Selection of appropriate state variables, and
Use of impedance.

- Obtain circuit models in transfer-function and block-diagram form.
- Apply impedance methods to obtain models of op-amp systems.
- Apply Newton's laws, electrical circuit laws, and electromagnetic principles to develop models of electromechanical systems.
- Analyze the performance of motors and amplifiers in motion control systems.
- Apply MATLAB and Simulink to analyze models of circuits and electromechanical systems in state-variable and transfer function form.

PROBLEMS

Section 6.1 Electrical Elements

6.1 Determine the equivalent resistance R_e of the circuit shown in Figure P6.1, such that $v_s = R_e i$. All the resistors are identical and have the resistance R.

6.2 Determine the voltage v_1 in terms of the supply voltage v_s for the circuit shown in Figure P6.2.

6.3 The Wheatstone bridge, like that shown in Figure P6.3, is used for various measurements. For example, a strain gage sensor utilizes the fact that the resistance of wire changes when deformed. If the sensor is one resistance leg of the bridge, then the deformation can be determined from the voltage v_1. Determine the relation between the voltage v_1 and the supply voltage v_s.

Figure P6.1

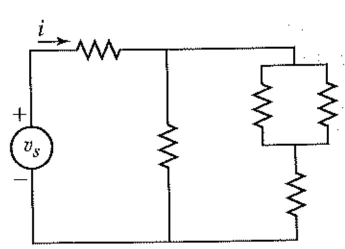


Figure P6.2

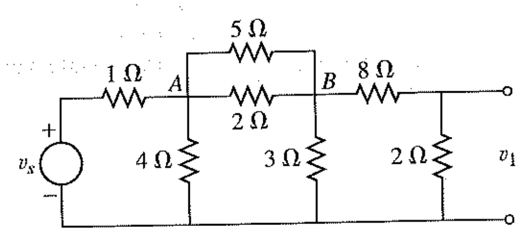
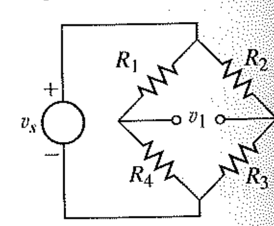


Figure P6.3



Section 6.2 Circuit Examples

6.4 The power supply of the circuit shown in Figure P6.4 supplies a voltage of 9 V. Compute the current i and the power P that must be supplied.

6.5 Obtain the model of the voltage v_1, given the current i_s, for the circuit shown in Figure P6.5.

Figure P6.4

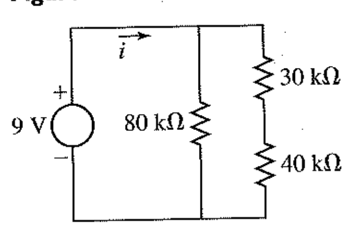
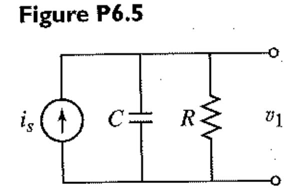


Figure P6.5



6.6 (a) Obtain the model of the voltage v_o, given the supply voltage v_s, for the circuit shown in Figure P6.6. (b) Suppose $v_s(t) = Vu_s(t)$. Obtain the expressions for the free and forced responses for $v_o(t)$.

6.7 (a) Obtain the model of the voltage v_o, given the supply voltage v_s, for the circuit shown in Figure P6.7. (b) Suppose $v_s(t) = Vu_s(t)$. Obtain the expressions for the free and forced responses for $v_o(t)$.

6.8 (a) Obtain the model of the voltage v_o, given the supply voltage v_s, for the circuit shown in Figure P6.8. (b) Suppose $v_s(t) = Vu_s(t)$. Obtain the expressions for the free and forced responses for $v_o(t)$.

6.9 (a) The circuit shown in Figure P6.9 is a model of a solenoid, such as that used to engage the gear of a car's starter motor to the engine's flywheel. The solenoid is constructed by winding wire around an iron core to make an electromagnet. The resistance R is that of the wire, and the inductance L is due to the electromagnetic effect. When the supply voltage v_s is turned on, the resulting current activates the magnet, which moves the starter gear. Obtain the model of the current i given the supply voltage v_s. (b) Suppose $v_s(t) = Vu_s(t)$ and $i(0) = 0$. Obtain the expression for the response for $i(t)$.

