Mechanical Systems Modeling and Design, Exam 2

Spring 2006

1. Find $x(t)$ for the following using:

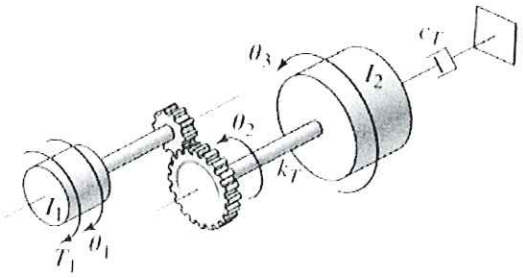
   (a) the Laplace transform method

   (b) the trial-solution method

$$2\ddot{x} + 6x = 0$$

   where $x(0) = 1$ and $\dot{x}(0) = 2$

2. Derive the equations of motion for the system below. Generate the simulink block diagram.



3. Presuming a model given by

$$10\ddot{x} + \dot{x}(1 - y^2) + 3x - 2y = f(t) \quad (1)$$

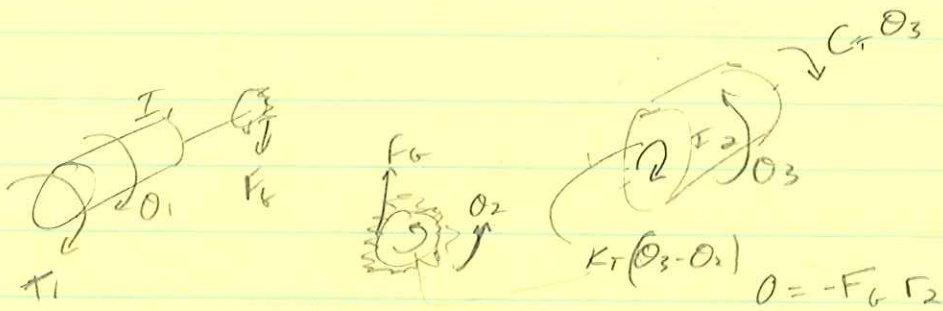
$$\ddot{y} + 3\dot{y} + 3y - 2x = 0 \quad (2)$$

   with a nominal force of $f(t) = 10u(t - 3)$, generate the simulink simulation for the system with results for the first 10 seconds.

4. The parameter values of a certain armature-controlled motor are $K_T = K_b = 0.2$ Nm/A, $c = 5 \times 10^{-4}$ Nms/rad, and $R_a = 0.8\Omega$. The manufacturer's data states that the motor's maximum speed is 3500 rpm, and the maximum armature current it can withstand without demagnitizing is 40 A. Compute the no-load speed, the no-load current, and the stall torque. Determine whether the motor can be used with an applied voltage of $v_a = 15$ V. Note:

$$\frac{I_a(s)}{V_a(s)} = \frac{Is+c}{L_aIs^2+(R_aI+cL_a)s+cR_a+K_bK_t},$$
$$\frac{I_a(s)}{T_L(s)} = \frac{K_b}{L_aIs^2+(R_aI+cL_a)s+cR_a+K_bK_t},$$
$$\frac{\Omega(s)}{V_a(s)} = \frac{K_T}{L_aIs^2+(R_aI+cL_a)s+cR_a+K_bK_t},$$
$$\frac{\Omega(s)}{T_L(s)} = -\frac{L_as+R_a}{L_aIs^2+(R_aI+cL_a)s+cR_a+K_bK_t},$$

2)



$$0 = -F_G r_2 + K_T(\theta_3 - \theta_2)$$

$$F_G = \frac{K_T}{r_2}(\theta_3 - \theta_2)$$

$$I_1\ddot{\theta}_1 = T + F_G r_1 \qquad I_2\ddot{\theta}_3 = -C_T\dot{\theta}_3 - K_T(\theta_3 - \theta_2)$$

Subst for F_G

$$I_1\ddot{\theta}_1 = T + \frac{K_T r_1}{r_2}(\theta_3 - \theta_2)$$

$$\theta_1 r_1 = \theta_2 r_2$$

$$\theta_2 = \frac{r_1}{r_2}\theta_1$$

$$I_1\ddot{\theta}_1 = T + \frac{K_T r_1}{r_2}\left(\theta_3 - \frac{r_1}{r_2}\theta_1\right)$$

$$I_2\ddot{\theta}_3 = -C_T\dot{\theta}_3 - K_T\left(\theta_3 - \frac{r_1}{r_2}\theta_1\right)$$

4) From eqn 3

a) $\Omega = 15 \dfrac{0.2}{5\times10^{-4}\cdot 0.8 + (0.2)^2} = 74$ rad/sec

$= 709$ RPM (ok)

b) No load current

$I = 15 \dfrac{5\times10^{-4}}{5\times10^{-4}\cdot 0.8 + (0.2)^2} = 0.186$ A

c) Stall Torque

From (3) and (4)

$$\Omega = \frac{V_a K_T + R_a T_L}{5\times10^{-4}\cdot 0.8 + (0.2)^2} = 0$$

$$T_L = \frac{15\cdot 0.2}{0.8} = 3.75 \text{ Nm}$$

d) Current is (add (1) and (2))

$$I_a = \frac{V_a C + F_b T}{5\times10^{-4}\cdot 0.8 + (0.2)^2} = 187.5 \text{ A}$$

This exceeds the 40 A limit.