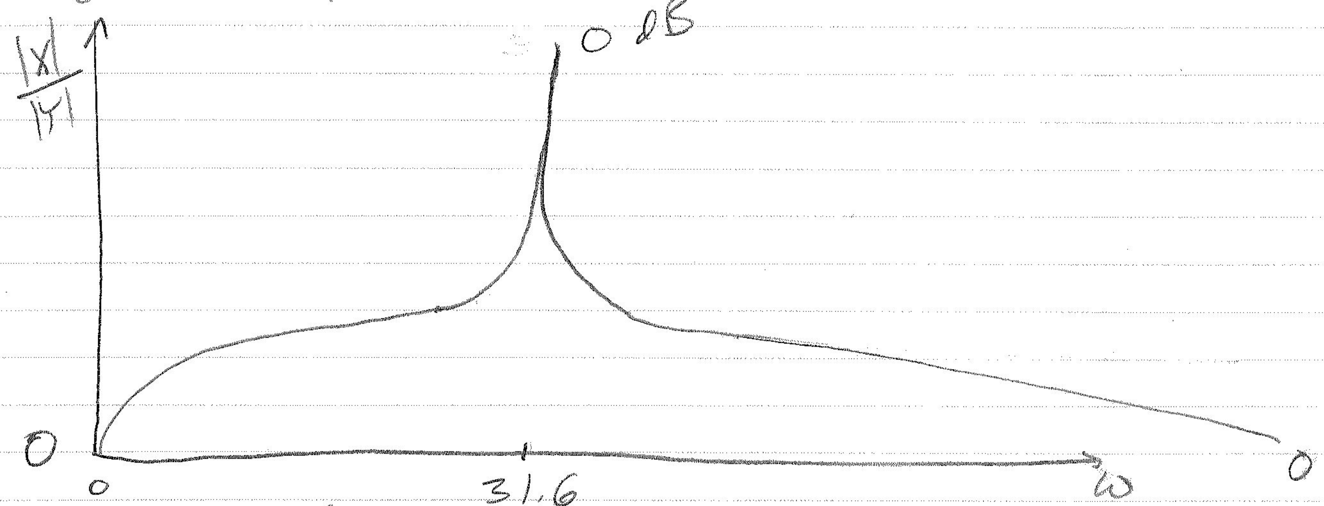
ME 460/660 Final Exam Soln Fa 2006

5A) See older exam for fill-ins

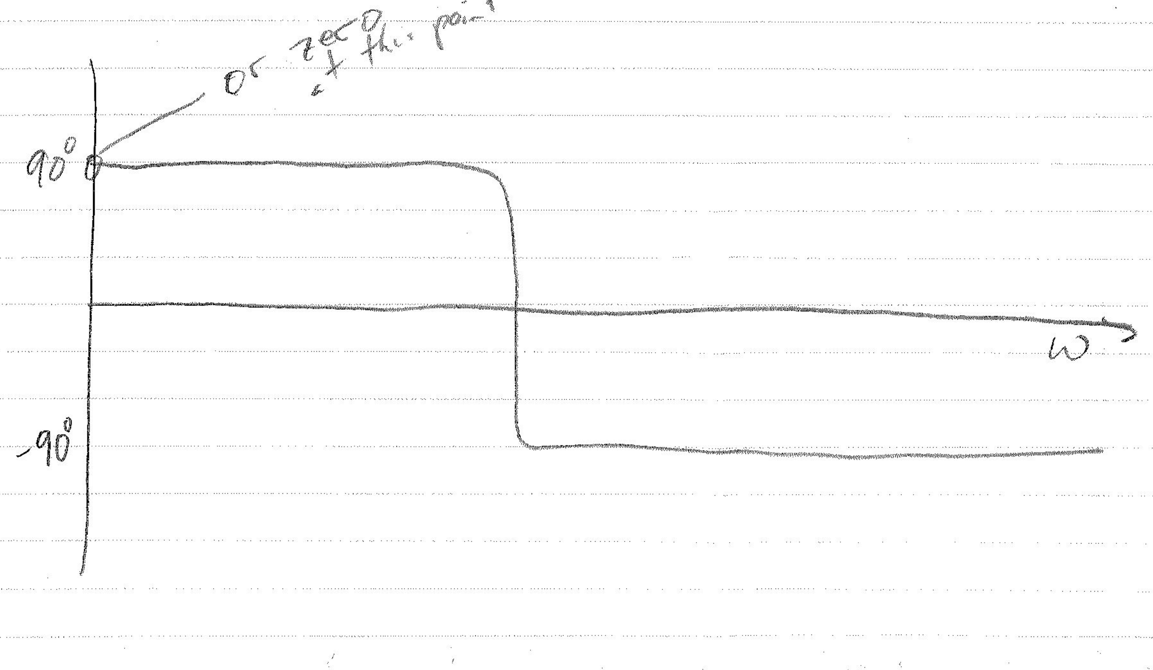
1) $|X| = |Y| \left| \dfrac{1\,j\,\omega_b}{10{,}000 - 10\omega_b^2 + 1\,\omega_b j} \right|$

Y	ω_b	$\|X\|$	$\angle X$
10	0	0	0, $\frac{\pi}{2}$ or undefined (90°)
2	31.42	0.468	76°
1	62.83	0.0021	−89.9

(You have a better chance at part a if you do part b first)



Note 0 is not defined in dB.



