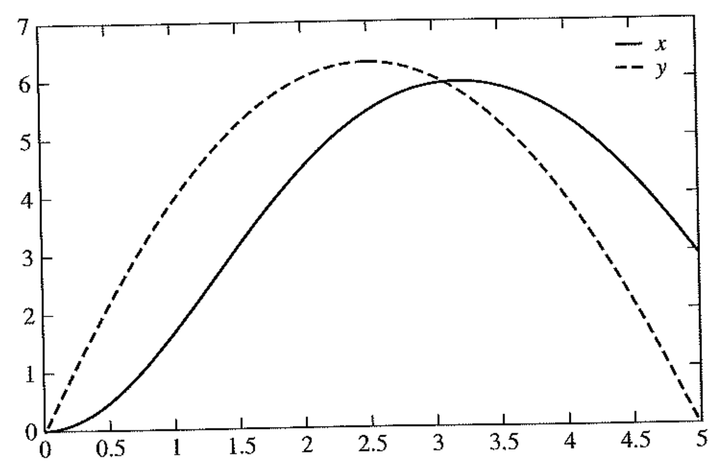


Figure 8.6.3 Plot of the desired trajectory (y) and the actual trajectory (x)

8.7 CHAPTER REVIEW

This chapter emphasizes understanding system behavior in the time domain. The forcing functions commonly used to model real inputs or to test a system's response in the time domain are the impulse, the step, and the ramp functions. The impulse models a suddenly applied and suddenly removed input. The step function models a suddenly applied input that remains constant. The ramp models an input that is changing at a constant rate.

Sections 8.1 and 8.2 treated the response of first- and second-order systems. The time constant τ, the damping ratio ζ, and the undamped natural frequency ω_n are important for assessing system response. In Section 8.3 we introduced the concepts of maximum overshoot M_p, peak time t_p, delay time t_d, rise time t_r, and settling time t_s. These are useful for describing and for specifying the step response. Section 8.4 treated parameter estimation in the time domain.

Section 8.5 showed how to use MATLAB to compute performance specifications, such as overshoot and rise time. Finally, Section 8.6 introduced several Simulink blocks that are useful for simulating nonlinear systems or systems with complicated inputs.

Now that you have completed Chapter 8, you should be able to do the following:

1. Obtain and interpret the free, step, ramp, and impulse response of linear models.
2. Compute and apply the time constant τ, the damping ratio ζ, and the undamped natural frequency ω_n to assess system response.
3. Compute and apply maximum overshoot M_p, peak time t_p, delay time t_d, rise time t_r, and settling time t_s to describe and assess system response.
4. Use time-domain response data to estimate coefficients in dynamic models.
5. Use MATLAB to compute performance specifications.
6. Use Simulink to simulate systems with complicated inputs.

PROBLEMS

Section 8.1 Response of First-Order Systems

8.1 A rocket sled has the following equation of motion: $6\dot{v} = 2700 - 24v$. How long must the rocket fire before the sled travels 2000 m? The sled starts from rest.

8.2 Suppose the rocket motor in Problem 8.1 takes 0.04 s to reach a constant thrust of 2700 N. Is a step function a good representation of this input? Support your answer with a calculation.

8.3 For each of the following models, obtain the free response and the time constant, if any.

a. $16\dot{x} + 14x = 0,\ x(0) = 6$
b. $12\dot{x} + 5x = 15,\ x(0) = 3$
c. $13\dot{x} + 6x = 0,\ x(0) = -2$
d. $7\dot{x} - 5x = 0,\ x(0) = 9$

8.4 For the model $2\dot{x} + x = 10f(t)$,

a. If $x(0) = 0$ and $f(t)$ is a unit step, what is the steady-state response x_{ss}? How long does it take before 98% of the difference between $x(0)$ and x_{ss} is eliminated?
b. Repeat part (a) with $x(0) = 5$.
c. Repeat part (a) with $x(0) = 0$ and $f(t) = 20u_s(t)$.

