ME 460 Test 2 Solutions, Fa '09

1) $\ddot{x} + 40\dot{x} + 10{,}000x = 2.5\times10^{-3}\, p(t)$

$$\frac{X(j\omega)}{P(j\omega)} = \frac{2.5\times10^{-3}}{(10000 - \omega^2 + 40j\omega)}$$

p(t) term	$\frac{X(j\omega)}{P(j\omega)}$	$\lvert X\rvert$	$\angle X$ (rad)
34083	2.5×10^{-7}	8.521×10^{-3}	0
$-26996\cos(52.36t)$	3.31×10^{-7}; $-.28$ rad	8.93×10^{-3}	2.86
$8307.7\sin(52.36t)$	"	2.75×10^{-3}	0.281
$1416.7\cos(104.72t)$	5.82×10^{-7}; -1.8 rad	8.24×10^{-4}	-1.8
$3608.3\sin(104.72t)$	"	2.10×10^{-3}	-1.8
$-5833.3\cos(157.08t)$	1.57×10^{-7}; -2.74	9.14×10^{-4}	0.405
$2333.3\sin(157.08t)$	"	3.65×10^{-4}	-2.74

$$x(t) = 8.521\times10^{-3} + 8.93\times10^{-3}\cos(52.36t + 2.86)$$
$$+ 2.75\times10^{-3}\sin(52.36t - 0.281)$$
$$+ 8.24\times10^{-4}\cos(104.72t - 1.8)$$
$$+ 2.10\times10^{-3}\sin(104.72t - 1.8)$$
$$+ 9.14\times10^{-4}\cos(157.08t + 0.405)$$
$$+ 3.65\times10^{-4}\sin(157.08t - 2.74)$$

note sign changes and 180° shift to match

2) For $t < 0.01$

$$x(t) = \frac{1}{\sqrt{10}} \int_0^t 300\tau \sin\frac{(t-\tau)}{\sqrt{10}}\, d\tau$$

For $0.01 < t < 0.02$

$$x(t) = \frac{1}{\sqrt{10}} \int_0^{0.01} 300\tau \sin\frac{(t-\tau)}{\sqrt{10}}\, d\tau + \frac{1}{\sqrt{10}} \int_{0.01}^{t} (6-300\tau) \sin\frac{(t-\tau)}{\sqrt{10}}\, d\tau$$

For $0.02 < t$

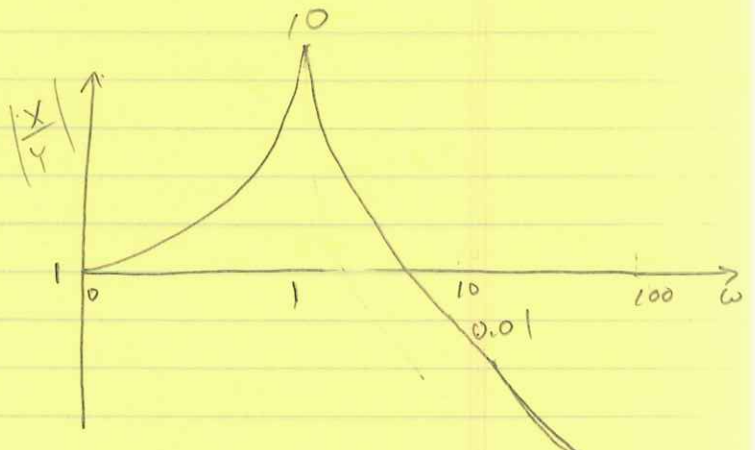
$$x(t) = \frac{1}{\sqrt{10}} \int_0^{0.01} 300\tau \sin\frac{(t-\tau)}{\sqrt{10}}\, d\tau + \frac{1}{\sqrt{10}} \int_{0.01}^{0.02} (6-300\tau) \sin\frac{(t-\tau)}{\sqrt{10}}\, d\tau$$

3) $$\frac{X}{Y} = \frac{0.01\,j\omega + 1}{1 - \omega^2 + 0.01\,j\omega} \qquad \omega_n = 1$$

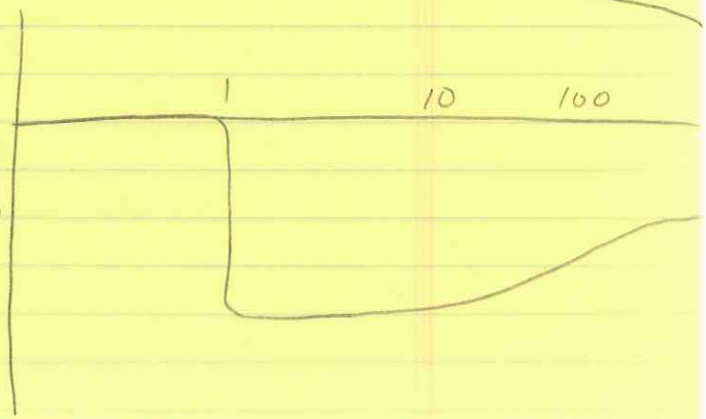
$\frac{X}{Y}(0.1) = 1.01, \angle 0^\circ$

