ME 460/660 Final, Spring '96

One equation sheet. Front and back. No examples. No derivations. It must be turned in with the exam. Each problem is worth 25 points.

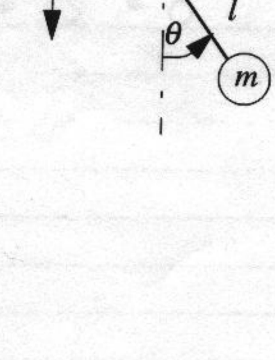
1) Find the spectral matrix, Λ, and eigenvector matrix, P, given the following:

$$M = \begin{bmatrix} 4 & 0 \\ 0 & 36 \end{bmatrix}, K = \begin{bmatrix} 3 & -3 \\ -3 & 27 \end{bmatrix}$$

2) Choose a suspension damping coefficient (c) for a 1000 kg car such that the settling time is less than 3 sec and the displacement transmissibility if less than 0.5 at 3 Hz ($r = 2$).

3) The force exerted by an eccentric (e = 0.22 mm) flywheel of 1000 kg is 600 cos(52.4t) N. Design a mounting to reduce the amplitude of the force exerted on the floor to 1% of the force generated. Also, use the suspension system to ensure that the maximum force transmitted to ground never exceeds twice the generated force (at any speed).

4) Derive the equations of motion for the following system using Lagrange's equations with x and θ as the generalized coordinates. The block can only move in the vertical direction. (Hint: Set the datums to be at the unstretched spring length and $\theta = 0$.)



1) $M^{1/2} = \begin{bmatrix} 2 & 0 \\ 0 & 6 \end{bmatrix}$

$M^{-1/2} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{6} \end{bmatrix}$

$M^{-1/2} K M^{-1/2} = \tilde{K} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{6} \end{bmatrix} \begin{bmatrix} 3 & -3 \\ -3 & 27 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{6} \end{bmatrix}$

$\tilde{K} = \begin{bmatrix} \frac{3}{2} & -\frac{3}{2} \\ -\frac{1}{2} & \frac{9}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{6} \end{bmatrix}$

$\tilde{K} = \begin{bmatrix} \frac{3}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{3}{4} \end{bmatrix}$ $+9$

$\det(\tilde{K} - \lambda I) = 0$

$\left(\frac{3}{4} - \lambda\right)^2 - \frac{1}{16} = 0$

$\frac{9}{16} - \frac{3}{2}\lambda + \lambda^2 - \frac{1}{16} = 0$

$\lambda^2 - \frac{3}{2}\lambda + \frac{1}{2} = 0$

$\lambda = \frac{1}{2}, 1$ $+8$

$\Lambda = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix}$

For $\lambda_1 = \frac{1}{2}$:

$$\begin{bmatrix} \frac{3}{4}-\frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{3}{4}-\frac{1}{2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\frac{1}{4} V_1 - \frac{1}{4} V_2 = 0$$

+4

$$-\frac{1}{4} V_1 + \frac{1}{4} V_2 = 0$$

$$\underline{V_1} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

For $\lambda_2 = 1$:

$$\begin{bmatrix} \frac{3}{4}-1 & -\frac{1}{4} \\ -\frac{1}{4} & \frac{3}{4}-1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

+4

$$\underline{V_2} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

2) $\ln .05 = -\xi \omega t_s$

$3 = \xi \omega t_s$

$t_s = 3/\xi\omega < 3$

$\xi\omega > 1$

Since $r = 2$ @ $3\ Hz$, $f = 1.5\ Hz$

$\omega = 2\pi f = 3\pi$ rad/s (5)

$\xi > \frac{1}{\omega} = \frac{1}{3\pi}$

$\xi > \frac{1}{3\pi} \approx \frac{1}{9.42}$

+10

Pick $\xi = .2$

Does this meet $TR < .5$ @ $r = 2$?

$$TR = \left[\frac{1 + (2\xi r)^2}{(1-r^2)^2 + (2\xi r)^2}\right]^{\frac{1}{2}} \overset{?}{<} .5$$

