Solutions Manual to accompany

Elementary Linear Programming with Applications

SECOND EDITION

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Chapter 0

Section 0.1, page 9

2. (a) $\begin{bmatrix} 4 & 2 & 1 \\ 4 & 4 & 11 \\ 4 & 4 & 4 \end{bmatrix}$ (b) $\mathbf{AB} = \begin{bmatrix} 14 & 8 \\ 11 & 6 \end{bmatrix}$ $\mathbf{BA} = \begin{bmatrix} 4 & 6 & 2 \\ 12 & 11 & 7 \\ 8 & 5 & 5 \end{bmatrix}$

(c) $\begin{bmatrix} 16 & -6 \\ 3 & -15 \end{bmatrix}$ (d) $\begin{bmatrix} 58 & -38 \\ 45 & -29 \end{bmatrix}$

4. (a) $\begin{bmatrix} 14 & 11 \\ 8 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 38 & 12 & 50 \\ 30 & 11 & 31 \end{bmatrix}$ (c) $\begin{bmatrix} 28 & -2 \\ 21 & -10 \\ 17 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 23 & 22 \\ -30 & -8 \\ 8 & -20 \end{bmatrix}$

8. (a) $\begin{bmatrix} 3 & 0 & 2 & 2 \\ 2 & 3 & 5 & -1 \\ 3 & 2 & 4 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}$. (b) $\begin{bmatrix} 3 & 0 & 2 & 2 \\ 2 & 3 & 5 & -1 \\ 3 & 2 & 4 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} -8 \\ 4 \\ 6 \\ -6 \end{bmatrix}$

(d) $\left[\begin{array}{cccc|c} 3 & 0 & 2 & 2 & -8 \\ 2 & 3 & 5 & -1 & 4 \\ 3 & 2 & 4 & 0 & 6 \\ 1 & 0 & 1 & 1 & -6 \end{array}\right]$

10. Denote the entries of the identity matrix by d_{ij}, so that

$$d_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

Then for $\mathbf{C} = \mathbf{AI_n}$, $c_{ij} = \sum_{k=1}^{p} a_{ik}d_{kj} = a_{ij}d_{jj}$ (all other d_{kj} are zero), $= a_{ij}$ and thus $\mathbf{C} = \mathbf{A}$. A similar argument shows that $\mathbf{I_m A} = \mathbf{A}$.

11. Let $\mathbf{B} = [b_{ij}] = (-1)\,\mathbf{A}$. Then $b_{ij} = -a_{ij}$.

12. $\mathbf{AB} = \begin{bmatrix} 16 & 17 & 12 & 26 & 20 \\ 6 & 19 & -4 & 18 & 17 \\ 26 & 33 & 18 & 52 & 46 \\ 5 & 16 & -5 & 15 & 9 \\ 3 & 15 & -9 & 17 & 11 \\ 18 & 29 & 12 & 38 & 33 \end{bmatrix}$

13. Let $\mathbf{A} = [a_{ij}]$ be $m \times p$ and $\mathbf{B} = [b_{ij}]$ be $p \times n$.

(a) Let the ith row of $\mathbf{A}$ consist entirely of zeros, so $a_{ik} = 0$ for $k = 1$, $2, \ldots, p$. Then the (i, j) entry in $\mathbf{AB}$ is

$$\sum_{k=1}^{p} a_{ik}b_{kj} = 0 \text{ for } j = 1, 2, \ldots, n.$$

(b) Let the jth column of $\mathbf{B}$ consist entirely of zeros, so $b_{kj} = 0$ for $k = 1, 2, \ldots, p$. Then again the (i, j) entry in $\mathbf{AB}$ is 0 for $i = 1, 2, \ldots, m$.

14. The jth column of $\mathbf{AB}$ is

$$\begin{bmatrix} \sum_k a_{1k}b_{kj} \\ \sum_k a_{2k}b_{kj} \\ \vdots \\ \sum_k a_{mk}b_{kj} \end{bmatrix}.$$

15. Suppose that $\mathbf{x_1}$ and $\mathbf{x_2}$ are solutions to $\mathbf{Ax} = \mathbf{b}$. Consider $\mathbf{x}_3 = r\mathbf{x}_1 + s\mathbf{x}_2$, where $r + s = 1$. Then

$$\begin{aligned} \mathbf{A}\mathbf{x}_3 &= \mathbf{A}(r\mathbf{x}_1 + s\mathbf{x}_2) \\ &= \mathbf{A}r\mathbf{x}_1 + \mathbf{A}s\mathbf{x}_2 \\ &= r\mathbf{A}\mathbf{x}_1 + s\mathbf{A}\mathbf{x}_2 \\ &= (r+s)\,\mathbf{b} = 1\mathbf{b} = \mathbf{b}. \end{aligned}$$

Therefore, $\mathbf{x_3}$ is a solution.

Section 0.2, page 20

2. $\begin{bmatrix} 1 & 0 & -7 & 2 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ 4. $\begin{bmatrix} 1 & 0 & 2 & 3 & 0 \\ 0 & 1 & 7/4 & 11/4 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

6. (a) No solution.

   (b) $x = -2 + 7r,\ y = 0,\ z = 3 - 4r,\ w = r.$

8. (a) $x = 4 - r + \frac{7}{5}s,\ y = -\frac{4}{5}s,\ z = r,\ w = s.$

   (b) $x = 2,\ y = -2,\ z = 1.$

10. (a) $x = 1,\ y = 2,\ z = -3.$

    (b) No solution.

12. (a) $x = -5r,\ y = -3r,\ z = r.$

    (b) $x = -\frac{20}{21}r,\ y = -\frac{4}{7}r,\ z = \frac{13}{42}r,\ w = r.$

14. (i) $a = -4$, (ii) $a =$ any real number except 4 and -4, (iii) $a = 4$.

15. If $\mathbf{A}$ has no rows of zeros, then each of its n rows has a leading 1 in some column. Since there are n columns, every column has a 1. Hence, $\mathbf{A} = \mathbf{I_n}$.

16. Suppose that $\mathbf{x}_1$ and $\mathbf{x}_2$ are solutions to $\mathbf{Ax} = \mathbf{0}$. Consider $\mathbf{x}_3 = r\mathbf{x}_1 + s\mathbf{x}_2$. Then

$$\begin{aligned}
\mathbf{Ax}_3 &= \mathbf{A}(r\mathbf{x}_1 + s\mathbf{x}_2) \\
&= \mathbf{A}r\mathbf{x}_1 + \mathbf{A}s\mathbf{x}_2 \\
&= r\mathbf{Ax}_1 + s\mathbf{Ax}_2 \\
&= r\mathbf{0} + s\mathbf{0} = \mathbf{0}.
\end{aligned}$$

Therefore, $\mathbf{x}_3$ is a solution.

17. (a) $\mathbf{A}$ is row equivalent to itself: the sequence of operations is the empty sequence.

(b) Each elementary row operation of types I, II, or III has a corresponding inverse operation of the same type that "undoes" the effect of the original operation. For example the inverse of the operation "add d times row r of $\mathbf{A}$ to its sth row" is " add $-d$ times row r of $\mathbf{A}$ to its sth row." Since $\mathbf{B}$ is assumed to be row equivalent to $\mathbf{A}$, there is a sequence of elementary row operations that gets from $\mathbf{A}$ to $\mathbf{B}$. Take those operations in the reverse order, and for each operation do its inverse, thereby taking $\mathbf{B}$ to $\mathbf{A}$. Thus, $\mathbf{A}$ is row equivalent to $\mathbf{B}$.

(c) Follow the operations that take $\mathbf{A}$ to $\mathbf{B}$ with those that take $\mathbf{B}$ to $\mathbf{C}$.

18. If $ad - bc = 0$, we show that the two rows of $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ are multiples of one another:

$$c\,[a \quad b] = [ac \quad bc] = [ac \quad ad] = a\,[c \quad d].$$

Thus, any elementary row operation applied to $\mathbf{A}$ will produce a matrix with rows that are multiples of each other. In particular, elementary row operations cannot produce $\mathbf{I}_2$, so $\mathbf{I}_2$ is not row equivalent to $\mathbf{A}$.

Conversely, if $ad - bc \neq 0$, then a and c are not both zero. Suppose $a \neq 0$.

$$\begin{array}{cc|cc}
a & b & 1 & 0 \\
c & d & 0 & 1 \\
\hline
1 & \frac{b}{a} & \frac{1}{a} & 0 \\
0 & d-\frac{bc}{a} & -c & 1 \\
\hline
1 & \frac{b}{a} & \frac{1}{a} & 0 \\
0 & 1 & \frac{-ac}{ad-bc} & \frac{1}{ad-bc} \\
\hline
1 & 0 & \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\
0 & 1 & \frac{-c}{ad-bc} & \frac{a}{ad-bc}
\end{array}$$

Section 0.3, page 27

2. $\begin{bmatrix} 9 & 13 \\ 13 & 11 \end{bmatrix}$

4. $k = 2$

6. (a) $\begin{bmatrix} -2/11 & 5/11 \\ 3/11 & -2/11 \end{bmatrix}$ (b) $\begin{bmatrix} 9/22 & -17/22 & -1/11 \\ 2/11 & -5/11 & 2/11 \\ -1/11 & 8/11 & -1/11 \end{bmatrix}$

   (c) No inverse

8. (a) $\begin{bmatrix} 3/14 & 1/7 \\ -2/7 & 1/7 \end{bmatrix}$ (b) $\begin{bmatrix} 8 & -2 & -5 \\ -3 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix}$

   (c) $\begin{bmatrix} 1/3 & 1/3 & -1/12 \\ -1/3 & 1/6 & 5/24 \\ 1/3 & -1/6 & 1/24 \end{bmatrix}$

10. (a) $\begin{bmatrix} 1/6 & 1/6 \\ 2/9 & -1/9 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1/2 & 0 \\ -4 & -7 & 3 \\ 3 & 9/2 & -2 \end{bmatrix}$

    (c) $\begin{bmatrix} 1/2 & 19/36 & -1/9 \\ 1/2 & 11/36 & -2/9 \\ -1 & -8/9 & 5/9 \end{bmatrix}$

11. For the case $a \neq 0$, we perform the following computations. Other cases are handled similarly.

$$\begin{array}{cc|cc} a & b & 1 & 0 \\ c & d & 0 & 1 \\ \hline 1 & \frac{b}{a} & \frac{1}{a} & 0 \\ c & d & 0 & 1 \\ \hline 1 & \frac{b}{a} & \frac{1}{a} & 0 \\ 0 & 1 & \frac{-c}{ad-bc} & \frac{a}{ad-bc} \\ \hline 1 & 0 & \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ 0 & 1 & \frac{-ac}{ad-bc} & \frac{a}{ad-bc} \end{array}$$

12. The elementary matrix $\mathbf{E}$ that results from $\mathbf{I}_n$ by a type I operation interchanging the ith and jth rows differs from $\mathbf{I}_n$ by having 1's in the (i,j) and (j,i) positions and 0's in the (i,i) and (j,j) positions. For this $\mathbf{E}$, $\mathbf{EA}$ has for its ith row, the jth row of $\mathbf{A}$ and for its jth row, the ith row of $\mathbf{A}$.

    The elementary matrix $\mathbf{E}$ which results from $\mathbf{I}_n$ by a type II operation differs from $\mathbf{I}_n$ by having $c \neq 0$ in the (i,i) position. Then $\mathbf{EA}$ has as its ith row, c times the ith row of $\mathbf{A}$.

    The elementary matrix $\mathbf{E}$ which results from $\mathbf{I}_n$ by a type III operation differs from $\mathbf{I}_n$ by having c in the (i,j) position. Then $\mathbf{EA}$ has as its jth row, the sum of the jth row and c times the ith row of $\mathbf{A}$.

13. If $\mathbf{A}$ is row equivalent to $\mathbf{B}$, then $\mathbf{B}$ results from $\mathbf{A}$ by a sequence of elementary row operations. This implies that there exist elementary matrices such that
    $$\mathbf{B} = \mathbf{E}_k\mathbf{E}_{k-1}\cdots\mathbf{E}_2\mathbf{E}_1\mathbf{A}.$$
    Conversely, if
    $$\mathbf{B}=\mathbf{E}_k\mathbf{E}_{k-1}\cdots\mathbf{E}_2\mathbf{E}_1\mathbf{A},$$
    where the $\mathbf{E}_i$ are elementary matrices, then $\mathbf{B}$ results from $\mathbf{A}$ by a sequence of elementary row operations, which implies that $\mathbf{A}$ is row equivalent to $\mathbf{B}$.

14. If $\mathbf{E}_1$ is an elementary matrix of type I, then $\mathbf{E}_1^{-1} = \mathbf{E}_1$. Let $\mathbf{E}_2$ be obtained from $\mathbf{I}_n$ by multiplying the ith row of $\mathbf{I}_n$ by $c \neq 0$. Let $\mathbf{E}_2^*$ be obtained from $\mathbf{I}_n$ by multiplying the ith row of $\mathbf{I}_n$ by $1/c$. Then $\mathbf{E}_2\mathbf{E}_2^* = \mathbf{I}_n$. Let $\mathbf{E}_3$ be obtained from $\mathbf{I}_n$ by adding c times the ith row

of $\mathbf{I}_n$ to the jth row of $\mathbf{I}_n$. Let $\mathbf{E}_3^*$ be obtained from $\mathbf{I}_n$ by adding $-c$ times the ith row of $\mathbf{I}_n$ to the jth row of $\mathbf{I}_n$. Then $\mathbf{E}_3\mathbf{E}_3^* = \mathbf{I}_n$.

15. Suppose that $\mathbf{A}$ is nonsingular. Then multiplying both sides of $\mathbf{Ax} = \mathbf{b}$ on the left by $\mathbf{A}^{-1}$, we obtain

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}.$$

Conversely, suppose that $\mathbf{Ax} = \mathbf{b}$ has a solution for every $n \times 1$ matrix $\mathbf{b}$. Let $\mathbf{e}_i$ be the $n \times 1$ matrix with a 1 in the ith row and 0's elsewhere. Then the linear system $\mathbf{Ax} = \mathbf{e}_i$ has a solution $\mathbf{x}_i$. Let $\mathbf{B}$ be the $n \times n$ matrix whose jth column is $\mathbf{e}_j$. The equations $\mathbf{Ax}_1 = \mathbf{e}_1, \mathbf{Ax}_2 = \mathbf{e}_2, \ldots,$ $\mathbf{Ax}_n = \mathbf{e}_n$ can be written in matrix form as

$$\mathbf{AB} = \mathbf{I_n}.$$

Hence, $\mathbf{B}$ is the inverse of $\mathbf{A}$ and thus $\mathbf{A}$ is nonsingular.

16. The matrices $\mathbf{A}$ and $\mathbf{B}$ are row equivalent if and only if

$$\mathbf{B} = \mathbf{E}_k\mathbf{E}_{k-1}\cdots\mathbf{E}_2\mathbf{E}_1\mathbf{A}.$$

Let $\mathbf{P} = \mathbf{E}_k\mathbf{E}_{k-1}\cdots\mathbf{E}_2\mathbf{E}_1$.

