Université de Montréal
Département d'informatique et de recherche opérationnelle

IFT6390
Fondements de l'apprentissage machine

Learning terminology

Professor: Ioannis Mitliagkas

Learning from examples

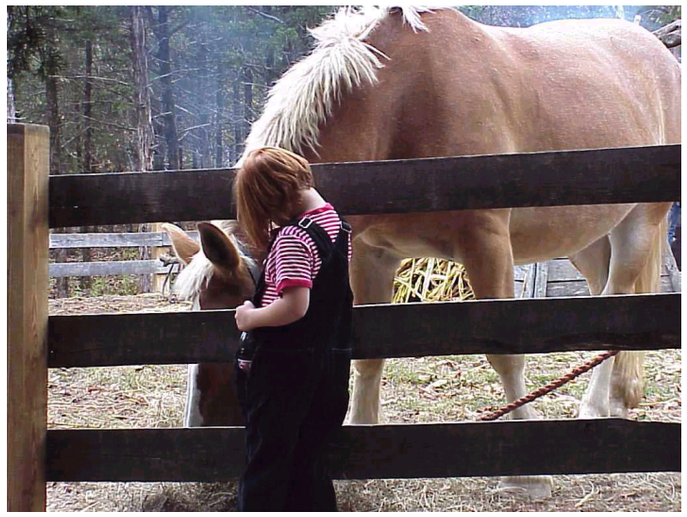
Learning from examples

Learning from examples

"horse"

Learning from examples

"horse"



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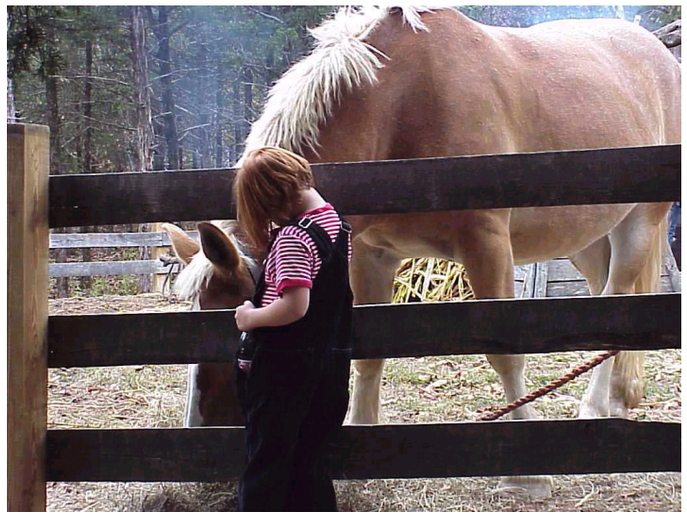

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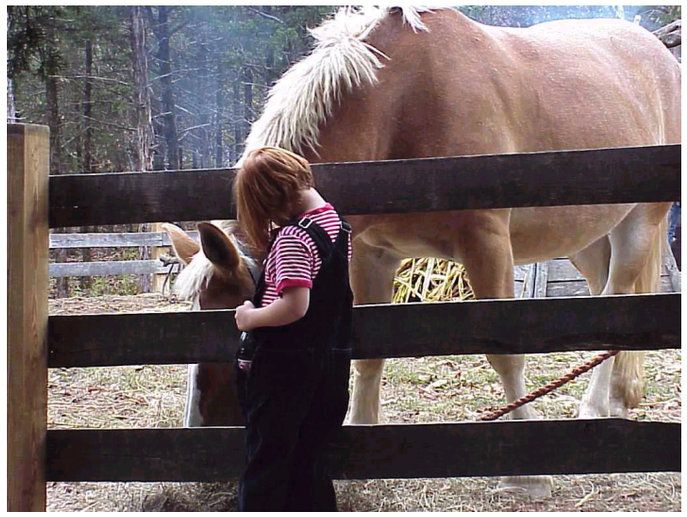

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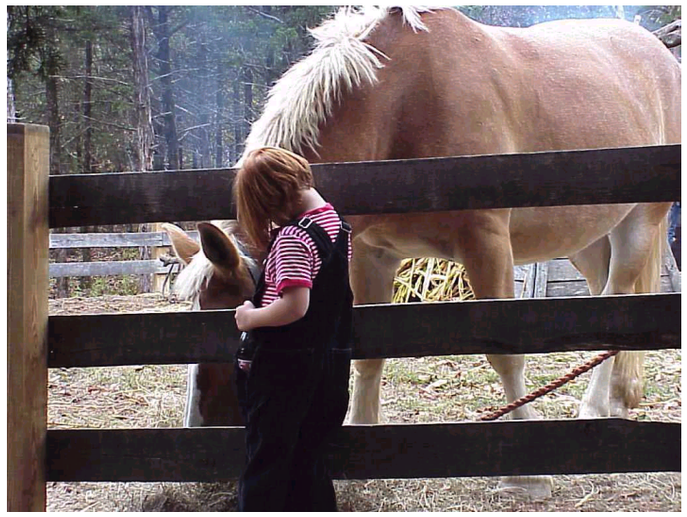

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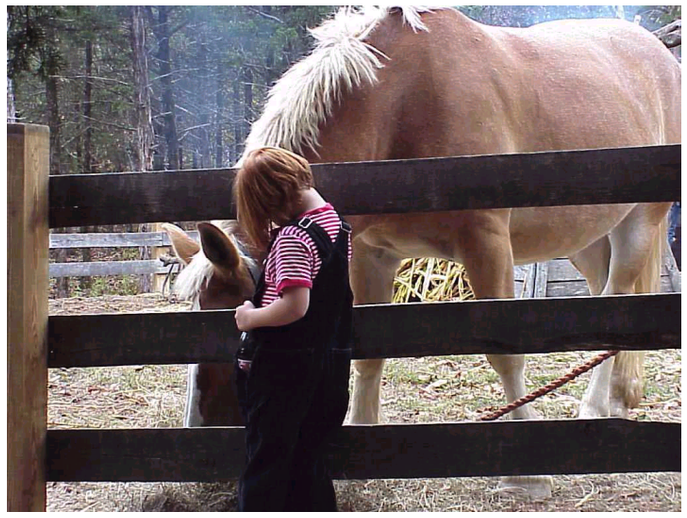


"horse"

"horse"

Principle much more general than to write by hand, starting from scratch, an algorithm to recognize a horse..

Categories of problems (tasks) in machine learning

Supervised learning

- Classification
- Regression

Non-supervised learning

- Density estimation (learn the distribution of examples)
- Clustering
- Dimensionality reduction
- Feature extraction

Reinforcement learning

(not scheduled for this class)

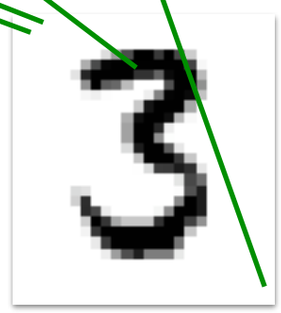
Example of classification problem

entry x_i

(vector representation)

$$x \in \mathbb{R}^d$$

$$x = (0, 0, \ldots, 54, 120, \ldots, 0, 0)$$



Test point:
(new x)

Example of classification problem

training set

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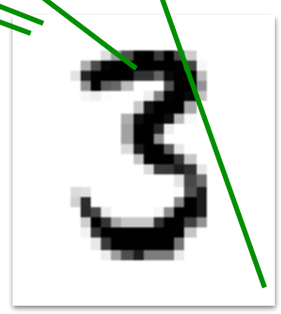

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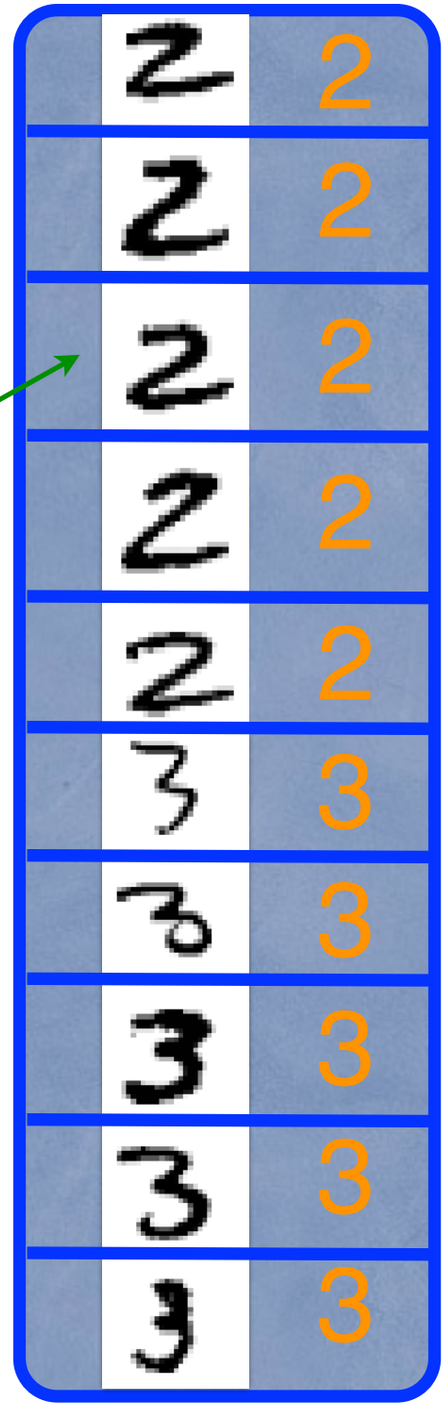

