ME 460/660 Formula Sheet for Exams

Please bring your own copy to exams and turn in your sheet with the exams. No additional information may be written on the formula sheet.

Euler Relations	$e^{j\beta} = \cos(\beta) + j\sin(\beta)$ $\sin(\beta) = \frac{e^{j\beta} - e^{-j\beta}}{2j}$ $\cos(\beta) = \frac{e^{j\beta} + e^{-j\beta}}{2}$
Expansion Envelope Slope	$\frac{f_0}{2\omega_n}$
Lagrange's Equation	$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} + \frac{\partial R}{\partial \dot{q}_i} = Q_i$
Fourier Series (Real Form)	$F(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n\cos(n\omega_T t) + b_n\sin(n\omega_T t)\right)$ where $\omega_T = 2\pi/T$, and T is the period of the function $a_0 = \frac{2}{T}\int_0^T F(t)dt = 2f_0$, $a_n = \frac{2}{T}\int_0^T F(t)\cos(n\omega_T t)dt = 2\,\Re(f_n)$, $\Re$ means real part, and $b_n = \frac{2}{T}\int_0^T F(t)\sin(n\omega_T t)dt = -2\,\Im(f_n)$, $\Im$ means imaginary part
Fourier Series (Complex Form)	$F(t) = \sum_{n=-\infty}^{\infty}\left(f_n e^{j\omega_T n t}\right)$ where $\omega_T = 2\pi/T$, and T is the period of the function $f_0 = \frac{1}{T}\int_0^T F(t)dt = \frac{a_0}{2}$ $f_n = \frac{1}{T}\int_0^T F(t)e^{-j\omega_T n t}dt = \frac{a_n}{2} - \frac{b_n}{2}j$
Convolution Integral	$x(t) = \frac{1}{m\omega_d}e^{-\zeta\omega_n t}\int_0^t\left[F(\tau)e^{\zeta\omega_n\tau}\sin\left(\omega_d(t-\tau)\right)\right]d\tau$ or $x(t) = \frac{1}{m\omega_d}\int_0^t\left[F(t-\tau)e^{-\zeta\omega_n\tau}\sin\left(\omega_d\tau\right)\right]d\tau$
Log Decrement	$\delta = \frac{1}{n}\ln\left(\frac{x(t)}{x(t+nT)}\right)$, $\zeta = \frac{\delta}{\sqrt{4\pi^2+\delta^2}}$