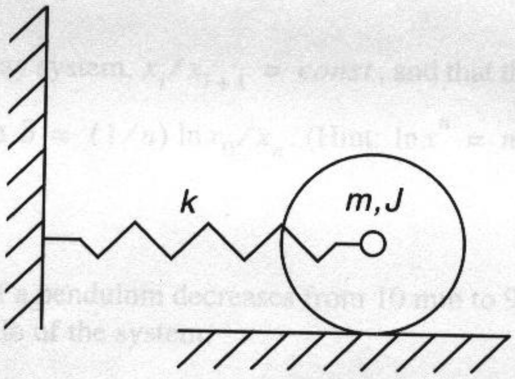
ME 460/660 Exam 1, Spring 1994

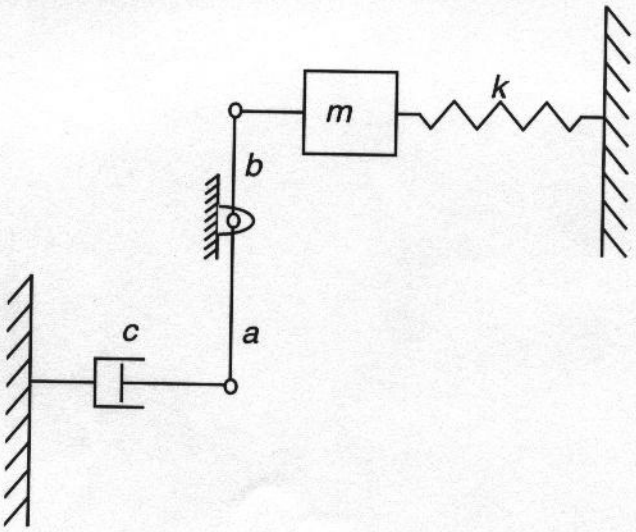
One problem may be skipped.

1) The damping ratio, ζ, and natural frequency, ω, of a single degree of freedom system are identified by examination of the free response to be 0.01 and 10 rad/sec. The spring stiffness is found to be 10 N/m by static analysis. What is the damping coefficient and the mass? Include correct units.

2) A cylinder of mass m and mass moment of inertia $1/2mr^2$ is free to roll without slipping but is restrained by a spring, k, as shown below. Determine the damping ratio and the natural frequency. (Hint: Use the energy method.)



3) Determine the equation of motion for the system below.



One problem may be skipped.

4) Given that the maximum value of A_0 occurs when the maximum of

$(A_0k)/F_0 = 1/\left(\sqrt{(1-r^2)^2 + (2\zeta r)^2}\right)$ occurs, derive the maximum of A_0 with respect to r.

5) a) Shown that for a free decay system, $x_i/x_{i+1} = const$ where x_i is the amplitude after i cycles.

b) Given that for a free decay system, $x_i/x_{i+1} = const$, and that the logarithmic decrement is $\delta = \ln x_i/x_{i+1}$, show that $\delta = (1/n)\ln x_0/x_n$. (Hint: $\ln x^n = n\ln x$)

6) The amplitude of motion of a pendulum decreases from 10 mm to 9 mm after 1000 cycles. Determine the damping ratio of the system.

ME 460/660 Exam 1 Solutions

1) $\omega = \sqrt{\frac{k}{m}}$

$$m = \frac{k}{\omega^2} = \frac{10\ N/m}{100\ rad/s} = \underline{.1\ kg}$$

$$c = 2\sqrt{km}\ \xi = 2 \cdot 1 \cdot .01 = \underline{.02\ kg \cdot s}$$

2) No damping. $\underline{\xi = 0.}$ Energy Method

$$T = \frac{1}{2} m v^2 + \frac{1}{2} J \omega^2$$

$$\omega = \frac{v}{r}$$

$$T = \frac{1}{2} m v^2 + \frac{1}{2} \cdot \frac{1}{2} m \cancel{r^2} \cdot \frac{v^2}{\cancel{r^2}}$$

$$= \frac{3}{4} m v^2 = \frac{3}{4} m \dot{x}^2$$

$$U = \frac{1}{2} k x^2$$

$$T + U = \text{const.}$$

$$\frac{d}{dt}(T + U) = 0 = \left(kX + \frac{3}{2} m \ddot{X}\right)\dot{x}$$

$$\omega = \sqrt{\frac{k}{\frac{3}{2}m}} = \underline{\sqrt{\frac{2k}{3m}}}$$

3) <u>For small displacements</u>

Define x as the displacement of the mass.
The resisting force on m due to the dashpot is $\left(\frac{a}{b}\right)^2 c\,\dot{x}$.

$$\therefore \quad m\ddot{x} + \left(\frac{a}{b}\right)^2 c\dot{x} + kx = 0.$$ See last page.

