IFT 2125 – Introduction à l'algorithmique – TP6

4 avril 2017, à remettre **au début** de la démo spéciale du 11 avril

Question 1. Faites l'exercice 10.4 du livre : Write a computer program to simulate Buffon's experiment to estimate the value of π. The challenge is that you are not allowed to use the value of π in your program. If you do not see why this is a difficulty, try it!

Question 2. Faites l'exercice 10.22 du livre : In Section 10.6.4 we studied amplification of the stochastic advantage of unbiased [1] Monte Carlo algorithms for decision problems. Here, we investigate the situation for problems that have more than two potential answers. For general problems, instances may have more than one *correct* answer. Think for example of the eight queens problem or the problem of finding an arbitrary nontrivial divisor of a composite integer. When such problems are solved by probabilistic algorithms, it may happen that different correct answers are obtained when the same algorithm is run several times on the same input. We saw in Section 10.7 that this is a virtue for Las Vegas algorithms, but it can be catastrophic when unbiased Monte Carlo algorithms are concerned.

Recall that a Monte Carlo algorithm is p–correct if it returns a correct answer with probability at least p, whatever the instance considered. The potential difficulty is that even though a p–correct algorithm returns a correct answer with high probability when p is large, it could happen that one systematic wrong answer is returned more often than any given correct answer. In this case, amplification of the stochastic advantage by majority voting would decrease the probability of being correct! Show that if algorithm MC is 75%–correct, it may happen that $MC3$ is not even 71%–correct, where $MC3$ returns the most frequent answer of three calls on MC, as in Section 10.6.4. (Ties are broken arbitrarily.) For what value of k could $RepeatMC(\cdot, k)$ be less than 50%–correct even though MC is 75%–correct?

Question 3. Faites l'exercice 10.28 du livre : Let A and B be two biased Monte Carlo algorithms for solving the same decision problem. Algorithm A is p–correct but its answer is guaranteed when it is *true*; algorithm B is q–correct but its answer is guaranteed when it is *false*. Show how to combine A and B into a Las Vegas algorithm $LV(x, y, success)$ to solve the same problem. One call on LV should not take significantly more time than a call on A followed by a call on B. If your Las Vegas algorithm succeeds with probability at least r whatever the instance, what is the best value of r you can obtain?

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[1] ≪ Unbiased ≫ veut dire qu'il n'y a *pas* une réponse qui a la propriété que si cette réponse est obtenue, alors elle est garantie correcte, comme c'était le cas pour *faux* dans le cas de l'algorithme probabiliste de primalité que nous avons étudié en cours. Pour amplifier l'avantage stochastique, il faut donc répéter l'exécution de l'algorithme un certain nombre de fois et retourner la réponse majoritaire.