ME 460/660, Mechanical Vibration — Final Exam, Fall 1999

Closed book, closed notes. Use one $8\frac{1}{2} \times 11$ formula sheet, front and back. Test books will be provided. Do all work on the exam pages with the exception of the full length problems.

Multiple choice. Circle the letter of the correct answers. 3 points each. *Read all questions very carefully*

1. What modeling method/s apply for determining the system equations of motion when viscous damping is not present in a system?
   a. The Energy Method
   b. Lagrange's Equations
   c. Newton's Law
   d. The Fourier method
   e. (a), (b) and (c)
   f. (a), (b) and (d)
2. A cyclic (not necessarily sinusoidal) force acts on a SDOF system. Given the equations of motion, what is the most appropriate way to approach solving for the system's steady state response?
   a. Apply Lagrange's equations
   b. Represent the force by a Fourier series and use the solution to sinusoidal excitation for each term in the series expansion
   c. Represent the response by a Fourier series and "be done with it"
   d. Use the convolution integral
3. Cyclic excitations generally occur under all of the following situations except…
   a. unbalanced shafts
   b. a bad bearing
   c. base excitation (i.e. road surface variation, floor motion induced by any source,…)
   d. piston motion in an engine
4. The suspension system of your car is designed to:
   a. Reduce displacement transmission at low frequencies
   b. Reduce displacement transmission at high frequencies
   c. Amplify displacement transmission at low frequencies
   d. Amplify displacement transmission at high frequencies
5. Single degree of freedom models are used to represent real world:
   a. ideal masses connected to ideal springs
   b. low frequency response of continuous systems
   c. high frequency response of multiple degree of freedom systems
   d. any complex system
6. The convolution integral
   a. can be used to obtain the response to an excitation of any form
   b. will not give the correct answer for a harmonically excited system

c. applies only for impulse excitations

d. only applies to free response

Circle the answer, 3 points each.

1. In order to provide good displacement isolation in an undamped base excited system, ω should ideally be (low, ω_{dr}, high).

2. In order to provide good force isolation in an undamped base excited system, ω should ideally be (low, ω_{dr}, high).

3. In order to maximize harmonic forced response of a system, ω should ideally be (low, ω_{dr}, high) and ζ should be (low, ω_{dr}, high)

4. In order to minimize the response of a system with rotating unbalance, ω should ideally be (low, ω_{dr}, high).

5. What is the value of the inertance function (amplitude of frequency divided by force amplitude) of a forced spring/mass/damper system governed by $m\ddot{x} + c\dot{x} + kx = F(t)$ as ω_{dr} goes to zero? (0,1,k,∞)

6. What is the value of the receptance function (amplitude of frequency divided by force amplitude) of a forced spring/mass/damper system governed by $m\ddot{x} + c\dot{x} + kx = F(t)$ as ω_{dr} goes to zero? (0,1,k,∞)

7. What is the value of the inertance function of a forced spring/mass/damper system as ω_{dr} goes to ∞? (0,1,k,∞)

Short Answer 4 points each

1. In one sentence, define what is meant by the settling time of a system.

2. What is the 5% settling time of a system defined by $m = 10$ kg, $c = 1$ kg/s, and $k = 1000$ N/m?

3. By substituting for ω and ζ, determine which parameter, m, c, or k, is irrelevant to determining the settling time?

Full Length Problems (20 points each)

1. Derive the equations of motion for the following system using Lagrange's Equations where $k_1 = 1$ N/m, $k_2 = 2$ N/m, $k_3 = 3$ N/m, $m_1 = 1$ kg, and $m_2 = 2$ kg.

2. Given $S = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$, $r_1(t) = \sin(t)$ and $r_2(t) = -\sin(t)$, find $\mathbf{x}(t)$.

3. Use the convolution integral to find x(t) for the following undamped single degree of freedom system: $m\ddot{x} + kx = at$.

4. Graduate Students Only: Solve the following equation for the steady state response $w(x,t)$ where $c = \sqrt{\tau/\rho}$.

$$w_{tt}(x,t) - c^2 w_{xx}(x,t) = 100\sin(3t)\delta(x - l/3)$$

Recall that the integral of a Dirac delta function times another function is equal to the "another function" evaluated when the argument of the argument of the Dirac delta function is zero.