Figure P6.6

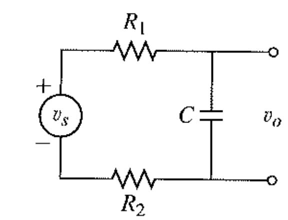


Figure P6.7

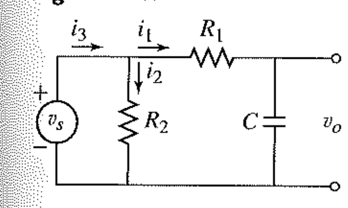


Figure P6.8

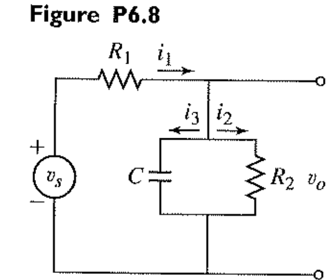
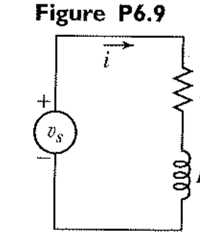


Figure P6.9



6.10 The resistance of a telegraph line is $R = 10\ \Omega$, and the solenoid inductance is $L = 5$ H. Assume that when sending a "dash," a voltage of 12 V is applied while the key is closed for 0.3 s. Obtain the expression for the current $i(t)$ passing through the solenoid. (See Figure 6.2.15)

6.11 Obtain the model of the voltage v_o, given the supply voltage v_s, for the circuit shown in Figure P6.11.

6.12 Obtain the model of the voltage v_o, given the supply voltage v_s, for the circuit shown in Figure P6.12.

6.13 Obtain the model of the current i, given the supply voltage v_s, for the circuit shown in Figure P6.13.

Figure P6.11

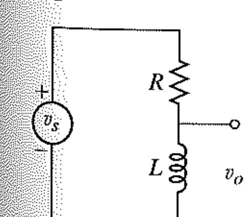


Figure P6.12

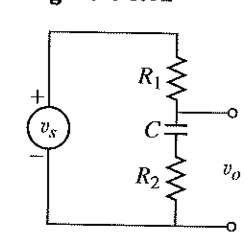
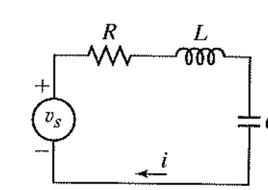


Figure P6.13



6.14 Obtain the model of the voltage v_o, given the supply current i_s, for the circuit shown in Figure P6.14.

Figure P6.14

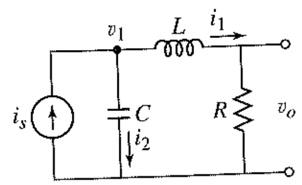
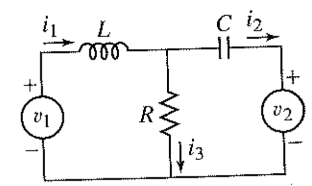


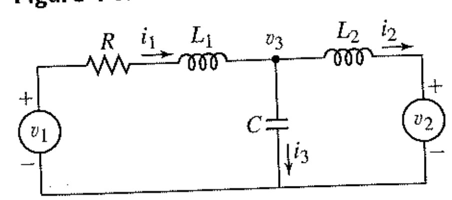
Figure P6.15



6.15 Obtain the model of the currents i_1, i_2, and i_3, given the input voltages v_1 and v_2, for the circuit shown in Figure P6.15.

6.16 Obtain the model of the currents i_1, i_2, and the voltage v_3, given the input voltages v_1 and v_2, for the circuit shown in Figure P6.16.