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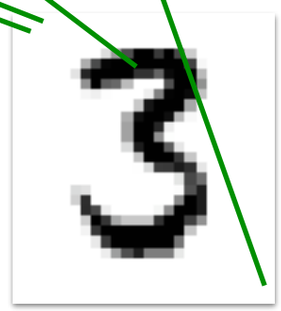

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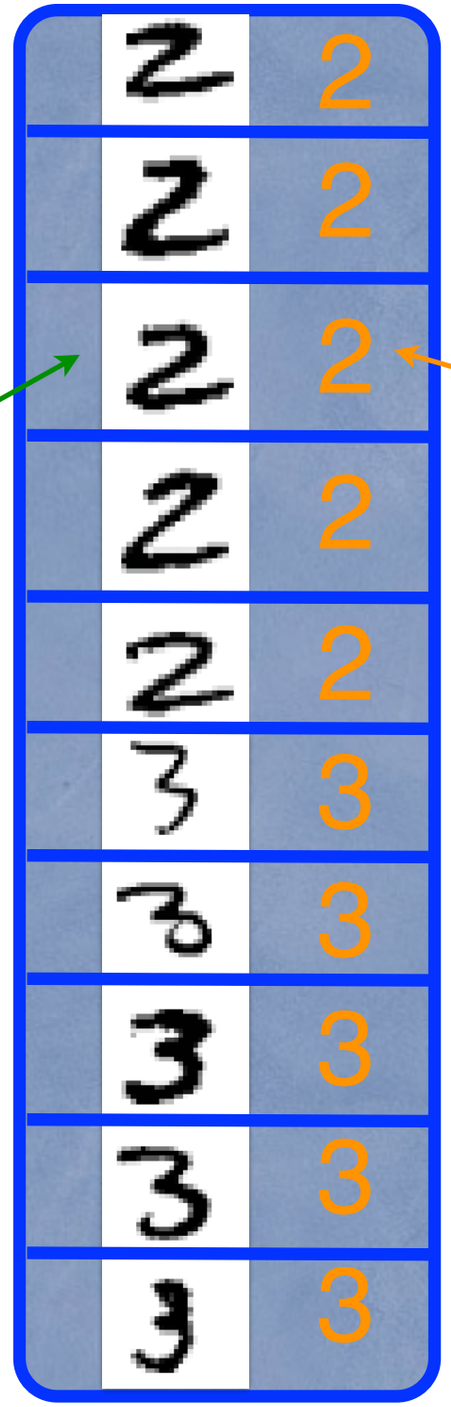

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label y_i

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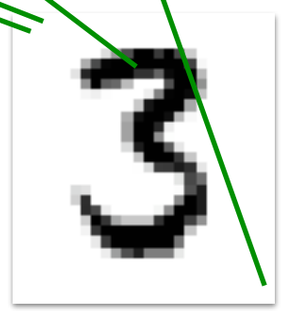

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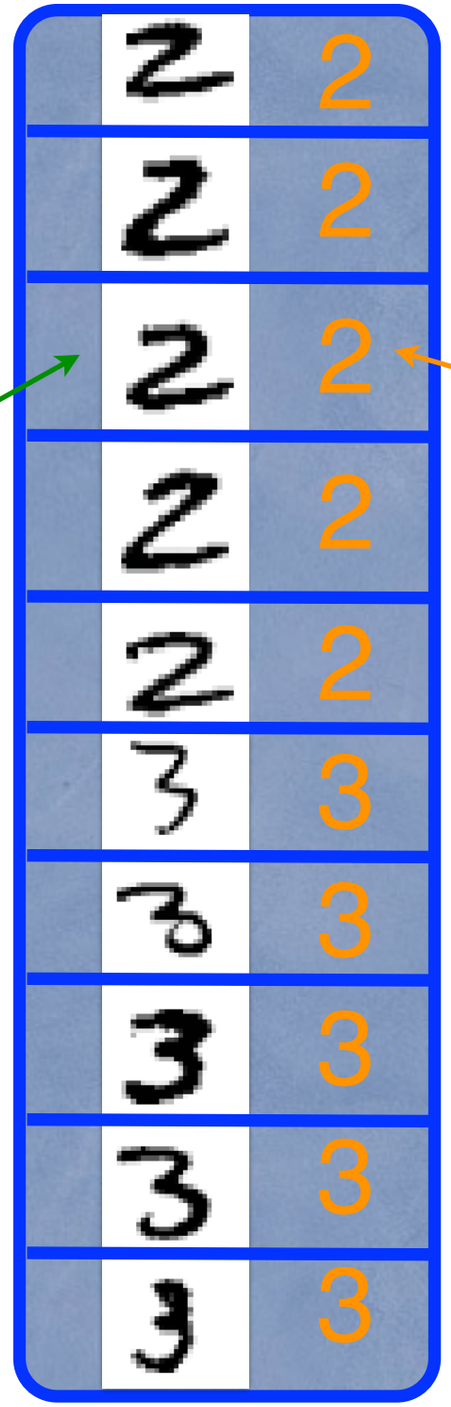

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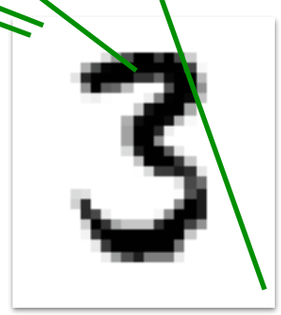

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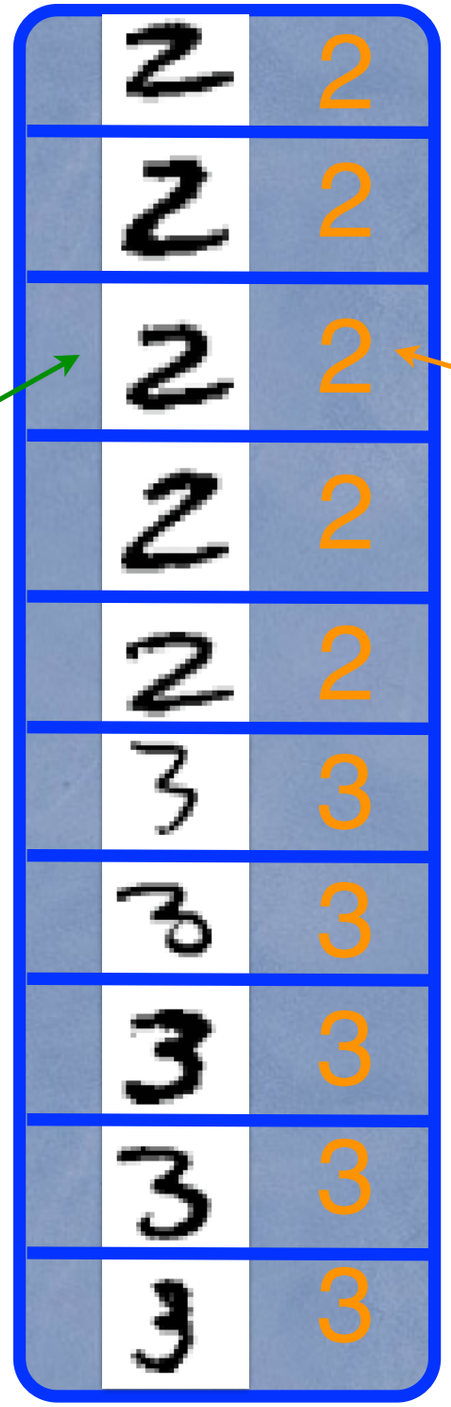

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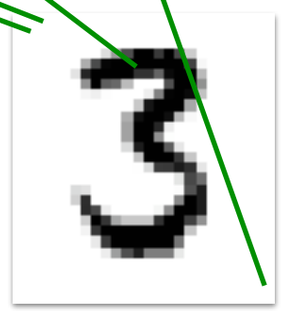

