ME 464/664 SP '06 Final Exam Solutions

1) Unknown magnitude of step.
Clear 1^{st} order response

$$a\dot{y} + by = b\,u(t)$$

τ appears to be ~ 2 sec (63% of final response)

$y(t) = 1 - e^{-t/2}$, $\dot{y} = \frac{t}{2}$

Subst into EOM

$$a\tfrac{1}{2}e^{-t/2} + \cancel{b} - be^{-t/2} = b\,\cancel{u(t)}$$

$\cancel{b} \to 0$, $b\,\cancel{u(t)} \to 0$

$$a = 2b$$

EOM is

$$2b\dot{y} + by = b\,u(t)$$

2) a) $10\dot{y} + 2y = 5t$

$y(t) = a + bt + ce^{-t/\tau}$

Sub into EOM

$10b - \frac{10}{\tau}ce^{-t/\tau} + 2a + 2bt + 2ce^{-t/\tau} = 5t$

1. $10b + 2a = 0$ $\qquad a = -5b$
2. $\frac{-10}{\tau} + 2 = 0$ $\qquad \tau = 5$
3. $2b = 5$ $\qquad b = 2.5$

Thus $a = -5 \cdot 2.5 = -12.5$

$y(t) = -12.5 + 2.5t + ce^{-t/\tau}$

If $y(0) = 1$, $\quad 1 = -12.5 + c$

$c = 13.5$

$y(t) = -12.5 + 2.5t + 13.5e^{-t/5}$

b) $10\dot{y} + 2y = 5u(t)$

From the previous problem, $y_h = ce^{-5t}$.

$y(t) = a + be^{-t/5}$

Subst into EOM

$-2be^{-t/5} + 2a + 2be^{-t/5} = 5u(t)$

$a = 2.5$

$y(t) = 2.5 + be^{-t/5}$

Since $y(0) = 1$

$1 = 2.5 + b$

$b = -1.5$

$y(t) = 2.5 - 1.5e^{-t/5}$

2c) $\varsigma = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} = \sqrt{2} \approx 1.4$ $\quad \omega = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}} \approx 0.707$

System is under damped $\quad \omega_d = \omega_n \sqrt{1 - \varsigma^2}$

$$= \frac{1}{\sqrt{2}}\sqrt{1-\frac{1}{2}} = \frac{1}{\sqrt{2}}\sqrt{\frac{1}{2}} = \frac{1}{2} \approx .5$$

$$x_h = A e^{-\varsigma\omega t} \sin(\omega_d t + \phi)$$

$$x_p = 2$$

$$x(t) = 2 + A e^{-t} \sin(\tfrac{1}{2}t + \phi)$$

$x(0) = 0 = 2 + A \sin\phi \qquad A = 0, \text{ or } \sin\phi = 0$

$\dot{x}(0) = 0 = -A\sin\phi + \frac{1}{2}A\cos\phi$

Clearly $A \neq 0$, so

$$\sin\phi = \tfrac{1}{2}\cos\phi$$

$$\tan\phi = \tfrac{1}{2} = 0.464 \text{ or } 3.605 \quad (26.6°, 206.6°)$$

Take $\phi = 26.6°$. From 1st ISC

$$A = \frac{-2}{\sin 26.6} = -4.472$$

$$x(t) = 2 - 4.472\, e^{-t} \sin(\tfrac{1}{2}t + 26.6)$$

3) a) $|T(j\omega)| = 0.0994$

$\measuredangle(j\omega) = -3.02$ rad (-173.7°)

b) -20.05 dB

c) $x(t) = 0.994 \sin(11t - 2.25)$

d)
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sys = tf (1, [10, 0.1, 1200])
bode(sys)
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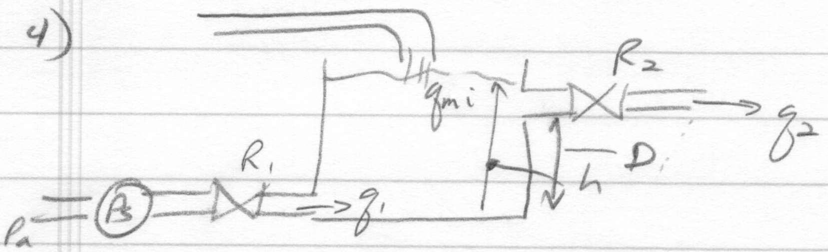
4)



