running the simulation, type the following in the Command window to set the values of the parameters.

```
>>KT = 0.04; Kb = KT; Ra = 0.6; La = 2e-3;
>>Ie = 1.802e-3; ce = 4.444e-4;
>>N = 1.5; KP = 0.63;
```

The To Workspace block puts the time variable `tout` and the array `simout` in the MATLAB workspace. The first column of `simout` contains the current, and the second column contains the speed in rpm. To plot the current response, in the Command window type `plot(tout,simout(:,1))`. Type `plot(tout,simout(:,2))` to obtain a plot of the speed. The plots are shown in Figure 10.8.11.

The model in Example 10.10.1 implements proportional control. To examine PID control in general, replace the Controller gain block in Figure 10.10.4 with the PID Controller block. This block enables you to specify the proportional, integral, and derivative gains.

10.11 CHAPTER REVIEW

This chapter introduced the basic concepts of feedback control. It showed how to model control system components and analyze control system performance.

When designing a control system, the control systems engineer is usually given the plant, the actuator, and the physical type of controller (electronic, pneumatic, hydraulic, or digital), and often is expected to develop a model of these components. The command and disturbance inputs might be specified, or the engineer might be expected to develop suitable test inputs based on the application. Step functions are the principal test inputs, because they are the most common and perhaps represent the severest test of system performance. Ramp, trapezoidal, and sinusoidal test inputs are also employed. The type to use should be made clear in the design specifications.

The engineer then proceeds to design the control system. Based on the system model and the performance specifications, a control action is chosen, and the output, error, and actuator equations are derived. These are then analyzed for stability. If the system cannot be made stable with a gain change, a different control action is tried. Using the given command and disturbance input functions (step, ramp, etc.), the steady-state response is evaluated with the final value theorem. Any constraints on the gain values required to satisfy the steady-state specifications are then determined.

The transient performance is then evaluated in light of the transient specifications, using the given input functions. These specifications often are stated in terms of the desired dominant time constant and damping ratio, but they can also be given in terms of overshoot, rise time, settling time, or bandwidth, for example. Other specifications, such as limits on the maximum available actuator output, are evaluated, and the system redesigned if necessary.

Now that you have finished this chapter you should be able to do the following:

1. Model common control system components.
2. Select an appropriate control algorithm of the PID type or one of its variations, for a given application and for given steady-state and transient performance specifications.
3. Analyze the performance of a control algorithm using transfer functions, block diagrams, and computer methods.

4. Compute the gain values to meet the specifications.
5. Use MATLAB and Simulink to analyze and simulate control systems.

REFERENCE

[Cannon, 1967] R. H. Cannon, Jr., *Dynamics of Physical Systems,* McGraw-Hill, New York, 1967.

PROBLEMS

Section 10.1 Closed-Loop Control

10.1 Discuss whether or not the following devices and processes are open-loop or closed-loop. If they are closed-loop, identify the sensing mechanism.

a. A traffic light.
b. A washing machine.
c. A toaster.
d. Cruise control.
e. An aircraft autopilot.
f. Temperature regulation in the human body.

10.2 Draw the block diagram of a system using proportional control and feedforward command compensation, for the plant $1/(4s^2 + 6s + 3)$. Determine the transfer function of the compensator. Discuss any practical limitations to its use.

10.3 Investigate the performance of proportional control using feedforward command compensation with a constant gain K_f and disturbance compensation with a constant gain K_d, applied to the plant $10/s$. Set the gains to achieve a closed-loop time constant of $\tau = 2$ and zero steady-state error for a step command and a step disturbance.

Section 10.2 Control System Terminology

10.4 Derive the output $C(s)$, error $E(s)$, and actuator $M(s)$ equations for the diagram in Figure P10.4, and obtain the characteristic polynomial.

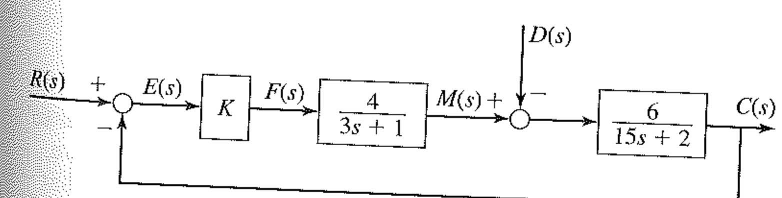
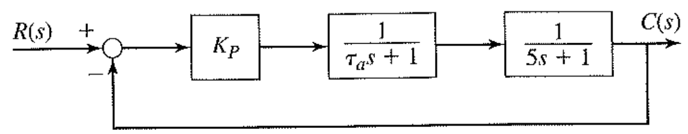


