Chapter 6: Temporal Difference Learning

Objectives of this chapter:

- Introduce Temporal Difference (TD) learning
- Focus first on policy evaluation, or prediction, methods
- Compare efficiency of TD learning with MC learning
- Then extend to control methods

TD methods bootstrap and sample

- Bootstrapping: update involves an estimate of the value function
  - TD and DP methods bootstrap
  - MC methods **do not** bootstrap
- Sampling: update **does not** involve an expected value
  - TD and MC method sample
  - Classical DP **does not** sample

TD Prediction

Policy Evaluation (the prediction problem):

for a given policy π, compute the state-value function v_π

Recall: Simple every-visit Monte Carlo method:

$$V(S_t) \leftarrow V(S_t) + \alpha\Big[G_t - V(S_t)\Big]$$

target: the actual return after time *t*

The simplest temporal-difference method TD(0):

$$V(S_t) \leftarrow V(S_t) + \alpha\Big[R_{t+1} + \gamma V(S_{t+1}) - V(S_t)\Big]$$

target: an estimate of the return

TD target for prediction

- The TD target: $R_{t+1} + \gamma v_\pi(S_{t+1})$
  - it is an *estimate* like MC target because it **samples** the expected value
  - it is an *estimate* like the DP target because it uses the current estimate of V instead of v_π

Tabular TD(0) for estimating v_π

Input: the policy π to be evaluated
Initialize $V(s)$ arbitrarily (e.g., $V(s) = 0$, for all $s \in \mathcal{S}^+$)
Repeat (for each episode):
    Initialize S
    Repeat (for each step of episode):
        $A \leftarrow$ action given by π for S
        Take action A, observe R, S'
        $V(S) \leftarrow V(S) + \alpha\big[R + \gamma V(S') - V(S)\big]$
        $S \leftarrow S'$
    until S is terminal

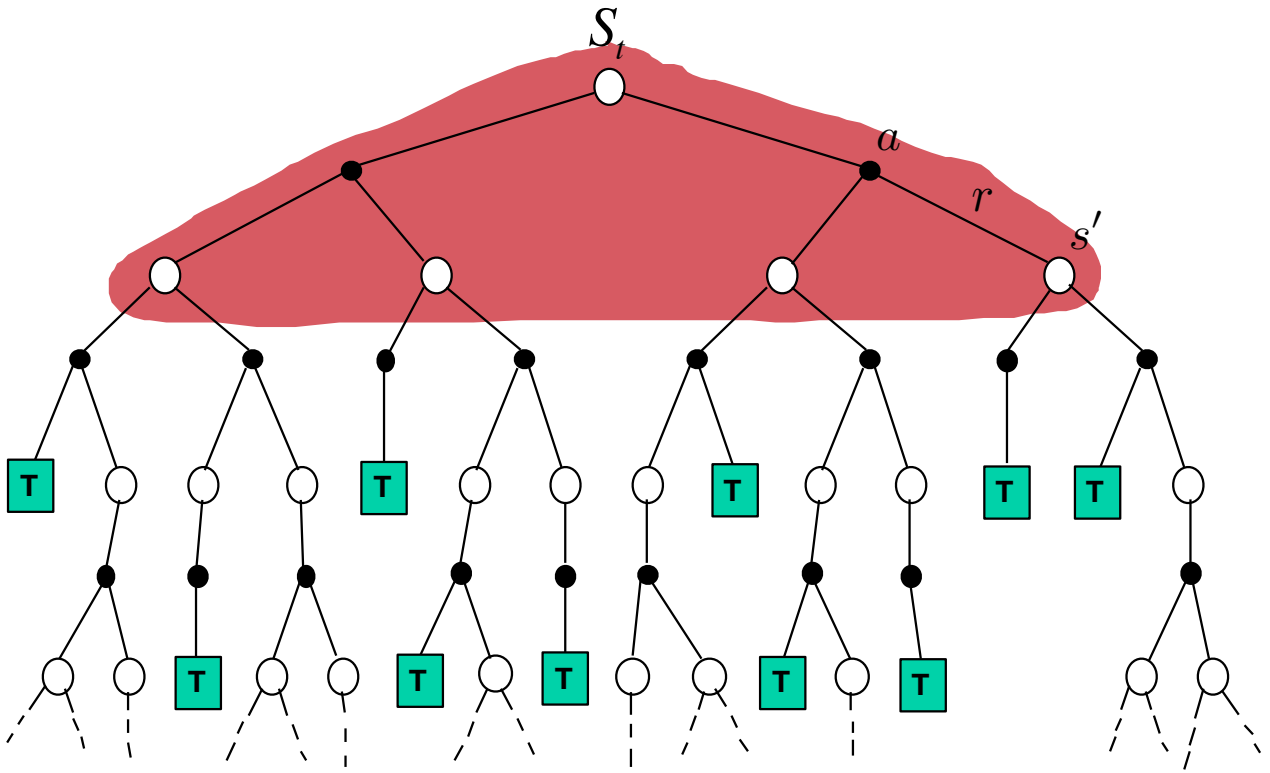
Agent program

Environment program

Experiment program

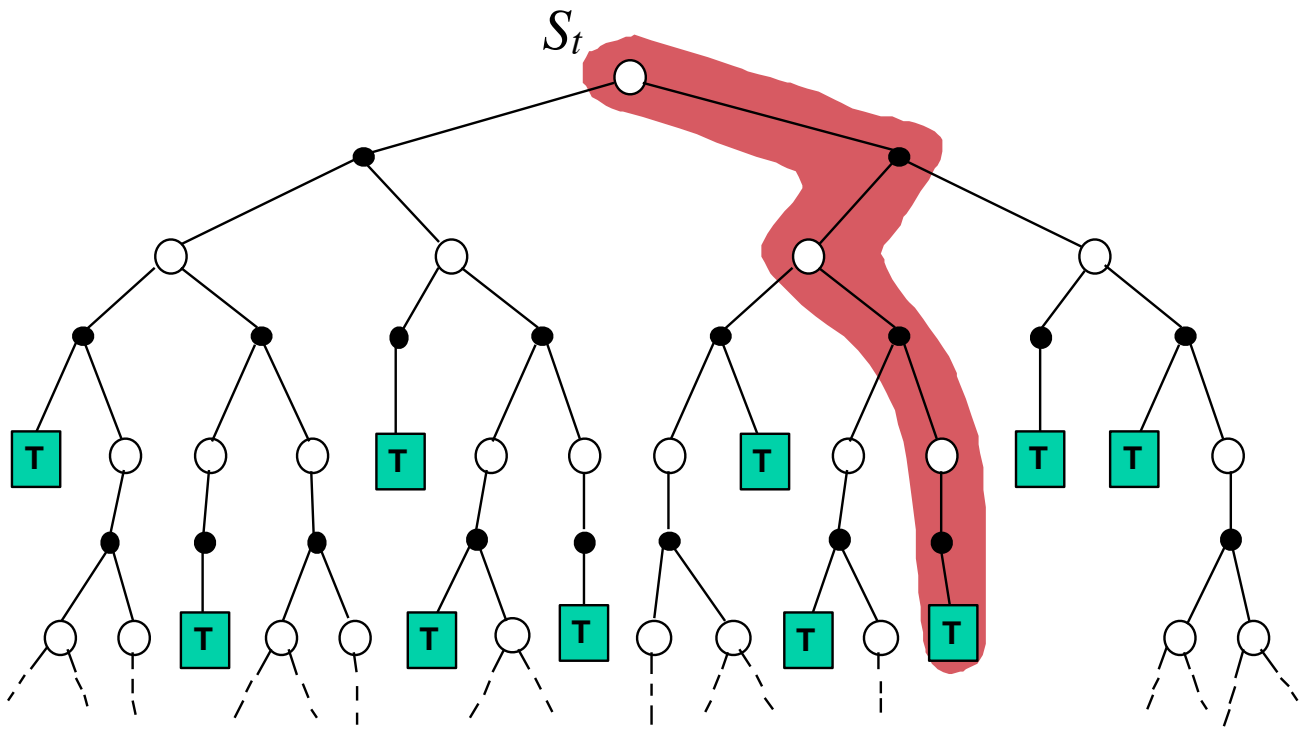
Dynamic programing

$$V(S_t) \leftarrow E_\pi\left[R_{t+1} + \gamma V(S_{t+1})\right] = \sum_a \pi(a|S_t) \sum_{s',r} p(s', r|S_t, a)[r + \gamma V(s')]$$



