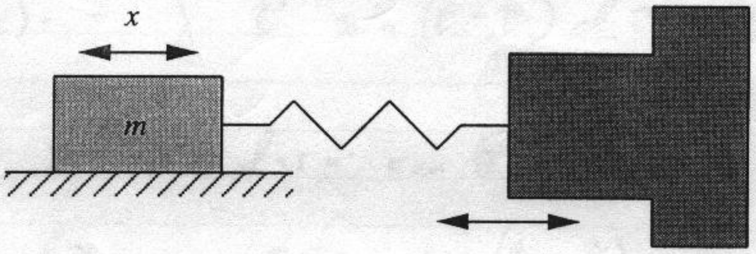
ME 460/660 Exam 2, Spring '95

1) Find the response of an undamped system with $m = 1$ kg and $k = 1$ N to a force $F(t) = -t$ using the convolution integral.

2) a) It is desired to position a mass of 1 kg within $\pm.01$ mm using a linear actuator. The dynamic and static coefficients of friction between the mass and the surface are μ=.07. Knowing that stiction occurs below $x = \mu N/k$, find the minimum stiffness allowable in the arm connecting the mass to the actuator. Use $g = 9.81$ m/s^2.

b) If the actuator moves a distance of 1 cm, how long will it take for the position of the mass to meet the design specification? Assume the motion of the actuator is instantaneous.

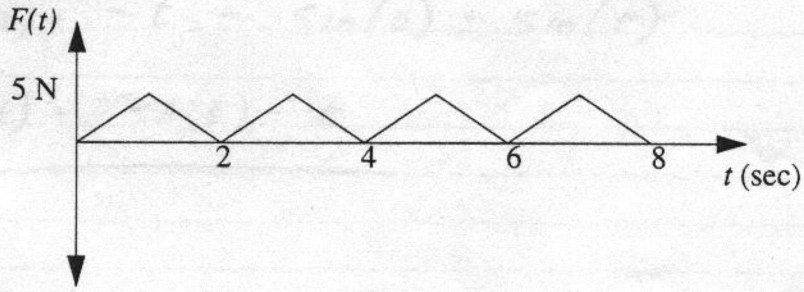


Linear Actuator

3) Find the eigenvectors and eigenvalues of the following matrix. Normalize the eigenvalues and determine if they form an orthonormal set.

$$A = \begin{bmatrix} 2 & -1 \\ -2 & 4 \end{bmatrix}$$

4) Find the steady state response of a SDOF system with $m = 5$ kg, $c = .01$ Ns/m, and $k = 20\pi^2$ N/m to the following force:



Which term in the Fourier series of the force is the most important one?

ME 460/660 Exam 2 Sp' 95, Solutions

1) Find the response of an undamped system with $m = 1$ kg, & $k = 1$ N/m to a force $F(t) = -t$ using the convolution integral.

$$h(t) = \sin(t-\tau)$$

$$x(t) = \int_0^t F(\tau)\, h(t-\tau)\, d\tau$$

$$x(t) = -\int_0^t \tau \sin(t-\tau)\, d\tau$$

$$u = \tau \qquad dv = \sin(t-\tau)\, d\tau$$

$$du = d\tau \qquad v = \cos(t-\tau)$$

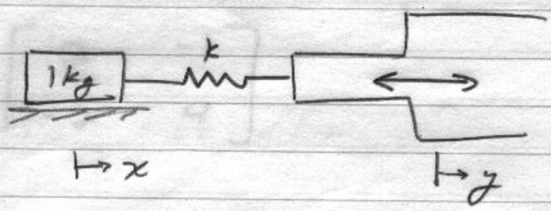
$$x(t) = -\left[\tau\cos(t-\tau)\Big|_0^t - \int_0^t \cos(t-\tau)\, d\tau\right]$$

$$x(t) = -\left[t + \sin(t-\tau)\Big|_0^t\right]$$

$$x(t) = -t - \sin(0) + \sin(t)$$

$$\underline{x(t) = \sin(t) - t}$$

2)



a) If $|x-y| < \frac{\mu N}{k}$ when the actuator is at rest, the mass will not slip.

