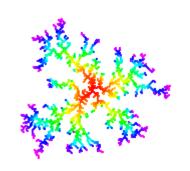
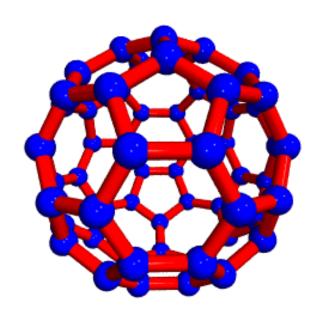
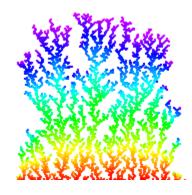


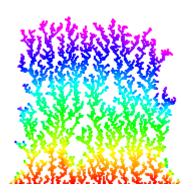
Data Structures and Algorithms







Elementary Data Structures: Stacks and Queues



Stack and Queue ADTs

Fundamental data types

- → Set of operations (add, remove, test if empty) on generic data.
- → Intent is clear when we insert.
- → Which object do we remove?
- Stack ("last in first out" or LIFO)
 - Remove the object most recently added.
 - → Analogy: cafeteria trays, surfing Web.
- Queue ("first in first out" or FIFO)
 - → Remove the object least recently added.
 - → Analogy: Registrar's line.

MultiSet

- -> Remove any object.
- Law professor calls on arbitrary student.

Stack

 A <u>stack</u> supports the insert and remove operations using a LIFO discipline. We name insert operation as <u>push</u> and the remove operation <u>pop</u>.



We also include a method is Empty to test whether or not the stack is empty. Each element in the collection can be any Java Object. Below we consider two different implementation of the following Stack interface:

```
public interface Stack {
    public boolean isEmpty();
    public Object peek();
    public void push(Object value);
    public Object pop();
}
```

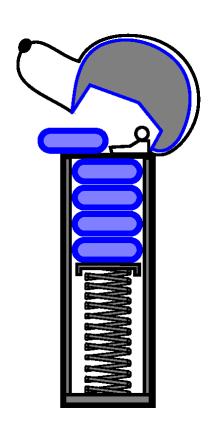
A simple application of stack

 Before we describe how to implement a stack, we give a simple client program that reads in an arbitrary sequence of values and prints them in reverse order.

```
public static void main(String[] args) {
   Stack stack = new StackArray();

   while (StdIn.hasNextString()) {
      String s = StdIn.nextString();
      stack.push(s);
   }

   while (!stack.isEmpty()) {
      String s = (String) stack.pop();
      StdOut.println(s);
   }
}
```



A Stack Interface in Java

```
public interface StackInterface {
   // accessor methods
                                      // see if the stack
   public boolean isEmpty();
                                      // is empty
                                      // return the top element
  public Object peek()
      throws StackEmptyException;
                                      // exception if called on
                                      // an empty stack
   // update methods
   public void push (Object element); // push an element
                                      // onto the stack
                                      // remove and return the
  public Object pop()
      throws StackEmptyException;
                                      // top element
                                      // exception if called on
                                      // an empty stack
```

An Array-Based Stack Implementation

- Create a stack using an array by specifying maximum size N for our stack, e.g. N = 1,000.
- The stack consists of an N-element array S and an integer variable t, the index of the top element in array S.



- Array indices start at 0, so we initialize t to -1
- The pseudo-code follows on the next slide

```
Algorithm isEmpty():
                                                Algorithm push(o):
   return (t < 0)
                                                    if size() = N then
                                                      throw a StackFullException
                                                    t \leftarrow t+1
Algorithm pop():
                                                    S[t] \leftarrow o
   if isEmpty() then
      throw a StackEmptyException
   e \leftarrow S[t]
                                                Algorithm peek():
   S[t] \leftarrow \text{null}
                                                    if isEmpty() then
                                                      throw a StackEmptyException
   t \leftarrow t - 1
                                                    return S[t]
   return e
```

An Array-Based Stack

- Each of the above methods runs in constant time
 - \rightarrow i.e. (O(1))



- The array implementation is simple and efficient.
- There is an upper bound, N, on the size of the stack.
 - → The arbitrary value N may be too small for a given application,
 - → or a waste of memory in other cases.

```
public class StackArray implements StackInterface {
   // Implementation of the Stack interface
   // using an array.
   // default capacity of the stack
  public static final int CAPACITY = 1000;
   // maximum capacity of the stack.
   private int capacity;
   // S holds the elements of the stack
   private Object S[];
   // the top element of the stack.
  private int top = -1;
```

```
public StackArray(int cap) {
   // Initialize the stack with given capacity
   capacity = cap;
   S = new Object[capacity];
public StackArray() {
   // Initialize the stack with default capacity
   this(CAPACITY);
public boolean isEmpty() {
   // Return true iff the stack is empty
   return (top < 0);
```







```
public void push(Object obj) throws StackFullException
   // Push a new object on the stack
   if (size() == capacity)
      throw new StackFullException("Stack overflow.");
   S[++top] = obj;
                          // ++top or top++
public Object peek() throws StackEmptyException
   // Return the top stack element
   if (isEmpty())
   throw new StackEmptyException("Stack is empty.");
   return S[top];
```



```
public Object pop()
    // Pop off the stack element
    throws StackEmptyException {
    Object elem;
    if (empty())
        throw new StackEmptyException("Stack is Empty");
    elem = S[top];
    // Dereference S[top] and decrement top
    S[top--] = null;
    return elem;
}
```



Performance

500,000 pop, push, and peek operations

	initial capacity	
Class	10	500,000
ArrayStack	0.44s	0.22s
DerivedArrayStack	0.60s	0.38s
DerivedArrayStackWithCatch	0.55s	0.33s
java.util.Stack	1.15s	-
DerivedLinkedStack	3.20s	3.20s
LinkedStack	2.96s	2.96s



Evaluation

- Code developed from scratch will run faster but will take more time (cost) to develop.
- Tradeoff between software development cost and performance.
- Tradeoff between time to market and performance.
- Could develop easy code first and later refine it to improve performance.



⇒ Stack stack = new StackArray();

N = 0 elements on stack



```
Stack stack = new StackArray();

⇒ stack.push("A");

N = 1

index 0

value A
```

Double size of array

```
Stack stack = new StackArray();
stack.push("A");
stack.push("B");
stack.push("C");

index 0 1 2 3
value A B C ?
```

Double size of array

```
Stack stack = new StackArray();
   stack.push("A");
                                                      N = 5
  stack.push("B");
   stack.push("C");
   stack.push("D");
                             index
                                      0
                                                 3
                                                     E
⇒ stack.push("E");
                              value
                                      A
                                                 D
                                                        ?
                                                                ?
```

Double size of array

```
Stack stack = new StackArray();
  stack.push("A");
                                                    N = 5
  stack.push("B");
  stack.push("C");
  stack.push("D");
                            index
                                               3
                                     0
  stack.push("E");
                            value
                                               D
                                                              ?
                                     A
  stack.push("F");
System.out.println(stack.pop());
```