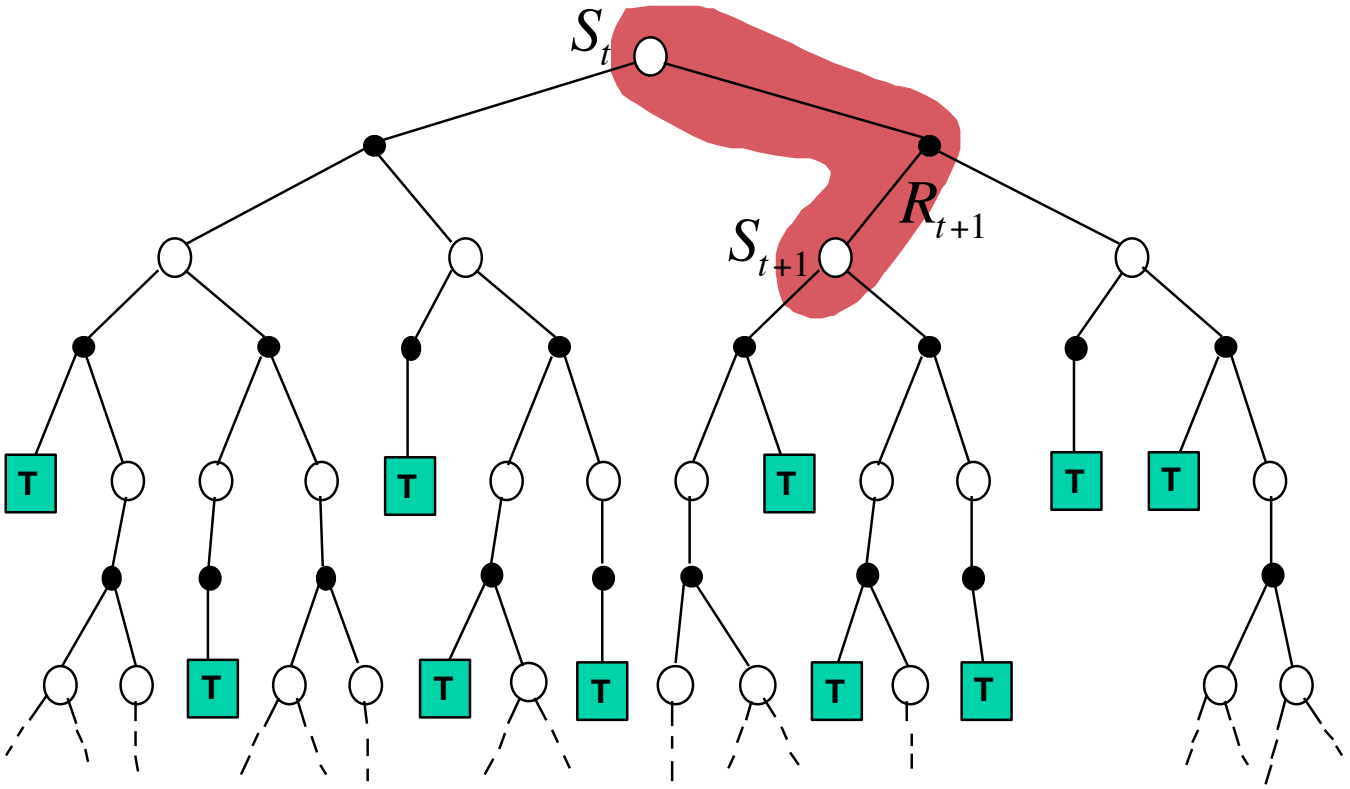
Simple Monte Carlo

$$V(S_t) \leftarrow V(S_t) + \alpha\left[G_t - V(S_t)\right]$$



Simplest TD method

$$V(S_t) \leftarrow V(S_t) + \alpha\left[R_{t+1} + \gamma V(S_{t+1}) - V(S_t)\right]$$



Example: Driving Home

- Consider driving home:
  - each day you drive home
  - your goal is to try and predict how long it will take at particular stages
  - when you leave office you note the time, day, & other relevant info
- Consider the policy evaluation or prediction task

Driving Home

State	*Elapsed Time (minutes)*	*Predicted Time to Go*	*Predicted Total Time*
leaving office, friday at 6	0	30	30
reach car, raining	5	35	40
exiting highway	20	15	35
2ndary road, behind truck	30	10	40
entering home street	40	3	43
arrive home	43	0	43

Driving home as an RL problem

- Rewards = 1 per step (if we were minimizing travel time what would reward be?)
- $\gamma = 1$
- G_t = time to go from state S_t
- $V(S_t)$ = expected time to get home from S_t

Updating our predictions

- Goal: update the prediction of total time leaving from office, while driving home
- With MC we would need to wait for a termination—until we get home—then calculate G_t for each step of episode, then apply our updates

Driving home

State	Elapsed Time (minutes)	R	V(s) Predicted Time to Go	V(office) Predicted Total Time
leaving office, friday at 6	0	5	30	30
reach car, raining	5	15	35	40
exiting highway	20	10	15	35
2ndary road, behind truck	30	10	10	40
entering home street	40	3	3	43
arrive home	43		0	43

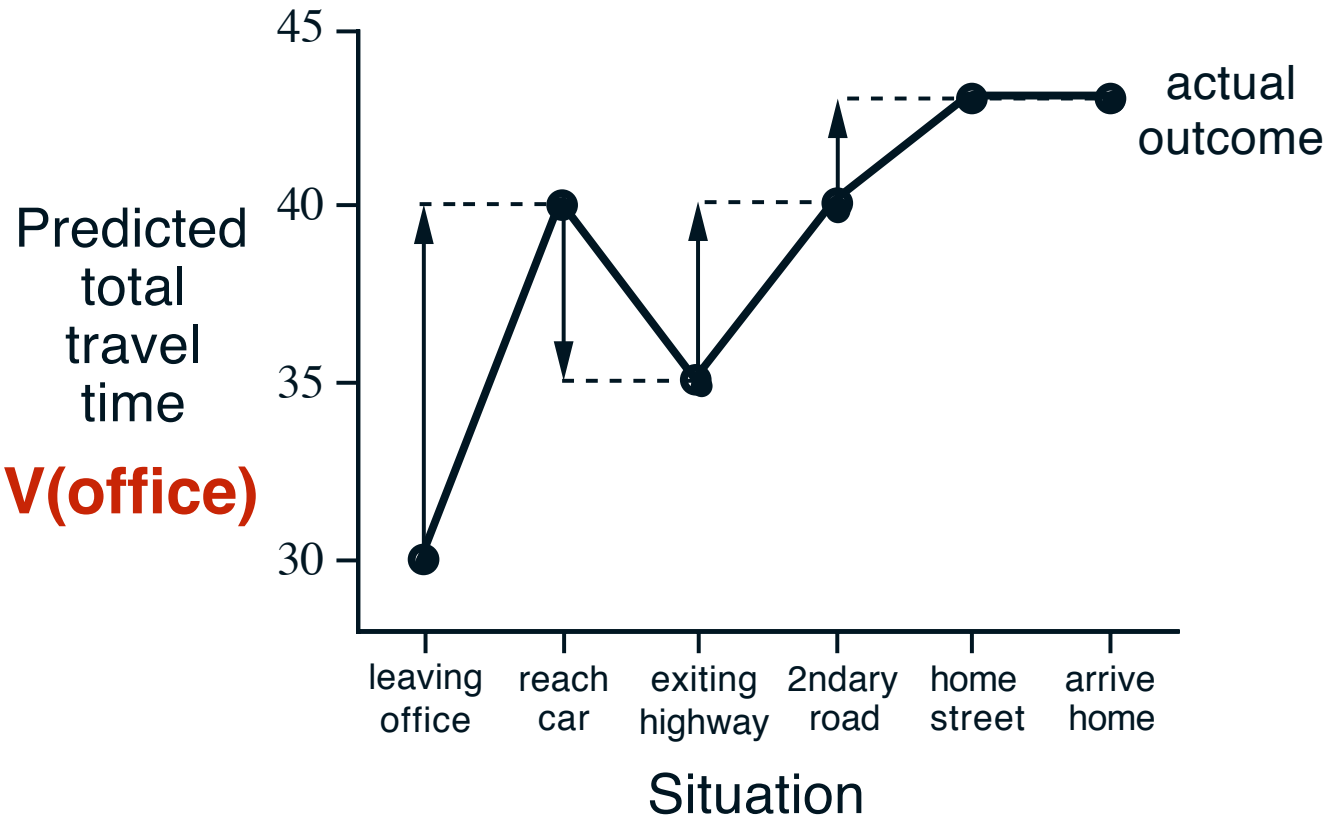
- Task: update the value function as we go, based on observed elapsed time—Reward column

