ME 712, Finite Element Method Applications Exam 1, Spring 2005

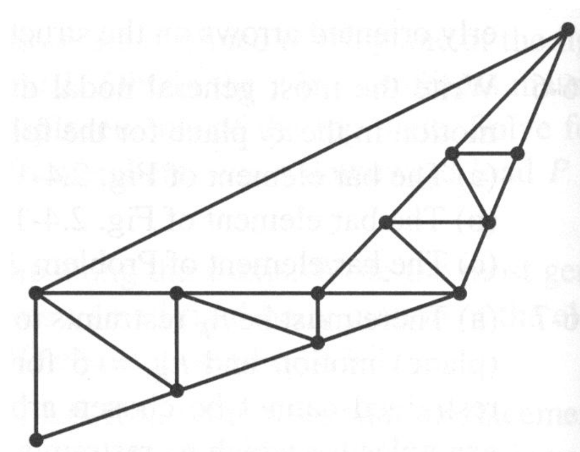
One formula sheet, closed notes. Test books will be provided. Two hours. *Problems must be done in order in the test books.* 10 points each.

1. A beam rests on a compliant foundation. The strain energy in the foundation due to deformation of the beam (per unit length) is $\frac{1}{2}k(x)v(x)^2$. Recall that the strain energy per unit length of the beam due to bending is $\frac{1}{2}EI\frac{dv(x)}{dx}^2$. Derive the *change* to the beam stiffness matrix element, K_{21} due to the addition of this foundation. The shape functions are

$$N = \left[1 - 3\frac{x^2}{l^2} + 2\frac{x^3}{l^3} \quad x - 2\frac{x^2}{l} + \frac{x^3}{l^2} \quad 3\frac{x^2}{l^2} - 2\frac{x^3}{l^3} \quad -\frac{x^2}{l} + \frac{x^3}{l^2}\right] \tag{1}$$

2. Determine the mass matrix for a rod in local coordinates (the rod being between $x = 0$ and $x = l$) presuming ρ is constant, but $A(x) = A_1 + (A_2 - A_1)\frac{x}{l}$. *Set up all math, but solve only for M_{11}.*

3. Assume that the structure shown has one DOF per node and that each straight line between the nodes is a two-node element. Try to assign a node numbering that minimizes the largest semi-bandwidth b_{max}. For this numbering, what is the semi-bandwidth?



Bonus (4 points): The rod mass matrix for a uniformly distributed rod is

$$M = \frac{\rho A \ell}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

What is the matrix in 3-D?