% java StackArray F

```
Stack stack = new StackArray();
   stack.push("A");
                                                   N = 4
   stack.push("B");
   stack.push("C");
   stack.push("D");
                              index
                                                  3
                                       0
                              value
                                                  D
                                                                 ?
   stack.push("E");
                                       A
   stack.push("F");
   System.out.println(stack.pop());
⇒ System.out.println(stack.pop());
                                                   java StackArray
                                                  \mathbf{E}
```

```
Stack stack = new StackArray();
   stack.push("A");
   stack.push("B");
                                                       N = 5
   stack.push("C");
   stack.push("D");
                              index
                                                  3
                                       0
                              value
                                                  D
                                                     G
                                                                 ?
   stack.push("E");
                                       A
   stack.push("F");
   System.out.println(stack.pop());
   System.out.println(stack.pop());
                                                   java StackArray
⇒ stack.push("G");
                                                 \mathbf{E}
```

```
Stack stack = new StackArray();
  stack.push("A");
  stack.push("B");
                                                 N
  stack.push("C");
  stack.push("D");
                             index
                                                3
                                     0
                             value
                                                   G
                                                               ?
  stack.push("E");
                                     A
                                                D
  stack.push("F");
  System.out.println(stack.pop());
  System.out.println(stack.pop());
                                                  java StackArray
  stack.push("G");
                                                F
⇒ System.out.println(stack.pop());
                                                Ε
```

```
Stack stack = new StackArray();
  stack.push("A");
                                              N = 3
  stack.push("B");
  stack.push("C");
  stack.push("D");
                             index
                                                3
                                     0
                             value
                                                    G
                                                               ?
  stack.push("E");
                                     A
                                                D
  stack.push("F");
  System.out.println(stack.pop());
  System.out.println(stack.pop());
                                                  java StackArray
  stack.push("G");
                                                F
  System.out.println(stack.pop());
                                                Ε
                                                G
⇒ System.out.println(stack.pop());
                                                D
```

```
Stack stack = new StackArray();
  stack.push("A");
                                          N = 2
  stack.push("B");
  stack.push("C");
  stack.push("D");
                             index
                                                3
                                     0
                             value
                                                   G
                                                              ?
  stack.push("E");
                                     A
                                                D
  stack.push("F");
  System.out.println(stack.pop());
  System.out.println(stack.pop());
                                                 java StackArray
  stack.push("G");
                                                F
  System.out.println(stack.pop());
                                               Ε
  System.out.println(stack.pop());
⇒ System.out.println(stack.pop());
```

Input





push 1

Input





push 1

Top of stack



1

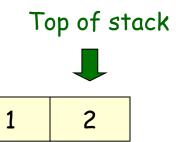
Input 2 3 4 5 * + 6 * + +

Top of stack

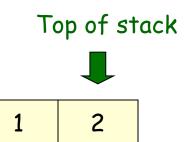


1

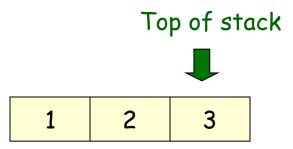
Input 1 2 3 4 5 * + 6 * + +



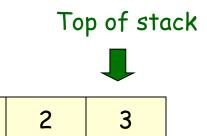
Input 1 2 3 4 5 * + 6 * + +



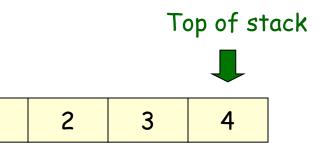
Input 1 2 3 4 5 * + 6 * + +

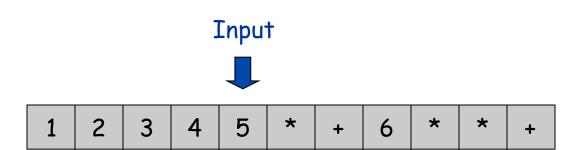


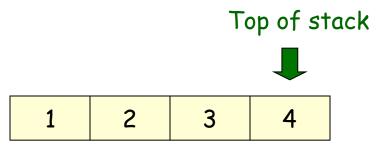


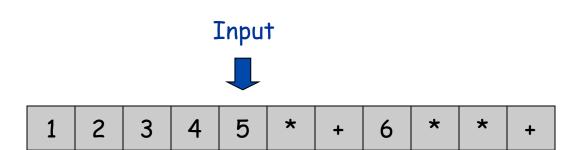


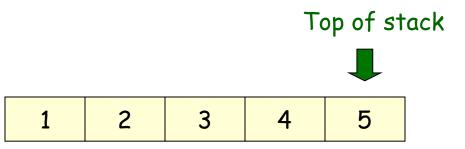


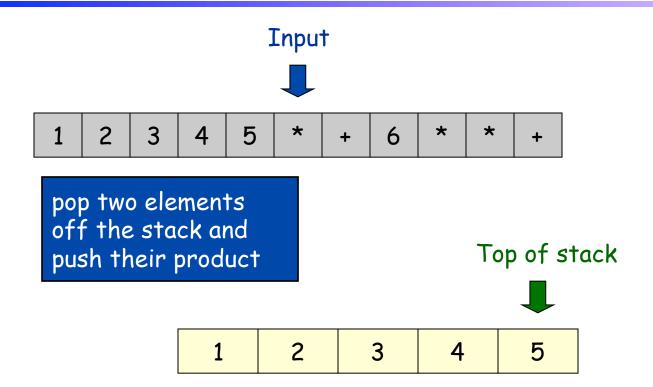


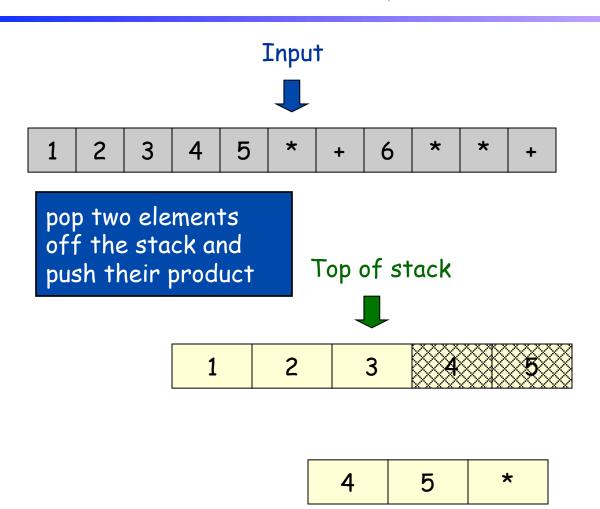


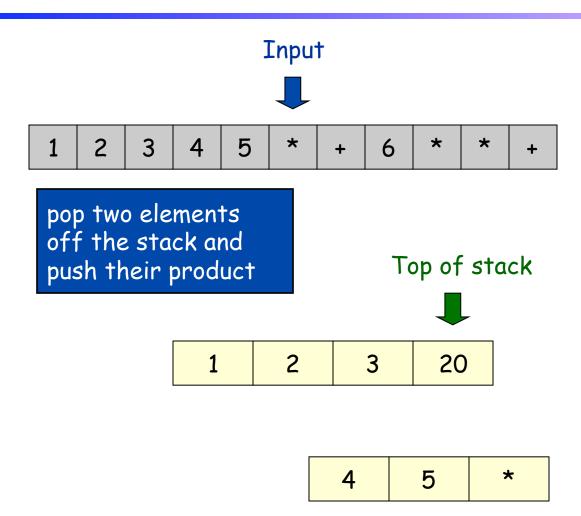


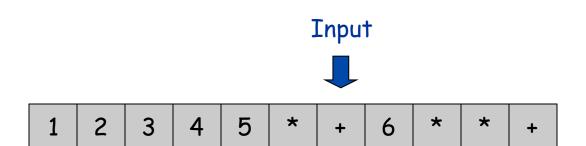






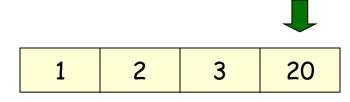


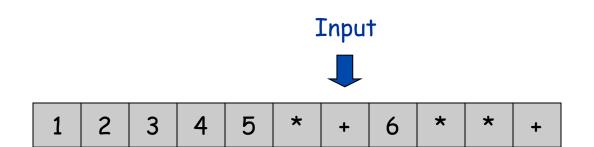




Top of stack

pop two elements off the stack and push their sum



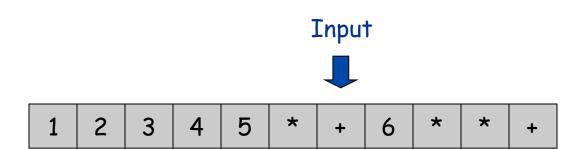


Top of stack



pop two elements off the stack and push their sum





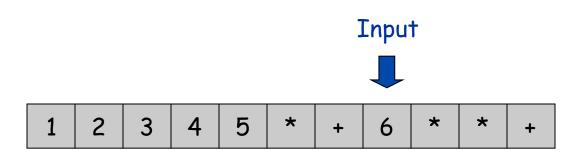
Top of stack

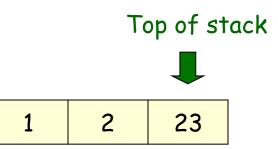


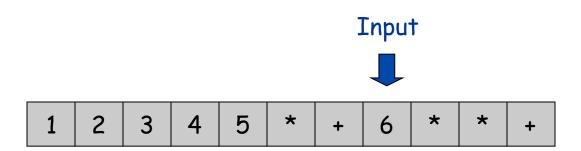
1 2 23

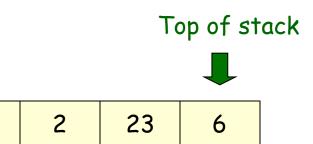
pop two elements off the stack and push their sum