17. If $\mathbf{A}$ is row equivalent to $\mathbf{I}_n$, then $\mathbf{I}_n = \mathbf{E}_k\mathbf{E}_{k-1}\cdots\mathbf{E}_2\mathbf{E}_1\mathbf{A}$, where $\mathbf{E}_1, \mathbf{E}_2, \ldots, \mathbf{E}_k$ are elementary matrices. Therefore, it follows that $\mathbf{A} = \mathbf{E}_1^{-1}\mathbf{E}_2^{-1}\cdots\mathbf{E}_k^{-1}$. Now the inverse of an elementary matrix is an elementary matrix. By Theorem 0.6, $\mathbf{A}$ is nonsingular.

Conversely, if $\mathbf{A}$ is nonsingular, then $\mathbf{A}$ is a product of elementary matrices, $\mathbf{A} = \mathbf{E}_k\mathbf{E}_{k-1}\cdots\mathbf{E}_2\mathbf{E}_1$ Now

$$\mathbf{A} = \mathbf{AI}_n = \mathbf{E}_k\mathbf{E}_{k-1}\cdots\mathbf{E}_2\mathbf{E}_1\mathbf{I}_n,$$

which implies that $\mathbf{A}$ is row equivalent to $\mathbf{I}_n$.

Section 0.4, page 32

1. $0 + 0 = 0$ and $r0 = 0$, where r is any real number.

2. (a)

4. (a) and (c)

6. $0 \cdot \mathbf{x} = 0 \cdot \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} 0 \cdot x_1 \\ 0 \cdot x_2 \\ \vdots \\ 0 \cdot x_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \mathbf{0}.$

8. $-(-\mathbf{x}) = -\begin{bmatrix} -x_1 \\ -x_2 \\ \vdots \\ -x_n \end{bmatrix} = \begin{bmatrix} -(-x_1) \\ -(-x_2) \\ \vdots \\ -(-x_n) \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \mathbf{x}.$

9. If

$$r \cdot \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \text{then} \begin{bmatrix} rx_1 \\ rx_2 \\ \vdots \\ rx_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

so $rx_i = 0$ for $i = 1, 2, ..., n$. Therefore, either $r = 0$, or $x_1 = x_2 = \cdots = x_n = 0$.

10.

$$-1\,(\mathbf{x}) = -1 \cdot \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} -x_1 \\ -x_2 \\ \vdots \\ -x_n \end{bmatrix} = -\mathbf{x}.$$

11. We have

$$\mathbf{x} + \mathbf{y} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} x_1 + y_1 \\ x_2 + y_2 \\ \vdots \\ x_n + y_n \end{bmatrix}$$

$$\mathbf{x} + \mathbf{z} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix} = \begin{bmatrix} x_1 + z_1 \\ x_2 + z_2 \\ \vdots \\ x_n + z_n \end{bmatrix}$$

$$\begin{aligned} x_1 + y_1 &= x_1 + z_1 \\ x_2 + y_2 &= x_2 + z_2 \\ &\vdots \\ x_n + y_n &= x_n + z_n \end{aligned}$$

Hence, $y_i = z_i, \quad i = 1, 2, \ldots n$, so $\mathbf{y} = \mathbf{z}$.

12. We have

$$\begin{aligned} (r_1\mathbf{x} + s_1\mathbf{y}) + (r_2\mathbf{x} + s_2\mathbf{y}) &= (r_1 + r_2)\mathbf{x} + (s_1 + s_2)\mathbf{y} \\ k(r\mathbf{x} + s\mathbf{y}) &= (kr)\mathbf{x} + (ks)\mathbf{y} \end{aligned}$$

Section 0.5, page 41

2. (a) and (d).

4. (a) and (c).

6. (a) $\begin{bmatrix} -5 \\ 5 \\ 6 \\ 1 \end{bmatrix} = 2\begin{bmatrix} -1 \\ 2 \\ 4 \\ 3 \end{bmatrix} - \begin{bmatrix} 3 \\ -1 \\ 2 \\ 5 \end{bmatrix}$

8. (a).

10. (b) and (c).

12. (a) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 3 \\ -8 \end{bmatrix}$ (c) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

14. (a) Let $S_1 = \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_k\}$ and $S_2 = \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_k, \mathbf{x}_{k+1}, \ldots, \mathbf{x}_n\}$. Then for some $a_1, a_2, \ldots, a_k$, not all zero, we have

$$a_1\mathbf{x}_1 + a_2\mathbf{x}_2 + \cdots + a_k\mathbf{x}_k = \mathbf{0}$$

Thus,

$$a_1\mathbf{x}_1 + a_2\mathbf{x}_2 + \cdots + a_k\mathbf{x}_k + 0\mathbf{x}_{k+1} + \cdots + 0\mathbf{x}_n = \mathbf{0}.$$

(b) If S_1 is linearly dependent, then by (a) S_2 is linearly dependent.

15. Let $S = \{\mathbf{0}, \mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_k\}$. Then

$$1 \cdot \mathbf{0} + 0 \cdot \mathbf{x}_1 + 0 \cdot \mathbf{x}_2 + \cdots + 0 \cdot \mathbf{x}_k = \mathbf{0}.$$

16. Since the dimension of R^n is n, any set of linearly independent vectors must have no more than n elements (Theorem 0.9).

17. If R^n were spanned by such a set, its dimension would be less than n.

18. 3.

20. Suppose that $\mathbf{A}$ is nonsingular. Then by Corollary 0.1, $\mathbf{A}$ is row equivalent to $\mathbf{I}_n$, so rank $\mathbf{A} = n$. Conversely, suppose that $\mathbf{A}$ is row equivalent to an $n \times n$ matrix $\mathbf{B}$ in reduced row echelon form with n nonzero rows. Hence, $\mathbf{B} = \mathbf{I}_n$. Thus, $\mathbf{A}$ is row equivalent to $\mathbf{I}_n$ and by Corollary 0.1, $\mathbf{A}$ is nonsingular.

Chapter 1

Section 1.1, page 57

2. Let

$$\begin{aligned} x &= \text{number of machines of model A} \\ y &= \text{number of machines of model B} \end{aligned}$$

Minimize $z = 15,000x + 20,000y$

subject to

$$\begin{aligned} 30x + 50y &\geq 320 \\ x + 2y &\leq 12 \end{aligned}$$

$$x \geq 0, \quad y \geq 0$$

To change to standard form, change the objective function to a maximum and the first constraint to $\leq$.

4. Let

$$\begin{aligned} x_1 &= \text{number of acres of corn} \\ x_2 &= \text{number of acres of soybeans} \\ x_3 &= \text{number of acres of oats} \end{aligned}$$

Maximize $z = 40x_1 + 30x_2 + 20x_3$

subject to

$$\begin{aligned} x_1 + x_2 + x_3 &\leq 12 \\ 6x_1 + 6x_2 + 2x_3 &\leq 48 \\ 36x_1 + 24x_2 + 18x_3 &\leq 360 \end{aligned}$$

$$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0.$$

This model is in standard form.

6. Let

$$\begin{aligned} x &= \text{number of barrels using four-field collector} \\ y &= \text{number of barrels using five-field collector} \end{aligned}$$

Minimize $z = 14x + 18y$

subject to

$$\begin{array}{rcrcr} x & + & y & \leq & 2,500,000 \\ 0.18x & - & 0.12y & \leq & 0 \end{array}$$

$x \geq 0, \quad y \geq 0$

To change to standard form, change the objective function to a maximum.

8. Let

$$\begin{aligned} x_1 &= \text{amount invested in utilities stock} \\ &\quad \text{(in tens of thousands of dollars)} \\ x_2 &= \text{amount invested in electronics stock} \\ &\quad \text{(in tens of thousands of dollars)} \\ x_3 &= \text{amount invested in bonds} \\ &\quad \text{(in tens of thousands of dollars)} \end{aligned}$$

Maximize $z = 0.09x_1 + 0.04x_2 + 0.05x_3$
subject to

$$\begin{array}{rcrcrcr} x_1 & + & x_2 & + & x_3 & \leq & 20 \\ \frac{1}{2}x_1 & + & \frac{1}{2}x_2 & - & \frac{1}{2}x_3 & \leq & 0 \\ x_1 & & & & & \leq & 4 \\ & & & & x_3 & \geq & 7 \end{array}$$

$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0.$

To change to standard form, change the fourth constraint to $\leq$.

10. Let x_{ij} =amount of ith component in jth mixture, where

component 1 = alkylate
component 2 = catalytic cracked
component 3 = straight run

component 4 = isopentane
mixture 1 = high octane
mixture 2 = low octane

Maximize $z = 6.5\sum_{i=1}^{4} x_{i1} + 7.5\sum_{i=1}^{4} x_{i2} - 7.2\,(x_{11} + x_{12}) -$
$4.35\,(x_{21} + x_{22}) - 3.8\,(x_{31} + x_{32}) - 4.3\,(x_{41} + x_{42})$
subject to

$$\begin{array}{rcrcrcrcl}
108x_{11} & + & 94x_{21} & + & 87x_{31} & + & 108x_{41} & = & 100\sum_{i=1}^{4} x_{i1} \\
99x_{12} & + & 87x_{22} & + & 80x_{32} & + & 100x_{42} & = & 90\sum_{i=1}^{4} x_{i2} \\
5x_{11} & + & 6.5x_{21} & + & 4x_{31} & + & 18x_{41} & = & 7\sum_{i=1}^{4} x_{i1} \\
5x_{12} & + & 6.5x_{22} & + & 4x_{32} & + & 18x_{42} & = & 7\sum_{i=1}^{4} x_{i2}
\end{array}$$

$$\begin{array}{rcrcl}
x_{11} & + & x_{12} & \leq & 700 \\
x_{21} & + & x_{22} & \leq & 600 \\
x_{31} & + & x_{32} & \leq & 900 \\
x_{41} & + & x_{42} & \leq & 500
\end{array}$$

$$\sum_{i=1}^{4} x_{i1} = 1300$$

$$\sum_{i=1}^{4} x_{i2} = 800$$

$$x_{ij} \geq 0, \quad i = 1,2,3,4;\ j = 1,2.$$

To change to standard form, change constraints 1, 2, 3, 4, 9, and 10 to $\leq$.

Project 1: Let

x_1 = fraction of mixture that is Wayne
x_2 = fraction of mixture that is Wayne TW
x_3 = fraction of mixture that is Purina Meal
x_4 = fraction of mixture that is Purina Chow
x_5 = fraction of mixture that is Purina HP
x_6 = fraction of mixture that is Gaines
x_7 = fraction of mixture that is Burgerbits
x_8 = fraction of mixture that is Agway 2000

(a)

$$\begin{array}{rcrcrcrcrcrcrcrcl}
25x_1 & + & 24x_2 & + & 27x_3 & + & 23.8x_4 & + & 26x_5 & + & 21x_6 & + & 23x_7 & + & 25.5x_8 & \geq & 20 \\
8x_1 & + & 9x_2 & + & 10.5x_3 & + & 9.4x_4 & + & 10x_5 & + & 8x_6 & + & 7x_7 & + & 10.5x_8 & \geq & 5 \\
2.1x_1 & + & 1.6x_2 & + & 1.6x_3 & + & 1.6x_4 & + & 1.6x_5 & + & .9x_6 & + & 1.5x_7 & + & 1.5x_8 & \geq & 1.4 \\
2.15x_1 & + & 1.2x_2 & + & 2.5x_3 & + & 1.75x_4 & + & 1.6x_5 & + & x_6 & + & 1.5x_7 & + & 1.5x_8 & \geq & 1 \\
1.43x_1 & + & x_2 & + & 1.4x_3 & + & 1.03x_4 & + & 1.2x_5 & + & .8x_6 & + & .8x_7 & + & 1.7x_8 & \geq & .8 \\
.73x_1 & + & .98x_2 & + & .8x_3 & + & .71x_4 & + & .9x_5 & + & .5x_6 & + & .5x_7 & + & .69x_8 & \geq & .5 \\
1.15x_1 & + & 1.15x_2 & + & .78x_3 & + & .64x_4 & + & 1.1x_5 & + & x_6 & + & 1.5x_7 & + & x_8 & \geq & 1 \\
.17x_1 & + & .22x_2 & + & .29x_3 & + & .27x_4 & + & .15x_5 & + & .036x_6 & + & .05x_7 & + & .23x_8 & \geq & .4 \\
45.77x_1 & + & 46.15x_2 & + & 41.83x_3 & + & 48.1x_4 & + & 41.45x_5 & + & 51.76x_6 & + & 47.15x_7 & + & 45.28x_8 & \geq & 25 \\
3.5x_1 & + & 4.7x_2 & + & 4.3x_3 & + & 3.7x_4 & + & 4x_5 & + & x_6 & + & 5x_7 & + & 2.9x_8 & \leq & 8 \\
10x_1 & + & 10x_2 & + & 9x_3 & + & 9x_4 & + & 12x_5 & + & 10x_6 & + & 12x_7 & + & 9.2x_8 & \leq & 5
\end{array}$$

(b) $\sum_{i=1}^{8} x_i = 1$

(c) Minimize $z = .17x_1 + .17x_2 + .17x_3 + .16x_4 + .21x_5 + .20x_6 + .17x_7 + .16x_8$

(d) Let x_1, $x_2, \ldots$, x_8 be nonnegative and assume that $\sum_{i=1}^{8} x_i = 1$. If $a_i \geq b$, for all i, then

$a_1x_1 + a_2x_2 + \cdots + a_8x_8 \geq bx_1 + bx_2 + \cdots + bx_8$

$= b \sum_{i=1}^{8} x_i = b$. If $a_i < b$ for all i, then

$a_1x_1 + a_2x_2 + \cdots + a_8x_8 < bx_1 + bx_2 + \cdots + bx_8$

$= b \sum_{i=1}^{8} x_i = b$.

(e) Redundant: 1, 2, 4, 5, 6, 9, 10

Impossible: 8, 11

Minimize

$z = .17x_1 + .17x_2 + +.17x_3 + .16x_4 + .21x_5 + .20x_6 + .17x_7 + .16x_8$

subject to

$$\begin{array}{rcrcrcrcrcl}
2.1x_1 & + & 1.6x_2 & + & 1.6x_3 & + & 1.6x_4 & + & 1.6x_5 & & \\
 & + & .9x_6 & + & 1.5x_7 & + & 1.5x_8 & & & \geq & 1.4 \\
1.15x_1 & + & 1.15x_2 & + & .78x_3 & + & .64x_4 & + & 1.1x_5 & & \\
 & + & x_6 & + & 1.5x_7 & + & x_8 & & & \geq & 1 \\
x_1 & + & x_2 & + & x_3 & + & x_4 & + & x_5 & & \\
 & + & x_6 & + & x_7 & + & x_8 & & & = & 1
\end{array}$$

$x_i \geq 0, \quad i = 1, 2, \ldots, 8$

(f) Because of the constraints in (b), the cheapest mixture of foods is some combination of foods 4 and 8. Using only food 8 satisfies the constraints in (e).

Project 2: Let

x_1 = number of advertising units in *TV Guide* /month
x_2 = number of advertising units in *Newsweek*/month
x_3 = number of advertising units in *Time*/month

Maximize $z = 19,089x_1 + 11,075x_2 + 10,813x_3$

subject to

$$\begin{array}{rcrcrcr}
55x_1 & + & 35.335x_2 & + & 49.48x_3 & \leq & 200 \\
2729x_1 & + & 3387x_2 & + & 3767x_3 & \geq & 12,000 \\
4312x_1 & + & 2808x_2 & + & 2714x_3 & \leq & 16,000 \\
x_1 & & & & & \leq & 2 \\
 & & x_2 & & & \leq & 4 \\
 & & & & x_3 & \leq & 4
\end{array}$$

$x_1, x_2, x_3 \geq 0$

where the objective function and first three constraints have been divided by 1000.

