

Computational Methods in Structural Dynamics, Final      Winter 2000  
One 8.5" by 11" cheat sheet.

1. The partially non-dimensionalized equation of motion of a tapered beam is given by:  
$$\frac{\partial^2}{\partial \xi^2} \left( \frac{2}{3} \xi^3 \frac{\partial^2 w}{\partial \xi^2} \right) + 2\xi \frac{\partial^2 w}{\partial t^2} = 0.$$
 Assuming a deflection form of  $W(\xi) = a_1 \sin(\pi\xi) + a_2 \sin(2\pi\xi)$ , estimate the first and second natural frequencies and mode shapes of the beam using both one and two term representations of the mode shape/s *using the collocation method*.
2. An system is defined by the operator  $L = EI \frac{\partial^4}{\partial x^4} + \beta \frac{\partial^3}{\partial x^3}$  with the boundary conditions  $x(0) = 0, \left. \frac{\partial^2 w}{\partial x^2} \right|_{x=0} = 0, x(l) = 0, \left. \frac{\partial^2 w}{\partial x^2} \right|_{x=l} = 0$ .
  - Is the system self adjoint?
  - If so, is the system positive definite?
3. Apply Cholesky decomposition to the following matrix:

$$\begin{bmatrix} 4 & 1 & 0 \\ 1 & 9 & 2 \\ 0 & 2 & 16 \end{bmatrix}$$

4. A beam/rod deforms in bending ( $v$ ) and extension ( $u$ ) and has a mass connected at its end by a rigid bar as shown. Assuming  $A, I, E$ , and  $\rho$  are constant, derive the equations of motion and the associated boundary conditions.