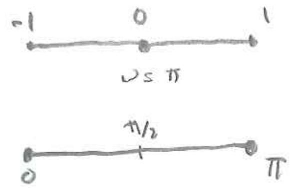
ME 7120, Fa 2012, Exam 2 Solns

1) $\epsilon_x = a_2 + a_4 y + 2a_5 x + 2a_6 x y$

Since strain can vary linearly across the element, and it can vary linearly along any side, it is capable of representing a series of straight-line approximations to the field. Because it can match adjacent elements, and matching will represent the solution better than not matching (satisfying equilibrium), the elements will exhibit strain continuity, even though it's not enforced.

$\omega = 2$



2. $\int_0^{\pi} \sin\theta \, d\theta$

10

$\theta(\xi) = \frac{\pi}{2} + \xi \frac{\pi}{2}$

$d\theta = \frac{\pi}{2} d\xi$

gauss pt $= \int_{-1}^{1} \sin\left(\frac{\pi}{2} + \xi \frac{\pi}{2}\right) \frac{\pi}{2} d\xi$

$f\left(\frac{1}{\sqrt{3}}\right)$ $\quad f\left(\frac{-1}{\sqrt{3}}\right)$

$\frac{\pi}{2}\left(\sin\left(\frac{\pi}{2} + \frac{\pi}{2\sqrt{3}}\right) + \sin\left(\frac{\pi}{2} - \frac{\pi}{2\sqrt{3}}\right)\right)$

$= 1.9358$

act $= \int_0^{\pi} \sin\theta \, d\theta = -\cos\theta \Big|_0^{\pi}$ $\quad -\cos\pi - {}^{-}\cos 0$

$-(-1) - {}^{-}1 = 1 + 1 = \underline{2}$

$\frac{2 - 1.9358}{1.9358} = .0332 = 3.32\%$ error

(10)

4. $[K] = \int_0^L N^T k(x) N \, dx$

$$\Delta k_{11} = \int_0^L N_1^2 \, k(x) \, dx$$

$$= \int_0^L \left(1 - 3\frac{x^2}{l^2} + 2\frac{x^3}{l^3}\right)^2 k(x) \, dx \qquad \left(1 - \frac{3x^2}{l^2} + \frac{2x^3}{l^3}\right)$$

+5

assume constant $k(x) = k$

$$= \int_0^L k\left(1 - 3\frac{x^2}{l^2} + 2\frac{x^3}{l^3} - \frac{3x^2}{l^2} + \frac{9x^4}{l^4} - \frac{6x^5}{l^5} + \frac{2x^3}{l^3} - \frac{6x^5}{l^5} + \frac{4x^6}{l^6}\right) dx$$

$$= k\int_0^L \left(1 - \frac{6x^2}{l^2} + \frac{4x^3}{l^3} + \frac{9x^4}{l^4} - \frac{12x^5}{l^5} + \frac{4x^6}{l^6}\right) dx$$

$$= k\left[x - \frac{2x^3}{l^2} + \frac{x^4}{l^3} + \frac{9}{5}\frac{x^5}{l^4} - \frac{2x^6}{l^5} + \frac{4}{7}\frac{x^7}{l^6}\right]_0^L$$

$$= kL\left(1 - 2 + 1 + \frac{9}{5} - 2 + \frac{4}{7}\right)$$

$$kL\left(\frac{-1}{5} + \frac{4}{7}\right)$$

$$kL\left(\frac{-7}{35} + \frac{20}{35}\right)$$

$$\Delta k_{11} = kL\left(13/35\right)$$

$$\Delta k_{21} = \int_0^L N_1 \, k(x) \, N_2 \, dx$$

$$\Delta k_{21} = \int_0^L \left(1 - 3\frac{x^2}{l^2} + 2\frac{x^3}{l^3}\right)\left(x - \frac{2x^2}{l} + \frac{x^3}{l^2}\right) k(x) \, dx$$

+5

assuming constant $k(x) = k$

$$= k\int_0^L \left(x - \frac{2x^2}{l} + \frac{x^3}{l^2} - \frac{3x^3}{l^2} + \frac{6x^4}{l^3} - \frac{3x^5}{l^4} + \frac{2x^4}{l^3} - \frac{4x^5}{l^4} + \frac{2x^6}{l^5}\right) dx$$

$$k\int_0^L \left(x - \frac{2x^2}{l} - \frac{2x^3}{l^2} + \frac{8x^4}{l^3} - \frac{7x^5}{l^4} + \frac{2x^6}{l^5}\right) dx$$

$$k\left[\frac{1}{2}x^2 - \frac{2}{3}\frac{x^3}{l} - \frac{1}{2}\frac{x^4}{l^2} + \frac{8}{5}\frac{x^5}{l^3} - \frac{7}{6}\frac{x^6}{l^4} + \frac{2}{7}\frac{x^7}{l^5}\right]_0^L$$

assuming $k(x) = $ constant

$$kL^2\left[\frac{1}{2} - \frac{4}{6} - \frac{1}{2} + \frac{8}{5} - \frac{7}{6} + \frac{2}{7}\right] = kL^2\left[\frac{-11}{6} + \frac{8}{5} + \frac{2}{7}\right] = kL^2\left(\frac{11}{210}\right)$$

$$\Delta k_{21} = kL^2\left(\frac{11}{210}\right)$$