Project 3: Let

x_1 = shovel dozer hours
x_2 = large backhoe hours
x_3 = backhoe I hours
x_4 = backhoe II hours
x_5 = crane with clamshell hours

Minimize $z = 17.5x_1 + 40x_2 + 27.5x_3 + 22x_4 + 47x_5$

subject to

$$\begin{array}{rcrcrcrcrcl}
28.23x_1 & + & 150x_2 & + & 90x_3 & + & 60x_4 & + & 40x_5 & = & 1000 \\
x_1 & & & & & & & & & \leq & 30 \\
& & x_2 & & & & & & & \leq & 30 \\
& & & & x_3 & & & & & \leq & 30 \\
& & & & & & x_4 & & & \leq & 40 \\
& & & & & & & & x_5 & \leq & 27.5
\end{array}$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

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2. Maximize $z = [2 \quad 3 \quad 4]\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

subject to

$$\begin{bmatrix} 3 & 2 & -3 \\ 2 & 3 & 2 \\ -3 & 1 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \leq \begin{bmatrix} 4 \\ 6 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

4. Maximize $z = [40 \quad 30 \quad 20]\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

subject to

$$\begin{bmatrix} 1 & 1 & 1 \\ 6 & 6 & 2 \\ 36 & 24 & 18 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \leq \begin{bmatrix} 12 \\ 48 \\ 360 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

6. Maximize $z = [2 \quad 5 \quad -5 \quad 0 \quad 0] \begin{bmatrix} x \\ y^+ \\ y^- \\ u \\ v \end{bmatrix}$

subject to

$$\begin{bmatrix} 3 & 2 & -2 & 1 & 0 \\ 2 & 9 & -9 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y^+ \\ y^- \\ u \\ v \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y^+ \\ y^- \\ u \\ v \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

8. Maximize $z = [-3 \quad -2.5 \quad 0 \quad 0] \begin{bmatrix} x \\ y \\ u \\ v \end{bmatrix}$

subject to

$$\begin{bmatrix} -30 & -40 & 1 & 0 \\ 40 & 20 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ u \\ v \end{bmatrix} \leq \begin{bmatrix} -120 \\ 80 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ u \\ v \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

10. Maximize $z = [300 \quad 500 \quad 400 \quad 0 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$

subject to

$$\begin{bmatrix} 4 & 4 & 2 & 1 & 0 \\ 2 & 3 & 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 80 \\ 50 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

11. (a) We have

$$\begin{bmatrix} 2 & 2 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 13 \end{bmatrix} \leq \begin{bmatrix} 8 \\ 15 \end{bmatrix}; \quad z = 340$$

and

$$\begin{bmatrix} 2 & 2 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 8 \\ 14 \end{bmatrix} \leq \begin{bmatrix} 8 \\ 15 \end{bmatrix}; \quad z = 420$$

Also,

$$\begin{bmatrix} 2 \\ 1 \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 \\ 3 \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(b) We have

$$\begin{bmatrix} 2 & 2 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 18 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ 16 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ 3 \end{bmatrix} \not\geq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

12. Maximize $z = [2 \quad 3 \quad 5 \quad 0 \quad 0 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}$

subject to

$$\begin{bmatrix} 3 & 2 & 1 & 1 & 0 & 0 \\ 1 & 1 & -2 & 0 & 1 & 0 \\ 2 & 5 & 4 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

14. (a) We have

$$\begin{bmatrix} 2 & 3 \\ 5 & 1 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ 7 \\ 11 \end{bmatrix} \leq \begin{bmatrix} 10 \\ 12 \\ 15 \end{bmatrix}$$

and $\begin{bmatrix} 1 \\ 2 \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

(b) $x_4 = 2, \quad x_5 = 5, \quad x_6 = 4$

15. If $\mathbf{x}_1$ and $\mathbf{x}_2$ are feasible solutions, then $\mathbf{x}_1 \geq 0$ and $\mathbf{x}_2 \geq 0$, so $\mathbf{x} = \frac{1}{3}\mathbf{x}_1 + \frac{2}{3}\mathbf{x}_2 \geq \mathbf{0}$.
Also, if $\mathbf{A}\mathbf{x}_1 \leq \mathbf{b}$ and $\mathbf{A}\mathbf{x}_2 \leq \mathbf{b}$, then

$$\begin{aligned} \mathbf{A}\mathbf{x} &= \mathbf{A}\left(\frac{1}{3}\mathbf{x}_1 + \frac{2}{3}\mathbf{x}_2\right) \\ &= \frac{1}{3}\mathbf{A}\mathbf{x}_1 + \frac{2}{3}\mathbf{A}\mathbf{x}_2 \\ &\leq \frac{1}{3}\mathbf{b} + \frac{2}{3}\mathbf{b} = \mathbf{b} \end{aligned}$$

16. We have

$$\begin{aligned} \mathbf{A}\mathbf{x} &= \mathbf{A}\left(r\mathbf{x}_1 + s\mathbf{x}_2\right) \\ &= r\mathbf{A}\mathbf{x}_1 + s\mathbf{A}\mathbf{x}_2 \\ &\leq r\mathbf{b} + s\mathbf{b} = (r+s)\,\mathbf{b} = \mathbf{b} \end{aligned}$$

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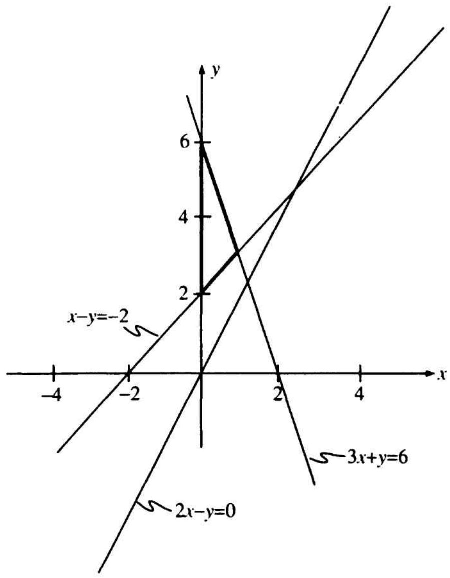
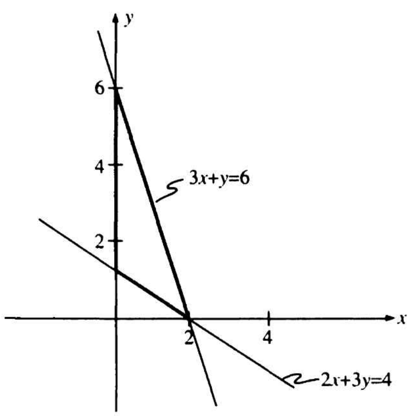
2.



4.



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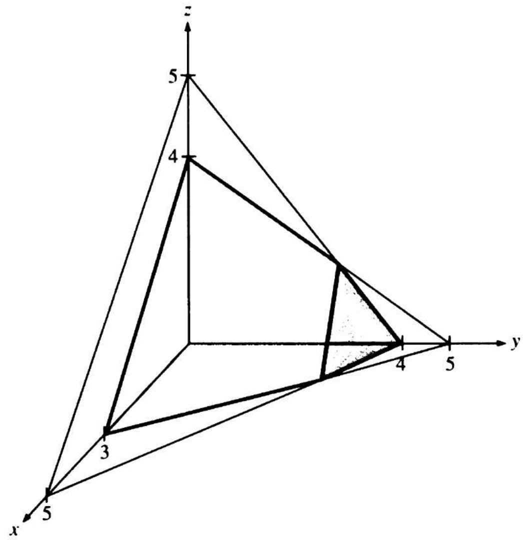

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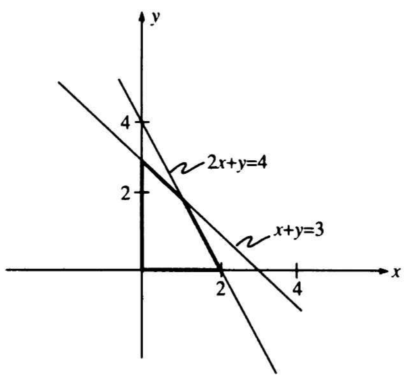
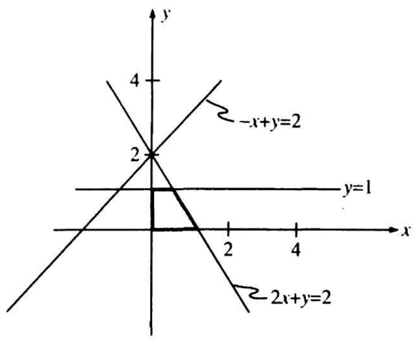
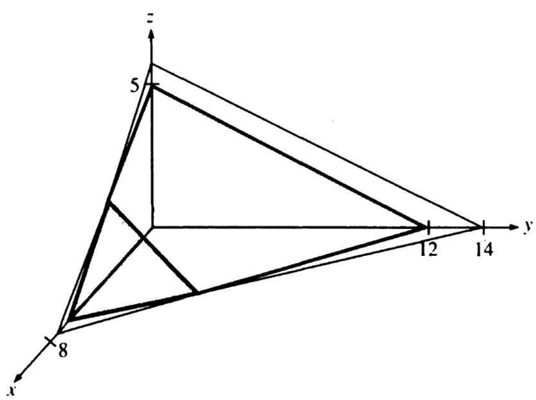


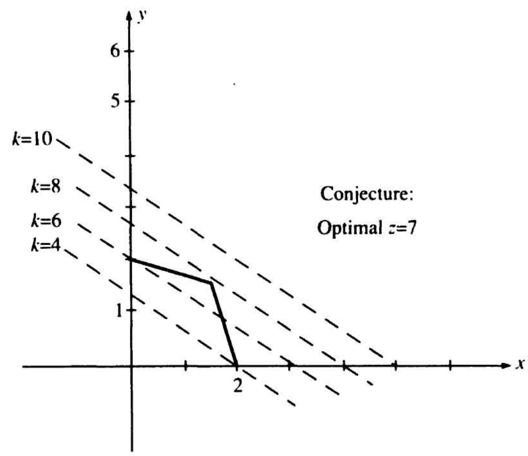
10.



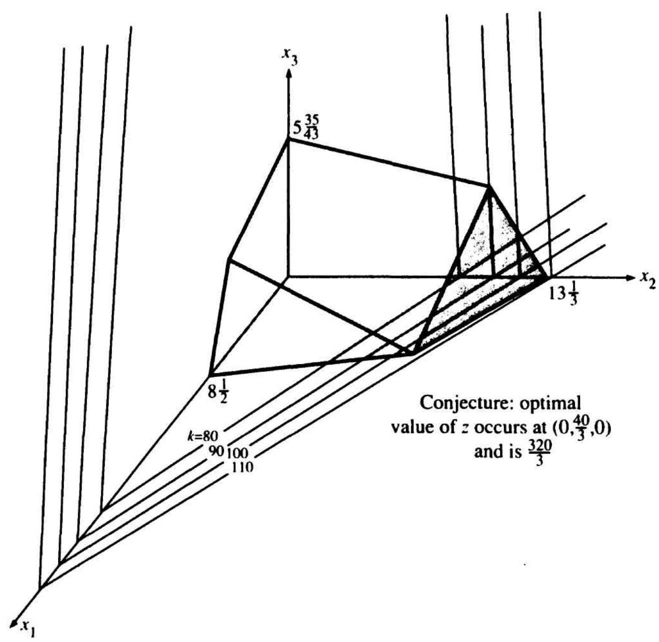
12.



14.



16.



18. No. 20. Yes. 22. No. 24. Yes

25. We have $R^n = \{\mathbf{x} \mid \mathbf{x} = (x_1, x_2, ..., x_n), x_i \text{ real numbers}\}$. Now let $\mathbf{v} = (v_1, v_2, ..., v_n)$ and $\mathbf{w} = (w_1, w_2, ..., w_n) \in R^n$. Then

$$\begin{aligned}\mathbf{x} &= \lambda\mathbf{v} + (1-\lambda)\mathbf{w} \\ &= (\lambda v_1 + (1-\lambda)w_1, \lambda v_2 + (1-\lambda)w_2, ..., \lambda v_n + (1-\lambda)w_n) \in R^n.\end{aligned}$$

Hence, R^n is a convex set.

26. Let V be a subspace of R^n and let $\mathbf{v}$ and $\mathbf{w} \in V$. Then since V is a subspace of R^n,

$$\mathbf{x} = \lambda\mathbf{v} + (1-\lambda)\mathbf{w} \in V \text{ for } 0 < \lambda < 1.$$

Hence, V is a convex set.

27. Let $R = \{\mathbf{x} \in R^n \mid a_i \le x_i \le b_i\}$ be a rectangle in R^n. Let $\mathbf{v}$ and $\mathbf{w} \in R$. Then

$$a_i \le v_i \le b_i \text{ and } a_i \le w_i \le b_i.$$

If

$$\mathbf{x} = \lambda\mathbf{v} + (1-\lambda)\mathbf{w}, \quad 0 < \lambda < 1,$$

then

$$x_i = \lambda v_i + (1-\lambda)w_i$$

We now have

$$a_i = \lambda a_i + (1-\lambda)a_i \le \lambda v_i + (1-\lambda)w_i \le \lambda b_i + (1-\lambda)b_i = b_i$$

or

$$a_i \le x_i \le b_i$$

Hence, $\mathbf{x} \in R$.

28. Let $\mathbf{v}$ and $\mathbf{w} \in H_2$ so that $\mathbf{c}^T\mathbf{v} \ge k$ and $\mathbf{c}^T\mathbf{w} \ge k$. If

$$\mathbf{x} = \lambda\mathbf{v} + (1-\lambda)\mathbf{w}, \quad 0 < \lambda < 1,$$

then

$$\begin{aligned}\mathbf{c}^T\mathbf{x} = \mathbf{c}^T(\lambda\mathbf{v} + (1-\lambda)\mathbf{w}) &= \mathbf{c}^T(\lambda\mathbf{v}) + (1-\lambda)\mathbf{c}^T\mathbf{w} \\ &\ge \lambda k + (1-\lambda)k = k\end{aligned}$$

Hence, $\mathbf{x} \in H_2$.

29. We have $H = \left\{ \mathbf{x} \in R^n \mid \mathbf{a}^T\mathbf{x} = \mathbf{b} \right\}$. Let $\mathbf{x}_1$ and $\mathbf{x}_2 \in H$ so that $\mathbf{a}^T\mathbf{x}_1 = \mathbf{b}$ and $\mathbf{a}^T\mathbf{x}_2 = \mathbf{b}$. If $\mathbf{x} = \lambda\mathbf{x}_1 + (1-\lambda)\mathbf{x}_2$, $0 < \lambda < 1$, then

$$\begin{aligned} \mathbf{a}^T\mathbf{x} &= \mathbf{a}^T(\lambda\mathbf{x}_1 + (1-\lambda)\mathbf{x}_2) \\ &= \lambda\mathbf{a}^T\mathbf{x}_1 + (1-\lambda)\mathbf{a}^T\mathbf{x}_2 \\ &= \lambda\mathbf{b} + (1-\lambda)\mathbf{b} = \mathbf{b}. \end{aligned}$$

Hence, $\mathbf{x} \in H$.

