ME 460/660 Final Exam Solutions

1) $m\ddot{x} + c\dot{x} + kx = 0$

$$\ddot{x} + 2\zeta\omega\dot{x} + \omega^2 x = 0$$

$$s^2 X(s) - s x_0 - v_0 + 2\zeta\omega\left(sX(s) - x_0\right) + \omega^2 X(s) = 0$$

+10 $$X(s) = \frac{s x_0 + v_0 + 2\zeta\omega x_0}{s^2 + 2\zeta\omega s + \omega^2}$$

$$X(s) = x_0 \frac{s + \frac{v_0}{x_0} + 2\zeta\omega}{s^2 + 2\zeta\omega s + \omega^2}$$

$$x(t) = x_0\left(e^{-\zeta\omega t}\cos\omega_d t + \frac{\frac{v_0}{x_0} + \zeta\omega}{\omega_d} e^{-\zeta\omega t}\sin\omega_d t\right)$$

+10 $$x(t) = e^{-\zeta\omega t}\left(x_0\cos\omega_d t + \frac{v_0 + \zeta\omega x_0}{\omega_d}\sin\omega_d t\right)$$

2) $m = 30\,kg$, $T.R. = 10^{-4}$, $\omega_{dr} = 200\,Hz$

$\zeta = .025$

$$TR = \sqrt{\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}}$$

Assume no damping + tighten design criteria to compensate. $TR = 1\times10^{-6}$

$$1\times10^{-6} = \frac{1}{r^2-1}$$

$$r^2 - 1 = 1\times10^{6}$$

$$r \approx 1000 \quad \text{choose} \quad r = 1000$$

See if $\zeta = .025$, $r = 1000$ meets original design criteria.

$$TR = \sqrt{\frac{1+(2\cdot .025\cdot 1000)^2}{(1000^2-1)^2+(2\cdot .025\cdot 1000)^2}}$$

$$= \sqrt{\frac{2501}{(1000^2-1)^2+2500}} = 5\times10^{-5} < 10^{-4}$$

$$r = \frac{\omega_{dr}}{\omega}, \quad \omega = \frac{400\pi}{1000} = 1.257\ rad/s$$

$$K = m\omega^2 = 47.4\ N/m$$

$$C = 2\zeta\omega m = 1.886\ kg\cdot s$$

($r \geq 500.4$ to meet design specifications, $K < 189\ N/m$)

3) The natural frequencies of the individual vibration absorbers must be equal to the driving frequencies.

$$\omega_1 = \frac{1000 \cdot 2\pi}{60} = 104.72 \text{ rad/s}$$

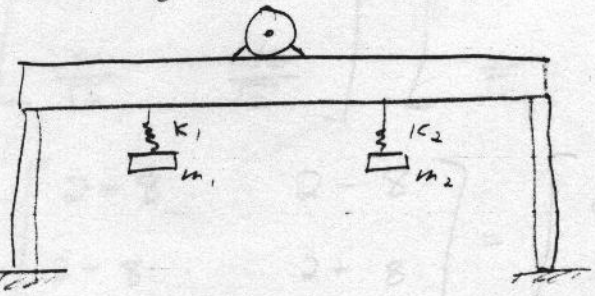
$$\omega_2 = 2\omega_1 = 209.4 \text{ rad/s}$$

Choose $\mu = .05$ for each absorber so that the total added mass is 10%

$$m_1 = m_2 = 3.75 \text{ kg}$$

$$k_1 = m_1 \omega_1^2 = 41100 \text{ N/m}$$

$$k_2 = m_2 \omega_2^2 = 4 m_1 \omega_1^2 = 164500 \text{ N/m}$$



They must be attached at points on the structure, not to each other.

Yes

In start-up, shut-down, and switching speeds, some of the modes of the table/isolator system will be excited for short periods of time.

4) $\Lambda = \begin{bmatrix} 4 & 0 \\ 0 & 16 \end{bmatrix}$

The mode shapes are <u>not</u> constrained, so we may choose them.

$$P = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

Because $M = I$, $P^T \tilde{K} P = P^T K P = \Lambda$, $K = P K P^T$

$(P^T)^{-1} K P^{-1}$

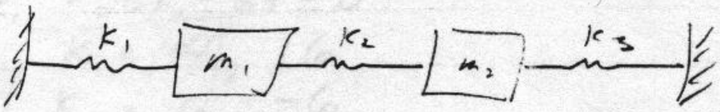
$$K = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 16 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{\sqrt{2}} & \frac{16}{\sqrt{2}} \\ \frac{4}{\sqrt{2}} & \frac{-16}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} 2+8 & 2-8 \\ 2-8 & 2+8 \end{bmatrix} = \begin{bmatrix} 10 & -6 \\ -6 & 10 \end{bmatrix}$$