Driving home

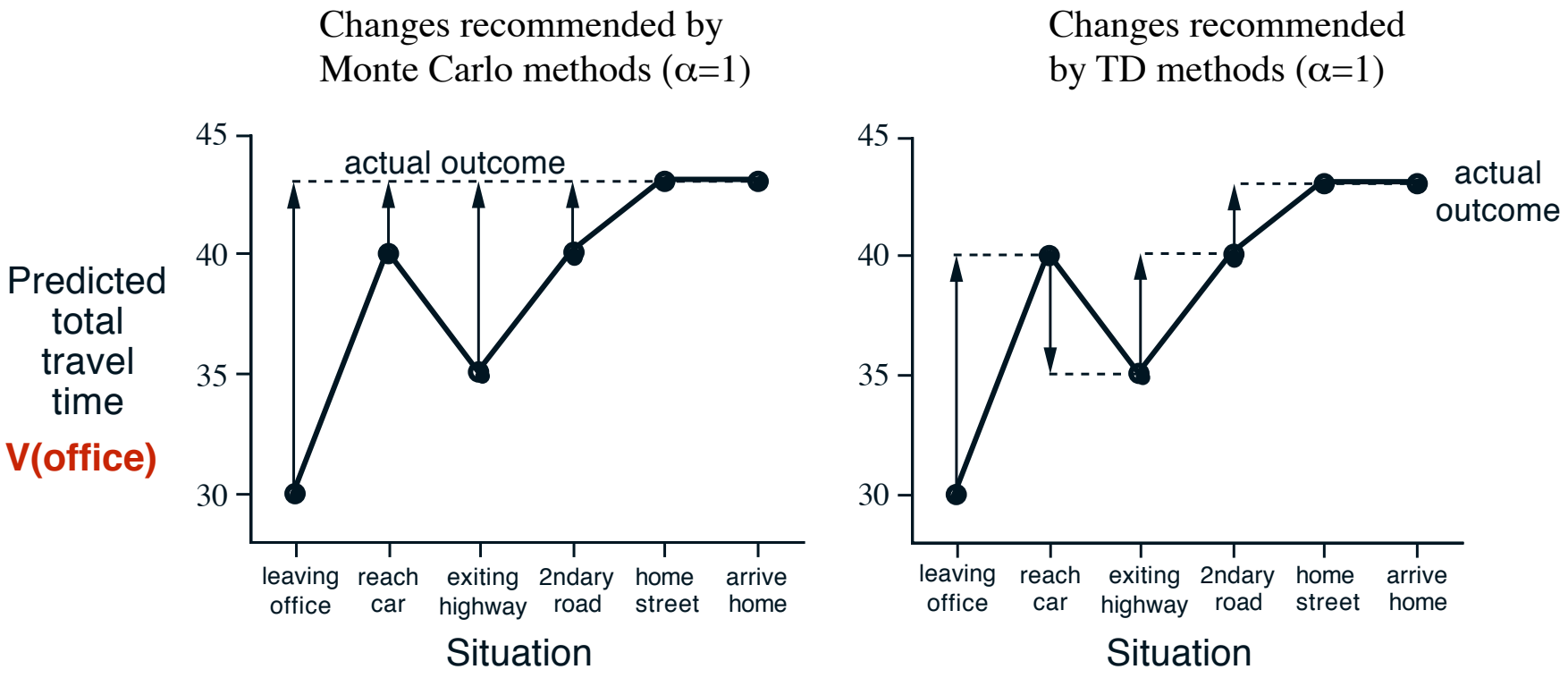
State	Elapsed Time (minutes)	R	V(s) Predicted Time to Go	V(office) Predicted Total Time
leaving office, friday at 6	0	5	30	30
reach car, raining	5	15	35	40
exiting highway	20	10	15	35
2ndary road, behind truck	30	10	10	40
entering home street	40	3	3	43
arrive home	43		0	43

- update V(office) with $\alpha = 1$?
  - $V(s) = V(s) + \alpha[R_{t+1} + \gamma V(s') - V(s)]$
  - $V(office) = V(office) + \alpha[R_{t+1} + \gamma V(car) - V(office)]$
  - new V(office) = 40; $\Delta = +10$
- update V(car)?
  - V(car) = 30; $\Delta = -5$
- update V(exit)?
  - V(exit) = 20; $\Delta = +5$

Changes recommended by TD methods ($\alpha = 1$)



Driving Home



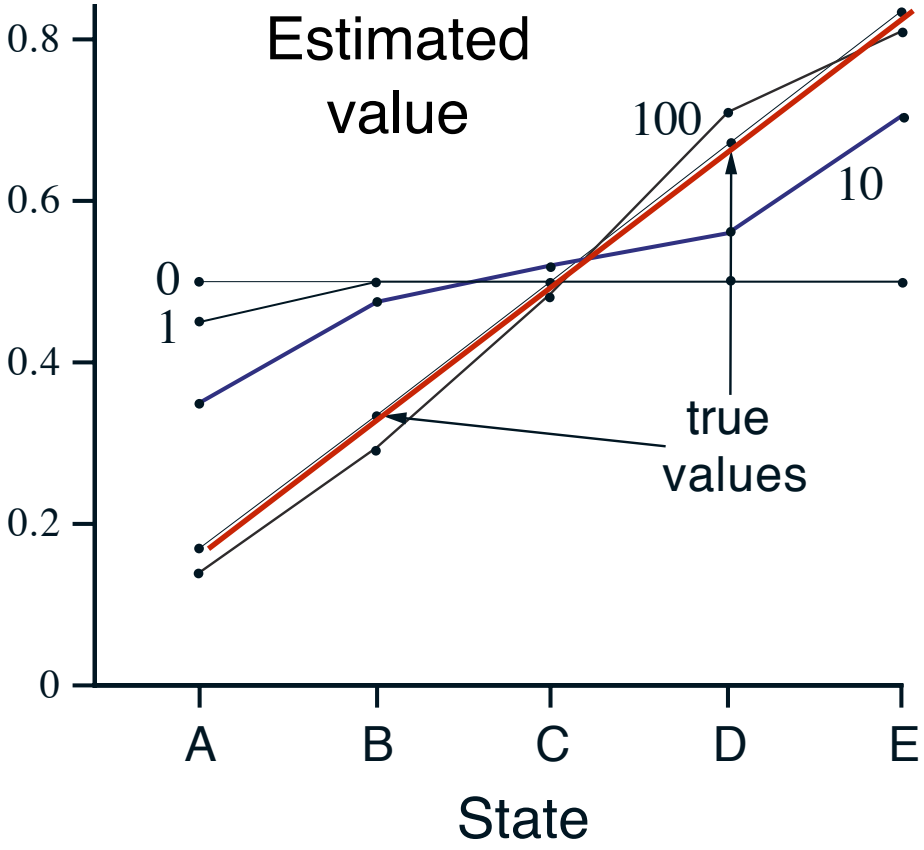
Advantages of TD learning

- TD methods do not require a model of the environment, only experience
- TD methods can be fully incremental
  - Make updates **before** knowing the final outcome
  - Requires less memory
  - Requires less peak computation
- You can learn **without** the final outcome, from incomplete sequences
- Both MC and TD converge (under certain assumptions to be detailed later), but which is faster?