30. Let $S_1, S_2, \ldots, S_n$ be a finite collection of convex sets, and let T be their intersection. Let $\mathbf{x}_1, \mathbf{x}_2 \in T$. Thus, $\mathbf{x}_1$ and $\mathbf{x}_2 \in S_i$ for each i. Then the line segment joining $\mathbf{x}_1$ and $\mathbf{x}_2$ is contained in S_i for each i. Hence, the line segment joining $\mathbf{x}_1$ and $\mathbf{x}_2$ is contained in T.

31. *Proof* I: Let $\mathbf{x}_1$ and $\mathbf{x}_2 \in$ solution set. If $\mathbf{x} = \lambda\mathbf{x}_1 + (1-\lambda)\mathbf{x}_2$, $0 < \lambda < 1$, then

$$\begin{aligned} \mathbf{A}\mathbf{x} &= \mathbf{A}(\lambda\mathbf{x}_1 + (1-\lambda)\mathbf{x}_2) \\ &= \lambda\mathbf{A}\mathbf{x}_1 + (1-\lambda)\mathbf{A}\mathbf{x}_2 \\ &= \lambda\mathbf{b} + (1-\lambda)\mathbf{b} = \mathbf{b}. \end{aligned}$$

Thus, $\mathbf{A}\mathbf{x} = \mathbf{b}$ and $\mathbf{x} \in$ solution set.

Proof II: Let $\mathbf{A}_1, \mathbf{A}_2, \ldots, \mathbf{A}_m$ be the m rows of $\mathbf{A}$, and let $\mathbf{b} = [b_1\ b_2 \ldots\ b_m]^T$. The set of solutions to $\mathbf{A}\mathbf{x} = \mathbf{b}$ is the intersection of the sets of solutions to $\mathbf{A}_i\mathbf{x} = b_i$, for $i = 1, 2, \ldots, m$. These sets are, by definition, hyperplanes, and hence are convex sets. Thus, the set of solutions to $\mathbf{A}\mathbf{x} = \mathbf{b}$ is the intersection of convex sets, which is a convex set.

32. We have $\mathbf{c}^T\mathbf{x}_1 = k$ and $\mathbf{c}^T\mathbf{x}_2 = k$. If $\mathbf{x} = \lambda\mathbf{x}_1 + (1-\lambda)\mathbf{x}_2$, $0 < \lambda < 1$, then

$$\begin{aligned} \mathbf{c}^T\mathbf{x} &= \mathbf{c}^T(\lambda\mathbf{x}_1 + (1-\lambda)\mathbf{x}_2) \\ &= \lambda\mathbf{c}^T\mathbf{x}_1 + (1-\lambda)\mathbf{c}^T\mathbf{x}_2 \\ &= \lambda k + (1-\lambda)k = k. \end{aligned}$$

33. Let S be the set of all solutions to $\mathbf{Ax} \leq \mathbf{b}$ and let $\mathbf{x}_1$ and $\mathbf{x}_2 \in S$. If $\mathbf{x} = \lambda\mathbf{x}_1 + (1-\lambda)\,\mathbf{x}_2$, $0 < \lambda < 1$, then

$$\begin{aligned}\mathbf{Ax} &= \mathbf{A}\,(\lambda\mathbf{x}_1 + (1-\lambda)\,\mathbf{x}_2)\\ &= \lambda\mathbf{Ax}_1 + (1-\lambda)\,\mathbf{Ax}_2\\ &\leq \lambda\mathbf{b} + (1-\lambda)\,\mathbf{b} = \mathbf{b}.\end{aligned}$$

Hence, $\mathbf{x} \in S$.

34. Let S be the set of all solutions to $\mathbf{Ax} > \mathbf{b}$ and let $\mathbf{x}_1$ and $\mathbf{x}_2 \in S$. If $\mathbf{x} = \lambda\mathbf{x}_1 + (1-\lambda)\,\mathbf{x}_2$, $0 < \lambda < 1$, then

$$\begin{aligned}\mathbf{Ax} &= \mathbf{A}\,(\lambda\mathbf{x}_1 + (1-\lambda)\,\mathbf{x}_2)\\ &= \lambda\mathbf{Ax}_1 + (1-\lambda)\,\mathbf{Ax}_2\\ &> \lambda\mathbf{b} + (1-\lambda)\,\mathbf{b} = \mathbf{b}.\end{aligned}$$

Hence, $\mathbf{x} \in S$.

35. Suppose that S is a convex set and let $\mathbf{v}'$ and $\mathbf{w}' \in f\,(S)$. Then $\mathbf{v}' = f\,(\mathbf{v})$ and $\mathbf{w}' = f\,(\mathbf{w})$ for some $\mathbf{v}$ and $\mathbf{w}$ in S. Now let

$$\mathbf{y} = \lambda\mathbf{v}' + (1-\lambda)\,\mathbf{w}'$$

Then

$$\begin{aligned}\mathbf{y} &= \lambda f\,(\mathbf{v}) + (1-\lambda)\,f\,(\mathbf{w})\\ &= f\,(\lambda\mathbf{v} + (1-\lambda)\,\mathbf{w})\end{aligned}$$

Since S is a convex set, $\mathbf{x} = \lambda\mathbf{v} + (1-\lambda)\,\mathbf{w} \in S$, so $\mathbf{y} = f\,(\mathbf{x}) \in f\,(S)$. Hence, $f\,(S)$ is a convex set.

36. We defined f to be convex if

$$f(\lambda\mathbf{x}_1 + (1-\lambda)\mathbf{x}_2) \leq \lambda f(\mathbf{x}_1) + (1-\lambda)f(\mathbf{x}_2).$$

The left side of this inequality is the second coordinate of the point $(\mathbf{x}, f(\mathbf{x}))$ on the graph of f, where $\mathbf{x} = \lambda\mathbf{x}_1 + (1-\lambda)\,\mathbf{x}_2$. The right side gives the points on the line segment joining the points $(\mathbf{x}_1, f(\mathbf{x}_1))$ and $(\mathbf{x}_2, f(\mathbf{x}_2))$ on the graph of f.

37. Let $\mathbf{x}_1$ and $\mathbf{x}_2 \in R^n$. Then

$$\begin{aligned} f(\lambda\mathbf{x}_1 + (1-\lambda)\mathbf{x}_2) &= \mathbf{c}^T\mathbf{x} \\ &= \mathbf{c}^T(\lambda\mathbf{x}_1 + (1-\lambda)\mathbf{x}_2) \\ &= \lambda\mathbf{c}^T\mathbf{x}_1 + (1-\lambda)\mathbf{c}^T\mathbf{x}_2 \\ &= \lambda f(\mathbf{x}_1) + (1-\lambda)f(\mathbf{x}_2) \end{aligned}$$

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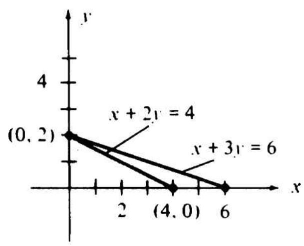
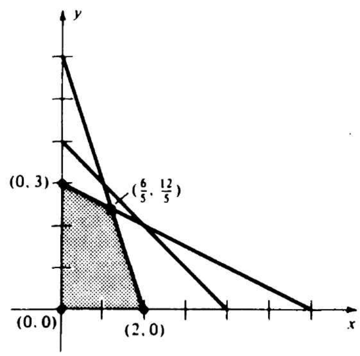
2. (a)



$\begin{bmatrix} 0 \\ 2 \end{bmatrix}$ (b) $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$; $z = -6$

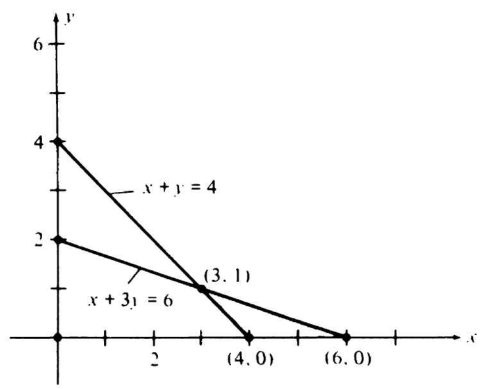
4. (a)



$$\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 2 \\ 6 \end{bmatrix}, \quad \begin{bmatrix} \frac{6}{5} \\ \frac{12}{5} \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

(b) $\begin{bmatrix} \frac{6}{5} \\ \frac{12}{5} \end{bmatrix}$; $z = \frac{48}{5}$

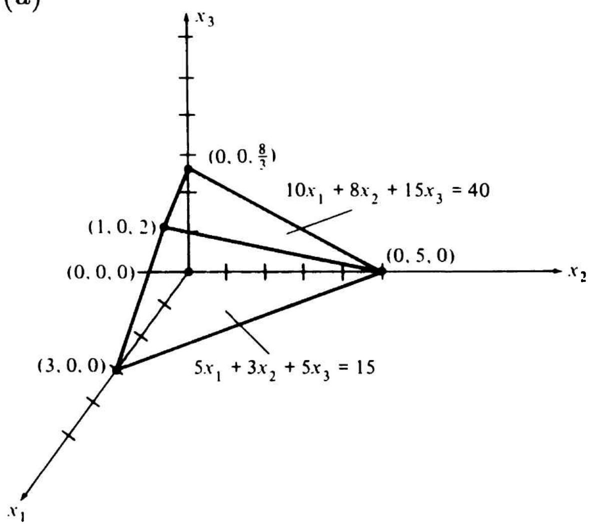
6. (a)



$$\begin{bmatrix} 6 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 4 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

(b) Any point on the line segment joining $\begin{bmatrix} 6 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$; $z = 3$

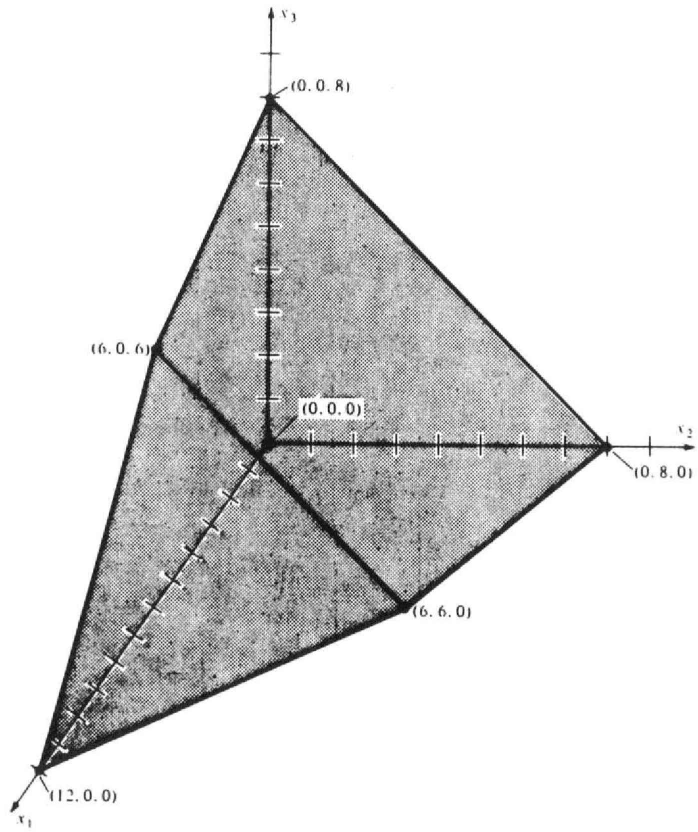
8. (a)



$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 0 \\ \frac{8}{3} \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix}$$

(b) $\begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix}$; $\quad z = 20$

10. (a)



$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 6 \\ 0 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 8 \end{bmatrix}, \begin{bmatrix} 0 \\ 8 \\ 0 \end{bmatrix}, \begin{bmatrix} 6 \\ 6 \\ 0 \end{bmatrix}$$

(b) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}; \quad z = 0$

12. No feasible solution.

13. Let $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_k \in R^n$, and let S be the set of all convex combinations

of $\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_k$. Let $\mathbf{u}, \mathbf{v} \in S$, and write

$$\mathbf{u} = \sum_{i=1}^{k} c_i \mathbf{x}_i, \quad \mathbf{v} = \sum_{i=1}^{k} d_i \mathbf{x}_i.$$

Suppose $\mathbf{x} = \lambda \mathbf{u} + (1 - \lambda) \mathbf{v}$, where $0 \leq \lambda \leq 1$. Then

$$\begin{aligned} \mathbf{x} &= \lambda \sum_{i=1}^{k} c_i \mathbf{x}_i + (1 - \lambda) \sum_{i=1}^{k} d_i \mathbf{x}_i \\ &= \sum_{i=1}^{k} [\lambda c_i + (1 - \lambda) d_i] \mathbf{x}_i \end{aligned}$$

Since

$$\begin{aligned} \sum_{i=1}^{k} [\lambda c_i + (1 - \lambda) d_i] &= \lambda \sum_{i=1}^{k} c_i + \sum_{i=1}^{k} d_i - \lambda \sum_{i=1}^{k} c_i \\ &= \lambda + 1 - \lambda = 1, \end{aligned}$$

and $[\lambda c_i + (1 - \lambda) d_i] \geq 0$, then $\mathbf{x} \in S$.

14. Suppose that $\mathbf{u}$ is an extreme point of S and that $\mathbf{u}$ is a convex combination of $\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_k$ where $\mathbf{u} \neq \mathbf{x}_j$, $j = 1, 2, ...k$. Then $\mathbf{u} = \sum_{i=1}^{k} c_i \mathbf{x}_i$ and $\sum_{i=1}^{k} c_i = 1$, $c_i \geq 0$, $i = 1, 2, ...k$. Suppose that $c_k \neq 0$. We can write

$$\mathbf{u} = (1 - c_k) \left(\sum_{i=1}^{k-1} \frac{c_i}{(1 - c_k)} \mathbf{x}_i \right) + c_k \mathbf{x}_k.$$

Thus, $\mathbf{u}$ is in the interior of the line segment joining $\mathbf{x}_k$ and $\left(\sum_{i=1}^{k-1} \frac{c_i}{(1-c_k)} \mathbf{x}_i \right)$, and hence we have a contradiction. The converse statement follows easily form the definition of extreme point.

15. Suppose that S is convex. Let $\mathbf{x}$ be a convex combination of k points in S. If $k = 2$, then $\mathbf{x} \in S$ because S is convex. For a proof by induction, assume that any convex combination of k points in S belongs to S. Now consider the convex combination of $k + 1$ points

$$\mathbf{x} = \sum_{i=1}^{k+1} c_i \mathbf{x}_i, \quad \sum_{i=1}^{k+1} c_i = 1, \quad c_i \geq 0$$

for $i = 1, 2, \ldots k+1$. Assume that $c_{k+1} \neq 1$ and write

$$\mathbf{x} = \left(\sum_{i=1}^{k} c_i\right)\left(\sum_{i=1}^{k} \frac{c_i}{\sum_{i=1}^{k} c_i}\mathbf{x}_i\right) + c_{k+1}\mathbf{x}_{k+1}.$$

Thus, $\mathbf{x}$ is a convex combination of two points, the first of which is in S because it is a convex combination of k points in S. Conversely, if every convex combination of a finite number of points of S is in S, then every convex combination of two points of S is in S.

