Computational Methods in Structural Dynamics, Final Winter 1999

One 8.5" by 11" cheat sheet.

1. Derive the equations of motion and allowable boundary conditions for a Timoshenko beam given the following.
   The bending strain energy is $V_b = \frac{1}{2}\int_0^L EI(\alpha')^2 dx$, the shear strain energy is $V_s = \frac{1}{2}\int_0^L \kappa GA(\alpha - \frac{\partial v}{\partial x})^2 dx$, the kinetic energy is $T = \frac{1}{2}\int_0^L \rho A\dot{v}^2 dx + \frac{1}{2}\int_0^L \rho I\dot{\alpha}^2 dx$, and the non-conservative variational work is $W_{nc} = \int_0^L p(x,t)\delta v(x,t)dx$. Of course the total potential energy is $V = V_b + V_s$. Note that for a Timoshenko beam the rotation parameter α is independent of the slope $\frac{\partial v}{\partial x}$, both being a function of x and t. As a result, you should expect to derive two coupled differential equations– one in α, and one in v. Assume A and I are functions of x as well. (20 points)

2. The partially non-dimensionalized equation of motion of a tapered beam is given by: $\frac{\partial^2}{\partial \xi^2}\left(\frac{2}{3}\xi^3\frac{\partial^2 w}{\partial \xi^2}\right) + 2\xi\frac{\partial^2 w}{\partial t^2} = 0$. Assuming a deflection form of $W(\xi) = a_1\left(1-\xi\right)^2 + a_2\xi\left(1-\xi\right)^2$, estimate the first and second natural frequencies of the beam using both one and two term representations of the mode shape/s. *Not using the collocation method*. (20 points)

   Choose one of the following

3. Find the natural frequencies and mode shapes for the system of problem 2 using the collocation method. Use the mid-point for the one term approximation, and the 1/3 and 2/3 points for the two term approximation. (20 points)

4. A system is modeled by the following equation. Assume m, c, and k are greater than zero. Is the system asymptotically stable? You must justify your answer using Liapunov stability. (20 points)

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\ddot{\mathbf{x}} + \begin{bmatrix} c & -c \\ -c & c \end{bmatrix}\dot{\mathbf{x}} + \begin{bmatrix} 2k & -k \\ -k & k \end{bmatrix}\mathbf{x} = \mathbf{0}$$