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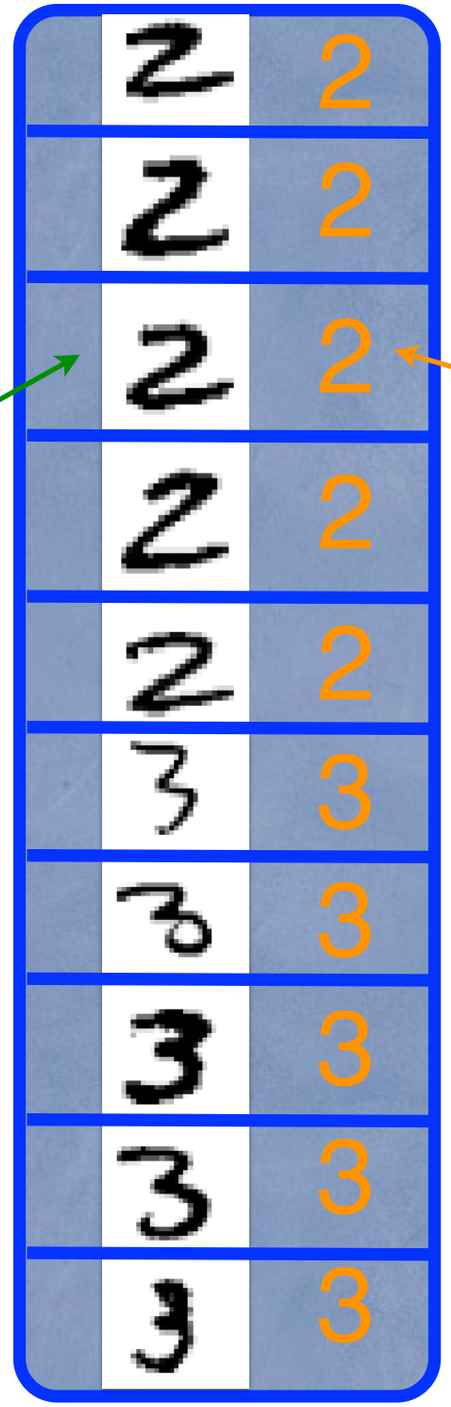

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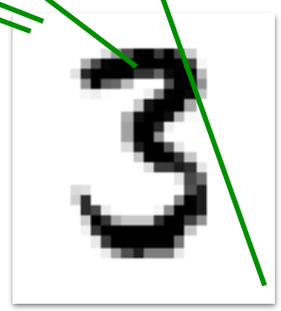

2 or 3?

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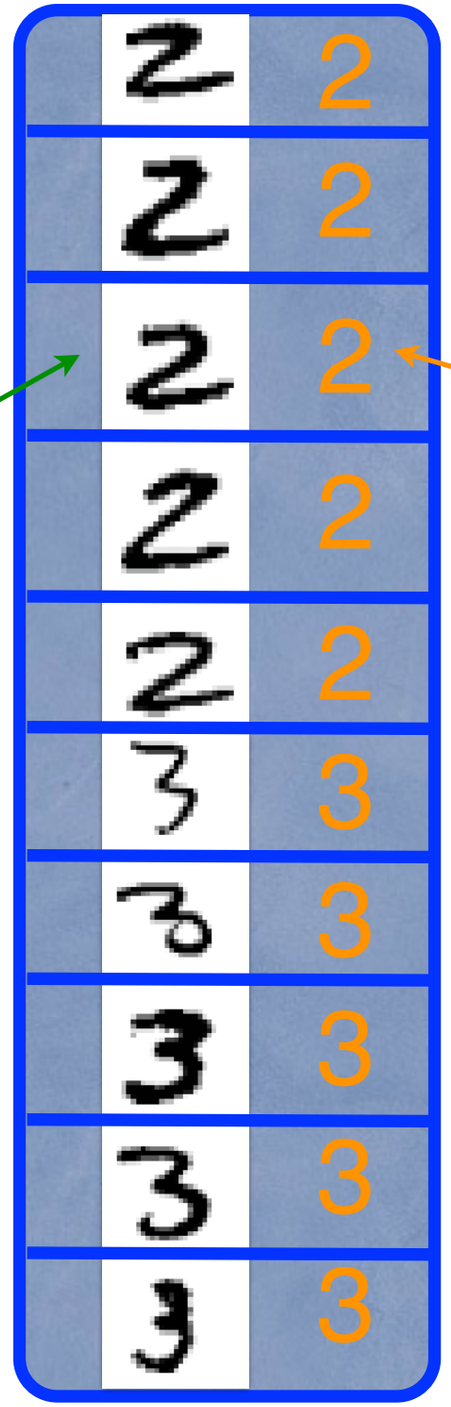

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Learning is not memorizing

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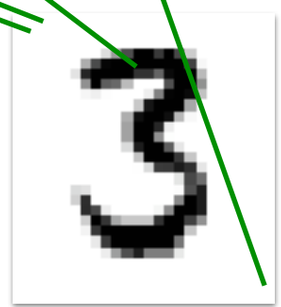

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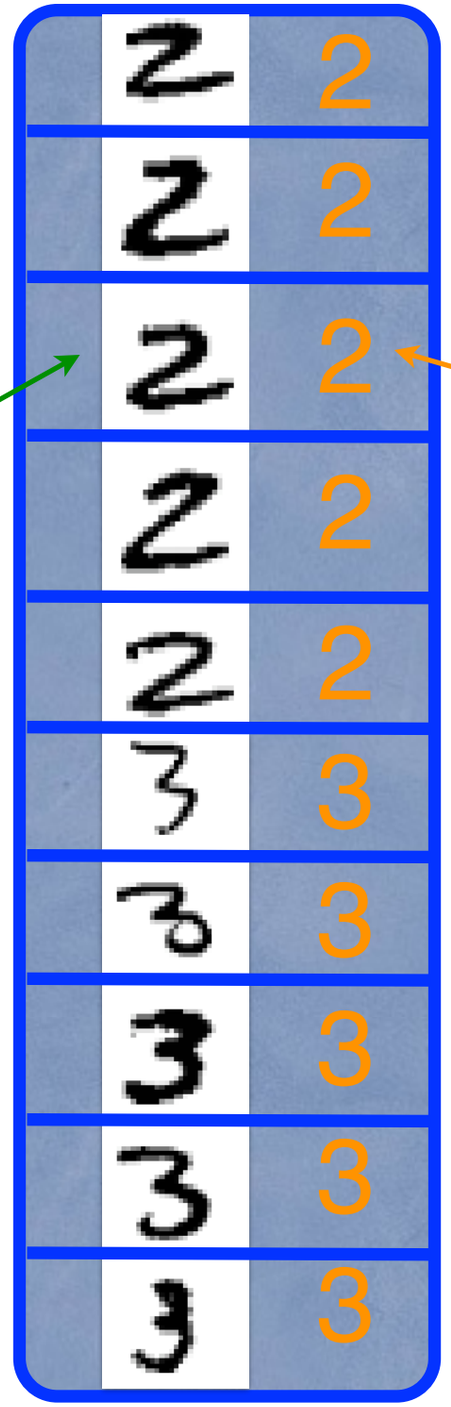

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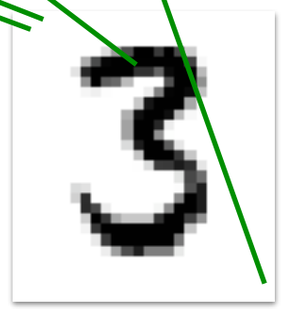


2 or 3?

It is the ability to generalize to unseen cases!

Data representation

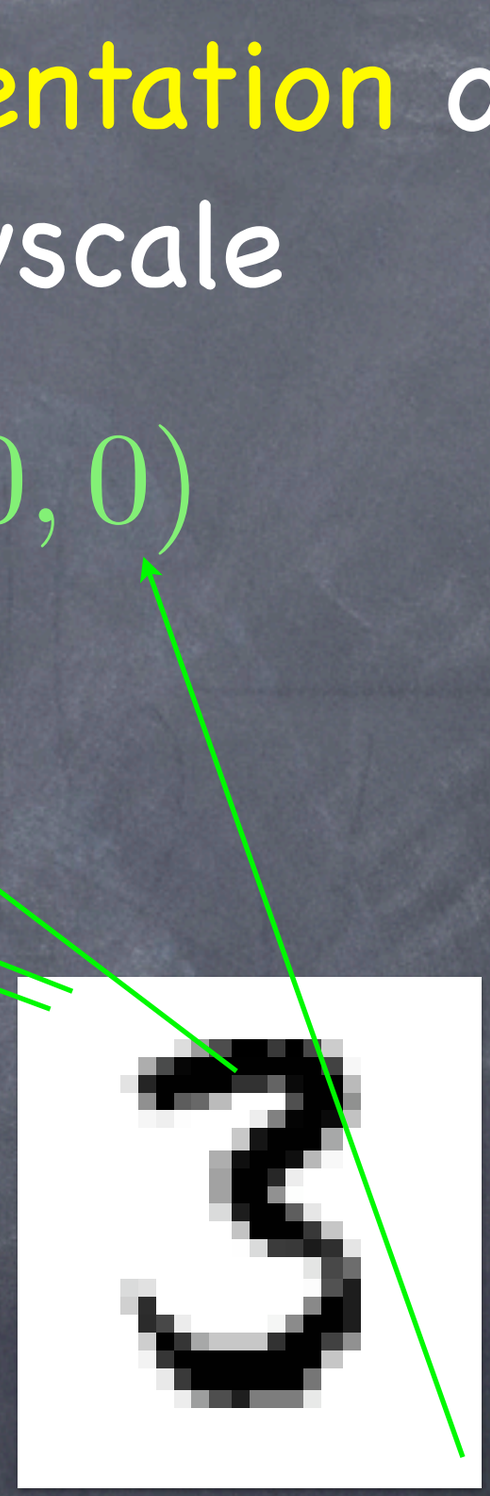
- Most learning algorithms require a **vector representation** of the examples
- Numerical vectors of fixed dimension **d**

