dof. Ignore transverse shear deformation.



For reference, the elemental stiffness matrix for a beam in the xy plane is

$$K = \frac{EI_x}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

4. Derive the stiffness matrix in the x-y plane where the beam runs along the y axis by using an orthonormal rotation. Hint: Consider an initial transformation that changes from using θ_z to $\theta_z L$ allowing the constants L to be removed from the matrix.

Table 1: Approximate Gauss point integration values

# points	ξ_i	w_i
1	0	2
2	$\pm\frac{1}{\sqrt{3}}$	1
3	0, $\pm\sqrt{0.6}$	$\frac{8}{9}$, $\frac{5}{9}$
4	-0.86, -0.34, 0.34, 0.86	0.35, 0.65, 0.65, 0.35