Faculté des arts et des sciences
Département d'informatique et de recherche opérationnelle

IFT 2125/6001 – Introduction to Algorithmics
Final Exam, Winter 2017

Date: Tuesday, 18 April 2017
Time: 12:30 to 15:20
Place: N–615, Pavillon Roger-Gaudry

Name: ....
Forename: ....
Matricule: ....
Signature: ....
Tick here ☐ if you are taking IFT6001 (predoc)

Directives:

1. *READ THE INSTRUCTIONS!*
2. **Do not detach pages from this questionnaire.**
3. No documentation allowed; no computer, no cell phone, no calculator.
4. **Write your answers on the questionnaire** in the free space following each question. (Use versos if you need more space, but if you do so, please mark it **clearly** on the recto of the corresponding page.)
5. If a question asks you to construct an object with a given property but you cannot do so, you can nevertheless, in *subsequent* questions (or subquestions), assume that you have an object with the stated property.

1	/20
2	/30
3	/15
4	/10
5	/20
6	/5
7	+
8	+

Professor: ______________________ Gilles Brassard

SOME REMINDERS

◇ The symbol «lg» is used to denote the base 2 logarithm.
Therefore, $\lg n = \log_2 n$ by definition.

◇ Let t_n be characterized by the following order-k recurrence:

$$a_0\, t_n + a_1\, t_{n-1} + \cdots a_k\, t_{n-k} = b_1^n\, p_1(n) + b_2^n\, p_2(n) + \cdots$$

where the a_i's are arbitrary constants subject to $a_0 \neq 0$, the b_i's are distinct constants, and each p_i is a degree-d_i polynomial in n. Then, the characteristic polynomial of this recurrence is:

$$(a_0\, x^k + a_1\, x^{k-1} + \cdots + a_k)\,(x - b_1)^{d_1+1}\,(x - b_2)^{d_2+1}\ldots$$

◇ If the roots (zeroes) of the characteristic polynomial for some order-k recurrence are $r_1,\, r_2,\, \ldots,\, r_\ell$, of multiplicity $m_1,\, m_2,\, \ldots,\, m_\ell$, respectively, where the sum of the m_i's is equal to the degree of the polynomial, then all the solutions to the recurrence are of the form

$$t_n = \sum_{i=1}^{\ell} \sum_{j=0}^{m_i-1} c_{ij}\, n^j r_i^n\,,$$

where k of the constants c_{ij}, $1 \leq i \leq \ell$, $0 \leq j \leq m_i - 1$, are determined by the k initial conditions, whereas all the other constants are determined by the recurrence itself, independently of the initial conditions.

◇ In a game graph, non-terminal configurations are labelled according to this rule:

- they are **winning** if *at least one* of their successors is losing;
- **losing** if *all* their successors are winning;
- **null** in all other cases.

◇ A *Monte Carlo* algorithm always provides an answer, but sometimes that answer is incorrect; A *Las Vegas* algorithm is allowed to admit ignorance of the answer (by setting $\mathsf{success} = \mathsf{false}$), but the returned answer is always correct when provided ($\mathsf{success} = \mathsf{true}$). Some Las Vegas algorithms always succeed.

◇ Any binary tree with k leaves has a minimal height of $\lceil \lg k \rceil$.