$$x \in \mathbb{R}^d$$

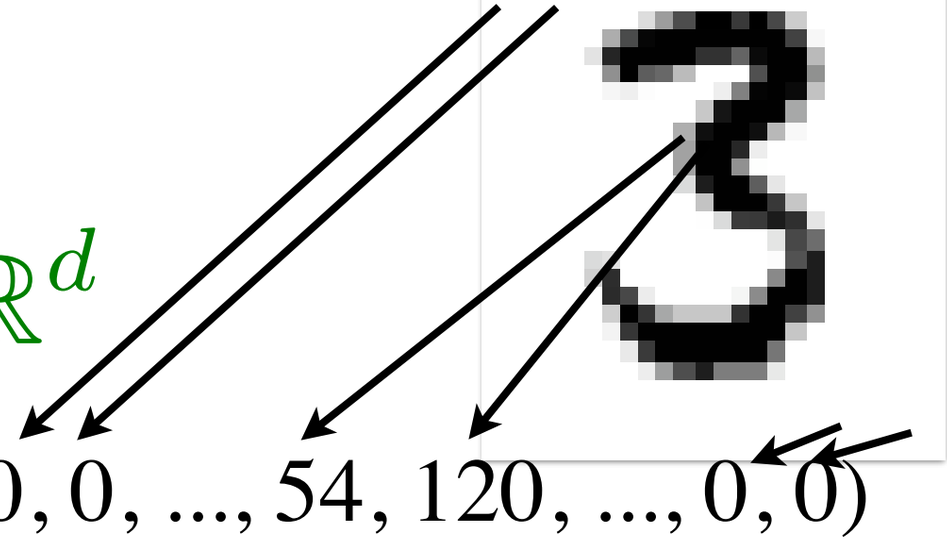
ex: $x = (0, 0, \ldots, 54, 120, \ldots, 0, 0)$

Example: raw vector representation of an image (bitmap) in grayscale

$$x = (0, 0, \ldots, 54, 120, \ldots, 0, 0)$$



Transform an example into a vector representation $\mathbf{x} \in \mathbb{R}^d$



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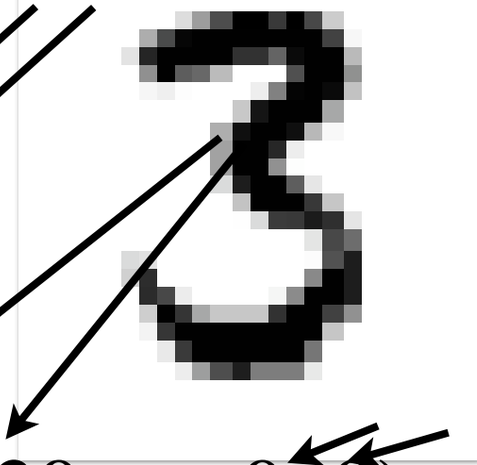
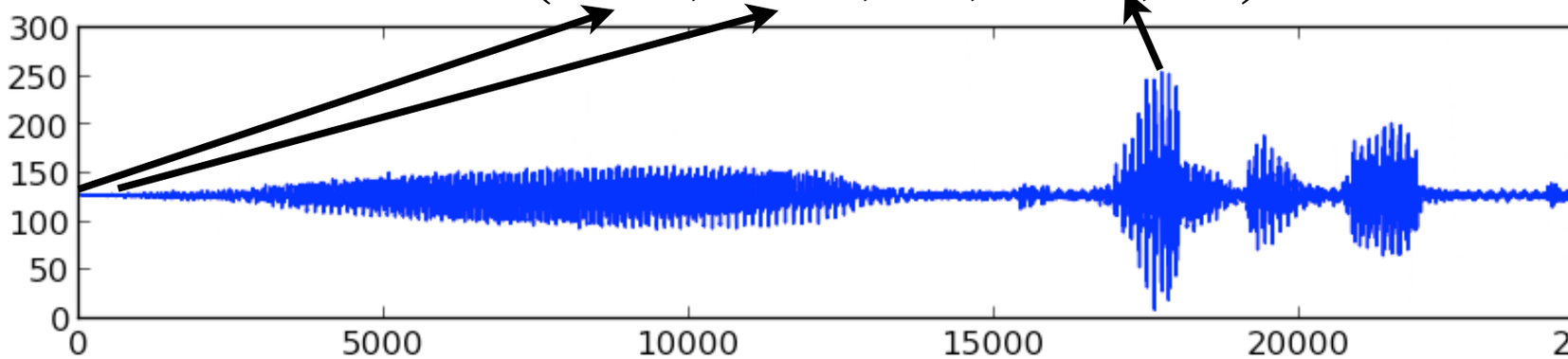


$x = (0, 0, ..., 54, 120, ..., 0, 0)$

Raw representation:

$x = (125, 125, ..., 250, ...)$



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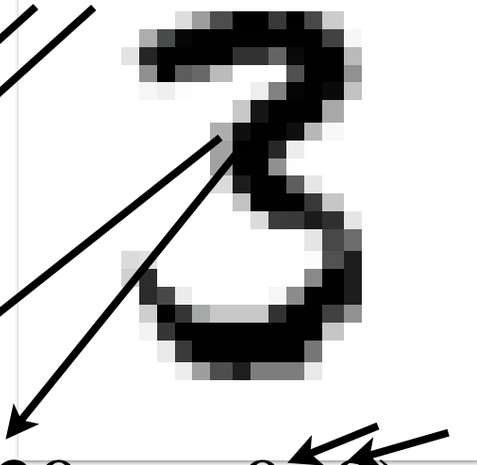

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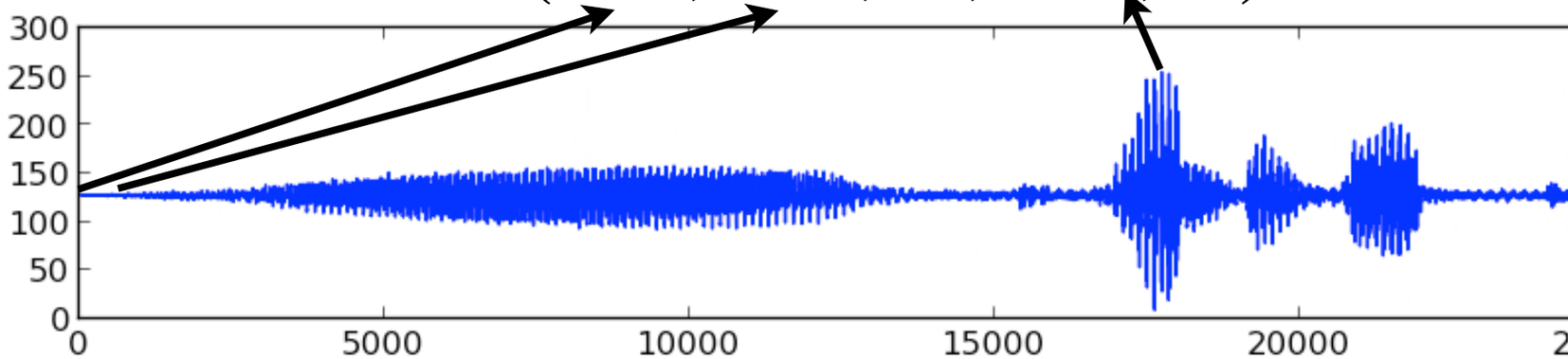
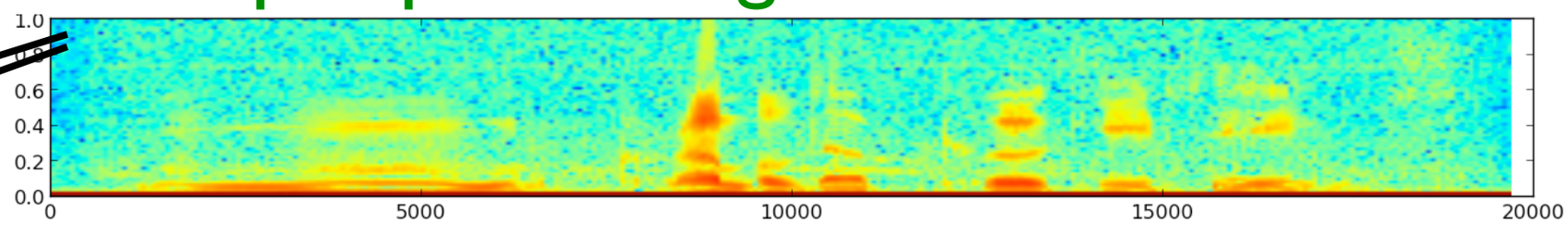


