ME 7210, Exam 1 Spring 2013

One 8.5" by 11" cheat sheet.

1. Prove that $\mathbf{x}^T A\mathbf{x}$ is bound by the eigenvalue of A when A is a symmetric real matrix and $|\mathbf{x}| = 1$.

2. A beam with constant properties E, I, ρ, A and l (where ρ is mass per volume) is supported in a pinned-pinned (simply supported) configuration. For an input a $\ell/3$ and an output at $\ell/4$, at what frequencies at the first 3 zeros in the frequency response function?

$$X(x) = A\cos(\beta_n x/\ell) + B\sin(\beta_n x/\ell) + Ce^{-\beta_n x/\ell} + De^{\beta_n x/\ell}$$

$$\omega_n = \frac{\beta_n^2}{\ell^2}\sqrt{\frac{EI}{\rho A}}$$

Boundary Conditions	Mode Number (n)	A	B	C	D	β_n
Free-free	1	1	0	0	0	0
	2	0^1	0	0	0	0
	3	1	-0.983	0.991	-0.009	4.730
	4	1	-1.001	1.000	0.000	7.853
	>4	1	-1.000	1.000	0.000	$\frac{(2n-3)\pi}{2}$
Clamped-free: cantilever	1	-1	0.734	0.867	0.133	1.875
	2	-1	1.019	1.009	-0.009	4.694
	3	-1	0.999	1.000	0.000	7.855
	>3	-1	1.000	1.000	0.000	$\frac{(2n-1)\pi}{2}$
Clamped-pinned	1	-1	1.001	1.000	0.000	3.927
	>1	-1	1.000	1.000	0.000	$\frac{(4n+1)\pi}{4}$
Clamped-sliding	1	-1	0.983	0.991	0.001	2.365
	>1	-1	1.000	1.000	0.000	$\frac{(4n-1)\pi}{4}$
Clamped-clamped	1	-1	0.983	0.991	0.001	4.730
	2	-1	1.001	1.000	0.000	7.853
	>2	-1	1.000	1.000	0.000	$\frac{(2n+1)\pi}{2}$
Pinned-pinned: simply supported	0	0	1	0	0	$n\pi$

3. A massless elastic beam with a rigid disk of mass M attached at midspan is rotating with the constant angular velocity Ω. Let Y, Z be a set of inertial axes and y, z a set of body axes rotating with the angular velocity Ω with respect to Y, Z. Derive the equations of motion of M using Lagrange's equation in terms of displacements y and z. Assume the beam has equivalent spring constants k_y and k_z for bending and that the system is subject to internal damping forces proportional to $\dot{y}$ and $\dot{z}$, where the constant of proportionality is c. Additionally presume external damping forces proportional to $\dot{Y}$ and $\dot{Z}$ with a proportionality of h.