Vibration Testing — Exam 1, Winter 2004

Closed book, closed notes. Test booklets will be provided. Formula sheet must be turned in with the exam. Formula sheet must be exactly the same as that posted on the web site.

1. Find the frequency response function for the system defined by the differential equation

$$10\ddot{x} + .01\dot{x} + 1000x = f(t)$$

2. Determine the Fourier Series representation of the function for which a single cycle is defined by

$$f(t) = \begin{cases} 0, & 0 < t < 2 \\ 1, & 2 < t < 3 \end{cases}$$

3. Given the matrix

$$\begin{aligned}
{}_1A = \boldsymbol{\psi}_1 \boldsymbol{\psi}_1^T &= \begin{bmatrix} \psi_{1,1}\psi_{1,1} & \psi_{1,1}\psi_{2,1} & \cdots & \psi_{1,1}\psi_{m,1} & \cdots & \psi_{1,1}\psi_{n,1} \\ \psi_{2,1}\psi_{1,1} & \psi_{2,1}\psi_{2,1} & \cdots & \psi_{2,1}\psi_{m,1} & \cdots & \psi_{2,1}\psi_{n,1} \\ \vdots & \vdots & \ddots & \vdots & \cdots & \vdots \\ \psi_{l,1}\psi_{1,1} & \psi_{l,1}\psi_{2,1} & \cdots & \psi_{l,1}\psi_{m,1} & \cdots & \psi_{l,1}\psi_{n,1} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \psi_{n,1}\psi_{1,1} & \psi_{n,1}\psi_{2,1} & \cdots & \psi_{n,1}\psi_{m,1} & \cdots & \psi_{n,1}\psi_{n,1} \end{bmatrix} \qquad (1) \\
&= \begin{bmatrix} 0.08450 & 0.00357 & 0.27811 \\ 0.00357 & 0.00015 & 0.01175 \\ 0.27811 & 0.01175 & 0.91535 \end{bmatrix}
\end{aligned}$$

find the first mode shape.

4. Can the matrix of the previous problem be used to extract the second and third modes? Explain.

5. Given the modal matrix

$$\Psi = \begin{bmatrix} 0.91174 & 0.40825 & 0.04546 \\ 0.38697 & -0.81650 & -0.42847 \\ 0.13780 & -0.40825 & 0.90241 \end{bmatrix}$$

and $\omega_1^2 = 1.58$, $\omega_2^2 = 4.00$, and $\omega_3^2 = 11.42$, determine the frequency response function between degree of freedom 1 and degree of freedom 2.

6. Given the state transition matrix,

$$\mathcal{S}(t) = e^{At} = \begin{bmatrix} \cos(\omega_n t) & \frac{\sin(\omega_n t)}{\omega_n} \\ -\omega_n \sin(\omega_n t) & \cos(\omega_n t) \end{bmatrix}, \qquad \text{and} \quad \boldsymbol{z}(0) = \begin{bmatrix} x(0) \\ \dot{x}(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

determine $x(t)$ and $\dot{x}(t)$.