```
Y = 2.1 × 10 ^ 11
```

2.1×10^{11}

```
k = 10 000 * 0
```

0

```
rho = 7800
```

7800

```
l = 1
```

1

```
h = .02
```

0.02

```
w = .03
```

0.03

```
Area = h * w
```

0.0006

```
Clear[A]
```

```
Izz = 1 / 12 * w * h ^ 3
```

$2. \times 10^{-8}$

```
X = A Sin[beta x] + B Cos[beta x] + C Sinh[beta x] + D Cosh[beta x]
```

B Cos[beta x] + D Cosh[beta x] + A Sin[beta x] + C Sinh[beta x]

```
eqn1 = (X /. x → 0) == 0
```

B + D == 0

```
eqn2 = (D[X, x] /. x → 0) == 0
```

A beta + beta C == 0

```
eqn3 = (D[X, {x, 2}] /. x → 1) == 0
```

$-B\ beta^2\ Cos[beta] + beta^2\ D\ Cosh[beta] - A\ beta^2\ Sin[beta] + beta^2\ C\ Sinh[beta] == 0$

```
eqn4 = ((Y Izz D[X, {x, 3}] - k X) /. x → 1) == 0
```

$4200. \left(-A\ beta^3\ Cos[beta] + beta^3\ C\ Cosh[beta] + B\ beta^3\ Sin[beta] + beta^3\ D\ Sinh[beta]\right) == 0$

```
eq1 = eqn1[[1]]
```

B + D

```
eq2 = eqn2[[1]]
```

A beta + beta C

```
eq3 = eqn3[[1]]
```

$$-B\,\text{beta}^2\,\text{Cos}[\text{beta}] + \text{beta}^2\,D\,\text{Cosh}[\text{beta}] - A\,\text{beta}^2\,\text{Sin}[\text{beta}] + \text{beta}^2\,C\,\text{Sinh}[\text{beta}]$$

```
eq4 = eqn4[[1]]
```

$$4200.\left(-A\,\text{beta}^3\,\text{Cos}[\text{beta}] + \text{beta}^3\,C\,\text{Cosh}[\text{beta}] + B\,\text{beta}^3\,\text{Sin}[\text{beta}] + \text{beta}^3\,D\,\text{Sinh}[\text{beta}]\right)$$

```
Coefficient[eq4, D]
```

$$4200.\,\text{beta}^3\,\text{Sinh}[\text{beta}]$$

```
M = {{Coefficient[eq1, A], Coefficient[eq1, B], Coefficient[eq1, C], Coefficient[eq1, D]},
  {Coefficient[eq2, A], Coefficient[eq2, B], Coefficient[eq2, C], Coefficient[eq2, D]},
  {Coefficient[eq3, A], Coefficient[eq3, B], Coefficient[eq3, C], Coefficient[eq3, D]},
  {Coefficient[eq4, A], Coefficient[eq4, B], Coefficient[eq4, C], Coefficient[eq4, D]}}
```

$$\left\{\{0, 1, 0, 1\}, \{\text{beta}, 0, \text{beta}, 0\},\right.$$
$$\left\{-\text{beta}^2\,\text{Sin}[\text{beta}], -\text{beta}^2\,\text{Cos}[\text{beta}], \text{beta}^2\,\text{Sinh}[\text{beta}], \text{beta}^2\,\text{Cosh}[\text{beta}]\right\},$$
$$\left.\left\{-4200.\,\text{beta}^3\,\text{Cos}[\text{beta}], 4200.\,\text{beta}^3\,\text{Sin}[\text{beta}], 4200.\,\text{beta}^3\,\text{Cosh}[\text{beta}], 4200.\,\text{beta}^3\,\text{Sinh}[\text{beta}]\right\}\right\}$$

```
MatrixForm[M]
```

$$\begin{pmatrix} 0 & 1 & 0 & 1 \\ \text{beta} & 0 & \text{beta} & 0 \\ -\text{beta}^2\,\text{Sin}[\text{beta}] & -\text{beta}^2\,\text{Cos}[\text{beta}] & \text{beta}^2\,\text{Sinh}[\text{beta}] & \text{beta}^2\,\text{Cosh}[\text{beta}] \\ -4200.\,\text{beta}^3\,\text{Cos}[\text{beta}] & 4200.\,\text{beta}^3\,\text{Sin}[\text{beta}] & 4200.\,\text{beta}^3\,\text{Cosh}[\text{beta}] & 4200.\,\text{beta}^3\,\text{Sinh}[\text{beta}] \end{pmatrix}$$

```
Det[M]
```

