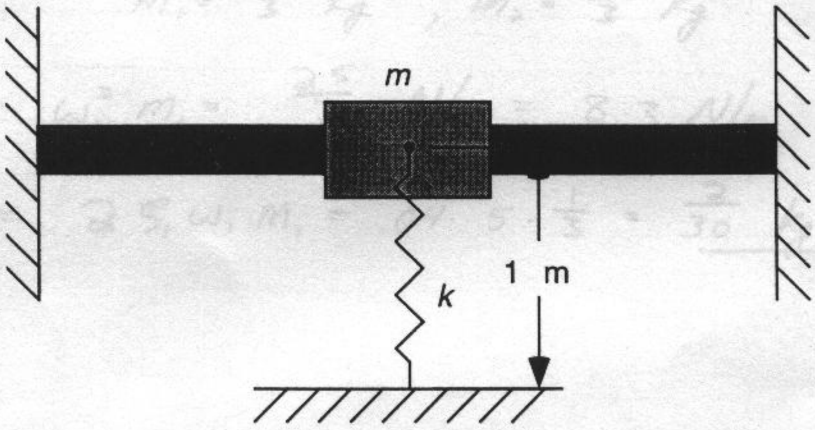
ME 460/660 Exam 1, Spring '95

1) The damping ratio, ζ, and natural frequency, ω, of a single degree of freedom (SDOF) system are identified by examination of the free response to be 0.02 and 5 rad/sec. If 1 kg is added to the SDOF system, the damping ratio is 0.01? What is the mass, damping coefficient, and stiffness of the original system? Use correct units. (20 points)

2) Design the suspension system for an automobile (choose the stiffness and damping value) subject to the following constraints: four wheels (four identical springs/dashpots), a maximum additional static displacement of 1 cm for each additional 80 kg passenger entering the car, and a response envelope that decays to 5% of its original value after 2 seconds.

3) Derive the equation of motion for a slider on a rod connected to a spring as shown below. Do **not** try to solve the differential equation.



4) A torsional system consists of a disk of mass moment of inertias $J = 10$ kg-m^2, a torsional damper of damping constant $c = 300$ N-m-s/rad, and a steel shaft of diameter 4 cm and length 1 m (fixed at one end with the disk at the other). A steady state angular oscillation of amplitude 2° is observed when a harmonic torque of magnitude 1000 N-m is applied to the disk. (a) Find the frequency of the applied torque, and (b) find the maximum torque transmitted to the support.

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Solutions

1) $\zeta_1 = .02, \quad \omega_1 = 5, \quad \zeta_2 = .01$

$$\zeta_1 = \frac{c}{c_{cr_1}} = \frac{c}{2\sqrt{k m_1}}$$

$$\zeta_2 = \frac{c}{c_{cr_2}} = \frac{c}{2\sqrt{k m_2}} \qquad m_2 = m_1 + 1$$

$$\frac{\zeta_1}{\zeta_2} = 2 = \sqrt{\frac{m_2}{m_1}}$$

$$4 m_1 = m_1 + 1$$

$$m_1 = \frac{1}{3}\ kg, \quad m_2 = \frac{4}{3}\ kg$$

$$k = \omega_1^2 m_1 = \underline{\frac{25}{3}\ N/m} = 8.3\ N/m$$

$$c = 2\zeta_1 \omega_1 m_1 = .04 \cdot 5 \cdot \frac{1}{3} = \underline{\frac{2}{30}\ kg \cdot s} = .0667\ kg \cdot s$$

2) Four spring, $K_t = 4 \cdot K$

Static deflection

$$9.8 \cdot 80 = K \cdot .01$$

$$K_t = 78400 \ N/m$$

$$\omega = \sqrt{\frac{K_t}{m}} = \sqrt{\frac{78400}{1000}} = \sqrt{78.4} = 8.85 \ rad/s$$

$$e^{-\zeta\omega t} = .05 \quad @ \quad t = 2$$

$$e^{-\zeta \cdot 8.85 \cdot 2} = .05$$

$$17.71 \ \zeta = 2.99$$

$$\zeta = .169$$

$$C_t = \zeta \cdot 2\sqrt{Km} = 2995 \ kg \cdot s$$

Check:

$$1000 \ddot{x} + 2995 \dot{x} + 78400 x = 0$$

$$\ddot{x} + 2.995 \dot{x} + 78.4 x = 0$$

$$\omega = \sqrt{78.4} = 8.85, \ \zeta = \frac{2.995}{2 \cdot 8.85} = .169$$

$$K = K_t/4 = 19600 \ N/m$$

$$C = C_t/4 = 748.75$$

Values greater than this meet the requirements, but if they are too great you are defeating the whole purpose of having a suspension.

3) Define x as positive motion to the right
Define y as the stretch of the spring

$$y = \sqrt{1^2 + x^2} - 1 \,, \quad y^2 = 1 + x^2 - 2(1+x^2)^{1/2} + 1$$

$$U = \frac{1}{2} k y^2 = \frac{1}{2} k \left(2 + x^2 - 2(1+x^2)^{1/2}\right)$$

$$T = \frac{1}{2} m \dot{x}^2$$

$$\frac{d}{dt}(T + U) = 0$$

$$\frac{d}{dt}\left(\frac{1}{2} m \dot{x}^2 + \frac{1}{2} k \left(2 + x^2 - 2(1+x^2)^{1/2}\right)\right) = 0$$

$$m \cancel{\dot{x}} \ddot{x} + \frac{1}{2} k \left(2x\cancel{\dot{x}} - 2\frac{1}{2} \cdot 2x\cancel{\dot{x}}(1+x^2)^{-1/2}\right) = 0$$

$$m\ddot{x} + k\left(x - \frac{x}{(1+x^2)^{1/2}}\right) = 0$$

4) $J = 10\ kg\text{-}m^2$, $C = 300\ N\text{-}m\ s/rad$

$d = .04\ m$, $G = 8 \times 10^{10}\ N/m^2$, $l = 1\ m$

$$J_p = \frac{\pi d^4}{32} = 2.513 \times 10^{-7}\ m^4$$

$$K = \frac{G J_p}{l} = 20104\ Nm/rad$$ (using incorrect value for G) 4 +4

$$\omega = \sqrt{\frac{K}{J}} = 44.837\ rad/s$$ 8 +4

$$\zeta = \frac{300}{2\sqrt{201040}} = .3345$$ 13 +5

Torque $T = 1000\ Nm$

$$\frac{A_0 k}{F_0} = \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$ 17 +4

$$2.03 = \left(\frac{F_0}{A_0 k}\right)^2 = (1-r^2)^2 + (2\zeta r)^2$$

If dynamic forces were neglected,

$F = 20104 \cdot \left(\frac{2\pi}{180}\right) = 701\ Nm$ +2

so the motion is attenuated. $\therefore r >> 1$ (see chart below) 19

r	$(1-r^2)^2 + (2\zeta r)^2$
.8	.416
.7	.479
.6	.570
1.2	.838
1.4	1.79
1.45	2.15
1.43	2.007 close enough

$r = 1.43 = \frac{\omega_{dr}}{\omega}$

$\omega_{dr} = 1.43 \; 44.837 = 64.26$ rad/s +2

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The torque transmitted to the base is

$T_t = k\theta + c\dot{\theta}$

The amplitude is the square root of the sum of the square of the components.

$T_t = \sqrt{(K A_0)^2 + (C A_0 \omega_{dr})^2} = \underline{972 \text{ N-m}}$ +4

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