ENTROPY: EXPECTED SURPRISE

One Bobsled Race To Go

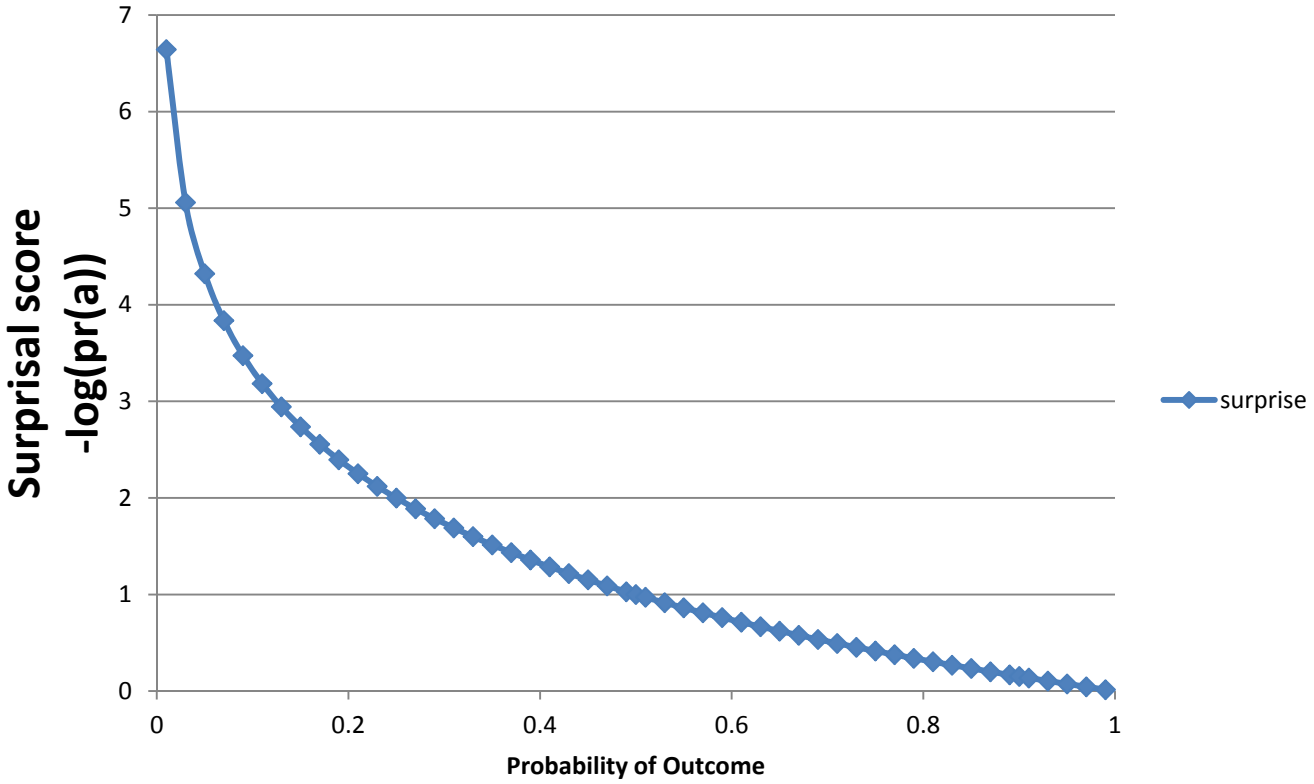
- Scenario 1
  - 2 teams tied for total time after 3 races
- Scenario 2
  - Swiss team is ahead by 22 seconds after 3 races
  - Announcer says there is only a 10% chance the American team will catch up and win gold
- NBC cuts to a commercial
- Comes back with a different sport!
- How much do you wonder about bobsled result?

Later You Find Out

- Scenario 1: teams were tied
  - 1a: American team got gold
  - 1b: American team got silver
- Scenario 2: Swiss team way ahead
  - 2a: American team got gold
  - 2b: American team got silver
- How surprising is each of these outcomes?
- On average: how much surprise do you expect in each scenario?
- How much “information” do you get from learning the result?