Or feature extraction from pre-processing:

$x = (, \quad , \quad , \quad , \quad ....)$



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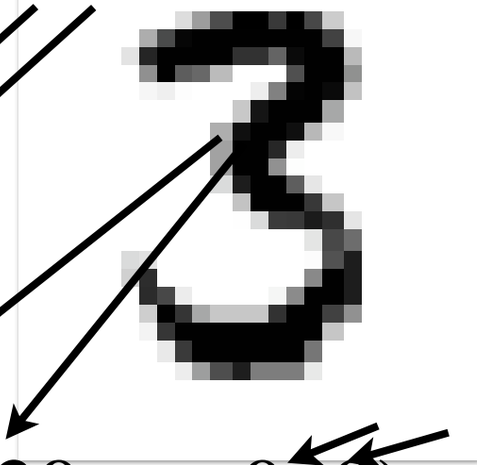

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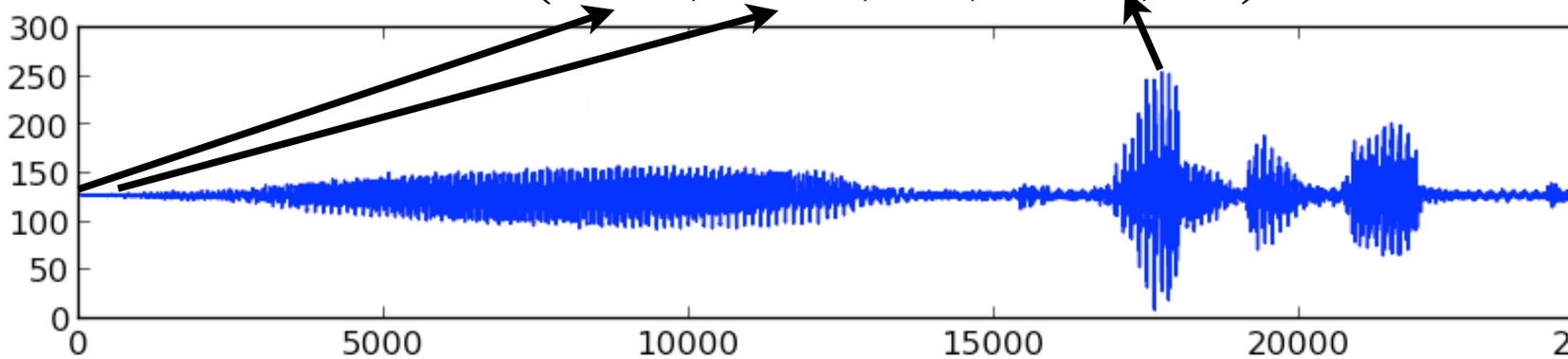

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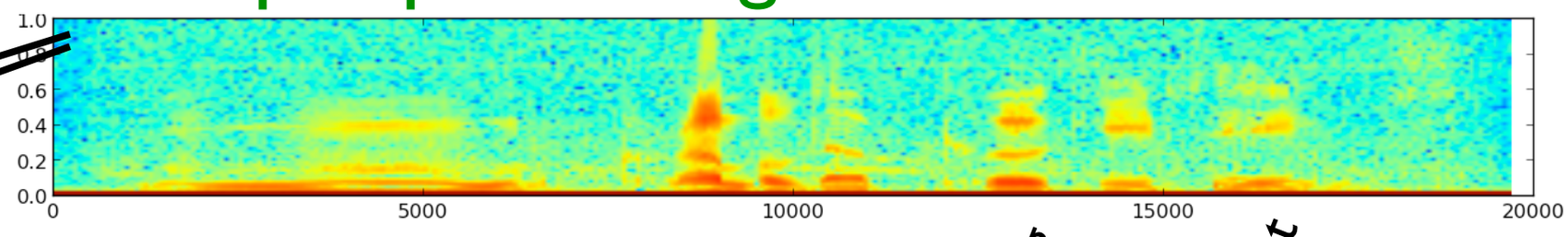
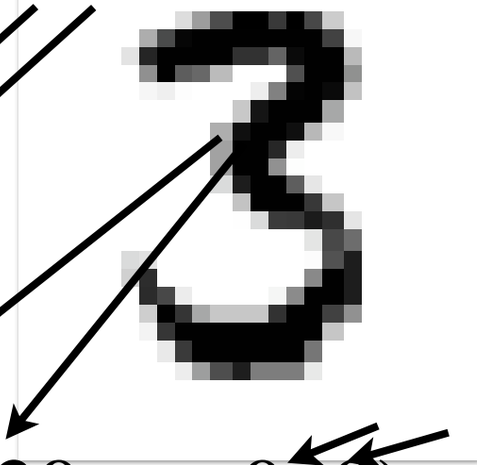


Bag of words for «The cat jumped»: $x = (... 0 ... , 0, 1, ... 0 ... , 1, 0, 0, ..., 0, 0, 1, 0, ... 0 ..$

we, the, jumped, jumping, run, elephant, dog, cat, horse

Transform an example into a vector representation $\mathbf{x} \in \mathbb{R}^d$



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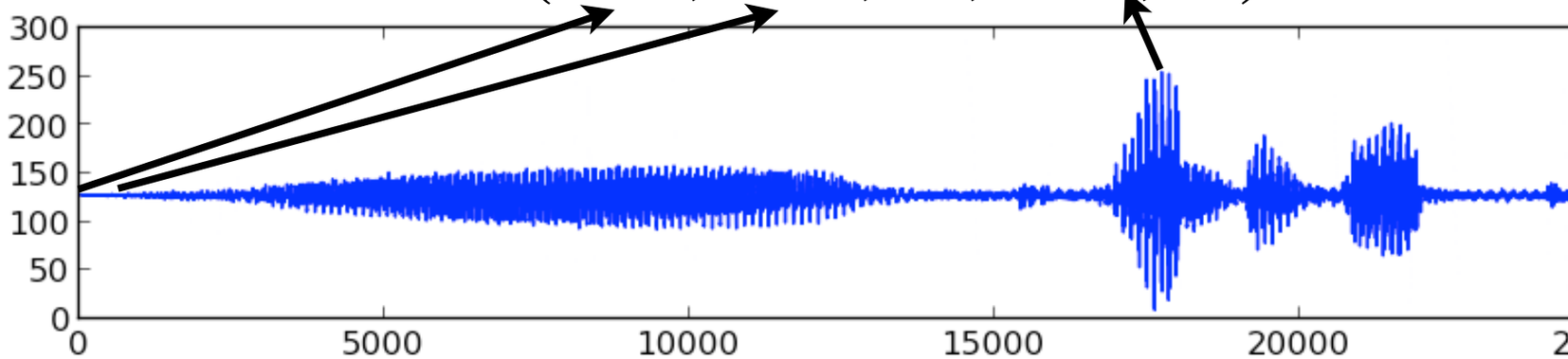

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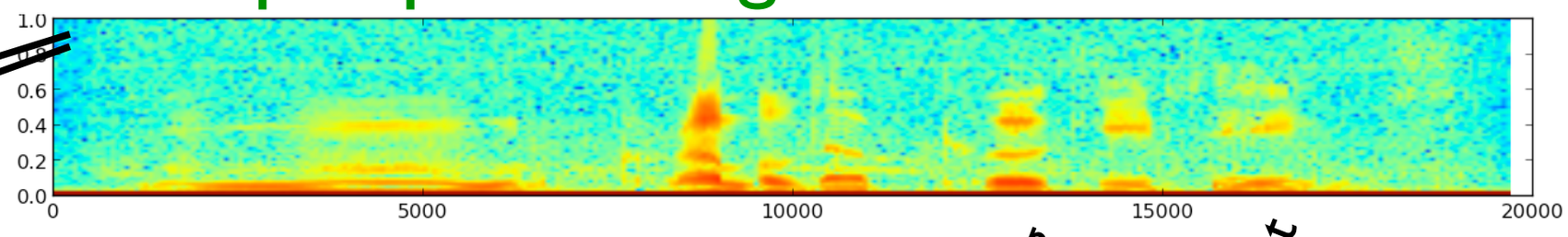

Bag of words for «The cat jumped»: $x = (... 0 ... , 0, 1, ... 0 ... , 1, 0, 0, ..., 0, 0, 1, 0, ... 0 ..$

we the jumped jumping run elephant dog cat horse

OR vector of handmade features:

$x = (\text{feature } 1, \ \ldots \ , \text{feature } d)$

ex: Histograms of Oriented Gradients

Another possibility:

Characteristic traits

describe the inputs

(features, descriptors)

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- eg. to represent a natural image (vision): SIFT descriptors.
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eg. feature k = how many times the word "plane" appears in the document.

