You can set the value of $qmax$ by editing the Relay block or by setting its value in the MATLAB Command window before running the simulation, for example, by typing `qmax = 4e+5` before the first simulation.

The simulation results show that when $q_{\max} = 4 \times 10^5$, the system is unable to keep the temperature from falling below 69°. When $q_{\max} = 8 \times 10^5$, the temperature stays within the desired band. The plot of $\int q\,dt$ versus t for this case shows that the energy used at the end of 24 hr is 9.6158×10^6 ft-lb. This value can be obtained by exporting the output of the second integrator to the workspace.

7.11 CHAPTER REVIEW

Part I of this chapter treated fluid systems, which can be divided into hydraulics and pneumatics. Hydraulics is the study of systems in which the fluid is incompressible; that is, its density stays approximately constant over a range of pressures. Pneumatics is the study of systems in which the fluid is compressible. Hydraulics and pneumatics share a common modeling principle: conservation of mass. It forms the basis of all our models of such systems.

Modeling pneumatic systems also requires application of thermodynamics, because the temperature of a gas can change when its pressure changes. Thus pneumatics provides a bridge to the treatment of thermal systems, which is the subject of Part II of the chapter. Thermal systems are systems that operate due to temperature differences. They thus involve the flow and storage of heat energy, and conservation of heat energy forms the basis of our thermal models.

Now that you have finished this chapter, you should be able to

1. Apply conservation of mass to model simple hydraulic and pneumatic systems.
2. Derive expressions for the capacitance of simple hydraulic and pneumatic systems.
3. Determine the appropriate resistance relation to use for laminar, turbulent, and orifice flow.
4. Develop a dynamic model of hydraulic and pneumatic systems containing one or more capacitances.
5. Determine the appropriate thermal resistance relation to use for conduction, convection, and radiation heat transfer.
6. Develop a model of a thermal process having one or more thermal storage compartments.
7. Apply MATLAB and Simulink to solve fluid and thermal system models.

REFERENCE

[Çengel, 2001] Y. A. Çengel and R. H. Turner, *Fundamentals of Thermal-Fluid Sciences*, McGraw-Hill, NY, 2001.

PROBLEMS

Section 7.1 Conservation of Mass

7.1 For the hydraulic system shown in Figure P7.1, given $A_1 = 10$ in.^{2}, $A_2 = 30$ in.^{2}, and $mg = 60$ lb, find the force f_1 required to lift the mass m a distance $x_2 = 6$ in. Also find the distance x_1 and the work done by the force f_1.

Figure P7.1

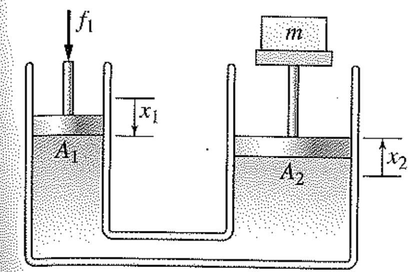
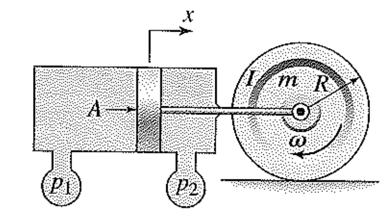


Figure P7.4



7.2 Refer to the water storage and supply system shown in Figure 7.1.2. The cylindrical tank has a radius of 11 ft, and the water height is initially 5 ft. Find the water height after 5 hr if 1000 gallons per minute are pumped out of the well and 800 gallons per minute are withdrawn from the tank. Note that 1 gallon is 0.13368 ft^3.

7.3 Consider the piston and mass shown in Figure 7.1.4a. Suppose there is dry friction acting between the mass m and the surface. Find the minimum area A of the piston required to move the mass against the friction force μmg, where $\mu = 0.6$, $mg = 1000$ N, $p_1 = 3 \times 10^5$ Pa, and $p_2 = 10^5$ Pa.

7.4 In Figure P7.4 the piston of area A is connected to the axle of the cylinder of radius R, mass m, and inertia I about its center. Given $p_1 - p_2 = 3 \times 10^5$ Pa, $A = 0.005$ m^2, $R = 0.4$ m, $m = 100$ kg, and $I = 7$ kg $\cdot$ m^2, determine the angular velocity $\omega(t)$ of the cylinder assuming that it starts from rest.

7.5 Refer to Figure 7.1.4a, and suppose that $p_1 - p_2 = 10$ lb/in.^{2}, $A = 3$ in.^{2}, and $mg = 600$ lb. If the mass starts from rest at $x(0) = 0$, how far will it move in 0.5 sec, and how much hydraulic fluid will be displaced?

7.6 Pure water flows into a mixing tank of volume $V = 300$ m^3 at the constant volume rate of 10 m^3/s. A solution with a salt concentration of s_i kg/m^3 flows into the tank at a constant volume rate of 2 m^3/s. Assume that the solution in the tank is well mixed so that the salt concentration in the tank is uniform. Assume also that the salt dissolves completely so that the volume of the mixture remains the same. The salt concentration s_o kg/m^3 in the outflow is the same as the concentration in the tank. The input is the concentration $s_i(t)$, whose value may change during the process, thus changing the value of s_o. Obtain a dynamic model of the concentration s_o.

7.7 Consider the mixing tank treated in Problem 7.6. Generalize the model to the case where the tank's volume is V m^3. For quality control purposes, we want to adjust the output concentration s_o by adjusting the input concentration s_i. How much volume should the tank have so that the change in s_o lags behind the change in s_i by no more than 20 s?