16. Suppose that the optimal value of z is k and that it is attained at the extreme points $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_r$. Let $\mathbf{x} = \sum_{i=1}^{r} c_i\mathbf{x}_i$, where $\sum_{i=1}^{r} c_i = 1, \quad c_i \geq 0$. Then

$$\begin{aligned}\mathbf{c}^T\mathbf{x} = \mathbf{c}^T\left(\sum_{i=1}^{r} c_i\mathbf{x}_i\right) &= \sum_{i=1}^{r} c_i\left(\mathbf{c}^T\mathbf{x}_i\right)\\ &= \sum_{i=1}^{r} c_i k = k.\end{aligned}$$

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2. 80 kg pizza-flavored potato chips

   0 kg chili-flavored chips

   Net profit = \$9.60

4. Let

   x_1 = Amount of pizza-flavored potato chips (in kg)
   x_2 = Amount of chili-flavored potato chips (in kg)

   Maximize $z = 0.12x_1 + 0.10x_2$

   subject to

$$\begin{array}{rcrcrcrcrcr} 3x_1 & + & 3x_2 & + & x_3 & & & & & = & 240 \\ 5x_1 & + & 4x_2 & + & & + & x_4 & & & = & 480 \\ 2x_1 & + & 3x_2 & + & & & & + & x_5 & = & 360 \end{array}$$

$$x_j \geq 0, \quad j = 1, 2, \ldots, 5$$

$$x_3 = 0, \quad x_4 = 80, \quad x_5 = 200$$

$x_3 =$ 0 minutes of unused time on the fryer.

$x_4 =$ 80 minutes of unused time on the flavorer.

$x_5 =$ 200 minutes of unused time on the packer.

The basic variables are x_1, x_4, and x_5.

6. (b) and (d)

8. (a)

$$\begin{array}{l} \text{Maximize } z = 3x + 2y \\ \text{subject to} \\ \begin{array}{rcrcrcrcr} 2x & - & y & + & u & & & = & 6 \\ 2x & + & y & + & & & v & = & 10 \end{array} \\ x \geq 0,\ y \geq 0,\ u \geq 0,\ v \geq 0 \end{array}$$

(b)

x	y	u	v	basic variables
0	0	6	10	u, v
0	10	16	0	y, u
3	0	0	4	x, v
4	2	0	0	x, y

(c) Optimal solution: $x = 0$, $y = 10$; $z = 20$

10.

$$\left[\begin{array}{cccc|cccc} a_{11} & a_{12} & \cdots & a_{1n} & 1 & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & a_{2n} & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & 0 & 0 & \cdots & 1 \end{array}\right] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \\ x'_1 \\ \vdots \\ x'_m \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

Multiplying these matrices we find that

$$\mathbf{Ax} + \mathbf{Ix}' = \mathbf{b}$$

11. Let $\mathbf{x}$ be an extreme point. By Theorem 1.9, the r colums of $\mathbf{A}$ corresponding to positive x_j form a linearly independent set of vectors in R^n. By Theorem 0.9, $r \leq m$. By Theorem 0.6, the set of columns $A_{i_1}, A_{i_2}, \ldots, A_{i_r}$ can be extended to a set of m linearly independent vectors in R^n by adjoining a suitably chosen set of $s - m$ columns of $\mathbf{A}$.

12. Let $\mathbf{x}$ be a basic feasible solution and number the columns of $\mathbf{A}$ so that the last m columns are linearly independent. By Theorem 1.8, $\mathbf{x}$ is an extreme point. Conversely, if $\mathbf{x}$ is an extreme point, then by Theorem 1.9, the columns of $\mathbf{A}$ corresponding to the positive components of $\mathbf{x}$ are linearly independent. By Corollary 1.1 these vectors can be extended to a linearly independent set of m vectors in R^m by adjoining other columns of $\mathbf{A}$. Hence, $\mathbf{A}$ is a basic feasible solution.

13. A standard linear programming problem may be transformed into a linear programming problem in canonical form by adding slack variables. By Theorems 1.11 and 1.12, the set of feasible solutions to the latter problem has a finite number of extreme points. By Theorem 1.13, these extreme points may be truncated to yield all extreme points of the set of feasible solutions to the given problem. Hence, the result follows.

Chapter 2

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2.

	x_1	x_2	x_3	x_4	x_5	
x_4	2	-5	1	1	0	3
x_5	1	4	0	0	1	5
	-1	-3	-5	0	0	0

4.

	x_1	x_2	x_3	x_4	
x_1	1	0	10/9	2/3	4
x_2	0	1	34/27	$-4/9$	16/3
	0	0	22/9	8/3	28

6. Using x_2 as the departing variable:

	x_1	x_2	x_3	x_4	x_5	
x_3	$-7/30$	$-3/5$	1	0	0	0
x_4	3/2	1	0	1	0	5/2
x_5	14/3	$-2/9$	0	0	1	1/9
	23/2	5	0	0	0	89/6

Using x_3 as the departing variable:

	x_1	x_2	x_3	x_4	x_5	
x_4	10/9	0	5/3	1	0	5/2
x_2	7/18	1	−5/3	0	0	0
x_5	385/81	0	−10/27	0	1	1/9
	86/9	0	25/3	0	0	89/6

8. $[4 \;\; 0 \;\; 0 \;\; 0 \;\; 4 \;\; 10 \;\; 0]^T$

10. Make 30 kg SWEET and 20 kg LO-SUGAR. Profit = \$12.

12. $[0 \;\; 0]^T$; $z = 12$

14. Plant 6 acres of corn, no soybeans, and 6 acres of oats. Profit = \$360.

16. Make 25,000 bags of GARDEN only. Profit = \$10,000.

18. Make 80 kg of pizza-flavored chips only. Net profit = \$9.60.

20. $\left[0 \;\; 4 \;\; \frac{4}{3} \;\; 0\right]^T$; $z = \frac{28}{3}$

22. No finite optimal solution

23. We have no means of finding an initial basic feasible solution.

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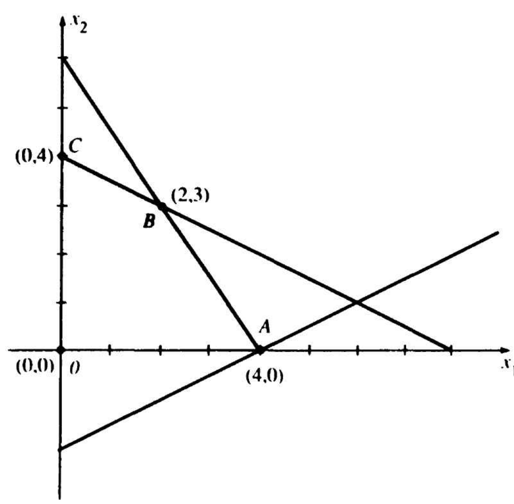
2. $[2 \;\; 3]^T$; $z = 22$

The simplex algorithm examines the following extreme points:

O, A, A, B



	↓ x_1	x_2	x_3	x_4	x_5	
x_3	1	2	1	0	0	8
← x_4	1	−2	0	1	0	4
x_5	3	2	0	0	1	12
	−5	−4	0	0	0	0

	x_1	x_2	x_3	x_4	x_5	
x_3	0	4	1	−1	0	4
x_1	1	−2	0	1	0	4
x_5	3	8	0	−3	1	0
	0	−14	0	5	0	20

	x_1	x_2	x_3	x_4	x_5	
x_3	0	0	1	1/2	−1/2	4
x_1	1	0	0	1/4	1/4	4
x_2	0	1	0	−3/8	1/8	0
	0	0	0	−1/4	7/4	20

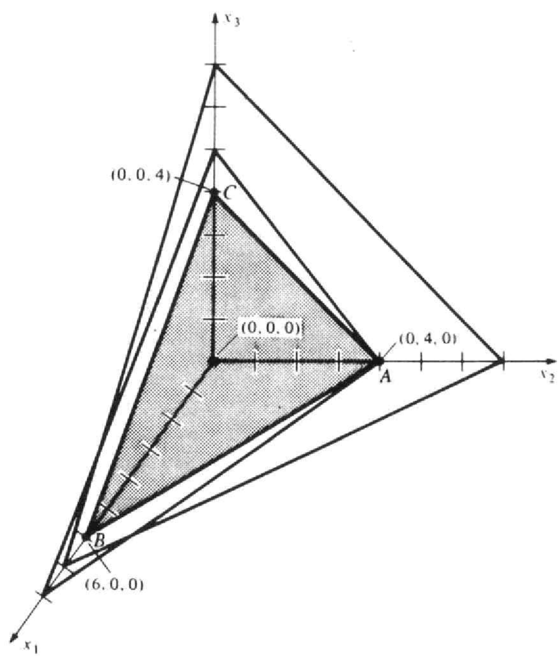
	x_1	x_2	x_3	x_4	x_5	
x_4	0	0	2	1	−1	8
x_1	1	0	−1/2	0	1/2	2
x_2	0	1	3/4	0	−1/4	3
	0	0	1/2	0	3/2	22

Note that in the first tableau, x_5 could also have been chosen as the departing variable.

4. $[0 \quad 4 \quad 0]^T$; $z = 32$

The simplex algorithm examines the following extreme points: O, A, A

	x_1	x_2	x_3	x_4	x_5	x_6	
x_4	1	1	1	1	0	0	7
x_5	2	3	3	0	1	0	12
x_6	3	6	5	0	0	1	24
	−5	−8	−1	0	0	0	0



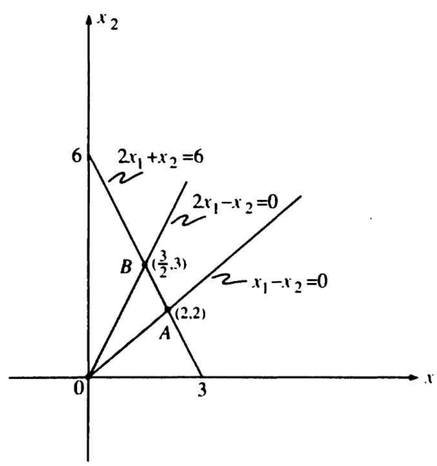
	x_1 ↓	x_2	x_3	x_4	x_5	x_6	
x_4	1/2	0	1/6	1	0	−1/6	3
← x_5	1/2	0	1/2	0	1	−1/2	0
x_2	1/2	1	5/6	0	0	1/6	4
	−1	0	17/3	0	0	4/3	32

	x_1	x_2	x_3	x_4	x_5	x_6	
x_4	0	0	−1/3	1	−1	1/2	3
x_1	1	0	1	0	2	−1	0
x_2	0	1	1/3	0	−1	2/3	4
	0	0	20/3	0	2	1/3	32

Note that in the second tableau, x_5 could also have been chosen as the departing variable.

6. $\begin{bmatrix} \frac{3}{2} & 3 \end{bmatrix}^T$; $z = \frac{33}{2}$

The simplex algorithm examines the following extreme points: O, O, A, B



	x_1 ↓	x_2	x_3	x_4	x_5	
x_3	2	1	1	0	0	6
x_4	−2	1	0	1	0	0
← x_5	1	−1	0	0	1	0
	−5	−3	0	0	0	0

	x_1	x_2 ↓	x_3	x_4	x_5	
← x_3	0	3	1	0	−2	6
x_4	0	−1	0	1	2	0
x_1	1	−1	0	0	1	0
	0	−8	0	0	5	0

	x_1	x_2	x_3	x_4	x_5	
x_2	0	1	1/3	0	−2/3	2
x_4	0	0	1/3	1	4/3	2
x_1	1	0	1/3	0	1/3	2
	0	0	8/3	0	−1/3	16

	x_1	x_2	x_3	x_4	x_5	
x_2	0	1	1/2	1/2	0	3
x_5	0	0	1/4	3/4	1	3/2
x_1	1	0	1/4	−1/4	0	3/2
	0	0	11/4	1/4	0	33/2

8. (a)

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
x_5	1	1	1	1	1	0	0	1
x_6	.5	−5.5	−2.5	9	0	1	0	0
x_7	.5	−1.5	−.5	1	0	0	1	0
	−1	7	1	2	0	0	0	0

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
x_5	0	12	6	−17	1	−2	0	1
x_1	1	−11	−5	18	0	2	0	0
x_7	0	4	2	−8	0	−1	1	0
	0	−4	−4	20	0	2	0	0

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
x_5	0	0	0	7	1	1	−3	1
x_1	1	0	.5	−4	0	−.75	2.75	0
x_2	0	1	.5	−2	0	−.25	.25	0
	0	0	−2	12	0	1	1	0

		x_1	x_2	x_3	x_4 ↓	x_5	x_6	x_7	
	x_5	0	0	0	7	1	1	−3	1
	x_3	2	0	1	−8	0	−1.5	5.5	0
←	x_2	−1	1	0	2	0	.5	−2.5	0
		4	0	0	−4	0	−2	12	0

		x_1	x_2	x_3	x_4	x_5	x_6 ↓	x_7	
	x_5	3.5	−3.5	0	0	1	−.75	5.75	1
←	x_3	−2	4	1	0	0	.5	−4.5	0
	x_4	−.5	.5	0	1	0	.25	−1.25	0
		2	2	0	0	0	−1	7	0

		x_1	x_2	x_3	x_4	x_5	x_6	x_7 ↓	
	x_5	.5	2.5	1.5	0	1	0	−1	1
	x_6	−4	8	2	0	0	1	−9	0
←	x_4	.5	−1.5	−.5	1	0	0	1	0
		−2	10	2	0	0	0	−2	0

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
x_5	1	1	1	1	1	0	0	1
x_6	.5	−5.5	−2.5	9	0	1	0	0
x_7	.5	−1.5	−.5	1	0	0	1	0
	−1	7	1	2	0	0	0	0

(b) In the sequence of tableaux leading to an optimal solution, the variables marked with a B, are chosen by Bland's rule:

Tableau	Entering variable	Departing variable
1	x_1	x_6 (B)
2	x_2 (B)	x_7
3	x_3	x_1 (B)
4	x_4 (B)	x_2
5	x_6	x_3 (B)
6	x_1 (B)	x_4

Optimal solution: $[0\ 0\ 0\ 0\ 1\ 0\ 0]^T$; $z = 0$

9. (a)

		x_1 ↓	x_2	x_3	x_4	x_5	x_6	x_7	
←	x_4	.6	−6.4	4.8	1	0	0	0	0
	x_5	.2	−1.8	.6	0	1	0	0	0
	x_6	.4	−1.6	.2	0	0	1	0	0
	x_7	0	1	0	0	0	0	1	1
		−.4	−.4	1.8	0	0	0	0	0

		x_1	x_2 ↓	x_3	x_4	x_5	x_6	x_7	
	x_1	1	−10.67	8	1.67	0	0	0	0
←	x_5	0	.33	−1	−.33	1	0	0	0
	x_6	0	2.67	−3	−.67	0	1	0	0
	x_7	0	1	0	0	0	0	1	1
		0	−4.67	5	.67	0	0	0	0

		x_1	x_2	x_3 ↓	x_4	x_5	x_6	x_7	
	x_1	1	0	−24	−9	32	0	0	0
	x_2	0	1	−3	−1	3	0	0	0
←	x_6	0	0	5	2	−8	1	0	0
	x_7	0	0	3	1	−3	0	1	1
		0	0	−9	−4	14	0	0	0

		x_1	x_2	x_3	x_4 ↓	x_5	x_6	x_7	
←	x_1	1	0	0	.6	−6.4	4.8	0	0
	x_2	0	1	0	.2	−1.8	.6	0	0
	x_3	0	0	1	.4	−1.6	.2	0	0
	x_7	0	0	0	−.2	1.8	−.6	1	1
		0	0	0	−.4	−.4	1.8	0	0