8.5 Obtain the steady-state response of each of the following models, and estimate how long it will take the response to reach steady-state.

a. $6\dot{x} + 5x = 20u_s(t),\ x(0) = 0$
b. $6\dot{x} + 5x = 20u_s(t),\ x(0) = 1$
c. $13\dot{x} - 6x = 18u_s(t),\ x(0) = -2$

8.6 Obtain the total response of the following models.

a. $6\dot{x} + 5x = 20u_s(t),\ x(0) = 0$
b. $6\dot{x} + 5x = 20u_s(t),\ x(0) = 1$
c. $13\dot{x} - 6x = 18u_s(t),\ x(0) = -2$

8.7 A certain rotational system has the equation of motion

$$100\frac{d\omega}{dt} + 5\omega = T(t)$$

where $T(t)$ is the torque applied by an electric motor, as shown in Figure 8.1.8. The model of the motor's field current i_f in amperes is

$$0.002\frac{di_f}{dt} + 4i_f = v(t)$$

where $v(t)$ is the voltage applied to the motor. The motor torque constant is $K_T = 15$ N · m/A. Suppose the applied voltage is $12u_s(t)$ V. Determine the steady-state speed of the inertia and estimate the time required to reach that speed.

8.8 The RC circuit shown in Figure 8.1.2c has the parameter values $R = 3 \times 10^6\ \Omega$ and $C = 1\ \mu$F. If the inital capacitor voltage is 6 V and the applied voltage is $v_s(t) = 12u_s(t)$, obtain the expression for the capacitor voltage response $v(t)$.

8.9 The liquid-level system shown in Figure 8.1.2d has the parameter values $A = 50\ \text{ft}^2$ and $R = 60\ \text{ft}^{-1}\text{sec}^{-1}$. If the inflow rate is $q_v(t) = 10u_s(t)$ ft³/sec, and the initial height is 2 ft, how long will it take for the height to reach 15 ft?

8.10 The immersed object shown in Figure 8.1.2e is steel and has a mass of 100 kg and a specific heat of $c_p = 500$ J/kg · °C. Assume the thermal resistance is $R = 0.09$°C · s/J. The inital temperature of the object is 20° when it is dropped into a large bath of temperature 80°C. Obtain the expression for the temperature $T(t)$ of the object after it is dropped into the bath.

8.11 Compare the responses of $2\dot{v} + v = \dot{g}(t) + g(t)$ and $2\dot{v} + v = g(t)$ if $g(t) = 10u_s(t)$ and $v(0) = 5$.

8.12 Compare the responses of $5\dot{v} + v = \dot{g} + g$ and $5\dot{v} + v = g$ if $v(0) = 5$ and $g = 10$ for $-\infty \le t \le \infty$.

8.13 Consider the following model:

$$6\dot{v} + 3v = \dot{g}(t) + g(t)$$

where $v(0) = 0$.

a. Obtain the response $v(t)$ if $g(t) = u_s(t)$.

b. Obtain the response $v(t)$ to the approximate step input $g(t) = 1 - e^{-5t}$ and compare with the results of part (a).

8.14 Obtain the response of the model $2\dot{v} + v = f(t)$, where $f(t) = 5t$ and $v(0) = 0$. Identify the transient and steady-state responses.

8.15 Obtain the response of the model $9\dot{v} + 3v = f(t)$, where $f(t) = 7t$ and $v(0) = 0$. Is steady-state response parallel to $f(t)$?

8.16 The resistance of a telegraph line is $R = 10\ \Omega$, and the solenoid inductance is $L = 5$ H. Assume that when sending a "dash," a voltage of 12 V is applied while the key is closed for 0.3 s. Obtain the expression for the current $i(t)$ passing through the solenoid.

Section 8.2 Response of Second-Order Systems

8.17 Obtain the oscillation frequency and amplitude of the response of the model $3\ddot{x} + 12x = 0$ for (a) $x(0) = 5$ and $\dot{x}(0) = 0$ and (b) $x(0) = 0$ and $\dot{x}(0) = 5$.