Random walk

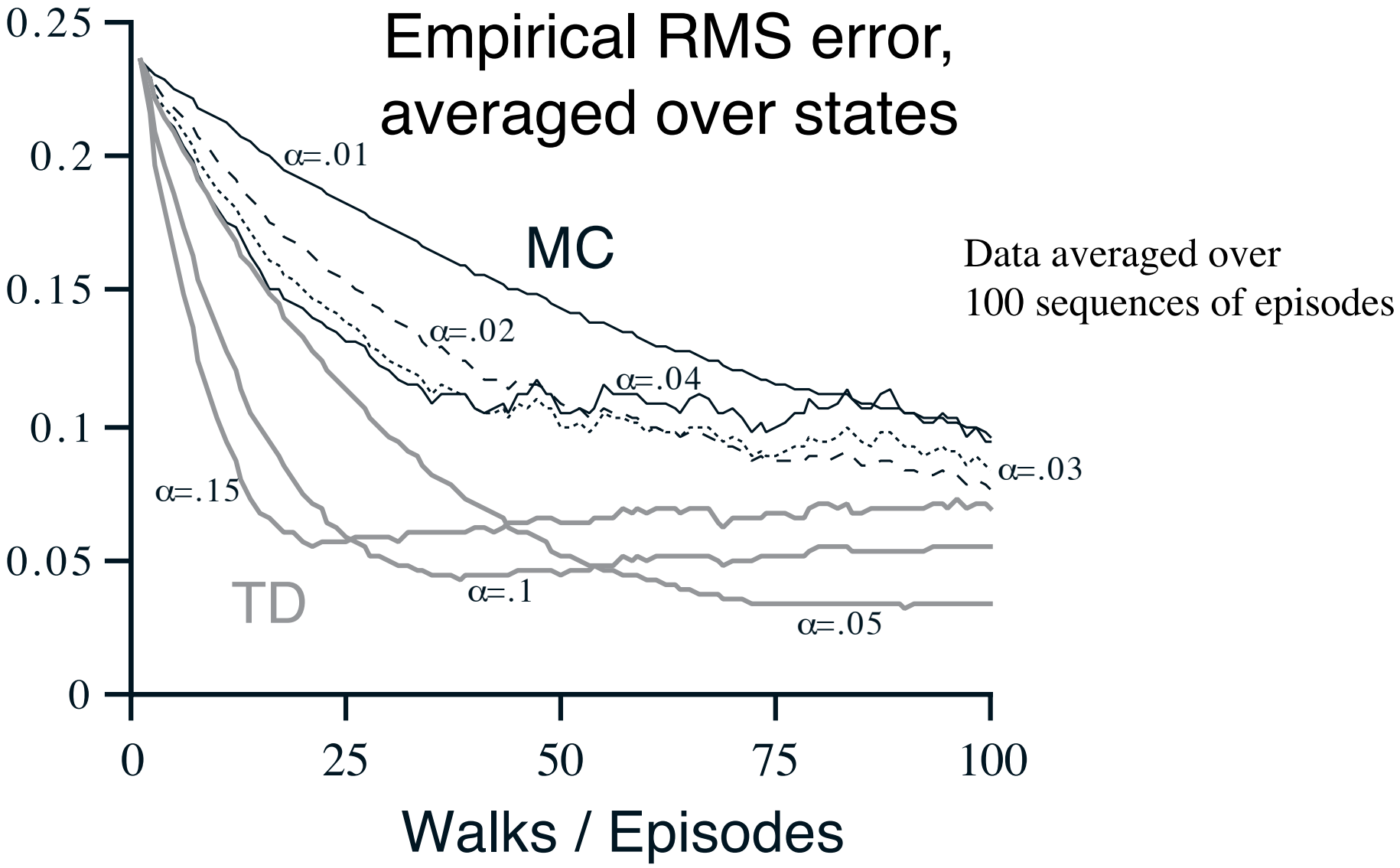
- C is start state, episodic, undiscounted γ = 1
- π is left or right with equal probability in all states
- termination at either end
- rewards +1 on **right** termination, 0 otherwise
- what does $v_\pi(s)$ tell us?
  - probability of termination on right side from each state, under random policy
  - what is v_π =[A B C D E]?
    - v_π = [1/6 2/6 3/6 4/6 5/6]
- Initialize V(s) = 0.5 $\forall\ s\ \in\ \mathcal{S}$

Values learned by TD from one run, after various numbers of episodes



$$V(S_t) \leftarrow V(S_t) + \alpha\Big[R_{t+1} + \gamma V(S_{t+1}) - V(S_t)\Big]$$

TD and MC on the Random Walk



Batch Updating in TD and MC methods

Batch Updating: train completely on a finite amount of data, e.g., train repeatedly on 10 episodes until convergence.

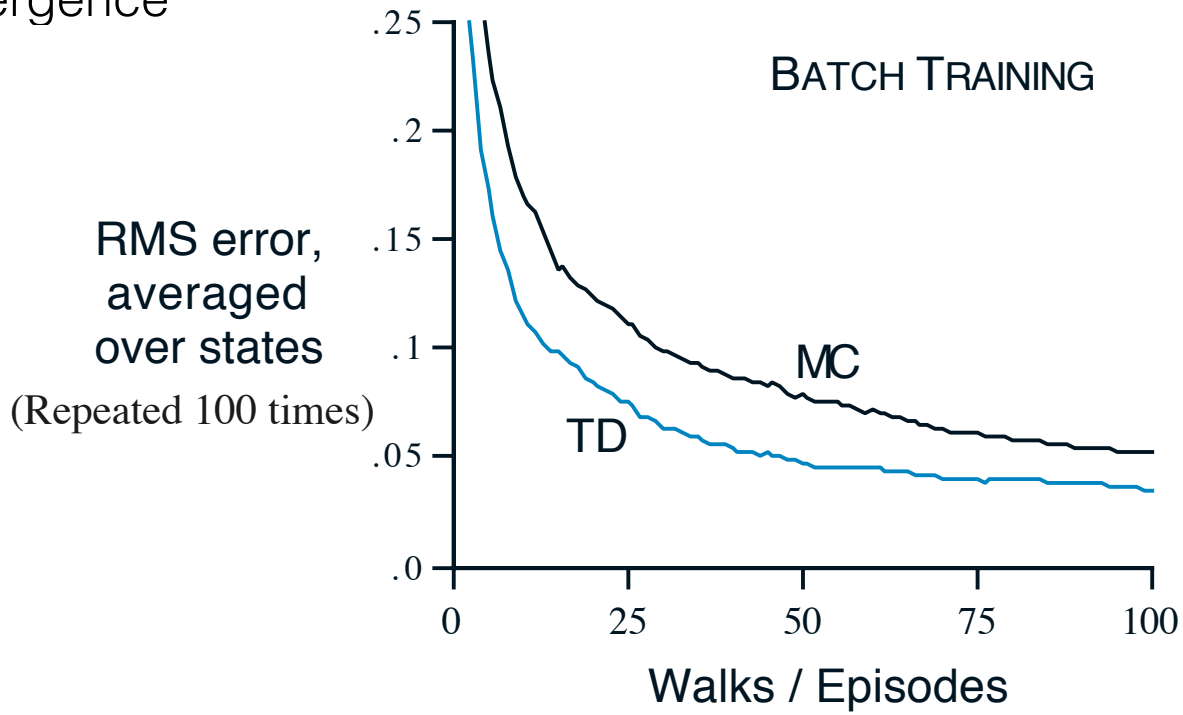
Compute updates according to TD or MC, but only update estimates after each complete pass through the data.

For any finite Markov prediction task, under batch updating, TD converges for sufficiently small α.

Constant-α MC also converges under these conditions, but to a difference answer!

Random Walk under Batch Updating

- After each new episode, all episodes seen so far are treated as a batch
- This growing batch is repeatedly processed by TD and MC until convergence



You are the Predictor

Suppose you observe the following 8 episodes:

A, 0, B, 0
B, 1
B, 1
B, 1
B, 1
B, 1
B, 1
B, 0

V(B)? 0.75

V(A)? 0?

Assume Markov states, no discounting ($\gamma = 1$)

Consider the following data

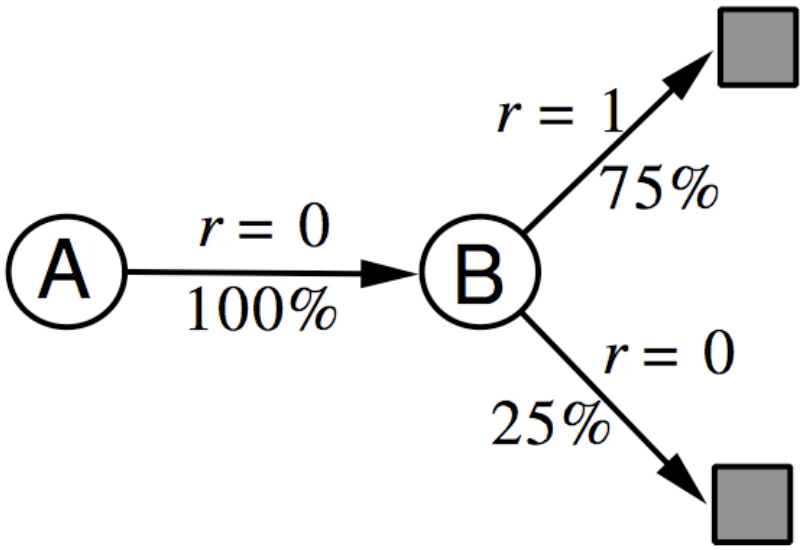
A, 0, B, 0
B, 1
B, 1
B, 1

B, 1
B, 1
B, 1
B, 0

- V(B)
  - 6 out of 8 times we saw a 1; V(B) = 3/4
- The batch MC prediction for V(A):
  - 100% of returns from A equal zero; V(A) = 0

You are the Predictor

- Modeling the Markov process based on the observed training data



$V(\mathrm{A})?$ 0.75

Optimality of TD(0)

- The prediction that best matches the training data is V(A)=0:
  - This minimizes the **mean-square-error** between V(s) and the sample returns in the training set. (**zero** MSE in our example)
  - Under batch training, this is what constant-α MC gets
- TD(0) achieves a different type of optimality, where V(A)=0.75
  - This is correct for the maximum likelihood estimate of the Markov model generating the data
  - i.e., if we do a best fit Markov model, and assume it is exactly correct, and then compute the predictions
  - This is called the **certainty-equivalence estimate**
  - This is what TD gets