Figure P10.4

Section 10.3 Modeling Control Systems

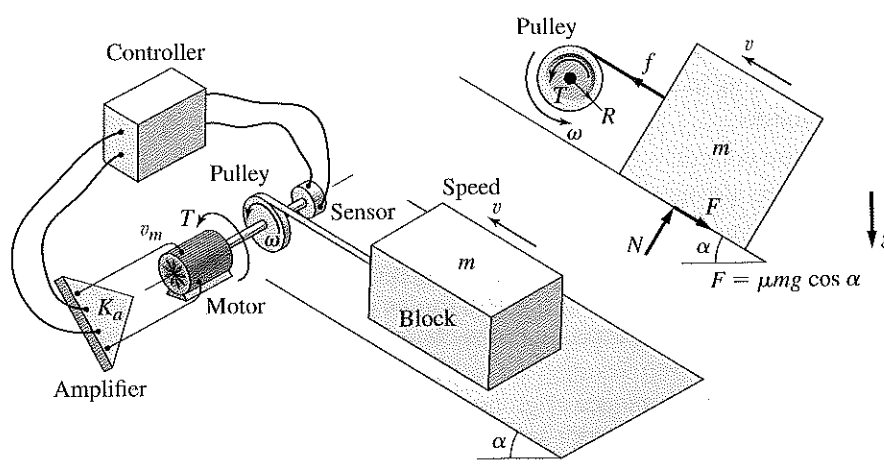
10.5 For the system shown in Figure P10.5, the plant time constant is 5 and the nominal value of the actuator time constant is $\tau_a = 0.05$. Investigate the effects of neglecting this time constant as the gain K_P is increased.

Figure P10.5



10.6 In Figure P10.6, the block is pulled up the incline by the tension force f in the inextensible cable. The motor torque T is controlled to regulate the speed v of the block to obtain some desired speed v_r. The precise value of the friction coefficient μ is unknown, as is the slope angle α, so we model them as a disturbance. Neglect all masses and inertias in the system except for the block mass m. Also neglect the field time constant of the field-controlled motor. Feedback of the block speed v is provided by a sensor that measures the pulley rotational speed ω, which is directly related to v by $v = R\omega$.

Figure P10.6



a. Obtain the equation of motion of the block speed v, with the voltage v_m, friction force $F = \mu mg \cos\alpha$, and the weight component $W_x = mg \sin\alpha$ as the inputs.

b. Draw a block diagram representing the control system, with the command input v_r, the output v, and the disturbance $D = F + W_x$. Model the speed sensor as directly sensing the speed ω. Show the necessary transfer functions for each block in the diagram.

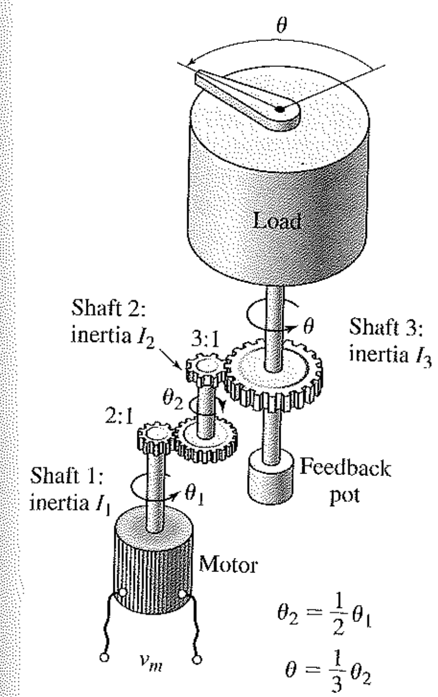
c. Obtain the output, error, and torque transfer functions from the block diagram.

10.7 The diagram in Figure P10.7 shows a system for controlling the angular position of a load, such as an antenna. There is no disturbance.

a. Draw the block diagram of a system using proportional control, similar to that shown in Figure 10.3.9 except that the command and the output are angular positions. Assume that the motor is armature-controlled and that its armature time constant is negligible. Let the inertia I_e be the equivalent inertia of the entire system, as felt on the motor shaft, and let N_e be the equivalent gear ratio of the entire system, as felt on the motor shaft. Show the necessary transfer functions for each block.

b. Determine the value for N_e, and determine I_e as a function of the inertias I_1, I_2, and I_3.

Figure P10.7



Section 10.4 The PID Control Algorithm

10.8 In the following controller transfer function, identify the values of K_P, K_I, K_P, T_I, and T_D.

$$G_c(s) = \frac{F(s)}{E(s)} = \frac{15s^2 + 6s + 4}{s}$$

10.9 Determine the resistance values required to obtain an op-amp PI controller with $K_P = 4$ and $K_I = 0.08$. Use a 1-μF capacitor.

10.10 a. Determine the resistance values to obtain an op-amp PD controller with $K_P = 2$, $T_D = 2$ s. The circuit should limit frequencies above 5 rad/s. Use a 1-μF capacitor.

b. Plot the frequency response of the circuit.

10.11 a. Determine the resistance values to obtain an op-amp PID controller with $K_P = 10$, $K_I = 1.4$, and $K_D = 4$. The circuit should limit frequencies above 100 rad/s. Take one capacitance to be 1 μF.

b. Plot the frequency response of the circuit.

Section 10.5 Control System Analysis

10.12 Obtain the steady-state response, if any, of the following models for the given input. If it is not possible to determine the response, state the reason.

a. $$\frac{Y(s)}{F(s)} = \frac{6}{7s + 3} \qquad f(t) = 14u_s(t)$$

b. $$\frac{Y(s)}{F(s)} = \frac{7s - 3}{10s^2 + 6s + 9} \qquad f(t) = 5u_s(t)$$

c. $\dfrac{Y(s)}{F(s)} = \dfrac{3s+5}{s^2-9} \qquad f(t) = 12u_s(t)$

d. $\dfrac{Y(s)}{F(s)} = \dfrac{4s+3}{s^2+2s-7} \qquad f(t) = 8u_s(t)$

10.13 For the following models, the error signal is defined as $e(t) = r(t) - c(t)$. Obtain the steady-state error, if any, for the given input. If it is not possible to determine the response, state the reason.