Section 7.2 Fluid Capacitance

7.8 Derive the expression for the fluid capacitance of the cylindrical tank shown in Figure P7.8.

7.9 Derive the expression for the capacitance of the container shown in Figure P7.9.

Figure P7.8

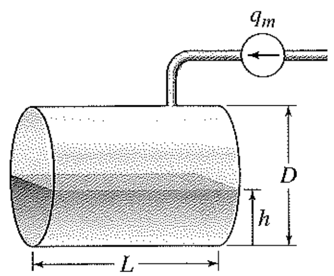
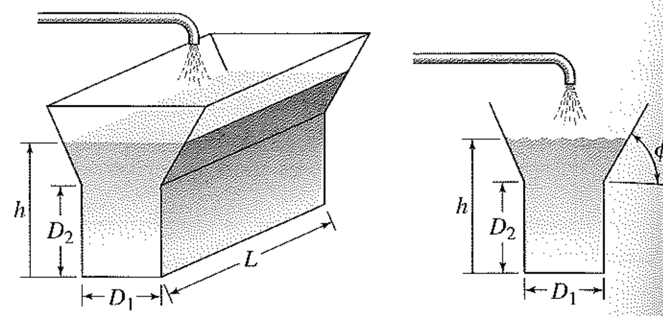


Figure P7.9



7.10 Consider the cylindrical tank shown in Figure P7.8. Derive the dynamic model of the height h, assuming that the input mass flow rate is $q_m(t)$.

7.11 Consider the tank shown in Figure P7.9. Derive the dynamic model of the height h, assuming that the input mass flow rate is $q_m(t)$.

Section 7.3 Fluid Resistance

7.12 Air flows in a certain cylindrical pipe 1 m long with an inside diameter of 1 mm. The pressure difference between the ends of the pipe is 0.1 atm. Compute the laminar resistance, the Reynolds number, the entrance length, and the mass flow rate. Comment on the accuracy of the resistance calculation. For air use $\mu = 1.58 \times 10^{-5}$ N · s/m^2 and $\rho = 1.2885$ kg/m^3.

7.13 Derive the expression for the linearized resistance due to orifice flow near a reference height h_r.

Section 7.4 Dynamic Models of Hydraulic Systems

7.14 Consider the cylindrical container treated in Example 7.4.3. Suppose the outlet flow is turbulent. Derive the dynamic model of the system (a) in terms of the gage pressure p at the bottom of the tank and (b) in terms of the height h.

7.15 A certain tank has a bottom area $A = 20$ m^2. The liquid level in the tank is initially 5 m. When the outlet is opened, it takes 200 s to empty by 98%.

a. Estimate the value of the linear resistance R.

b. Find the steady-state height if the inflow is $q = 3$ m^3/s.

7.16 A certain tank has a circular bottom area $A = 20$ ft^2. It is drained by a pipe whose linear resistance is $R = 150$ m^{-1}sec^{-1}. The tank contains water whose mass density is 1.94 slug/ft^3.

a. Estimate how long it will take for the tank to empty if the water height is initially 30 ft.

b. Suppose we dump water into the tank at a rate of 0.1 ft^3/sec. If the tank is initially empty and the outlet pipe remains open, find the steady-state height and the time to reach one-third that height, and estimate how long it will take to reach the steady-state height.

7.17 The water inflow rate to a certain tank was kept constant until the water height above the orifice outlet reached a constant level. The inflow rate was then measured, and the process repeated for a larger inflow rate. The data are given in the table. Find the effective area $C_d A_o$ for the tank's outlet orifice.

Inflow rate (liters/min)	Liquid height (cm)
98	30
93	27
91	24
86	21
81	18
75	15
68	12
63	9
56	6
49	3

7.18 In the system shown in Figure P7.18, a component such as a valve has been inserted between the two lengths of pipe. Assume that turbulent flow exists throughout the system. Use the resistance relation 7.3.7. (a) Find the total turbulent resistance. (b) Develop a model for the behavior of the liquid height h, with the mass flow rate q_{mi} as the input.

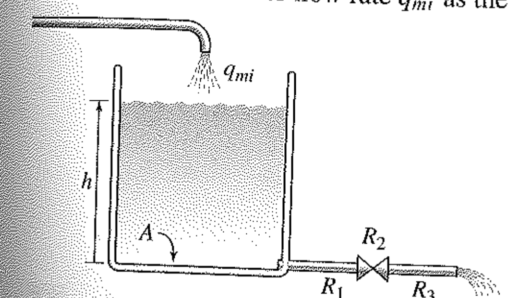


Figure P7.18

7.19 The cylindrical tank shown in Figure 7.4.3 has a circular bottom area A. The mass inflow rate from the flow source is $\hat{q}_{mi}(t)$, a given function of time. The flow through the outlet is *turbulent,* and the outlet discharges to atmospheric pressure p_a. Develop a model of the liquid height h.

7.20 In the liquid level system shown in Figure P7.20, the resistances R_1 and R_2 are linear, and the input is the pressure source p_s. Obtain the differential equation model for the height h, assuming that $\hat{h} > D$.

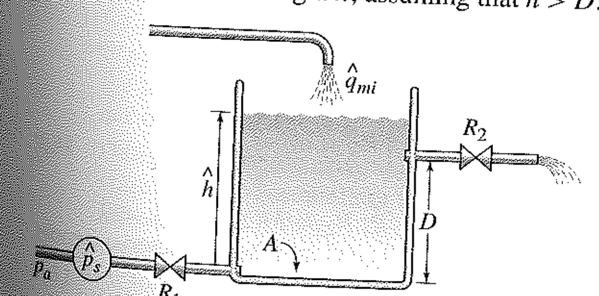


Figure P7.20

7.21 The water height in a certain tank was measured at several times with no inflow applied. See Figure 7.4.3. The resistance R is a linearized resistance. The data are given in the table. The tank's bottom area is $A = 6$ ft^2.