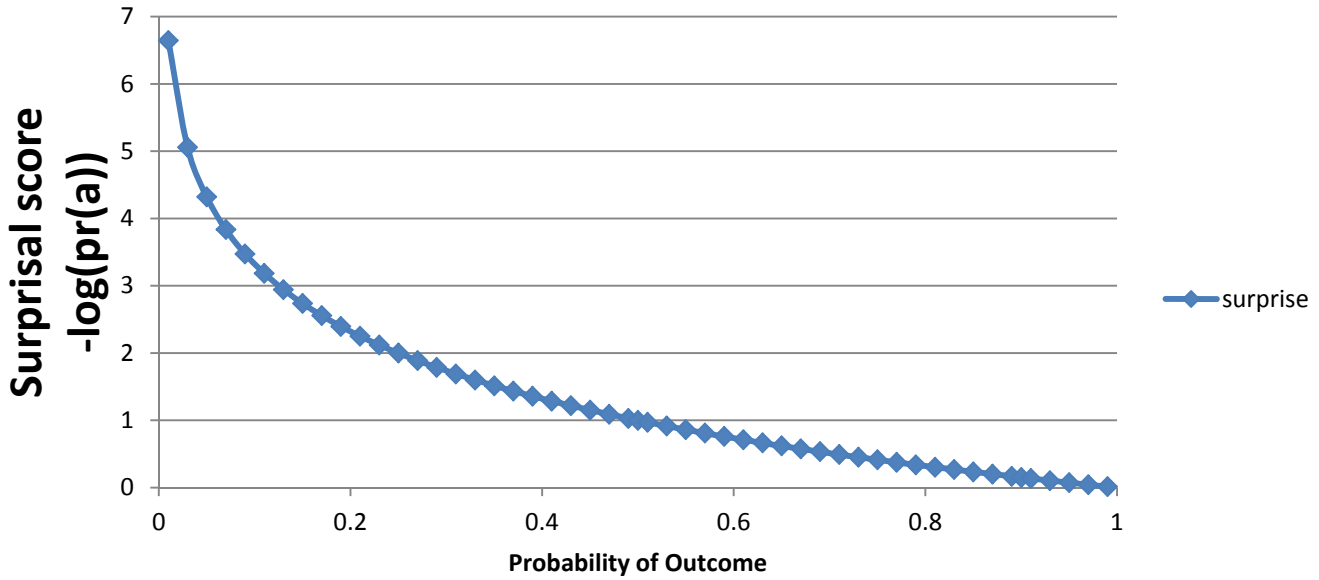
Measuring Surprisal

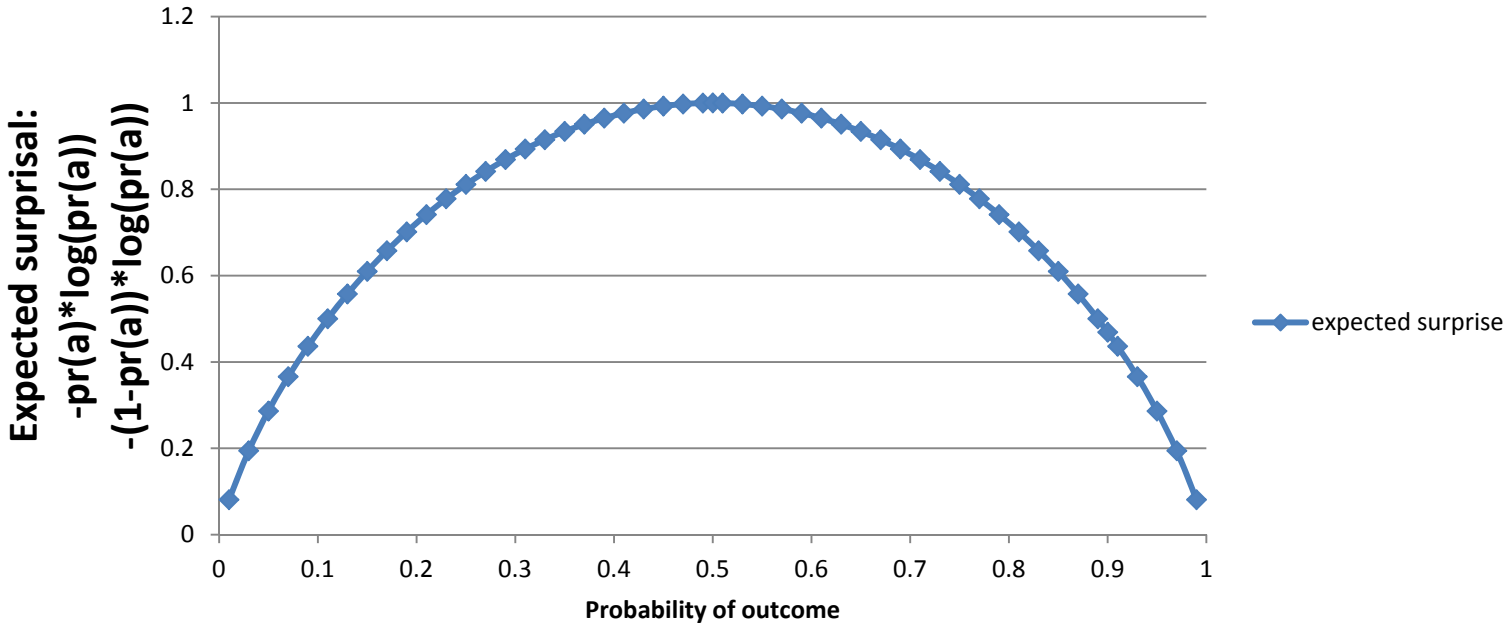
- If outcome is unexpected, surprise is higher
- Assign probabilities to possible outcomes
  - Each probability is in (0, 1)
  - Sum of probabilities is 1
- Quantify surprise of outcome a as
  - -log(pr(a))