a. $\dfrac{C(s)}{R(s)} = \dfrac{1}{3s+1} \qquad r(t) = 6t$

b. $\dfrac{C(s)}{R(s)} = \dfrac{5}{3s+1} \qquad r(t) = 6t$

c. $\dfrac{C(s)}{R(s)} = \dfrac{4}{3s^2+5s+4} \qquad r(t) = 12t$

d. $\dfrac{C(s)}{R(s)} = \dfrac{10}{2s^2+4s+5} \qquad r(t) = 8t$

10.14 Given the model

$$3\ddot{x} - (3b+6)\dot{x} + (6b+15)x = 0$$

a. Find the values of the parameter b for which the system is
   1. Stable.
   2. Neutrally stable.
   3. Unstable.

b. For the stable case, for what values of b is the system
   1. Underdamped?
   2. Overdamped?

10.15 A certain system has the characteristic equation $s^3 + 9s^2 + 26s + K = 0$. Find the range of K values for which the system will be stable.

10.16 For the characteristic equation $s^3 + 9s^2 + 26s + K = 0$, use the Routh-Hurwitz criterion to compute the range of K values required so that the dominant time constant is no larger than $1/2$.

10.17 For the following characteristic equations, use the Routh-Hurwitz criterion to determine the range of K values for which the system is stable, where a and b are assumed to be known.

a. $2s^3 + 2as^2 + Ks + b = 0$

b. $5s^3 + 5as^2 + bs + 5K = 0$

c. $4s^3 + 12s^2 + 12s + 4 + K = 0$

10.18 The parameter values for a certain armature-controlled motor, load, and tachometer are

$$\begin{array}{lll}
K_T = K_b = 0.2\ \mathrm{N\cdot m/A} & & \\
c_m = 5\times 10^{-4}\ \mathrm{N\cdot m\cdot s/rad} & c_L = 2\times 10^{-3} & \\
R_a = 0.8\ \Omega & L_a = 4\times 10^{-3}\ \mathrm{H} & \\
I_m = 5\times 10^{-4} & I_t = 10^{-4} & I_L = 5\times 10^{-3}\ \mathrm{kg\cdot m^2} \\
N = 2 & K_a = 10\ \mathrm{V/V} & \\
K_{\mathrm{tach}} = 20\ \mathrm{V\cdot s/rad} & K_{\mathrm{pot}} = 10\ \mathrm{V/rad} & K_d = 2\ \mathrm{rad/(rad/s)}
\end{array}$$

For the control system whose block diagram is given by Figure 10.3.9, determine the value of the proportional gain K_P required for the load speed to be within 10% of the desired speed of 2000 rpm at steady state, and use the characteristic roots to evaluate the resulting transient response. For this value of K_P, evaluate the resulting steady-state deviation of the load speed caused by a load torque $T_L = 0.2\ \mathrm{N\cdot m}$.

Section 10.6 Controlling First-Order Plants

10.19 Suppose the plant shown in Figure 10.6.1 has the parameter values $I = 2$ and $c = 3$. Find the smallest value of the gain K_P required so that the steady-state offset error will be no greater than 0.2 if ω_r is a unit-step input. Evaluate the resulting time constant and steady-state response due to the disturbance if T_d is also a unit step.

10.20 Suppose the plant shown in Figure 10.6.1 has the parameter values $I = 2$ and $c = 3$. The command input and the disturbance are unit-ramp functions. Evaluate the response of the proportional controller with $K_P = 12$.

10.21 For the control system shown in Figure 10.6.2, $I = 20$, and suppose that only I action is used, so that $K_P = 0$. The performance specifications require the steady-state errors due to step command and disturbance inputs to be zero. Find the required gain value K_I so that $\zeta = 1$. Evaluate the resulting time constant. Do this for each of the following values of c:

a. $c = 10$

b. $c = 0.2$

10.22 Suppose that $I = c = 4$ for the PI controller shown in Figure 10.6.2. The performance specifications require that $\tau = 0.2$. (a) Compute the required gain values for each of the following cases.
1. $\zeta = 0.707$
2. $\zeta = 1$
3. A root separation factor of 10

(b) Use a computer method to plot the unit-step command responses for each of the cases in part (a). Compare the performance of each case.

10.23 For the designs obtained in part (a) of Problem 10.22, use a computer method to plot the actuator torque versus time. Compare the peak torque values for each case.

10.24 For the designs found in part (a) of Problem 10.22, evaluate the steady-state error due to a unit-ramp command and due to a unit-ramp disturbance.

10.25 Consider the PI speed control system shown in Figure 10.6.2, where $I = c = 2$. The desired time constant is $\tau = 0.1$. (a) Compute the required values of the gains for the following three sets of root locations.
1. $s = -10, -15$ (root separation factor is 1.5)
2. $s = -10, -20$ (root separation factor is 2)
3. $s = -10, -50$ (root separation factor is 5)

(b) Use a computer method to plot the response of the speed $\omega(t)$ for a unit-step command for each of the cases in part (a). Discuss the effects of the root separation factor on the rise time, the overshoot, and the maximum required torque.

10.26 Suppose that $I = c = 4$ for the I controller with internal feedback shown in Figure 10.6.6. The performance specifications require that $\tau = 0.2$.
(a) Compute the required gain values for each of the following cases.
1. $\zeta = 0.707$
2. $\zeta = 1$
3. A root separation factor of 10

(b) Use a computer method to plot the unit-step command responses for each of the cases in part (a). Compare the performance of each case.