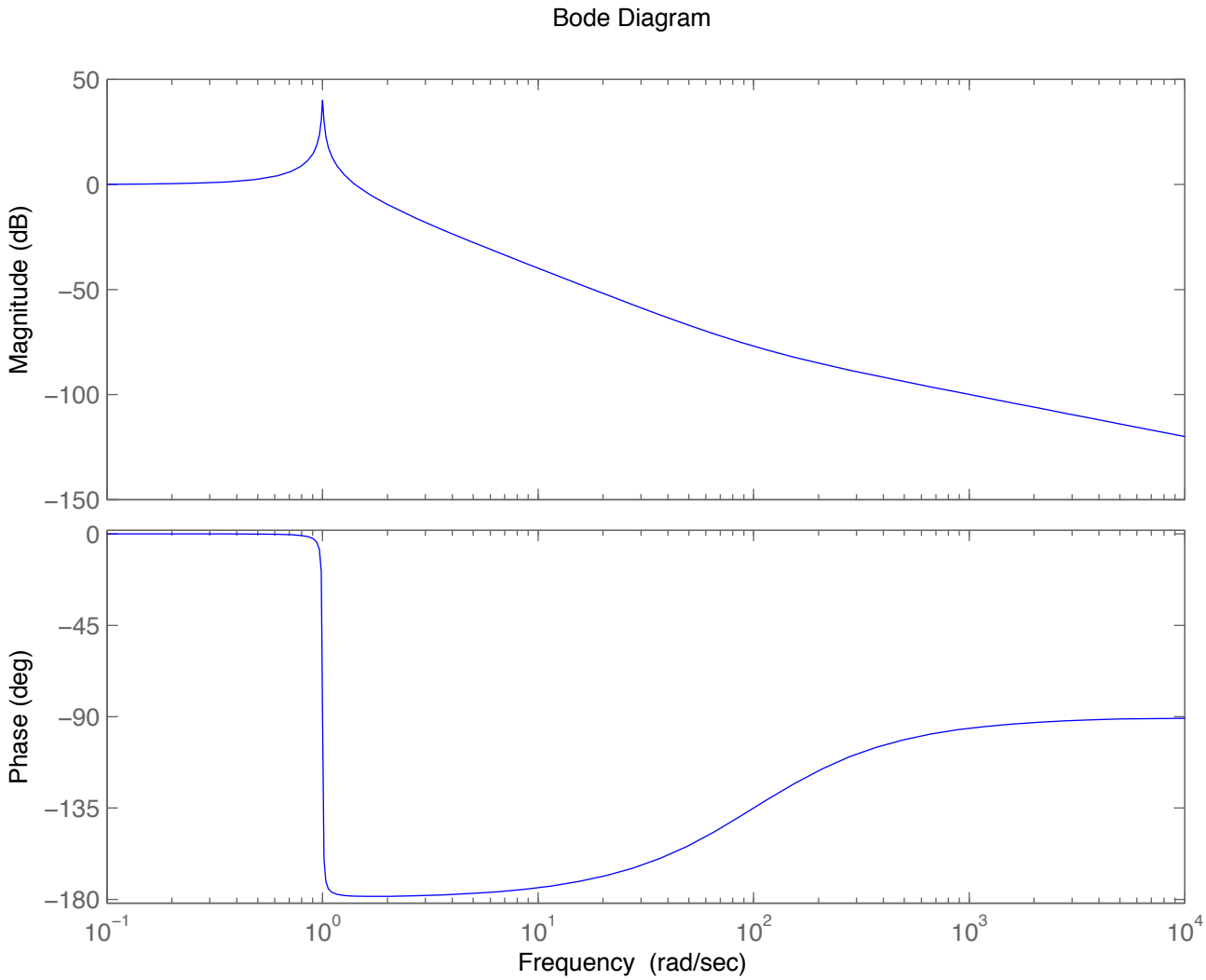
$\frac{X}{Y}(1) = 10.0, \angle -89.4^\circ$

$\frac{X}{Y}(10) = 0.01, \angle -174^\circ$



You can plot on most calculators to get this, but use log scale




Bode Diagram
Magnitude (dB)
50
0
−50
−100
−150
Phase (deg)
0
−45
−90
−135
−180
10^−1
10^0
10^1
10^2
10^3
10^4
Frequency (rad/sec)