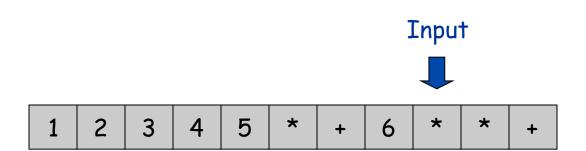




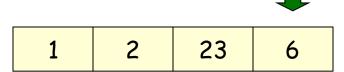


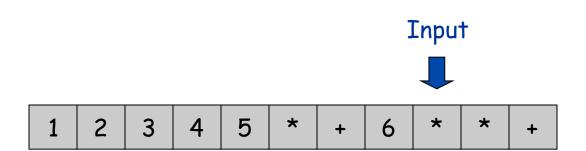


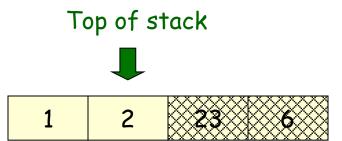




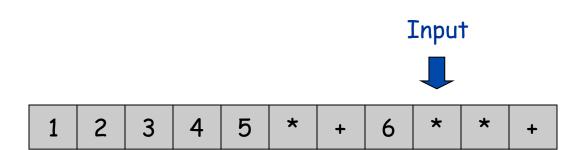
Top of stack

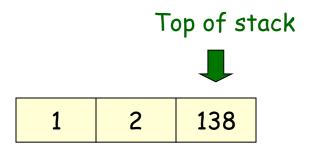




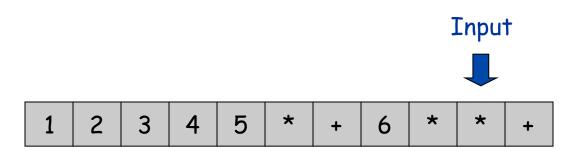


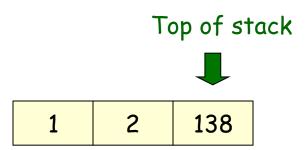










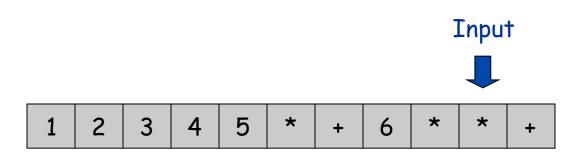


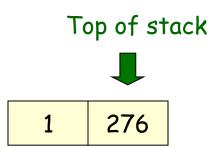
Input 1 2 3 4 5 * + 6 * + +

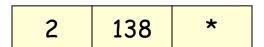
Top of stack



2 138 *

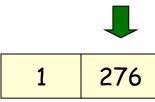






Input 1 2 3 4 5 * + 6 * * +

Top of stack



Input 1 2 3 4 5 * + 6 * * +

Top of stack







Input 1 2 3 4 5 * + 6 * * +

Top of stack



277



Input

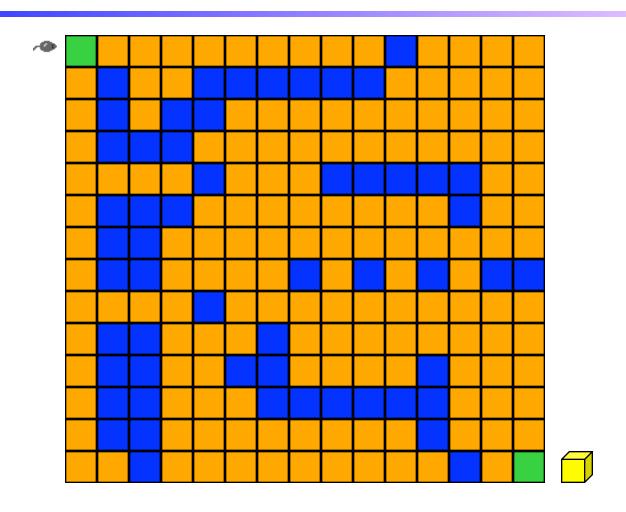




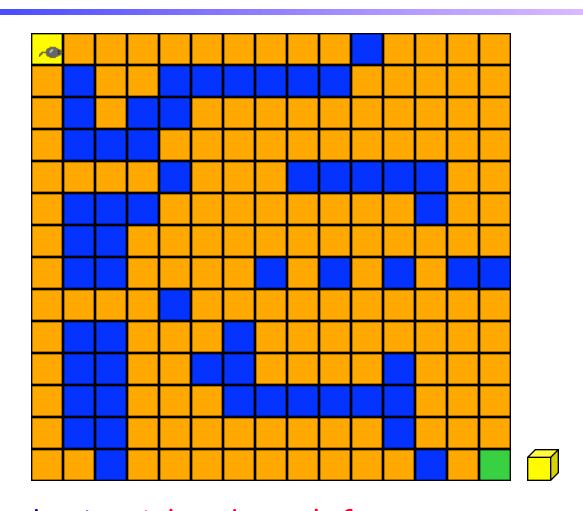
Top of stack



277

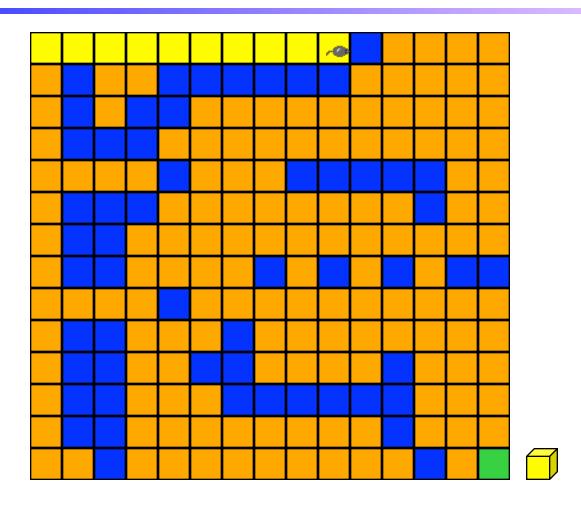




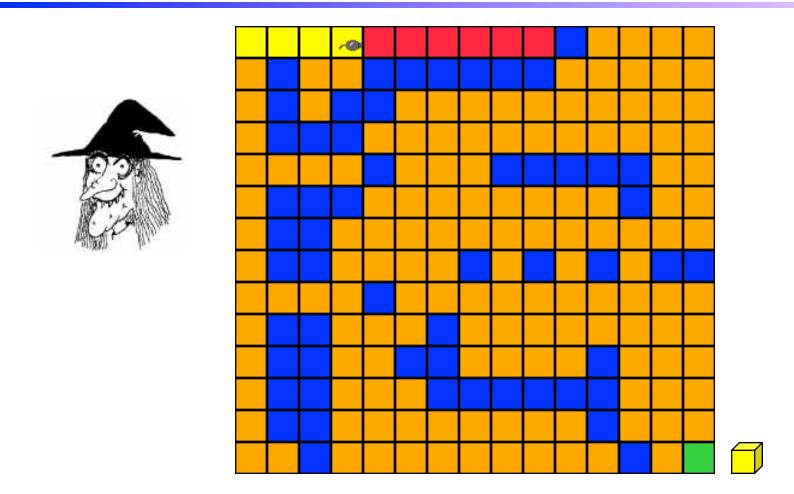


- Move order is: right, down, left, up
- Block positions to avoid revisit.