Figure P6.16



6.17 For the circuit shown in Figure P6.14, determine a suitable set of state variables, and obtain the state equations.

6.18 For the circuit shown in Figure P6.15, determine a suitable set of state variables, and obtain the state equations.

6.19 For the circuit shown in Figure P6.16, determine a suitable set of state variables, and obtain the state equations.

Section 6.3 Transfer Functions and Impedance

6.20 Use the impedance method to obtain the transfer function $V_o(s)/V_s(s)$ for the circuit shown in Figure P6.20.

6.21 Use the impedance method to obtain the transfer function $I(s)/V_s(s)$ for the circuit shown in Figure P6.21.

6.22 Use the impedance method to obtain the transfer function $V_o(s)/V_s(s)$ for the circuit shown in Figure P6.22.

6.23 Use the impedance method to obtain the transfer function $V_o(s)/I_s(s)$ for the circuit shown in Figure P6.23.

Figure P6.20

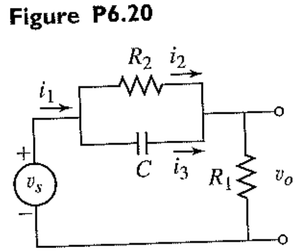


Figure P6.21

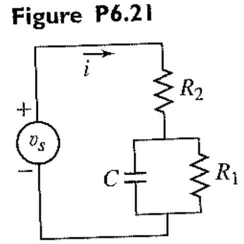


Figure P6.22

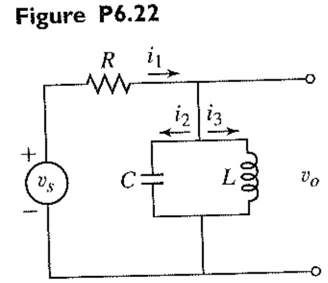
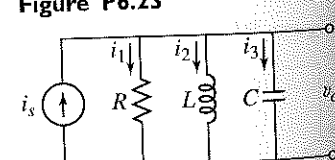


Figure P6.23



6.24 Use the impedance method to obtain the transfer function $V_o(s)/V_s(s)$ for the circuit shown in Figure P6.24.

Figure P6.24

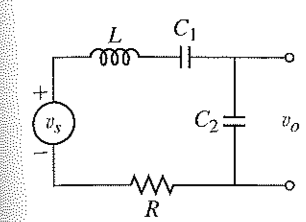


Figure P6.25

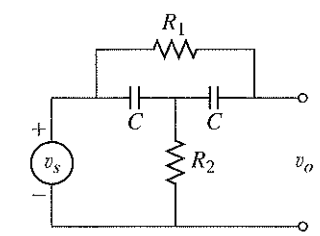
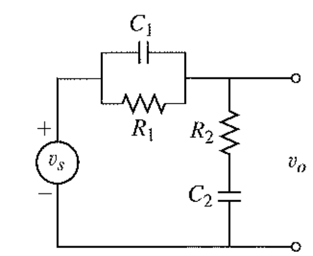


Figure P6.26



6.25 Use the impedance method to obtain the transfer function $V_o(s)/V_s(s)$ for the circuit shown in Figure P6.25.

6.26 Use the impedance method to obtain the transfer function $V_o(s)/V_s(s)$ for the circuit shown in Figure P6.26.

6.27 Draw a block diagram of the circuit shown in Figure P6.15. The inputs are v_1 and v_2. The output is i_2.

6.28 Draw a block diagram of the circuit shown in Figure P6.16. The inputs are v_1 and v_2. The output is v_3.

Section 6.4 Operational Amplifiers

6.29 Obtain the transfer function $V_o(s)/V_i(s)$ for the op-amp system shown in Figure P6.29.

6.30 Obtain the transfer function $V_o(s)/V_i(s)$ for the op-amp system shown in Figure P6.30.

Figure P6.29

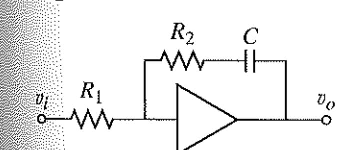
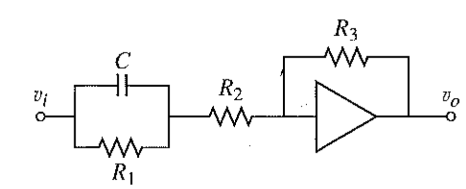