$.1061 < \xi < .322$

$$\frac{1 + .8^2}{9 + .8^2} \overset{?}{<} .25$$

$2000 < C < 6070$

$.17 < .25$

+10

$K = m\omega^2 = 1000 \cdot 9\pi^2 = 88826\ N/m$

$C = \xi\, 2\sqrt{Km} = 3770\ kg/s$

3) a) $TR = .01 = \left[\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}\right]^{1/2}$

b) At resonance, $TR = 2$

$$2 = \left(\frac{1+(2\zeta)^2}{(2\zeta)^2}\right)^{1/2}$$

$$4 \cdot 4\zeta^2 = 1 + 4\zeta^2$$

$$12\zeta^2 = 1$$

$$\zeta \geq .2887$$

+10

$$\text{pick } \zeta = .3$$

$$.0001 = \frac{1+(.6r)^2}{(1-r^2)^2+(.6r)^2}$$

$$.0001(1-2r^2+r^4+.36r^2) = 1+.36r^2$$

$$.0001r^2 \approx .36$$

$$r = 60 = \frac{52.4}{\omega} \qquad \omega = .873 \text{ rad/s}$$

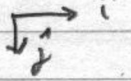
+10

$$K = m\omega^2 = 1000 \cdot .873^2 = 762.7 \text{ N/m}$$

$$C = 2\zeta\sqrt{Km} = 524 \text{ kg/s}$$

+5

4) The position of the pendulum mass is



$$x_p = x\hat{j} + l\cos\theta\,\hat{j} + l\sin\theta\,\hat{i}$$

It's velocity is then

$$V_p = \dot{x}\,\hat{j} - \dot{\theta}\,l\sin\theta\,\hat{j} + \dot{\theta}\,l\cos\theta\,\hat{i}$$

The total kinetic energy is then

$$T = \tfrac{1}{2}m\dot{x}^2 + \tfrac{1}{2}m\left(\dot{x} - \dot{\theta}\,l\sin\theta\right)^2 + \tfrac{1}{2}m\dot{\theta}^2 l^2\cos^2\theta$$

+10

The potential energy is

$$U = \underbrace{\tfrac{1}{2}kx^2 - mgx}_{\text{Top mass}} \underbrace{- mgx + mgl(1-\cos\theta)}_{\text{pendulum mass}}$$

+10

$$U = \tfrac{1}{2}kx^2 - 2mgx + mgl(1-\cos\theta)$$

Lagrange's Equation

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} = 0$$

+5

For $q_i = x$

$$\frac{\partial T}{\partial \dot{x}} = m\dot{x} + m\left(\dot{x} - \dot{\theta}\,l\sin\theta\right)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) = 2m\ddot{x} - m\ddot{\theta}\,l\sin\theta - m\dot{\theta}^2 l\cos\theta$$

$$\frac{\partial T}{\partial x} = 0$$

$$\frac{\partial U}{\partial x} = kx - 2mg$$

(1) $$2m\ddot{x} - m\ddot{\theta}\,l\sin\theta - m\dot{\theta}^2 l\cos\theta + kx - 2mg = 0$$

For $q_i = \theta$

$$\frac{\partial T}{\partial \dot\theta} = -ml\sin\theta\left(\dot x - l\dot\theta\sin\theta\right) + m\dot\theta l^2\cos^2\theta$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot\theta}\right) = -ml\dot\theta\cos\theta\left(\dot x - l\dot\theta\sin\theta\right) - ml\sin\theta\left(\ddot x - l\ddot\theta\sin\theta - l\dot\theta^2\cos\theta\right) + m\ddot\theta l^2\cos^2\theta - 2m\dot\theta^2 l^2\cos\theta\sin\theta$$

$$\frac{\partial T}{\partial \theta} = -m\dot\theta l\cos\theta\left(\dot x - \dot\theta l\sin\theta\right) - m\dot\theta^2 l^2\sin\theta\cos\theta$$

$$\frac{\partial U}{\partial \theta} = mgl\sin\theta$$

$$\cancel{-ml\dot\theta\cos\theta\left(\dot x - l\dot\theta\sin\theta\right)}^{1} - ml\sin\theta\left(\ddot x - l\ddot\theta\sin\theta - l\dot\theta^2\cos\theta\right)^{2} + m\ddot\theta l^2\cos^2\theta \cancel{- 2m\dot\theta^2 l^2\cos\theta\sin\theta}^{2}$$
$$\cancel{+m\dot\theta l\cos\theta\left(\dot x - \dot\theta l\sin\theta\right)}^{1} \cancel{+ m\dot\theta^2 l^2\sin\theta\cos\theta}^{2}$$
$$+mgl\sin\theta = 0$$

$$-ml\sin\theta\,\ddot x + \underbrace{ml^2\sin^2\theta\,\ddot\theta + m\ddot\theta l^2\cos^2\theta}_{ml^2\ddot\theta} + mgl\sin\theta = 0$$

(2) $$l\ddot\theta - \sin\theta\,\ddot x + g\sin\theta = 0$$