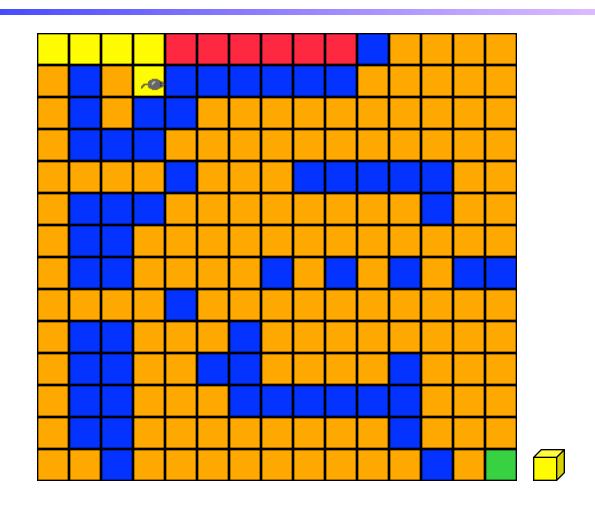


 Move backward until we reach a square from which a forward move is possible.

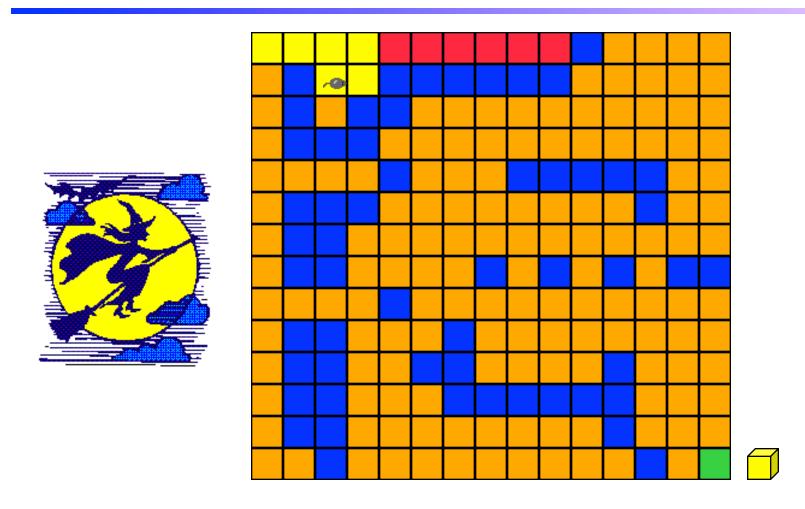


□ Move down.

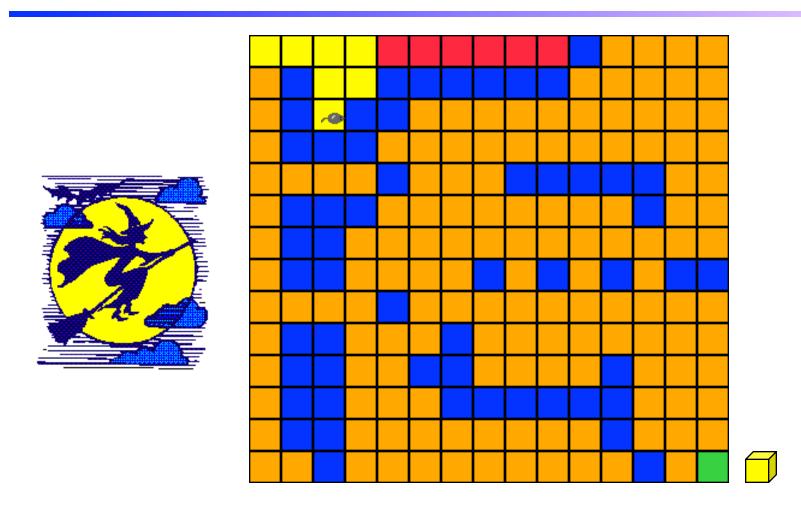




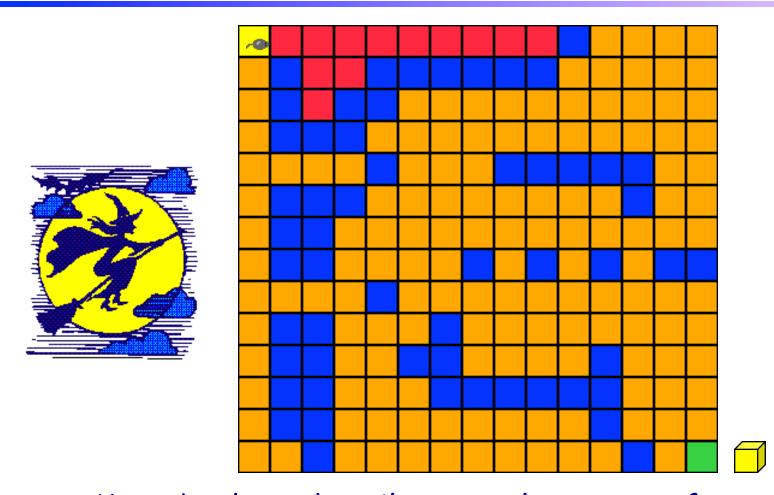
□ Move left.



□ Move down.

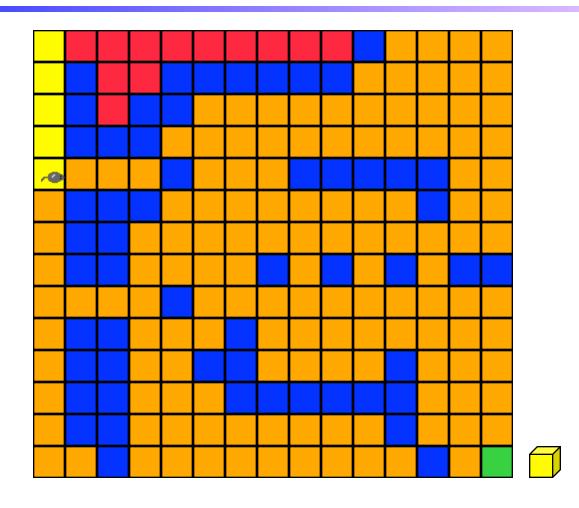


 Move backward until we reach a square from which a forward move is possible.

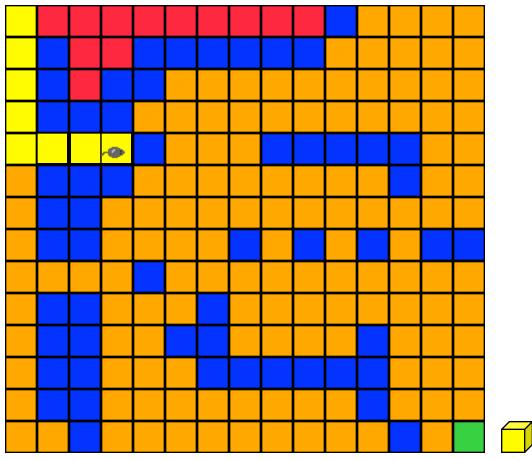


- Move backward until we reach a square from which a forward move is possible.
- Move downward

