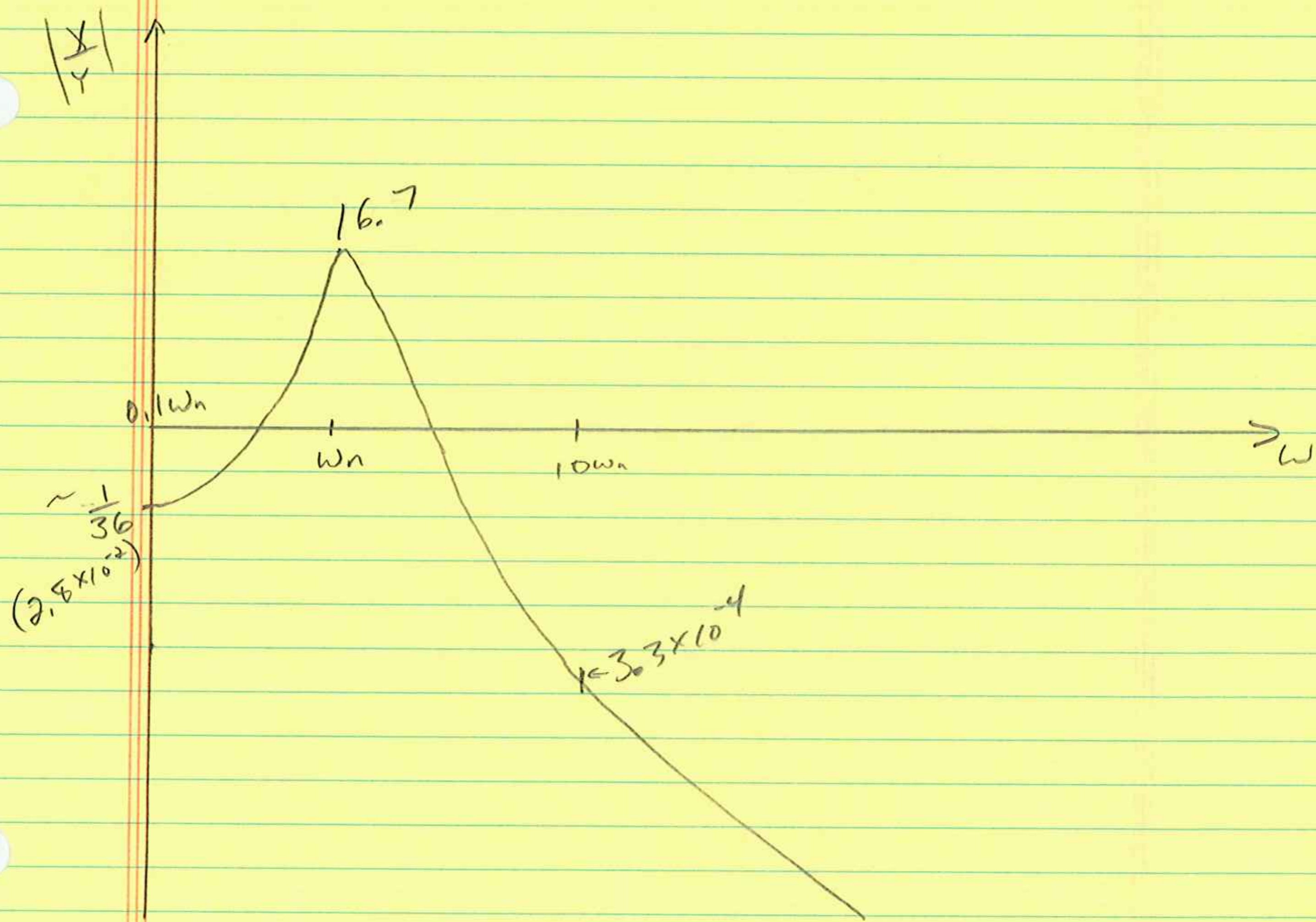
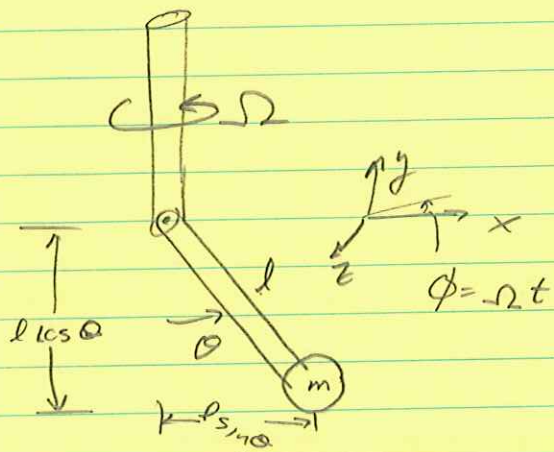
ME 460/660 Exam 1 Solution Spring 2011