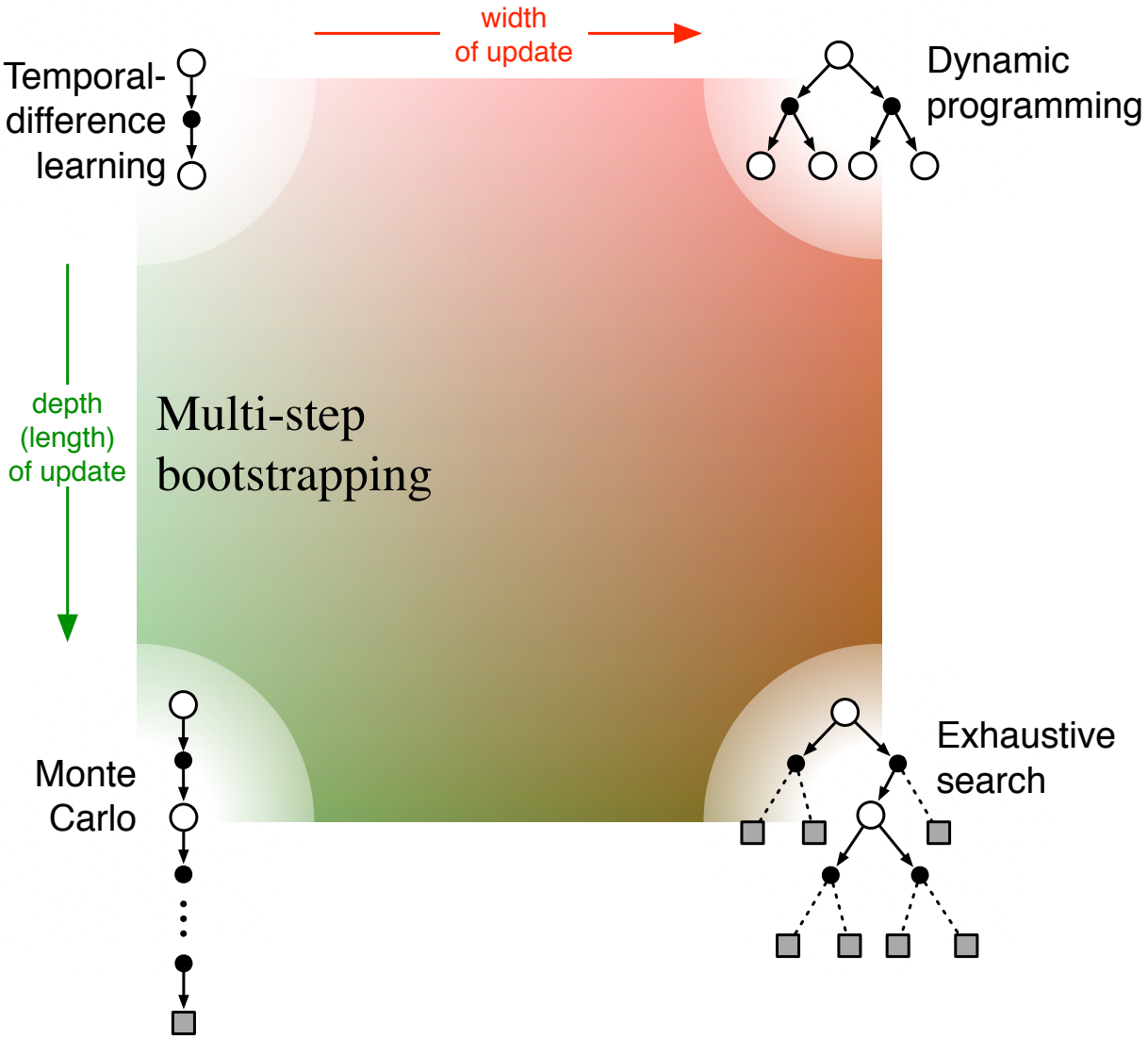
Advantages of TD

- If the process is Markov, then we expect the TD estimate to produce lower error on future data
- This helps explain why TD methods converge more quickly than MC in the batch setting
- TD(0) makes progress towards the certainty-equivalence estimate without explicitly building the model!

Summary so far

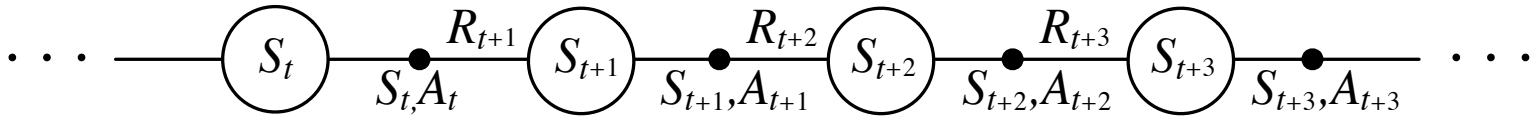
- Introduced *one-step tabular model-free TD methods*
- These methods bootstrap and sample, combining aspects of DP and MC methods
- TD methods are *computationally congenial*
- If the world is truly Markov, then TD methods will learn faster than MC methods
- MC methods have lower error on past data, but higher error on future data

Unified View



Learning An Action-Value Function

Estimate q_π for the current policy π



After every transition from a nonterminal state, S_t, do this:

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha\left[R_{t+1} + \gamma Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t)\right]$$

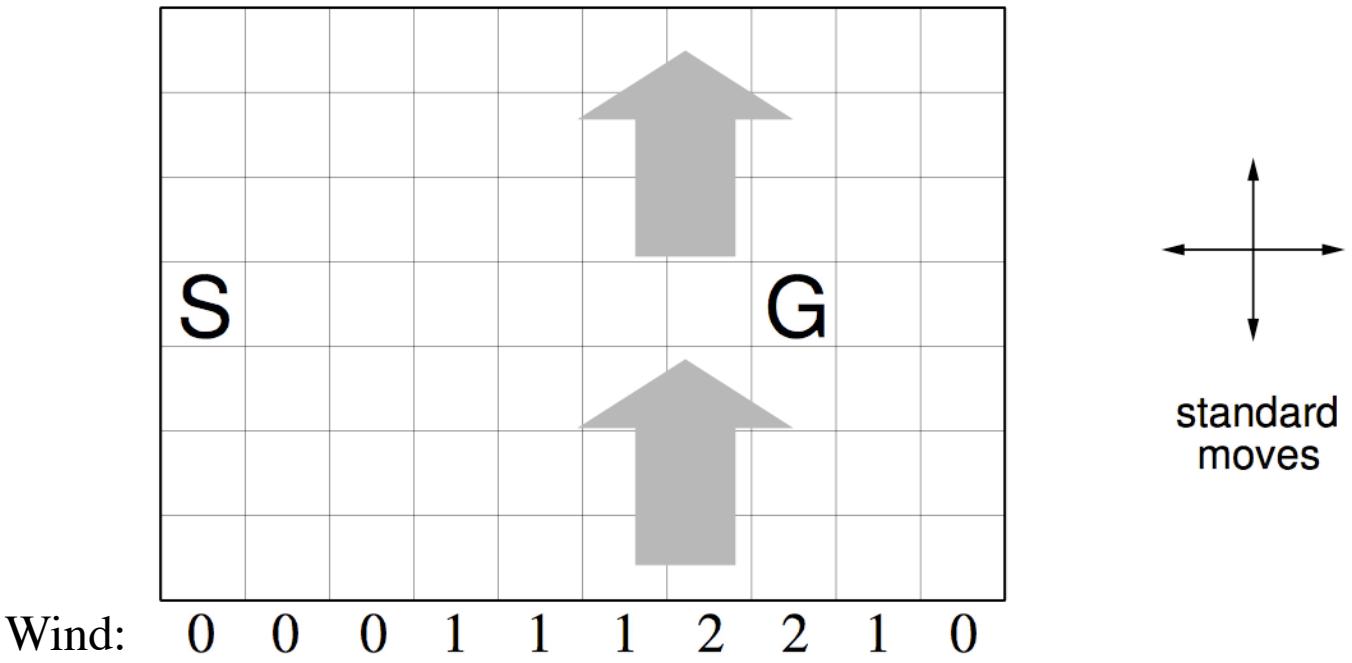
If S_{t+1} is terminal, then define $Q(S_{t+1}, A_{t+1}) = 0$

Sarsa: On-Policy TD Control

Turn this into a control method by always updating the policy to be greedy with respect to the current estimate:

