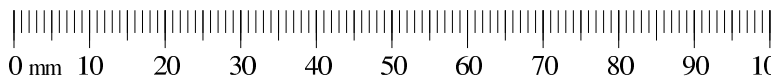
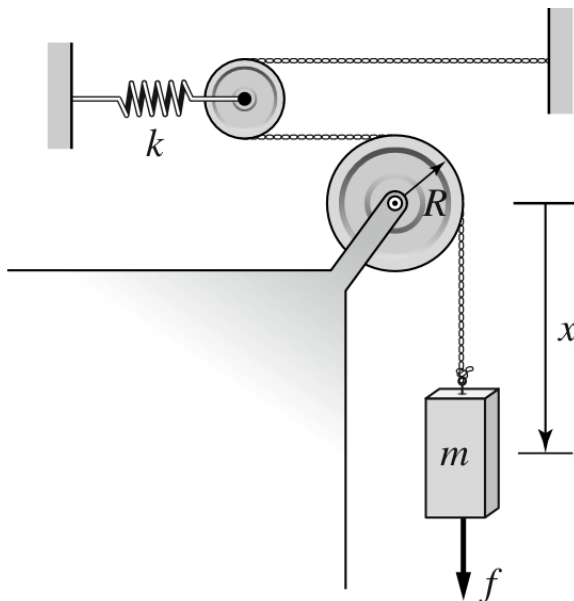


Closed book, closed notes. Test booklets will be provided. Formula sheet must be turned in with the exam. Formula sheet must be exactly the same as that posted on the web site. Calculators allowed. Problem 4 is required for graduate students.

1. (a) Find the forced response ($x(t)$, particular solution) of $10\ddot{x} + 0.1\dot{x} + 1000x = 100y(t) + 0.1\dot{y}(t)$ if $y(t) = Y \sin \omega t$ for

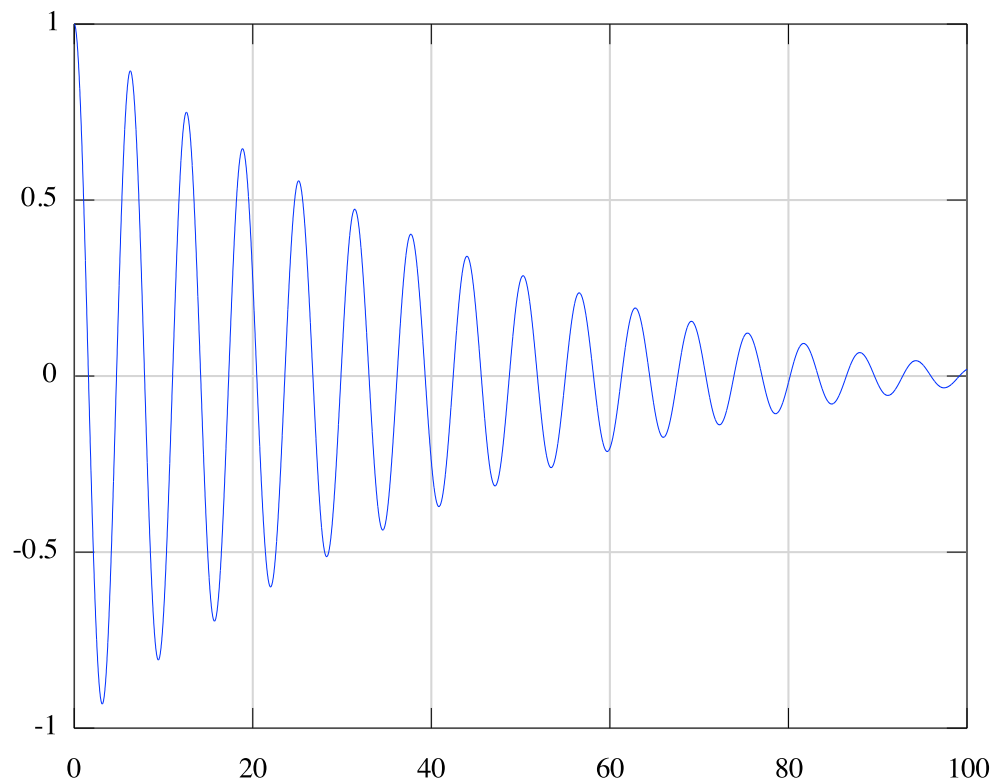
Y	ω (rad/s)
10	1
0.1	10
0.001	100

 - (b) Sketch the magnitude (*using 1/2 of a page for scale!*) of $X(i\omega)/Y(i\omega)$ for the system above for $0 < \omega < 10\omega_n$ on a dB scale ($20 * \log_{10}(\frac{x}{y})$) labeling values (not in dB) at $\omega = (0.1, \omega_n, 10\omega_n)$ where ω_n is the natural frequency in Hz.
 - (c) Sketch the force and equation terms in the complex plane for the values in the first row of the table.
2. Presume the mass of the pulley connected to the spring is $0.1m$ and the mass of the pulley of radius R is m . Neglect the mass moment of inertia of the pulley connected to the spring, and presume the pulley of radius R is a uniform disk. Derive the governing equation/s.



(This is a ruler you can use on the next problem)

3. Find the natural frequency of the systems whose free response is shown below. What kind of damping is exhibited in the response?



4. *Grad student/bonus:* Determine the natural frequencies and mode shapes for a clamped-free circular cross section extension rod. The equation of motion of a rod is $\left(\frac{E}{\rho}\right) \frac{\partial^2 w(x,t)}{\partial x^2} = \frac{\partial^2 w(x,t)}{\partial t^2}$. (20% of other points)