Computational Methods in Structural Dynamics, Midterm Winter 2008

One 8.5" by 11" cheat sheet.

1. A beam with constant properties E, I, ρ, A and l (where ρ is mass per volume) is clamped at the left end and pinned at the right end with an unknown torsional spring stiffness of k. Find the upper and lower bounds of the 3rd natural frequency using the provided page.

2. Find the natural frequencies and mode shapes of the beam of the previous problem if $E = 2.1E11$ Pa, $\rho = 7.8E3$ kg/m^3, $G = 8.0E10$ Pa, $l = 1$ m. The cross section is rectangular with $h = 2$ cm, $w = 3$cm. The spring stiffness is $k = 10$ Nm.

3. Determine the steady state (non-transient) response of the system

$$EI\frac{\partial^4 w}{\partial x^4} + \rho A\frac{\partial^2 w}{\partial t^2} = \sin(t)\delta\left(x - \frac{l}{3}\right)$$

   for a pinned-pinned (simply supported) boundary condition. Plot the frequency response function between the input at $l/3$ and $l/3$. Presume constants as provided in the previous problem. Hint: Plot using dB.

4. Given

$$M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \qquad K = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \tag{1}$$

   Define the stability of the system. Explain why.

5. Derive the relationship between M (moment) and w (deflection) for a beam.