Figure P6.30



6.31 Obtain the transfer function $V_o(s)/V_i(s)$ for the op-amp system shown in Figure P6.31.

6.32 Obtain the transfer function $V_o(s)/V_i(s)$ for the op-amp system shown in Figure P6.32.

Figure P6.31

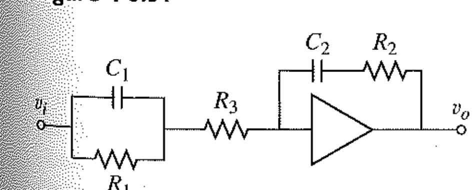
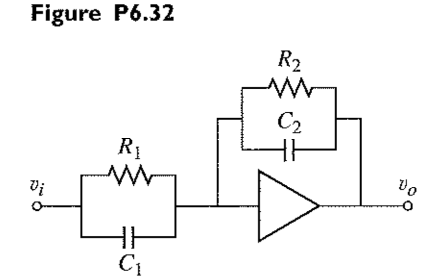
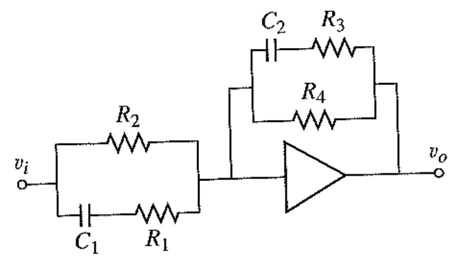


Figure P6.32



6.33 Obtain the transfer function $V_o(s)/V_i(s)$ for the op-amp system shown in Figure P6.33.

Figure P6.33



Section 6.5 Electric Motors

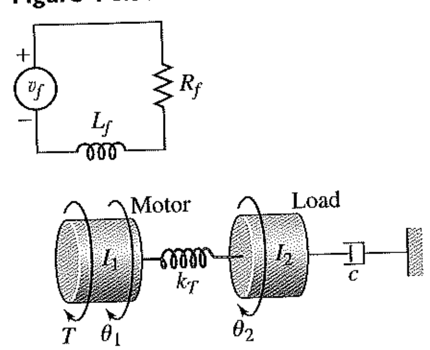
6.34 (a) Obtain the transfer function $\Theta(s)/V_i(s)$ for the D'Arsonval meter. (b) Use the final value theorem to obtain the expression for the steady-state value of the angle θ if the applied voltage v_i is a step function.

6.35 (a) Obtain the transfer function $\Omega(s)/T_L(s)$ for the field-controlled motor of Example 6.5.2. (b) Modify the field-controlled motor model in Example 6.5.2 so that the output is the angular displacement θ, rather than the speed ω, where $\omega = \dot{\theta}$. Obtain the transfer functions $\Theta(s)/V_f(s)$ and $\Theta(s)/T_L(s)$.

6.36 Modify the motor model given in Example 6.5.2 to account for a gear pair between the motor shaft and the load. The ratio of motor speed to load speed ω_L is N. The motor inertia is I_m and the motor damping is c_m. The load inertia is I_L and the load damping is c_L. The load torque T_L acts directly on the load inertia. Obtain the transfer functions $\Omega_L(s)/V_f(s)$ and $\Omega_L(s)/T_L(s)$.

6.37 The derivation of the field-controlled motor model in Section 6.5 neglected the elasticity of the motor-load shaft. Figure P6.37 shows a model that includes this elasticity, denoted by its equivalent torsional spring constant k_T. The motor inertia is I_1, and the load inertia is I_2. Derive the differential equation model with θ_2 as output and v_f as input.

Figure P6.37



6.38 Figure P6.38 is the circuit diagram of a speed-control system in which the dc motor voltage v_a is supplied by a generator driven by an engine. This system has been used on locomotives whose diesel engine operates most efficiently at one speed. The efficiency of the electric motor is not as sensitive to speed and thus can be used to drive the locomotive at various speeds. The motor voltage v_a is varied by changing the generator input voltage v_f. The voltage v_a is related to the generator field current i_f by $v_a = K_f i_f$.

Derive the system model relating the output speed ω to the voltage v_f, and obtain the transfer function $\Omega(s)/V_f(s)$.