10.27 For the designs obtained in part (a) of Problem 10.26, use a computer method to plot the actuator torque versus time. Compare the peak torque values for each case.

10.28 For the designs found in part (a) of Problem 10.26, evaluate the steady-state error due to a unit-ramp command and due to a unit-ramp disturbance.

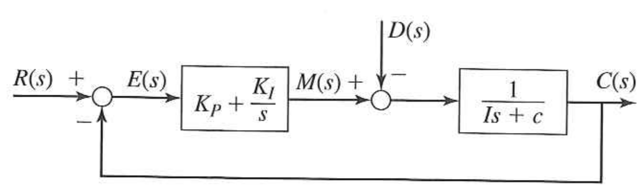
10.29 Consider the speed control system using I control with internal feedback shown in Figure 10.6.6, where $I = c = 2$. The desired time constant is $\tau = 0.1$.

a. Compute the required values of the gains for the following three sets of root locations.
   1. $s = -10, -8$ (root separation factor is $10/8 = 1.25$)
   2. $s = -10, -20$ (root separation factor is 2)
   3. $s = -10, -50$ (root separation factor is 5)

b. Use a computer method to plot the response of the speed $\omega(t)$ for a unit-step command for each of the cases in part (a). Discuss the effects of the root separation factor on the rise time, the overshoot, and the maximum required torque.

10.30 Modify the diagram shown in Figure P10.30 to include feedforward command compensation with a constant compensator gain K_f. Determine whether such compensation can eliminate steady-state error for step and ramp commands.

Figure P10.30



10.31 Suppose that $I = 10$ and $c = 5$ for the PI controller shown in Figure 10.6.2. The performance specifications require that $\tau = 2$. (a) Compute the required gain values for each of the following cases.
1. $\zeta = 0.707$
2. $\zeta = 1$
3. A root separation factor of 5

(b) Use a computer method to plot the unit-step command responses for each of the cases in part (a). Compare the performance of each case.

10.32 For the designs obtained in part (a) of Problem 10.31, use a computer method to plot the actuator torque versus time. Compare the peak torque values for each case.

10.33 For the designs found in part (a) of Problem 10.31, evaluate the steady-state error due to a unit-ramp command and due to a unit-ramp disturbance.

10.34 Consider the PI speed control system shown in Figure 10.6.2, where $I = 5$ and $c = 4$. The desired time constant is $\tau = 0.5$. (a) Compute the required values of the gains for the following three sets of root locations.
1. $s = -2, -20$ (root separation factor is 10)
2. $s = -2, -10$ (root separation factor is 5)
3. $s = -2, -4$ (root separation factor is 2)

(b) Use a computer method to plot the response of the speed $\omega(t)$ for a unit-step command for each of the cases in part (a). Discuss the effects of the root separation factor on the rise time, the overshoot, and the maximum required torque.

10.35 Suppose that $I = 15$ and $c = 5$ for the I controller with internal feedback shown in Figure 10.6.6. The performance specifications require that $\tau = 0.5$.
(a) Compute the required gain values for each of the following cases.
1. $\zeta = 0.707$
2. $\zeta = 1$
3. A root separation factor of 5

(b) Use a computer method to plot the unit-step command responses for each of the cases in part (a). Compare the performance of each case.

10.36 For the designs obtained in part (a) of Problem 10.35, use a computer method to plot the actuator torque versus time. Compare the peak torque values for each case.

10.37 For the designs found in part (a) of Problem 10.35, evaluate the steady-state error due to a unit-ramp command and due to a unit-ramp disturbance.

10.38 Consider the speed control system using I control with internal feedback shown in Figure 10.6.6, where $I = 15$ and $c = 5$. The desired time constant is $\tau = 0.5$.

a. Compute the required values of the gains for the following three sets of root locations.
   1. $s = -2, -20$ (root separation factor is 10)
   2. $s = -2, -10$ (root separation factor is 5)
   3. $s = -2, -4$ (root separation factor is 2)

b. Use a computer method to plot the response of the speed $\omega(t)$ for a unit-step command for each of the cases in part (a). Discuss the effects of the root separation factor on the rise time, the overshoot, and the maximum required torque.

Section 10.7 Controlling Second-Order Plants

10.39 Consider the PD control system shown in Figure 10.7.1. Suppose that $I = 20$ and $c = 10$. The specifications require the steady-state error due to a unit-step command to be zero and the steady-state error due to a unit-step disturbance to be no greater than 0.1 in magnitude. In addition, we require that $\zeta = 0.707$.

Compute the required values of the gains, and evaluate the resulting time constant.

10.40 Suppose that $I = 10$ and $c = 3$ in the PD control system shown in Figure 10.7.1. The performance specifications require that $\tau = 1$ and $\zeta = 0.707$. Compute the required gain values.

10.41 Figure 10.7.2 shows a system using proportional control with velocity feedback. Suppose that $I = 20$ and $c = 10$. The specifications require the steady-state error due to a unit-step command to be zero and the steady-state error due to a unit-step disturbance to be no greater than 0.1 in magnitude. In addition, we require that the time constant be $\tau = 0.1$. Compute the required values of the gains and evaluate the resulting damping ratio.

10.42 For the system discussed in Problem 10.40,

a. Use a computer method to plot the output $\theta(t)$ and the actuator response $T(t)$ for a unit-ramp command input.

b. Use a computer method to plot the disturbance frequency response. Determine the peak response and the bandwidth.