$$10^{-5} > \frac{.07 \cdot 9.81 \cdot 1}{k}$$

$$k > 6.867 \times 10^{4} \ N/m$$

b) $x_0 = .01 \ m$

The decay envelope is

$$x = x_0 - \frac{2 \mu N \omega}{\pi k} t$$

$$10^{-5} = .01 - \frac{2 \cdot .07 \cdot 9.81 \cdot}{\pi \sqrt{6.867 \times 10^{4}}} t_s$$

$$t_s = \boxed{5.99} \text{ sec}$$

3) $A = \begin{bmatrix} 2 & -1 \\ -2 & 4 \end{bmatrix}$

$\det(A - \lambda I) = 0$

$(2-\lambda)(4-\lambda) - 2 = 0$

$8 - 6\lambda + \lambda^2 - 2 = 0$

$\lambda^2 - 6\lambda + 6 = 0$

$\lambda_{1,2} = 1.268, 4.732$

$(A - \lambda I)\underline{v} = \underline{0}$

For $\lambda_1 = 1.268$

$\begin{bmatrix} .732 & -1 \\ -2 & 2.732 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$.732 v_1 = v_2$

$\underline{v_1} = \begin{bmatrix} 1 \\ .732 \end{bmatrix}$ normalized $\underline{v_1} = \begin{bmatrix} .807 \\ .591 \end{bmatrix}$

For $\lambda_2 = 4.732$:

$\begin{bmatrix} -2.732 & -1 \\ -2 & -.732 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$2.732 v_1 = -v_2$

$\underline{v_2} = \begin{bmatrix} 1 \\ -2.732 \end{bmatrix}$ normalized $\underline{v_2} = \begin{bmatrix} .344 \\ -.939 \end{bmatrix}$

$\underline{v_1}' \underline{v_2} = .277$ so they are not an orthonormal set.

4)

$$a_0 = \frac{2}{T}\int_0^T f(t)\,dt$$

$$= 1\int_0^1 5t\,dt + 1\int_1^2 5(2-t)\,dt$$

$$= 1\left(2.5t^2\Big|_0^1 + \left(10t - 2.5t^2\right)\Big|_1^2\right)$$

$$= 1\left(2.5 + (20-10) - (10-2.5)\right)$$

$$= 1\left(2.5 + 10 - 7.5\right) = 5$$

$b_n = 0$ because the function is even

$$a_n = \int_0^1 5t\cos\frac{2\pi n t}{2}\,dt + \int_1^2 5(2-t)\cos\frac{2\pi n t}{2}\,dt$$

$$\int_0^1 5t\cos\frac{2\pi n t}{2}\,dt$$

$u = 5t \qquad dv = \cos\frac{2\pi n t}{2}\,dt$

$du = 5\,dt \qquad v = \frac{1}{\pi n}\sin \pi n t$

$$\int_0^1 5t\cos\frac{2\pi n t}{2}\,dt = \frac{5t}{\pi n}\sin\pi n t\Big|_0^1 - 5\int_0^1 \frac{1}{\pi n}\sin\pi n t\,dt$$

$$= \frac{-5}{\pi n}\cdot\frac{-1}{\pi n}\cos\pi n t\Big|_0^1$$

$$= \frac{5}{\pi^2 n^2}\left(\cos\pi n - 1\right)$$

$$\int_1^2 10 \cos \pi n t \, dt$$

$$= \frac{10}{\pi n} \sin n \pi t \Big|_1^2$$

$$= 0$$

$$\int_1^2 -5t \cos \pi n t \, dt$$

$$= \frac{-5}{\pi^2 n^2} \cos \pi n t \Big|_1^2$$

$$= \frac{+5}{\pi^2 n^2} \left(-\cos 2\pi n + \cos \pi n \right)$$

$$a_n = \frac{5}{\pi^2 n^2} \left(\cos \pi n - 1 - \underbrace{\cos 2\pi n}_{1} + \cos \pi n \right)$$

$$a_n = \frac{5}{\pi^2 n^2} \left(2 \cos \pi n - 2 \right)$$

$$= \frac{5}{\pi^2 n^2} \left(\cos \pi n - 1 \right) \begin{cases} \dfrac{-10}{\pi^2 n^2} & n = 1, 3, 5, \dots \\ 0 & n = 2, 4, 6, \dots \end{cases}$$

The Fourier series is then

$$F(t) = 2.5 - \frac{10}{\pi^2} \cos \pi t - \frac{10}{9\pi^2} \cos 3\pi t - \dots$$

Since none of the terms drive the system at resonance, the most significant term is the one with the largest coefficient, or term 1.