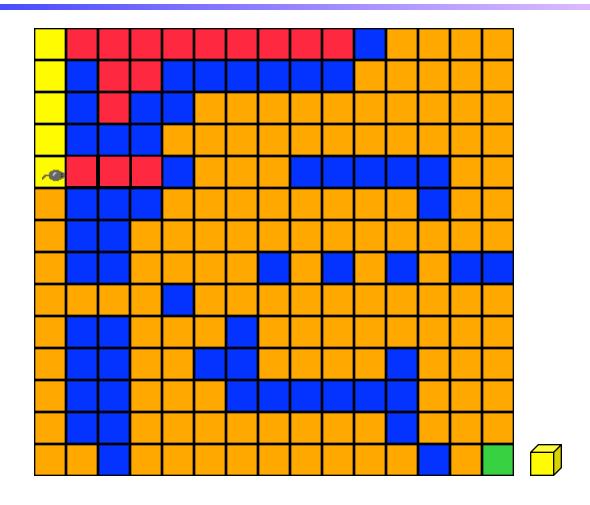


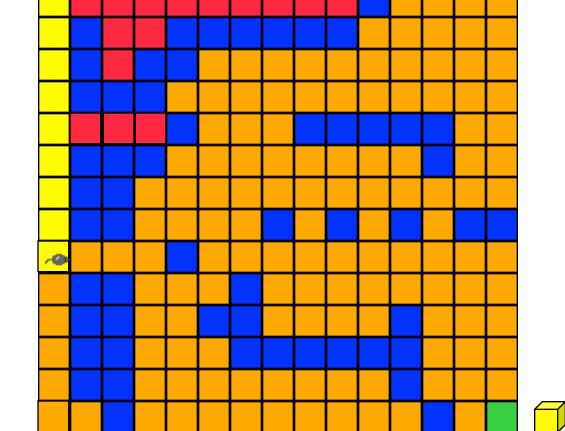






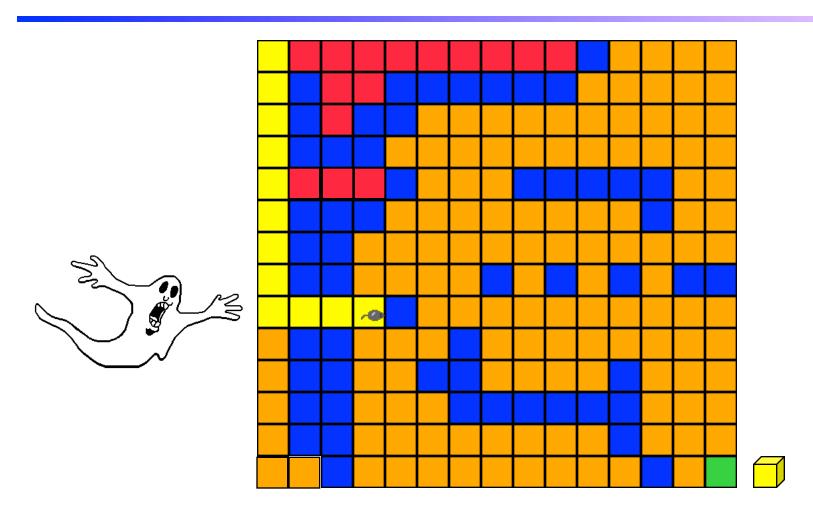




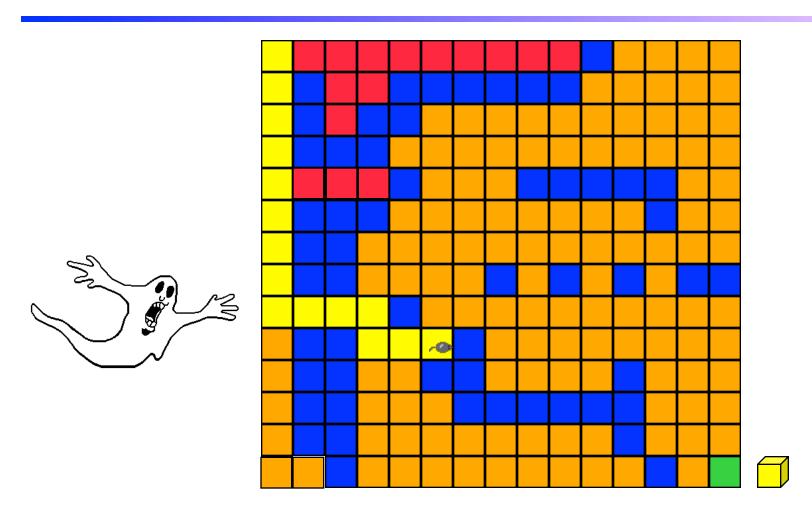




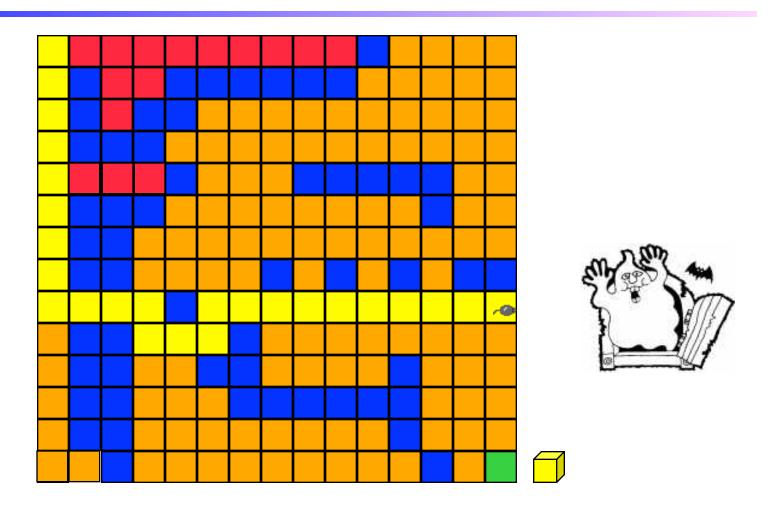
- □ Move right.
- Backtrack.



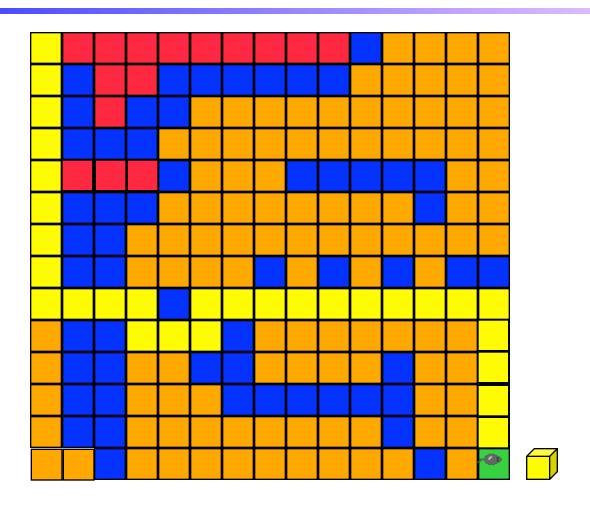
Move one down and then right.

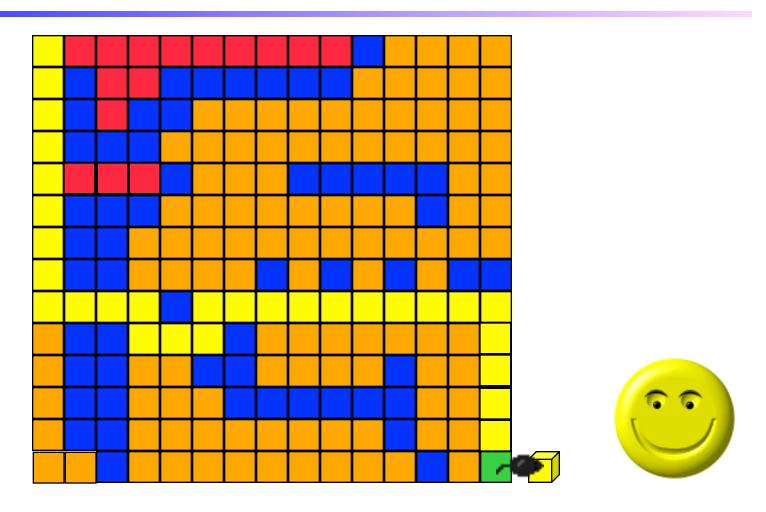


Move one up and then right.