2) $F(t) = \frac{-1}{2} + \sum_{n=-\infty}^{\infty} a_n e^{j\omega_T n t}$

$$a_n = \frac{1}{T}\left[\int_0^{T/2} e^{-j\frac{2\pi n t}{T}}\,dt - \int_{T/2}^{T} 2e^{-j\frac{2\pi n t}{T}}\,dt\right]$$

The 1st integral is

$$\frac{-T}{2\pi n j} e^{-j\frac{2\pi n t}{T}}\Big|_0^{T/2}$$

$$= \frac{-T}{2\pi n j}\left(e^{-j\pi n} - \cos 0\right)$$

$$= \frac{-T}{2\pi n j}\left(\cos n\pi - j\sin n\pi - \cos 0\right)$$

$$= \frac{-T}{2\pi n j}\left(\cos n\pi - 1\right)$$

The 2nd integral is

$$\frac{-2T}{2\pi n j}\left(-e^{-j\frac{2\pi n t}{T}}\Big|_{T/2}^{T}\right)$$

$$= \frac{-2T}{2\pi n j}(-1)\left(\underbrace{\cos 2\pi n}_{1} - \overset{0}{\sin 2\pi n} - \left(\cos\pi n - \overset{0}{\sin \pi n}\right)\right)$$

Substituting

$$a_n = \frac{-j}{2\pi n}\left(\cos n\pi - 1 - 2 + 2\cos \pi n\right)$$

$$= \frac{-j3}{2\pi n}\left(1 - \cos n\pi\right)$$

In real form

$$F(t) = \frac{-1}{2} + \sum_{n=1}^{\infty} \frac{3(1-(-1)^n)}{n\pi} \sin\frac{2\pi n t}{T}$$

3) $\det(K - M\omega^2) = 0$

$\omega = 0, 5.4 \text{ rad/s}$

$\underline{u}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \underline{u}_2 = \begin{bmatrix} 1 \\ -\frac{2}{3} \end{bmatrix}$

4) $\underline{f} = S^T \underline{F}$

$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \frac{1}{\sqrt{2}} \delta(t) \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$= \frac{1}{2} \delta(t) \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} \delta(t) \\ 0 \end{bmatrix}$

$\ddot{r}_1 + 9r = \delta(t)$

$r_1(t) = \frac{1}{3} \sin 3t$

$r_2(t) = 0$

$\underline{x}(t) = S\underline{r} = \frac{1}{3\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \sin 3t \\ 0 \end{bmatrix}$

$= \frac{1}{3\sqrt{2}} \sin 3t \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

5) $U = \frac{1}{2} kx^2 + mgl(1 - \cos\theta)$

$T = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} m_2 \left((\dot{x} + \dot{\theta} l \cos\theta)^2 + (\dot{\theta} l \sin\theta)^2 \right)$

$(\dot{x}^2 + 2\dot{x}\dot{\theta} l \cos\theta + \dot{\theta}^2 l^2)$

$\frac{d}{dt} \frac{\partial T}{\partial \dot{x}} = m_1 \ddot{x} + m_2 \ddot{x} + m_2 \ddot{\theta} l \cos\theta + m_2 \dot{\theta}^2 l \sin\theta$

$\frac{\partial U}{\partial x} = kx$ Eqn 1: $(m_1 + m_2)\ddot{x} + m_2(\ddot{\theta} l \cos\theta + \dot{\theta}^2 l \sin\theta) + kx = 0$

$\frac{d}{dt} \frac{\partial T}{\partial \dot{\theta}} = m_2 \ddot{x} l \cos\theta - m_2 \dot{x} l \dot{\theta} \sin\theta + m_2 \ddot{\theta} l^2$

$\frac{\partial U}{\partial \theta} = mgl \sin\theta$ Eqn 2: $m_2 \ddot{x} l \cos\theta - m_2 \dot{x} l \dot{\theta} \sin\theta + m_2 \ddot{\theta} l^2 + mgl \sin\theta = 0$

6) $X_n(x) = A_n \sin \frac{n\pi x}{l}$ (to satisfy BC)

$\omega_n = \frac{n\pi}{l}\sqrt{\frac{\tau}{\rho}}$ (see text)

$$w(x,t) = \sum_{n=1}^{\infty} T_n(t) X_n(x)$$

Subst into EOM

$$\sum_{n=1}^{\infty}\left(\ddot{T}_n + \frac{\tau}{\rho}\left(\frac{n\pi}{l}\right)^2\right) \sin\frac{n\pi x}{l} = 100\,\delta(t)\sin\frac{3\pi x}{l}$$

Mult both sides by $\sin\frac{m\pi x}{l}$, integrate over l

$$\frac{l}{2}\left(\ddot{T}_m + \frac{\tau}{\rho}\left(\frac{m\pi}{l}\right)^2 T_m\right) = 0 \quad \text{for } m \neq 3, \; T_m = 0 \text{ for } m \neq 3$$

$$\frac{l}{2}\left(\ddot{T}_3 + \frac{\tau}{\rho}\left(\frac{3\pi}{l}\right)^2 T_3\right) = 100\,\delta(t)\,\frac{l}{2}$$

$$\ddot{T}_3 + \frac{\tau}{\rho}\left(\frac{3\pi}{l}\right)^2 T_3 = 100\,\delta(t)$$

Using convolution integral formula on formula sheet to get $h(t)$

$$T_3 = \underbrace{\frac{100\,l}{3\pi}\sqrt{\frac{\rho}{\tau}}}_{\frac{\hat{F}}{m\omega_n}} \sin\sqrt{\frac{\tau}{\rho}}\,\frac{3\pi}{l}\,t$$

$$w(x,t) = \frac{100\,l}{3\pi}\sqrt{\frac{\rho}{\tau}}\,\sin\frac{3\pi}{l}\sqrt{\frac{\tau}{\rho}}\,t\,\sin\frac{3\pi x}{l}$$