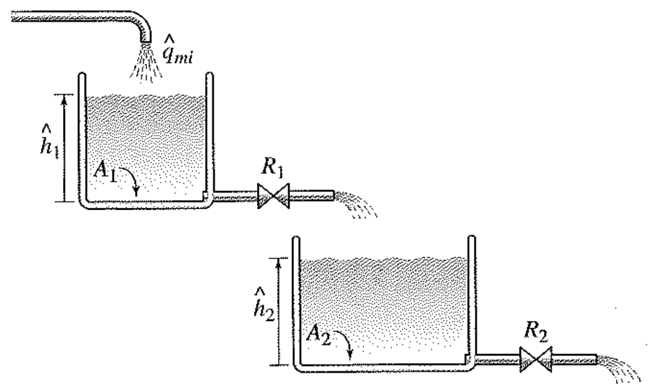
a. Estimate the resistance R.

b. Suppose the initial height is known to be exactly 20.2 ft. How does this change the results of part (a)?

Time (sec)	Height (ft)
0	20.2
300	17.26
600	14.6
900	12.4
1200	10.4
1500	9.0
1800	7.6
2100	6.4
2400	5.4

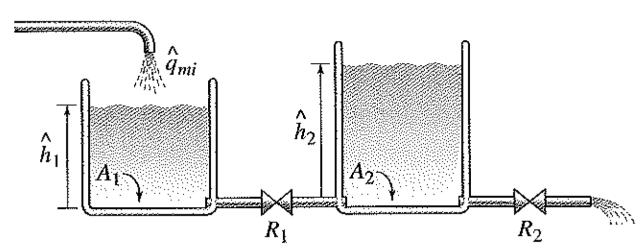
7.22 Derive the model for the system shown in Figure P7.22. The flow rate $\hat{q}_{mi}$ is a mass flow rate and the resistances are linearized.

Figure P7.22



7.23 (a) Develop a model of the two liquid heights in the system shown in Figure P7.23. The inflow rate $q_{mi}(t)$ is a mass flow rate. (b) Using the values $R_1 = R$, $R_2 = 3R$, $A_1 = A$, and $A_2 = 4A$, find the transfer function $H_2(s)/Q_{mi}(s)$.

Figure P7.23



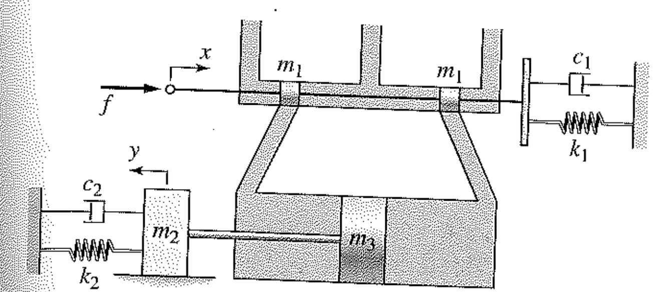
7.24 Consider Example 7.4.6. Suppose that $R_1 = R$, $R_2 = 3R$, $A_1 = A$, and $A_2 = 2A$. Find the transfer function $H_1(s)/Q_{mi}(s)$ and the characteristic roots.

7.25 Design a piston-type damper using an oil with a viscosity at 20°C of $\mu =$ 0.9 kg/(m · s). The desired damping coefficient is 2000 N · s/m. See Figure 7.4.4.

7.26 For the damper shown in Figure 7.4.7, assume that the flow through the hole is turbulent, and neglect the term $m\ddot{y}$. Develop a model of the relation between the force f and $\dot{x}$, the relative velocity between the piston and the cylinder.

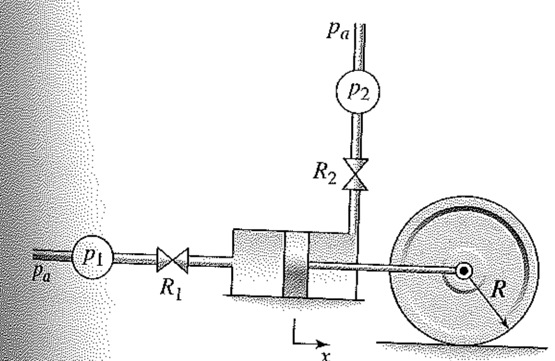
7.27 An electric motor is sometimes used to move the spool valve of a hydraulic motor. In Figure P7.27 the force f is due to an electric motor acting through a rack-and-pinion gear. Develop a model of the system with the load displacement y as the output and the force f as the input. Consider two cases: (a) $m_1 = 0$ and (b) $m_1 \neq 0$.

Figure P7.27



7.28 In Figure P7.28 the piston of area A is connected to the axle of the cylinder of radius R, mass m, and inertia I about its center. Develop a dynamic model of the axle's translation x, with the pressures p_1 and p_2 as the inputs.

Figure P7.28



7.29 Figure P7.29 shows a pendulum driven by a hydraulic piston. Assuming small angles θ and a concentrated mass m a distance L_1 from the pivot, derive the equation of motion with the pressures p_1 and p_2 as inputs.

7.30 Figure P7.30 shows an example of a *hydraulic accumulator*, which is a device for reducing pressure fluctuations in a hydraulic line or pipe. The fluid density is ρ, the plate mass is m, and the plate area is A. Develop a dynamic model of the pressure p with the pressures p_1 and p_2 as the given inputs. Assume that $m\ddot{x}$ of the plate is small, and that the hydrostatic pressure $\rho g h$ is small.