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
x_4	1.67	0	0	1	−10.67	8	0	0
x_2	−.33	1	0	0	.33	−1	0	0
x_3	−.67	0	1	0	2.67	−3	0	0
x_7	.33	0	0	0	−.33	1	1	1
	.67	0	0	0	−4.67	5	0	0

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
x_4	−9	32	0	1	0	−24	0	0
x_5	−1	3	0	0	1	−3	0	0
x_3	2	−8	1	0	0	5	0	0
x_7	0	1	0	0	0	0	1	1
	−4	0	0	0	0	−9	0	0

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
x_4	.6	−6.4	4.8	1	0	0	0	0
x_5	.2	−1.8	.6	0	1	0	0	0
x_6	.4	−1.6	.2	0	0	1	0	0
x_7	0	1	0	0	0	0	1	1
	−.4	−.4	1.8	0	0	0	0	0

(b) In the sequence of tableaux leading to an optimal solution, the variables marked with a B, are chosen by Bland's rule:

Tableau	Entering variable	Departing variable
1	x_1 (B)	x_4 (B)
2	x_2	x_5 (B)
3	x_3 (B)	x_6
4	x_4 (B)	x_1 (B)
5	x_5	x_2 (B)
6	x_1 (B)	x_3
7	x_2	x_7

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2. (a)

	x_1	x_2	x_3	x_4	x_5	x_6	y_1	y_2	
x_5	1	3	−1	1	1	0	0	0	5
y_1	1	7	1	0	0	−1	1	0	4
y_2	4	2	0	1	0	0	0	1	3
	−5	−9	−1	−1	0	1	0	0	−7

(b)

	x_1	x_2	x_3	x_4	x_5	x_6	y_1	y_2	
x_5	1	3	−1	1	1	0	0	0	5
y_1	1	7	1	0	0	−1	1	0	4
y_2	4	2	0	1	0	0	0	1	3
	$-1-5M$	$-2-9M$	$-M$	$-1-M$	0	M	0	0	$-7M$

4. (a)

	x_1	x_2	x_3	x_4	x_5	y_1	y_2	
y_1	3	1	−2	−1	0	1	0	2
y_2	2	4	7	0	−1	0	1	3
	−5	−5	−5	1	1	0	0	−5

(b)

	x_1	x_2	x_3	x_4	x_5	y_1	y_2	
y_1	3	1	−2	−1	0	1	0	2
y_2	2	4	7	0	−1	0	1	3
	$-1-5M$	$2-5M$	$7-5M$	$-1+M$	M	0	0	$-5M$

6.

	x_1	x_2	x_3	x_4	x_5	x_6	y_1	y_2	
x_1	1	−1/2	0	1/2	0	1/2	1/2	−1/2	3
x_5	0	−25/6	0	−5/6	1	1/2	−1/6	−1/2	10/3
x_3	0	5/6	1	1/6	0	−1/2	−1/6	1/2	1/3
	0	0	0	0	0	0	1	1	0

8. (a)

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
x_7	-3	0	-2	-1	0	$-3/4$	1	0
x_2	0	1	1	3	0	$1/2$	0	2
x_5	1	0	3	0	1	$-1/2$	0	4
	0	0	-2	2	0	$1/2$	0	2

(b) $\begin{bmatrix}0 & \frac{2}{3} & \frac{4}{3} & 0 & 0 & \frac{8}{3}\end{bmatrix}^T; \quad z = \frac{14}{3}$

10. Use 3 oz of food A and 4 oz of food B; $z = \$1.60$

12. $[0 \quad 7 \quad 0 \quad 3]^T; \quad z = 2$

14. $\begin{bmatrix}\frac{19}{13} & 0 & \frac{5}{13} & 0 & 0\end{bmatrix}^T; \quad z = \frac{43}{13}$

16. Chewy should consist of 53 1/3 kg of sunflower seeds and 80 kg of raisins. Crunchy should consist of 46 2/3 kg of sunflower seeds and 31 1/9 kg of peanuts. Nutty should consist of 28 8/9 kg of of peanuts only. Profit is \$157 7/9.

18. Make 1000 glazed doughnuts and 400 powdered sugar doughnuts. Profit is \$90.

20. No feasible solutions.

22. No finite optimal solution.

24. Assume that $\mathbf{x}$ is a feasible solution to (10), (11), and (12). Then for the ith constraint in (14) we have

$$\sum_{j=1}^{s} a_{ij}x_j + 0 = \sum_{j=1}^{s} a_{ij}x_j = b_i$$

Also, $\begin{bmatrix}\mathbf{x}\\0\end{bmatrix} \geq \begin{bmatrix}\mathbf{0}\\0\end{bmatrix}$ so that $\begin{bmatrix}\mathbf{x}\\0\end{bmatrix}$ is a feasible solution to (13), (14), and (15). Conversely, if $\begin{bmatrix}\mathbf{x}\\0\end{bmatrix}$ is a feasible solution to (13), (14), and (15), then $\mathbf{x} \geq \mathbf{0}$. Furthermore, for the ith constraint in (14) we have

$$\sum_{j=1}^{s} a_{ij}x_j + 0 = b_i = \sum_{j=1}^{s} a_{ij}x_j$$

Chapter 3

Section 3.1, page 165

2. Maximize $z' = 3w_1 + 8w_2$
subject to

$$\begin{array}{rcrcr} w_1 & + & 2w_2 & \leq & 6 \\ 2w_1 & + & w_2 & \leq & 6 \\ w_1 & + & 4w_2 & \leq & 8 \\ w_1 & + & 9w_2 & \leq & 9 \end{array}$$

$w_1 \geq 0,\ w_2 \geq 0$

4. Maximize $z' = 10w_1 - 5w_2 - 8w_3 + 15w_4 + 20w_5$
subject to

$$\begin{array}{rcrcrcrcrcr} 4w_1 & - & 4w_2 & - & 3w_3 & + & 3w_4 & + & w_5 & \geq & 2 \\ 2w_1 & - & 2w_2 & - & 5w_3 & + & 5w_4 & + & w_5 & \geq & 1 \\ 5w_1 & - & 5w_2 & - & 4w_3 & + & 4w_4 & + & w_5 & \geq & 3 \\ 5w_1 & - & 5w_2 & - & w_3 & + & w_4 & + & w_5 & \geq & 4 \end{array}$$

$w_i \geq 0,\ i = 1 \ldots 4;\ \ w_5$ unrestricted

6. Maximize $z' = 12w_1 - 6w_2$
subject to

$$\begin{array}{rcrcr} 4w_1 & - & 3w_2 & \leq & 5 \\ 2w_1 & - & 2w_2 & \leq & 2 \\ w_1 & - & 3w_2 & = & 6 \end{array}$$

$w_1 \geq 0,\ w_2 \geq 0$

8. Maximize $z' = 320w_1 - 12w_2$

subject to

$$\begin{aligned} 30w_1 - w_2 &\leq 15,000 \\ 50w_1 - 2w_2 &\leq 20,000 \end{aligned}$$

$w_1 \geq 0, \ w_2 \geq 0$

10. Minimize $z' = 420w_1 + 600w_2$
subject to

$$\begin{aligned} 2w_1 + 4w_2 &\geq 0.5 \\ 2w_1 + 6w_2 &\geq 0.8 \\ 3w_1 + 10w_2 &\geq 1.2 \end{aligned}$$

$w_1 \geq 0, \ w_2 \geq 0$

where w_1 and w_2 represent the marginal values of the sewing and gluing processes, respectively.

11. Let $\mathbf{x}' = \mathbf{u} - \mathbf{v}, \ \mathbf{u} \geq \mathbf{0}, \ \mathbf{v} \geq \mathbf{0}$. The given problem can be written as

$$\text{Maximize } z = [\mathbf{c}^T \ \ \mathbf{d}^T \ \ -\mathbf{d}^T] \begin{bmatrix} \mathbf{x} \\ \mathbf{u} \\ \mathbf{v} \end{bmatrix}$$

subject to

$$[\mathbf{A} \ \ \mathbf{B} \ \ -\mathbf{B}] \begin{bmatrix} \mathbf{x} \\ \mathbf{u} \\ \mathbf{v} \end{bmatrix} \leq \mathbf{b}$$

$$\begin{bmatrix} \mathbf{x} \\ \mathbf{u} \\ \mathbf{v} \end{bmatrix} \geq \mathbf{0}$$

By definition the dual is

Minimize $z' = \mathbf{b}^T\mathbf{w}$
subject to

$$\begin{bmatrix} \mathbf{A}^T \\ \mathbf{B}^T \\ -\mathbf{B}^T \end{bmatrix} \mathbf{w} \geq \begin{bmatrix} \mathbf{c} \\ \mathbf{d} \\ -\mathbf{d} \end{bmatrix}$$

$\mathbf{w} \geq \mathbf{0}$

or multiplying out we have

Minimize $z' = \mathbf{b}^T\mathbf{w}$
subject to

$$\begin{array}{rcr} \mathbf{A}^T\mathbf{w} & \geq & \mathbf{c} \\ \mathbf{B}^T\mathbf{w} & \geq & \mathbf{d} \\ -\mathbf{B}^T\mathbf{w} & \geq & -\mathbf{d} \\ \mathbf{w} & \geq & \mathbf{0} \end{array}$$

Combining the second and third constraints, we have

Minimize $z' = \mathbf{b}^T\mathbf{w}$
subject to

$$\begin{array}{rcl} \mathbf{A}^T\mathbf{w} & \geq & \mathbf{c} \\ \mathbf{B}^T\mathbf{w} & = & \mathbf{d} \\ \mathbf{w} & \geq & \mathbf{0} \end{array}$$

Section 3.2, page 182

2. The primal problem has an optimal solution with objective function value 125. Moreover, the slack variables for the second, third, and fifth constraints must be zero at an optimal solution.

4. The primal problem has no feasible solutions.

6. $[\frac{4}{5} \;\; \frac{7}{5}]^T;\quad z = \frac{58}{5}$

   Dual problem:

   Maximize $z' = 5w_1 + 3w_2$
   subject to

   $$\begin{array}{rcrcl} w_1 & + & 2w_2 & \leq & 4 \\ 3w_1 & + & w_2 & \leq & 6 \end{array}$$
   $$w_1 \geq 0,\; w_2 \geq 0$$

   Solution: $[\frac{8}{5} \;\; \frac{6}{5}]^T;\quad z = \frac{58}{5}$

8. Use $\frac{18}{11}$ ounces of walnuts, $\frac{48}{11}$ ounces of pecans, and no almonds. Cost = 58.9 cents.

9. Dual problem is:

Minimize $z' = 6w_1 + 12w_2 + 5w_3$
subject to

$$\begin{array}{rcrcrcr} 2w_1 & + & 5w_2 & & & = & 9 \\ w_1 & + & 4w_2 & + & 2w_3 & = & 14 \\ 3w_1 & + & w_2 & & & = & 7 \end{array}$$

$w_1 \geq 0,\ w_2 \geq 0,\ w_3 \geq 0$

Solving the constraints we obtain $[2\ 1\ 4]^T$ as a feasible solution to the dual problem. Also $z' = 44$ and $z = 44$. Hence, by the Duality Theorem, the given solution is optimal, since it is feasible.

10. Solution to the given problem is $[7\ \ 1]^T$; $z = 25$. The slack in the first constraint at the optimal solution is 1. By complementary slackness, $w_1 = 0$.

11. The slack variables of the given problem have values $x_4 = 4$, $x_5 = 0$, $x_6 = 0$ at the optimal solution, and $z = 20$. By complementary slackness, $w_1 = 0$. Also using complementary slackness, there can be no slack in the first and third constraints of the dual problem at an optimal solution. The dual problem is:

    Minimize $z' = 12w_1 + 10w_2 + 10w_3$
    subject to

$$\begin{array}{rcrcrcr} 2w_1 & + & w_2 & + & 3w_3 & \geq & 4 \\ 3w_1 & + & 4w_2 & + & w_3 & \geq & 2 \\ w_1 & + & 2w_2 & + & w_3 & \geq & 3 \end{array}$$

    $w_1 \geq 0,\ w_2 \geq 0,\ w_3 \geq 0$

    Therefore, we solve

$$\begin{array}{rcrcr} w_2 & + & 3w_3 & = & 4 \\ 2w_2 & + & w_3 & = & 3 \end{array}$$

    for w_2 and w_3.

    Solution to the dual: $[0\ 1\ 1]^T$; $z' = 20$

12. The dual objective function is $z' = \mathbf{b}^T\mathbf{w}$. Hence for $\mathbf{w} = (\mathbf{B}^{-1})^T\mathbf{c}_B$, $z' = \mathbf{b}^T(\mathbf{B}^{-1})^T\mathbf{c}_B = (\mathbf{B}^{-1}\mathbf{b})^T\mathbf{c}_B = \mathbf{c}_B^T(\mathbf{B}^{-1}\mathbf{b}) = \mathbf{c}_B^T\mathbf{x}_B = \mathbf{c}_B^T\mathbf{x}_0$.

Section 3.3, page 202

2.

		4	5/3	4/3	3	−1	0	−2/3	
$\mathbf{c}_B$		x_1	x_2	x_3	x_4	x_5	x_6	x_7	$\mathbf{x}_B$
−1	x_5	0	4/3	2/3	0	1	0	−1/3	4
0	x_6	0	1/3	2/3	1	0	1	−1/3	10
4	x_1	1	1/3	1/6	1/2	0	0	1/6	4
		0	−5/3	−4/3	−1	0	0	5/3	12

4.

		3	1	3	0	0	0	0	
$\mathbf{c}_B$		x_1	x_2	x_3	x_4	x_5	x_6	y_1	$\mathbf{x}_B$
3	x_1	1	1	0	2	0	2/3	0	2
3	x_3	0	−1	1	−1	0	−1	0	5/2
0	x_5	0	2	0	3	1	1	0	2/3
0	y_1	0	3/2	0	1/2	0	2	1	0
		0	−1	0	3	0	−1	0	27/2

$[2 \;\; 0 \;\; \frac{5}{2}]^T$; $z = \frac{27}{2}$

6. $[\frac{10}{3} \;\; \frac{5}{3} \;\; \frac{1}{3}]^T$; $z = \frac{28}{3}$

8. $[\frac{1}{4} \;\; \frac{3}{4} \;\; 0]^T$; $z = \frac{13}{2}$

10. No feasible solutions

12. No feasible solutions

14. *Exercise 6:* $[\frac{1}{7} \;\; \frac{8}{21} \;\; \frac{2}{3}]^T$; $z' = \frac{28}{3}$
    Exercise 8: $[0 \;\; \frac{1}{2} \;\; 0 \;\; 0 \;\; \frac{11}{2}]^T$; $z' = \frac{13}{2}$

16. Buy 2 advertising units in *TV Guide* per month, 2.54 advertising units in *Newsweek* per month, and no advertising units in *Time*. Total number of male readers = 66,386,025.