Surprisal score
-log(pr(a))
7
6
5
4
3
2
1
0
0
0.2
0.4
0.6
0.8
1
Probability of Outcome
surprise

Expected Surprise

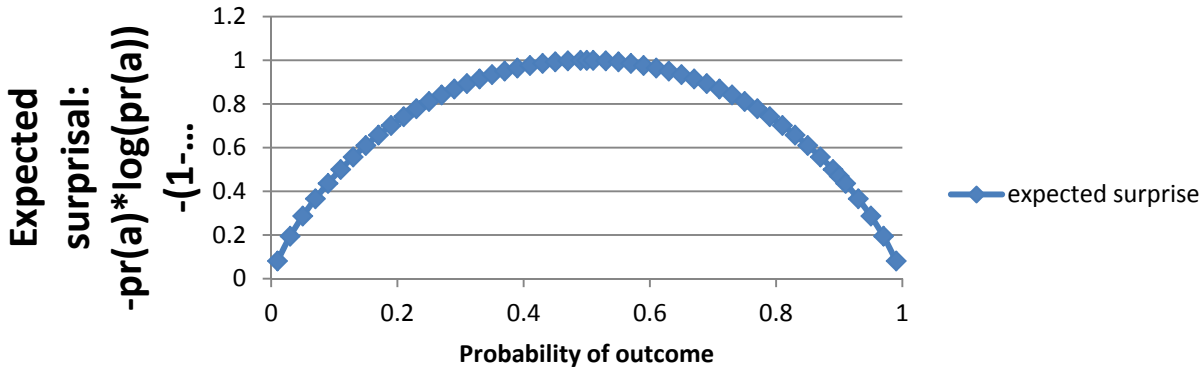
- Scenario 1: teams were tied
  - With probability ½ Americans win
    - Surprise –log(½)
  - With probability ½ Americans lose
    - Surprise –log(½))
  - Expected surprise
    - ½(-log(½)) + ½(-log(½))
- Scenario 2: Swiss way ahead
  - With probability .1 Americans win
    - Surprise –log(.1)
  - With probability .9 Americans lose
    - Surprise –log(.9)
  - Expected surprise
    - .1(-log(.1))) + .9* -log(.9))
- Which has greater expected surprise?


Surprisal score
-log(pr(a))
7
6
5
4
3
2
1
0
0
0.2
0.4
0.6
0.8
1
Probability of Outcome
surprise


Expected surprisal:
-pr(a)*log(pr(a))
-(1-pr(a))*log(pr(a))
1.2
1
0.8
0.6
0.4
0.2
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Probability of outcome
expected surprise

The General Formula

- Mutually exclusive outcomes $x_1, x_2, \ldots, x_n$
- $-\sum_{i=1}^{n} pr(x_i) * log(x_i)$
  - (x_1, x_2) = .5, .5
    - -½log(½) - ½log(½) = 1
  - (x_1, x_2) = .1, .9
    - -.1*log(.1) - .9* log(.9) = 0.469



Summary

- Surprisal is a measure of an outcome
- Expected (average) surprisal is a measure of a process with uncertain outcome
- Shannon defined “information” as expected surprisal, the uncertainty that will be resolved once outcome is revealed
- How much information in a message depends on what you knew before you got the message