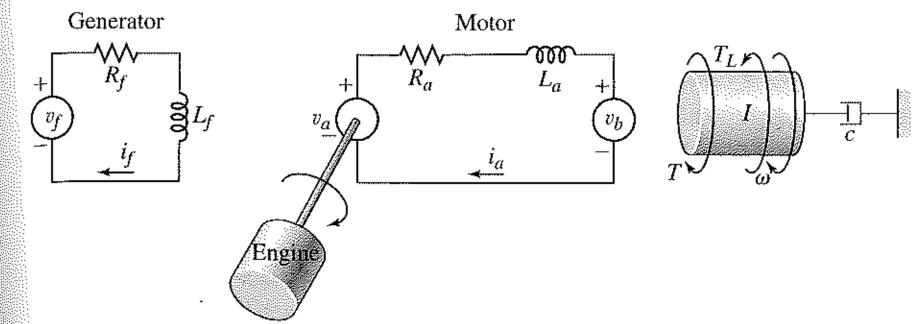


Figure P6.38

Section 6.6 Analysis of Motor Performance

6.39 The parameter values for a certain armature-controlled motor are

$$K_T = K_b = 0.2 \text{ N} \cdot \text{m/A}$$

$$c = 5 \times 10^{-4} \text{ N} \cdot \text{m} \cdot \text{s/rad} \qquad R_a = 0.8\ \Omega$$

The manufacturer's data states that the motor's maximum speed is 3500 rpm, and the maximum armature current it can withstand without demagnetizing is 40 A.

Compute the no-load speed, the no-load current, and the stall torque. Determine whether the motor can be used with an applied voltage of $v_a = 15$ V.

6.40 The parameter values for a certain armature-controlled motor are

$$K_T = K_b = 0.05 \text{ N} \cdot \text{m/A}$$

$$R_a = 0.8\ \Omega$$

$$L_a = 3 \times 10^{-3} \text{ H} \qquad I = 8 \times 10^{-5} \text{ kg} \cdot \text{m}^2$$

where I includes the inertia of the armature and that of the load. Investigate the effect of the damping constant c on the motor's characteristic roots and on its response to a step voltage input. Use the following values of c (in N · m · s/rad): $c = 0$, $c = 0.01$, and $c = 0.1$. For each case, estimate how long the motor's speed will take to become constant, and discuss whether or not the speed will oscillate before it becomes constant.

6.41 The parameter values for a certain armature-controlled motor are

$$K_T = K_b = 0.2 \text{ N} \cdot \text{m/A}$$

$$c = 5 \times 10^{-4} \text{ N} \cdot \text{m} \cdot \text{s/rad} \qquad R_a = 0.8\ \Omega$$

$$L_a = 4 \times 10^{-3} \text{ H} \qquad I = 5 \times 10^{-4} \text{ kg} \cdot \text{m}^2$$

where c and I include the effect of the load.

a. Obtain the step response of $i_a(t)$ and $\omega(t)$ if the applied voltage is $v_a = 10$ V.
b. Obtain the step response of $i_a(t)$ and $\omega(t)$ if the load torque is $T_L = 0.2$ N · m.

6.42 The following measurements were performed on a permanent magnet motor when the applied voltage was $v_a = 20$ V. The measured stall current was 25 A. The no-load speed was 2400 rpm and the no-load current was 0.6 A. Estimate the values of K_b, K_T, R_a, and c.

6.43 A single link of a robot arm is shown in Figure P6.43. The arm mass is m and its center of mass is located a distance L from the joint, which is driven by a motor torque T_m through spur gears. Suppose that the equivalent inertia felt at the motor shaft is 0.215 kg · m^2. As the arm rotates, the effect of the arm weight generates an opposing torque that depends on the arm angle, and is therefore nonlinear. For this problem, however, assume that the effect of the opposing torque is a constant 4.2 N · m at the motor shaft. Neglect damping in the system. It is desired to have the motor shaft rotate through $3\pi/4$ rad in a total time of 2 s, using a trapezoidal speed profile with $t_1 = 0.3$ s and $t_2 = 1.7$ s.

The given motor parameters are $R_a = 4\ \Omega$, $L_a = 3 \times 10^{-3}$ H, and $K_T = 0.3$ N · m/A. Compute the energy consumption per cycle; the maximum required torque, current, and voltage; the rms torque; and the rms current.

