ME 712, Finite Element Method Applications Exam 1, Spring 2007

One formula sheet, closed notes. Test books will be provided. Two hours. *Problems must be done in order in the test books.* 10 points each.

1. Strain ϵ_x is given by $\epsilon_x = \frac{\partial u}{\partial x}$. What expression for ϵ_x is given when u in a four-noded element is given by $u = a_1 + a_2 x + a_3 y + a_4 xy + a_5 x^2 + a_6 x^2 y$? For a mesh of such elements, what can you say about the interelement continuity of ϵ_x? How does the strain vary across the element

2. A beam rests on a compliant foundation. The strain energy in the foundation due to deformation of the beam (per unit length) is $\frac{1}{2}k(x)v(x)^2$. Recall that the strain energy per unit length of the beam due to bending is $\frac{1}{2}EI\frac{dv(x)}{dx}^2$. Derive the *change* (**I really mean it, only the CHANGE**) to the beam stiffness matrix element, K_{21}, due to the addition of this foundation. The shape functions are

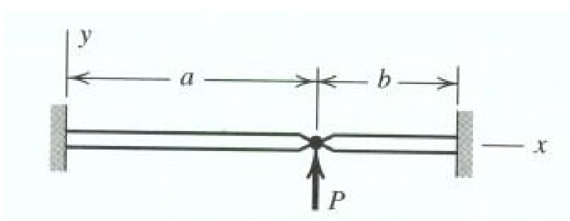
$$N = \begin{bmatrix} 1 - 3\frac{x^2}{l^2} + 2\frac{x^3}{l^3} & x - 2\frac{x^2}{l} + \frac{x^3}{l^2} & 3\frac{x^2}{l^2} - 2\frac{x^3}{l^3} & -\frac{x^2}{l} + \frac{x^3}{l^2} \end{bmatrix} \tag{1}$$

3. Determine the 1,3 element of the stiffness matrix for a beam in local coordinates (the beam being between $x = 0$ and $x = l$) presuming E is constant, but $I(x) = I_1 + (I_2 - I_1)\frac{x}{l}$. *Set up all math in matrix form without multiplying out the matrices, but solve only for* K_{13}.

$$N = \begin{bmatrix} 1 - 3\frac{x^2}{l^2} + 2\frac{x^3}{l^3} & x - 2\frac{x^2}{l} + \frac{x^3}{l^2} & 3\frac{x^2}{l^2} - 2\frac{x^3}{l^3} & -\frac{x^2}{l} + \frac{x^3}{l^2} \end{bmatrix} \tag{2}$$

4. Two collinear cantilever beams are connected by a frictionless hinge as shown. Flexural stiffness, EI_z, is the same for both beams. Load P and deformations are confined to the xy plane. Write the stiffness matrix that operates on the "active" DOFs. Ignore transverse shear deformation. The elemental stiffness matrix is given by

$$K = \frac{EI_x}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$



5. Define *isoparametric element.*

6. Sketch the rigid and flexible body modes of a basic beam element in 2-D. How do the eigenvalues of the stiffness matrix change when the element is rotated 45 degrees?