8.18 Obtain the response of the following models with the initial conditions: $x(0) = 0$ and $\dot{x}(0) = 1$.

a. $\ddot{x} + 4\dot{x} + 8x = 0$

b. $\ddot{x} + 8\dot{x} + 12x = 0$

c. $\ddot{x} + 4\dot{x} + 4x = 0$

8.19 Obtain the response of the following models with zero initial conditions:

a. $\ddot{x} + 4\dot{x} + 8x = 2u_s(t)$

b. $\ddot{x} + 8\dot{x} + 12x = 2u_s(t)$

c. $\ddot{x} + 4\dot{x} + 4x = 2u_s(t)$

8.20 Obtain the response of the following models with the initial conditions: $x(0) = 1$ and $\dot{x}(0) = -1$.

a. $3\ddot{x} + 21\dot{x} + 30x = 0$

b. $5\ddot{x} + 20\dot{x} + 20x = 0$

c. $2\ddot{x} + 8\dot{x} + 58x = 0$

8.21 Obtain the unit-step response of the following models with zero initial conditions:

a. $3\ddot{x} + 21\dot{x} + 30x = f(t)$

b. $5\ddot{x} + 20\dot{x} + 20x = f(t)$

c. $2\ddot{x} + 8\dot{x} + 58x = f(t)$

8.22 Find the response for the following models. The initial conditions are zero.

a. $3\ddot{x} + 21\dot{x} + 30x = 4t$

b. $5\ddot{x} + 20\dot{x} + 20x = 7t$

c. $2\ddot{x} + 8\dot{x} + 58x = 5t$

8.23 Suppose the input $f(t)$ to the following model is a ramp function: $f(t) = at$. Assuming that the model is stable, for what values of a, m, c, and k will the steady-state response be parallel to the input? For this case, what is the steady-state difference between the input and the response?

$$m\ddot{x} + c\dot{x} + kx = f(t)$$

8.24 If applicable, compute ζ, τ, ω_n, and ω_d for the following roots, and find the corresponding characteristic polynomial.

1. $s = -2 \pm 6j$
2. $s = 1 \pm 5j$
3. $s = -10, -10$
4. $s = -10$

8.25 If applicable, compute ζ, τ, ω_n, and ω_d for the dominant root in each of the following sets of characteristic roots.

1. $s = -2, -3 \pm j$
2. $s = -3, -2 \pm 2j$

8.26 A certain fourth-order model has the roots

$$s = -2 \pm 4j, -10 \pm 7j$$

Identify the dominant roots and use them to estimate the system's time constant, damping ratio, and oscillation frequency.

8.27 Given the model

$$\ddot{x} - (\mu + 2)\dot{x} + (2\mu + 5)x = 0$$

a. Find the values of the parameter μ for which the system is
   1. Stable
   2. Neutrally stable
   3. Unstable

b. For the stable case, for what values of μ is the system
   1. Underdamped?
   2. Overdamped?

8.28 The characteristic equation of the system shown in Figure 8.2.3 for $m = 3$ and $k = 27$ is $3s^2 + cs + 27 = 0$. Obtain the free response for the following values of damping: $c = 0, 9, 18,$ and 22. Use the initial conditions $x(0) = 1$ and $\dot{x}(0) = 0$.

8.29 The characteristic equation of a certain system is $4s^2 + 6ds + 25d^2 = 0$, where d is a constant. (a) For what values of d is the system stable? (b) Is there a value of d for which the free response will consist of decaying oscillations?

8.30 The characteristic equation of a certain system is $s^2 + 6bs + 5b - 10 = 0$, where b is a constant. (a) For what values of b is the system stable? (b) Is there a value of b for which the free response will consist of decaying oscillations?

8.31 A certain system has two coupled subsystems. One subsystem is a rotational system with the equation of motion:

$$50\frac{d\omega}{dt} + 10\omega = T(t)$$

where $T(t)$ is the torque applied by an electric motor, Figure 8.1.8. The second subsystem is a field-controlled motor. The model of the motor's field current i_f in amperes is

$$0.001\frac{di_f}{dt} + 5i_f = v(t)$$

where $v(t)$ is the voltage applied to the motor. The motor torque constant is $K_T = 25$ N · m/A. Obtain the damping ratio ζ, time constants, and undamped natural frequency ω_n of the combined system.

8.32 A certain armature-controlled dc motor has the characteristic equation

$$L_a I s^2 + (R_a I + cL_a)s + cR_a + K_b K_T = 0$$

Using the following parameter values:

$$K_b = K_T = 0.1 \text{ N} \cdot \text{m/A} \qquad I = 6 \times 10^{-5} \text{ kg} \cdot \text{m}^2$$

$$R_a = 0.6\ \Omega \qquad L_a = 4 \times 10^{-3} \text{ H}$$

obtain the expressions for the damping ratio ζ and undamped natural frequency ω_n in terms of the damping c. Assuming that $\zeta < 1$, obtain the expression for the time constant.