10.43 Suppose that $I = 10$ and $c = 3$ for the PID control system shown in Figure 10.7.3. The performance specifications require that $\tau = 1$ and $\zeta = 0.707$.

a. Compute the required gain values.

b. Use a computer method to plot the disturbance frequency response. Determine the peak response and the bandwidth.

10.44 Consider the PD control system shown in Figure 10.7.1. Suppose that $I = 20$ and $c = 10$. The specifications require the steady-state error due to a unit-step command to be zero and the steady-state error due to a unit-step disturbance to be no greater than 0.1 in magnitude. In addition, we require that the time constant be $\tau = 0.1$.

Compute the required values of the gains, and evaluate the resulting damping ratio.

10.45 Modify the PD system diagram shown in Figure 10.7.1 to include feedforward compensation with a compensator gain of K_f. Determine whether such compensation can reduce the steady-state error for step and ramp commands.

10.46 Consider a plant whose transfer function is $1/(20s + 0.2)$. The performance specifications are

1. The magnitude of the steady-state command error must be no more than 0.01 for a unit-ramp command.
2. The damping ratio must be unity.
3. The dominant time constant must be no greater than 0.1.

a. Select a control algorithm to meet the first specification, and compute the required values of its gains. What is the damping ratio that results? What is the time constant?

b. Select a control algorithm to meet the first two specifications, and compute the required values of its gains. Evaluate the resulting time constant.

c. Select a control algorithm to meet all three specifications.

10.47 For the system shown in Figure 10.7.1, $I = c = 1$. Derive the expressions for the steady-state errors due to a unit-ramp command and to a unit-ramp disturbance.

10.48 For the PD control system shown in Figure 10.7.1, $I = c = 2$. Compute the values of the gains K_P and K_D to meet all of the following specifications:

1. No steady-state error with a step input
2. A damping ratio of 0.9
3. A dominant time constant of 1

10.49 Consider the PID position control system shown in Figure 10.7.3, where $I = 10$ and $c = 2$. The desired time constant is $\tau = 2$.

a. Compute the required values of the gains for the following two sets of root locations.

1. $s = -0.5, s = -5 \pm 5j$
2. $s = -0.5, s = -1, s = -2$.

b. For both cases, use a computer method to plot the command response to a unit step. Discuss the effects of the root separation factor on the response. Compare the results with those of Example 10.7.4, where the roots are $s = -5$ and $s = -0.5 \pm 0.5j$.

c. For both cases, use a computer method to plot the disturbance frequency response. Discuss the effects of the root separation factor on the frequency response. Compare the results with those of Example 10.7.4.

10.50 Derive the expression for $T(s)$ in Figure 10.7.6. Using the values given and computed in Example 10.7.5, use MATLAB to plot $T(t)$ for a unit-step command input. Determine the maximum value of $T(t)$.

10.51 Integral control of the plant

$$G_p(s) = \frac{3}{5s + 1}$$

results in a system that is too oscillatory. Will D action improve this situation?

10.52 Modify the system diagram shown in Figure P10.52 to include feedforward compensation with a compensator gain K_f. Determine whether such compensation can reduce the steady-state error for step and ramp commands.

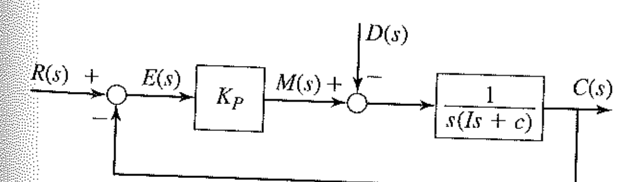


Figure P10.52

10.53 Consider the PD control system shown in Figure 10.7.1. Suppose that $I = 25$ and $c = 5$. The specifications require the steady-state error due to a unit-step command to be zero and the steady-state error due to a unit-step disturbance to be no greater than 0.2 in magnitude. In addition, we require that $\zeta = 0.707$. Compute the required values of the gains, and evaluate the resulting time constant.

10.54 Suppose that $I = 15$ and $c = 10$ in the PD control system shown in Figure 10.7.1. The performance specifications require that $\tau = 2$ and $\zeta = 0.707$. Compute the required gain values.

10.55 For the system discussed in Problem 10.54,

a. Use a computer method to plot the output $\theta(t)$ and the actuator response $T(t)$ for a unit-ramp command input.

b. Use a computer method to plot the disturbance frequency response. Determine the peak response and the bandwidth.

10.56 Suppose that $I = 15$ and $c = 5$ for the PID control system shown in Figure 10.7.3. The performance specifications require that $\tau = 2$ and $\zeta = 0.707$.

a. Compute the required gain values.

b. Use a computer method to plot the disturbance frequency response. Determine the peak response and the bandwidth.

10.57 Consider the PD control system shown in Figure 10.7.1. Suppose that $I = 15$ and $c = 3$. The specifications require the steady-state error due to a unit-step command to be zero and the steady-state error due to a unit-step disturbance to be no greater than 0.2 in magnitude. In addition, we require that the time constant be $\tau = 0.5$.

Compute the required values of the gains, and evaluate the resulting damping ratio.

