STAC Test Sheet - Multiple Regression

$y \in \mathbb{R}^{n \times 1}$ $\qquad$ $X \in \mathbb{R}^{n \times p+1}$ $\qquad$ $B \in \mathbb{R}^{p+1 \times 1}$

$y = X'B + \varepsilon$

$$y'y = \left[\sum y_i^2 \right]$$

$$X'X = \begin{bmatrix} n & \sum X_1 & \cdots & \sum X_p \\ \sum X_1 & \sum X_1^2 & \cdots & \sum X_1 X_p \\ \vdots & \vdots & \ddots & \vdots \\ \sum X_p & \sum X_p X_1 & \cdots & \sum X_p^2 \end{bmatrix}$$

$$X'y = \begin{bmatrix} \sum y_i \\ \sum X_{1i} y_i \\ \vdots \\ \sum X_{pi} y_i \end{bmatrix}$$

$Var(y) = \sigma^2 I$

$E(Y) = XB$ $\qquad\qquad\qquad$ $y \sim N(XB, \sigma^2 I)$

$\hat{B} = (X'X)^{-1} X'y$

$H = X(X'X)^{-1} X'$

$e = (I-H)y$

$$E(\hat{B}) = B$$

$$Var(\hat{B}) = (X'X)^{-1}\sigma^2 \qquad \hat{B} \sim N(B, (X'X)^{-1}\sigma^2)$$

$$\hat{y} = X\hat{B} = X(X'X)^{-1}X'y = HY$$

$$E(\hat{y}) = X\hat{B}$$

$$Var(\hat{y}) = X(X'X)^{-1}\sigma^2 X' = H\sigma^2 \qquad \hat{y} \sim N(X\hat{B}, H\sigma^2)$$

$$Var(\hat{y}_r) = X_r(X'X)^{-1}X'_r\sigma^2$$

$$E(e) = 0$$

$$Var(e) = (I-H)\sigma^2 \qquad e \sim N(0, (I-H)\sigma^2)$$

$$Var(Y) = Var(\hat{y}) + Var(e)$$
$$= \sigma^2 H + (I-H)\sigma^2$$
$$= I\sigma^2$$

$$\hat{Y}_{pred_0} = X'_0\hat{B} \qquad X'_0 \in \mathbb{R}^{P+1 \times 1}$$

$$E(\hat{Y}_{pred_0}) = X'_0 B$$

$$Var(\hat{Y}_{pred_0}) = (1 + X'_0(X'X)^{-1}X_0)\sigma^2$$

$$\hat{y}_{pred_0} \sim N(X_0'B, (1 + X_0'(X'X)^{-1}X_0)\sigma^2)$$

$$\hat{y}_0 = X_0'B$$

$$Var(\hat{y}_0) = X_0'(X'X)^{-1}X_0\sigma^2$$

$$\hat{y}_0 \sim N(X_0'B, X_0'(X'X)^{-1}X_0\sigma^2)$$

Use r-code : $Vcov(fit)$ for var-covar matrix