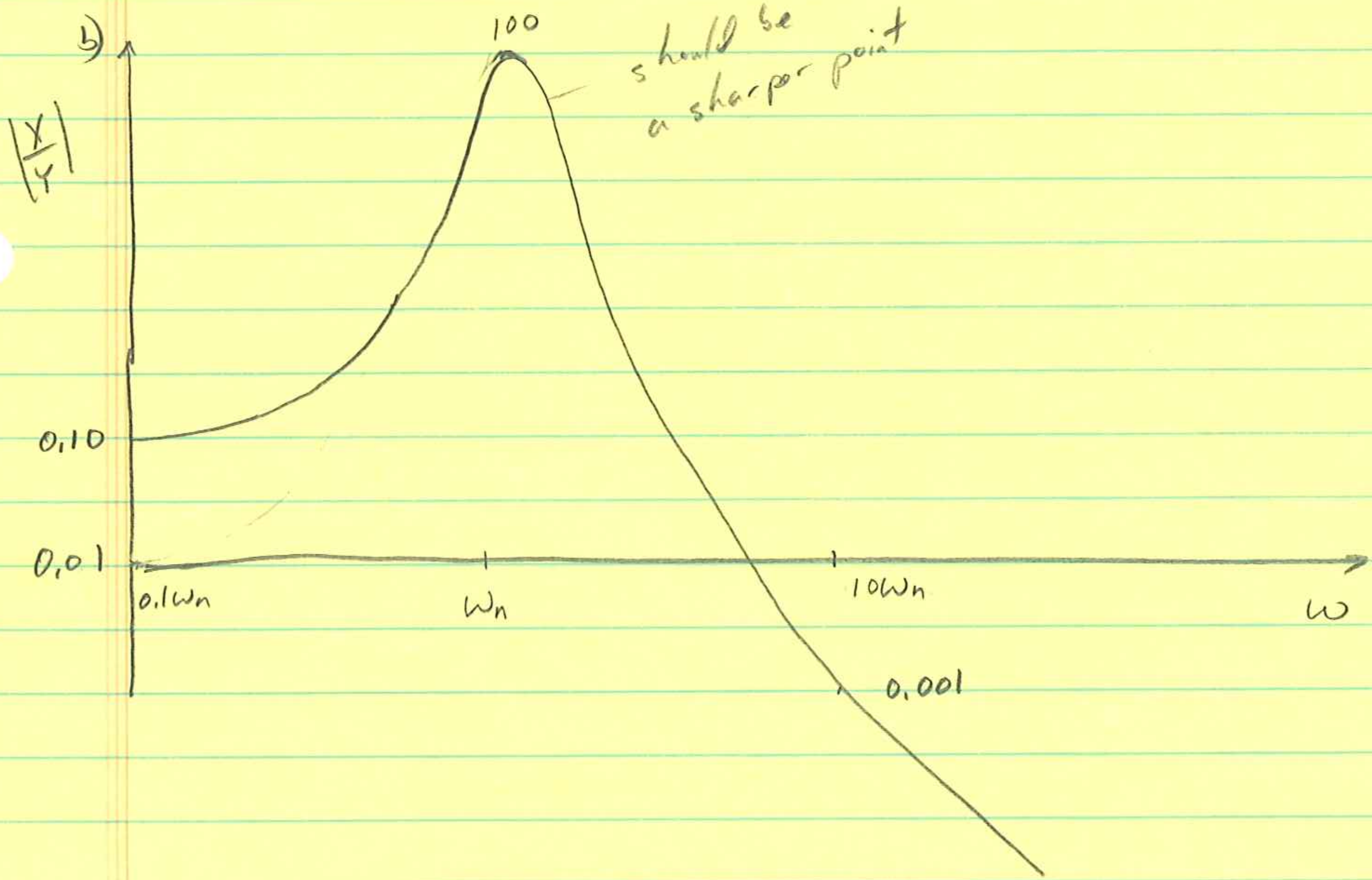
Fall 2011 Solutions ME460 Exam 1

1) $\frac{X}{Y} = \frac{100 + 0.1j\omega}{1000 - 10\omega^2 + 0.1j\omega}$ $\omega_n = 10$ rad/s

a)

	Y	ω	$\left\lvert\frac{X}{Y}\right\rvert$	$\measuredangle \frac{X}{Y}$ (rad)	x(t)
$0.1\omega_n$	10	1	0.101	9×10^{-4}	$1.01\sin(t + 9\times10^{-4})$
ω_n	0.10	10	100	-1.561	$10\sin(10t - 1.561)$
$10\omega_n$	0.001	100	1×10^{-3}	-3.042	$1\times10^{-6}\sin(100t - 3.042)$