7.31 Design a hydraulic accumulator of the type shown in Figure P7.30. The liquid volume in the accumulator should increase by 30 in.^{3} when the pressure p increases by 1.5 lb/in.^{2}. Determine suitable values for the plate area A and the spring constant k.

7.32 Consider the liquid-level system treated in Example 7.4.10 and shown in Figure 7.4.12. The pump curve and the line for the steady-state flow through both valves are shown in Figure P7.32. It is known that the bottom area of the tank is 2 m^2 and the outlet resistance is $R_2 = 400$ 1/(m · s). (a) Compute the pump resistance R_1 and the steady-state height. (b) Derive a linearized dynamic model of the height deviation δh in the tank.

Figure P7.29

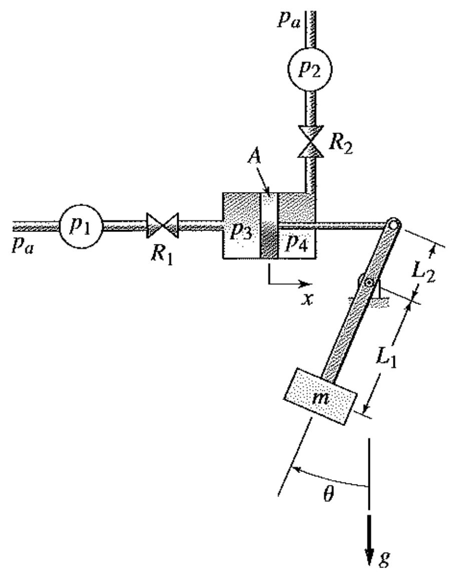


Figure P7.30

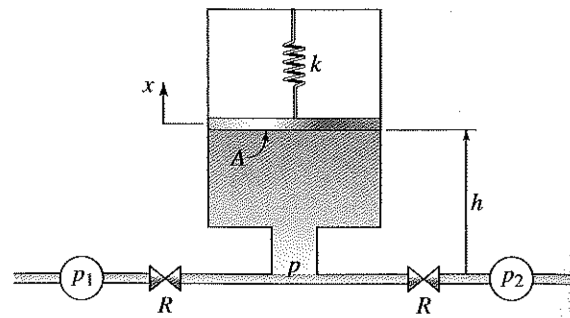
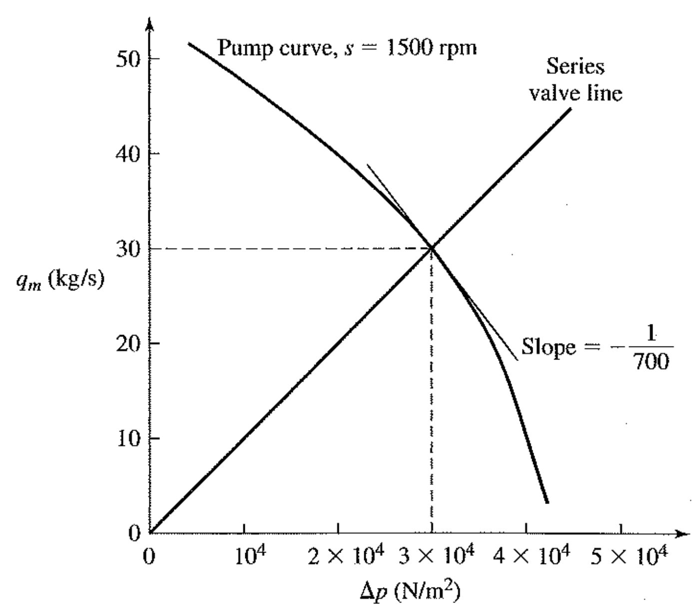


Figure P7.32



7.33 Consider the V-shaped container treated in Example 7.2.2, whose cross section is shown in Figure P7.33. The outlet resistance is linear. Derive the dynamic model of the height h.

7.34 Consider the V-shaped container treated in Example 7.2.2, whose cross section is shown in Figure P7.34. The outlet is an orifice of area A_o and discharge coefficient C_d. Derive the dynamic model of the height h.

7.35 Consider the cylindrical container treated in Problem 7.8. In Figure P7.35 the tank is shown with a valve outlet at the bottom of the tank. Assume that the flow through the valve is turbulent with a resistance R. Derive the dynamic model of the height h.

Figure P7.33

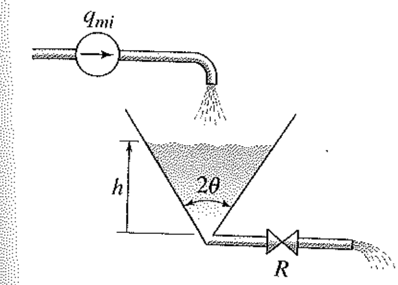


Figure P7.34

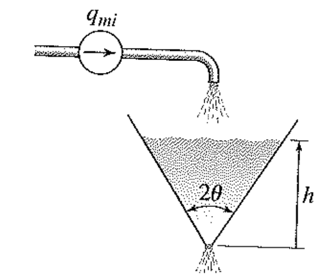
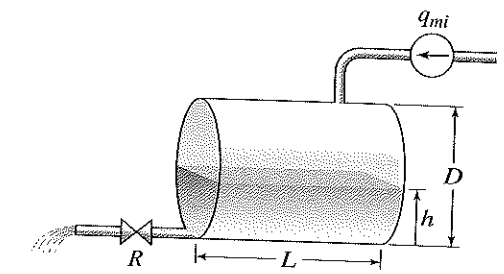


Figure P7.35



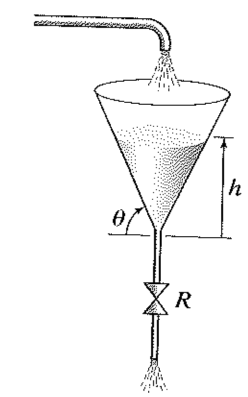
7.36 A certain tank contains water whose mass density is 1.94 slug/ft^3. The tank's circular bottom area is $A = 100$ ft^2. It is drained by an orifice in the bottom. The effective cross-sectional area of the orifice is $C_d A_o = 0.5$ ft^2. A pipe dumps water into the tank at the *volume* flow rate q_v.

a. Derive the model for the tank's height h with the input q_v.
b. Compute the steady-state height if the input flow rate is $q_v = 5$ ft^3/sec.
c. Estimate the tank's time constant when the height is near the steady-state height.

