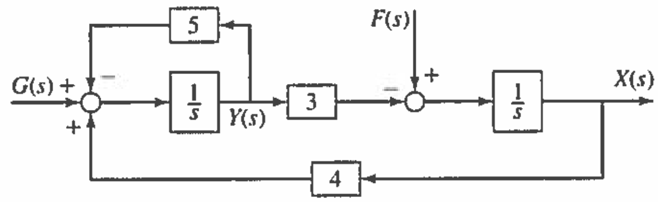
System Dynamics Exam 2 Fall 2015

The FE reference book, calculator, and 1 formula sheet may be used during this exam. Exam books provided. 10 points each.

1. Find $\frac{Y(s)}{G(s)}$ for the system shown below.



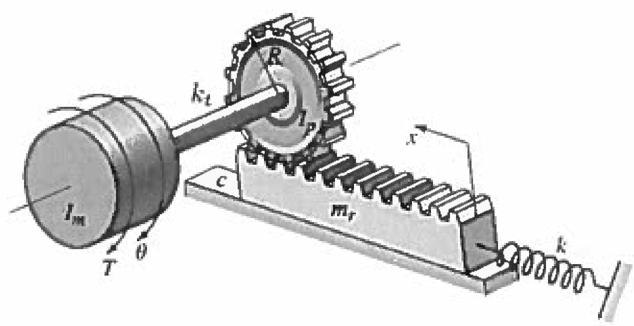
(b)

2. Write the state space equation matrices A, B, C, and D for the following systems:

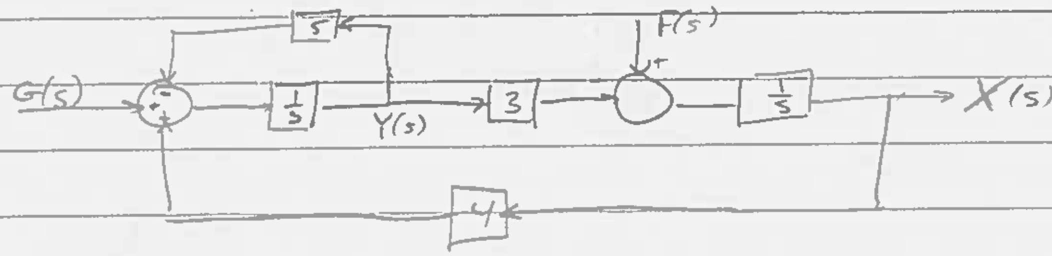
   (a) $y(t)$ and $\dot{y}(t)$ are the outputs:

$$\frac{Y(s)}{F(s)} = \frac{6}{3s^3 + 63s^2 + 10}$$

3. **Note the torsional flexibility of the shaft** k_t and derive the differential equation, or equations, of motion for the system below in terms of the unique degrees of freedom. *Note the viscous friction c between the surfaces.*



ME3210 System Dynamics, Fall 2015 Test 2 Solutions

1) 

Find $\frac{Y(s)}{G(s)}$.

Note that F(s) is not relevant

First sum block:

$$\underbrace{\left(G(s) + 4X(s) - 5Y(s)\right)}_{\text{block}} \frac{1}{s} = Y(s) \qquad (1)$$

Second sum block.

$$-\frac{3}{s} Y(s) = X(s) \qquad (2)$$

Substitute (2) into (1)

$$G(s) - \left(\frac{12}{s} + 5\right) Y(s) = s\,Y(s)$$

$$G(s) = \left(s + 5 + \frac{12}{s}\right) Y(s)$$

$$s\,G(s) = \left(s^2 + 5s + 12\right) Y(s)$$

$$\boxed{\frac{Y(s)}{G(s)} = \frac{s}{s^2 + 5s + 12}}$$

2)

$$3s^3 Y(s) + 63s^2 Y(s) + 0.5 + 10 = 6F(s)$$

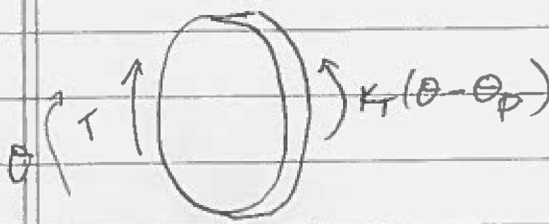
$$\dddot{y} = -\frac{10}{3} y - 21\ddot{y} + 2F(t)$$

Define the states to be y, $\dot{y}$ and $\ddot{y}$

$$\dot{\underline{z}} = \begin{bmatrix} \dot{y} \\ \ddot{y} \\ \dddot{y} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\frac{10}{3} & 0 & -21 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} y \\ \dot{y} \\ \ddot{y} \end{bmatrix}}_{\underline{z}} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}}_{B} F(t)$$

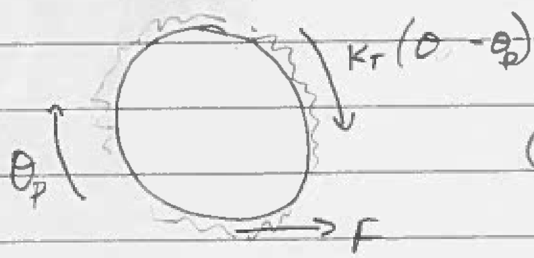
$$\underline{y} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} y \\ \dot{y} \\ \ddot{y} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{D} F(t)$$

3) FBD Inertia



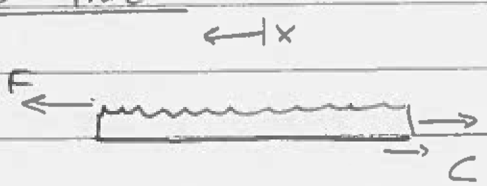
① $\Sigma M = I_m \ddot{\theta} = -K_T(\theta - \theta_p) + T$

FBD Pinion:



② $\Sigma M = I_p \ddot{\theta}_p = K_T(\theta - \theta_p) - FR$

FBD Rack



③ $\Sigma F = m_r \ddot{x} = F - Kx - c\dot{x}$

From Kinematics $x = R\theta_p$

③ Becomes ④ $m_r R \ddot{\theta}_p + KR\theta_p + cR\dot{\theta}_p = F$

Substitute ④ into ②

⑤ $I_p \ddot{\theta}_p + K_T\theta_p - K_T\theta = -m_r R^2 \ddot{\theta}_p - KR^2\theta_p - cR^2\dot{\theta}_p$

5 simplifies to

$$\underline{(I_p + m_r R^2)\ddot{\theta}_p + cR^2\ddot{\theta}_p + (KR^2 + K_T)\theta_p - K_T\theta = 0}$$

Equation 1 and equation 5 are the minimum governing equations

$$\underline{I_m\ddot{\theta} + K_T\theta - K_T\theta_p = T}$$