18. If $t_{i_r k} < 0$, the normalized row must be multiplied by a positive number to zero this entry. The right hand side of the pivotal row is nonnegative. Hence multiplying this number by a positive number and adding the

result to zero (the right hand side of the i_rth row) may yield a positive result. Thus, an artificial variable could take on a positive value.

Section 3.4, page 214

2.

		5	6	0	0	0	0	
$\mathbf{c}_B$		x_1	x_2	x_3	x_4	x_5	x_6	$\mathbf{x}_B$
5	x_1	1	0	0	-1	0	3	3
6	x_2	0	1	0	1	0	-2	4
0	x_3	0	0	1	7	0	-24	10
0	x_5	0	0	0	0	1	-3	1
		0	0	0	1	0	3	39

4. No feasible solutions.

8. $\left[\frac{45}{13} \;\; 0 \;\; \frac{12}{13}\right]; \quad z = -\frac{45}{13}$

9. *Possibility 1:* The new problem has no feasible solutions.

   Possibility 2: The new problem has the same feasible solutions as the old problem.

10. "Row" and "column" are interchanged. "Feasibility" and "optimality" are interchanged.

Section 3.5, page 223

2. $$\begin{bmatrix} 1 & -2/3 & 0 & 0 & 0 \\ 0 & 4/3 & 0 & 0 & 0 \\ 0 & 4/9 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & -5/9 & 0 & 0 & 1 \end{bmatrix}$$

4. $$\begin{bmatrix} 1 & -1 & 2 & 1 \\ 0 & 1/4 & 0 & 1/4 \\ -1 & 7/2 & 1 & -5/2 \\ 2 & -7/6 & 2 & 23/6 \end{bmatrix}$$

6. $[0\ \ 0\ \ 12\ \ 0]^T$; $z = 36$

10.

$$\mathbf{M}\mathbf{M}^{-1} = \begin{bmatrix} 1 & -\mathbf{c}_B^T \\ \mathbf{0} & \mathbf{B} \end{bmatrix} \begin{bmatrix} 1 & \mathbf{c}_B^T\mathbf{B}^{-1} \\ \mathbf{0} & \mathbf{B}^{-1} \end{bmatrix} = \begin{bmatrix} 1 & \mathbf{c}_B^T\mathbf{B}^{-1} - \mathbf{c}_B^T\mathbf{B}^{-1} \\ \mathbf{0} & \mathbf{B}\mathbf{B}^{-1} \end{bmatrix} = \begin{bmatrix} 1 & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_m \end{bmatrix}$$

11. (a) Let $\mathbf{x}'$ = the vector of slack variables. Then $\mathbf{A}\mathbf{x} + \mathbf{I}\mathbf{x}' = \mathbf{b}$ and $\mathbf{x}' \geq 0$, or

$$[\mathbf{A} \mid \mathbf{I}] \begin{bmatrix} \mathbf{x} \\ \mathbf{x}' \end{bmatrix} = \mathbf{b} \text{ where } \begin{bmatrix} \mathbf{x} \\ \mathbf{x}' \end{bmatrix} \geq \mathbf{0}$$

The objective function coefficients for $\mathbf{x}'$ are $\mathbf{c}'^T = [0 \ \cdots \ 0]$. Thus, the objective function is $z = \mathbf{c}^T\mathbf{x} + \mathbf{c}'^T\mathbf{x}'$.

(b) We rewrite the objective function as the equation

$$z - \mathbf{c}^T\mathbf{x} - \mathbf{c}'^T\mathbf{x}' = 0,$$

and multiply out the constraint equations to obtain $\mathbf{A}\mathbf{x}+\mathbf{I}\mathbf{x}' = \mathbf{b}$. In matrix form these equations can be written as

$$\begin{bmatrix} 1 & -\mathbf{c}^T & -\mathbf{c}'^T \\ \mathbf{0} & \mathbf{A} & \mathbf{I} \end{bmatrix} \begin{bmatrix} z \\ \mathbf{x} \\ \mathbf{x}' \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{b} \end{bmatrix}.$$

(c) Multiply both sides of (b) by $\mathbf{M}^{-1}$ on the left obtaining

$$\begin{bmatrix} 1 & \mathbf{c}_B^T\mathbf{B}^{-1} \\ \mathbf{0} & \mathbf{B}^{-1} \end{bmatrix} \begin{bmatrix} 1 & -\mathbf{c}^T & -\mathbf{c}'^T \\ \mathbf{0} & \mathbf{A} & \mathbf{I} \end{bmatrix} \begin{bmatrix} z \\ \mathbf{x} \\ \mathbf{x}' \end{bmatrix} = \begin{bmatrix} 1 & \mathbf{c}_B^T\mathbf{B}^{-1} \\ \mathbf{0} & \mathbf{B}^{-1} \end{bmatrix} \begin{bmatrix} 0 \\ \mathbf{b} \end{bmatrix}$$

or

$$\begin{bmatrix} 1 & \mathbf{c}_B^T\mathbf{B}^{-1}\mathbf{A} - \mathbf{c}^T & \mathbf{c}_B^T\mathbf{B}^{-1} - \mathbf{c}'^T \\ \mathbf{0} & \mathbf{B}^{-1}\mathbf{A} & \mathbf{B}^{-1} \end{bmatrix} \begin{bmatrix} z \\ \mathbf{x} \\ \mathbf{x}' \end{bmatrix} = \begin{bmatrix} \mathbf{c}_B^T\mathbf{B}^{-1}\mathbf{b} \\ \mathbf{B}^{-1}\mathbf{b} \end{bmatrix}$$

(d) Once an optimal solution is found, then $z_j - c_j \geq 0$ for all j. For the original variables, we have from (c) that the vector of $z_j - c_j$'s is $\mathbf{c}_B^T\mathbf{B}^{-1}\mathbf{A}-\mathbf{c}^T$. For the slack variables, this vector is $\mathbf{c}_B^T\mathbf{B}^{-1}-\mathbf{c}'^T$. Thus, at an optimal solution $\mathbf{c}_B^T\mathbf{B}^{-1}\mathbf{A} - \mathbf{c}^T \geq 0$ or $\mathbf{c}_B^T\mathbf{B}^{-1}\mathbf{A} \geq \mathbf{c}^T$ and likewise, $\mathbf{c}_B^T\mathbf{B}^{-1} \geq \mathbf{c}'^T$

12. (a) Minimize $z = \mathbf{b}^T\mathbf{w}$
subject to
$$\begin{bmatrix} \mathbf{A}^T \\ \mathbf{I} \end{bmatrix} \mathbf{w} \geq \begin{bmatrix} \mathbf{c} \\ \mathbf{c}' \end{bmatrix}$$
$\mathbf{w}$ unrestricted

(b) We must show that $\mathbf{A}^T(\mathbf{B}^{-1})^T\mathbf{c}_B \geq \mathbf{c}$ and $\mathbf{I}(\mathbf{B}^{-1})^T\mathbf{c}_B \geq \mathbf{c}'$. Taking transposes of both sides, we must show $\mathbf{c}_B^T\mathbf{B}^{-1}\mathbf{A} \geq \mathbf{c}^T$ and $\mathbf{c}_B^T\mathbf{B}^{-1} \geq \mathbf{c}'^T$, which hold from Exercise 11(d).

(c) Since by part (d), $\mathbf{w} = (\mathbf{B}^{-1})^T\mathbf{c}_B$ is a feasible solution to the dual problem and, by Exercise 9, the objective functions of the dual and primal problems have the same value, we conclude that $\mathbf{w}$ is an optimal solution to the dual problem, using the Duality Theorem.

Section 3.6, page 233

2. (a) $[3 \;\; 2 \;\; 0 \;\; 0]^T$; $z = 13$
(b) $[0 \;\; 8 \;\; 0 \;\; 0]^T$; $z = 16$
(c) No change
(d) No change

4. (a) $[0 \;\; 3 \;\; 3 \;\; 0 \;\; 0 \;\; 9]^T$; $z = 24$
(b) No change
(c) No feasible solution
(d) $[6 \;\; 0 \;\; 3 \;\; 0 \;\; 0 \;\; 16]^T$; $z = 29$

6. (a) Invest \$40,000 in utility stock, \$0 in electronics stock, and \$160,000 in bond. (No change.) Return = \$11,920.
(b) Invest same amounts as in (a). Return = \$10,800.
(c) Invest same amounts as in (a). Return = \$11,600.

7.
$$\hat{\mathbf{x}}_B = \mathbf{B}^{-1}\hat{\mathbf{b}} = \mathbf{B}^{-1}(\mathbf{b} + \Delta\mathbf{b}) = \mathbf{B}^{-1}\mathbf{b} + \mathbf{B}^{-1}\Delta\mathbf{b} = \mathbf{x}_B + \mathbf{B}^{-1}\Delta\mathbf{b}$$
Thus, $\hat{\mathbf{x}}_B$ is feasible if and only if $\hat{\mathbf{x}}_B + \mathbf{B}^{-1}\Delta\mathbf{b} \geq \mathbf{0}$

Chapter 4

Section 4.1, page 259

2. Let

x_1 = number of solid-back chairs made per month
x_2 = number of ladder-back chairs made per month
x_3 = number of vinyl-covered chairs made per month

Maximize $z = 10x_1 + 13x_2 + 8x_3$
subject to

$$\begin{array}{rcrcrcl} x_1 & + & 1.2x_2 & + & 0.7x_3 & \leq & 600 \\ 0.5x_1 & + & 0.5x_2 & + & 0.3x_3 & \leq & 300 \\ 0.7x_1 & + & 0.7x_2 & + & 0.3x_3 & \leq & 300 \\ & & & & 0.7x_3 & \leq & 140 \end{array}$$

$x_j \geq 0$ and integer, $\quad j = 1, 2, 3$

4. Let x_i, $i = 1, 2, \ldots, 7$ be the number of bottles of cola, root beer, cherry, lemon, orange, grape, and ginger ale, respectively.

Minimize $z = \sum_{i=1}^{7} c_i x_i$,
where c_i is the cost of the ith drink
subject to

$$\begin{array}{rcl} \sum_{i=1}^{7} x_i & \geq & 12 \\ x_7 & \geq & 2 \\ x_1 & \geq & 2 \\ x_3 + x_4 + x_5 + x_6 & \leq & 3 \end{array}$$

$x_i \geq 0$, integer, $i = 1, 2, \ldots, 7$

6. Let

$$\begin{aligned}
\text{rest state} &= \text{job } 7 \\
c_{ij} &= \text{time needed to switch from job } i \text{ to job } j \\
x_{ij} &= \begin{cases} 1 & \text{if schedule includes going from job } i \text{ to job } j \\ 0 & \text{otherwise} \end{cases}
\end{aligned}$$

Minimize $z = \sum_{i=1}^{7} \sum_{j=1}^{7} c_{ij} x_{ij}$
subject to

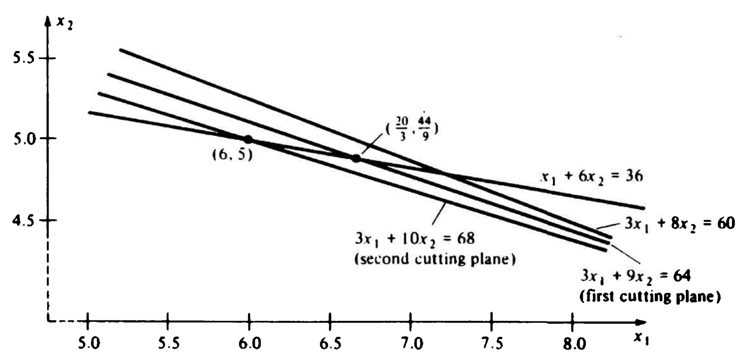
$$\begin{aligned}
\textstyle\sum_{i=1}^{7} x_{ij} &= 1, \; j = 1, \ldots, 7 \\
\textstyle\sum_{j=1}^{7} x_{ij} &= 1, \; i = 1, \ldots, 7
\end{aligned}$$

$u_i - u_j + 7x_{ij} \le 6, \; 2 \le i, j \le 7$ and $i \ne j$
$u_i \ge 0$, integer, $i = 1, 2, \ldots, 7$

Section 4.2, page 274

2. $-\frac{3}{4}x_5 - \frac{11}{12}x_6 - \frac{1}{12}x_7 + u_1 = -\frac{1}{3}$

4. $x = 6, y = 5; z = 26$



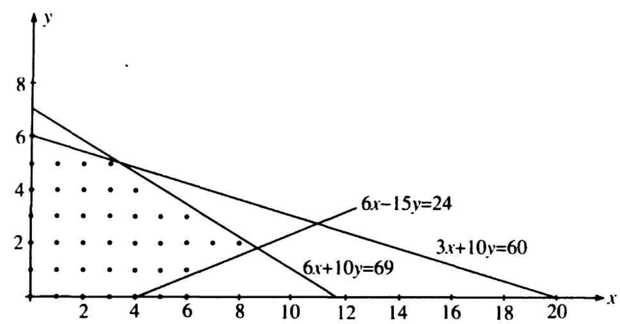
6. $[0 \;\; 2 \;\; 2]^T; z = 10$

8. $[0 \;\; 4 \;\; 10]^T; z = 9$

10. $[0 \;\; \frac{5}{2} \;\; 2]^T; z = \frac{21}{2}$

12. $[0 \;\; 2 \;\; \frac{7}{3}]^T; z = \frac{34}{3}$

Project 2:



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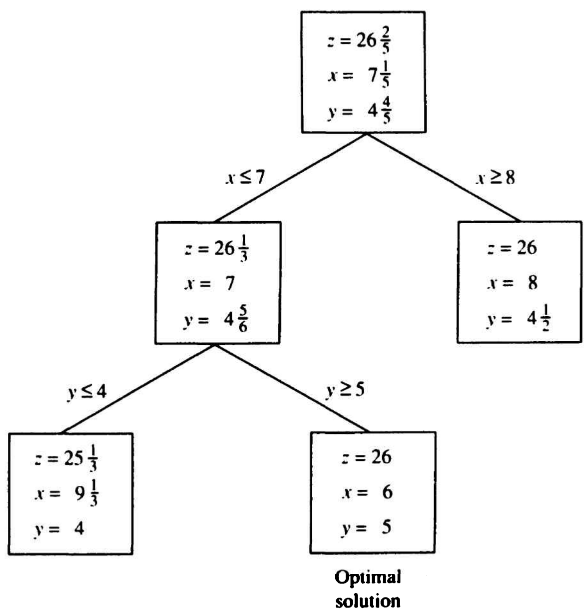
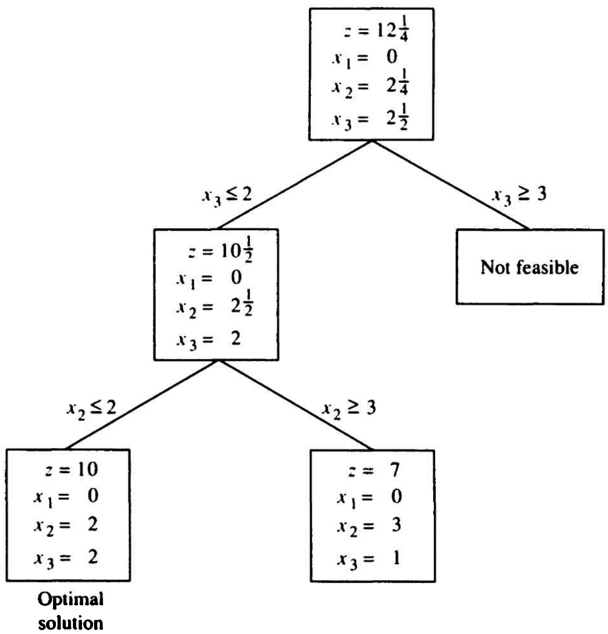
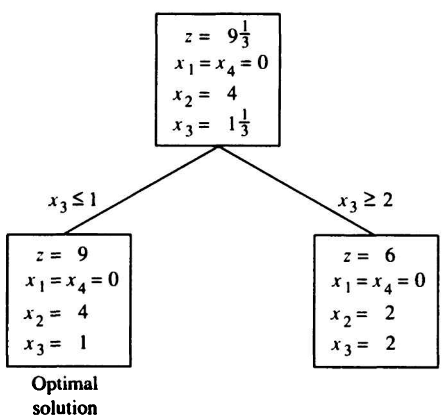
2.