7.37 (a) Derive the expression for the fluid capacitance of the conical tank shown in Figure P7.37. The cone angle θ is a constant and should appear in your answer as a parameter. (b) Derive the dynamic model of the liquid height h. The mass inflow rate is $q_{mi}(t)$. The resistance R is linear.

7.38 (a) Determine the capacitance of a spherical tank of radius R, shown in Figure P7.38. (b) Obtain a model of the pressure at the bottom of the tank, given the mass flow rate q_{mi}.

Figure P7.37



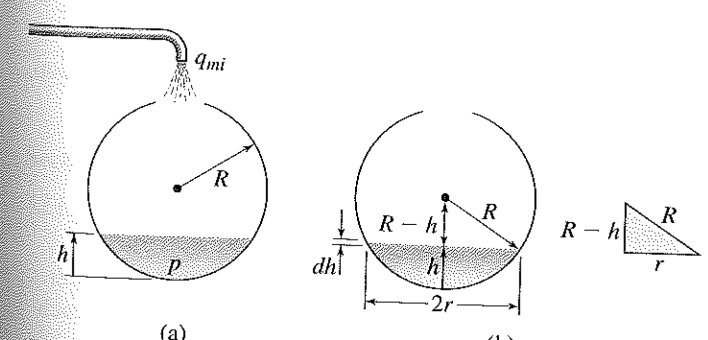
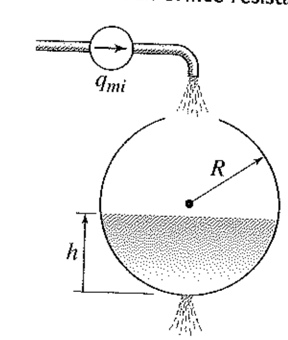


Figure P7.38 A spherical tank.

7.39 Obtain the dynamic model of the liquid height h in a spherical tank of radius R, shown in Figure P7.39. The mass inflow rate through the top opening is q_{mi} and the orifice resistance is R_o.

Figure P7.39 A spherical tank with an orifice resistance.



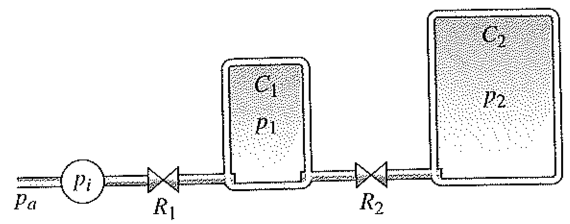
Section 7.5 Pneumatic Systems

7.40 A rigid container has a volume of 20 ft^3. The air inside is initially at 70°F. Find the pneumatic capacitance of the container for an isothermal process.

7.41 Consider the pneumatic system treated in Example 7.5.2. Derive the linearized model for the case where $p_i < p$.

7.42 Figure P7.42 shows two rigid tanks whose pneumatic capacitances are C_1 and C_2. The variables δp_i, δp_1, and δp_2 are small deviations around a reference steady-state pressure p_{ss}. The pneumatic lines have linearized resistances R_1 and R_2. Assume an isothermal process. Derive a model of the pressures δp_1 and δp_2 with δp_i as the input.

Figure P7.42



Section 7.6 Thermal Capacitance

7.43 (a) Compute the thermal capacitance of 250 ml of water, for which $\rho = 1000$ kg/m^3 and $c_p = 4.18 \times 10^3$ J/kg · °C. Note that 1 ml $= 10^{-6}$ m^3. (b) How much energy does it take to raise the water temperature from room temperature (20°C) to 99°C (just below boiling).

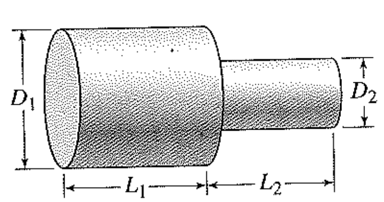
7.44 A certain room measures 15 ft by 10 ft by 8 ft. (a) Compute the thermal capacitance of the room air using $c_p = 6.012 \times 10^3$ ft-lb/slug-°F and $\rho = 0.0023$ slug/ft^3. (b) How much energy is required to raise the air temperature from 68°F to 72°F, neglecting heat transfer to the walls, floor, and ceiling?

7.45 Liquid initially at 20°C is pumped into a mixing tank at a constant volume flow rate of 0.5 m^3/s. See Figure 7.6.1. At time $t = 0$ the temperature of the incoming liquid suddenly is changed to 80°C. The tank walls are perfectly insulated. The tank volume is 12 m^3, and the liquid within is well-mixed so that its temperature is uniform throughout, and denoted by T. The liquid's specific heat and mass density are c_p and ρ. Given that $T(0) = 20$°C, develop and solve a dynamic model for the temperature T as a function of time.