6.44 A conveyor drive system to produce translation of the load is shown in Figure P6.44. Suppose that the equivalent inertia felt at the motor shaft is 0.05 kg · m^2, and that the effect of Coulomb friction in the system produces an opposing torque of 3.6 N · m at the motor shaft. Neglect damping in the system. It is desired to have the motor shaft rotate through 11 revolutions in a total time of 3 s, using a trapezoidal speed profile with $t_1 = 0.5$ s and $t_2 = 2.5$ s.

The given motor parameters are $R_a = 3\ \Omega$, $L_a = 4 \times 10^{-3}$ H, and $K_T = 0.4$N · m/A. Compute the energy consumption per cycle; the maximum required torque, current, and voltage; the rms torque; and the rms current.

Figure P6.43

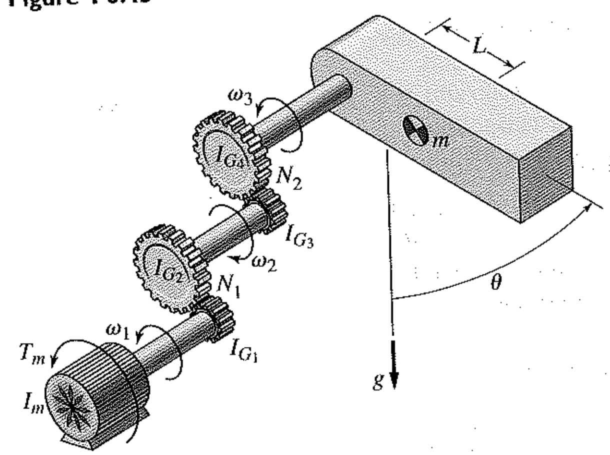
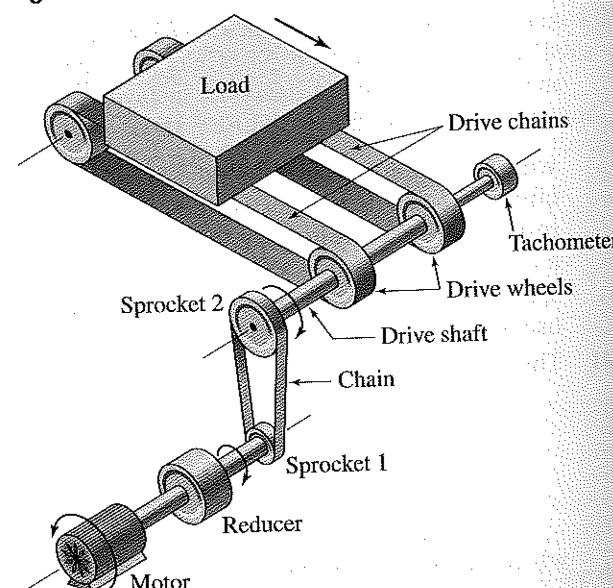


Figure P6.44



Section 6.7 Sensors and Electroacoustic Devices

6.45 Consider the accelerometer model in Section 6.7. Its transfer function can be expressed as

$$\frac{Y(s)}{Z(s)} = -\frac{s^2}{s^2 + (c/m)s + k/m}$$

Suppose that the input displacement is $z(t) = 10 \sin 120t$ mm. Consider two cases, in SI units: (a) $k/m = 100$ and $c/m = 18$ and (b) $k/m = 10^6$ and $c/m = 1800$. Obtain the steady-state response $y(t)$ for each case. By comparing the amplitude of $y(t)$ with the amplitudes of $z(t)$ and $\ddot{z}(t)$, determine which case can be used as a vibrometer (to measure displacement) and which can be used as an accelerometer (to measure acceleration).

6.46 An electromagnetic microphone has a construction similar to that of the speaker shown in Figure 6.7.2, except that there is no applied voltage and the sound waves are incoming rather than outgoing. They exert a force f_s on the diaphragm whose mass is m, damping is c, and stiffness is k. Develop a model of the microphone, whose input is f_s and output is the current i in the coil.

