Tutorial 9 Mar 19 2021

Bayesian Regression

We find the probability distribution of y given a test input x and data D.

1) $p(y|x,D) = \int p(y|w,x)\, p(w|D)\, dw$

$y = b_x^T w + \eta \quad , \quad \eta \sim N(0, \sigma^2)$

2) $p(y|w,x) = G(y; b_x^T w, \sigma^2)$

3) $p(w|D) = G(w; \hat{w}, K)$, K = covariance matrix.

($\hat{w}$: Map estimate of $\hat{w}$.)

Since 2), 3) is Gaussian, 1), the marginal of product of two Gaussian is also Gaussian.

$p(y|x,D) = G(y; b_x^T \hat{w}, \sigma^2 + b_x^T K b_x)$

$E(y) = E(b_x^T w + \eta) = b_x^T E(w) + 0 = b_x^T \hat{w}$

$$\mathrm{Var}(y) = \mathrm{Var}(b_x^T u + \eta) = b_x^T \mathrm{Var}(u) b_x^T + \sigma^2$$
$$= b_x^T K b_x + \sigma^2$$