

Sp '09 Vibes Exam I Soln

1) a) $\sqrt{\frac{1000}{10}} = 10 \text{ rad/s} = 1.59 \text{ Hz}$

b) $2\zeta\omega = \frac{5}{10}$

$$\zeta = \frac{5}{10 \cdot 2 \cdot \omega} = \frac{5}{200} = 0.025$$

c) $\frac{x}{y} = H(j\omega) = \frac{1}{-10\omega^2 + 1000 + 5j\omega}$

$$H(10j) = 0.2$$

$$x(t) = 2 \sin 10t$$

d) $H(100j) = \frac{5.1 \times 10^{-6} - 1.0 \times 10^{-3}j}{1.0 \times 10^{-3}}$
 $|H(100j)| = 1.0 \times 10^{-3} \quad \angle H(100j) = -1.57 \text{ rad}$

$$x(t) = 0.01 \sin(100t - 1.57)$$

e) $F_s = |1000 X| = 2000 \text{ (no units provided)}$

2) $T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} \left(\frac{1}{2} m_2 R^2 \right) \dot{\theta}^2 \quad \left(\dot{\theta} = \frac{\dot{x}}{R} \right)$

$$= \frac{1}{2} \left(m + \frac{m_2}{2} \right) \dot{x}^2$$

$$U = -mgx + \frac{1}{2} k y^2 \quad y = \frac{x}{2}$$

$$= -mgx + \frac{1}{2} \frac{k}{4} x^2$$

Using Lagrange, $\frac{d}{dt} \frac{\partial T}{\partial \dot{x}} + \frac{\partial U}{\partial x} = F$

$$\left(m + \frac{m_2}{2} \right) \ddot{x} + \frac{k}{4} x - mg = F$$

3) 50 cycles in 10 seconds

$$T = 0.2 \text{ sec}$$

$$f_n = 5 \text{ Hz}$$

$$\omega_n = 5 \cdot 2\pi = 10\pi \text{ rad/s} = 31.4 \text{ rad/s}$$

$$\delta = \frac{1}{50} \ln \frac{x(0)}{x(10)} = \frac{1}{50} \ln \frac{0.5}{0.04} = 0.0505$$

$$\zeta = \frac{0.05}{2\pi} = 0.008$$

$$(\text{using } \zeta = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} \text{ gives same answer to 4 places})$$

$$\frac{K}{m} = \omega_n^2$$

$$K = m\omega_n^2 = (31.4)^2 \cdot 10 = 9900 \text{ N/m}$$

$$C = 2\zeta\omega_n m = 5 \text{ kg/s}$$

(Truth is actually the same numbers as problem 1)

4) See old solns for more details

$$\frac{G}{\rho} X'' + \omega_n^2 X = 0$$

$$X_n(x) = A_n \sin \omega_n \sqrt{\frac{\rho}{G}} x + B_n \cos \omega_n \sqrt{\frac{\rho}{G}} x$$

$$X'_n(0) = 0 = A_n \omega_n + 0, \quad A_n = 0$$

$$X(l) = 0 = B_n \cos \omega_n \sqrt{\frac{\rho}{G}} l$$

$$\omega_n \sqrt{\frac{\rho}{G}} l = \frac{\pi}{2}, \frac{3\pi}{2}, \dots$$

$$\omega_n \sqrt{\frac{\rho}{G}} l = \frac{(2n-1)\pi}{2}, \quad \omega_n = \frac{(2n-1)\pi}{2} \frac{1}{l} \sqrt{\frac{G}{\rho}}$$

$$X_n(x) = B_n \cos \frac{(2n-1)\pi}{2} \frac{x}{l}$$