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ME 460/660
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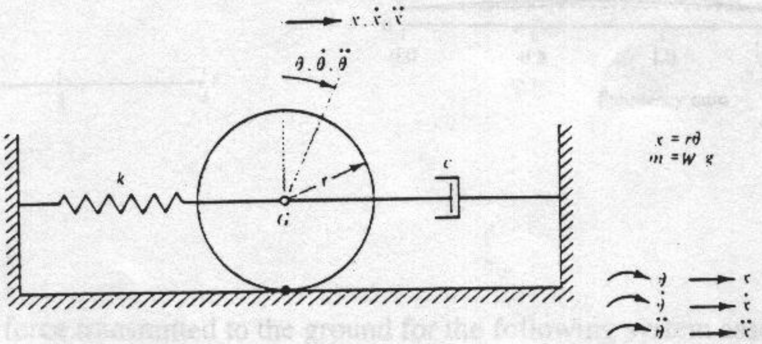
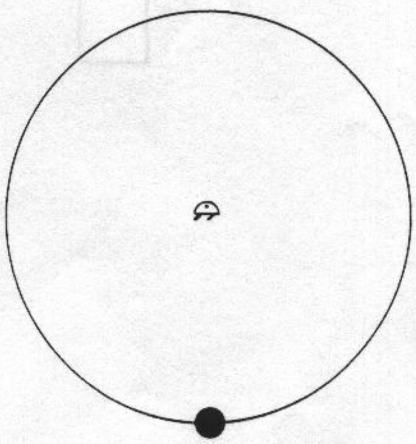
# ME 460/660 Exam 1, Fall 1994

1) The damping ratio, $\zeta$, and natural frequency, $\omega$, of a single degree of freedom (SDOF) system are identified by examination of the free response to be 0.01 and 10 rad/sec. The spring stiffness is found to be 10 N/m by static analysis. If 1 kg is added to the SDOF system, what arethe new natural frequency and damping ratio? What are the final damping coefficient and mass? Use correct units.

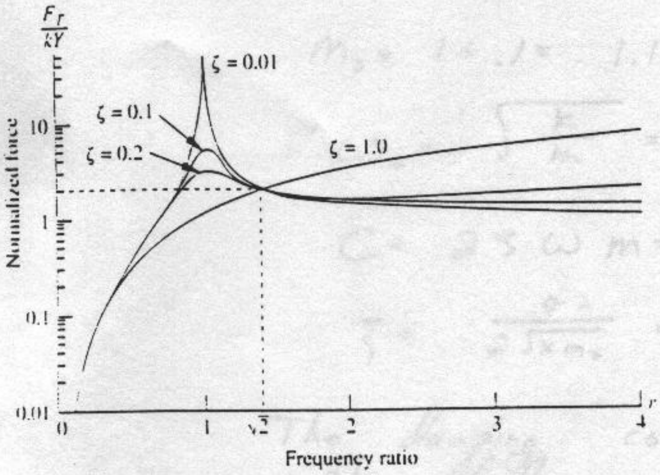
2) A cylinder of mass *m* and mass moment of inertia $1/2mr^2$ is free to roll without slipping but is restrained by a spring, *k*, and damper, *c*, as shown below. Determine the damping ratio and the natural frequency.

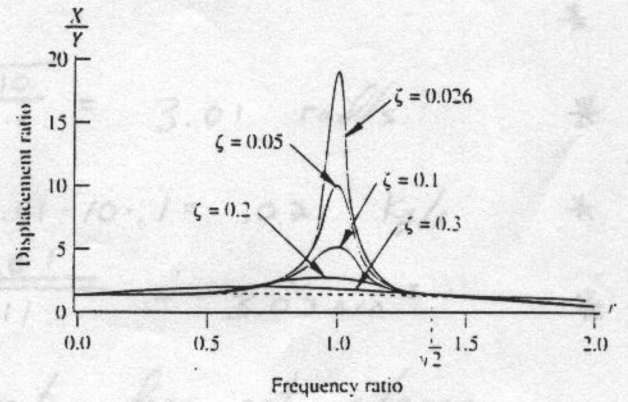



3) Determine the equation of motion for the system below. The disk is solid with mass *m* and the point mass attached to the disk also hass mass *m*. The disk is pinned at the center so that it can rotate freely.

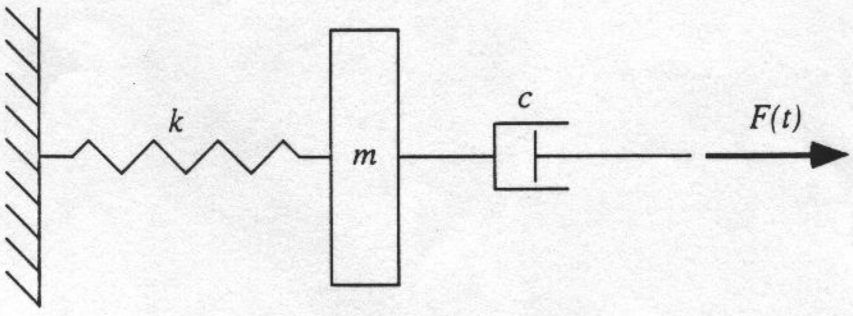


4) A piece of equipment weighing 10 kg is subject to a base excitation of 10 N with a frequency of 10 rad/sec. Using the figures below, choose a spring stiffness and damping coefficient that will minimize displacement and force transmissibility. Justify (explain) your solution.




5) Find the amplitude of the force transmitted to the ground for the following system assuming a harmonic excitation.