6.47 Consider the speaker model developed in Example 6.7.1. The model, whose transfer function is given by equation (3) in that example, is third order and therefore we cannot obtain a useful expression for the characteristic roots. Sometimes inductance L and damping c are small enough to be ignored. If $L = 0$, the model becomes second order. (a) Obtain the transfer function $X(s)/V(s)$ for the case where $L = c = 0$, and obtain the expressions for the two roots. (b) Compare the results with the third-order case where

$$\begin{aligned} m &= 0.002 \text{ kg} & k &= 4 \times 10^5 \text{ N/m} \\ K_f &= 16 \text{ N/A} & K_b &= 13 \text{ V} \cdot \text{s/m} \\ R &= 12\ \Omega & L &= 10^{-3} \text{ H} \\ c &= 0 \end{aligned}$$

Section 6.8 MATLAB Applications

6.48 The parameter values for a certain armature-controlled motor are

$$\begin{aligned} K_T = K_b &= 0.2 \text{ N} \cdot \text{m/A} \\ c &= 5 \times 10^{-4} \text{ N} \cdot \text{m} \cdot \text{s/rad} & R_a &= 0.8\ \Omega \\ L_a &= 4 \times 10^{-3} \text{ H} & I &= 5 \times 10^{-4} \text{ kg} \cdot \text{m}^2 \end{aligned}$$

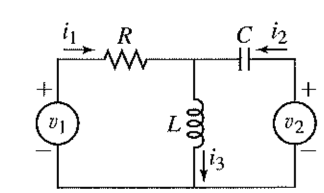
where c and I include the effect of the load. The load torque is zero.

Use MATLAB to obtain a plot of the step response of $i_a(t)$ and $\omega(t)$ if the applied voltage is $v_a = 10$ V. Determine the peak value of $i_a(t)$.

6.49 Consider the motor whose parameters are given in Problem 6.48. Use MATLAB to obtain a plot of the response of $i_a(t)$ and $\omega(t)$ if the applied voltage is the modified step $v_a(t) = 10(1 - e^{-100t})$ V. Determine the peak value of $i_a(t)$.

6.50 Consider the circuit shown in Figure P6.50. The parameter values are $R = 10^3\ \Omega$, $C = 2 \times 10^{-6}$ F, and $L = 2 \times 10^{-3}$ H. The voltage v_1 is a step input of magnitude 5 V, and the voltage v_2 is sinusoidal with frequency of 60 Hz and an amplitude of 4 V. The initial conditions are zero. Use MATLAB to obtain a plot of the current response $i_3(t)$.

Figure P6.50



6.51 The parameter values for a certain armature-controlled motor are

$$\begin{aligned} K_T = K_b &= 0.2 \text{ N} \cdot \text{m/A} \\ c &= 3 \times 10^{-4} \text{ N} \cdot \text{m} \cdot \text{s/rad} & R_a &= 0.8\ \Omega \\ L_a &= 4 \times 10^{-3} \text{ H} & I &= 4 \times 10^{-4} \text{ kg} \cdot \text{m}^2 \end{aligned}$$

The system uses a gear reducer with a reduction ratio of 3:1. The load inertia is 10^{-3} kg · m^2, the load torque is 0.04 N · m, and the load damping constant is 1.8×10^{-3} N · m · s/rad.

Use MATLAB to obtain a plot of the step response of $i_a(t)$ and $\omega(t)$ if the applied voltage is $v_a = 20$ V. Determine the peak value of $i_a(t)$.

6.52 The parameter values for a certain armature-controlled motor are

$$K_T = K_b = 0.05 \text{ N}\cdot\text{m/A}$$
$$c = 0 \qquad R_a = 0.8\ \Omega$$
$$L_a = 3 \times 10^{-3}\text{ H} \qquad I = 8 \times 10^{-5}\text{ kg}\cdot\text{m}^2$$

where I includes the inertia of the armature and that of the load. The load torque is zero. The applied voltage is a trapezoidal function defined as follows.

$$v(t) = \begin{cases} 60t & 0 \le t \le 0.5 \\ 30 & 0.5 < t < 2 \\ 60(2.5 - t) & 2 \le t \le 2.5 \\ 0 & 2.5 < t \le 4 \end{cases}$$

a. Use MATLAB to obtain of plot of the response of $i_a(t)$ and $\omega(t)$.

b. Compute the energy consumption per cycle; the maximum required torque, current, and voltage; the rms torque; and the rms current.