Move down to exit and eat cheese.





 Path from maze entry to current position operates as a stack.

Rat In A Maze - Java code (1)

```
/** find a path from (1,1) to the exit (size, size)
    * @return true if successful, false if impossible */
 private static boolean findPath()
    path = new StackArray();
    // initialize offsets
    Position [] offset = new Position [4];
    offset[0] = new Position(0, 1); // right
    offset[1] = new Position(1, 0); // down
    offset[2] = new Position(0, -1); // left
    offset[3] = new Position(-1, 0); // up
    // initialize wall of obstacles around maze
     for (int i = 0; i \le size + 1; i++)
       maze[0][i] = maze[size + 1][i] = 1; // bottom and top
       maze[i][0] = maze[i][size + 1] = 1; // left and right
```

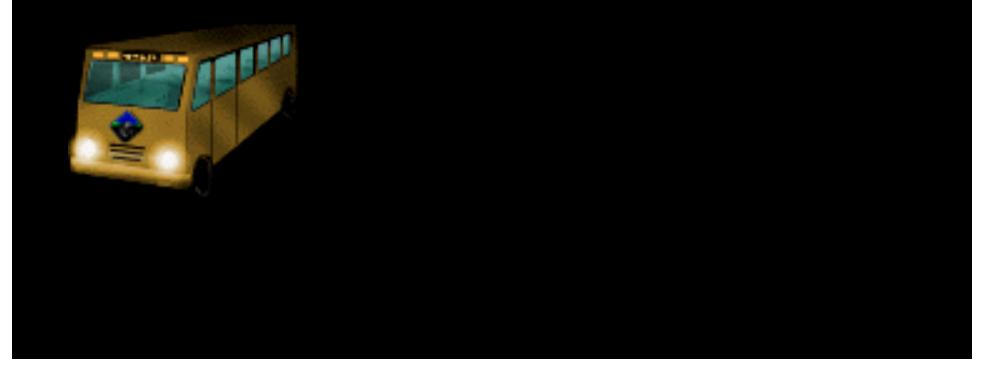
Rat In A Maze - Java code (2)

```
Position here = new Position(1, 1);
maze[1][1] = 1; // prevent return to entrance
int option = 0; // next move
int lastOption = 3;
// search for a path
while (here.row != size || here.col != size)
{ // not at exit
    // find a neighbor to move to
    // won't compile without explicit initialization
    int r = 0, c = 0; // row and column of neighbor
    while (option <= lastOption)</pre>
       r = here.row + offset[option].row;
       c = here.col + offset[option].col;
       if (maze[r][c] == 0) break;
       option++; // next option
```

Rat In A Maze - Java code (3)

```
// was a neighbor found?
if (option <= lastOption) // yes</pre>
{ // move to maze[r][c]
   path.push(here);
   here = new Position(r, c);
   // set to 1 to prevent revisit
   maze[r][c] = 1;
   option = 0;
else
{ // no neighbor to move to, back up
   if (path.isEmpty()) return false; // no place to
                                       // back up to
   Position next = (Position) path.pop();
   if (next.row == here.row)
      option = 2 + next.col - here.col;
   else
      option = 3 + next.row - here.row;
   here = next;
return true; // at exit
```















rear











front

rear







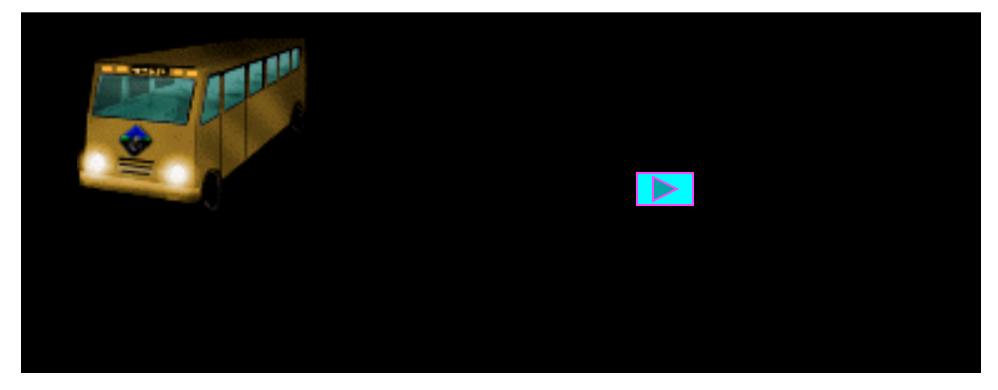






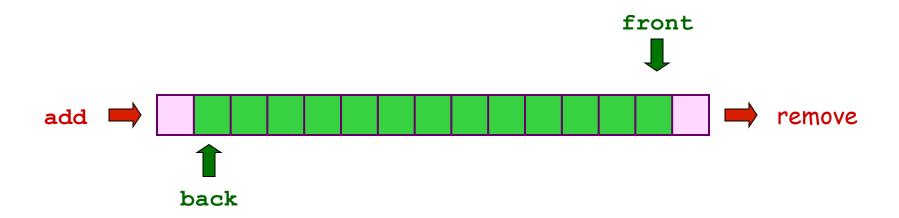
front

rear



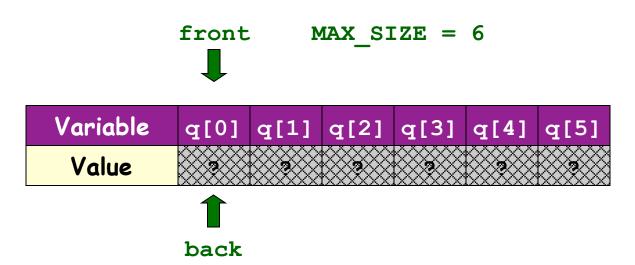
Queue

- Queue operations.
 - → add (enqueue) Insert a new object onto queue.
 - → remove (dequeue) Delete and return the object least recently added.
 - → isEmpty Is the queue empty?



q.init();

q.init();



```
q.init();
q.add('A');
```





Variable	q[0]	q[1]	q[2]	q[3]	q[4]	q[5]
Value				2		



```
q.init();
q.add('A');
```





Variable	q[0]	q[1]	q[2]	q[3]	q[4]	q[5]
Value	A			9		,



```
q.init();
q.add('A');
q.add('D');
```

front



Variable	q[0]	q[1]	q[2]	q[3]	q[4]	q[5]
Value	A			9		,



```
q.init();
q.add('A');
q.add('D');
```

front



Variable	q[0]	q[1]	q[2]	q[3]	q[4]	q[5]
Value	A	D	2		•	,



```
q.init();
q.add('A');
q.add('D');
q.add('T');
```

front



Variable	q[0]	q[1]	q[2]	q[3]	q[4]	q[5]
Value	A	D	2		•	,



```
q.init();
q.add('A');
q.add('D');
q.add('T');
```

front



Variable	q[0]	q[1]	q[2]	q[3]	q[4]	q[5]
Value	A	D	Т			,



```
q.init();
q.add('A');
q.add('D');
q.add('T');
q.add('E');
```

front



Variable	q[0]	q[1]	q[2]	q[3]	q[4]	q[5]
Value	A	D	Т	2	?	



```
q.init();
q.add('A');
q.add('D');
q.add('T');
q.add('E');
```

front



Variable	q[0]	q[1]	q[2]	q[3]	q[4]	q[5]
Value	A	D	T	E	9	2



```
q.init();
q.add('A');
q.add('D');
q.add('T');
q.add('E');
q.add();
```



Variable	q[0]	q[1]	q[2]	q[3]	q[4]	q[5]
Value	A	D	T	E	2	?



```
q.init();
q.add('A');
q.add('D');
q.add('T');
q.add('E');
q.remove();
```



Variable	q[0]	q[1]	q[2]	q[3]	q[4]	q[5]
Value	24	D	T	E	?	