$$4200.\,\text{beta}^6\,\text{Cos}[\text{beta}]^2 + 8400.\,\text{beta}^6\,\text{Cos}[\text{beta}]\,\text{Cosh}[\text{beta}] + 4200.\,\text{beta}^6\,\text{Cosh}[\text{beta}]^2 + 4200.\,\text{beta}^6\,\text{Sin}[\text{beta}]^2 + 0.\,\text{beta}^6\,\text{Sin}[\text{beta}]\,\text{Sinh}[\text{beta}] - 4200.\,\text{beta}^6\,\text{Sinh}[\text{beta}]^2$$

```
chareqn = Simplify[Det[M]] == 0
```

$$4200.\,\text{beta}^6 + 8400.\,\text{beta}^6\,\text{Cos}[\text{beta}]\,\text{Cosh}[\text{beta}] + 4200.\,\text{beta}^6\,\text{Cosh}[\text{beta}]^2 + 0.\,\text{beta}^6\,\text{Sin}[\text{beta}]\,\text{Sinh}[\text{beta}] - 4200.\,\text{beta}^6\,\text{Sinh}[\text{beta}]^2 == 0$$

```
betasolved = FindRoot[chareqn, {beta, 5}]
```

$$\{\text{beta} \to 4.69409113297\}$$

```
soln = Solve[{eqn1 /. A → 1, eqn2 /. A → 1, eqn3 /. A → 1}, {B, C, D}] /. betasolved
```

$$\{\{B \to -0.981867539175, D \to 0.981867539175, C \to -1\}\}$$

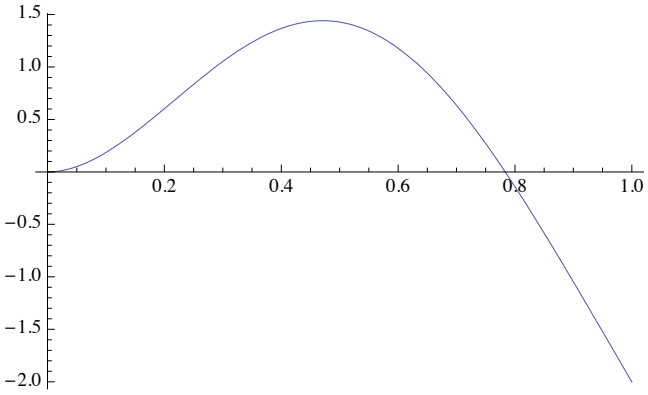
```
ms = X /. soln[[1]] /. {A → 1} /. betasolved
```

$$-0.981867539175\,\text{Cos}[4.69409113297\,x] + 0.981867539175\,\text{Cosh}[4.69409113297\,x] + \text{Sin}[4.69409113297\,x] - \text{Sinh}[4.69409113297\,x]$$

```
ms = ms / Sqrt[Integrate[ms^2, {x, 0, 1}]]
```

$$1.01846731876\,(-0.981867539175\,\text{Cos}[4.69409113297\,x] + 0.981867539175\,\text{Cosh}[4.69409113297\,x] + \text{Sin}[4.69409113297\,x] - \text{Sinh}[4.69409113297\,x])$$

```
Plot[ms, {x, 0, l}]
```




```
wn = beta^2 / l^2 Sqrt[Y Izz / rho / Area] /. betasolved
```

```
660.092430357
```

```
fn = wn / 2 / Pi // N
```

```
105.056973189
```

```
betasolved1 = FindRoot[chareqn, {beta, 2}]
```

```
{beta → 1.87510406871}
```

```
betasolved2 = FindRoot[chareqn, {beta, 5}]
```

```
{beta → 4.69409113297}
```

```
betasolved3 = FindRoot[chareqn, {beta, 8}]
```

FindRoot::lstol :
The line search decreased the step size to within tolerance specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the merit function. You may need more than MachinePrecision digits of working precision to meet these tolerances. ≫

```
{beta → 7.85475743824}
```

```
betasolved4 = FindRoot[chareqn, {beta, 11}]
```

