ME 460/660, Mechanical Vibration — Final Exam, Fall 2003

Closed book, closed notes. Use one provided $8\frac{1}{2} \times 11$ formula sheet and turn in with exam. Test books will be provided. **Do all work on the exam pages** with the exception of the full length problems. Full length problems are to be done in the test book.

1 Formulae

Euler Relations	$e^{j\beta} = \cos(\beta) + j\sin(\beta)$ $\sin(\beta) = \frac{e^{j\beta} - e^{-j\beta}}{2j}$ $\cos(\beta) = \frac{e^{j\beta} + e^{-j\beta}}{2}$
Lagrange's Equation	$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} + \frac{\partial R}{\partial \dot{q}_i} = Q_i$
Fourier Series (Real Form)	$F(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos(n\omega_T t) + b_n \sin(n\omega_T t))$ where $\omega_T = 2\pi/T$, and T is the period of the function $a_0 = \frac{2}{T}\int_0^T F(t)dt$, $a_n = \frac{2}{T}\int_0^T F(t)\cos(n\omega_T t)dt$, and $b_n = \frac{2}{T}\int_0^T F(t)\sin(n\omega_T t)dt$
Fourier Series (Complex Form)	$F(t) = \sum_{n=-\infty}^{\infty} \left(a_n e^{j\omega_T n t}\right)$ where $\omega_T = 2\pi/T$, and T is the period of the function and $a_n = \frac{1}{T}\int_0^T F(t)e^{-j\omega_T n t}dt$
Convolution Integral	$x(t) = \frac{1}{m\omega_d}e^{-\zeta\omega_n t}\int_0^t \left[F(\tau)e^{\zeta\omega_n \tau}\sin\left(\omega_d(t-\tau)\right)\right]d\tau$ or $x(t) = \frac{1}{m\omega_d}\int_0^t \left[F(t-\tau)e^{\zeta\omega_n(t-\tau)}\sin\left(\omega_d \tau\right)\right]d\tau$
Log Decrement	$\delta = \frac{1}{n}\ln\left(\frac{x(t)}{x(t+nT)}\right)$, $\zeta = \frac{\delta}{\sqrt{4\pi^2+\delta^2}}$

2 Final Exam

Problems are 10 points each.

1. A forcing function due to a bearing fault is approximately represented by $f(t) = \sum_{m=-\infty}^{\infty} \delta(t-m)$.
   (a) Find the Fourier Series representation of the forcing function. (4 points)
   (b) Find the forced response of an undamped system to this Fourier Series representation. (3 points)
   (c) Recognizing that the Dirac delta function is an approximation of reality, and knowing that each impulse has a finite period of Δt, discuss for which frequencies the Dirac delta function should, and for which frequencies it should not, be used in a multiple degree of freedom system with an infinite number of degrees of freedom (*the system* ***does*** *have infinite degrees of freedom, just like a real system does for all practical purposes.*). (3 points)

2. Find $x(t)$ for the system defined by $10\ddot{x} + 4000x = \delta(t)$, given $x(0) = 1$, and $\dot{x}(0) = -0.1$ (just prior to the impulse). Sketch your solution.

3. Given

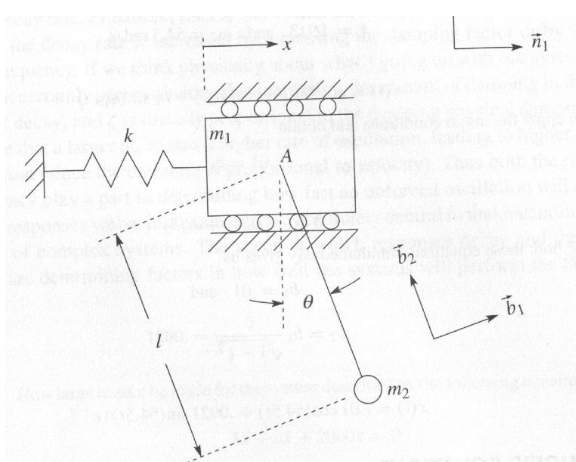
$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}, \text{and} \qquad K = \begin{bmatrix} 65 & -35 & 0 \\ -35 & 65 & -15 \\ 0 & -15 & 30 \end{bmatrix}$$

   and the *non-mass normalized* mode shapes

$$\mathbf{u}_1 = \begin{bmatrix} 1.00000 \\ 1.69193 \\ 3.69392 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} 1.00000 \\ 1.26369 \\ -0.35708 \end{bmatrix}, \quad \mathbf{u}_3 = \begin{bmatrix} 1.00000 \\ -0.38419 \\ 0.02031 \end{bmatrix}$$

   find the natural frequencies of the system.

4. Obtain the equations of motion for the system below. Assume no friction. Write you governing equations in terms of x and θ. Assume m_2 is a point mass.



5. Graduate Students/Undergraduate Bonus (20%): Solve for the steady-state (particular) response of the following system if the boundary conditions are presumed to be clamped-clamped where $c = \sqrt{\tau/\rho}$.

$$w_{tt}(x,t) - c^2 w_{xx}(x,t) = 100\sin(t)\delta(x - l/2)$$

   Recall that the integral of a Dirac delta function times another function is equal to the "another function" evaluated when the argument of the argument of the Dirac delta function is zero.