Items dequeued: A

```
q.init();
q.add('A');
q.add('D');
q.add('T');
q.add('E');
q.remove();
q.remove();
```



Variable	q[0]	q[1]	q[2]	q[3]	q[4]	q[5]
Value	A	D	Т	E		2



Items dequeued: A

```
q.init();
q.add('A');
q.add('D');
q.add('T');
q.add('E');
q.remove();
q.remove();
```



Variable	q[0]	q[1]	q[2]	q[3]	q[4]	q[5]
Value	2	D	T	E	2	2



```
q.init();
q.add('A');
q.add('D');
q.add('T');
q.add('E');
q.remove();
q.remove();
q.remove();
```



Variable	q[0]	q[1]	q[2]	q[3]	q[4]	q[5]
Value	2	b	T	E	2	



```
q.init();
q.add('A');
q.add('D');
q.add('T');
q.add('E');
q.remove();
q.remove();
q.remove();
```



Variable	q[0]		q[2]	q[3]	q[4]	q[5]
Value	2	D	T	E	2	?



```
q.init();
q.add('A');
q.add('D');
q.add('T');
q.add('E');
q.remove();
q.remove();
q.remove();
q.add('X');
```



Variable	q[0]	q[1]	q[2]	q[3]	q[4]	q[5]
Value	A	D	T	F	?	2



```
q.init();
q.add('A');
q.add('D');
q.add('T');
q.add('E');
q.remove();
q.remove();
q.remove();
q.add('X');
```



Variable	q[0]	q[1]	q[2]	q[3]	q[4]	q[5]
Value	24	D	T	E	Х	



```
q.init();
q.add('A');
q.add('D');
q.add('T');
q.add('E');
q.remove();
q.remove();
q.remove();
q.add('X');
q.add('A');
```



Variable	q[0]	q[1]	q[2]	q[3]	q[4]	q[5]
Value	24	D	T	E	Х	



```
q.init();
q.add('A');
q.add('D');
q.add('T');
q.add('E');
q.remove();
q.remove();
q.remove();
q.add('X');
q.add('A');
```

```
// modulo function takes care of
// wrapping around front & back
back = (back + 1) % q.size
```

front



Variable	q[0]	q[1]	q[2]	q[3]	q[4]	q[5]
Value	2	D	T	E	x	A



back

```
q.init();
q.add('A');
q.add('D');
q.add('T');
q.add('E');
q.remove();
q.remove();
q.remove();
q.add('X');
q.add('A');
q.add('M');
```



Variable	q[0]			q[3]	q[4]	q[5]
Value	A	D	F	E	X	A



```
q.init();
q.add('A');
q.add('D');
q.add('T');
q.add('E');
q.remove();
q.remove();
q.remove();
q.add('X');
q.add('A');
q.add('M');
```

wrap-around



Variable	q[0]	q[1]	q[2]	q[3]	q[4]	q[5]
Value	M	D	F.	E	X	A



```
q.init();
q.add('A');
q.add('D');
q.add('T');
q.add('E');
q.remove();
q.remove();
q.remove();
q.add('X');
q.add('A');
q.add('M');
q.remove();
```



Variable	q[0]			q[4]	q[5]
Value	M	D	T.	x	A



```
q.init();
q.add('A');
q.add('D');
q.add('T');
q.add('E');
q.remove();
q.remove();
q.remove();
q.add('X');
q.add('A');
q.add('M');
q.remove();
```



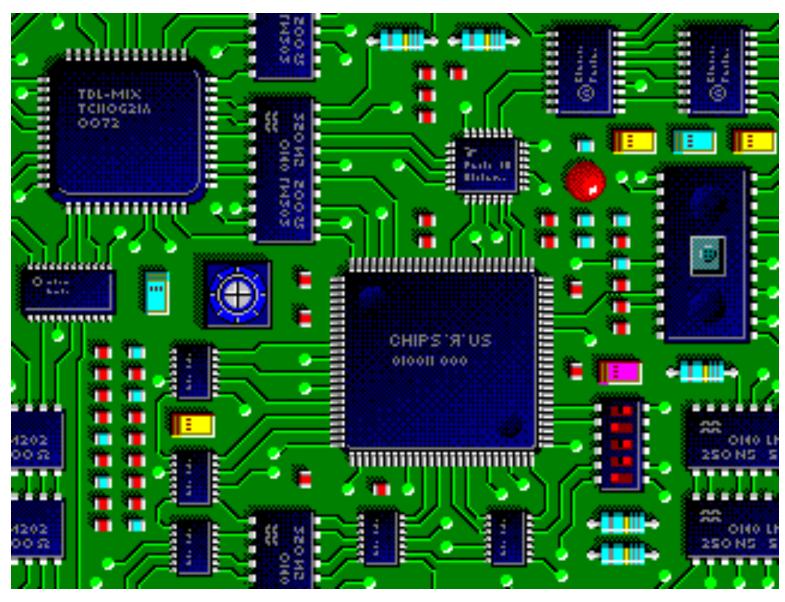
Variable	q[0]	_	1		q[4]	q[5]
Value	M	D	T	E	х	A



Queue: Linked List Implementation

```
public class QueueList implements QueueInterface {
                               reference to first element of queue
   private List first;
                                  reference to last element of queue
   private List last;
   private class List { String item; List next; } tem nested class
   public boolean isEmpty() { return (first == null); }
   public void add(String anItem) {
      List x = new List();
      x.item = anItem;
      x.next = null;
      if (isEmpty()) { head = x; last = x; }
      else
                   { last.next = x; last = x; }
   public String remove() {
      String val = first.item;
      first = first.next;
                                     delete first element
      return val;
```

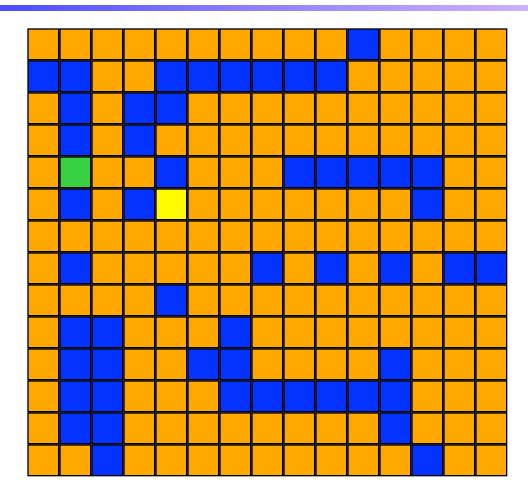
Lee's Wire Router



Lee's Wire Router





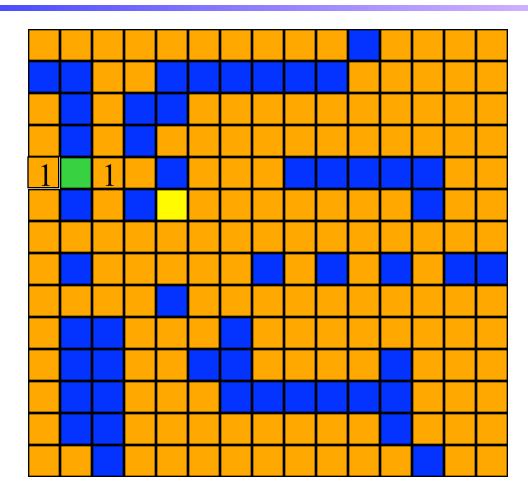


Label all reachable squares 1 unit from start.

Lee's Wire Router



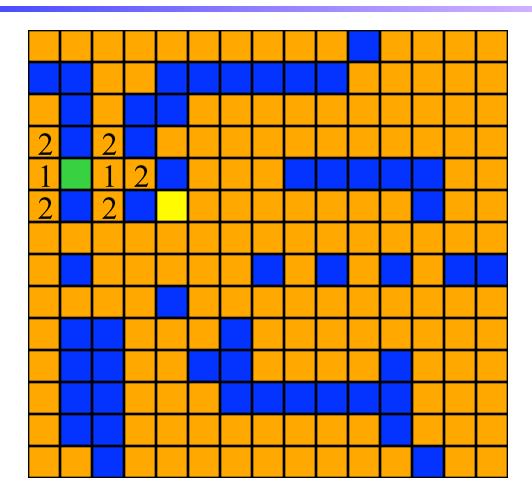
end pin



Label all reachable unlabeled squares 2 units from start.