FindRoot::lstol :
The line search decreased the step size to within tolerance specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the merit function. You may need more than MachinePrecision digits of working precision to meet these tolerances. ≫

```
{beta → 10.9955407349}
```

```
betasolved5 = FindRoot[chareqn, {beta, 14}]
```

```
{beta → 14.137168391}
```

```
beta1 = beta /. betasolved1
```

```
1.87510406871
```

```
soln = Solve[{eqn1 /. A → 1, eqn2 /. A → 1, eqn3 /. A → 1}, {B, C, D}] /. betasolved
```

```
{{B → -0.981867539175, D → 0.981867539175, C → -1}}
```

```
ms1 = X /. soln[[1]] /. {A → 1} /. betasolved1
```

```
-0.981867539175 Cos[1.87510406871 x] +
 0.981867539175 Cosh[1.87510406871 x] + Sin[1.87510406871 x] - Sinh[1.87510406871 x]
```

```
beta2 = beta /. betasolved2
```

```
4.69409113297
```

```
soln = Solve[{eqn1 /. A → 1, eqn2 /. A → 1, eqn3 /. A → 1}, {B, C, D}] /. betasolved2
```

```
{{B → -0.981867539175, D → 0.981867539175, C → -1}}
```

```
ms2 = X /. soln[[1]] /. {A → 1} /. betasolved2
```

```
-0.981867539175 Cos[4.69409113297 x] +
 0.981867539175 Cosh[4.69409113297 x] + Sin[4.69409113297 x] - Sinh[4.69409113297 x]
```

```
beta3 = beta /. betasolved3
```

```
7.85475743824
```

```
soln = Solve[{eqn1 /. A → 1, eqn2 /. A → 1, eqn3 /. A → 1}, {B, C, D}] /. betasolved3
```

```
{{B → -1.00077610535, D → 1.00077610535, C → -1}}
```

```
ms3 = X /. soln[[1]] /. {A → 1} /. betasolved3
```

```
-1.00077610535 Cos[7.85475743824 x] +
 1.00077610535 Cosh[7.85475743824 x] + Sin[7.85475743824 x] - Sinh[7.85475743824 x]
```

```
beta4 = beta /. betasolved4
```

```
10.9955407349
```

```
soln = Solve[{eqn1 /. A → 1, eqn2 /. A → 1, eqn3 /. A → 1}, {B, C, D}] /. betasolved4
```

```
{{B → -0.999966447874, D → 0.999966447874, C → -1}}
```

```
ms4 = X /. soln[[1]] /. {A → 1} /. betasolved4
```

```
-0.999966447874 Cos[10.9955407349 x] +
 0.999966447874 Cosh[10.9955407349 x] + Sin[10.9955407349 x] - Sinh[10.9955407349 x]
```

```
beta5 = beta /. betasolved5
```

```
14.137168391
```

```
soln = Solve[{eqn1 /. A → 1, eqn2 /. A → 1, eqn3 /. A → 1}, {B, C, D}] /. betasolved5
```

```
{{B → -1.00000144989, D → 1.00000144989, C → -1}}
```

```
ms5 = X /. soln[[1]] /. {A → 1} /. betasolved5
```

```
-1.00000144989 Cos[14.137168391 x] +
 1.00000144989 Cosh[14.137168391 x] + Sin[14.137168391 x] - Sinh[14.137168391 x]
```

```
K11 = Integrate[Y Izz D[ms1, {x, 2}] D[ms1, {x, 2}], {x, 0, l}]
```

```
47 707.1637307
```

```
K12 = Integrate[Y Izz D[ms1, {x, 2}] D[ms2, {x, 2}], {x, 0, l}]
```

```
117 044.926962 + 0. i
```

```
K13 = Integrate[Y Izz D[ms1, {x, 2}] D[ms3, {x, 2}], {x, 0, l}]
```

$-221\,754.769501 + 7.37904425631 \times 10^{-12}\,i$

```
K14 = Integrate[Y Izz D[ms1, {x, 2}] D[ms4, {x, 2}], {x, 0, l}]
```

$328\,530.034404 - 1.05201702793 \times 10^{-11}\,i$

```
K15 = Integrate[Y Izz D[ms1, {x, 2}] D[ms5, {x, 2}], {x, 0, l}]
```

$-435\,593.734046 + 0.\,i$

```
K22 = Integrate[Y Izz D[ms2, {x, 2}] D[ms2, {x, 2}], {x, 0, l}]
```

$1.96589882352 \times 10^{6}$

```
K23 = Integrate[Y Izz D[ms2, {x, 2}] D[ms3, {x, 2}], {x, 0, l}]
```

$0.0000467797948264 + 0.\,i$

```
K24 = Integrate[Y Izz D[ms2, {x, 2}] D[ms4, {x, 2}], {x, 0, l}]
```

$-0.000469059158041 + 2.36836094925 \times 10^{-20}\,i$

```
K25 = Integrate[Y Izz D[ms2, {x, 2}] D[ms5, {x, 2}], {x, 0, l}]
```

$-3.2041694832 \times 10^{-10} - 1.12932304314 \times 10^{-26}\,i$

```
K33 = Integrate[Y Izz D[ms3, {x, 2}] D[ms3, {x, 2}], {x, 0, l}]
```

