Existence (constructive proof) [edit]

Existence may be established by an explicit construction of x.[14] This construction may be split into two steps, firstly by solving the problem in the case of two moduli, and the second one by extending this solution to the general case by induction on the number of moduli.

Case of two moduli [edit]

We want to solve the system

$$\begin{aligned} x &\equiv a_1 \pmod{n_1} \\ x &\equiv a_2 \pmod{n_2}, \end{aligned}$$

where n_1 and n_2 are coprime.

Bézout's identity asserts the existence of two integers m_1 and m_2 such that

$$m_1 n_1 + m_2 n_2 = 1.$$

The integers m_1 and m_2 may be computed by Extended Euclidean algorithm.

A solution is given by

$$x = a_1 m_2 n_2 + a_2 m_1 n_1.$$

Indeed,

$$\begin{aligned} x &= a_1 m_2 n_2 + a_2 m_1 n_1 \\ &= a_1 (1 - m_1 n_1) + a_2 m_1 n_1 \\ &= a_1 + (a_2 - a_1) m_1 n_1, \end{aligned}$$

implying that $x \equiv a_1 \pmod{n_1}$. The second congruence is proved similarly.