Section 7.7 Thermal Resistance

7.46 The copper shaft shown in Figure P7.46 consists of two cylinders with the following dimensions: $L_1 = 10$ mm, $L_2 = 5$ mm, $D_1 = 2$ mm, and $D_2 = 1.5$ mm. The shaft is insulated around its circumference so that heat transfer occurs only in the axial direction. (a) Compute the thermal resistance of each section of the shaft and of the total shaft. Use the following value for the conductivity of copper: $k = 400$ W/m · °C. (b) Compute the heat flow rate in the axial direction if the temperature difference across the endpoints of the shaft is 30°C.

Figure P7.46



7.47 A certain radiator wall is made of copper with a conductivity $k = 47$ lb/sec-°F at 212°F. The wall is 3/16 in. thick and has circulating water on one side with a convection coefficient $h_1 = 85$ lb/sec-ft-°F. A fan blows air over the other side, which has a convection coefficient $h_2 = 15$ lb/sec-ft-°F. Find the thermal resistance of the radiator on a square foot basis.

7.48 A particular house wall consists of three layers and has a surface area of 30 m^2. The inside layer is 10 mm thick and made of plaster board with a thermal conductivity of $k = 0.2$ W/(m · °C). The middle layer is made of fiberglass insulation with $k = 0.04$ W/(m · °C). The outside layer is 20 mm thick and made of wood siding with $k = 0.1$ W/(m · °C). The inside temperature is 20°C, and the convection coefficient for the inside wall surface is $h_i = 40$ W/(m^2 · °C). The convection coefficient for the outside wall surface is $h_o = 70$ W/(m^2 · °C). How thick must the insulation layer be so that the heat loss is no greater than 400 W if the outside temperature is −20°C?

7.49 A certain wall section is composed of a 12 in. by 12 in. brick area 4 in. thick. Surrounding the brick is a 36 in. by 36 in. concrete section, which is also 4 in. thick. The thermal conductivity of the brick is $k = 0.086$ lb/sec-°F. For the concrete, $k = 0.02$ lb/sec-°F. (a) Determine the thermal resistance of the wall section. (b) Compute the heat flow rate through (1) the concrete, (2) the brick, and (3) the wall section if the temperature difference across the wall is 40°F.

7.50 Water at 120°F flows in an iron pipe 10 ft long, whose inner and outer radii are 1/2 in. and 3/4 in. The temperature of the surrounding air is 70°F. (a) Assuming that the water temperature remains constant along the length of the pipe, compute the heat loss rate from the water to the air in the radial direction, using the following values. For iron, $k = 10.1$ lb/sec-°F. The convection coefficient at the inner surface between the water and the iron is $h_i = 16$ lb/sec-ft-°F. The convection coefficient at the outer surface between the air and the iron is $h_o = 1.1$ lb/sec-ft-°F. (b) Suppose the water is flowing at 0.5 ft/sec. Check the validity of the constant-temperature assumption. For water, $\rho = 1.94$ slug/ft^3 and $c_p = 25{,}000$ ft-lb/slug-°F.

Section 7.8 Dynamic Models of Thermal Systems

7.51 Consider the water pipe treated in Example 7.7.4. Suppose now that the water is not flowing. The water is initially at 120°F. The copper pipe is 6 ft long, with inner and outer radii of 1/4 in. and 3/8 in. The temperature of the surrounding air is constant at 70°F. Neglect heat loss from the ends of the pipe, and use the following values. For copper, $k = 50$ lb/sec-°F. The convection coefficient at the inner surface between the water and the copper is now different because the water is standing. Use $h_i = 6$ lb/sec-ft-°F. The convection coefficient at the outer surface between the air and the copper is $h_o = 1.1$ lb/sec-ft-°F. Develop and solve a dynamic model of the water temperature $T(t)$ as a function of time.

7.52 A steel tank filled with water has a volume of 1000 ft^3. Near room temperature, the specific heat for water is $c = 25{,}000$ ft-lb/slug-°F, and its mass density is $\rho = 1.94$ slug/ft^3.

a. Compute the thermal capacitance C_1 of the water in the tank.

b. Denote the total thermal resistance (convective and conductive) of the tank's steel wall by R_1. The temperature of the air surrounding the tank is T_o. The tank's water temperature is T_1. Assume that the thermal capacitance of the steel wall is negligible. Derive the differential equation model for the water's temperature, with T_o as the input.

7.53 Consider the tank of water discussed in Problem 7.52. A test was performed in which the surrounding air temperature T_o was held constant at 70°F. The tank's water temperature was heated to 90° and then allowed to cool. The following data show the tank's water temperature as a function of time. Use these data to estimate the value of the thermal resistance R_1.

Time t (sec)	Water temperature T_1 (°F)
0	90
500	82
1000	77
1500	75
2000	73
2500	72
3000	71
4000	70

7.54 The oven shown in Figure P7.54 has a heating element with appreciable capacitance C_1. The other capacitance is that of the oven air C_2. The corresponding temperatures are T_1 and T_2, and the outside temperature is T_o. The thermal resistance of the heater-air interface is R_1; that of the oven wall is R_2. Develop a model for T_1 and T_2, with input q_i, the heat flow rate delivered to the heater mass.

7.55 A simplified representation of the temperature dynamics of two adjacent masses is shown in Figure P7.55. The mass with capacitance C_2 is perfectly insulated on all sides except one, which has a convective resistance R_2. The thermal capacitances of the masses are C_1 and C_2, and their representative uniform temperatures are T_1 and T_2. The thermal capacitance of the surroundings is very large and the temperature is T_o. (a) Develop a model of the behavior of T_1 and T_2. (b) Discuss what happens if the thermal capacitance C_2 is very small.