$1.60123199083 \times 10^{7}$

```
K34 = Integrate[Y Izz D[ms3, {x, 2}] D[ms4, {x, 2}], {x, 0, l}]
```

$0.114913956724 + 9.05830221756 \times 10^{-18}\,i$

```
K35 = Integrate[Y Izz D[ms3, {x, 2}] D[ms5, {x, 2}], {x, 0, l}]
```

$-0.880059677104 + 0.\,i$

```
K44 = Integrate[Y Izz D[ms4, {x, 2}] D[ms4, {x, 2}], {x, 0, l}]
```

$6.1388428418 \times 10^{7}$

```
K45 = Integrate[Y Izz D[ms4, {x, 2}] D[ms5, {x, 2}], {x, 0, l}]
```

$-110.228244511 + 0.\,i$

```
K55 = Integrate[Y Izz D[ms5, {x, 2}] D[ms5, {x, 2}], {x, 0, l}]
```

1.677648×10^{8}

```
M11 = Integrate[(rho Area + DiracDelta[x - l] rho Area l) ms1 ms1, {x, 0, l}]
```

6.89816581474

```
M12 = Integrate[(rho Area + DiracDelta[x - l] rho Area l) ms1 ms2, {x, 0, l}]
```

-5.89381704889

```
M22 = Integrate[(rho Area + DiracDelta[x - l] rho Area l) ms2 ms2, {x, 0, l}]
```

13.5354566574

```
ams1 = .9 ms1 + .1 ms2
```

```
0.9 (-0.981867539175 Cos[1.87510406871 x] +
    0.981867539175 Cosh[1.87510406871 x] + Sin[1.87510406871 x] - Sinh[1.87510406871 x]) +
 0.1 (-0.981867539175 Cos[4.69409113297 x] + 0.981867539175 Cosh[4.69409113297 x] +
    Sin[4.69409113297 x] - Sinh[4.69409113297 x])
```

```
bb = 0.01
```

```
0.01
```

```
D[ms1 + bb ms2, {x, 2}]
```

```
3.45226125938 Cos[1.87510406871 x] + 3.45226125938 Cosh[1.87510406871 x] -
 3.5160152685 Sin[1.87510406871 x] - 3.5160152685 Sinh[1.87510406871 x] +
 0.01 (21.6349520096 Cos[4.69409113297 x] + 21.6349520096 Cosh[4.69409113297 x] -
    22.0344915647 Sin[4.69409113297 x] - 22.0344915647 Sinh[4.69409113297 x])
```

```
RQn = Integrate[Y Izz D[ms1 + bb ms2, {x, 2}] D[ms1 + bb ms2, {x, 2}], {x, 0, 1}]
```

```
50244.6521523 + 0. i
```

```
Rqd =
 Integrate[(rho Area + DiracDelta[x - l + .001] rho Area l) (ms1 + bb ms2) (ms1 + bb ms2), {x, 0, l}]
```

```
$Aborted
```

```
RQ = Integrate[Y Izz D[ms1, {x, 2}] D[ms1, {x, 2}], {x, 0, l}] /
  Integrate[(rho Area + DiracDelta[x - l] rho Area l) (ms1) (ms1), {x, 0, l}]
```

```
6915.92011731
```

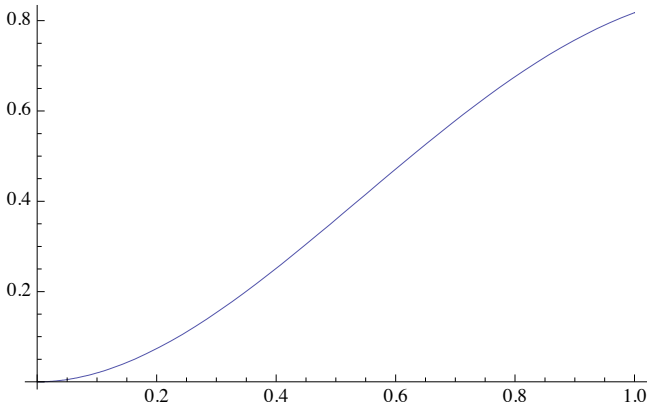
```
K11 / M11
```

```
6915.92011731
```

```
ms1normalized = ms1 / Sqrt[Integrate[rho Area ms1^2, {x, 0, l}]]
```

```
0.609744705522 (-0.981867539175 Cos[1.87510406871 x] +
   0.981867539175 Cosh[1.87510406871 x] + Sin[1.87510406871 x] - Sinh[1.87510406871 x])
```

```
Plot[ms1normalized, {x, 0, l}]
```



```
Plot[ms1, {x, 0, l}]
```