1) $$\frac{X}{Y} = \frac{0.1j\omega + 100}{3600 - \omega^2 + 0.1j\omega}$$

Y	ω (rad/s)	|X|	∡ X (rad)	x(t)
10	2π	0.281	6×10^{-3}	$0.281 \sin 6.28t$
0.1	20π	0.0288	−3.06	$0.0288 \sin 62.8t - 3.06$
0.001	200π	3.02×10^{-7}	−2.58	$3.02\times10^{-7} \sin 628t - 2.58$
		in deg	0.35	
			−175	
			−148	



2)



Side note:

Since we can break v into latitude/longitude,

$$v^2 = \dot\theta^2 l^2 + (l\sin\theta\,\Omega)^2$$

$$U = mgl(1-\cos\theta)$$

$$\vec r = -l\cos\theta\,\hat j + l\sin\theta\cos\phi\,\hat i - l\sin\theta\sin\phi\,\hat k$$

$$\vec v = \dot{\vec r} = \dot\theta l\sin\theta\,\hat j + (\dot\theta l\cos\theta\cos\phi - \underset{\dot\phi}{\Omega}\, l\sin\theta\sin\phi)\,\hat i$$
$$-(\dot\theta l\cos\theta\sin\phi + \Omega l\sin\theta\cos\phi)\,\hat k$$

$$v^2 = \dot\theta^2 l^2\sin^2\theta + \dot\theta^2 l^2\cos^2\theta\cos^2\phi - 2\dot\theta\Omega l^2\cos\theta\cos\phi\sin\theta\sin\phi$$
$$+ \Omega^2 l^2\sin^2\theta\sin^2\phi + \dot\theta^2 l^2\cos^2\theta\sin^2\phi$$
$$+ 2\dot\theta\Omega l^2\cos\theta\sin\phi\sin\theta\cos\phi + \Omega^2 l^2\sin^2\theta\cos^2\phi$$
$$= \dot\theta^2 l^2\sin^2\theta + \overbrace{\dot\theta^2 l^2\cos^2\theta}^{2} + \overbrace{\Omega^2 l^2\sin^2\theta}^{3} + \overbrace{0}^{1}$$
$$= \dot\theta^2 l^2 + \Omega^2 l^2\sin^2\theta$$

$$T = \frac{1}{2}m(\dot\theta^2 l^2 + \Omega^2 l^2\sin^2\theta)$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\theta}} = m l^2 \ddot{\theta}$$

$$\frac{\partial T}{\partial \theta} = m \Omega^2 l^2 \sin\theta \cos\theta$$

$$\frac{\partial U}{\partial \theta} = m g l \sin\theta$$

$$m l^2 \ddot{\theta} + m l (g - \Omega^2 l \cos\theta) \sin\theta = 0$$

If $\Omega = 0$, $\quad \ddot{\theta} + \frac{g}{l} \sin\theta = 0$

3) $\frac{50 \text{ cycles}}{9.9 \text{ seconds}} = \underline{5.05 \text{ Hz}} = 31.\underline{37 \text{ rad/s}}$

$\delta = \frac{1}{50} \ln \frac{0.5}{0.04} = 0.05$

$\zeta = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} = 0.008$ (using $\zeta = \frac{\delta}{2\pi}$ gives 0.008)

$\omega_n^2 = \frac{k}{m}$

$m = \frac{k}{\omega_n^2} = \underline{0.102 \text{ kg}}$

$c = 2\zeta\omega_n m = \underline{0.051 \text{ kg/s}}$

4) $\frac{E}{\rho} X''(x) = -\omega_n^2 X(x)$

$$X''(x) + \omega_n^2 \frac{\rho}{E} X(x) = 0$$

$X(x) = A_n \sin \sigma_n x + B_n \cos \sigma_n x$, where $\sigma_n = \omega_n \sqrt{\frac{\rho}{E}}$

$X(0) = 0$ gives $B_n = 0$

$$X'(x)\Big|_{x=\ell} = \sigma_n A_n \cos \sigma_n x \Big|_{x=\ell} = 0$$

$$\sigma_n \ell = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots \frac{(2n-1)}{2}\pi$$

$$\sigma_n = \frac{1}{\ell} \frac{2n-1}{2} \pi$$

$$\omega_n = \frac{(2n-1)\pi}{2\ell} \sqrt{\frac{E}{\rho}} \quad , \quad X_n = A_n \sin \frac{(2n-1)\pi x}{\ell}$$