

ME 712 Sp '08 Midterm Solns

1) See code

- a) $I = 4$
- b) $I = 1.52$
- c) $I = 1.62$

c is exact to reported accuracy.

2)

$$K = \int_0^l \frac{d^2}{dx^2} N^T E I(x) \frac{d^2}{dx^2} N dx$$

where N is as given

$$\frac{d^2}{dx^2} N = \left[-\frac{6}{l^2} + \frac{12x}{l^3}, -\frac{4}{l} + \frac{6x}{l^2}, \frac{6}{l^2} - \frac{12x}{l^3}, -\frac{2}{l} + \frac{6x}{l^2} \right]$$

$$K_{14} = \int_0^l \left(-\frac{6}{l^2} + \frac{12x}{l^3} \right) E (I_1 + (I_2 - I_1) \frac{x}{l}) \left(-\frac{2}{l} + \frac{6x}{l^2} \right) dx$$

Using Mathematica or Matlab

$$K_{14} = \frac{2E}{l^2} (I_1 + 2I_2)$$

3) For top member

$$\{K\} = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & 12b & -6bL & 0 \\ 0 & -6bL & 4bL^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{where } b = \frac{EI}{L^3}, \quad a = \frac{EA}{L}$$

bottom member:

$$\{K\} = \begin{bmatrix} 12b & 0 & 6bL & 6bL \\ 0 & a & 0 & 0 \\ 6bL & 0 & 4bL^2 & 2bL^2 \\ 6bL & 0 & 2bL^2 & 4bL^2 \end{bmatrix}$$

Sym

Together

$$\{K\} = \begin{bmatrix} a+12b & 0 & 6bL & 6bL \\ 0 & a+12b & -6bL & 0 \\ 6bL & -6bL & 8bL^2 & 2bL^2 \\ 6bL & 0 & 2bL^2 & 4bL^2 \end{bmatrix}$$

Sym

4) See example 4 and plot

$$N_3 = I + -2\xi^2 + \xi^4$$

```

xi=0
eta=0
w=2
I=0;
for i=1:1
    for j=1:1
        I=I+w(i)*w(j)*cos(xi(i)*pi/2)*cos(eta(j)*pi/2);
    end
end
disp('Integral for single point integration')
I

```

```

xi=[-1 1]/sqrt(3)
eta=xi
w=[ 1 1]
I=0;
for i=1:2
    for j=1:2
        I=I+w(i)*w(j)*cos(xi(i)*pi/2)*cos(eta(j)*pi/2);
    end
end
disp('Integral for two point integration')
I

```

```

xi=[ -sqrt(.6) 0 sqrt(.6)];
eta=xi
w=[5 8 5]/9
I=0;
for i=1:3
    for j=1:3
        I=I+w(i)*w(j)*cos(xi(i)*pi/2)*cos(eta(j)*pi/2);
    end
end
disp('Integral for three point integration')
I

```

% w=a+bxi+cxi^2+dxi^3+exi^4+fxi^5

b=[1 -1 1 -1 1 -1; % w(-1) x^n
1 0 0 0 0 0; % w(0)
1 1 1 1 1 1 ;%w(1)
0 1 -2 3 -4 5; % w'(-1) x^n
0 1 0 0 0 0;% w'(0)
0 1 2 3 4 5] %w'(-1)]

a=[0 1 0 0 0 0]'

coefs= b\ a

xi=-1:.01:1;
plot(xi,polyval(fliplr(coefs'),xi))