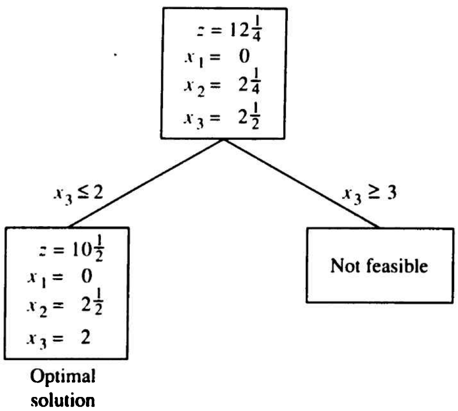
4.


$z = 12\frac{1}{4}$
$x_1 = 0$
$x_2 = 2\frac{1}{4}$
$x_3 = 2\frac{1}{2}$
$x_3 \leq 2$
$x_3 \geq 3$
$z = 10\frac{1}{2}$
$x_1 = 0$
$x_2 = 2\frac{1}{2}$
$x_3 = 2$
Not feasible
$x_2 \leq 2$
$x_2 \geq 3$
$z = 10$
$x_1 = 0$
$x_2 = 2$
$x_3 = 2$
Optimal solution
$z = 7$
$x_1 = 0$
$x_2 = 3$
$x_3 = 1$

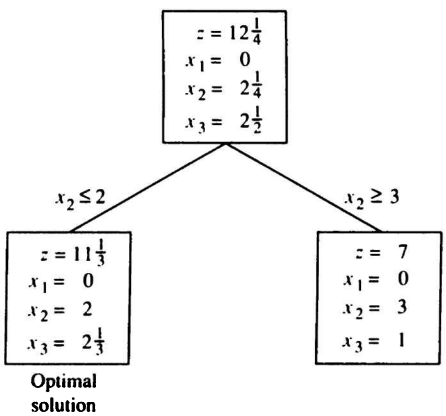
6.


$z = 9\frac{1}{3}$
$x_1 = x_4 = 0$
$x_2 = 4$
$x_3 = 1\frac{1}{3}$
$x_3 \leq 1$
$x_3 \geq 2$
$z = 9$
$x_1 = x_4 = 0$
$x_2 = 4$
$x_3 = 1$
Optimal solution
$z = 6$
$x_1 = x_4 = 0$
$x_2 = 2$
$x_3 = 2$

8.



10.



12. Make no solid-back chairs, 343 ladder-back chairs, and 199 vinyl-covered chairs. Profit = \$6051.

14. Do every project except projects 2 and 5. Value = 295.

Chapter 5

Section 5.1, page 324

2\. (a)

	60	40	
60			20
		40	100

(b)

	20	80	
			80
60	40		40

(c)

		80	20
			80
60	60		20

4\. (a)

40	10		50
	60	0	
		50	
		70	

(b)

40		10	50
	20	40	
	50		
		70	

(c)

0		50	50
40	20		
	50		
		70	

6.

100			20
	60	80	

$z = \$1720$

8.

70	30		
			90
	20	80	30

$z = \$800$

10.

50		20		
	50	30		
				60
10			70	40

$z = \$1150$

12.

		20	50	5
45		5		
	50			10

$z = \$715$

↑ Dummy demand

14. Summing the m supply constraints

$$\sum_{j=1}^{n} x_{ij} \leq s_i \qquad i = 1, 2, \ldots, m$$

we obtain

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} \leq \sum_{i=1}^{m} s_i$$

Likewise, summing the n demand constraints we obtain

$$\sum_{j=1}^{n} \sum_{i=1}^{m} x_{ij} \geq \sum_{j=1}^{n} d_j$$

By hypothesis,

$$\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$$

Therefore,

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} \leq \sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j \leq \sum_{j=1}^{n}\sum_{i=1}^{m} x_{ij}$$

Hence,

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} = \sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$$

and the inequalities must be equalities.

15. Summing the supply constraints we obtain by Exercise 14

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} = \sum_{i=1}^{m} s_i$$

Summing the first $n-1$ demand constraints, we have by Exercise 14

$$\sum_{j=1}^{n-1}\sum_{i=1}^{m} x_{ij} = \sum_{j=1}^{n-1} d_j$$

Subtracting the second equation from the first one we have

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} - \sum_{j=1}^{n-1}\sum_{i=1}^{m} x_{ij} = \sum_{i=1}^{m} s_i - \sum_{j=1}^{n-1} d_j = \sum_{j=1}^{n} d_j - \sum_{j=1}^{n-1} d_j = d_n$$

Interchanging the order of the first summation, the left hand side may be written as

$$\sum_{j=1}^{n}\sum_{i=1}^{m} x_{ij} - \sum_{j=1}^{n-1}\sum_{i=1}^{m} x_{ij} = \sum_{i=1}^{m} x_{in}$$

Thus, we have obtained the last demand constraint

$$\sum_{i=1}^{m} x_{in} = d_n$$

16. Maximize

$$z' = \sum_{i=1}^{m} s_i v_i + \sum_{j=1}^{n} d_j w_j$$

subject to

$v_i + w_j < c_{ij}, \quad i = 1, 2, \ldots, m; \; j = 1, 2, \ldots, n, \quad v_i$ and w_j unrestricted

17. Since s_i, d_j, and S are positive, $x_{ij} > 0$ for all i, j. Furthermore, for each j

$$\sum_{i=1}^{m} x_{ij} = \sum_{i=1}^{m} \frac{s_i d_j}{S} = \frac{d_j}{S} \sum_{i=1}^{m} s_i = \frac{d_j}{S} S = d_j$$

and for each i

$$\sum_{j=1}^{n} x_{ij} = \sum_{j=1}^{n} \frac{s_i d_j}{S} = \frac{s_i}{S} \sum_{j=1}^{n} d_j = \frac{s_i}{S} S = s_i$$

18. Since $\sum_{i=1}^{m} x_{ij} = d_j$, we must have $x_{ij} \leq d_j$ for each i and j. Similarly, since $\sum_{j=1}^{m} x_{ij} = s_i$, we must have $x_{ij} \leq s_i$ for each i and j. Hence, $0 \leq x_{ij} \leq \min\{s_i, d_j\}$.

Section 5.2, page 338

2.

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$z = 11$

4.

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$z = 13$

6.

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$z = 10$

Section 5.3, page 344

2. (a)

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

(b) Path 1: (1,3), (3,4)
Path 2: (1,5), (5,4)
Path 3: (1,3), (3,6), (6,2), (2,4)

(c) Cycle 1: (2,4), (4,2)
Cycle 2: (2,4), (4,3), (3,6), (6,2)

4.

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

6. (a)

$$\begin{bmatrix} 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 3 & 0 & 0 & 0 & 0 \\ 10 & 0 & 0 & 0 & 0 & 7 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 11 \\ 0 & 0 & 0 & 6 & 0 & 0 & 0 \end{bmatrix}$$

(b) 5

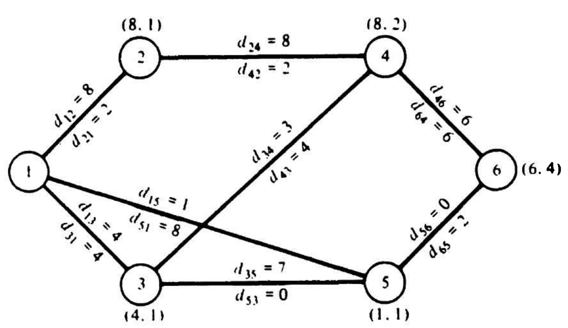
(c) 3

7. A node is a source if and only if no arcs lead into it. Arcs leading into nodes have nonzero entries in the columns of the capacity matrix. Thus, the column corresponding to a source would contain only zeros.

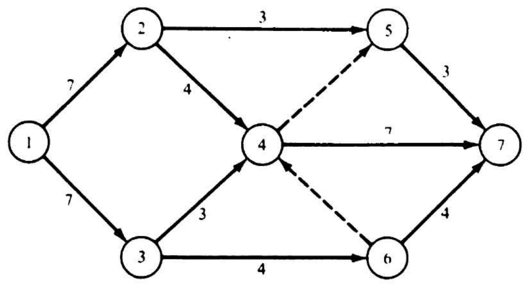
8. A node is a sink if and only if no arcs lead away from it. Arcs leading away from a node have nonzero entries in a row of the capacity matrix. Thus, the row corresponding to a sink would contain only zeros.

Section 5.4, page 363

2.

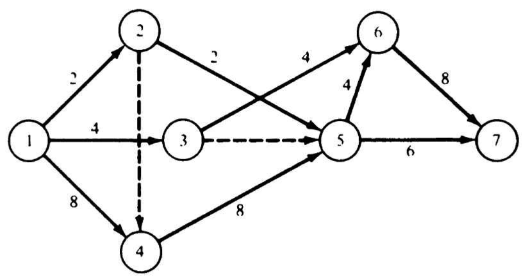


4.



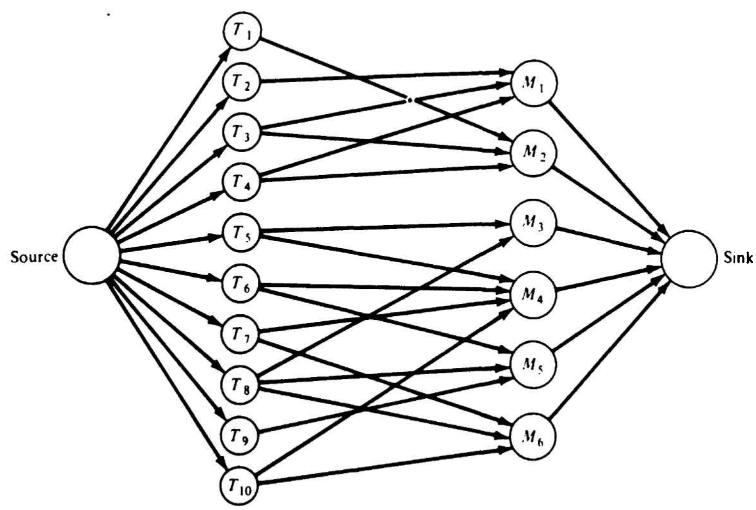
maximum flow = 14

6.



maximum flow = 14

8. (a) Capacities are all 1.



(b) 6 tasks, namely $T_1, T_2, T_5, T_6, T_7, T_8$

Section 5.5, page 375

2. Path: $1 \to 3 \to 5 \to 8 \to 9$ Length $= 19$

4. Path: $1 \to 2 \to 6 \to 8 \to 11$ Length $= 25$

6. Replace equipment at beginning of third and seventh years. Total equipment cost = \$311,000.

Section 5.6, page 385

2. (a,b)

Node	Early event time	Late event time
1	0	0
2	5	5
3	12	15
4	8	8
5	17	17
6	18	19
7	19	19
8	25	25

(c) $1 \to 4 \to 5 \to 7 \to 8$

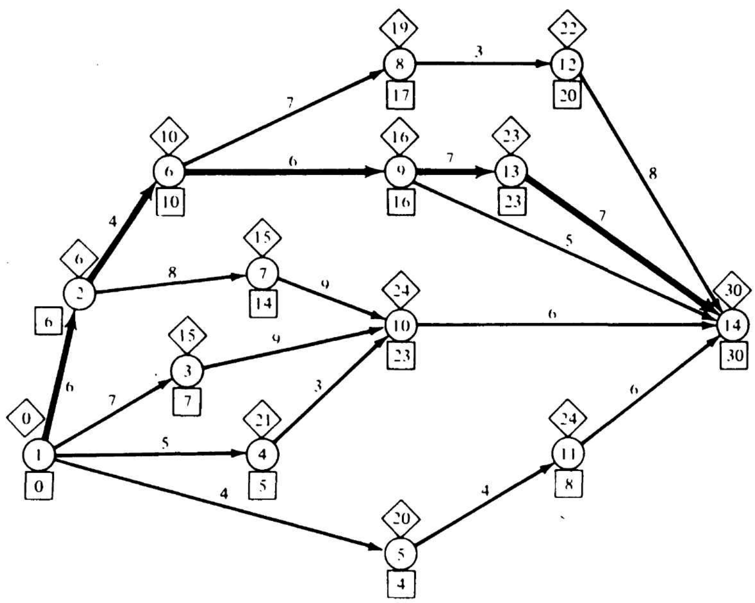
4. (a,b)

Node	Early event time	Late event time
1	0	0
2	3	6
3	5	6
4	4	4
5	7	12
6	5	8
7	5	5
8	10	13
9	9	10
10	9	14
11	10	10
12	11	19
13	17	17
14	12	20
15	19	22
16	23	23
17	27	27

(c) $1 \to 4 \to 7 \to 11 \to 13 \to 16 \to 17$

6. Insert dummy activity between nodes 2 and 6.
   Delete dummy activity (7,9).

8.

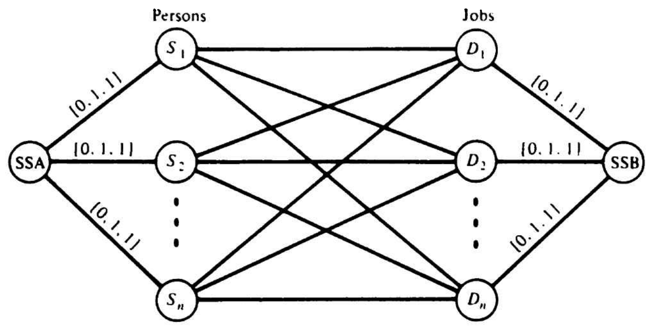


$1 \to 2 \to 6 \to 9 \to 13 \to 14$

Section 5.7, page 393

1. Connecting each demand point to a supersink provides arcs whose capacities guarantee that the required amount is shipped to each demand point. The supersink represents the total demand of the shipper's area of operation.

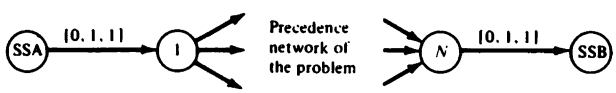
2.



All arcs connecting persons and jobs should be labeled $[c, 1, 0]$ where c is obtained from the cost matrix.

4.

Label each arc in the precedence network by $[-c, 1, 0]$ where c is the time estimate for each arc.



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