

## IFT 2125/6001 – Introduction to Algorithmics Midterm Exam, Winter 2017

Date de l'examen : Friday, 17 February 2017  
Time : 10:30 to 12:30  
Place : 1360, Pavillon André-Aisenstadt

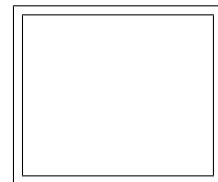
Name : .....  
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Signature : .....  
Tick here ☐ if you are taking IFT6001 (predoc)

### Directives :

1. *READ THE INSTRUCTIONS!*
2. No documentation allowed; no computer, no cell phone, no calculator.
3. **Write your answers on the questionnaire** in the free space following each question. (Use versos if you need more space, but if you do so, please mark it **clearly** on the recto of the corresponding page.)
4. If a question asks you to construct an object with a given property but you cannot do so, you can nevertheless, in *subsequent* questions (or subquestions), assume that you have an object with the stated property.

1	/20
2	/15
3	/25
4	/15
5	/10
6	/15
7	+

Professeur : \_\_\_\_\_ Gilles BRASSARD



## SOME REMINDERS

- ◇ The symbol  $\ll \lg \gg$  is used to denote the base 2 logarithm.  
Therefore,  $\lg n = \log_2 n$  by definition.

- ◇ For any real numbers  $b > 1$  and  $x > 0$ ,  $\log_b x = \frac{\lg x}{\lg b}$ .

- ◇ Let  $t_n$  be characterized by the following order- $k$  recurrence:

$$a_0 t_n + a_1 t_{n-1} + \cdots + a_k t_{n-k} = b_1^n p_1(n) + b_2^n p_2(n) + \cdots$$

where the  $a_i$ 's are arbitrary constants subject to  $a_0 \neq 0$ , the  $b_i$ 's are distinct constants, and each  $p_i$  is a degree- $d_i$  polynomial in  $n$ . Then, the characteristic polynomial of this recurrence is:

$$(a_0 x^k + a_1 x^{k-1} + \cdots + a_k) (x - b_1)^{d_1+1} (x - b_2)^{d_2+1} \dots$$

- ◇ If the roots (zeroes) of the characteristic polynomial for some order- $k$  recurrence are  $r_1, r_2, \dots, r_\ell$ , de multiplicité  $m_1, m_2, \dots, m_\ell$ , respectively, where the sum of the  $m_i$ 's is equal to the degree of the polynomial, then all the solutions to the recurrence are of the form

$$t_n = \sum_{i=1}^{\ell} \sum_{j=0}^{m_i-1} c_{ij} n^j r_i^n,$$

where  $k$  of the constants  $c_{ij}$ ,  $1 \leq i \leq \ell$ ,  $0 \leq j \leq m_i - 1$ , are determined by the  $k$  initial conditions, whereas all the other constants are determined by the recurrence itself, independently of the initial conditions.

- ◇ Let  $\ell \geq 1$ ,  $b \geq 2$  and  $k \geq 0$  be integers and consider some function  $T$  characterized by recurrence  $T(n) = \ell T(\lfloor n/b \rfloor) + g(n)$ , where  $g(n) \in \Theta(n^k)$ , then

$$T(n) \in \begin{cases} \Theta(n^k) & \text{si } \ell < b^k \\ \Theta(n^k \log n) & \text{si } \ell = b^k \\ \Theta(n^{\log_b \ell}) & \text{si } \ell > b^k. \end{cases}$$

This conclusion remains valid even if the  $\ell$  occurrences of  $\ll \lfloor n/b \rfloor \gg$  in the recurrence are replaced by integer value that are within an additive constant of  $n/b$ , provided they are strictly smaller than  $n$ .