Across R_1 ① $R_1 q_1 = P_s - \rho g h$, $q_1 = \dfrac{P_s - \rho g h}{R_1}$

Across R_2 ② $R_2 q_2 = \rho g (h - D)$ $q_2 = \dfrac{\rho g (h-D)}{R_2}$

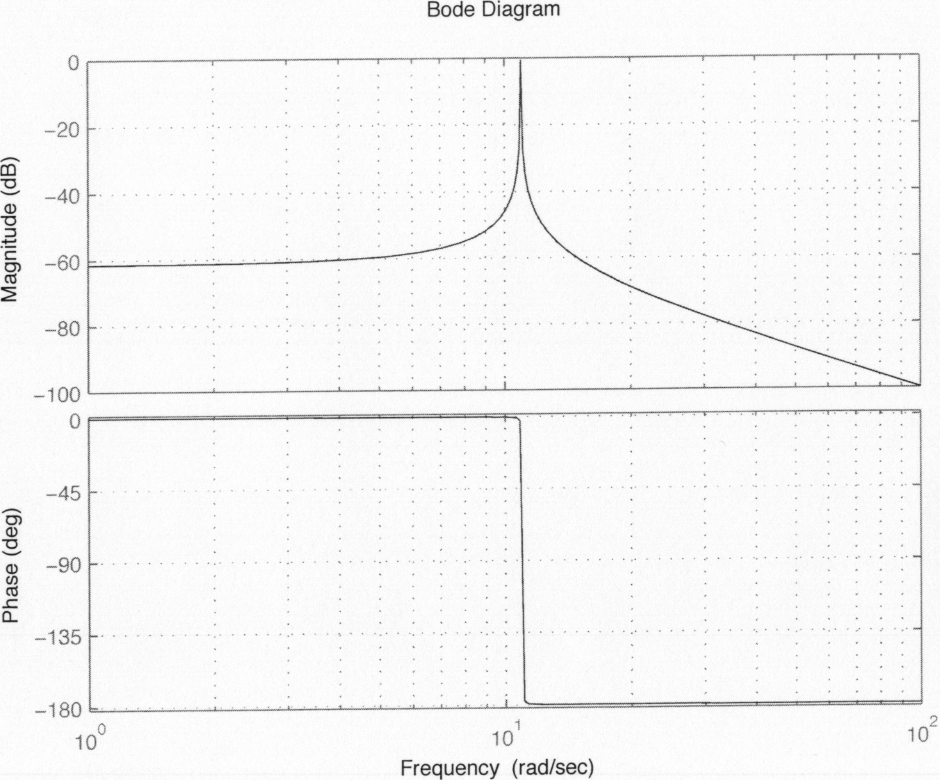
Mass in tank ③ $\rho A \dot{h} = q_1 - q_2 + q_{mi}$

Subst ① and ② into ③

$$\rho A \dot{h} = \frac{P_s}{R_1} - \frac{\rho g h}{R_1} - \frac{\rho g h}{R_2} + \frac{\rho g D}{R_2} + q_{mi}$$

$$\rho A \dot{h} = -\rho g\left(\frac{1}{R_1} + \frac{1}{R_2}\right)h + \frac{\rho g D}{R_2} + \frac{P_s}{R_1} + q_{mi}$$

problem 3



Bode Diagram
Magnitude (dB)
0
-20
-40
-60
-80
-100
Phase (deg)
0
-45
-90
-135
-180
10^0
10^1
10^2
Frequency (rad/sec)

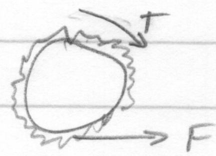
5) $v_a = R_a i + L_a \frac{di}{dt} + v_b$

