Computational Methods in Structural Dynamics, Final Winter 2006

One 8.5" by 11" cheat sheet. Log all work and print out your computer work. Show all work. 120 points total.

Short answer (4 points each)

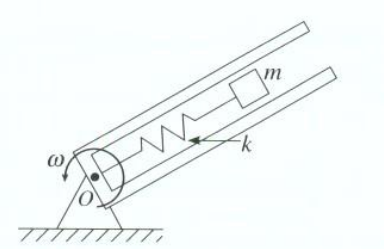
1. What does inclusion principle mean? i.e. how can you use this to your advantage as an engineer?
2. Why is Householder's method preferred over Given's method?
3. On what fact is the Rayleigh-Ritz method based?
4. One what principle does subspace iteration operate? (what method is it similar to and how?)
5. What types of problems require the use of weighted residual methods in place of the Rayleigh-Ritz method? (hint: 2 types).

Short problem. 10 points each

1. Given $y_1(x) = sin(\pi x)$ and $y_1(x) = x$ over $0 < x < 1$, find an orthonormal set of functions that span the same domain.
2. Knowing that $\sigma = E\epsilon$, that the curvature of a beam is approximated well by $\frac{d^2w}{dx^2}$ and presuming that plane sections remain plane during deformation, derive the stress at a point a distance y from the neutral axis, from that the strain energy in a cross section of a beam, and then the total strain energy in a beam (Euler Bernoulli).

Full-length problems. 20 points each.

1. For the figure below, a mass is constrained to ride in a tube that is rotating in the vertical plane at a constant rate ω. Derive the equation of motion and discuss the stability of the system.



2. Determine the mode shapes and natural frequencies of a clamped-clamped beam given the equation of a beam

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + \rho A\frac{\partial^2 w(x,t)}{\partial t^2} = p(x,t)$$

Obtain numerical values (as much as is possible) for the first 2 natural frequencies.

3. Given the matrix

$$K = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 3 & -2 \\ 0 & -2 & 5 \end{bmatrix}$$

perform one complete cycle of the QR eigen-solution method.

4. A constant-value compressive load of P is applied to a simply supported beam of length l. Derive the equation of motion.
Bonus: Determine the value of P for which the first natural frequency is zero. What physically happens if P exceeds this value?