ME 460/660 Sp '11 Exam 2 Solutions

1) $H(jw) = \dfrac{1}{25000 - 10w^2 + 0.1jw}$, $w = \dfrac{2\pi n}{T} = \dfrac{2\pi n}{2\pi} 10 = 10n$

n	w	a_n	$\lvert H(jw) \rvert$	$\angle H(jw)$ (rad)	$\lvert X_n \rvert$
2	20	$\frac{1}{2}$ (0.5)	4.76×10^{-5}	-8×10^{-5} (0)	2.38×10^{-5}
4	40	$\frac{1}{4}$ (0.25)	1.11×10^{-4}	-4×10^{-4} (0)	2.78×10^{-5}
6	60	$\frac{1}{6}$ ($0.16\overline{16}$)	9.09×10^{-5}	-3.14	1.5×10^{-5}
8	80	$\frac{1}{8}$ (0.125)	2.56×10^{-5}	-3.14	3.2×10^{-6}
10	100	$\frac{1}{10}$ (0.1)	1.33×10^{-5}	-3.14	1.33×10^{-6}

$x(t) = 2.38\times10^{-5}\cos 20t + 2.78\times10^{-5}\cos 40t - 1.5\times10^{-5}\cos 60t$
$\quad - 3.2\times10^{-6}\cos 80t - 1.33\times10^{-6}\, 100t$

There is no good answer for a single term.

$2.78\times10^{-5}\cos 40t$ is the best, but insufficient to meet the intent of the request.

2) $x(t) = \int_0^t \frac{1}{m\omega_d} e^{-\zeta\omega_n(t-\tau)} \sin\omega_d(t-\tau)\, F(\tau)\, d\tau$

$\zeta = 0,\ \omega_d = \omega_n$

$x(t) = 0 = \int_0^T \frac{1}{m\omega_n} \sin\omega_n(t-\tau)\, d\tau \qquad (F(\tau) = 1 \quad 0 < t < T)$

$0 = \frac{1}{m\omega_n^2} \cos\omega_n(t-\tau) \Big|_0^T$

$0 = \cos\omega_n(t-T) - \cos\omega_n t$

If $\omega_n T = 2\pi n$ where $n = 1, 2, 3 \ldots$ then
$\cos\omega_n(t-T) = \cos\omega_n t$

$T = \frac{2\pi}{\omega_n} = 0.126$ sec

3) $T = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2$

$U = \frac{1}{2} K_1 X_1^2 + \frac{1}{2} k_2 (x_2 - x_1)^2 + \frac{1}{2} k_3 X_2^2$

$R = \frac{1}{2} C_1 \dot{X}_1^2 + \frac{1}{2} C_2 (\dot{x}_2 - \dot{x}_1)^2 + \frac{1}{2} C_3 \dot{x}_2^2$

For DOF 1:

$\frac{d}{dt} \frac{\partial T}{\partial \dot{x}_1} = m_1 \ddot{x}_1$

$\frac{\partial U}{\partial x_1} = (K_1 + k_2) x_1 - k_2 X_2$

$\frac{\partial R}{\partial \dot{x}_1} = (C_1 + C_2) \dot{X}_1 - C_2 \dot{X}_2$

$$\boxed{m_1 \ddot{x}_1 + (C_1 + C_2) \dot{X}_1 - C_2 \dot{X}_2 + (K_1 + K_2) X_1 - K_2 X_2 = 0}$$

For DOF 2:

$\frac{d}{dt} \frac{\partial T}{\partial \dot{x}_2} = m_2 \ddot{X}_2$

$\frac{\partial U}{\partial x_2} = (k_2 + k_3) X_2 - k_2 X_1$

$\frac{\partial R}{\partial \dot{x}_2} = (C_2 + C_3) \dot{x}_2 - C_2 \dot{x}_1$

$$\boxed{m_2 \ddot{X}_2 + (C_2 + C_3) \dot{x}_2 - C_2 \dot{x}_1 + (k_2 + k_3) \dot{x}_2 - k_2 \dot{x}_1 = 0}$$

4) $\frac{EI}{\rho A} w'''' + \ddot{w} = 0$

$w(x,t) = X(x)\,T(t)$ $\qquad\qquad$ $\ddot{w} = -\omega^2 X(x)\,T(t)$

$\frac{EI}{\rho A \omega^2} X'''' = X$

$X'''' = \sigma^4 X$ where $\sigma = \frac{\rho A \omega^{\frac{1}{2}}}{EI}$

$X(x) = A \sin\sigma x + B\cos\sigma x + C\sinh\sigma x + D\cosh\sigma x$

$X(0) = 0 = B + D$ $\qquad\qquad$ $D = -B$

$X'(0) = 0 = A + C$ $\qquad\qquad$ $C = -A$

$X(x) = A\sin\sigma x + B\cos\sigma x - A\sinh\sigma x - B\cosh\sigma x$

$X(l) = 0 = A\left(\sin\sigma l - \sinh\sigma l\right) + B\left(\cos\sigma l - \cosh\sigma l\right)$

$X'(l) = 0 = \left[A\left(\cos\sigma l - \cosh\sigma l\right) + B\left(-\sin\sigma l - \sinh\sigma l\right)\right]\sigma$

$\sigma\left[\left(\sin\sigma l - \sinh\sigma l\right)\left(-\sin\sigma l - \sinh\sigma l\right) - \left(\cos\sigma l - \cosh\sigma l\right)\left(\cos\sigma l - \cosh\sigma l\right)\right]$
$= 0$

$\sigma = 0$ (would mean $\omega = 0$)

$\underbrace{-\sin^2\sigma l + \sinh^2\sigma l - \left[\cos^2\sigma l - 2\cos\sigma l\cosh\sigma l + \cosh^2\sigma l\right]}_{-1 \qquad -1} = 0$

$-2 = -2\cos\sigma l\cosh\sigma l$

$\underline{\cos\sigma l\cosh\sigma l = 1}$ $\qquad$ (σ is called β in text)

Since

$$X(l)=0 = A(\sin\sigma l-\sinh\sigma l)+B(\cos\sigma l-\cosh\sigma l)$$

$$\frac{A}{B}=\frac{-\cos\sigma l+\cosh\sigma l}{\sin\sigma l-\sinh\sigma l} \qquad (\text{called } -\sigma \text{ in text})$$

$$\underline{\omega_n=\sigma^2\sqrt{\frac{EI}{\rho A}}}$$

$$X(x)=(\cos\sigma x-\cosh\sigma x)+\frac{A}{B}(\sin\sigma x-\sinh\sigma x)$$