10.58 For the PD control system shown in Figure 10.7.1, $I = 25$ and $c = 5$. Compute the values of the gains K_P and K_D to meet all of the following specifications:

1. No steady-state error with a step input
2. A damping ratio of 0.5
3. A dominant time constant of 4

Section 10.8 Additional Examples

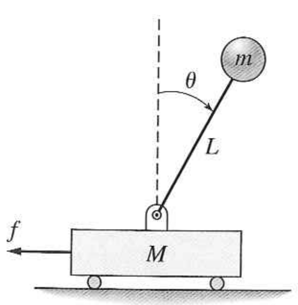
10.59 We need to stabilize the plant $3/(s^2 - 4)$ with a feedback controller. The closed-loop system should have a damping ratio of $\zeta = 0.707$ and a dominant time constant $\tau = 0.1$.

a. Use PD control and compute the required values of the gains.

b. Use P control with rate feedback and compute the required values of the gains.

c. Compare the unit-step command responses of the two designs.

Figure P10.60



10.60 The system shown in Figure P10.60 represents the problem of stabilizing the attitude of a rocket during takeoff or controlling the balance of a personal transporter. The applied force f represents that from the side thrusters of the rocket or the tangential force on the transporter wheels. For small angles, Newton's law for the system reduces to

$$ML\ddot{\theta} - (M + m)g\theta = f$$

where f is the control variable. Design a control law to maintain θ near zero. The specifications are $\zeta = 0.707$ and a 2% settling time of 8 sec. The parameter values are $M = 40$ slugs, $m = 8$ slugs, $L = 20$ ft, and $g = 32.2$ ft/sec^2.

10.61 Figure P10.61 shows PD control applied to an unstable plant. The gains have been computed so that the damping ratio is $\zeta = 0.707$ and the time constant is 2.5 sec, assuming that the transfer functions of the actuator and the feedback sensor are unity. Suppose that the actuator has the transfer function

$$G_a(s) = \frac{1}{\tau s + 1}$$

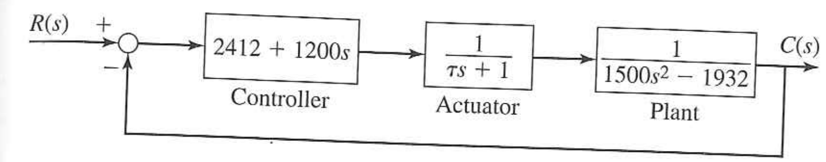


Figure P10.61

This lag in the response of the actuator might affect the system's stability, its overshoot in response to a step input, or its 2% settling time.

a. What effect does this lag have on the system's performance if $\tau = 0.1$ sec?

b. What effect does this lag have on the system's performance if $\tau = 1$ sec?

10.62 Figure P10.62 shows PD control applied to an unstable plant. The gains have been computed so that the damping ratio is $\zeta = 0.707$ and the time constant is 2.5 sec, assuming that the transfer functions of the actuator and the feedback sensor are unity. Suppose that the feedback sensor has the transfer function

$$G_s(s) = \frac{1}{\tau s + 1}$$

This lag in the response of the feedback elements might affect the system's stability, its overshoot in response to a step input, or its 2% settling time.

a. What effect does this lag have on the system's performance if $\tau = 0.1$ sec?

b. What effect does this have on the system's performance if $\tau = 1$ sec?

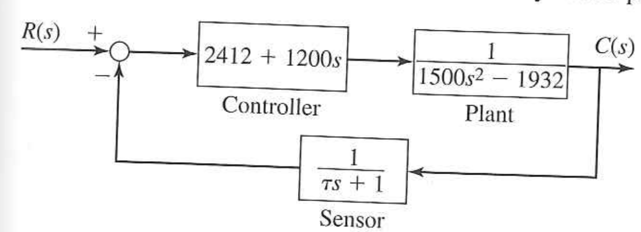
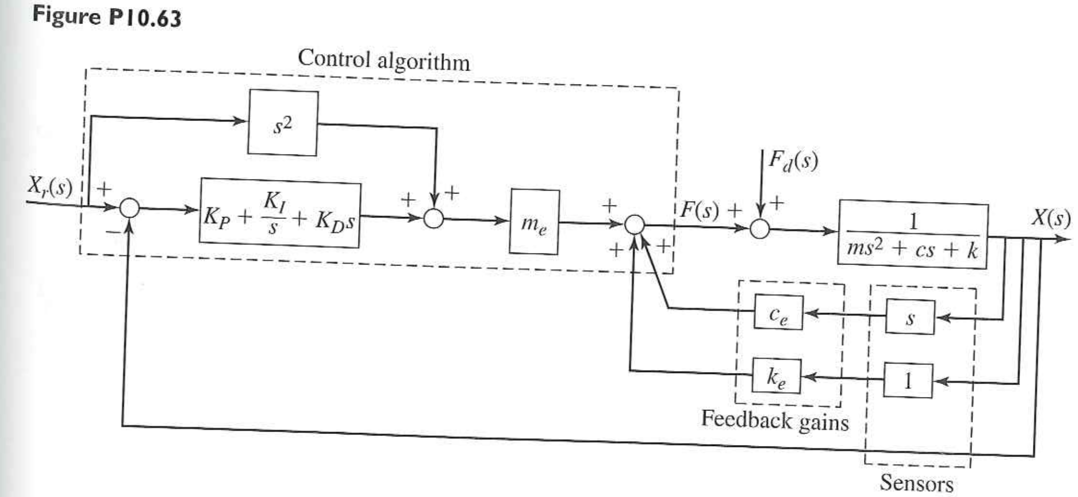


Figure P10.62

10.63 Figure P10.63 shows a proposed scheme for controlling the position of a mechanical system such as a link in a robot arm. It uses two feedback loops—one for position and one for velocity—and a feedforward compensator transfer function s^2.

a. Suppose that the estimates of the mass, damping, and stiffness are accurate so that $m_e = m$, $c_e = c$, and $k_e = k$. Derive the expressions for the transfer

Figure P10.63



functions $X(s)/X_r(s)$ and $X(s)/F_d(s)$. How well does this control scheme work? Obtain the expressions for the PID gains to achieve a damping ratio of $\zeta = 1$ and a closed-loop time constant of specified value τ_d.

b. Discuss any practical limitations to this scheme.