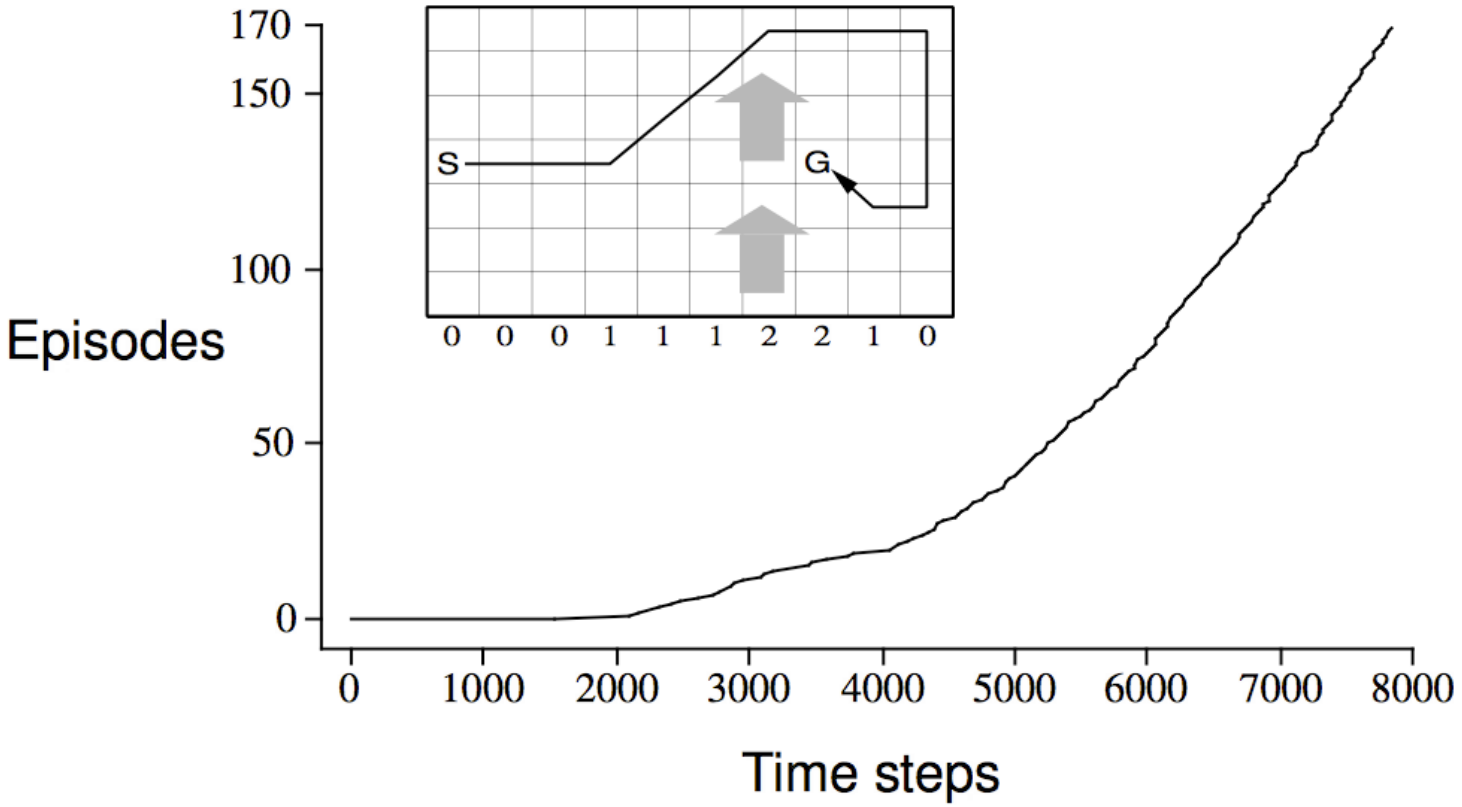
Initialize $Q(s,a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\textit{terminal-state}, \cdot) = 0$
Repeat (for each episode):
  Initialize S
  Choose A from S using policy derived from Q (e.g., ε-greedy)
  Repeat (for each step of episode):
    Take action A, observe R, S'
    Choose A' from S' using policy derived from Q (e.g., ε-greedy)
    $Q(S,A) \leftarrow Q(S,A) + \alpha[R + \gamma Q(S',A') - Q(S,A)]$
    $S \leftarrow S'$; $A \leftarrow A'$;
  until S is terminal

Windy Gridworld



undiscounted, episodic, reward = –1 until goal

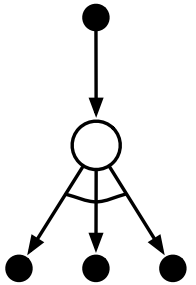
Results of Sarsa on the Windy Gridworld



Q-Learning: Off-Policy TD Control

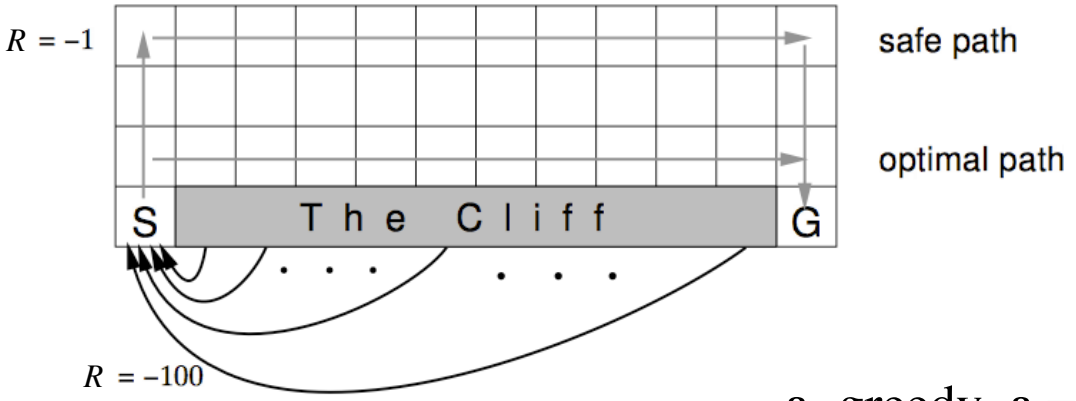
One-step Q-learning:

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha\Big[R_{t+1} + \gamma \max_a Q(S_{t+1}, a) - Q(S_t, A_t)\Big]$$

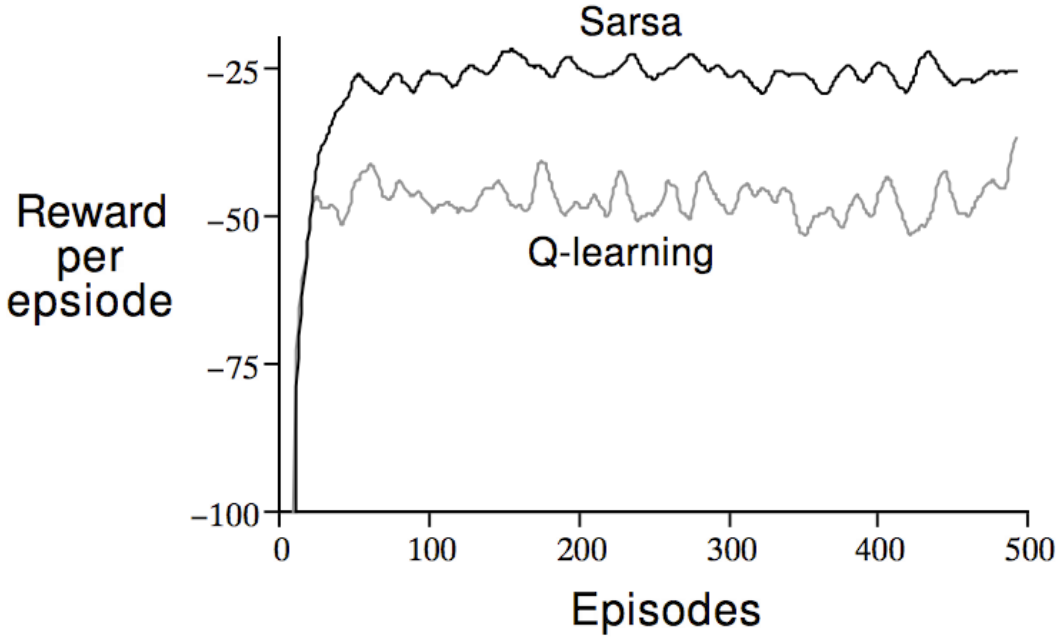


Initialize $Q(s,a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\textit{terminal-state}, \cdot) = 0$
Repeat (for each episode):
    Initialize S
    Repeat (for each step of episode):
        Choose A from S using policy derived from Q (e.g., ε-greedy)
        Take action A, observe R, S'
        $Q(S,A) \leftarrow Q(S,A) + \alpha[R + \gamma \max_a Q(S',a) - Q(S,A)]$
        $S \leftarrow S'$;
    until S is terminal

Cliffwalking



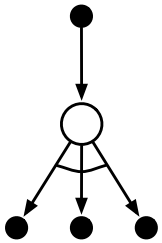
ε−greedy, ε = 0.1



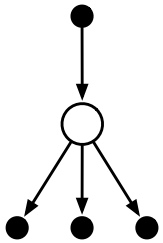
Expected Sarsa

- Instead of the *sample* value-of-next-state, use the expectation!

