

Membership testing problem

From Groupprops

(Redirected from Generating set to membership test)

Contents

- 1 Definition
 - 1.1 Given data
 - 1.2 Goal
- 2 Relation with other problems
 - 2.1 Problems it reduces to
 - 2.2 Problems that are solved using it
- 3 Algorithms
 - 3.1 Black-box group algorithms
 - 3.2 Permutation group algorithms
 - 3.3 Linear group algorithms

Definition

Given data

G is a finite group equipped with an encoding. H is a subgroup of G and we are given a generating set B for H .

Goal

We need to describe a test for membership in H , i.e., we need to construct an algorithm that can take as input the code-word for any $g \in G$ and outputs whether or not $g \in H$.

Relation with other problems

Problems it reduces to

Problem ♦	Nature of problem ♦	Description of reduction of membership testing to the problem ♦
Order-finding problem	Suppose G is a finite group described by means of an encoding. Consider a subset B of G . The goal is to find the order of the subgroup $\langle B \rangle$.	Compute the orders of the subgroups $H = \langle B \rangle$ and $\langle H, g \rangle = \langle B \cup \{g\} \rangle$. If the orders are the same, then $g \in H$. If the orders are different, then $g \notin H$.

Problems that are solved using it

- Subgroup testing problem: For this problem, we are given sets A and B inside U and we are asked whether the group G generated by A contains the group H generated by B . The subgroup testing problem reduces to the membership testing problem via a positive truth-table reduction. The idea of the reduction is to check, for each element in B , whether it is a member of G .

- Normality testing problem: Given generating sets A for G and B for H , the problem asks whether H is a normal subgroup of G . The normality testing problem reduces to the membership testing problem via a positive truth-table reduction. The idea is to first use the subgroup testing problem and to then check whether every conjugate of an element in B by an element in A must be in B .
- Normal closure-finding: This is solved using the normality testing problem
- Subnormality testing problem: This is solved using the normal closure-finding algorithm

Algorithms

Black-box group algorithms

These work for a group specified by means of an encoding.

Algorithm ↕	Additional information needed for algorithm, if any ↕	Time taken, where N is the order of the group and s is the size of the generating set ↕	Type of algorithm ↕
Black-box group algorithm for finding the subgroup generated by a subset		$O(Ns)$ times the time for the group operations.	deterministic
Nondeterministic black-box group algorithm for membership testing			nondeterministic

Permutation group algorithms

- Permutation group algorithm for membership testing

Linear group algorithms

- Linear group algorithm for membership testing

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