Exam 2, Fall 2011 Solutions ME460/660

1)

$$\frac{X}{F} = \frac{1}{1000-100\omega^2 + j\omega}$$

ω	$\frac{X}{F}$	$\lvert X\rvert$	$\angle X$ (rad)	$\angle X$ (deg)	
$\frac{2\pi}{5}$	$1.187\times10^{-3} - 1.70\times10^{-6}j$	9.626×10^{-4}	3.140	179.9	$\sim \pi$ or 180°
$\frac{6\pi}{5}$	$-2.374\times10^{-3} - 2.125\times10^{-5}j$	2.138×10^{-4}	0.009	0.513	~ 0
2π	$-3.392\times10^{-4} - 7.231\times10^{-7}j$	1.100×10^{-5}	0.002	0.122	~ 0

$$x(t) = 9.626\times10^{-4}\cos\left(\frac{2\pi}{5}t + 3.14\right) + 2.138\times10^{-4}\cos\left(\frac{6\pi}{5}t + 0.009\right)$$
$$+ 1.100\times10^{-5}\cos\left(2\pi t + 0.002\right)$$

2) $\underline{0 < t < 0{,}02}$

$$x(t) = \frac{1}{10\sqrt{10}} \int_0^t \sin\sqrt{10}\,(t-\tau)\,d\tau$$

$$= \frac{1}{10\sqrt{10}}\;\frac{1}{\sqrt{10}}\quad \cos\sqrt{10}\,(t-\tau)\Big|_0^t$$

$$= \frac{1}{100}\left(\cos 0 \; - \; \cos\sqrt{10}\,t\right)$$

$$= \frac{1}{100}\left(1 - \cos\sqrt{10}\,t\right)$$

$\underline{0{,}02 < t}$

$$x(t) = \frac{1}{10\sqrt{10}} \int_0^{0{,}02} \sin\sqrt{10}\,(t-\tau)\,d\tau \quad + \quad 0$$

$$= \frac{1}{100}\left(\cos\sqrt{10}\,(t-\tau)\right)\Big|_0^{0{,}02}$$

$$= \frac{1}{100}\left(\cos\sqrt{10}\,(t-0{,}02) - \cos\sqrt{10}\,t\right)$$

3) a) $300\ddot{x} + c\dot{x} + 40000x = 40000y + c\dot{y}$

$$\frac{X}{Y} = \frac{40000 + cj\omega}{40000 - 300\omega^2 + cj\omega}$$

At resonance $\frac{X}{Y} = 40$, $\omega = \sqrt{\frac{40000}{300}} = 11.55$ rad/s

$$\left| \frac{40000 - cj\,11.55}{11.55\,cj} \right| = 40$$

Let's be honest. I solved this using the solver in my calculator.

$$\frac{40000^2 + c^2\,133.3}{c^2\,133.3} = 1600$$

For entertainment,

$$40000^2 + c^2\,133.3 = 1600 \cdot 133.3\,c^2$$

$$40000^2 = 1599 \cdot 133.3\,c^2$$

$$\underline{c = 86.61 \text{ kg/s}}$$

b) $F_{spring} = \underset{K}{40000} \, |X - Y| = 40000 \, |0.1e^{-1.5j} - 0.0025| = \underline{4004\text{ N}}$

$F_{dashpot} = c\omega X = 86.61 \cdot 11.55 \, |X - Y| = \underline{100.1\text{ N}}$

4)

$$\boxed{m\ddot{u}\Big|_{\ell} = -ku\Big|_{\ell} - EA\frac{du}{dx}\Big|_{\ell}}$$