10.64 Refer to Figure 10.3.9, which shows a speed control system using an armature-controlled dc motor. The motor has the following parameter values.

$K_b = 0.199$ V-sec/rad $\quad R_a = 0.43\ \Omega$
$K_T = 0.14$ lb-ft/A $\quad c_e = 3.6 \times 10^{-4}$ lb-ft-sec/rad
$I_e = 2.08 \times 10^{-3}$ slug-ft^2 $\quad L_a = 2.1 \times 10^{-3}$ H
$N = 1$

a. Compute the time constants of the plant transfer function $\Omega_L(s)/V_m(s)$.

b. Modify Figure 10.3.9 to use PI control instead of P control. Compute the PI control gains required to give a response having a dominant time constant of no less than 0.05 sec and a dominant damping ratio in the range $0.5 \le \zeta \le 1$.

Section 10.9 MATLAB Applications

10.65 Using the value of K_P computed in Problem 10.18, obtain a plot of the current versus time for a step-command input of 209.4 rad/s (2000 rpm).

10.66 Consider Example 10.6.3. Modify the diagram in Figure 10.6.2 to show an actuator transfer function $T(s)/M(s) = 1/(0.1s + 1)$. Use the same gain values computed for the three cases in that example.

a. Use MATLAB to plot the command response and the actuator response to a unit-step command. Identify the peak actuator values for each case.

b. Use MATLAB to plot the disturbance frequency response.

c. Compare the results in parts (a) and (b) with those of Example 10.6.3.

10.67 Consider Example 10.6.3. Use the same gain values computed for the three cases in that example.

a. Use MATLAB to plot the command response and the actuator response to the modified unit-step command $r(t) = 1 - e^{-20t}$. Identify the peak actuator values for each case.

b. Compare the results in part (a) with those of Example 10.6.3.

10.68 Consider Example 10.6.4. Modify the diagram in Figure 10.6.6 to show an actuator transfer function $T(s)/M(s) = 1/(0.1s + 1)$. Use the same gain values computed for the three cases in that example.

a. Use MATLAB to plot the command response and the actuator response to a unit-step command. Identify the peak actuator values for each case.

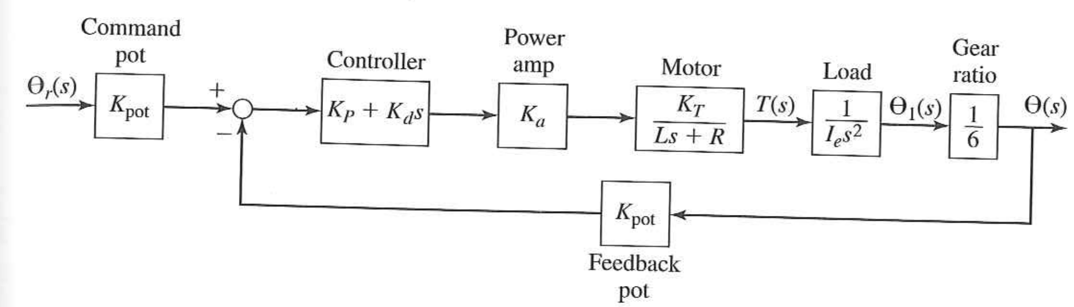
b. Use MATLAB to plot the disturbance frequency response.

c. Compare the results in parts (a) and (b) with those of Example 10.6.4.

10.69 Figure P10.7 shows a system for controlling the angular position of a load, such as an antenna. Figure P10.69 shows the block diagram for PD control of this system using a field-controlled motor. Use the following values:

$K_a = 1$ V/V $\quad R = 0.3\ \Omega \quad K_T = 0.6$ N·m/A
$K_{pot} = 2$ V/rad $\quad I_1 = 0.01$ kg·m^2
$I_2 = 5 \times 10^{-4}$ kg·m^2 $\quad I_3 = 0.2$ kg·m^2

Figure P10.69



The inertia I_e in the block diagram is the equivalent inertia of the entire system, as felt on the *motor* shaft.

a. Assume that the motor inductance is very small and set $L = 0$. Compute I_e, obtain the transfer function $\Theta(s)/\Theta_r(s)$, and compute the values of the control gains K_P and K_d to meet the following specifications: $\zeta = 1$ and $\tau = 0.5$ s.

b. Use the MATLAB `tf` function to create the model `sys1` from this transfer function.

c. Using the values of K_P and K_d computed in part (a), and the value $L = 0.015$ H, obtain the transfer function $\Theta(s)/\Theta_r(s)$ and use the MATLAB `tf` function to create the model `sys2` from this transfer function.

d. Use the MATLAB `step(sys1,sys2)` function to plot the unit step response of both transfer functions. Right-click on the plots to obtain the maximum percent overshoot and settling time for each. How close are the two responses? What is the effect of neglecting the inductance?