Vibrations Exam 1, Fa 1994 Solutions

1) $\zeta = .01$, $\omega = 10$ rad/s, $K = 10$ N/m

$\omega = \sqrt{\frac{k}{m}}$ $\quad m = \frac{K}{\omega^2} = .1$ kg

$m_2 = 1 + .1 = 1.1$ kg * 5

$\omega_2 = \sqrt{\frac{k}{m}} = \sqrt{\frac{10}{1.1}} = 3.01$ rad/s * 5

$C = 2\zeta\omega m = 2 \cdot .01 \cdot 10 \cdot .1 = .02$ kg/s * 5

$\zeta = \frac{.02}{2\sqrt{k m_2}} = \frac{.01}{\sqrt{11}} = 3.02 \times 10^{-3}$ * 5

The damping coefficient does not change with added mass.

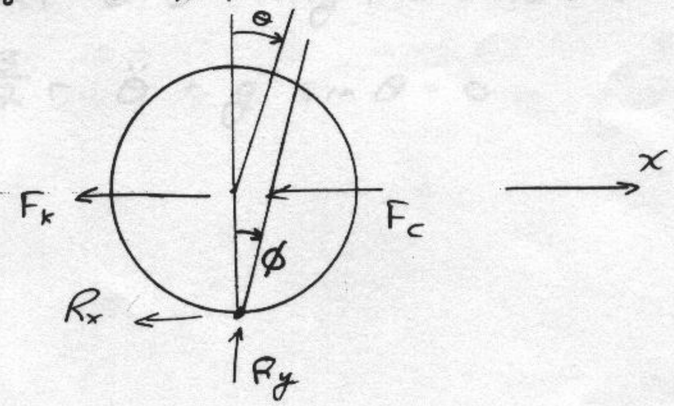
2) For pure rolling without slipping, pure rotation occurs about the point of contact.

From the parallel-axis theorem.

$$J_o = J_G + mr^2$$

where $_o$ is the mass moment of inertia about the base.

$$\therefore \quad J_o = \frac{1}{2} mr^2 + mr^2 = \frac{3}{2} mr^2$$




The forces on the cylinder at its center are $-kx$ and $-c\dot{x}$. Thus applying Neton's Law for rotation about the base

$$\sum M = (-F_c - F_k) r = {}_o\ddot{\phi}$$

$$x = r\phi \quad , \quad \dot{x} = r\dot{\phi}$$

$$J_o\ddot{\phi} + cr^2\dot{\phi} + kr^2\phi = 0$$

$$\ddot{\phi} + \frac{cr^2}{J_o}\dot{\phi} + \frac{kr^2}{J_o}\phi = 0$$

$$\omega = \sqrt{\frac{kr^2}{\frac{3}{2}mr^2}} = \sqrt{\frac{2k}{3m}}$$

$$\zeta = \frac{cr^2}{2J_o\omega} = \frac{c\cancel{r^2}}{\cancel{2}\cdot\frac{3}{\cancel{2}}m\cancel{r^2}}\sqrt{\frac{3m}{2k}} = \frac{c}{3m}\sqrt{\frac{3m}{2k}} = \frac{c}{\sqrt{6km}}$$

3) $J = \frac{1}{2} m r^2 + m r^2 = \frac{3}{2} m r^2$

$$T = \frac{1}{2} J \dot{\theta}^2 = \frac{3}{4} m r^2 \dot{\theta}^2$$

$$U = mgr(1 - \cos\theta)$$

$$T + U = \frac{3}{4} m r^2 \dot{\theta}^2 + mgr(1 - \cos\theta)$$

$$\frac{d}{dt}(T + U) = 0$$

$$\frac{3}{2} m r^2 \dot{\theta} \ddot{\theta} + mgr \dot{\theta} \sin\theta = 0$$

$$\frac{3}{2} r \ddot{\theta} + g \sin\theta = 0 \qquad *$$

4) Multiple answers are acceptable for this problem.

From the graphs, the displacement is amplified for $r < \sqrt{2}$. The force is amplified for $.8 < r < 1$ and varies depending on $\xi$. Clearly, r must not be near 1. As a matter of practicality, r cannot be very high. Also notice that r very high leads to large transmitted forces for high damping. Light damping is also very difficult to design, so we will choose no damping, and make the design robust to variations in $\xi$. Thus $\xi = 0$ and $r = 4$ will give good results.

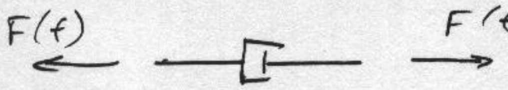
$$r = \frac{\omega_{dr}}{\omega} \qquad \omega = \frac{10}{4} = 2.5 = \sqrt{\frac{k}{m}}$$

$$k = 2.5^2 m = 62.5 \text{ N/m}$$

$$C = 2\xi\omega m = 2 \cdot 0 \cdot 2.5 \cdot 10 = 0 \text{ kg} \cdot \text{s}$$

$k = 62.5$ N/m *
$C = 0$ kg·s *

5) The force applied to the dashpot is directly applied to the mass as well.




Thus, the force transmitted is $kX$ where $X$ is the amplitude of the forced response for an undamped system.

$$F_{trans} = k \frac{f_0}{\omega^2 - \omega_{dr}^2} \quad \text{but} \quad f_0 = \frac{F}{m}, \quad \omega = \sqrt{\frac{k}{m}}$$

$$F_{trans} = \frac{Fk}{k - m\omega_{dr}^2} \quad *$$

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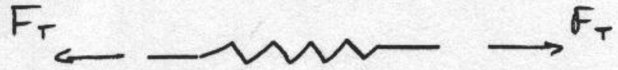
FBD of Spring




$$F_T = kx$$