4)
$$\frac{A_0 k}{F_0} = \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

$$\frac{d}{dr}\left((1-r^2)^2 + (2\zeta r)^2\right)^{-1/2} = 0$$

$$-\frac{1}{2}\,\frac{d}{dr}\left((1-r^2)^2 + (2\zeta r)^2\right)\left((1-r^2)^2 + (2\zeta r)^2\right)^{-3/2} = 0$$

$$2(-2r)(1-r^2) + 2\cdot 2\zeta(2\zeta r) = 0$$

$$-\cancel{r}(1-r^2) + 2\zeta^2 \cancel{r} = 0$$

$$r^2 = 1-2\zeta^2$$

$$\underline{r = \sqrt{1-2\zeta^2}}$$

$$\frac{Ak}{F_0} = \frac{1}{\sqrt{(1-1+2\zeta^2)^2 + 4\zeta^2(1-2\zeta^2)}}$$

$$= \frac{1}{\sqrt{4\zeta^4 + 4\zeta^2 - 8\zeta^4}}$$

$$= \underline{\frac{1}{2\zeta\sqrt{1-\zeta^2}}}$$

5) $x_i = X_0 e^{-\zeta\omega(t+iT)} \sin(\omega_d(t+iT)+\phi)$

$x_{i+1} = X_0 e^{-\zeta\omega(t+iT+T)} \sin(\omega_d(t+iT)+\phi+\omega_d T)$

Since $\omega_d i T$ where i is an integer $= i \cdot 2\pi$, both sin functions are equal to $\sin(\omega_d t + \phi)$

$$\frac{x_i}{x_{i+1}} = \frac{\cancel{X_0} e^{-\zeta\omega(t+iT)}}{\cancel{X_0} e^{-\zeta\omega(t+iT+T)}} \frac{\cancel{\sin(\omega_d t+\phi)}}{\cancel{\sin(\omega_d t+\phi)}}$$

$$\frac{x_i}{x_{i+1}} = e^{-\zeta\omega T}$$

a) Since $e, \zeta, \omega, + T$ are constants, $\frac{x_i}{x_{i+1}}$ is a constant.

$$\ln\frac{x_0}{x_n} = \ln\left(\frac{x_0}{x_1}\frac{x_1}{x_2}\frac{x_2}{x_3}\cdots\frac{x_{n-2}}{x_{n-1}}\frac{x_{n-1}}{x_n}\right)$$

$$= \ln\left(\frac{x_i}{x_{i+1}}\right)^n$$

$$= n \ln\frac{x_i}{x_{i+1}}$$

Since $\ln\frac{x_i}{x_{i+1}} = \delta$

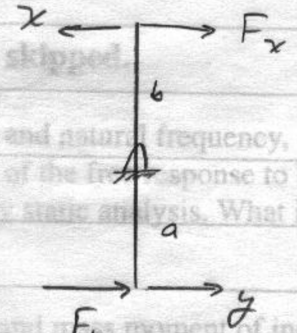
$$\ln\frac{x_0}{x_n} = n\delta$$

b) $$\underline{\delta = \frac{1}{n}\ln\frac{x_0}{x_n}}$$

6) $\delta = \frac{1}{n} \ln \frac{10}{9}$

$$\delta = \frac{1}{1000} \ln \frac{10}{9} = 1.054 \times 10^{-4}$$

$$\xi = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} = 1.677 \times 10^{-5}$$



Taking moments about the pivot point

$$F_y \, a = F_x \, b$$

Since F_y is caused by a dashpot

$$F_y = -c\dot{y}$$

Thus $F_x = \frac{-a}{b} c\dot{y}$

$$\dot{y} = \frac{a}{b} \cdot \dot{x}$$

$$F_x = -\left(\frac{a}{b}\right)^2 c\dot{x}$$

is the resisting force due to the dashpot.