Vibration Testing — Midterm, Winter 2005

Closed book, closed notes. Test booklets will be provided. Formula sheet must be turned in with the exam. Formula sheet must be exactly the same as that posted on the web site.

1. A single degree of freedom system is defined by the equation

$$10\ddot{x} + .01\dot{x} + 1000x = f(t)$$

   Presuming you have an accelerometer, what is the frequency response function?

2. Given the mass-normalized mode shapes $\psi_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\psi_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and the modal frequency response functions $\tilde{h}_1 = \frac{1}{4.0+0.04\omega j-\omega^2}$ and $\tilde{h}_2 = \frac{1}{9.0+0.05\omega j-\omega^2}$, determine the natural frequencies, damping ratios, and frequency response function between degree of freedom 1 and 2. Sketch the modal contributions to the FRF and use them to sketch the net FRF. Write the residue matrices.

3. Determine the Fourier Series representation of the function for which a single cycle is defined by

$$f(t) = \begin{cases} 1, & 0 < t < 2 \\ -1, & 2 < t < 4 \end{cases}$$

4. Write the state space representation of the system defined by

$$9\dot{x} + 16x = f(t)$$

   presuming that you have a displacement sensor. What is the state transition matrix?

5. Consider a the state space system for which

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & 1 & 0 & 0 \\ 1 & -3 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix}, \quad \text{and} \quad D = [0]$$

   Determine whether the system is controllable. Use any method, and do no more calculations than are necessary (points will be given only for the answer and calculations necessary for proof of your point).