Training set

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input:

targets:

"horse"

"cat"

etc...

"horse"

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number of examples

n

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preprocessing, feature extraction

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inputs: X
(vector of features)

targets: Y

X_1 (3.5, -2, ... , 127, 0, ...) +1 Y_1

(-9.2, 32, ... , 24, 1, ...) -1

etc...

$X_{n,2}$

X_n (6.8, 54, ... , 17, -3, ...) +1 Y_n

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$x =$ (5.7, -27, ... , 64, 0, ...) →

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→ ?

$x =$ (5.7, -27, ... , 64, 0, ...) → ?

Importance of the problem dimension

⇨ Determines which learning algorithms are applicable or not (because of their algorithmic complexity).

- Number of examples: **n**
  (often in the millions of more)
- Dimension of input: **d**
  how many features we are using
  (often in the order of 100 to 1000, sometimes in the 10s of thousands)
- Dimensions of the target we want to predict eg., number of classes m (often low, sometimes thousands)

→ A dataset will often be organized
in a matrix n x (d+1) or n x (d+m)

Terminology of supervised learning

One input generally represented by a vector of dimension d.

$x \in \mathbb{R}^d$

dimension of entry

1	observation, *input*, x_1	*target*, desired output, y_1
2	observation, *input*, x_2	*target*, desired output, y_2
3	observation, *input*, x_3	*target*, desired output, y_3
⋮	*etc…* ⋮	⋮
n	observation, *input*, x_n	*target*, desired output, y_n

size of the dataset, number of examples, or "samples"

? ? ?

Terminology of supervised learning

Training set

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size of the dataset, number of examples, or "samples"

test point  ? ? ?

We are looking for an algorithm that produces an **output** that is a good prediction of the target.

This algorithm finds a good **function x -> y**

Terminology of supervised learning

- When the target is a class label, a categorical variable (indicating which class or category the entry belongs to, among many) is said to be a **CLASSIFICATION** problem. (we often use an integer as a label).
- When the target is one (or more) real values to predict, we talk about a **REGRESSION** problem.

When we do not have an explicit target, we are in the context of **unsupervised learning.**

Learning tasks

Supervised learning = predict a target y from the input x

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Unsupervised learning: no explicit target (label) y

Learning tasks

Supervised learning = predict a target y from the input x

- y represents a category or "class"
  ➠classification (binary or multiclass)
- y is a real value
  ➠ regression

Unsupervised learning: no explicit target (label) y

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Unsupervised learning: no explicit target (label) y

- modeling the distribution of x
  ➠density estimation

- discover an underlying structure in x
  ➠ clustering
  ➠ dimensionality reduction (eg. for visualisation)

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} predictive models

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  ➠ dimensionality reduction (eg. for visualisation)

} descriptive models

Learning tasks

Semi-supervised learning:
same goal as supervised learning, but using both labeled (with target) and unlabeled examples.

Reinforcement learning
an artificial agent must learn to decide what actions to perform in a changing environment to maximize a total reward.

Data set seen as a scatter plot in a high-dimensional vector space

(input) $\mathbf{x} \in \mathbb{R}^d$ | (label) y

n examples

$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$	$\mathbf{x}_4$	$\mathbf{x}_5$	
0.32	-0.27	+1	0	0.82	1
-0.12	0.42	-1	1	0.22	0
0.06	0.35	-1	1	-0.37	1
0.91	-0.72	+1	0	-0.63	1
...	...	...	...	...	...

Each example (row) is a vector of d+1-dimensions

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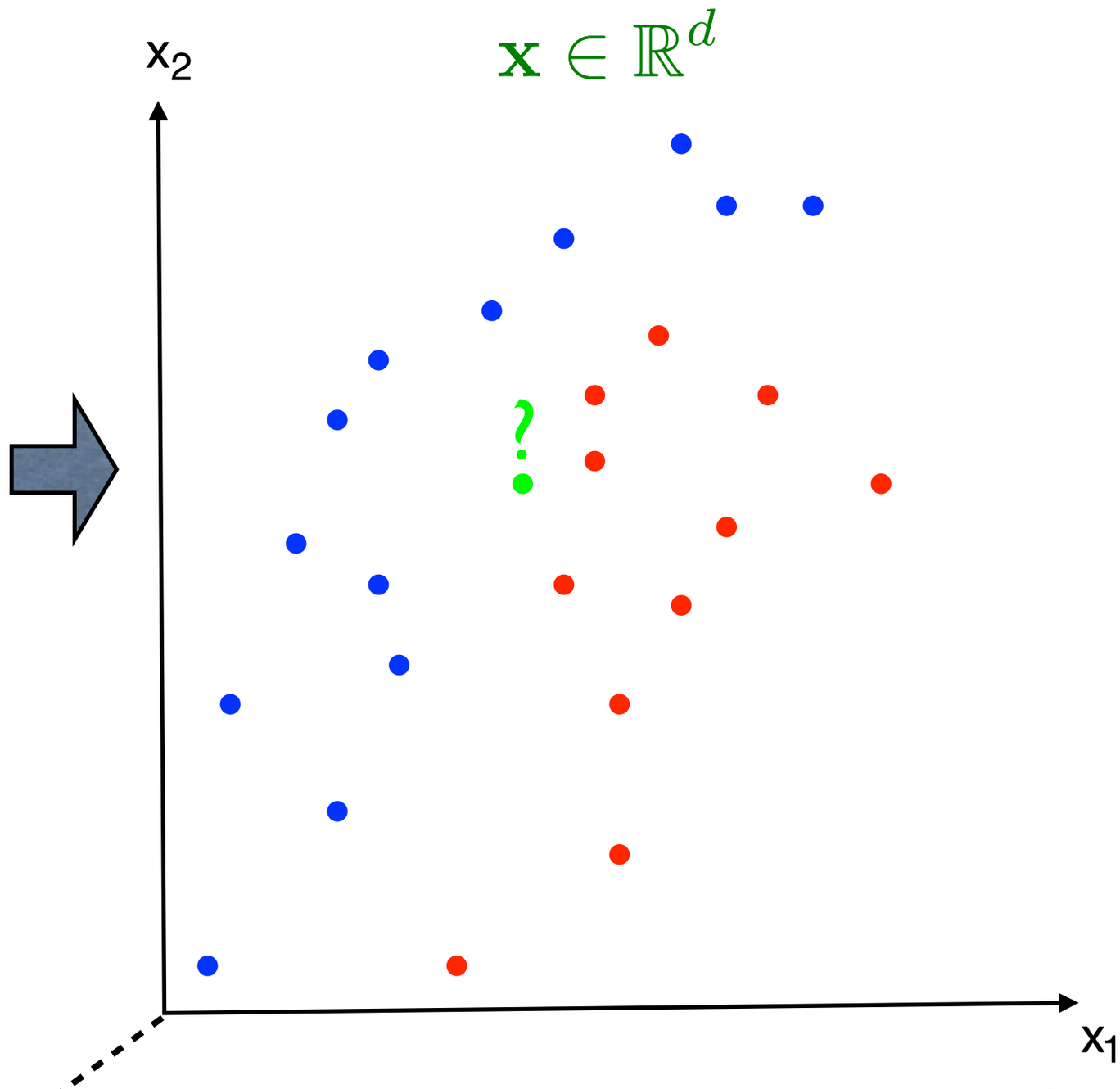
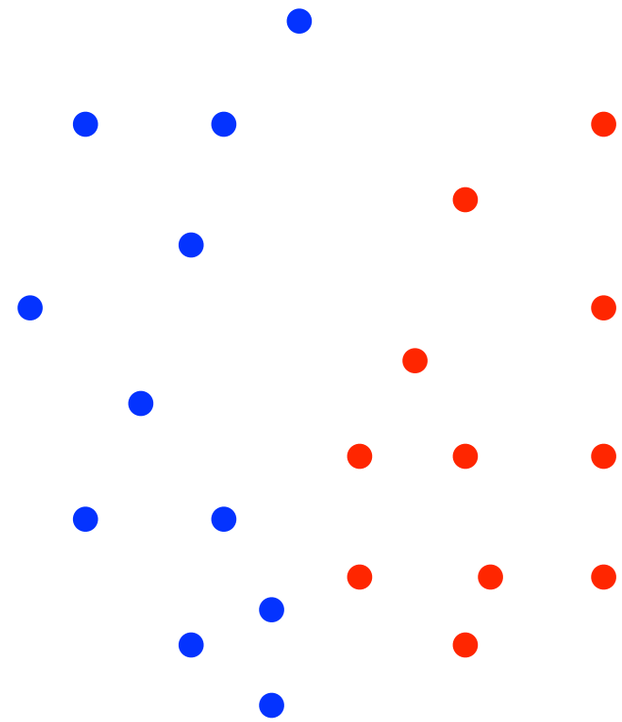


