ME 460 Final Exam solutions, Fall 2009

1) $\frac{X}{Y} = \frac{10000 + jw}{10000 - 10w^2 + jw}$

ω	$\lvert Y \rvert$	$\frac{X}{Y}$ (mag, phase) phase in radians	X
0	$\frac{1}{2}$	1, 0	$\frac{1}{2}$
π	$\frac{-1}{\pi}$	$1.01, -3.1\times10^{-6}$	-0.321
2π	$\frac{-1}{2\pi}$	$1.04, -2.6\times10^{-5}$	-0.166
3π	$\frac{-1}{3\pi}$	$1.10, -9.2\times10^{-5}$	-0.116
4π	$\frac{-1}{4\pi}$	$1.19, -2.4\times10^{-4}$	-0.0945

$\measuredangle$ in deg

$x(t) = \frac{1}{2} - 0.321\sin(\pi t - 3.1\times10^{-6})$ $\quad -1.78\times10^{-4\circ}$

$- 0.166\sin(2\pi t - 2.6\times10^{-5})$ $\quad -1.49\times10^{-3\circ}$

$- 0.116\sin(3\pi t - 9.2\times10^{-5})$ $\quad -5.27\times10^{-3\circ}$

$- 0.0945\sin(4\pi t - 2.4\times10^{-4})$ $\quad -0.0138^\circ$

(Only 4 terms requested, although 5 presented here)

Phases could alternatively be flipped by 180° if signs are changed.

2) $(K - M\omega^2) = \begin{bmatrix} 27 - 4\omega^2 & -4 \\ -4 & 10 - 4\omega^2 \end{bmatrix}$

$\det(K - M\omega^2) = 0$

$\omega^4 - 9.25\omega^2 + 15.875 = 0$

$\omega^2 = 2.28,\ 6.97$

a) $\boxed{\omega_1 = 1.51 \text{ rad/s},\ \omega_2 = 2.64 \text{ rad/s}}$

My calculator gives $\underline{u}_1 = \begin{bmatrix} 0.224 \\ 1 \end{bmatrix}$

$\underline{u}_1^T M \underline{u}_1 = \alpha^2 = 4.2$

$\frac{1}{\alpha} = 0.488$

$\therefore \boxed{\underline{u}_1 = \begin{bmatrix} 0.109 \\ 0.488 \end{bmatrix}}$ $\qquad \underline{u}_1^T K \underline{u}_1 = \omega_1^2$ ✓

My calculator gives $\underline{u}_2 = \begin{bmatrix} 1 \\ -0.224 \end{bmatrix}$

$\underline{u}_2^T M \underline{u}_2 = \alpha^2 = 4.2$

$\boxed{\underline{u}_2 = \begin{bmatrix} 0.488 \\ -0.109 \end{bmatrix}}$ $\qquad \underline{u}_2^T K \underline{u}_2 = \omega_2^2$ ✓

$\underline{u}_1^T K \underline{u}_2 = 0$ ✓ $\qquad \underline{u}_1^T M \underline{u}_2 = 0$ ✓

4 checks available

3) $T = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} m v_u^2$

Presuming X up

$U = \frac{1}{2} k_1 x^2 + \frac{1}{2} k_2 x^2 + m_1 g x + m g h_u$

Position of unbalance

$$\vec{r}_u = (\underbrace{x - e\sin\omega t}_{h_u})\hat{i} + e\cos\omega t\,\hat{j}$$

$$\vec{V}_u = (\dot{x} - e\omega\cos\omega t)\hat{i} - e\omega\sin\omega t\,\hat{j}$$

$$V_u^2 = \dot{x}^2 - 2e\omega\cos\omega t\,\dot{x} + e^2\omega^2\cos^2\omega t + e^2\omega^2\sin^2\omega t$$

$$= \dot{x}^2 - 2e\omega\cos\omega t\,\dot{x} + e^2\omega^2$$

$R = \frac{1}{2} c\dot{x}^2$

$\frac{d}{dt}\frac{\partial T}{\partial \dot{x}} = m_1\ddot{x} + m\ddot{x} + me\omega^2\sin\omega t$

$\frac{\partial T}{\partial x} = 0$, $\frac{\partial U}{\partial x} = (k_1 + k_2)x + m_1 g + mg$

$\frac{\partial R}{\partial \dot{x}} = c\dot{x}$

Sub into Lagrange's

$$(m_1 + m)\ddot{x} + c\dot{x} + (k_1 + k_2)x = -(m_1 + m)g - me\omega^2\sin\omega t$$

Problem did not state if m is part of m_1. If so, $(m_1 + m)$ becomes m_1.

4)

$$w_{tt} - c^2 w_{xx} = 0$$

$X(x) = \sin \sigma x$ ($X(x) = 0$, so cos term must not exist)

$X'(x)\Big|_{x=l} = 0 = \cos \sigma l$

$$\sigma l = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots \frac{(2n-1)\pi}{2}$$

$$\sigma_n = \frac{(2n-1)\pi}{2l}$$

$$w(x,t) = \sum_{n=1}^{\infty} T_n(t) X_n(x)$$

Substituting into EOM

$$\sum_{n=1}^{\infty} (\ddot{T}_n + c^2 \sigma_n^2 T_n) \cos \sigma_n l = 100\, \delta(x - \tfrac{l}{2}) \sin 3t$$

Multiplying both sides by $\cos \sigma_m l$ and integrating, noting

$$\int_0^l \cos^2 \frac{(2n-1)\pi}{2l}\, dt = \frac{l}{2}$$

$$\frac{l}{2}(\ddot{T}_m + c^2 \sigma_m^2 T_m) = \underbrace{100 \cos \sigma_m \tfrac{l}{2}}_{f_m} \sin 3t$$

$f_m = 100 \times \left(\cos\frac{\pi}{4},\ \cos\frac{3\pi}{4},\ \cos\frac{5\pi}{4} \cdots \cos\frac{(2n-1)\pi}{4} \right.$

Using section 2.1

$$x(t) = \sum_{n=1}^{\infty} \frac{200\cos\frac{(2n-1)\pi}{4}}{\ell\left[c^2\left(\frac{(2n-1)\pi}{\ell}\right)^2 - 3^2\right]} \sin 3t \, \cos\frac{(2n-1)\pi}{\ell}x$$

$\cos\frac{(2n-1)\pi}{4}$ can be simplified since the magnitude never changes, but sign does.