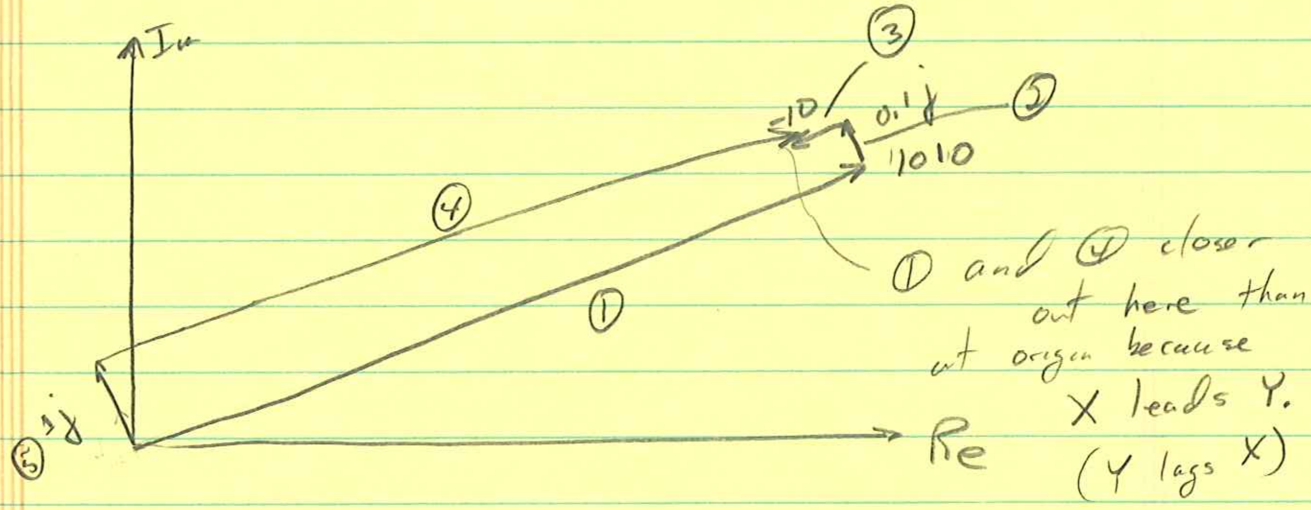
b)



c) $-10X + 0.1jX + 1000X = 100Y + 0.1jY$

$X = 1.01, \quad Y = 10$

$$\underset{①}{-10} + \underset{②}{0.1j} + \underset{③}{1010} = \underset{④}{1000} + \underset{⑤}{.1j}$$



② $U = -mg\,X + \frac{1}{2} k \delta^2 \qquad \delta = \frac{1}{2} X$

$= -mg\,X + \frac{1}{8} k X^2$

$T = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} J \underset{\text{2nd pulley}}{\dot{\theta}^2} + \frac{1}{2} \underset{\text{1st pulley}}{(0.1m)\,\dot{\delta}^2}$

$J = \frac{1}{2} m R^2 \quad , \dot{\delta} = \frac{1}{2}\dot{x} \quad , \theta = \frac{X}{R}$

$T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2}\left(\frac{1}{2} m R^2\right)\frac{\dot{x}^2}{R^2} + \frac{1}{2}\left(0.1 m \frac{1}{4}\right)\dot{x}^2$

$= \frac{1}{2}\left(m + \frac{1}{2} m + 0.025 m\right)\dot{x}^2$

$= \frac{1}{2}\left(1.525\, m\right)\dot{x}^2$

$\frac{d}{dt}\frac{\partial T}{\partial \dot{x}} + \frac{\partial U}{\partial x} = 0$ (other terms are zero

$1.525 m\ddot{x} + -mg + \frac{1}{4} k x = 0$

$$\boxed{1.525\, m\ddot{x} + \frac{1}{4} k x = mg}$$

3) For high amplitudes

$$\delta = \frac{1}{6} \ln \frac{47}{19} = 0.15$$

For low amplitudes

$$\delta = \frac{1}{6} \ln \frac{11}{2} = 0.28$$

Damping ratio ζ clearly increasing.
Implies Coulomb.
Not straight decay, so Coulomb plus viscous

$$T = \frac{93}{15} \text{ sec}, \quad \omega_n = \frac{2\pi \cdot 15}{93} = 1.0 \text{ rad/s}$$

4) $\frac{E}{\rho} w'' = \ddot{w}$ $\qquad \ddot{w} = -\omega_n^2 w$

$-\omega_n^2 X(x) + \frac{E}{\rho} X'' = 0$ $\qquad (\ddot{T}(t) X(x) = -\omega_n^2 T(t) X(x))$

$X(x) = A \sin \beta_n x + B \cos \beta_n x$

$X(0) = 0 = B, \quad B = 0$ $\qquad X'(l) = 0 = A\beta_n \cos \beta_n l$

$X(x) = A \sin \beta_n x$ $\qquad \beta_n = \frac{(2n-1)\pi}{2l}, \; n = 1, 2, \ldots \infty$

$$\omega_n^2 A \sin \beta x - \frac{E}{\rho} \beta_n^2 \sin \beta_n x = 0$$

$$\boxed{\omega_n = \beta_n \sqrt{\frac{E}{\rho}} = \frac{(2n-1)\pi}{2l} \sqrt{\frac{E}{\rho}} \qquad X_n(x) = A \sin \frac{(2n-1)x}{2l}}$$