Section 8.3 Description and Specification of Step Response

8.33 Compute the maximum percent overshoot, the maximum overshoot, the peak time, the 100% rise time, the delay time, and the 2% settling time for the following model. The initial conditions are zero. Time is measured in seconds.

$$\ddot{x} + 4\dot{x} + 8x = 2u_s(t)$$

8.34 A certain system is described by the model

$$\ddot{x} + c\dot{x} + 4x = u_s(t)$$

Set the value of the damping constant c so that both of the following specifications are satisfied. Give priority to the overshoot specification. If both cannot be satisfied, state the reason. Time is measured in seconds.

1. Maximum percent overshoot should be as small as possible and no greater than 20%.
2. 100% rise time should be as small as possible and no greater than 3 s.

8.35 A certain system is described by the model

$$9\ddot{x} + c\dot{x} + 4x = u_s(t)$$

Set the value of the damping constant c so that both of the following specifications are satisfied. Give priority to the overshoot specification. If both cannot be satisfied, state the reason. Time is measured in seconds.

1. Maximum percent overshoot should be as small as possible and no greater than 20%.
2. 100% rise time should be as small as possible and no greater than 3 s.

8.36 Derive the fact that the peak time is the same for all characteristic roots having the same imaginary part.

8.37 For the two systems shown in Figure 8.3.8, the displacement $y(t)$ is a given input function. Obtain the response for each system if $y(t) = u_s(t)$ and $m = 3$, $c = 18$, and $k = 10$, with zero initial conditions.

Section 8.4 Parameter Estimation in the Time Domain

8.38 Suppose that the resistance in the circuit of Figure 8.4.1a is $3 \times 10^6\ \Omega$. A voltage is applied to the circuit and then is suddenly removed at time $t = 0$. The measured voltage across the capacitor is given in the following table. Use the data to estimate the value of the capacitance C.

Time t (s)	Voltage v_C (V)
0	12.0
2	11.2
4	10.5
6	9.8
8	9.2
10	8.6
12	8.0
14	7.5
16	7.0
18	6.6
20	6.2

8.39 The temperature of liquid cooling in a cup at room temperature (68°F) was measured at various times. The data are given next.

Time t (sec)	Temperature T (°F)
0	178
500	150
1000	124
1500	110
2000	97
2500	90
3000	82

Develop a model of the liquid temperature as a function of time, and use it to estimate how long it will take the temperature to reach 135°F.

8.40 Figure P8.40 shows the response of a system to a step input of magnitude 1000 N. The equation of motion is

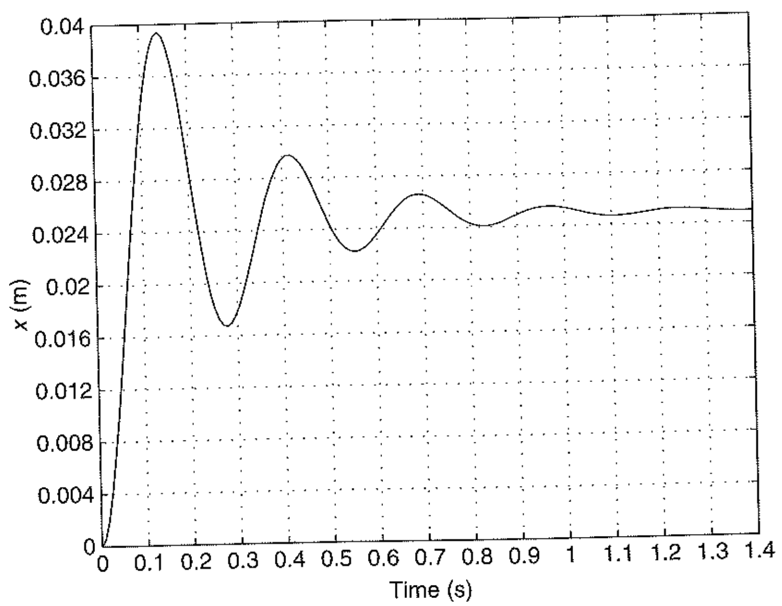
$$m\ddot{x} + c\dot{x} + kx = f(t)$$

Estimate the values of m, c, and k.

8.41 A mass-spring-damper system has a mass of 100 kg. Its free response amplitude decays such that the amplitude of the 30th cycle is 20% of the amplitude of the 1st cycle. It takes 60 s to complete 30 cycles. Estimate the damping constant c and the spring constant k.

