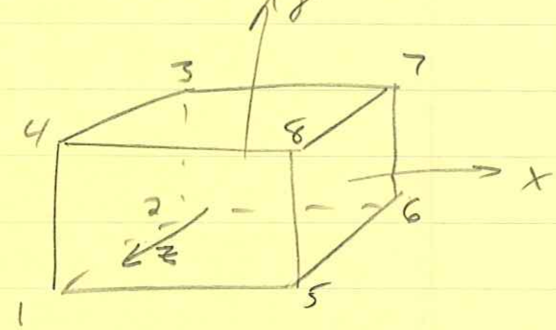
ME712 Final Exam Solution Sp 2012

2) DoF BC



1) Fixed x y z
2) Fixed x, y
3) Fixed x
4) Fixed x
5-8 Free

3) First, Find shape function

$$w = a_1 + a_2 \xi + a_3 \xi^2 + a_4 \xi^3$$
$$w' = a_2 + 2 a_3 \xi + 3 a_4 \xi^2$$

$$\begin{array}{c} w(-1) \\ w'(-1) \\ w(1) \\ w'(1) \end{array} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & -2 & 3 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

Solving for a_1, a_2, a_3, a_4 gives

$$N_1 = 0.5 - 0.75 \xi + 0.25 \xi^3$$

For the change in K_{11}, we only need N_1

$$U = \frac{1}{2}\int_0^{l} \gamma w^2 \, dx$$

$$= \frac{1}{2}\gamma \int_{-1}^{1} \underline{d}^T \underline{N}^T \underline{N}\, \underline{d} \; \det(J)\, d\xi \qquad \det J = \frac{l}{2}$$

We only need K_{11}, so

$$K_{11} = \frac{\gamma l}{2}\int_{-1}^{1} N_1^2 \, d\xi$$

N_1^2 is 6^{th} order polynomial
Need 4 point Gauss
point integration.

Table gives 3 point. You need 4.
Extra 2 points if you recognize this.

Using 3 points gives $K_{11} = 0.3708 \gamma l$
The correct answer is $K_{11} = 0.371 \gamma l$

The error in under integrating is 0.4%!

4) Using subspace, to find 1 mode, I need to start by guessing 2.

$$\bar{X}_1 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$X_1 = K^{-1} M \bar{X}_1 =$ K is singular!

So, first, $\tilde{K} = K + M$ (shift is I)

$$X_1 = \tilde{K}^{-1} \underset{\underset{I}{\uparrow}}{M} \bar{X}_1 = \begin{bmatrix} 1 & 0.25 \\ 1 & 0.5 \\ 1 & 0.25 \end{bmatrix}$$

$$K_{red} = \begin{bmatrix} 3 & 1 \\ 1 & 0.5 \end{bmatrix} \qquad M_{red} = \begin{bmatrix} 3 & 1 \\ 1 & 0.375 \end{bmatrix}$$

eig($M^{-1}K$) gives

$$\Lambda = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} \qquad Q = \begin{bmatrix} -1 & 0.316 \\ 0 & -0.949 \end{bmatrix}$$

$$\bar{X}_2 = X_1 Q_1 = \begin{bmatrix} -1 & 0.0791 \\ -1 & -0.1581 \\ -1 & 0.0791 \end{bmatrix}$$

$$X_2 = \bar{K}^{-1} M \bar{X}_2 = \begin{bmatrix} -1 & 0.0198 \\ -1 & -0.0395 \\ -1 & 0.0198 \end{bmatrix}$$

$$K_{2red} = \begin{bmatrix} 3 & 0 \\ 0 & 0.0094 \end{bmatrix} \quad M_{2red} = \begin{bmatrix} 3 & 0 \\ 0 & 0.0023 \end{bmatrix}$$

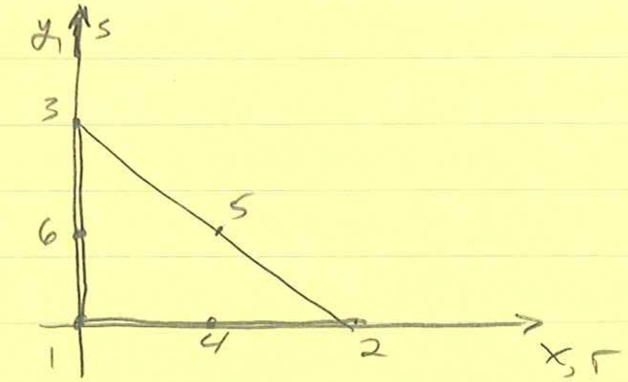
These eqns are already diagonalized,
so $\lambda_1 = 1$, $\lambda_2 = \sqrt{\frac{0.0094}{0.0023}} = 4$

Taking out the shift

$$\omega_1 = \sqrt{\lambda_1 - 1} = 0 \text{ rad/s}$$

$$\underline{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

5)



Since nodes are nicely spaced, we can use formulas for CST Triangle

$$N_1 = 1 - r - s \qquad J = \begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} \\ \frac{\partial x}{\partial s} & \frac{\partial y}{\partial s} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$N_2 = r$$

$$N_3 = s \qquad x = r, \quad y = s$$

$\frac{\det(J)}{2}$

$$A = \int_A 1\, dx\, dy = 1\left(\sum_{i=1}^{1} 1\right)\frac{1}{2} = \frac{1}{2}$$

$$\int_A x^2\, dx\, dy = \sum_{i=1}^{3} w_i r_i^2 \, \frac{J}{2} = \frac{1}{2} \cdot \frac{1}{3}\left(\left(\frac{2}{3}\right)^2 + \left(\frac{1}{6}\right)^2 + \left(\frac{1}{6}\right)^2\right)$$

$$= \frac{1}{6} \cdot \left(\frac{4}{9} + \frac{1}{18}\right)$$

$$= \frac{1}{6}\ \frac{9}{18} = \frac{1}{12}$$

$$K_x = \sqrt{\frac{\frac{1}{12}}{\frac{1}{2}}} = \sqrt{\frac{1}{6}} = \frac{1}{\sqrt{6}}$$