Each entry is a point in a space of d-dimensions

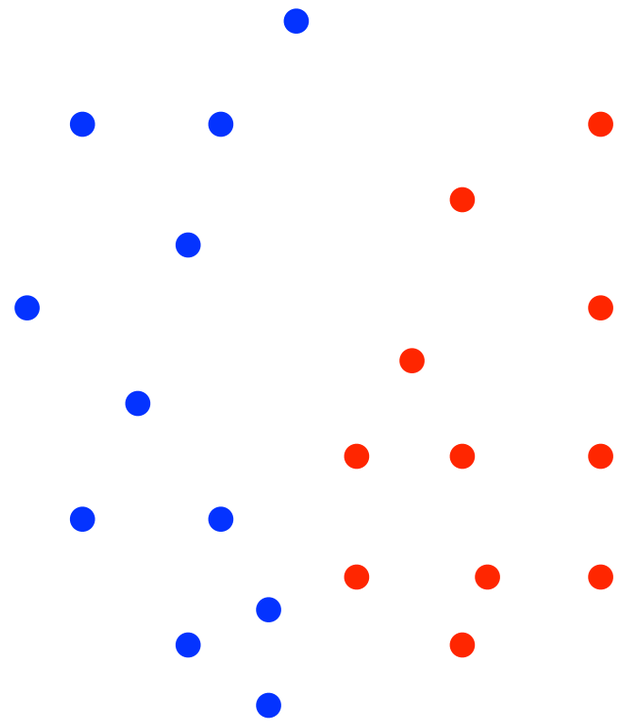
The nearest neighbor algorithm



Training set

The nearest neighbor algorithm

For a test point **x**:



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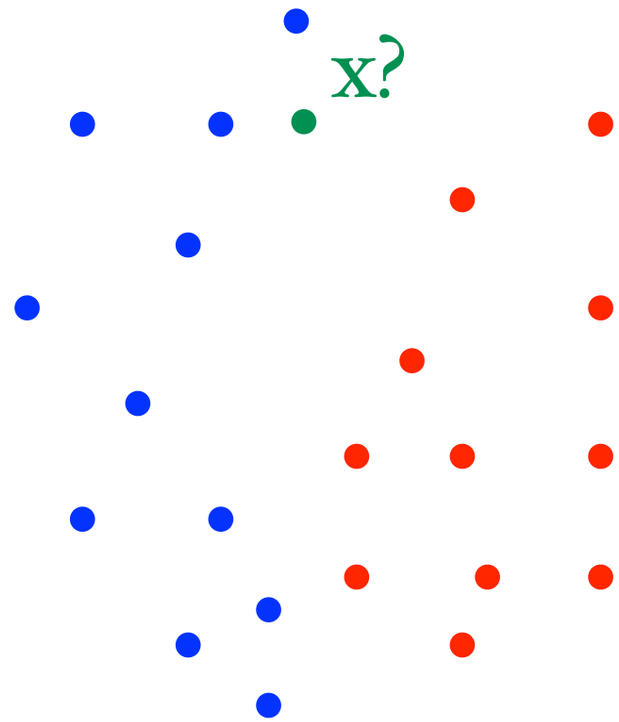
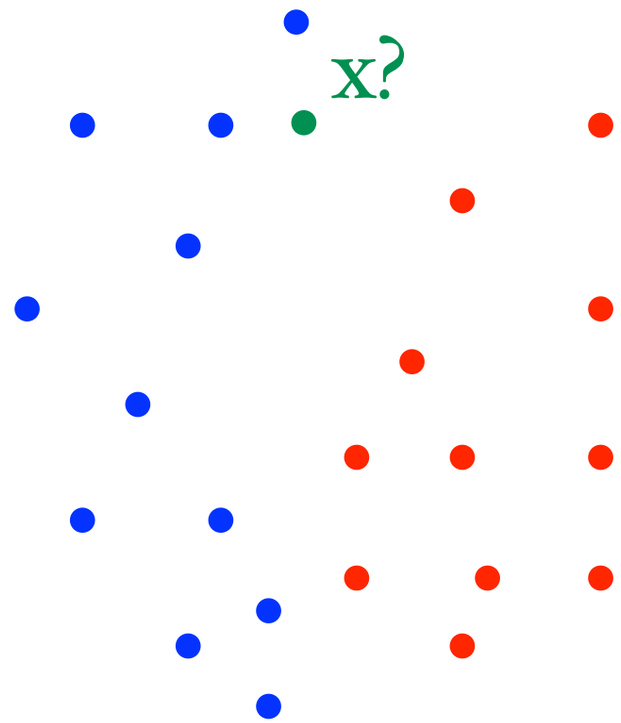


Training set

The nearest neighbor algorithm

For a test point x:

- We find the **nearest neighbour** of x within the training set by some measure of distance (eg Euclidean distance).



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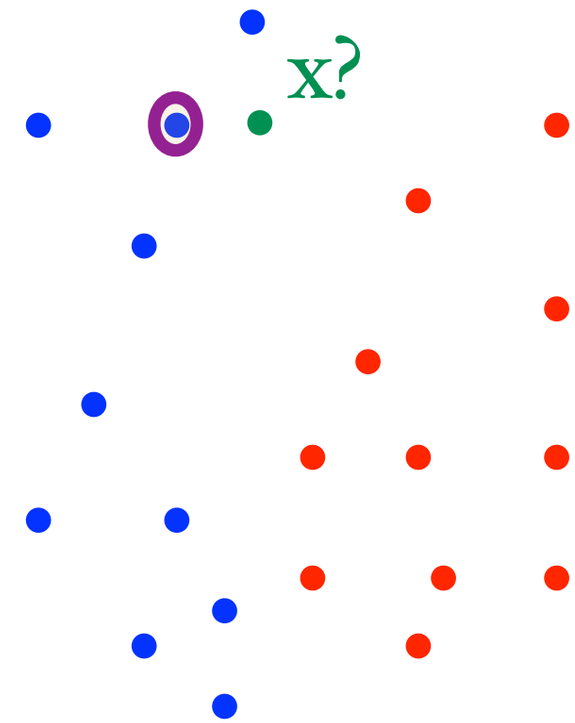

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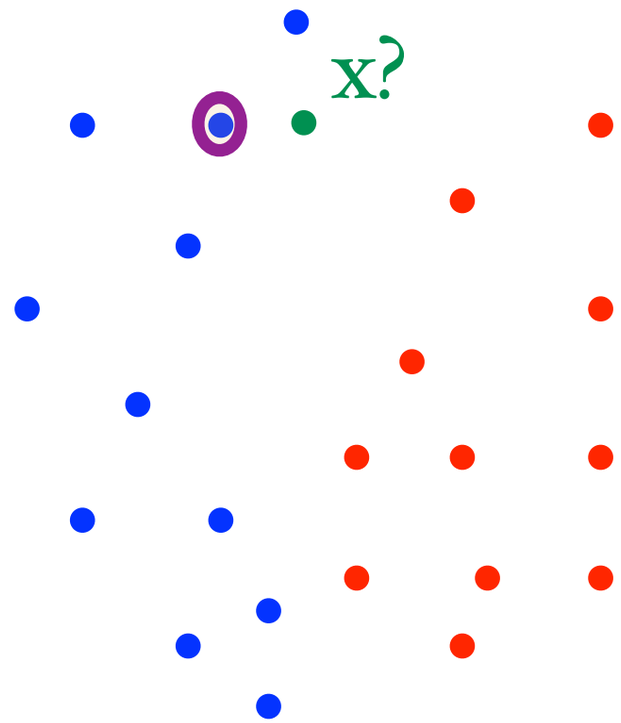

