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Homework 15

CISP440

Section 10.3

In Exercises 1-6, determine whether the given grammar is context-sensitive, context-free, regular or none of these. Give all characterizations that apply.

2. $T = \{a, b, c\}$, $N = \{\sigma, A, B\}$, with productions

$$\begin{array}{lll} \sigma \rightarrow b\sigma, & AB \rightarrow BA, & A \rightarrow a, \\ \sigma \rightarrow aA, & A \rightarrow bA, & \sigma \rightarrow b, \end{array}$$

and σ as a starting point.

This grammar is context context-sensitive, as evidenced by the production that has more than one symbol on the left-hand-side.

3. $T = \{a, b\}$, $N = \{\sigma, A, B\}$, with productions

$$\begin{array}{lll} \sigma \rightarrow A, & A \rightarrow aa, & AB \rightarrow ABB, \\ \sigma \rightarrow AAB, & Bb \rightarrow ABb, & B \rightarrow b \\ Aa \rightarrow ABa, & & \end{array}$$

and σ as a starting point.

This grammar is also context-sensitive, because of the $Bb \rightarrow ABb$, $Aa \rightarrow ABa$ and $AB \rightarrow ABB$ productions.

In Exercises 7-11, show that the given string α is in $L(G)$ for the given grammar G by giving a definition of α

8. *abab*, Exercise 2

$$\begin{array}{l} \sigma \rightarrow AB \\ AB \rightarrow aAB \\ aAB \rightarrow aABb \\ aABb \rightarrow aBAb \\ aBAb \rightarrow abAb \\ abAb \rightarrow abab \end{array}$$

11. *abaabbabba*, Exercise 5

$$\begin{array}{l} \langle S \rangle \rightarrow a \langle A \rangle \rightarrow ab \langle B \rangle \rightarrow aba \langle S \rangle \rightarrow abaa \langle A \rangle \rightarrow abaab \langle B \rangle \\ \rightarrow abaabb \langle A \rangle \rightarrow abaabba \langle S \rangle \rightarrow abaabbab \langle S \rangle \rightarrow abaabbabb \langle S \rangle \rightarrow abaabbabba \end{array}$$

12. Write the grammars of Examples 10.3.4 and 10.3.9 and Exercises 1-4 and 6 in BNF

10.3.4:

$\langle \sigma \rangle ::= b \langle \sigma \rangle \mid a \langle S \rangle$

$\langle S \rangle ::= b \langle S \rangle \mid b$

10.3.9:

$\langle \sigma \rangle ::= a \langle A \rangle \langle B \rangle \mid a \langle B \rangle$

$\langle A \rangle ::= a \langle A \rangle \langle C \rangle \mid a \langle C \rangle$

$\langle B \rangle ::= \langle D \rangle c$

$\langle D \rangle ::= b$

$\langle C \rangle \langle D \rangle ::= \langle C \rangle \langle E \rangle$

$\langle C \rangle \langle E \rangle ::= \langle D \rangle \langle E \rangle$

$\langle D \rangle \langle E \rangle ::= \langle D \rangle \langle C \rangle$

$\langle C \rangle c ::= \langle D \rangle cc$

Exercise 1

$\langle \sigma \rangle ::= b \langle \sigma \rangle \mid a \langle A \rangle \mid b$

$\langle A \rangle ::= b \langle A \rangle \mid a \langle \sigma \rangle \mid a$

Exercise 2

$\langle \sigma \rangle ::= \langle A \rangle \langle B \rangle \mid a$

$\langle A \rangle \langle B \rangle ::= \langle B \rangle \langle A \rangle$

$\langle A \rangle ::= a\sigma \mid b \langle A \rangle \mid a$

Exercise 3

$\langle \sigma \rangle ::= \langle A \rangle \mid \langle A \rangle \langle A \rangle \langle B \rangle$

$\langle A \rangle a ::= \langle A \rangle \langle B \rangle a$

$\langle A \rangle ::= aa$

$\langle B \rangle b ::= \langle A \rangle \langle B \rangle b$

$\langle A \rangle \langle B \rangle ::= \langle A \rangle \langle B \rangle \langle B \rangle$

$\langle B \rangle ::= b$

Exercise 4

$\langle \sigma \rangle ::= \langle B \rangle \langle A \rangle \langle B \rangle \mid \langle A \rangle \langle B \rangle \langle A \rangle$

$\langle A \rangle ::= \langle A \rangle \langle B \rangle \mid a \langle A \rangle \mid ab$

$\langle B \rangle ::= \langle B \rangle \langle A \rangle \mid b$

Exercise 6

$\langle \sigma \rangle ::= \langle A \rangle \langle A \rangle \sigma$

$\langle A \rangle \langle A \rangle ::= \langle B \rangle$

$\langle B \rangle ::= b \langle B \rangle$

$\langle A \rangle ::= a$

In Exercises 15-24, write a grammar that generates the strings having the given property.

16. Strings over $\{a, b\}$ starting with a .

$T = \{a, b\}, N = \{\sigma, A, B\}$
 $\langle \sigma \rangle ::= \langle A \rangle \langle B \rangle \mid \langle B \rangle$
 $\langle A \rangle ::= a \mid b$
 $\langle B \rangle ::= ab$

17. Strings over $\{a, b\}$ containing a .

$T = \{a, b\}, N = \{\sigma, A, B, C\}$
 $\langle \sigma \rangle ::= \langle \sigma \rangle \langle C \rangle \mid \langle C \rangle \langle \sigma \rangle$
 $\langle A \rangle ::= a \mid b$
 $\langle B \rangle ::= ab$
 $\langle C \rangle ::= \langle A \rangle \langle B \rangle \mid \langle B \rangle \langle A \rangle$

19. Integers with no leading 0's.

$T = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}, N = \{\sigma, NONZERO, INT\}$
 $\langle \sigma \rangle ::= \langle NONZERO \rangle \mid \langle NONZERO \rangle \langle INT \rangle \mid 0$
 $\langle NONZERO \rangle ::= 1, 2, 3, 4, 5, 6, 7, 8, 9$
 $\langle INT \rangle ::= 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$

21. Exponential numbers.

$T = \{0-9, ., E, -, +\}, N = \{\sigma, I, B, S\}$
 $\langle \sigma \rangle ::= 0 \mid \langle B \rangle \mid \langle B \rangle E \langle I \rangle$
 $\langle I \rangle ::= \langle S \rangle 0-9 \mid \langle S \rangle \langle I \rangle 0-9$
 $\langle B \rangle ::= \langle I \rangle \mid \langle I \rangle . \langle I \rangle \mid$
 $\langle S \rangle ::= + \mid - \mid \lambda$

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Yes, any string generated would have equal numbers of a's and b's. You can tell because the only productions each add one a and one b, just in different places in the string.