Faculté des arts et des sciences
Département d'informatique et de recherche opérationnelle

IFT 2125/6001 – Introduction to Algorithmics
Midterm Exam, Winter 2017

Date de l'examen : Friday, 17 February 2017
Time : 10:30 to 12;30
Place : 1360, Pavillon André-Aisenstadt

Name : . . . .
Matricule : . . . .
Signature : . . . .
Tick here ☐ if you are taking IFT6001 (predoc)

Directives :

1. *READ THE INSTRUCTIONS!*
2. No documentation allowed; no computer, no cell phone, no calculator.
3. **Write your answers on the questionnaire** in the free space following each question. (Use versos if you need more space, but if you do so, please mark it **clearly** on the recto of the corresponding page.)
4. If a question asks you to construct an object with a given property but you cannot do so, you can nevertheless, in *subsequent* questions (or subquestions), assume that you have an object with the stated property.

1	/20
2	/15
3	/25
4	/15
5	/10
6	/15
7	+

Professeur : ______________________ Gilles Brassard

SOME REMINDERS

◇ The symbol « lg » is used to denote the base 2 logarithm.
Therefore, $\lg n = \log_2 n$ by definition.

◇ For any real numbers $b > 1$ and $x > 0$, $\log_b x = \dfrac{\lg x}{\lg b}$.

◇ Let t_n be characterized by the following order-k recurrence:

$$a_0\, t_n + a_1\, t_{n-1} + \cdots a_k\, t_{n-k} = b_1^n\, p_1(n) + b_2^n\, p_2(n) + \cdots$$

where the a_i's are arbitrary constants subject to $a_0 \neq 0$, the b_i's are distinct constants, and each p_i is a degree-d_i polynomial in n. Then, the characteristic polynomial of this recurrence is:

$$(a_0\, x^k + a_1\, x^{k-1} + \cdots + a_k)\, (x - b_1)^{d_1+1}\, (x - b_2)^{d_2+1} \ldots$$

◇ If the roots (zeroes) of the characteristic polynomial for some order-k recurrence are r_1, r_2, …, r_ℓ, de multiplicité m_1, m_2, …, m_ℓ, respectively, where the sum of the m_i's is equal to the degree of the polynomial, then all the solutions to the recurrence are of the form

$$t_n = \sum_{i=1}^{\ell} \sum_{j=0}^{m_i-1} c_{ij}\, n^j r_i^n\ ,$$

where k of the constants c_{ij}, $1 \le i \le \ell$, $0 \le j \le m_i - 1$, are determined by the k initial conditions, whereas all the other constants are determined by the recurrence itself, independently of the initial conditions.

◇ Let $\ell \ge 1$, $b \ge 2$ and $k \ge 0$ be integers and consider some function T characterized by recurrence $T(n) = \ell\, T(\lfloor n/b \rfloor) + g(n)$, where $g(n) \in \Theta(n^k)$, then

$$T(n) \in \begin{cases} \Theta(n^k) & \text{si } \ell < b^k \\ \Theta(n^k \log n) & \text{si } \ell = b^k \\ \Theta(n^{\log_b \ell}) & \text{si } \ell > b^k\,. \end{cases}$$

This conclusion remains valid even if the ℓ occurrences of « $\lfloor n/b \rfloor$ » in the recurrence are replaced by integer value that are within an additive constant of n/b, provided they are strictly smaller than n.