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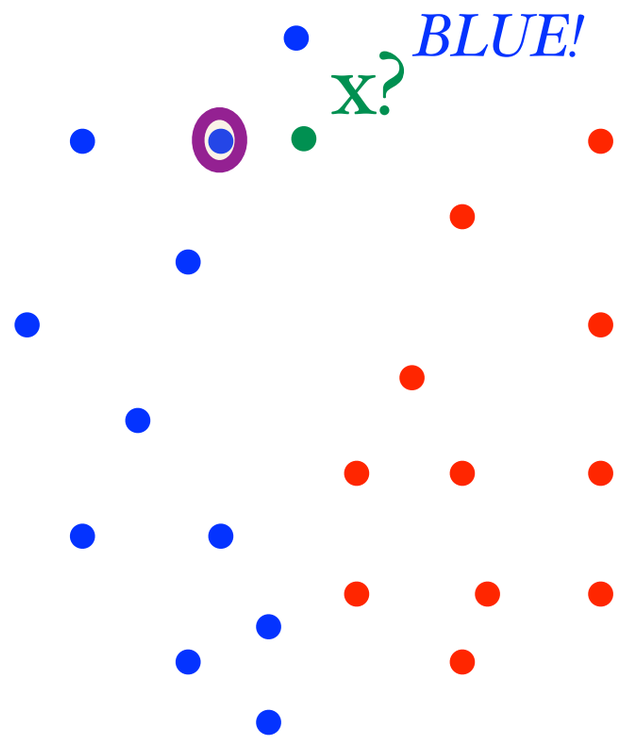


Training set

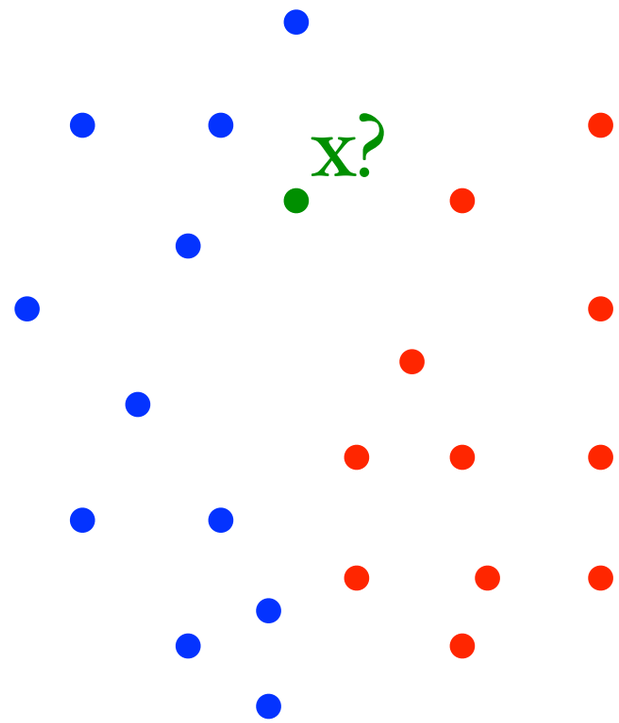
The k-nearest neighbors algorithm (k-NN)

For a test point x:

k=3

- We find the **k nearest neighbors** of x within the training set apprentissage by some measure of distance (eg Euclidean distance).

- We associate x with the class of the **majority** of neighbors.



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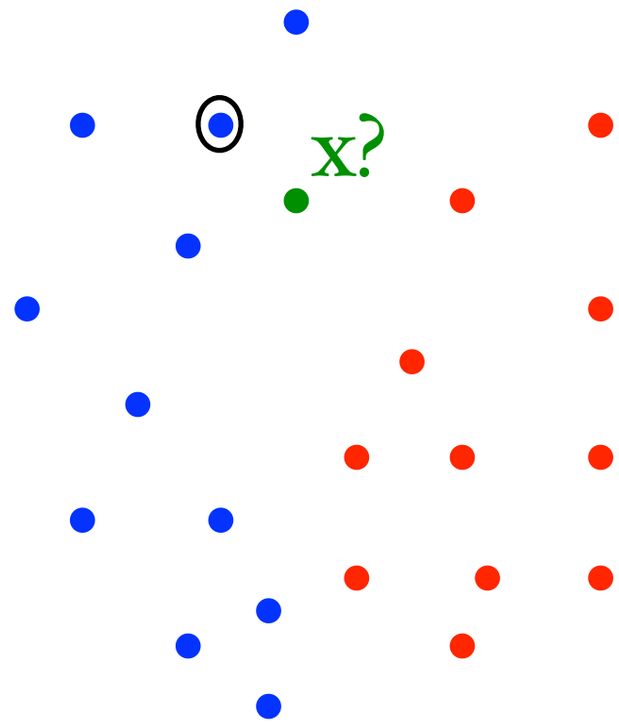

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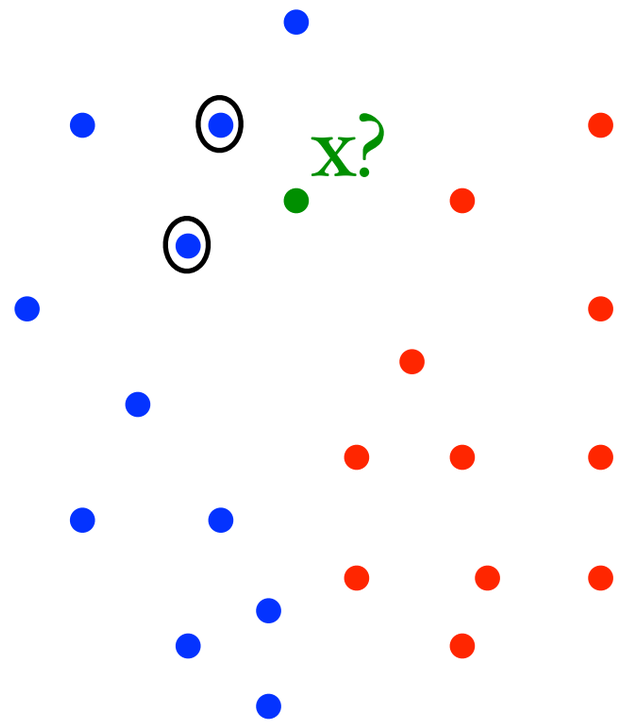

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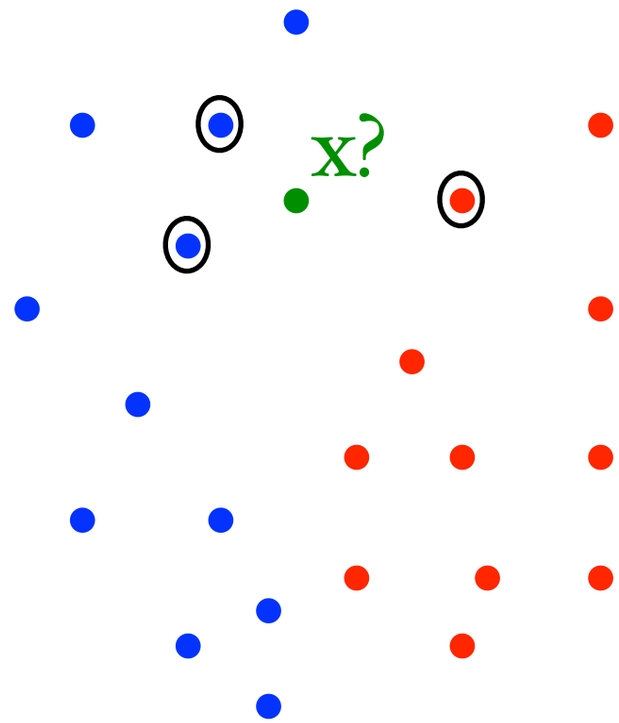

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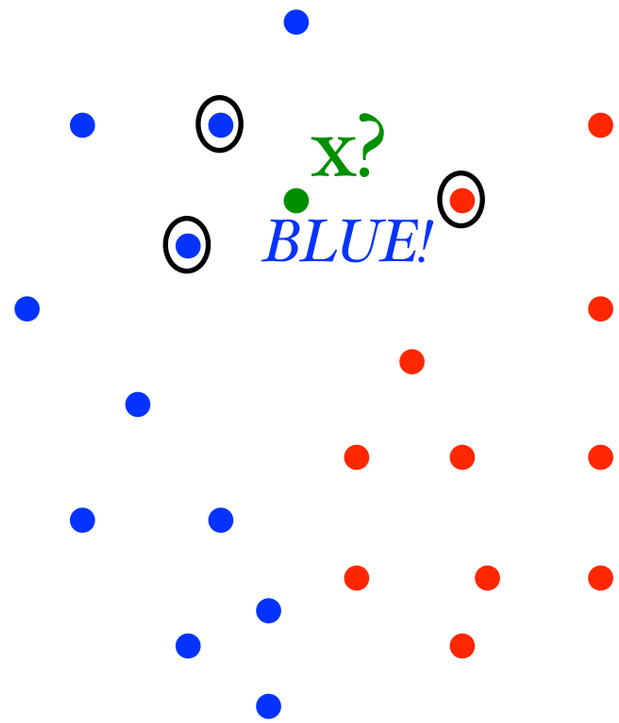

Training set

Short review of vectors

(of dimension d)

- what is a vector? mathematical, computer, graphic representations ...
- Euclidean distance
- norms
- scalar product
- efficient calculation of the distance from a point to a set of points

The notion of level of representation