6.53 A single link of a robot arm is shown in Figure P6.43. The arm mass is m and its center of mass is located a distance L from the joint, which is driven by a motor torque T_m through spur gears. Suppose that the equivalent inertia felt at the motor shaft is 0.215 kg · m^2. As the arm rotates, the effect of the arm weight generates an opposing torque that depends on the arm angle, and is therefore nonlinear. The effect of the opposing torque at the motor shaft is $4.2 \sin\theta$ N · m. Neglect damping in the system. It is desired to have the motor shaft rotate through $3\pi/4$ rad in a total time of 2 s, using a trapezoidal speed profile with $t_1 = 0.3$ s and $t_2 = 1.7$ s.

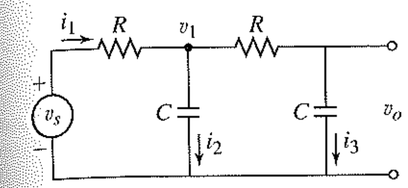
The given motor parameters are $R_a = 4\ \Omega$, $L_a = 3 \times 10^{-3}$ H, and $K_T = 0.3$ N · m/A. Use MATLAB to obtain of plot of the response of the motor current and the motor speed.

Section 6.9 Simulink Applications

6.54 Consider the circuit shown in Figure P6.50. The parameter values are $R = 10^4\ \Omega$, $C = 2 \times 10^{-6}$ F, and $L = 2 \times 10^{-3}$ H. The voltage v_1 is a single pulse of magnitude 5 V and duration 0.05 s, and the voltage v_2 is sinusoidal with frequency of 60 Hz and an amplitude of 4 V. The initial conditions are zero. Use Simulink to obtain a plot of the current response $i_3(t)$.

6.55 Consider the circuit shown in Figure P6.55. The parameter values are $R = 2 \times 10^4\ \Omega$ and $C = 3 \times 10^{-6}$ F. The voltage v_s is $v_s(t) = 12u_s(t) + 3 \sin 120\pi t$ V. The initial conditions are zero. Use Simulink to obtain a plot of the responses $v_o(t)$ and $v_1(t)$.

Figure P6.55



6.56 The parameter values for a certain armature-controlled motor are

$$K_T = K_b = 0.2 \text{ N}\cdot\text{m/A}$$
$$c = 5 \times 10^{-4}\text{ N}\cdot\text{m}\cdot\text{s/rad} \qquad R_a = 0.8\ \Omega$$
$$L_a = 4 \times 10^{-3}\text{ H} \qquad I = 5 \times 10^{-4}\text{ kg}\cdot\text{m}^2$$

where c and I include the effect of the load. The load torque is zero.

a. Use Simulink to obtain a plot of the step response of the motor torque and speed if the applied voltage is $v_a = 10$ V. Determine the peak value of the motor torque.

b. Now suppose that the motor torque is limited to one-half the peak value found in part (a). Use Simulink to obtain a plot of the step response of the motor torque and speed if the applied voltage is $v_a = 10$ V.

6.57 The parameter values for a certain armature-controlled motor are

$$K_T = K_b = 0.05 \text{ N}\cdot\text{m/A}$$
$$c = 0 \qquad R_a = 0.8\ \Omega$$
$$L_a = 3 \times 10^{-3}\text{ H} \qquad I = 8 \times 10^{-5}\text{ kg}\cdot\text{m}^2$$

where I includes the inertia of the armature and that of the load. The load torque is zero. The applied voltage is a trapezoidal function defined as follows.

$$v(t) = \begin{cases} 60t & 0 \le t \le 0.5 \\ 30 & 0.5 < t < 2 \\ 60(2.5 - t) & 2 \le t \le 2.5 \\ 0 & 2.5 < t \le 4 \end{cases}$$

A trapezoidal profile can be created by adding and subtracting ramp functions starting at different times. Use several Ramp source blocks and Sum blocks in Simulink to create the trapezoidal input. Obtain a plot of the response of $i_a(t)$ and $\omega(t)$.