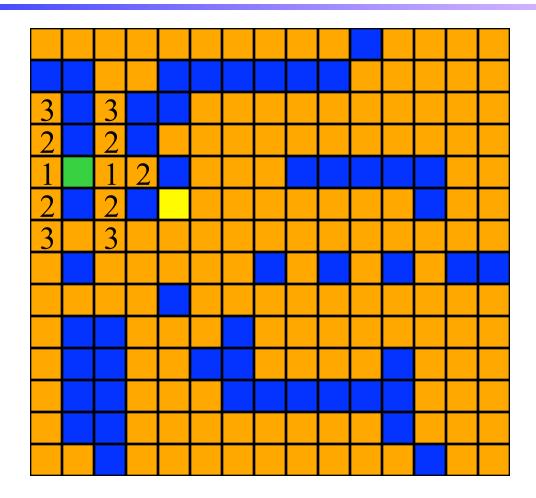




Label all reachable unlabeled squares 3 units from start.



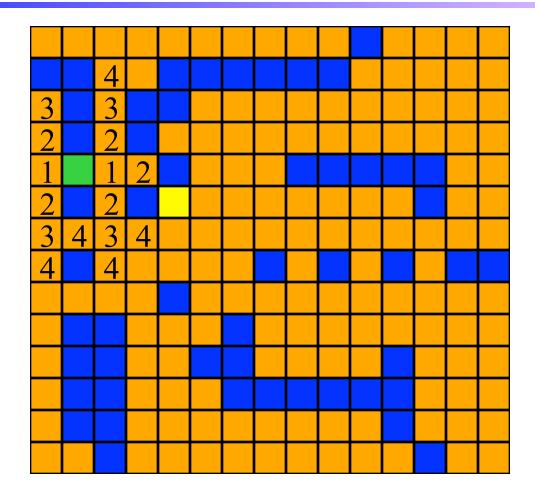
end pin



Label all reachable unlabeled squares 4 units from start.



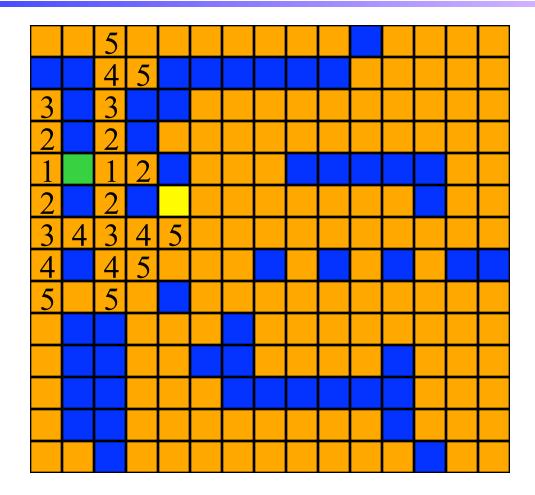




Label all reachable unlabeled squares 5 units from start.



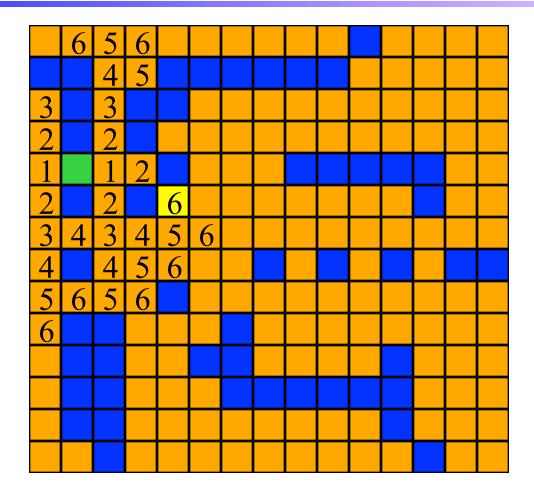




Label all reachable unlabeled squares 6 units from start.



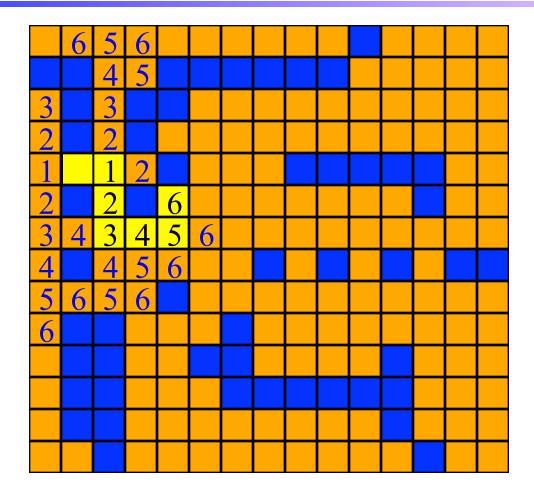
end pin



End pin reached. Traceback.







End pin reached. Traceback.

Find a Wire Route - Java Code (1)

```
/** find a shortest path from start to finish
    * @return true if successful, false if impossible */
 private static boolean findPath()
  {
    if ((start.row == finish.row) && (start.col == finish.col))
     { // start == finish
        pathLength = 0;
        return true;
     }
    // initialize offsets
    Position [] offset = new Position [4];
    offset[0] = new Position(0, 1); // right
    offset[1] = new Position(1, 0); // down
    offset[2] = new Position(0, -1); // left
    offset[3] = new Position(-1, 0); // up
```

Find a Wire Route - Java Code (2)

```
// initialize wall of blocks around the grid
   for (int i = 0; i <= size + 1; i++)
   {
      grid[0][i] = grid[size + 1][i] = 1; // bottom and top
      grid[i][0] = grid[i][size + 1] = 1; // left and right
   }

Position here = new Position(start.row, start.col);
   grid[start.row][start.col] = 2; // block
   int numOfNbrs = 4; // neighbors of a grid position</pre>
```

Find a Wire Route - Java Code (3)

```
// label reachable grid positions
     QueueList q = new QueueList();
     Position nbr = new Position(0, 0);
     do
     { // label neighbors of 'here'
         for (int i = 0; i < numOfNbrs; i++)</pre>
         { // check out neighbors of 'here'
            nbr.row = here.row + offset[i].row;
            nbr.col = here.col + offset[i].col;
            if (grid[nbr.row][nbr.col] == 0)
            { // unlabeled nbr, label it
               grid[nbr.row][nbr.col]
                  = grid[here.row][here.col] + 1;
               if ((nbr.row == finish.row) &&
                   (nbr.col == finish.col)) break; // done
               // enqueue - put on queue for later expansion
              q.add(new Position(nbr.row, nbr.col));
```

Find a Wire Route - Java Code (4)

```
// have we reached finish?
       if ((nbr.row == finish.row) &&
            (nbr.col == finish.col)) break; // done
       // finish not reached, can we move to a nbr?
       if ( q.isEmpty() ) return false;  // no path
       // dequeue next position
       here = (Position) q.remove();
     } while(true);
    // construct path
    pathLength = grid[finish.row][finish.col] - 2;
    path = new Position [pathLength];
```

Find a Wire Route - Java Code (5)

```
// trace backwards from finish
     here = finish;
     for (int j = pathLength - 1; j >= 0; j--)
        path[j] = here;
        // find predecessor position
        for (int i = 0; i < numOfNbrs; i++)</pre>
           nbr.row = here.row + offset[i].row;
           nbr.col = here.col + offset[i].col;
           if (grid[nbr.row][nbr.col] == j + 2) break;
        // move to predecessor
        here = new Position(nbr.row, nbr.col);
     return true;
```