8.42 a. For the following model, find the steady-state response and use the dominant-root approximation to find the dominant response (how long will it take to reach steady-state? does it oscillate?). The initial conditions

Figure P8.40



are zero.

$$\frac{d^3x}{dt^3} + 22\frac{d^2x}{dt^2} + 131\frac{dx}{dt} + 110x = u_s(t)$$

b. Obtain the exact solution for the response of the above model, and use it to check the prediction based on the dominant-root approximation.

8.43 The following model has a dominant root of $s = -3 \pm 5j$ as long as the parameter μ is no less than 3.

$$\frac{d^3y}{dt^3} + (6+\mu)\frac{d^2y}{dt^2} + (34+6\mu)\frac{dy}{dt} + 34\mu y = u_s(t)$$

Investigate the accuracy of the estimate of the maximum overshoot, the peak time, the 100% rise time, and the 2% settling time based on the dominant root, for the following three cases: (a) $\mu = 30$, (b) $\mu = 6$, and (c) $\mu = 3$. Discuss the effect of μ on these predictions.

8.44 Estimate the maximum overshoot, the peak time, and the rise time of the unit-step response of the following model if $f(t) = 5000u_s(t)$ and the initial conditions are zero.

$$\frac{d^4y}{dt^4} + 26\frac{d^3y}{dt^3} + 269\frac{d^2y}{dt^2} + 1524\frac{dy}{dt} + 4680y = f(t)$$

Its roots are

$$s = -3 \pm 6j, \qquad -10 \pm 2j$$

8.45 What is the form of the unit step response of the following model? Find the steady-state response. How long does the response take to reach steady state?

$$2\frac{d^4y}{dt^4} + 52\frac{d^3y}{dt^3} + 6250\frac{d^2y}{dt^2} + 4108\frac{dy}{dt} + 1.1202 \times 10^4 y = 5 \times 10^4 f(t)$$

8.46 Use a software package such as MATLAB to plot the step response of the following model for three cases: $a = 0.2$, $a = 1$, and $a = 10$. The step input has a magnitude of 2500.

$$\frac{d^4y}{dt^4} + 24\frac{d^3y}{dt^3} + 225\frac{d^2y}{dt^2} + 900\frac{dy}{dt} + 2500y = f + a\frac{df}{dt}$$

Compare the response to that predicted by the maximum overshoot, peak time, 100% rise time, and 2% settling time calculated from the dominant roots.

Section 8.5 MATLAB Applications

8.47 Use MATLAB to find the maximum percent overshoot, peak time, 2% settling time, and 100% rise time for the following equation. The initial conditions are zero.

$$\ddot{x} + 4\dot{x} + 8x = 2u_s(t)$$

8.48 Use MATLAB to compare the maximum percent overshoot, peak time, and 100% rise time of the following models where the input $f(t)$ is a unit step function. The initial conditions are zero.

a. $3\ddot{x} + 18\dot{x} + 10x = 10f(t)$

b. $3\ddot{x} + 18\dot{x} + 10x = 10f(t) + 10\dot{f}(t)$

8.49 a. Use MATLAB to find the maximum percent overshoot, peak time, and 100% rise time for the following equation. The initial conditions are zero.

$$\frac{d^3x}{dt^3} + 22\frac{d^2x}{dt^2} + 113\frac{dx}{dt} + 110x = u_s(t)$$

b. Use the dominant root pair to compute the maximum percent overshoot, peak time, and 100% rise time, and compare the results with those found in part (a).

8.50 a. Use MATLAB to find the maximum percent overshoot, peak time, and 100% rise time for the following equation. The initial conditions are zero.

$$\frac{d^4y}{dt^4} + 26\frac{d^3y}{dt^3} + 269\frac{d^2y}{dt^2} + 1524\frac{dy}{dt} + 4680y = 5000u_s(t)$$

b. The characteristic roots are $s = -3 \pm 6j, -10 \pm 2j$. Use the dominant root pair to compute the maximum percent overshoot, peak time, and 100% rise time, and compare the results with those found in part (a).

8.51 a. Use MATLAB to find the maximum percent overshoot, peak time, and 100% rise time for the following equation. The initial conditions are zero.

$$\frac{d^4y}{dt^4} + 14\frac{d^3y}{dt^3} + 127\frac{d^2y}{dt^2} + 426\frac{dy}{dt} + 962y = 926u_s(t)$$

b. Use the dominant root pair to compute the overshoot, peak time, and 100% rise time, and compare the results with those found in part (a).