10.70 Consider the P, PI, and modified I control systems discussed in Examples 10.6.2, 10.6.3, and 10.6.4. The plant transfer function is $1/(Is + c)$, where $I = 10$ and $c = 3$. Investigate the performance of these systems for a trapezoidal command input having a slew speed of 1 rad/s, an acceleration time of 4 s, a slew time of 6 s, a deceleration time of 4 s, and a rest time of 5 s.

10.71 A speed control system using an armature-controlled motor with proportional control action was discussed in Section 10.3. Its block diagram is shown in Figure 10.3.8 with a simplified version given in Figure 10.3.9. The given parameter values for a certain motor, load, and tachometer are

$K_T = K_b = 0.04$ N·m/A
$c_m = 0 \quad c_L = 10^{-3}$ N·m·s/rad
$R_a = 0.6\ \Omega \quad L_a = 2 \times 10^{-3}$ H
$I_m = 2 \times 10^{-5} \quad I_t = 10^{-5} \quad I_L = 4 \times 10^{-3}$ kg·m^2
$N = 1.5 \quad K_a = 5$ V/V
$K_{tach} = 10$ V/(rad/s) $\quad K_{pot} = 5$ V/rad $\quad K_d = 2$ rad/(rad/s)

where the subscript m refers to the motor, L refers to the load, and t refers to the tachometer. (a) Determine the value of the proportional gain K_P required for the load speed to be within 10% of the desired speed of 1000 rpm at steady-state, and plot the resulting transient response. (b) For this value of

K_P, plot the resulting deviation of the load speed caused by a load torque $T_L = 1$ N · m.

10.72 Consider the control system of Problem 10.71. Use MATLAB to evaluate the following performance measures: energy consumption, maximum current, maximum speed error, rms current, and rms speed error.

Section 10.10 Simulink Applications

10.73 Consider Example 10.6.3. Use the diagram in Figure 10.6.2 to create a Simulink model. Modify the model to use an actuator saturation with the limits 0 and 20. Use the same gain values computed in that example for the three cases.

a. Plot the command response and the actuator response to a unit-step command.

b. Compare the results in part (a) with those of Example 10.6.3.

10.74 Consider Example 10.7.4. Use the diagram in Figure 10.7.3 to create a Simulink model using the same gain values computed in that example. Set the initial position to 3. Plot the command response to a unit-step command and compare the results with those of Example 10.7.4.

10.75 Consider Example 10.7.4. Use the diagram in Figure 10.7.3 to create a Simulink model. Modify the model to use an actuator saturation with the limits 0 and 20. Use the same gain values computed in that example.

a. Plot the command response and the actuator response to a unit-step command.

b. Compare the results in part (a) with those of Example 10.7.4.

10.76 Consider Example 10.7.4. Use the diagram in Figure 10.7.3 to create a Simulink model. Modify the model to use an actuator transfer function $G_a(s) = 1/(0.2s + 1)$. Use the same gain values computed in that example.

a. Plot the command response and the actuator response to a unit-step command.

b. Compare the command response with that of Example 10.7.4.

10.77 Refer to Figure 10.3.9, which shows a speed control system using an armature-controlled dc motor. The motor has the following parameter values. Create a Simulink model by modifying Figure 10.3.9 to use PI control instead of P control. Use the PI control gains computed in Problem 10.50 part (b).

$K_b = 0.199$ V-sec/rad $R_a = 0.43\ \Omega$
$K_T = 0.14$ lb-ft/A $c_e = 3.6 \times 10^{-4}$ lb-ft-sec/rad
$I_e = 2.08 \times 10^{-3}$ slug-ft^2 $L_a = 2.1 \times 10^{-3}$ H
$N = 1$

a. Run the simulation using a unit-step command starting at $t = 0$ and a unit-step disturbance starting at $t = 4$ sec. Plot the speed and the motor current versus time.

b. Run the simulation for $0 \le t \le 2$ sec using a unit-ramp command. Plot the speed error and the motor current versus time.

10.78 For the system in Problem 10.77 part (a), create a Simulink model that has a current limiter of ± 10 A. Run the simulation for a step-command input of 104.7 rad/s (1000 rpm). Plot the current and the speed.

CHAPTER 11

Control System Design and the Root Locus Plot

CHAPTER OUTLINE

CHAPTER OBJECTIVES

When you have finished this chapter, you should be able to

1. Sketch the root locus plot for lower-order models, and use MATLAB to obtain the plot for higher-order models.
2. Interpret and use the root locus plot to determine the location of dominant roots and roots having desired properties such as damping ratio and time constant.
3. Determine the major features of a root locus plot, and use it to assess the effectiveness of a proposed control scheme.
4. Use the Ziegler-Nichols methods to design controllers.
5. Design a control system to avoid actuator saturation.
6. Design a controller incorporating state-variable feedback.
7. Apply MATLAB and Simulink to analyze and design control systems using the concepts presented in this chapter.

Chapter 10 introduced the basic concepts of feedback control. It showed how to choose an appropriate control action for a first- or second-order plant, and how to compute the control gains required to meet a simple set of performance specifications. This chapter shows how the root locus plot can be used to develop a more systematic approach to designing a control system. Such an approach is usually needed when the plant order is greater than two or where it is not clear how to select the gains to meet the performance specifications.