1) This is a variation of problem 2.1 in your text.

The position of the C.G of mass 1 is

$$\vec{r}_1 = \sin\Omega t \frac{L_1}{2}\cos\theta\, \hat{i} + \frac{L_1}{2}\sin\theta\, \hat{j} + \cos\Omega t \frac{L_1}{2}\cos\theta\, \hat{k}$$

For the C.G of mass 2

$$\vec{r}_2 = 2\vec{r}_1 + \frac{L_2}{2}\cos\phi\, \hat{j} - \sin\Omega t \frac{L_2}{2}\sin\phi\, \hat{i} - \cos\Omega t \frac{L_2}{2}\sin\phi\, \hat{k}$$

The velocities are then

$$\vec{V}_1 = \dot{\vec{r}}_1 = \frac{L_1}{2}\Big[\left(\Omega\cos\Omega t\cos\theta - \sin(\Omega t)\dot{\theta}\sin\theta\right)\hat{i} + \dot{\theta}\cos\theta\,\hat{j}$$
$$+ \left(-\Omega\sin\Omega t\cos\theta - \cos(\Omega t)\dot{\theta}\sin\theta\right)\hat{k}\Big]$$

$$\vec{V}_2 = \dot{\vec{r}}_2 = 2\vec{V}_1 - \frac{L_2}{2}\dot{\phi}\sin\phi\,\hat{j} + \left(-\Omega\cos\Omega t\frac{L_2}{2}\sin\phi - \sin\Omega t\frac{L_2}{2}\dot{\phi}\cos\phi\right)\hat{i}$$
$$-\left(-\Omega\sin\Omega t\frac{L_2}{2}\sin\phi + \cos\Omega t\frac{L_2}{2}\dot{\phi}\cos\phi\right)\hat{k}$$

The translational kinetic energies are then

$$T_1 = \frac{1}{2}m_1\left(V_x^2 + V_y^2 + V_z^2\right) = \frac{1}{2}m_1\left(\frac{L_1}{2}\right)^2\left(\Omega^2\cos^2\theta + \dot{\theta}^2\right)$$

Where $V_x = \frac{L_1}{2}\left(\Omega\cos\Omega t\cos\theta - \sin\Omega t\,\dot{\theta}\sin\theta\right)$

$V_y = \frac{L_1}{2}\dot{\theta}\cos\theta$

$V_z = \frac{L_1}{2}\left(-\Omega\sin\Omega t\cos\theta - \cos\Omega t\,\dot{\theta}\sin\theta\right)$

$$T_2 = \frac{1}{2} m_2 \left(V_x^2 + V_y^2 + V_z^2 \right)$$

where: $V_x = L_1(\Omega \cos\Omega t \cos\theta - \sin\Omega t \, \dot{\theta} \sin\theta)$
$- \frac{L_2}{2}(\Omega \cos\Omega t \sin\phi + \sin\Omega t \, \dot{\phi} \cos\phi)$

$V_y = L_1 \dot{\theta} \cos\theta - \frac{L_2}{2} \dot{\phi} \sin\phi$

$V_z = L_1(-\Omega \sin\Omega t \cos\theta - \cos\Omega t \, \dot{\theta} \sin\theta)$
$- \frac{L_2}{2}(-\Omega \sin\Omega t \sin\phi + \cos\Omega t \, \dot{\phi} \cos\phi)$

The rotational kinetic energies are

$$T_3 = \frac{1}{2} J_1 \dot{\theta}^2 + \frac{1}{2} J_1' \Omega^2$$

where $J_1 = \frac{1}{12} m_1 L_1^2$, $J_1' = J_1 \cos^2\theta$

and

$$T_4 = \frac{1}{2} J_2 \dot{\phi}^2 + \frac{1}{2} J_2' \Omega^2$$

where $J_2 = \frac{1}{12} m_2 L_2^2$, $J_2' = J_2 \sin^2\phi$

The potential energy is given by

$$U = \frac{1}{2} K_1 \theta^2 + \frac{1}{2} K_2 \phi^2$$

FBD for positive θ, θ'

2) $$GJ \frac{d^2\theta}{dx^2} = \rho J \frac{d^2\theta}{dt^2}$$

B.C. are $\theta(0) = 0$

$$-GJ\theta' - K_T\theta = 0$$

Assume

$$\theta = \sum X(x)\,T(t)$$

$$\ddot{T} = -\omega^2 T$$

$$\therefore \quad GJX'' + \rho J \omega^2 X = 0$$

$$X'' = \sigma^2 X$$

$$\sigma = \omega\sqrt{\frac{\rho}{G}}$$

$$X(x) = A \sin\sigma x + B\cos\sigma x$$

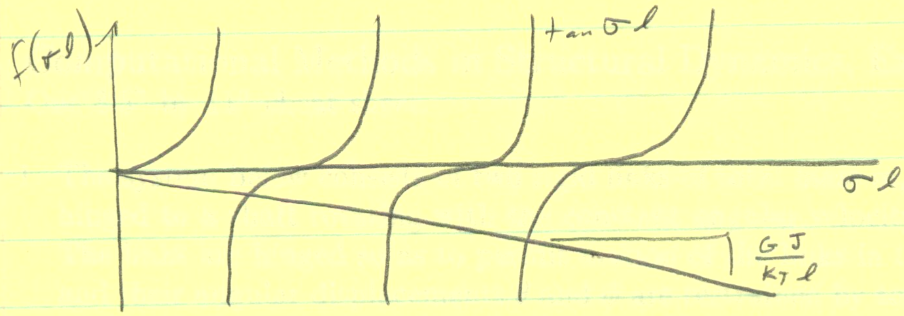
Applying B.C.,

$B = 0$

$$GJA\sigma\cos\sigma l + K_T A\sin\sigma l = 0$$

$$\tan\sigma l = \frac{-GJ\sigma}{K_T}$$

Which can be solved numerically for σ.
Nondimensionally $\tan\sigma l = \frac{-GJ\sigma l}{K_T l}$ would be solved using



$f(\sigma l)$
$\tan \sigma l$
σl
$\frac{GJ}{K_T l}$