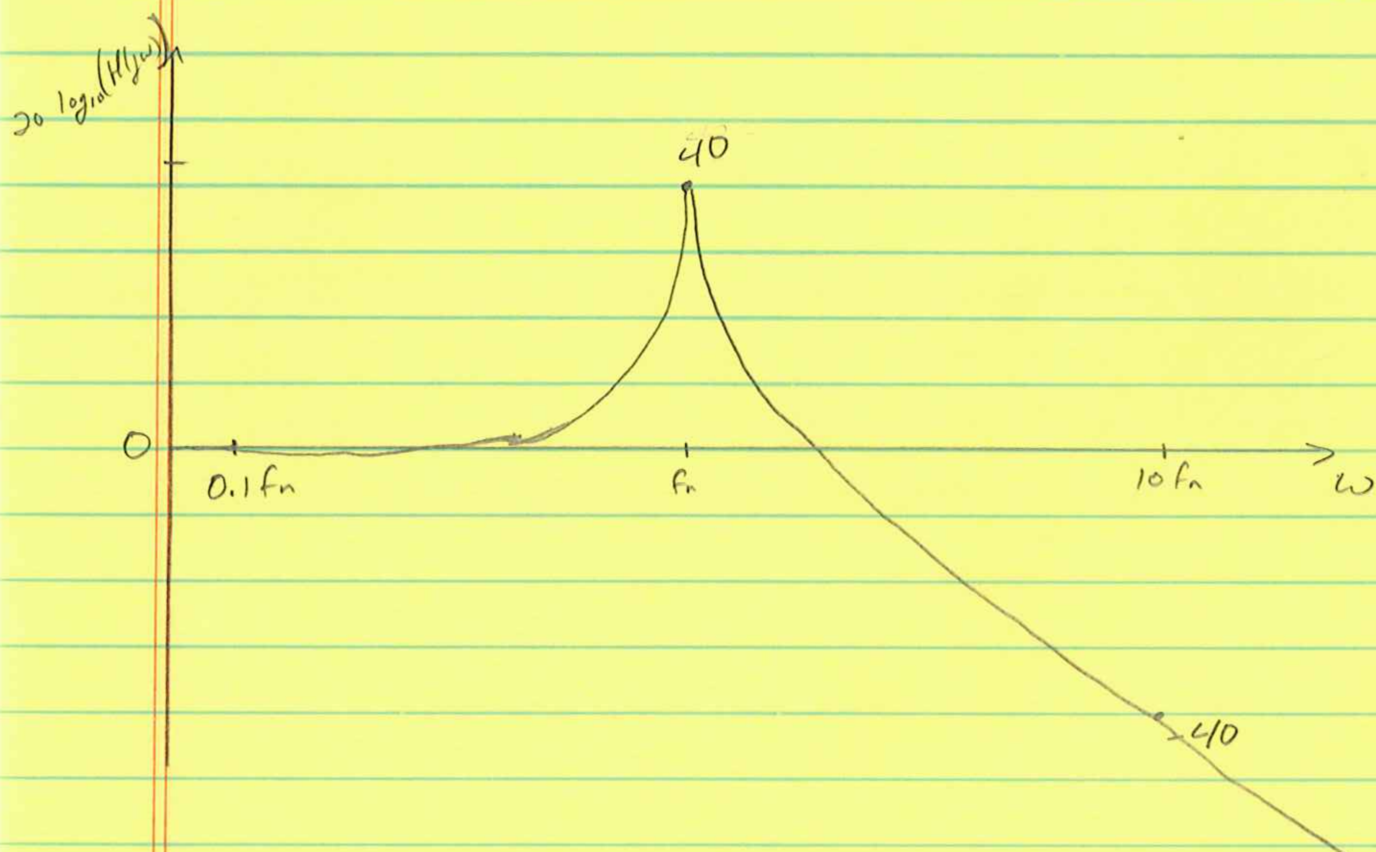
ME 460/660 Exam 1 Fall 2010 Solutions

1) $H(j\omega) = \dfrac{100 + 0.1 j\omega}{100 - \omega^2 + 0.1 j\omega}$

Y	ω	$x(t)$	
10	2π	$16.5 \sin(2\pi t - 0.24^\circ)$	(-0.004 rad)
0.1	20π	$2.61 \times 10^{-3} \sin(20\pi t - 176^\circ)$	(-3 rad)
0.001	200π	$2.99 \times 10^{-7} \sin(200\pi t - 148^\circ)$	(-2.58 rad)

$Y|H(j\omega)|$ ω $\measuredangle H(j\omega)$



2) $T = \frac{1}{2} m v^2 + \frac{1}{2} J \dot{\phi}^2$

$v = (R - R_1)\dot{\theta}$, $\dot{\phi} R_1 = \dot{\theta}(R - R_1)$

$\dot{\phi} = \dot{\theta}\left(\frac{R - R_1}{R_1}\right)$

$T = \frac{1}{2} m (R - R_1)^2 \dot{\theta}^2 + \frac{1}{2} m R_1^2 \left(\frac{R - R_1}{R_1}\right)^2 \dot{\theta}^2$

$= \frac{1}{2} J_{eff} \dot{\theta}^2$ $\qquad J_{eff} = 2m(R - R_1)^2$

$U = mgh = mg(R - R_1)(1 - \cos\theta)$

$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) + \frac{\partial U}{\partial \theta} = 0$ (other neglected terms are zero)

$2m(R - R_1)^2 \ddot{\theta} + mg(R - R_1)\theta = 0$

$\omega_n = \sqrt{\frac{g}{2(R - R_1)}}$

3) $A\omega^2_{max} = 18 \cdot 9.81 = 176 \ m/s^2$

max accel with safety factor is $161.0 \, m/s^2$

max velocity is $\frac{161}{10 \cdot 2\pi} = 2.55 \ m/s$

max displacement is $\frac{2.55}{10 \cdot 2\pi} = 0.041 \ m$ (4.1 cm)

I doubt this analysis is valid. The 18g value is for constant accel. An oscillating accel at this amplitude will likely be worse than a constant accel.

4) $\frac{E}{\rho} w'' - \ddot{w} = 0$

$w(x,t) = X(x)\,T(t)$

$T(t) = \sin \omega_n t \quad , \quad \ddot{T}(t) = -\omega_n^2\, T(t)$

Substituting for $w(x,t)$

$\left(\frac{E}{\rho} X'' + \omega_n^2 X\right) T = 0$

$X = A \sin \sigma_n x + B \cos \sigma_n x$

$X'' = -\sigma_n^2 X$

$\sigma_n^2 = \omega_n^2 \sqrt{\frac{\rho}{E}}$

From $X(0) = 0, \quad B = 0$

From $X(l) = 0 \quad \sigma_n l = n\pi \quad , \quad \sigma_n = \frac{n\pi}{l}$

$X_n(x) = \underline{A \sin \frac{n\pi}{l} x}$

$\omega_n = \sigma_n \sqrt{\frac{E}{\rho}} = \underline{\frac{n\pi}{l} \sqrt{\frac{E}{\rho}}}$