Figure P7.54

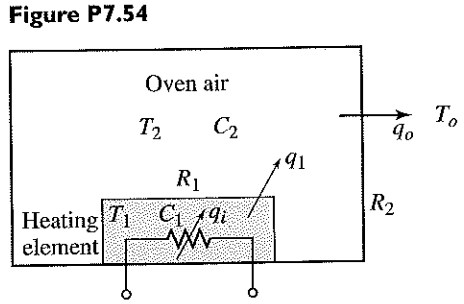
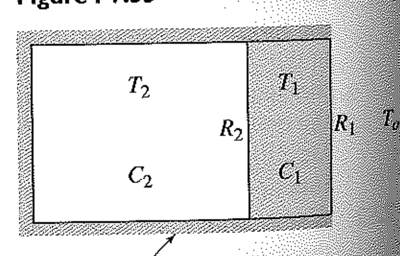


Figure P7.55



7.56 A metal sphere 25 mm in diameter was heated to 95°C, and then suspended in air at 22°C. The mass density of the metal is 7920 kg/m^3, its specific heat at 30°C is c_p = 500 J/(kg · °C), and its thermal conductivity at 30°C is 400 W/(m · °C). The following sphere temperature data were measured as the sphere cooled.

t (s)	T (°C)	t (s)	T (°C)	t (s)	T (°C)
0	95	120	85	540	67
15	93	135	84	600	65
30	92	180	82	660	62
45	90	240	79	720	61
60	89	300	76	780	59
75	88	360	73	840	57
90	87	420	71	900	56
105	86	480	69	960	54

a. Assume that the sphere's heat loss rate is due entirely to convection. Estimate the convection coefficient h.

b. Compute the Biot number and discuss the accuracy of the lumped-parameter model used in part (a).

c. Discuss whether some of the heat loss rate could be due to radiation. Give a numerical reason for your answer.

7.57 A copper sphere is to be quenched in an oil bath whose temperature is 50°C. The sphere's radius is 30 mm, and the convection coefficient is h = 300 W/(m^2 · °C). Assume the sphere and the oil properties are constant. These properties are given in the following table. The sphere's initial temperature is 400°C.

Property	Sphere	Oil
Density ρ (kg/m^3)	8900	7900
Specific heat c_p J/(kg · °C)	385	400
Thermal conductivity k [W/(m · °C)]	400	—

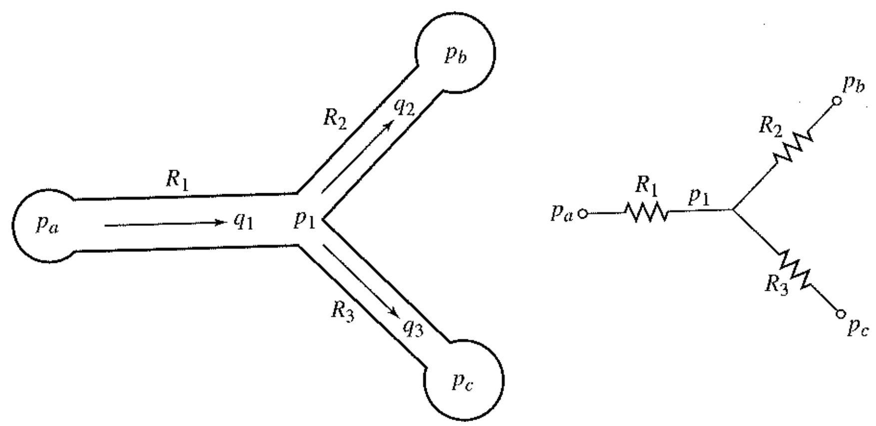
Assume that the volume of the oil bath is large enough so that its temperature does not change when the sphere is immersed in the bath. Obtain the dynamic model of the sphere's temperature T. How long will it take for T to reach 130°C?

7.58 Consider the quenching process discussed in Problem 7.57. Suppose the oil bath volume is 0.1 m^3. Neglect any heat loss to the surroundings and develop a dynamic model of the sphere's temperature and the bath temperature. How long will it take for the sphere temperature to reach 130°C?

Section 7.9 MATLAB Applications

7.59 Consider Example 7.7.1. The MATLAB left division operator can be used to solve the set of linear algebraic equations **AT** = **b** as follows: `T = A\b`. Use this method to write a script file to solve for the three steady-state temperatures T_1, T_2, and T_3, given values for the resistances and the temperatures T_i and T_o. Use the results of Example 7.7.1 to test your file.

Figure P7.60



(a) (b)

7.60 Fluid flows in pipe networks can be analyzed in a manner similar to that used for electric resistance networks. Figure P7.60a shows a network with three pipes, which is analogous to the electrical network shown in part (b) of the figure. The volume flow rates in the pipes are q_1, q_2, and q_3. The pressures at the pipe ends are p_a, p_b, and p_c. The pressure at the junction is p_1.

a. Assuming that the linear resistance relation applies, we have

$$q_1 = \frac{1}{R_1}(p_a - p_1)$$

Obtain the equations for q_2 and q_3.

b. Note that conservation of mass gives $q_1 = q_2 + q_3$. Set up the equations in a matrix form $\mathbf{Aq} = \mathbf{b}$ suitable for solving for the three flow rates q_1, q_2, and q_3, and the pressure p_1, given the values of the pressures p_a, p_b, and p_c, and the values of the resistances R_1, R_2, and R_3. Find the expressions for matrix **A** and the column vector **b**.

c. Use MATLAB to solve the matrix equations obtained in part (b) for the case: $p_a = 30$ psi, $p_b = 25$ psi, and $p_c = 20$ psi. Use the resistance values $R_1 = 10{,}000$, $R_2 = R_3 = 14{,}000$ 1/(ft-sec). These values correspond to fuel oil flowing through pipes 2 ft long, with 2 in. and 1.4 in. diameters, respectively. The units of the answers should be ft^3/sec for the flow rates, and lb/ft^2 for pressure.

