Wi '09 System Dynamics Exam I solns.

1) $4.912 = A e^{-1/\tau}$ $\qquad 3.293 = A e^{-3/\tau}$

$\ln 4.912 = \ln A + \frac{-1}{\tau}$ ① $\qquad \ln 3.293 = \ln A + \frac{-3}{\tau}$ ②

① - ②

$\ln 4.912 - \ln 3.293 = \frac{2}{\tau}$

$\tau = \frac{2}{\ln 4.912 - \ln 3.293}$

$\therefore \tau = 5$

From ①

$\ln A = \ln 4.912 + \frac{1}{5} = 1.79$

$A = 6$

2) a) $(T_2 + N T_1) = (I_2 + I_1 N^2)\,\dot{\omega}_2$

using equivalent torques / inertias

b) $\dot{\omega}_2 = \left(\frac{T_2 + N T_1}{I_2 + N^2 I_1}\right)$

$\frac{d\dot{\omega}_2}{dN} = 0 = \frac{I_2 T_1 - I_1 N (N T_1 + 2 T_2)}{(I_2 + I_1 N^2)^2}$

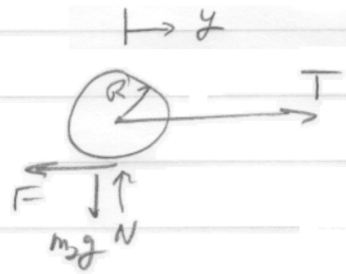
$I_1 T_1 N^2 + 2 I_1 T_2 N - I_2 T_1 = 0$

$N = \frac{-I_1 T_2 + \sqrt{(I_1 T_2)^2 + I_1 T_1^2 I_2}}{I_1 T_1}$

$= -\frac{T_2}{T_1} + \sqrt{\left(\frac{T_2}{T_1}\right)^2 + \frac{I_2}{I_1}}$

Notes of interest, for $I_2 = 0$, $N = 0$ // for $T_2 = 0$, $N = \sqrt{\frac{I_2}{I_1}}$ or $I_1 = I_2/N^2$. This means the inertia 'felt' by motor is equal to the motor's inertia.

3) FBD 1



$\Sigma F = m_2 \ddot{y}$ $\quad \Sigma M = I\alpha = \left(\frac{1}{2} m_2 R^2\right)\frac{\ddot{y}}{R}$

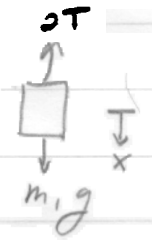
$FR = \frac{1}{2} m_2 R \ddot{y}$

$T - F = m_2 \ddot{y}$ $\quad F = \frac{1}{2} m_2 \ddot{y}$ ②

② into ①

$T = \frac{3}{2} m_2 \ddot{y}$ ③

FBD 2



$\Sigma F = m_1 \ddot{x}$

$m_1 g - 2T = m_1 \ddot{x}$ ④

Since $y = 2x$, ③ becomes

$T = \frac{3}{2} m_2 2\ddot{x} = 3 m_2 \ddot{x}$ ⑤

sub ⑤ into ④

$m_1 g - 6 m_2 \ddot{x} = m_1 \ddot{x}$

$\boxed{(m_1 + 6 m_2)\ddot{x} = m_1 g}$

(If I not $\frac{1}{2} m_2 R^2$,

$\left(m_1 + 4 m_2 + \frac{4I}{R^2}\right)\ddot{x} = m_1 g$