$$
\begin{aligned}
Q(S_t, A_t) &\leftarrow Q(S_t, A_t) + \alpha\Big[R_{t+1} + \gamma \mathbb{E}[Q(S_{t+1}, A_{t+1}) \mid S_{t+1}] - Q(S_t, A_t)\Big] \\
&\leftarrow Q(S_t, A_t) + \alpha\Big[R_{t+1} + \gamma \sum_a \pi(a|S_{t+1}) Q(S_{t+1}, a) - Q(S_t, A_t)\Big]
\end{aligned}
$$



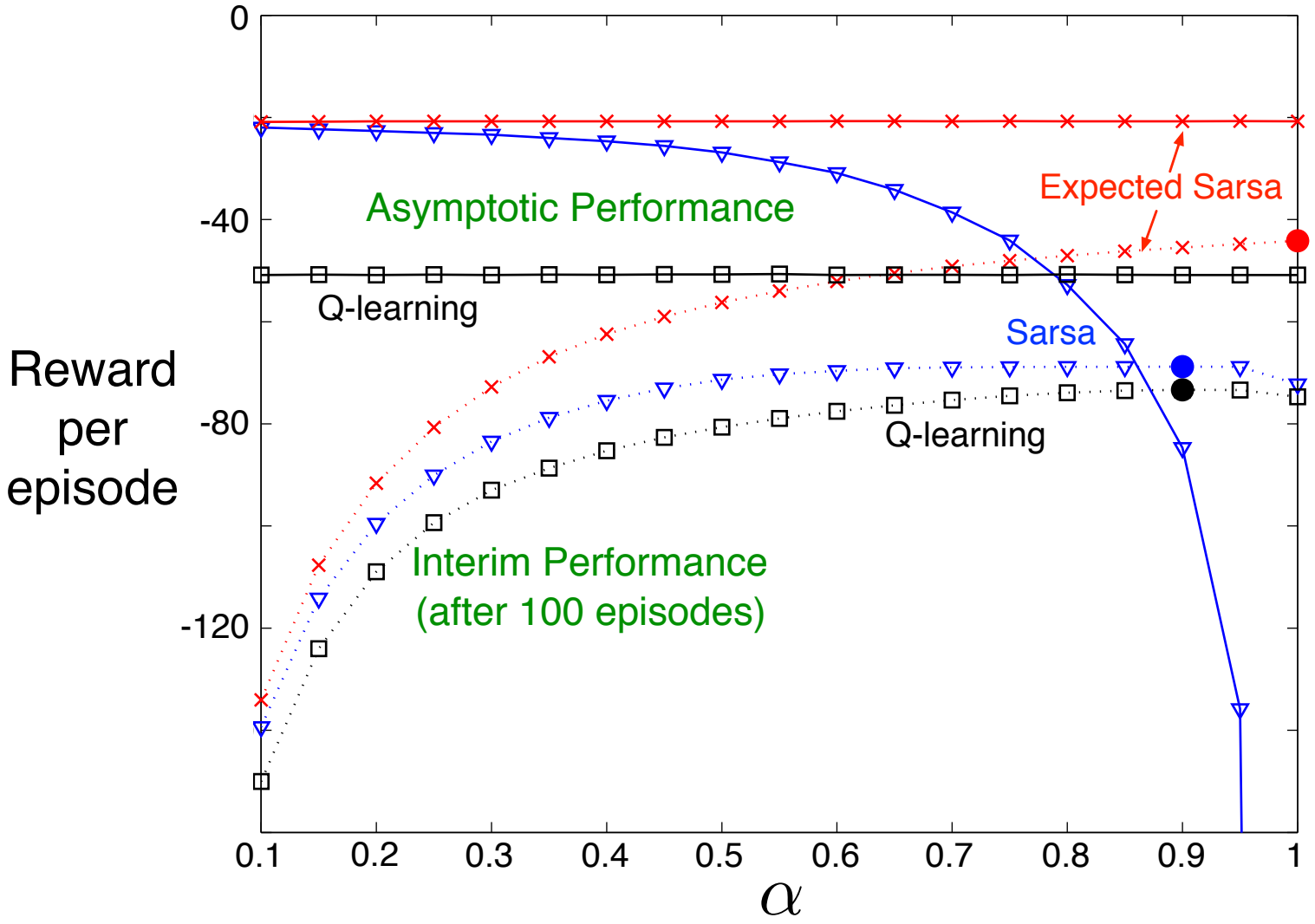
Q-learning



Expected Sarsa

- Expected Sarsa's performs better than Sarsa (but costs more)

Performance on the Cliff-walking Task

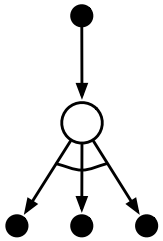


Off-policy Expected Sarsa

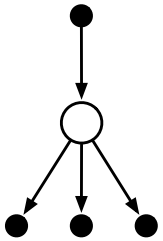
- Expected Sarsa generalizes to arbitrary behavior policies μ
  - in which case it includes Q-learning as the special case in which π is the greedy policy

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha\Big[R_{t+1} + \gamma\mathbb{E}[Q(S_{t+1}, A_{t+1}) \mid S_{t+1}] - Q(S_t, A_t)\Big]$$
$$\leftarrow Q(S_t, A_t) + \alpha\Big[R_{t+1} + \gamma\sum_a \pi(a|S_{t+1})Q(S_{t+1}, a) - Q(S_t, A_t)\Big]$$

Nothing changes here



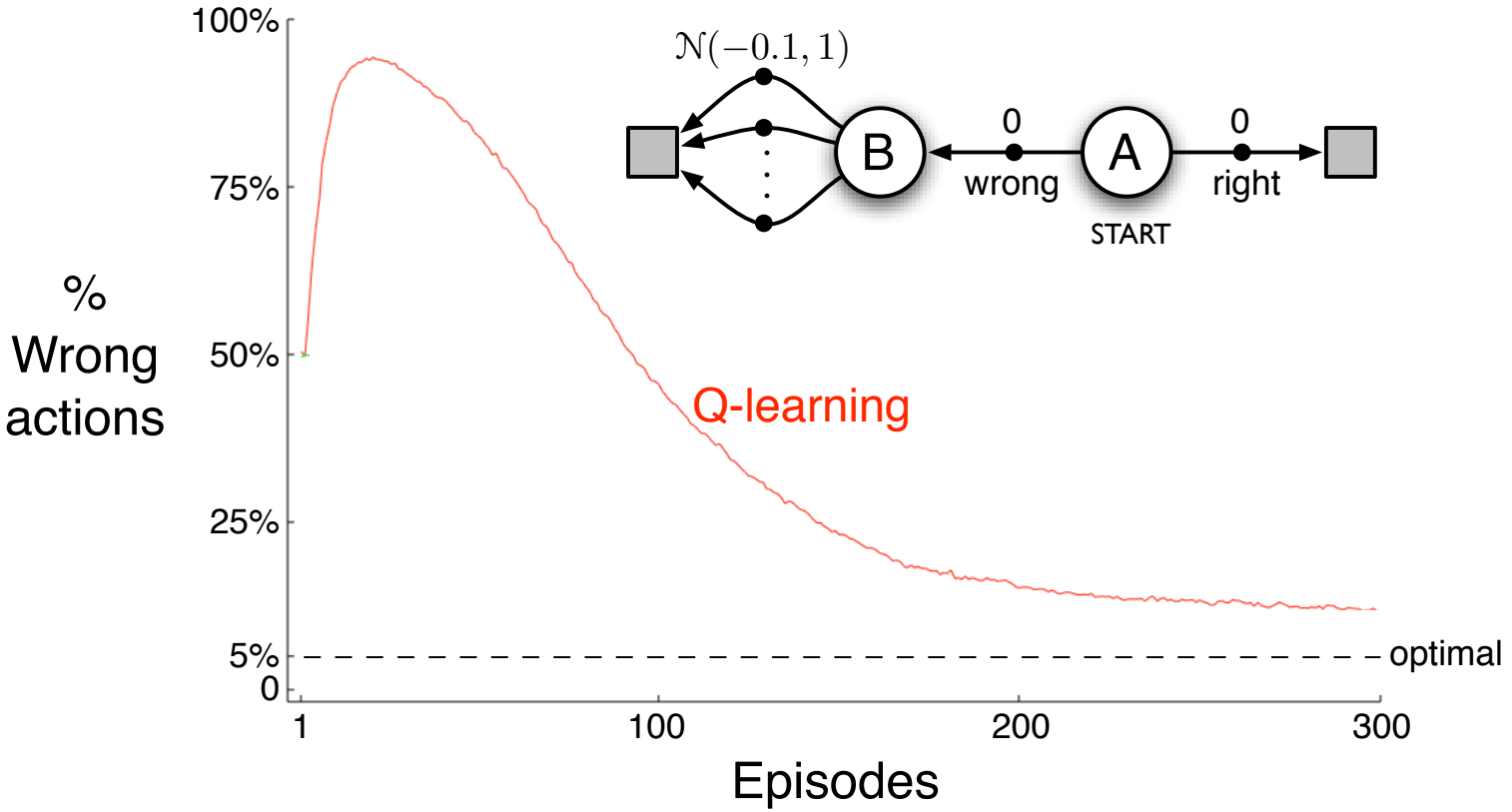
Q-learning



Expected Sarsa

- This idea seems to be new

Maximization Bias Example



Tabular Q-learning: $$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha\Big[R_{t+1} + \gamma \max_a Q(S_{t+1}, a) - Q(S_t, A_t)\Big]$$