7.61 The equation describing the water height h in a spherical tank with a drain at the bottom is

$$\pi(2rh - h^2)\frac{dh}{dt} = -C_d A_o \sqrt{2gh}$$

Suppose the tank's radius is $r = 3$ m and that the circular drain hole has a radius of 2 cm. Assume that $C_d = 0.5$, and that the initial water height is $h(0) = 5$ m. Use $g = 9.81$ m/s^2.

a. Use an approximation to estimate how long it takes for the tank to empty.

b. Use MATLAB to solve the nonlinear equation and plot the water height as a function of time until $h(t)$ is not quite zero.

7.62 The following equation describes a certain dilution process, where $y(t)$ is the concentration of salt in a tank of fresh water to which salt brine is being added.

$$\frac{dy}{dt} + \frac{2}{10 + 2t}y = 4$$

Suppose that $y(0) = 0$.

a. Use MATLAB to solve this equation for $y(t)$ and to plot $y(t)$ for $0 \le t \le 10$.

b. Check your results by using an approximation that converts the differential equation into one having constant coefficients.

7.63 A tank having vertical sides and a bottom area of 100 ft^2 is used to store water. To fill the tank, water is pumped into the top at the rate given in the following table. Use MATLAB to solve for and plot the water height $h(t)$ for $0 \le t \le 10$ min.

Time (min)	0	1	2	3	4	5	6	7	8	9	10
Flow Rate (ft^3/min)	0	80	130	150	150	160	165	170	160	140	120

7.64 A cone-shaped paper drinking cup (like the kind used at water fountains) has a radius R and a height H. If the water height in the cup is h, the water volume is given by

$$V = \frac{1}{3}\pi\left(\frac{R}{H}\right)^2 h^3$$

Suppose that the cup's dimensions are $R = 1.5$ in. and $H = 4$ in.

a. If the flow rate from the fountain into the cup is 2 in.^{3}/sec, use MATLAB to determine how long will it take to fill the cup to the brim.

b. If the flow rate from the fountain into the cup is given by $2(1 - e^{-2t})$ in.^{3}/sec, use MATLAB to determine how long will it take to fill the cup to the brim.

Section 7.10 Simulink Applications

7.65 Refer to Figure 7.10.1. Assume that the resistances obey the linear relation, so that the mass flow q_l through the left-hand resistance is $q_l = (p_l - p)/R_l$, with a similar linear relation for the right-hand resistance.

a. Create a Simulink subsystem block for this element.

b. Use the subsystem block to create a Simulink model of the system discussed in Example 7.4.3 and shown in Figure 7.4.3a. Assume that the mass inflow rate q_{mi} is a step function.

c. Use the Simulink model to obtain plots of $h_1(t)$ and $h_2(t)$ for the following parameter values: $A_1 = 2$ m^2, $A_2 = 5$ m^2, $R_1 = 400$ 1/(m · s), $R_2 = 600$ 1/(m · s), $\rho = 1000$ kg/m^3, $q_{mi} = 50$ kg/s, $h_1(0) = 1.5$ m, and $h_2(0) = 0.5$ m.

7.66 Use Simulink to solve Problem 7.61(b).

7.67 Use Simulink to solve Problem 7.63.

7.68 Use Simulink to solve Problem 7.64. Plot $h(t)$ for both parts (a) and (b).

7.69 Refer to Example 7.10.1. Use the simulation with $q = 8 \times 10^5$ to compare the energy consumption and the thermostat cycling frequency for the two temperature bands $(69°, 71°)$ and $(68°, 72°)$.

7.70 Consider the liquid-level system shown in Figure 7.3.3. Suppose that the height h is controlled by using a relay to switch the flow rate q_{mi} between the values 0 and 50 kg/s. The flow rate is switched on when the height is less than 4.5 m and is switched off when the height reaches 5.5 m. Create a Simulink model for this application using the values $A = 2\ \mathrm{m}^2$, $R = 400\ 1/(\mathrm{m} \cdot \mathrm{s})$, $\rho = 1000\ \mathrm{kg/m}^3$, and $h(0) = 1$ m. Obtain a plot of $h(t)$.

CHAPTER 8

System Analysis in the Time Domain

CHAPTER OUTLINE

CHAPTER OBJECTIVES

When you have finished this chapter, you should be able to

1. Obtain and interpret the free, step, ramp, and impulse response of linear models.
2. Compute and use the time constant τ, the undamped natural frequency ω_n, and other parameters to describe and assess system response.
3. Use time-domain response data to estimate coefficient values in dynamic models.
4. Use Simulink to simulate nonlinear systems and systems with inputs more complicated than the impulse, step, and ramp functions.

Now that we have seen how to model the various types of physical systems (mechanical, electrical, fluid, and thermal), it is appropriate at this point to pull together our modeling knowledge and our analytical and computer solution methods. The purpose of this chapter is to integrate this knowledge with emphasis on understanding system behavior in the *time domain*. The forcing functions commonly used to model real inputs or to test a system's response in the time domain are the *impulse*, the *step*, and the *ramp* functions. The impulse models a suddenly applied and suddenly removed input. The step function models a suddenly applied input that remains constant. The ramp models an input that is changing at a constant rate. In this chapter we will show how to analyze systems subjected to these inputs. In Chapter 9, we will treat system response in the *frequency domain* by analyzing the response to the other commonly used input, the sinusoid.

The transfer function and the characteristic polynomial are the principal tools for analyzing linear system response, and so it is now appropriate to review these concepts.