$v_b = K_b \omega$, so $\quad v_a = R_a i + L_a \frac{di}{dt} + K_b \omega$

$T = K_T i$

$I_{eff} = I_m + I_b + m_r R^2$

$x = R\theta$



Resisting Force of spring is Kx
Moment is $R_1 K x$, so

$$T = I_{eff}\ddot{\theta} + R K x$$

since $\theta = \frac{x}{R_1}$

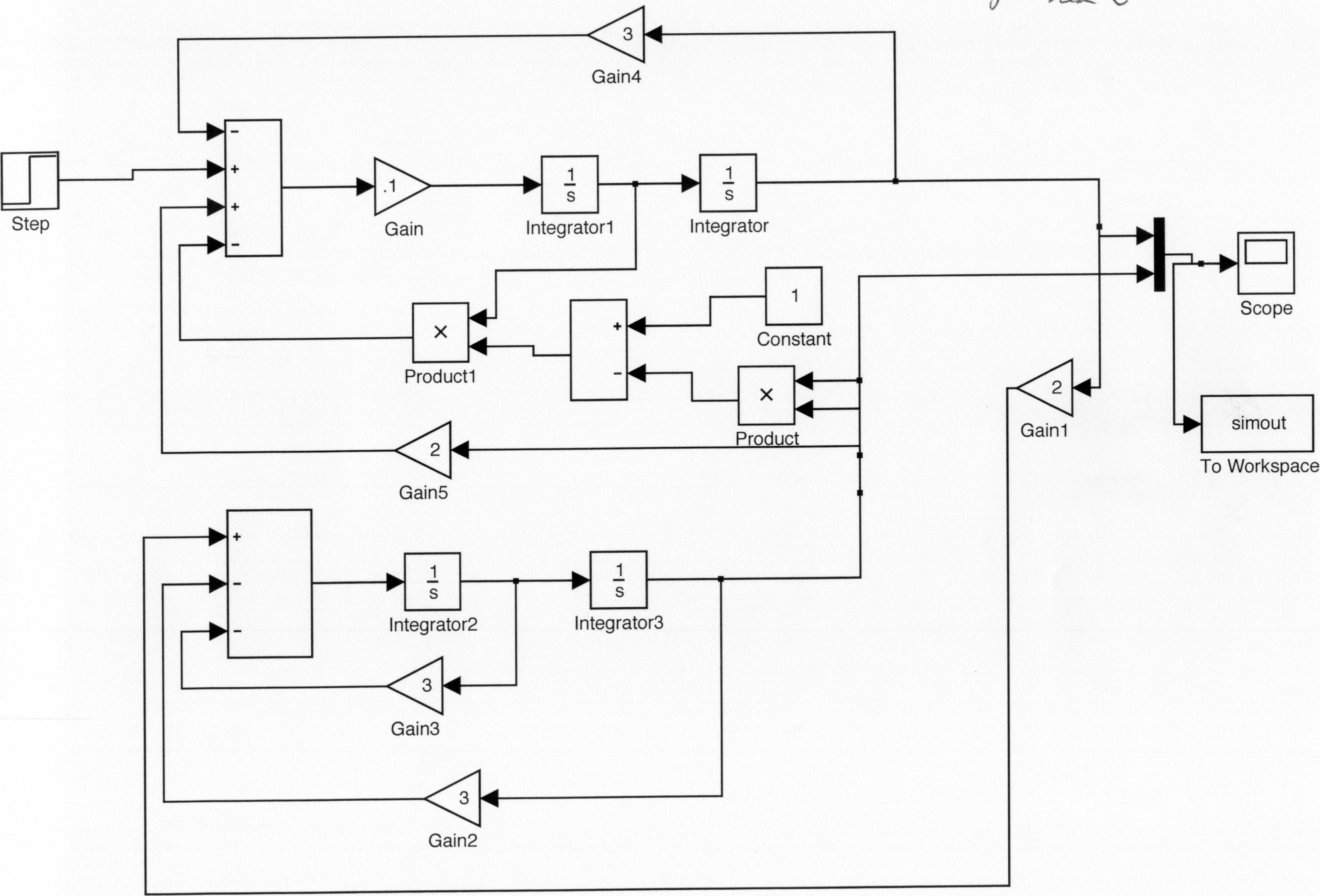
$$T = \frac{I_{eff}}{R_1}\ddot{x} + R K x, \quad K_T i = \frac{I_{eff}}{R}\ddot{x} + R K x$$

