Sp '09 Exam 2 Soln

1) $$\frac{X_n}{F} = \frac{1}{8.8826\times10^4 - 10\omega_n^2 + j\omega_n}$$

For $n=3$, $X_n = 0$

For $n=4$, $\omega_n = 40\pi$, $a_n = \frac{-1}{2\pi^2}$

$$X_n = \frac{-1}{2\pi^2}\left(\frac{1}{8.88\times10^4 - 10(40\pi)^2 + 40\pi j}\right)$$

$$= 7.33\times10^{-7}\, e^{1.8\times10^{-3} j}$$

$$x_3(t) = 0$$

$$x_4(t) = 7.33\times10^{-7}\cos\left(40\pi + 1.8\times10^{-3}\right)$$

2) For $0 < t < 1$

$$x(t) = \frac{1}{2\pi}\int_0^t \sin 2\pi(t-\tau)\, d\tau$$

$$= \frac{1}{2\pi}\left(\frac{+1}{2\pi}\cos 2\pi(t-\tau)\Big|_0^t\right)$$

$$= \left(\frac{1}{2\pi}\right)^2\left(\cos 0 - \cos 2\pi t\right)$$

$$\boxed{= \left(\frac{1}{2\pi}\right)^2\left(1 - \cos 2\pi t\right)}$$

For $1 < t$

$$x(t) = \frac{1}{2\pi}\int_0^1 \sin 2\pi(t-\tau)\, d\tau = \left(\frac{1}{2\pi}\right)^2 \cos 2\pi(t-\tau)\Big|_0^1$$

$$= \left(\frac{1}{2\pi}\right)^2\left(\underbrace{\cos 2\pi(t-1)}_{\cos 2\pi t} - \cos 2\pi t\right)$$

$$\boxed{= 0}$$

3) a) Not straight-line decay envelope. ∴ Not Coulomb

b) High aplitude log dec ≠ low amplitude log dec
Eg for $0 < t < 4$ and $4 < t < 8$

$$\delta_1 = \frac{1}{13} \ln \frac{1}{0.23} = 0.11 \quad , \quad \delta_2 = \frac{1}{13} \ln \frac{0.23}{0.13} = 0.04$$

Therefor not viscous (log decrement is constant for viscous)

∴ Air damping

4) See text

$$X_n = \sin \frac{n\pi x}{\ell} \quad , \quad \omega_n = \left(\frac{n\pi}{\ell}\right)^2 \sqrt{\frac{EI}{\rho A}}$$