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Double Q-Learning

- Train 2 action-value functions, Q_1 and Q_2
- Do Q-learning on both, but
  - never on the same time steps (Q_1 and Q_2 are indep.)
  - pick Q_1 or Q_2 at random to be updated on each step
- If updating Q_1, use Q_2 for the value of the next state:

$$Q_1(S_t, A_t) \leftarrow Q_1(S_t, A_t) + \alpha\Big(R_{t+1} + Q_2\big(S_{t+1}, \operatorname*{arg\,max}_a Q_1(S_{t+1}, a)\big) - Q_1(S_t, A_t)\Big)$$

- Action selections are (say) ε-greedy with respect to the sum of Q_1 and Q_2

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Double Q-Learning

Initialize $Q_1(s, a)$ and $Q_2(s, a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily
Initialize $Q_1(\textit{terminal-state}, \cdot) = Q_2(\textit{terminal-state}, \cdot) = 0$
Repeat (for each episode):
  Initialize S
  Repeat (for each step of episode):
    Choose A from S using policy derived from Q_1 and Q_2 (e.g., ε-greedy in $Q_1 + Q_2$)
    Take action A, observe R, S'
    With 0.5 probabilility:

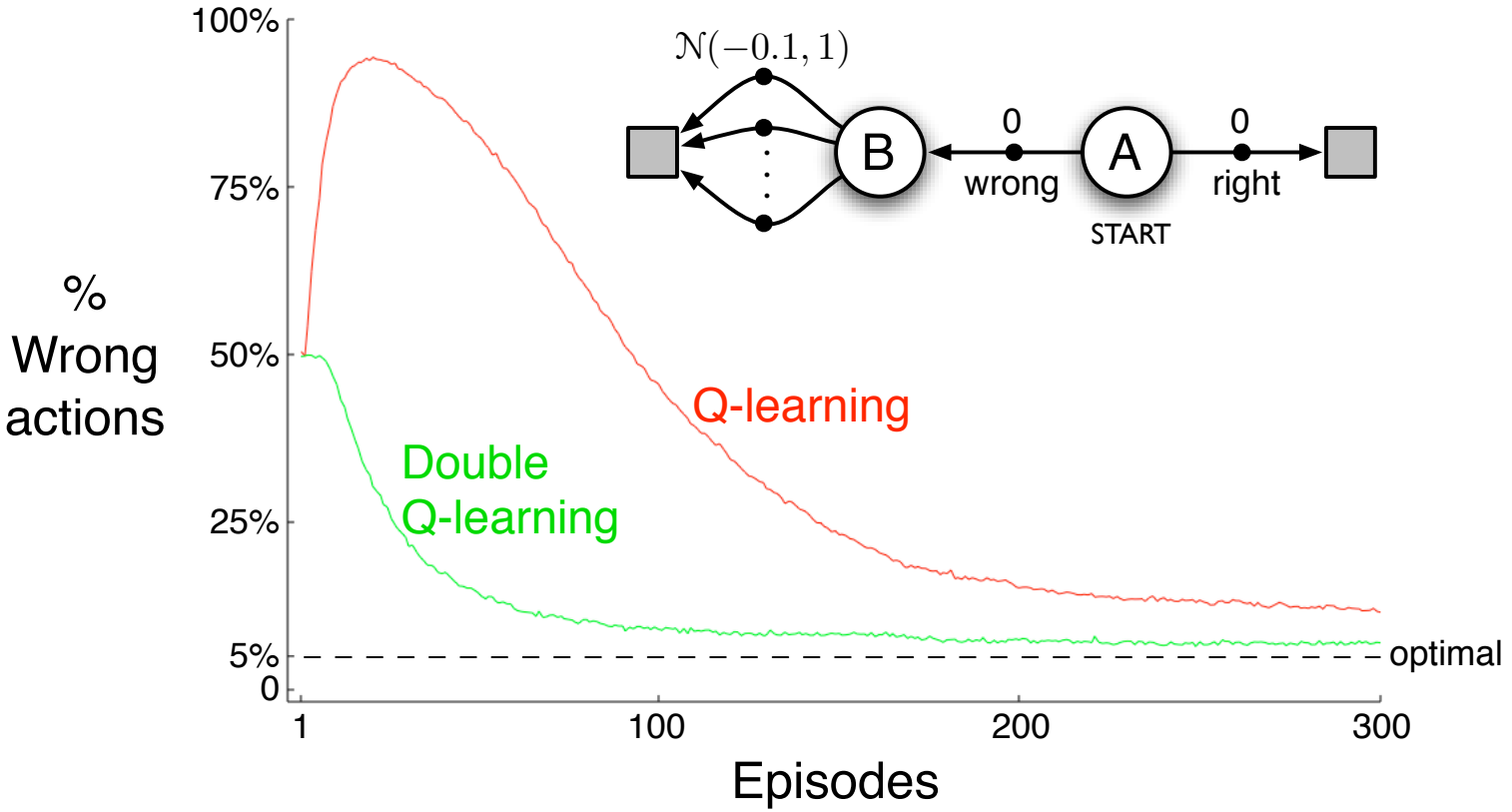
$$Q_1(S, A) \leftarrow Q_1(S, A) + \alpha\Big(R + \gamma Q_2\big(S', \operatorname{arg\,max}_a Q_1(S', a)\big) - Q_1(S, A)\Big)$$

    else:

$$Q_2(S, A) \leftarrow Q_2(S, A) + \alpha\Big(R + \gamma Q_1\big(S', \operatorname{arg\,max}_a Q_2(S', a)\big) - Q_2(S, A)\Big)$$

    $S \leftarrow S'$;
  until S is terminal

Example of Maximization Bias

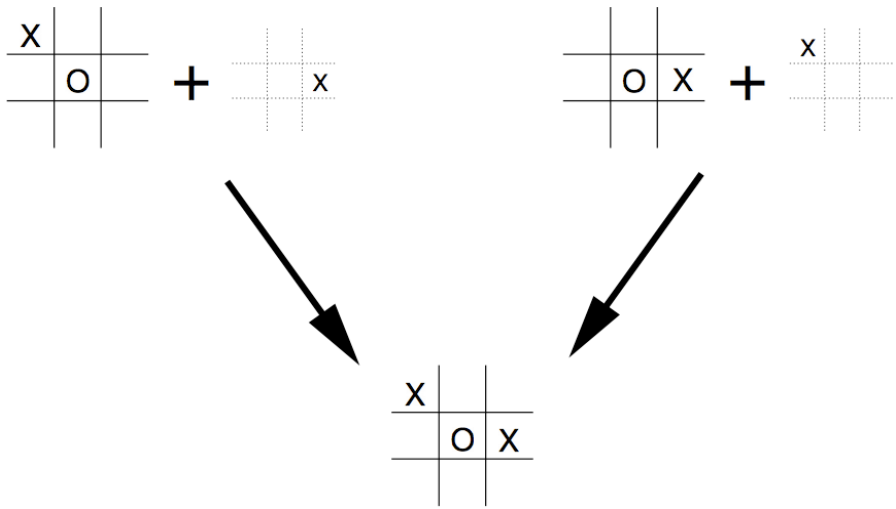


Double Q-learning:

$$Q_1(S_t, A_t) \leftarrow Q_1(S_t, A_t) + \alpha\Big[R_{t+1} + \gamma Q_2\big(S_{t+1}, \arg\max_a Q_1(S_{t+1}, a)\big) - Q_1(S_t, A_t)\Big]$$

Afterstates

- Usually, a state-value function evaluates states in which the agent can take an action.
- But sometimes it is useful to evaluate states after agent has acted, as in tic-tac-toe.
- Why is this useful?



- What is this in general?

Summary

- Introduced *one-step tabular model-free TD methods*
- These methods bootstrap and sample, combining aspects of DP and MC methods
- TD methods are *computationally congenial*
- If the world is truly Markov, then TD methods will learn faster than MC methods
- MC methods have lower error on past data, but higher error on future data
- Extend prediction to control by employing some form of GPI
  - On-policy control: Sarsa, Expected Sarsa
  - Off-policy control: Q-learning, Expected Sarsa
- Avoiding maximization bias with Double Q-learning