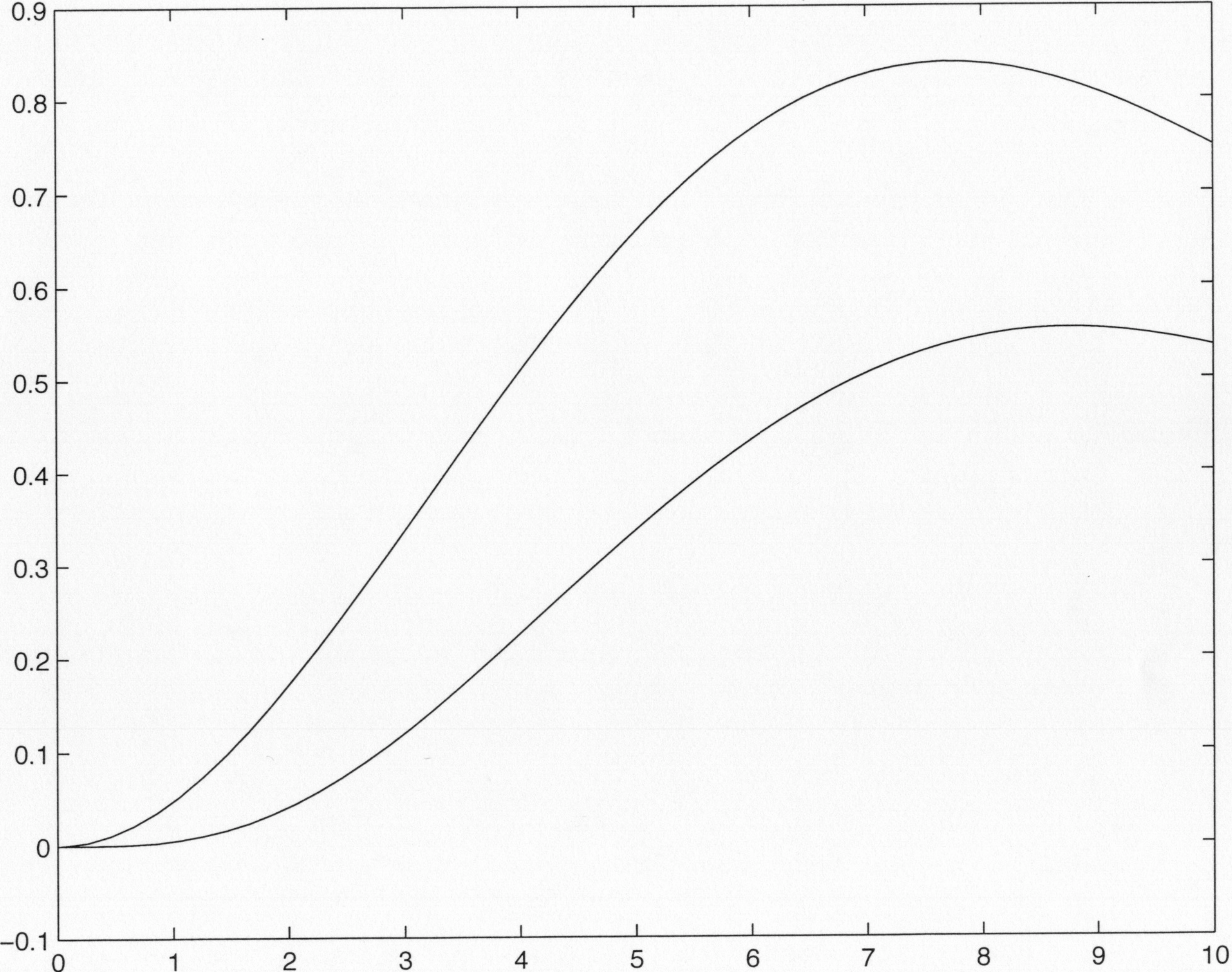
States are i, x, and $\dot{x}$

$$\begin{bmatrix} \frac{di}{dt} \\ \dot{x} \\ \ddot{x} \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{-R_a}{L_a} & 0 & \frac{-K_b}{L_a R} \\ 0 & 0 & 1 \\ \frac{K_T R}{I_{eff}} & \frac{-K R^2}{I_{eff}} & 0 \end{bmatrix}}_{A} \begin{bmatrix} i \\ x \\ \dot{x} \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{1}{L_a} \\ 0 \\ 0 \end{bmatrix}}_{B} v_a$$

$$y = \underbrace{\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} i \\ x \\ \dot{x} \end{bmatrix} + \underbrace{0}_{D} \cdot v_a$$

Problem 6


3
Gain4
Step
.1
Gain
1/s
Integrator1
1/s
Integrator
×
Product1
1
Constant
×
Product
2
Gain1
Scope
simout
To Workspace
2
Gain5
1/s
Integrator2
1/s
Integrator3
3
Gain3
3
Gain2



Problem 6