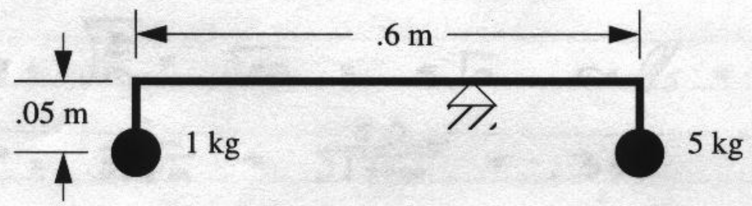
ME 460/660 Exam 1, Fall '96

One formula sheet. Front and back. No examples. No derivations. It must be turned in with the exam.

1) Find the free response of the following system: m = 10 kg, k = 500 N/m, c = 50 kg/s. Assume initial conditions of $x(0)$ = 0 m and $v(0)$ = 0.1 m/s. 20 points

2) Can the frequency of oscillation (free response frequency) for a viscously damped system be changed without changing the decay envelope? How? Derive the constraint on the system parameters (m, c, and k). 20 points

3) Derive the equation of motion for the following balance. Assume that the weights always hang straight down and that they are balanced. 20 points



Bonus: What is the frequency of the maximum response of a damped SDOF system to a harmonic excitation? Use correct units. 5 points.

ME 460/660 Exam 1 Solutions

1) Given: $m = 10\ kg$, $k = 500\ N/m$, $c = 50\ kg/s$. $x_0 = 0\ m$, $v_0 = .1\ m/s$.

Find: Free response

$$x(t) = A e^{-\xi\omega t} \sin(\omega_d t + \phi)$$ + 8

$$A = \sqrt{\frac{(v_0 + \xi\omega x_0)^2 + (\omega_d x_0)^2}{\omega_d^2}}, \quad \phi = \operatorname{atan} \frac{x_0 \omega_d}{v_0 + \xi\omega x_0}$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{50} = 5\sqrt{2} \text{ rad/s} = 7.071$$ + 2

$$\xi = \frac{c}{2\sqrt{km}} = \frac{50}{2\sqrt{5000}} = .354$$ + 2

$$\omega_d = \omega\sqrt{1-\xi^2} = 6.614 \text{ rad/s}$$ + 2

$$A = \frac{v_0}{\omega_d} = 1.51 \times 10^{-2}\ m$$ + 2

$$\phi = 0$$ + 2

$$x(t) = 1.51 \times 10^{-2} e^{-2.5t} \sin 6.614t\ m$$ + 2

2) The decay envelope is defined by $e^{-\xi\omega t}$

If $\xi\omega$ remains unchanged, the decay envelope does as well.

By definition

$$\xi = \frac{c}{2\sqrt{km}} \quad , \quad \omega = \sqrt{\frac{k}{m}}$$

So $$\xi\omega = \frac{c}{2\sqrt{km}}\sqrt{\frac{k}{m}} = \frac{c}{2m}$$

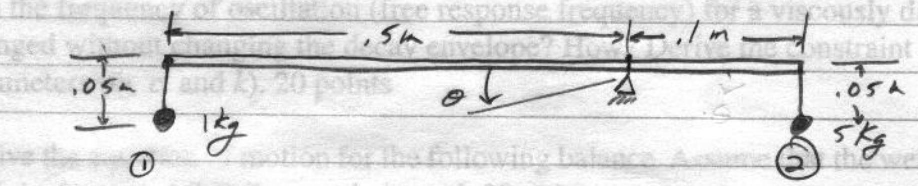
The frequency of oscillation is $\omega_d = \omega\sqrt{1-\xi^2}$

$$\omega_d = \sqrt{\frac{k}{m}}\sqrt{1-\frac{c}{2m}}$$

So, by holding $\frac{c}{m}$ constant, ω_d can be changed independently.
(Note that simply changing k changes the period of oscillation without changing the decay envelope)

3) Note: If the masses always hang straight down, their velocity is exactly the same as that of the corners above them.

For the balance to be in balance: +4 (to balance)



The position of mass 1 is

$$x = -.5\cos\theta, \quad y = -.5\sin\theta$$

The velocity is then

$$\dot{x} = +.5\dot{\theta}\sin\theta, \quad \dot{y} = -.5\dot{\theta}\cos\theta$$

Likewise for mass 2

$$x = .1\cos\theta, \quad \dot{x} = -.1\dot{\theta}\sin\theta$$
$$y = .1\sin\theta, \quad \dot{y} = .1\dot{\theta}\cos\theta$$

The potential energy is mgy for each mass

Thus

$$U = 1g\,{-.5}\sin\theta + 5g\,.1\sin\theta = 0 \qquad +6$$

The kinetic energy is $\frac{1}{2}m v_{total}^2$ for each mass

Thus

$$T = \frac{1}{2}\cdot 1\cdot\left(.5^2\dot{\theta}^2\sin^2\theta + .5^2\dot{\theta}^2\cos^2\theta\right)$$
$$+ \frac{1}{2}5\left(.1^2\dot{\theta}^2\sin^2\theta + .1^2\dot{\theta}^2\cos^2\theta\right) \qquad +6$$
$$= \frac{1}{2}\,1\cdot .5^2\dot{\theta}^2 + \frac{1}{2}\cdot 5\cdot .1^2\dot{\theta}^2 = .15\dot{\theta}^2$$

$$\frac{d}{dt}(T+U) = .3\dot{\theta}\ddot{\theta} = 0 \qquad +4$$
$$\ddot{\theta} = 0$$