The stiffness matrix is $\begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 + k_3 \end{bmatrix}$

where



<u>Thus, $k_2 = 6$, $k_1 = k_3 = 4$</u>

$$\det \begin{bmatrix} k_1+k_2-\lambda & -k_2 \\ -k_2 & k_2+k_3-\lambda \end{bmatrix} = 0$$

If $k_1 = k_3$

$$(k_1+k_2-\lambda)^2 - k_2^2 = 0$$

$$(k_1+k_2)^2 - 2\lambda(k_1+k_2) + \lambda^2 - k_2^2 = 0$$

$$\lambda^2 - 2(k_1+k_2)\lambda + k_1^2 + 2k_1k_2 = 0$$

$$\lambda = 4,\ 16$$

$$(\lambda-4)(\lambda-16) = 0$$

$$\lambda^2 - 20\lambda + 64 = 0$$

① $k_1 + k_2 = 10$, ② $k_1^2 + 2k_1k_2 = 64$

$k_2 = 10 - k_1$

① into ②

$$k_1^2 + 2k_1(10-k_1) = 64$$

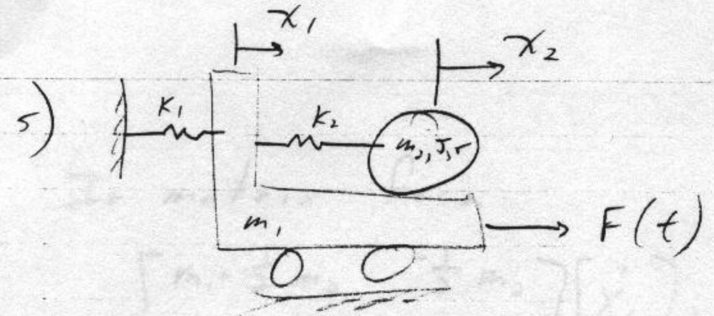
$$k_1^2 + 20k_1 - 2k_1^2 = 64$$

$$-k_1^2 + 20k_1 - 64 = 0$$

$$\boxed{\begin{matrix} k_1 = 4, & 16 \\ k_2 = 6, & -6 \end{matrix}}$$

$k_1 = 4$ N/m , $k_2 = 6$ N/m , $k_3 = k_1 = 4$ N/m

5)



x_2 is absolute motion of disk

$$U = \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k_2 (x_2 - x_1)^2$$ +6

$$T = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 + \frac{1}{2} J \dot{\theta}^2$$

$$r\dot{\theta} = \dot{x}_2 - \dot{x}_1 \quad , \quad J = \frac{1}{2} m_2 r^2$$

$$T = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 + \frac{1}{4} m_2 (\dot{x}_2 - \dot{x}_1)^2$$ +6

Lagrange's Eqn

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \cancel{\frac{\partial T}{\partial q_i}}^{\,0} + \frac{\partial U}{\partial q_i} = Q_i$$ +4

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_1}\right) = m_1 \ddot{x}_1 - \frac{1}{2} m_2 (\ddot{x}_2 - \ddot{x}_1)$$

$$= \left(m_1 + \frac{1}{2} m_2\right) \ddot{x}_1 - \frac{1}{2} m_2 \ddot{x}_2$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_2}\right) = m_2 \ddot{x}_2 + \frac{1}{2} m_2 (\ddot{x}_2 - \ddot{x}_1)$$

$$= -\frac{1}{2} m_2 \ddot{x}_1 + \frac{3}{2} m_2 \ddot{x}_2$$

<u>$i=1$:</u>

$$\left(m_1 + \frac{1}{2} m_2\right) \ddot{x}_1 - \frac{1}{2} m_2 \ddot{x}_2 + k_1 x_1 + k_2 (x_1 - x_2) = F(t)$$

<u>$i=2$:</u> +4

$$-\frac{1}{2} m_2 \ddot{x}_1 + \frac{3}{2} m_2 \ddot{x}_2 + k_2 (x_2 - x_1) = 0$$

In matrix form

$$\begin{bmatrix} m_1 + \frac{1}{2} m_2 & -\frac{1}{2} m_2 \\ -\frac{1}{2} m_2 & \frac{3}{2} m_2 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} F(t) \\ 0 \end{bmatrix}$$