Section 8.6 Simulink Applications

8.52 The following equation has a polynomial input.

$$0.125\ddot{x} + 0.75\dot{x} + x = y(t) = -\frac{27}{800}t^3 + \frac{270}{800}t^2$$

Use Simulink to plot $x(t)$ and $y(t)$ on the same graph. The initial conditions are zero.

8.53 The following model has a polynomial input.

$$\dot{x}_1 = -3x_1 + 4x_2$$

$$\dot{x}_2 = -5x_1 - x_2 + f(t)$$

$$f(t) = -\frac{5}{3}t^2 + \frac{25}{3}t$$

The initial conditions are $x_1(0) = 3$ and $x_2(0) = 7$. Use Simulink to plot $x_1(t)$, $x_2(t)$, and $f(t)$ on the same graph.

8.54 First create a Simulink model containing an LTI System block to plot the unit-step response of the following equation for $k = 4$. The initial conditions are zero.

$$5\frac{d^3x}{dt^3} + 3\frac{d^2x}{dt^2} + 7\frac{dx}{dt} + kx = u_s(t)$$

Then create a script file to run the Simulink model. Use the file to experiment with the value of k to find the largest possible value of k such that the system remains stable.

8.55 Figure P8.55 shows an engine valve driven by an overhead camshaft. The rocker arm pivots about the fixed point O. For particular values of the parameters shown, the valve displacement $x(t)$ satisfies the following equation.

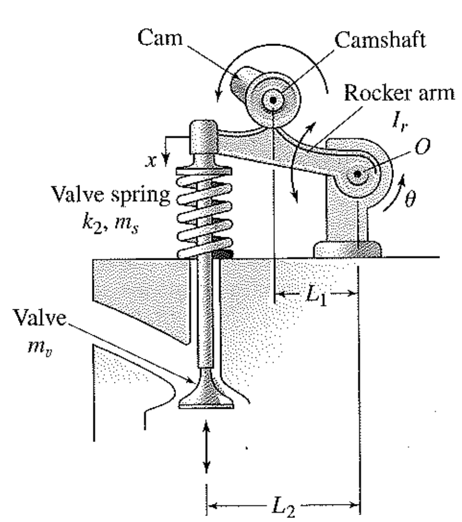
$$10^{-6}\ddot{x} + 0.3x = 5\theta(t)$$

where $\theta(t)$ is determined by the cam shaft speed and the cam profile. A particular profile is

$$\theta(t) = 16 \times 10^3(10^4t^4 - 200t^3 + t^2) \qquad 0 \le t \le 0.01$$

Use Simulink to plot $x(t)$. The initial conditions are zero.

Figure P8.55



CHAPTER 9

System Analysis in the Frequency Domain

CHAPTER OUTLINE

CHAPTER OBJECTIVES

When you have finished this chapter, you should be able to

1. Sketch frequency response plots for a given transfer function, and use the plots or the transfer function to determine the steady-state response to a sinusoidal input.
2. Compute the frequencies at which resonance occurs, and determine a system's bandwidth.
3. Analyze vibration isolation systems and the effects of base motion and rotating unbalance.
4. Determine the steady-state response to a periodic input, given the Fourier series description of the input.
5. Estimate the form of a transfer function and its parameter values, given frequency response data.
6. Use MATLAB as an aid in the preceding tasks.

The term *frequency response* refers to how a system responds to a periodic input, such as a sinusoid. An input $f(t)$ is periodic with a period P if $f(t + P) = f(t)$ for all values of time t, where P is a constant called the *period*. Periodic inputs are commonly found in many applications. The most common perhaps is ac voltage, which is sinusoidal. For the common ac frequency of 60 Hz, the period is $P = 1/60$ s. Rotating unbalanced machinery produces periodic forces on the supporting structures, internal combustion engines produce a periodic torque, and reciprocating pumps produce hydraulic and pneumatic pressures that are periodic.

Frequency response analysis focuses on harmonic inputs, such as sines and cosines. A sine function has the form $A \sin \omega t$, where A is its *amplitude* and ω is its frequency in radians per unit time. Note that a cosine is simply a sine shifted by 90° or $\pi/2$ rad, as